Computer Algebra Independent Integration Tests

Summer 2023 edition

4-Trig-functions/4.5-Secant/126-4.5.7-d-trig-^m-a+b-c-sec-^n-^p

Nasser M. Abbasi

September 5, 2023 Compiled on September 5, 2023 at 10:34pm

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [470]. This is test number [126].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.57 (468)	0.43 (2)
Mathematica	93.19 (438)	6.81 (32)
Maple	91.70 (431)	8.30 (39)
Fricas	88.51 (416)	11.49 (54)
Maxima	61.70 (290)	38.30 (180)
Giac	53.19 (250)	46.81 (220)
Mupad	51.70 (243)	48.30 (227)
Sympy	4.68 (22)	95.32 (448)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

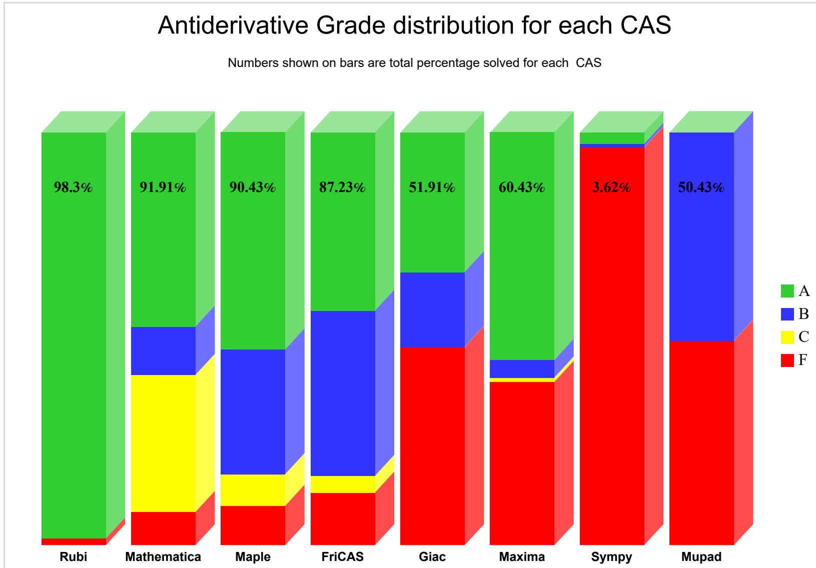
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

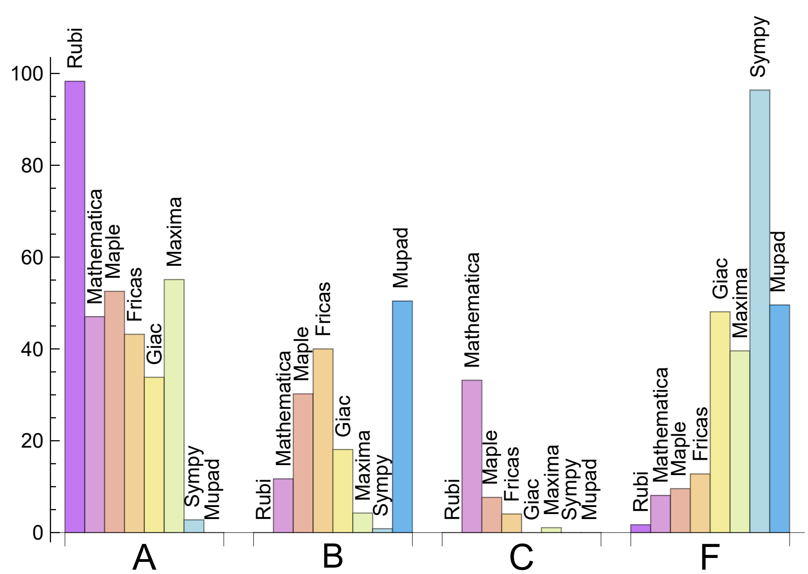
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.298	0.000	0.000	1.702
Maxima	55.106	4.255	1.064	39.574
Maple	52.553	30.213	7.660	9.574
Mathematica	47.021	11.702	33.191	8.085
Fricas	43.191	40.000	4.043	12.766
Giac	33.830	18.085	0.000	48.085
Sympy	2.766	0.851	0.000	96.383
Mupad	0.000	50.426	0.000	49.574

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	2	100.00	0.00	0.00
Mathematica	32	100.00	0.00	0.00
Maple	39	100.00	0.00	0.00
Fricas	54	100.00	0.00	0.00
Maxima	180	88.33	11.67	0.00
Giac	220	97.73	0.00	2.27
Mupad	227	0.00	100.00	0.00
Sympy	448	72.99	27.01	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.20
Maxima	0.37
Giac	0.53
Fricas	0.93
Mathematica	4.12
Maple	4.99
Sympy	11.83
Mupad	18.49

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	73.41	1.68	63.50	1.31
Rubi	123.78	1.00	103.50	1.00
Maxima	223.76	2.81	101.50	1.17
Giac	256.05	2.64	149.50	1.36
Mathematica	428.96	3.68	168.00	1.55
Fricas	607.82	4.42	401.00	3.55
Mupad	1563.74	10.76	105.00	1.17
Maple	1754.82	9.02	147.00	1.29

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

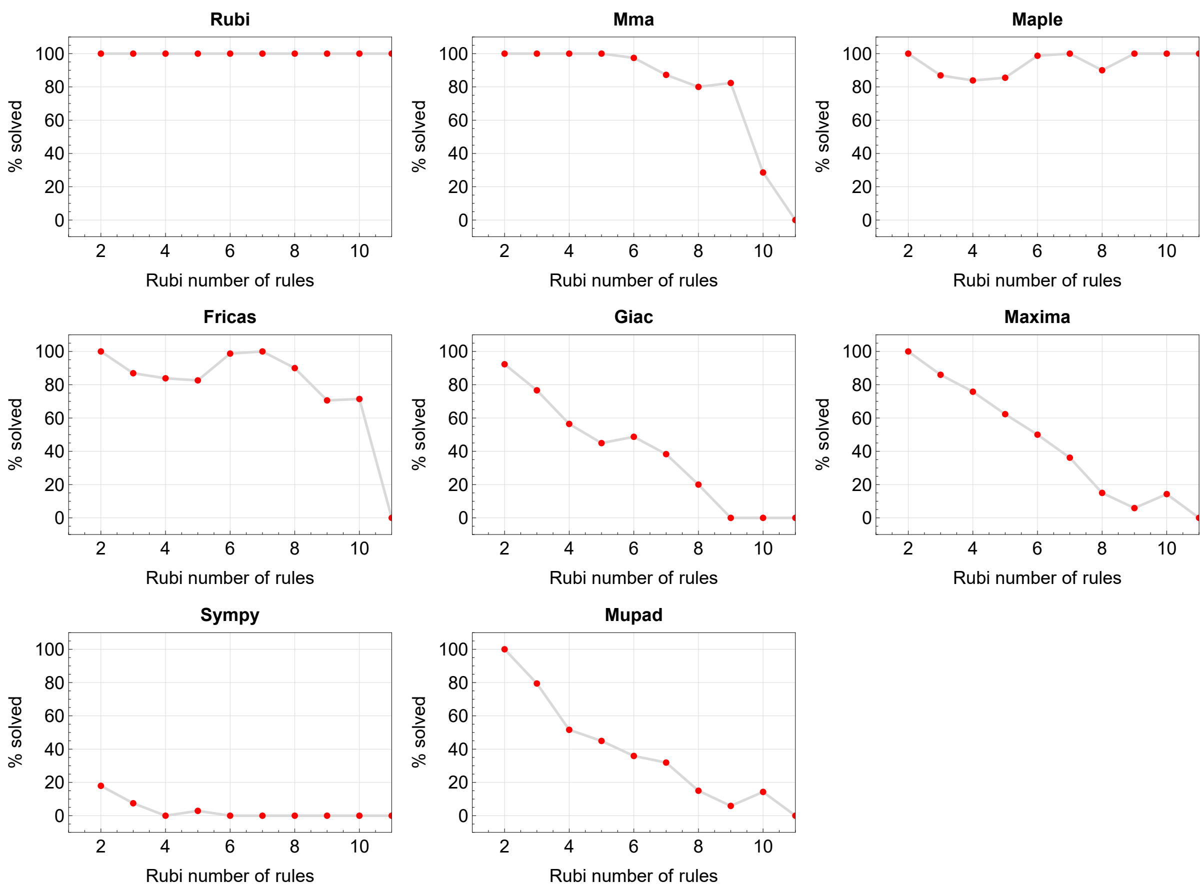


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

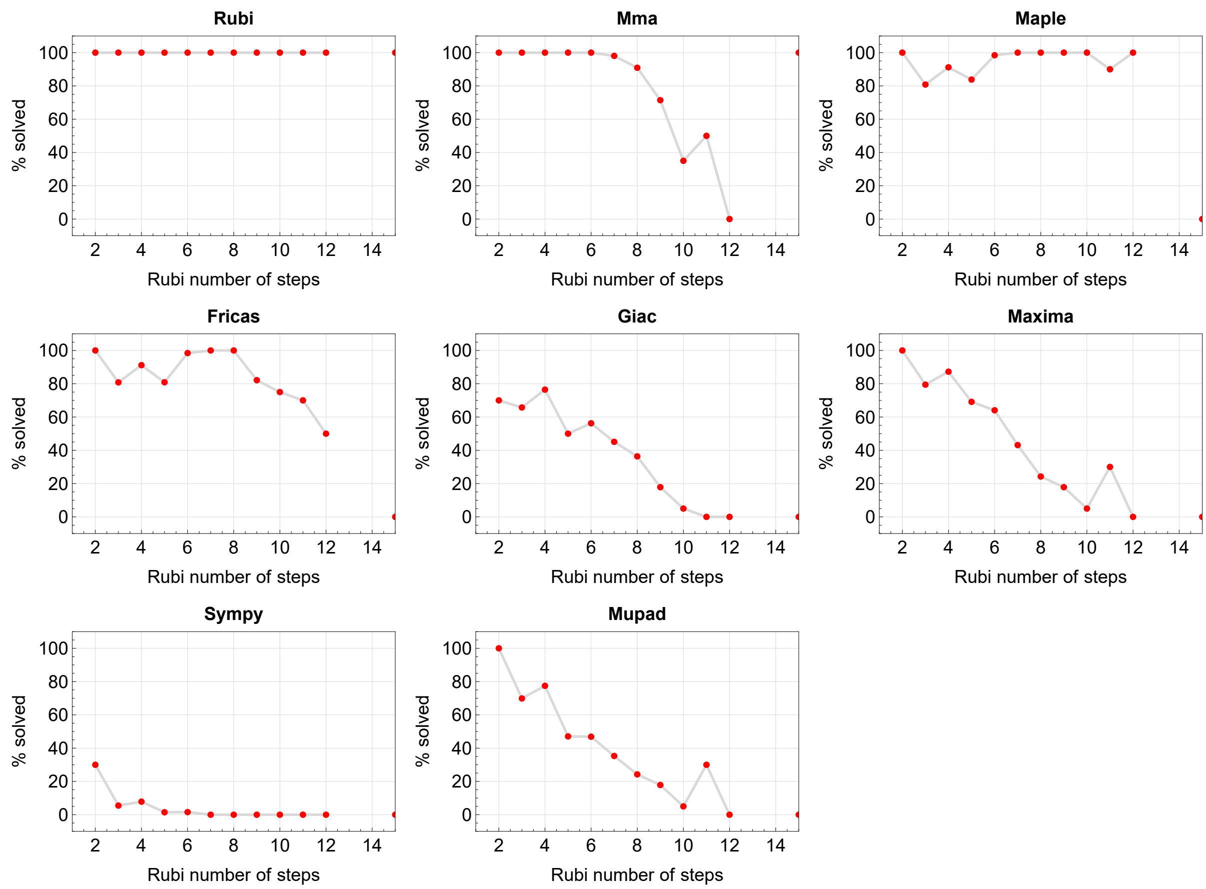


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

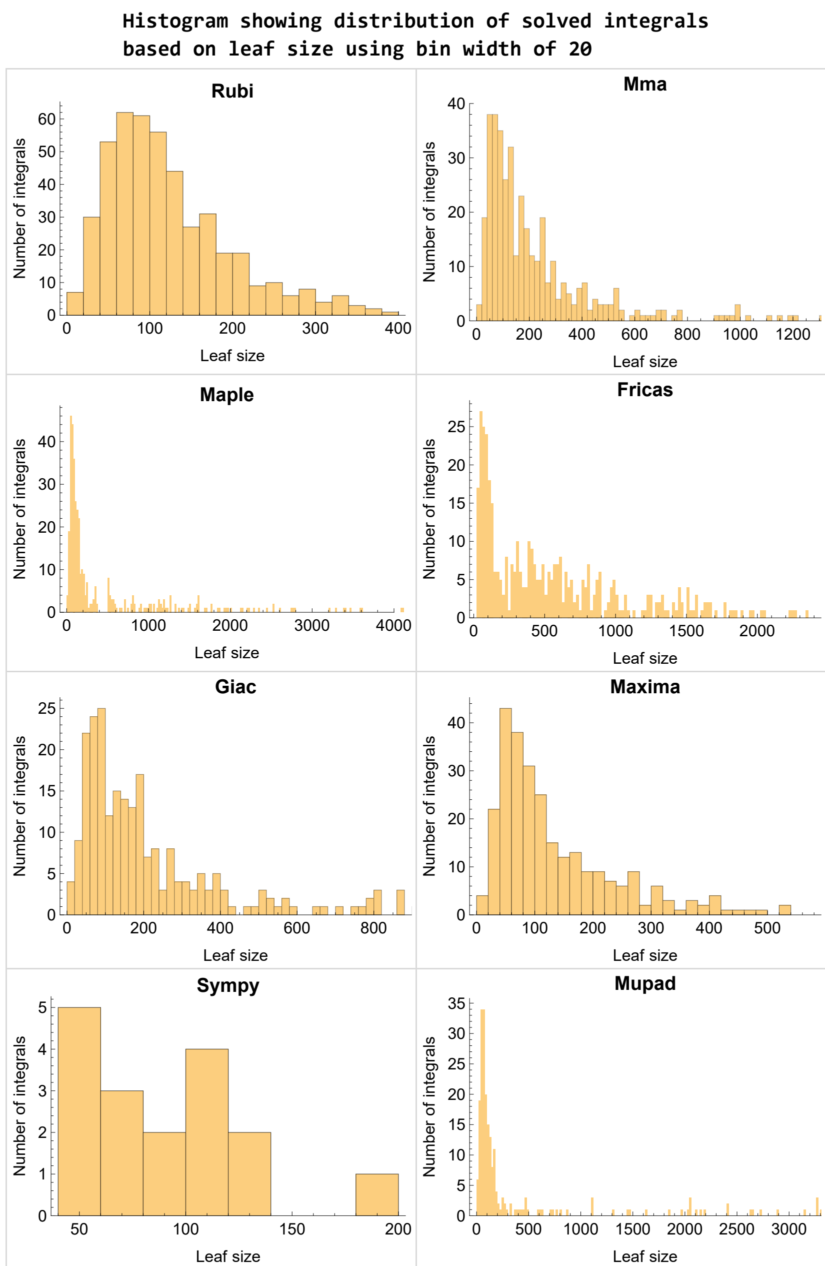


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

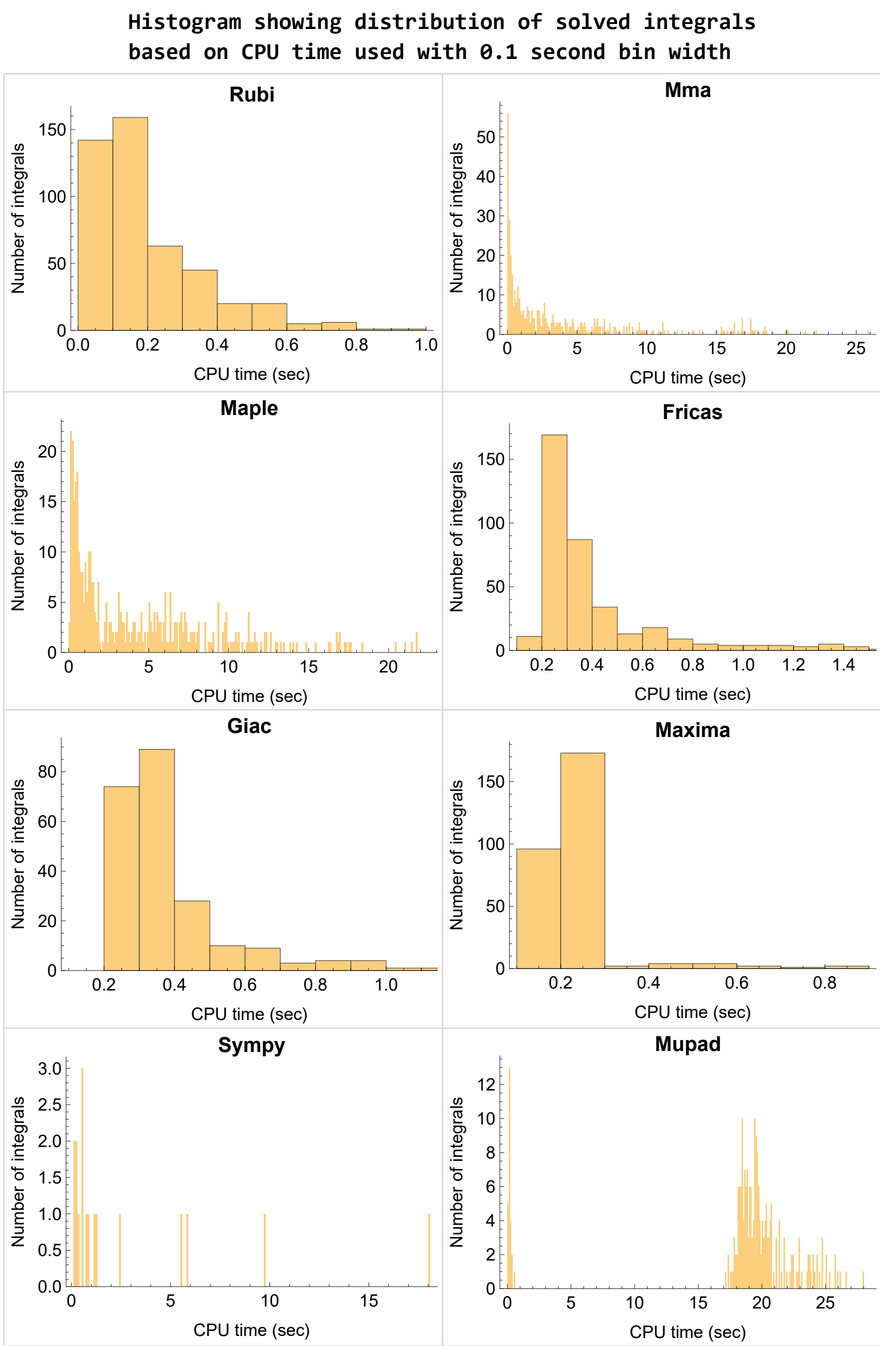


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

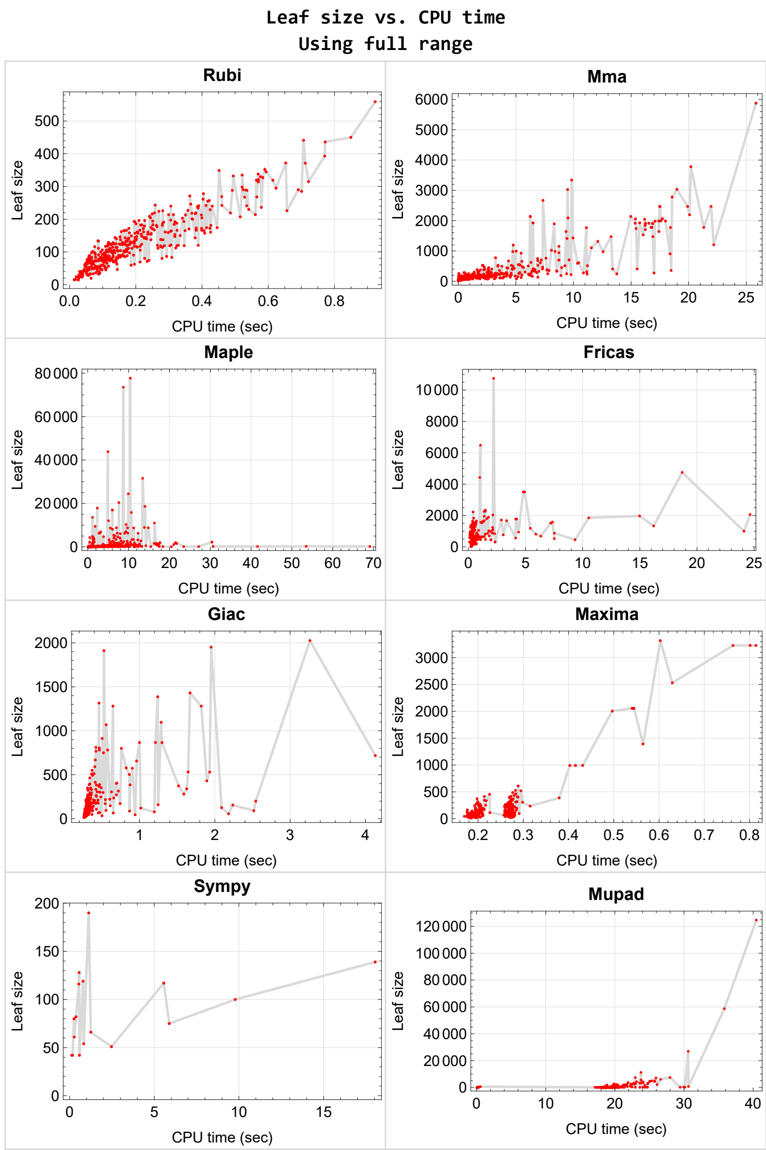


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{462, 466, 467, 468, 469, 470}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {28, 29, 30, 34, 35, 36, 39, 40, 41, 42, 43, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 78, 79, 88, 89, 125, 126, 128, 132, 136, 137, 138, 139, 140, 180, 193, 199, 202, 205, 208, 211, 213, 214, 215, 217, 218, 219, 235, 240, 247, 248, 250, 251, 254, 264, 268, 269, 276, 277, 280, 281, 282, 284, 289, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 307, 308, 309, 310, 348, 349, 356, 357, 358, 359, 360, 361, 362, 369, 370, 371, 372, 373, 374, 375, 380, 384, 392, 393, 394, 397, 398, 399, 428, 429, 430, 437, 441, 447, 448, 449, 450, 451, 463, 464}

Maple {72, 73, 74, 86, 87, 88, 89, 97, 111, 123, 124, 228, 229, 230, 231, 232, 233, 241, 244, 246, 247, 248, 250, 251, 254, 257, 258, 260, 261, 262, 265, 270, 271, 272, 273, 274, 275, 277, 283, 284, 285, 286, 287, 288, 380, 381, 392, 393, 394, 398, 399, 406, 407, 410, 418, 419, 420, 422, 425, 426, 427, 431, 432, 433, 434, 438, 439, 440}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

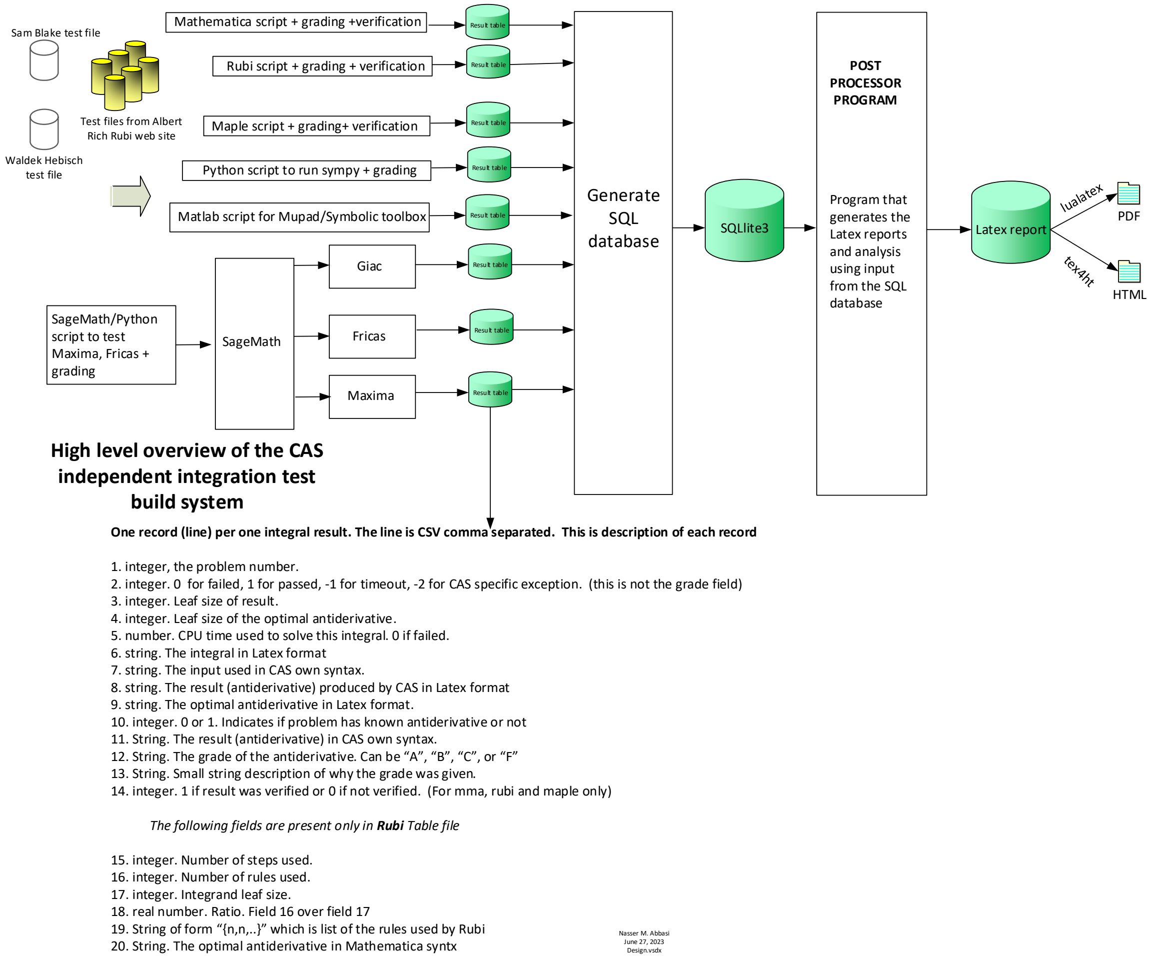
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 463, 464, 465 }

B grade { }

C grade { }

F normal fail { 132, 298 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 22, 23, 24, 26, 67, 68, 69, 70, 71, 72, 80, 81, 83, 84, 85, 87, 93, 94, 95, 96, 97, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 112, 113, 114, 116, 117, 118, 119, 120, 121, 129, 130, 131, 133, 134, 135, 141, 142, 143, 144, 145, 146, 147, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 188, 190, 191, 192, 194, 195, 196, 197, 198, 200, 203, 204, 206, 207, 209, 210, 212, 216, 220, 221, 222, 223, 224, 225, 226, 227, 231, 238, 239, 245, 252, 253, 259, 267, 271, 278, 283, 290, 304, 305, 306, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 324, 325, 326, 327, 328, 329, 333, 337, 338, 339, 340, 341, 342, 350, 351, 352, 353, 354, 355, 363, 364, 365, 366, 367, 368, 376, 377, 379, 382, 383, 386, 387, 388, 389, 390, 395, 396, 402, 403, 404, 408, 409, 412, 413, 414, 415, 416, 421, 422, 425, 426, 427, 438, 439, 440, 442, 443, 444, 445, 446, 452, 453, 454, 455, 456, 459, 463, 464, 465 }

B grade { 5, 6, 7, 18, 19, 21, 25, 27, 102, 115, 125, 126, 127, 132, 136, 137, 138, 139, 140, 236, 256, 265, 266, 279, 291, 297, 298, 299, 300, 301, 302, 303, 307, 308, 309, 310, 330, 331, 332, 334, 335, 336, 378, 411, 423, 424, 434, 435, 436, 441, 447, 448, 449, 450, 451 }

C grade { 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 75, 76, 77, 78, 79, 82, 88, 89, 90, 91, 92, 98, 110, 111, 122, 123, 124, 128, 148, 149, 150, 151, 180, 186, 187, 189, 193, 199, 201, 202, 205, 208, 211, 213, 214, 215, 217, 218, 219, 232, 234, 235, 237, 240, 246, 247, 248, 249, 250, 251, 254, 255, 260, 263, 264, 268, 269, 272, 276, 277, 280, 281, 282, 284, 289, 292, 293, 294, 295, 296, 321, 322, 323, 343, 344, 345, 346, 347, 348, 349, 356, 357, 358, 359, 360, 361, 362, 369, 370, 371, 372, 373, 374, 375, 380, 384, 385, 391, 392, 393, 394, 397, 398, 399, 400, 401, 405, 410, 417, 428, 429, 430, 437, 457, 458, 460, 461 }

F normal fail { 73, 74, 86, 228, 229, 230, 233, 241, 242, 243, 244, 257, 258, 261, 262, 270, 273, 274, 275, 285, 286, 287, 288, 381, 406, 407, 418, 419, 420, 431, 432, 433 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 69, 82, 93, 94, 95, 103, 104, 105, 106, 107, 108, 116, 117, 118, 119, 120, 121, 129, 130, 131, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 278, 290, 291, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 378, 391, 404, 417, 430, 452, 453, 454, 455, 456, 459, 461 }

B grade { 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 96, 97, 98, 99, 100, 101, 102, 109, 110, 111, 112, 113, 114, 115, 122, 123, 124, 125, 126, 127, 128, 234, 235, 236, 237, 238, 239, 240, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 263, 264, 265, 266, 267, 268, 269, 276, 277, 279, 280, 281, 282, 289, 292, 293, 294, 295, 296, 297, 376, 377, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440 }

C grade { 26, 144, 145, 228, 229, 230, 231, 232, 233, 241, 242, 243, 244, 245, 246, 257, 258, 259, 260, 261, 262, 270, 271, 272, 273, 274, 275, 283, 284, 285, 286, 287, 288, 457, 458, 460 }

F normal fail { 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 463, 464, 465 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 8, 9, 10, 12, 13, 14, 15, 16, 17, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 41, 42, 43, 47, 48, 49, 54, 55, 56, 60, 61, 67, 68, 69, 70, 71, 73, 74, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 112, 113, 116, 117, 118, 119, 120, 121, 125, 126, 129, 144, 145, 146, 147, 148, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 189, 190, 191, 192, 193, 194, 195, 196, 198, 203, 204, 205, 218, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 239, 240, 247, 248, 249, 253, 263, 268, 269, 276, 281, 282, 290, 295, 311, 312, 313, 314, 315, 316, 317, 318, 319, 321, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 337, 338, 339, 340, 341, 345, 346, 350, 351, 352, 353, 363, 364, 365, 382, 395, 396, 452, 453, 454, 455, 456, 459 }

B grade { 5, 6, 7, 11, 18, 19, 20, 33, 39, 40, 44, 45, 46, 50, 51, 52, 53, 57, 58, 59, 62, 63, 64, 65, 66, 72, 75, 76, 77, 78, 79, 88, 89, 101, 102, 109, 110, 111, 114, 115, 122, 123, 124, 127, 128, 130, 131, 149, 150, 151, 161, 186, 187, 188, 197, 199, 200, 201, 202, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 219, 236, 237, 238, 250, 251, 252, 254, 255, 256, 264, 265, 266, 267, 277, 278, 279, 280, 289, 291, 292, 293, 294, 296, 297, 320, 322, 323, 335, 336, 342, 343, 344, 347, 348, 349, 354, 355, 356, 357, 358, 359, 360, 361, 362, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440 }

C grade { 228, 229, 230, 241, 242, 243, 257, 258, 259, 270, 271, 272, 283, 284, 285, 457, 458, 460, 461 }

F normal fail { 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 231, 232, 233, 244, 245, 246, 260, 261, 262, 273, 274, 275, 286, 287, 288, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 463, 464, 465 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 66, 67, 68, 69, 77, 78, 79, 80, 81, 82, 90, 91, 92, 93, 94, 95, 103, 104, 105, 106, 107, 108, 116, 117, 118, 119, 120, 121, 129, 130, 131, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 247, 248, 249, 263, 264, 265, 276, 277, 278, 290, 291, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 369, 370, 371, 372, 373, 374, 375, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461 }

B grade { 33, 46, 58, 59, 64, 65, 102, 115, 219, 266, 279, 289, 297, 355, 367, 368, 411, 416, 423, 424 }

C grade { 76, 237, 256, 379, 385 }

F normal fail { 70, 71, 72, 73, 74, 75, 83, 84, 85, 86, 87, 88, 89, 96, 97, 98, 99, 100, 101, 109, 112, 113, 114, 122, 125, 126, 127, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 228, 229, 230, 231, 232, 233, 238, 239, 240, 241, 242, 243, 244, 245, 246, 250, 251, 252, 253, 254, 255, 257, 258, 259, 260, 261, 262, 267, 268, 269, 270, 271, 272, 273, 274, 275, 280, 281, 282, 283, 284, 285, 286, 287, 288, 293, 294, 295, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 376, 377, 378, 380, 381, 382, 383, 384, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 412, 413, 414, 415, 417, 418, 422, 425, 430, 431, 434, 438, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 463, 464, 465 }

F(-1) timedout fail { 110, 111, 123, 124, 128, 292, 296, 419, 420, 421, 426, 427, 428, 429, 432, 433, 435, 436, 437, 439, 440 }

F(-2) exception fail { }

Giac

A grade { 3, 4, 8, 9, 10, 11, 12, 13, 14, 16, 17, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 34, 35, 36, 37, 38, 39, 40, 42, 43, 47, 48, 49, 50, 51, 52, 53, 55, 56, 60, 61, 62, 63, 64, 65, 66, 69, 82, 95, 108, 121, 144, 145, 146, 147, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 225, 226, 227, 317, 318, 319, 320, 330, 331, 332, 333, 334, 343, 344, 345, 346, 347, 348, 349, 356, 357, 358, 359, 360, 361, 362, 369, 370, 371, 372, 373, 374, 375, 455 }

B grade { 1, 2, 5, 6, 7, 15, 18, 19, 20, 28, 32, 33, 41, 44, 45, 46, 54, 57, 58, 59, 67, 68, 70, 71, 72, 80, 81, 96, 109, 122, 148, 149, 150, 151, 222, 223, 311, 312, 313, 314, 315, 316, 321, 322, 323, 324, 325, 326, 327, 328, 329, 335, 336, 337, 338, 339, 340, 341, 342, 350, 351, 352, 353, 354, 355, 363, 364, 365, 366, 367, 368, 376, 377, 378, 379, 380, 381, 389, 390, 391, 392, 452, 453, 454, 456 }

C grade { }

F normal fail { 73, 74, 75, 76, 77, 78, 79, 86, 87, 88, 89, 90, 91, 92, 93, 94, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 224, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 382, 383, 384, 385, 386, 387, 388, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 457, 458, 459, 460, 461, 463, 464, 465 }

F(-1) timedout fail { }

F(-2) exception fail { 83, 84, 85, 393, 394 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 69, 82, 95, 103, 104, 105, 108, 116, 117, 121, 129, 135, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 278, 290, 291, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 378, 391, 404, 417, 430, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461 }

C grade { }

F normal fail { }

F(-1) timedout fail { 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 97, 98, 99, 100, 101, 102, 106, 107, 109, 110, 111, 112, 113, 114, 115, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 376, 377, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 463, 464, 465 }

F(-2) exception fail { }

Sympy

A grade { 162, 312, 313, 317, 318, 319, 325, 326, 417, 430, 452, 453, 454 }

B grade { 311, 324, 339, 459 }

C grade { }

F normal fail { 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 16, 17, 18, 19, 23, 24, 25, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 43, 44, 45, 48, 49, 50, 51, 52, 63, 69, 70, 71, 72, 74, 75, 76, 77, 78, 82, 83, 89, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 121, 122, 123, 124, 126, 127, 128, 129, 130, 135, 140, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 165, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 199, 200, 201, 202, 203, 204, 206, 207, 208, 212, 213, 214, 215, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 234, 235, 236, 237, 238, 239, 241, 242, 243, 244, 247, 248, 249, 250, 254, 255, 256, 257, 258, 259, 260, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 289, 290, 291, 292, 293, 294, 296, 297, 298, 299, 300, 301, 306, 307, 314, 315, 316, 320, 321, 322, 323, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 353, 354, 355, 356, 357, 358, 359, 360, 361, 369, 370, 371, 372, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 395, 396, 397, 398, 399, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 443, 444, 445, 448, 449, 455, 456, 457, 458, 460, 461, 464, 465 }

F(-1) timedout fail { 1, 8, 14, 15, 20, 21, 22, 26, 27, 28, 29, 41, 42, 46, 47, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 73, 79, 80, 81, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 106, 107, 119, 120, 125, 131, 132, 133, 134, 136, 137, 138, 139, 141, 142, 143, 164, 170, 177, 184, 185, 197, 198, 205, 209, 210, 211, 216, 217, 218, 219, 220, 232, 233, 240, 245, 246, 251, 252, 253, 261, 262, 274, 275, 287, 288, 295, 302, 303, 304, 305, 308, 309, 310, 351, 352, 362, 363, 364, 365, 366, 367, 368, 373, 374, 375, 394, 400, 401, 441, 442, 446, 447, 450, 451, 463, 467 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	120	98	73	75	0	266	70
N.S.	1	1.00	1.45	1.18	0.88	0.90	0.00	3.20	0.84
time (sec)	N/A	0.079	0.404	4.138	0.195	0.251	0.000	0.325	0.090

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	88	81	58	60	0	197	55
N.S.	1	1.00	1.33	1.23	0.88	0.91	0.00	2.98	0.83
time (sec)	N/A	0.064	0.034	0.256	0.188	0.268	0.000	0.323	17.328

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	53	62	40	42	0	57	39
N.S.	1	1.00	1.20	1.41	0.91	0.95	0.00	1.30	0.89
time (sec)	N/A	0.048	0.028	0.175	0.195	0.257	0.000	0.291	0.071

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	35	25	25	27	0	26	25
N.S.	1	1.00	1.46	1.04	1.04	1.12	0.00	1.08	1.04
time (sec)	N/A	0.027	0.042	0.079	0.193	0.239	0.000	0.274	0.052

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	84	40	44	60	0	60	29
N.S.	1	1.00	3.11	1.48	1.63	2.22	0.00	2.22	1.07
time (sec)	N/A	0.039	0.073	0.119	0.194	0.248	0.000	0.276	0.101

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	236	90	76	124	0	193	62
N.S.	1	1.00	4.45	1.70	1.43	2.34	0.00	3.64	1.17
time (sec)	N/A	0.060	0.874	0.216	0.190	0.251	0.000	0.298	17.323

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	198	120	101	178	0	262	86
N.S.	1	1.00	2.44	1.48	1.25	2.20	0.00	3.23	1.06
time (sec)	N/A	0.093	6.264	0.300	0.178	0.257	0.000	0.310	17.538

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	78	89	111	86	0	104	105
N.S.	1	1.00	0.80	0.91	1.13	0.88	0.00	1.06	1.07
time (sec)	N/A	0.131	0.424	0.445	0.281	0.250	0.000	0.306	18.465

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	54	71	82	68	0	82	79
N.S.	1	1.00	0.77	1.01	1.17	0.97	0.00	1.17	1.13
time (sec)	N/A	0.072	0.371	0.273	0.269	0.253	0.000	0.294	17.106

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	54	46	47	50	0	47	35
N.S.	1	1.00	1.29	1.10	1.12	1.19	0.00	1.12	0.83
time (sec)	N/A	0.053	0.164	0.141	0.278	0.257	0.000	0.282	17.630

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	31	0	15	17
N.S.	1	1.00	1.00	1.07	1.00	2.07	0.00	1.00	1.13
time (sec)	N/A	0.014	0.010	0.147	0.185	0.242	0.000	0.260	17.710

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	36	43	26	39	0	26	28
N.S.	1	1.00	1.38	1.65	1.00	1.50	0.00	1.00	1.08
time (sec)	N/A	0.040	0.069	0.227	0.179	0.234	0.000	0.280	19.045

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	84	73	43	66	0	50	46
N.S.	1	1.00	1.83	1.59	0.93	1.43	0.00	1.09	1.00
time (sec)	N/A	0.051	0.054	0.487	0.179	0.254	0.000	0.288	19.230

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	128	89	64	91	0	76	60
N.S.	1	1.00	1.88	1.31	0.94	1.34	0.00	1.12	0.88
time (sec)	N/A	0.063	0.726	0.664	0.179	0.251	0.000	0.315	18.929

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	118	155	89	90	0	414	87
N.S.	1	1.00	1.22	1.60	0.92	0.93	0.00	4.27	0.90
time (sec)	N/A	0.105	2.333	0.460	0.177	0.258	0.000	0.412	18.291

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	83	69	67	67	0	91	66
N.S.	1	1.00	1.15	0.96	0.93	0.93	0.00	1.26	0.92
time (sec)	N/A	0.079	1.882	0.265	0.179	0.257	0.000	0.368	18.215

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	75	42	42	44	0	44	45
N.S.	1	1.00	1.63	0.91	0.91	0.96	0.00	0.96	0.98
time (sec)	N/A	0.044	0.103	0.175	0.175	0.253	0.000	0.311	0.093

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	108	94	82	101	0	172	53
N.S.	1	1.00	2.08	1.81	1.58	1.94	0.00	3.31	1.02
time (sec)	N/A	0.077	0.911	0.218	0.181	0.260	0.000	0.318	0.159

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	1021	163	126	193	0	341	96
N.S.	1	1.00	9.82	1.57	1.21	1.86	0.00	3.28	0.92
time (sec)	N/A	0.133	8.090	0.379	0.176	0.262	0.000	0.352	18.508

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	218	211	165	286	0	493	135
N.S.	1	1.00	1.55	1.50	1.17	2.03	0.00	3.50	0.96
time (sec)	N/A	0.177	9.790	0.558	0.177	0.257	0.000	0.376	18.296

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	499	178	164	131	0	183	163
N.S.	1	1.00	3.37	1.20	1.11	0.89	0.00	1.24	1.10
time (sec)	N/A	0.218	3.433	0.608	0.260	0.263	0.000	0.406	19.110

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	153	123	120	107	0	123	116
N.S.	1	1.00	1.34	1.08	1.05	0.94	0.00	1.08	1.02
time (sec)	N/A	0.140	2.230	0.407	0.270	0.250	0.000	0.372	18.780

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	126	71	67	81	0	67	94
N.S.	1	1.00	1.73	0.97	0.92	1.11	0.00	0.92	1.29
time (sec)	N/A	0.121	2.189	0.253	0.257	0.261	0.000	0.334	19.134

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	41	46	44	58	0	49	42
N.S.	1	1.00	1.02	1.15	1.10	1.45	0.00	1.22	1.05
time (sec)	N/A	0.036	0.373	0.126	0.170	0.240	0.000	0.281	18.820

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	109	96	54	71	0	60	56
N.S.	1	1.00	2.18	1.92	1.08	1.42	0.00	1.20	1.12
time (sec)	N/A	0.071	2.618	0.214	0.176	0.252	0.000	0.327	18.617

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	151	130	80	101	0	98	85
N.S.	1	1.00	1.99	1.71	1.05	1.33	0.00	1.29	1.12
time (sec)	N/A	0.101	5.183	0.293	0.183	0.238	0.000	0.345	18.494

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	353	154	107	139	0	141	108
N.S.	1	1.00	3.43	1.50	1.04	1.35	0.00	1.37	1.05
time (sec)	N/A	0.123	2.667	0.453	0.198	0.249	0.000	0.388	18.749

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	425	115	102	229	0	349	123
N.S.	1	1.00	4.34	1.17	1.04	2.34	0.00	3.56	1.26
time (sec)	N/A	0.137	4.852	1.006	0.261	0.284	0.000	0.309	18.106

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	376	67	63	154	0	85	76
N.S.	1	1.00	5.30	0.94	0.89	2.17	0.00	1.20	1.07
time (sec)	N/A	0.110	2.207	0.486	0.276	0.268	0.000	0.292	0.117

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	329	44	40	118	0	42	39
N.S.	1	1.00	7.00	0.94	0.85	2.51	0.00	0.89	0.83
time (sec)	N/A	0.052	1.712	0.180	0.260	0.269	0.000	0.288	17.885

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	239	71	64	156	0	85	123
N.S.	1	1.00	4.35	1.29	1.16	2.84	0.00	1.55	2.24
time (sec)	N/A	0.083	1.239	0.144	0.264	0.281	0.000	0.292	0.247

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	371	115	128	327	0	209	392
N.S.	1	1.00	4.31	1.34	1.49	3.80	0.00	2.43	4.56
time (sec)	N/A	0.120	1.915	0.244	0.261	0.299	0.000	0.318	18.893

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	549	181	231	693	0	389	870
N.S.	1	1.00	4.26	1.40	1.79	5.37	0.00	3.02	6.74
time (sec)	N/A	0.192	5.212	0.720	0.261	0.325	0.000	0.332	22.212

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	357	170	209	428	0	237	1448
N.S.	1	1.00	2.15	1.02	1.26	2.58	0.00	1.43	8.72
time (sec)	N/A	0.383	3.575	2.537	0.261	0.298	0.000	0.306	19.652

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	303	119	137	332	0	152	494
N.S.	1	1.00	2.59	1.02	1.17	2.84	0.00	1.30	4.22
time (sec)	N/A	0.196	1.385	1.262	0.262	0.284	0.000	0.302	18.783

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	245	78	77	257	0	92	111
N.S.	1	1.00	3.22	1.03	1.01	3.38	0.00	1.21	1.46
time (sec)	N/A	0.115	0.607	0.574	0.261	0.285	0.000	0.298	18.391

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	182	46	44	231	0	65	460
N.S.	1	1.00	4.04	1.02	0.98	5.13	0.00	1.44	10.22
time (sec)	N/A	0.054	1.413	0.167	0.262	0.282	0.000	0.276	18.479

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	189	52	50	271	0	71	46
N.S.	1	1.00	3.50	0.96	0.93	5.02	0.00	1.31	0.85
time (sec)	N/A	0.079	0.886	0.293	0.263	0.295	0.000	0.295	18.116

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	226	69	82	397	0	103	80
N.S.	1	1.00	2.97	0.91	1.08	5.22	0.00	1.36	1.05
time (sec)	N/A	0.109	1.863	0.507	0.265	0.295	0.000	0.315	18.335

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	318	93	137	587	0	173	112
N.S.	1	1.00	3.03	0.89	1.30	5.59	0.00	1.65	1.07
time (sec)	N/A	0.172	1.777	0.785	0.263	0.294	0.000	0.339	19.026

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	454	159	148	405	0	509	195
N.S.	1	1.00	2.82	0.99	0.92	2.52	0.00	3.16	1.21
time (sec)	N/A	0.204	5.661	4.609	0.261	0.303	0.000	0.364	0.164

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	403	102	104	297	0	136	130
N.S.	1	1.00	3.54	0.89	0.91	2.61	0.00	1.19	1.14
time (sec)	N/A	0.130	2.914	2.459	0.263	0.284	0.000	0.337	0.138

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	393	69	70	201	0	72	72
N.S.	1	1.00	4.68	0.82	0.83	2.39	0.00	0.86	0.86
time (sec)	N/A	0.060	2.584	1.030	0.259	0.285	0.000	0.309	18.816

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	384	103	138	390	0	272	2188
N.S.	1	1.00	3.88	1.04	1.39	3.94	0.00	2.75	22.10
time (sec)	N/A	0.127	1.633	0.578	0.262	0.321	0.000	0.331	20.176

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	468	148	231	698	0	550	1845
N.S.	1	1.00	3.18	1.01	1.57	4.75	0.00	3.74	12.55
time (sec)	N/A	0.202	2.259	0.805	0.258	0.327	0.000	0.371	18.841

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	450	213	369	1202	0	669	4338
N.S.	1	1.00	2.28	1.08	1.87	6.10	0.00	3.40	22.02
time (sec)	N/A	0.305	2.637	1.221	0.273	0.365	0.000	0.412	23.158

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	2468	204	302	674	0	294	1461
N.S.	1	1.00	9.24	0.76	1.13	2.52	0.00	1.10	5.47
time (sec)	N/A	0.529	19.912	5.733	0.267	0.326	0.000	0.362	20.400

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	1105	147	205	522	0	193	435
N.S.	1	1.00	5.79	0.77	1.07	2.73	0.00	1.01	2.28
time (sec)	N/A	0.284	11.557	3.168	0.260	0.306	0.000	0.343	19.422

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	699	109	126	441	0	149	816
N.S.	1	1.00	5.38	0.84	0.97	3.39	0.00	1.15	6.28
time (sec)	N/A	0.193	8.734	1.685	0.265	0.300	0.000	0.329	18.499

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	240	90	106	435	0	114	2056
N.S.	1	1.00	2.61	0.98	1.15	4.73	0.00	1.24	22.35
time (sec)	N/A	0.109	2.679	0.409	0.260	0.312	0.000	0.275	20.406

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	242	78	117	407	0	127	91
N.S.	1	1.00	2.66	0.86	1.29	4.47	0.00	1.40	1.00
time (sec)	N/A	0.106	2.543	0.576	0.262	0.309	0.000	0.352	18.054

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	303	106	193	663	0	185	141
N.S.	1	1.00	2.46	0.86	1.57	5.39	0.00	1.50	1.15
time (sec)	N/A	0.218	5.803	0.907	0.267	0.307	0.000	0.387	19.580

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	777	124	268	987	0	254	198
N.S.	1	1.00	4.13	0.66	1.43	5.25	0.00	1.35	1.05
time (sec)	N/A	0.324	3.241	1.326	0.262	0.322	0.000	0.429	20.341

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	1641	190	204	579	0	781	255
N.S.	1	1.00	7.67	0.89	0.95	2.71	0.00	3.65	1.19
time (sec)	N/A	0.299	9.356	10.368	0.271	0.324	0.000	0.467	17.898

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	1153	125	149	439	0	174	172
N.S.	1	1.00	7.49	0.81	0.97	2.85	0.00	1.13	1.12
time (sec)	N/A	0.216	8.741	5.737	0.260	0.300	0.000	0.417	0.177

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	656	83	103	299	0	92	105
N.S.	1	1.00	5.66	0.72	0.89	2.58	0.00	0.79	0.91
time (sec)	N/A	0.076	5.522	2.949	0.260	0.289	0.000	0.380	0.154

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	447	156	261	779	0	592	3557
N.S.	1	1.00	2.90	1.01	1.69	5.06	0.00	3.84	23.10
time (sec)	N/A	0.236	4.217	1.471	0.260	0.396	0.000	0.407	21.902

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-1)	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	532	198	399	1332	0	750	2728
N.S.	1	1.00	2.50	0.93	1.87	6.25	0.00	3.52	12.81
time (sec)	N/A	0.350	5.356	1.760	0.269	0.430	0.000	0.522	19.873

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-1)	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	549	259	528	1833	0	1316	5613
N.S.	1	1.00	2.14	1.01	2.05	7.13	0.00	5.12	21.84
time (sec)	N/A	0.422	5.318	2.312	0.283	0.468	0.000	0.462	23.778

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	1639	247	418	930	0	351	2117
N.S.	1	1.00	5.22	0.79	1.33	2.96	0.00	1.12	6.74
time (sec)	N/A	0.570	17.416	11.695	0.272	0.359	0.000	0.492	22.332

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	2469	193	299	803	0	306	1317
N.S.	1	1.00	10.37	0.81	1.26	3.37	0.00	1.29	5.53
time (sec)	N/A	0.386	21.952	6.369	0.267	0.342	0.000	0.438	21.111

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	1915	155	272	815	0	208	2628
N.S.	1	1.00	10.41	0.84	1.48	4.43	0.00	1.13	14.28
time (sec)	N/A	0.316	15.357	4.212	0.273	0.334	0.000	0.406	22.316

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	332	147	231	819	0	196	3271
N.S.	1	1.00	2.31	1.02	1.60	5.69	0.00	1.36	22.72
time (sec)	N/A	0.225	6.393	1.535	0.266	0.327	0.000	0.288	22.736

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	749	97	219	615	0	177	146
N.S.	1	1.00	6.04	0.78	1.77	4.96	0.00	1.43	1.18
time (sec)	N/A	0.134	6.880	1.373	0.266	0.308	0.000	0.452	19.490

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	994	140	323	1009	0	264	207
N.S.	1	1.00	6.06	0.85	1.97	6.15	0.00	1.61	1.26
time (sec)	N/A	0.307	5.026	1.681	0.271	0.331	0.000	0.423	20.644

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	479	180	434	1423	0	367	267
N.S.	1	1.00	1.98	0.74	1.79	5.88	0.00	1.52	1.10
time (sec)	N/A	0.460	5.566	2.549	0.280	0.359	0.000	0.463	21.389

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	152	572	171	295	0	1281	0
N.S.	1	1.00	1.09	4.12	1.23	2.12	0.00	9.22	0.00
time (sec)	N/A	0.178	1.071	3.313	0.267	0.414	0.000	0.647	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	120	340	116	232	0	805	0
N.S.	1	1.00	1.20	3.40	1.16	2.32	0.00	8.05	0.00
time (sec)	N/A	0.117	0.364	0.381	0.264	0.386	0.000	0.467	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	99	93	88	182	0	55	87
N.S.	1	1.00	1.50	1.41	1.33	2.76	0.00	0.83	1.32
time (sec)	N/A	0.066	0.143	0.067	0.266	0.394	0.000	0.278	18.363

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	119	542	0	496	0	405	0
N.S.	1	1.00	1.45	6.61	0.00	6.05	0.00	4.94	0.00
time (sec)	N/A	0.105	0.298	0.560	0.000	0.337	0.000	0.699	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	163	2129	0	867	0	578	0
N.S.	1	1.00	1.31	17.17	0.00	6.99	0.00	4.66	0.00
time (sec)	N/A	0.150	0.662	0.444	0.000	0.396	0.000	0.824	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	183	183	209	4318	0	1476	0	867	0
N.S.	1	1.00	1.14	23.60	0.00	8.07	0.00	4.74	0.00
time (sec)	N/A	0.251	1.717	0.507	0.000	0.658	0.000	1.211	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	240	240	0	1180	0	1715	0	0	0
N.S.	1	1.00	0.00	4.92	0.00	7.15	0.00	0.00	0.00
time (sec)	N/A	0.431	0.000	12.638	0.000	2.887	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	A	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	181	181	0	791	0	1565	0	0	0
N.S.	1	1.00	0.00	4.37	0.00	8.65	0.00	0.00	0.00
time (sec)	N/A	0.262	0.000	9.838	0.000	1.085	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	432	516	0	1417	0	0	0
N.S.	1	1.00	3.51	4.20	0.00	11.52	0.00	0.00	0.00
time (sec)	N/A	0.161	4.291	7.378	0.000	0.606	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	C	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	284	353	3227	1227	0	0	0
N.S.	1	1.00	3.59	4.47	40.85	15.53	0.00	0.00	0.00
time (sec)	N/A	0.065	2.888	6.387	0.763	0.478	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	61	326	50	306	0	0	0
N.S.	1	1.00	0.90	4.79	0.74	4.50	0.00	0.00	0.00
time (sec)	N/A	0.103	0.198	5.342	0.180	0.310	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	285	1037	82	436	0	0	0
N.S.	1	1.00	2.71	9.88	0.78	4.15	0.00	0.00	0.00
time (sec)	N/A	0.135	2.761	7.325	0.184	0.473	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	941	1680	143	656	0	0	0
N.S.	1	1.00	6.32	11.28	0.96	4.40	0.00	0.00	0.00
time (sec)	N/A	0.187	8.732	7.177	0.187	1.387	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	188	802	277	351	0	1952	0
N.S.	1	1.00	0.96	4.09	1.41	1.79	0.00	9.96	0.00
time (sec)	N/A	0.209	1.155	7.029	0.265	0.527	0.000	1.951	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	164	552	250	278	0	1388	0
N.S.	1	1.00	1.01	3.41	1.54	1.72	0.00	8.57	0.00
time (sec)	N/A	0.156	0.557	6.934	0.270	0.497	0.000	1.242	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	73	121	142	231	0	87	61
N.S.	1	1.00	0.73	1.21	1.42	2.31	0.00	0.87	0.61
time (sec)	N/A	0.094	0.483	0.089	0.259	0.413	0.000	0.311	20.409

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	171	1437	0	715	0	0	0
N.S.	1	1.00	1.40	11.78	0.00	5.86	0.00	0.00	0.00
time (sec)	N/A	0.160	0.451	6.762	0.000	0.413	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	202	3470	0	984	0	0	0
N.S.	1	1.00	1.25	21.55	0.00	6.11	0.00	0.00	0.00
time (sec)	N/A	0.237	1.115	6.921	0.000	0.427	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	262	5131	0	1511	0	0	0
N.S.	1	1.00	1.20	23.54	0.00	6.93	0.00	0.00	0.00
time (sec)	N/A	0.381	2.604	6.610	0.000	0.468	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	298	298	0	1602	0	1855	0	0	0
N.S.	1	1.00	0.00	5.38	0.00	6.22	0.00	0.00	0.00
time (sec)	N/A	0.521	0.000	16.254	0.000	10.531	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	217	217	225	1186	0	1667	0	0	0
N.S.	1	1.00	1.04	5.47	0.00	7.68	0.00	0.00	0.00
time (sec)	N/A	0.380	3.567	16.911	0.000	3.352	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	161	161	493	900	0	1535	0	0	0
N.S.	1	1.00	3.06	5.59	0.00	9.53	0.00	0.00	0.00
time (sec)	N/A	0.224	3.784	11.543	0.000	1.180	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	118	118	527	709	0	1457	0	0	0
N.S.	1	1.00	4.47	6.01	0.00	12.35	0.00	0.00	0.00
time (sec)	N/A	0.113	3.456	9.876	0.000	0.615	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	64	741	98	370	0	0	0
N.S.	1	1.00	0.61	7.06	0.93	3.52	0.00	0.00	0.00
time (sec)	N/A	0.122	0.202	10.167	0.195	0.468	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	369	1141	243	472	0	0	0
N.S.	1	1.00	2.15	6.63	1.41	2.74	0.00	0.00	0.00
time (sec)	N/A	0.179	7.692	10.654	0.192	1.502	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	512	1851	273	682	0	0	0
N.S.	1	1.00	2.45	8.86	1.31	3.26	0.00	0.00	0.00
time (sec)	N/A	0.241	9.263	11.454	0.205	6.336	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	93	94	162	87	0	0	0
N.S.	1	1.00	0.76	0.76	1.32	0.71	0.00	0.00	0.00
time (sec)	N/A	0.168	0.972	1.170	0.199	0.269	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	64	58	83	57	0	0	0
N.S.	1	1.00	0.86	0.78	1.12	0.77	0.00	0.00	0.00
time (sec)	N/A	0.103	0.176	1.056	0.193	0.248	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	48	31	28	37	0	31	46
N.S.	1	1.00	1.60	1.03	0.93	1.23	0.00	1.03	1.53
time (sec)	N/A	0.053	0.101	0.271	0.188	0.248	0.000	0.299	17.825

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	86	247	0	140	0	305	0
N.S.	1	1.00	2.00	5.74	0.00	3.26	0.00	7.09	0.00
time (sec)	N/A	0.084	0.108	1.217	0.000	0.308	0.000	0.628	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	87	87	140	1215	0	305	0	0	0
N.S.	1	1.00	1.61	13.97	0.00	3.51	0.00	0.00	0.00
time (sec)	N/A	0.132	0.734	1.283	0.000	0.335	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	78	1760	0	491	0	0	0
N.S.	1	1.00	0.57	12.75	0.00	3.56	0.00	0.00	0.00
time (sec)	N/A	0.202	0.150	1.141	0.000	0.364	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	163	1136	0	639	0	0	0
N.S.	1	1.00	0.84	5.89	0.00	3.31	0.00	0.00	0.00
time (sec)	N/A	0.345	1.114	6.756	0.000	1.280	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	145	809	0	565	0	0	0
N.S.	1	1.00	1.07	5.99	0.00	4.19	0.00	0.00	0.00
time (sec)	N/A	0.196	0.335	5.557	0.000	0.565	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	125	509	0	497	0	0	0
N.S.	1	1.00	1.47	5.99	0.00	5.85	0.00	0.00	0.00
time (sec)	N/A	0.132	0.139	4.486	0.000	0.408	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	87	138	992	408	0	0	0
N.S.	1	1.00	2.23	3.54	25.44	10.46	0.00	0.00	0.00
time (sec)	N/A	0.038	0.060	2.726	0.431	0.387	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	55	48	33	47	0	0	74
N.S.	1	1.00	1.67	1.45	1.00	1.42	0.00	0.00	2.24
time (sec)	N/A	0.090	0.095	2.250	0.206	0.259	0.000	0.000	18.195

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	74	77	96	96	0	0	123
N.S.	1	1.00	0.95	0.99	1.23	1.23	0.00	0.00	1.58
time (sec)	N/A	0.123	0.156	3.635	0.210	0.303	0.000	0.000	22.952

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	100	120	191	171	0	0	723
N.S.	1	1.00	0.76	0.91	1.45	1.30	0.00	0.00	5.48
time (sec)	N/A	0.174	0.271	5.022	0.211	0.588	0.000	0.000	30.640

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	130	169	250	136	0	0	0
N.S.	1	1.00	0.76	0.99	1.46	0.80	0.00	0.00	0.00
time (sec)	N/A	0.223	6.908	5.043	0.217	0.326	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	93	115	141	98	0	0	0
N.S.	1	1.00	0.82	1.01	1.24	0.86	0.00	0.00	0.00
time (sec)	N/A	0.143	2.612	1.459	0.209	0.306	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	64	59	57	67	0	48	155
N.S.	1	1.00	1.03	0.95	0.92	1.08	0.00	0.77	2.50
time (sec)	N/A	0.073	0.941	0.101	0.211	0.277	0.000	0.369	24.732

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	113	1061	0	344	0	503	0
N.S.	1	1.00	1.41	13.26	0.00	4.30	0.00	6.29	0.00
time (sec)	N/A	0.117	0.565	1.177	0.000	0.330	0.000	0.864	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	97	1714	0	547	0	0	0
N.S.	1	1.00	0.77	13.60	0.00	4.34	0.00	0.00	0.00
time (sec)	N/A	0.184	0.263	1.086	0.000	0.376	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	177	177	100	3612	0	873	0	0	0
N.S.	1	1.00	0.56	20.41	0.00	4.93	0.00	0.00	0.00
time (sec)	N/A	0.264	0.383	1.234	0.000	0.419	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	256	1162	0	813	0	0	0
N.S.	1	1.00	1.06	4.80	0.00	3.36	0.00	0.00	0.00
time (sec)	N/A	0.412	6.295	9.328	0.000	5.907	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	229	833	0	703	0	0	0
N.S.	1	1.00	1.31	4.76	0.00	4.02	0.00	0.00	0.00
time (sec)	N/A	0.246	3.333	7.286	0.000	1.825	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	190	532	0	607	0	0	0
N.S.	1	1.00	1.57	4.40	0.00	5.02	0.00	0.00	0.00
time (sec)	N/A	0.176	1.102	5.306	0.000	0.674	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	168	515	2055	601	0	0	0
N.S.	1	1.00	2.18	6.69	26.69	7.81	0.00	0.00	0.00
time (sec)	N/A	0.054	1.052	3.687	0.545	0.430	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	76	84	64	102	0	0	2151
N.S.	1	1.00	1.12	1.24	0.94	1.50	0.00	0.00	31.63
time (sec)	N/A	0.104	1.274	3.986	0.210	0.343	0.000	0.000	26.102

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	102	142	156	189	0	0	124682
N.S.	1	1.00	0.83	1.15	1.27	1.54	0.00	0.00	1013.67
time (sec)	N/A	0.154	0.534	4.528	0.208	0.667	0.000	0.000	40.457

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	126	217	282	314	0	0	0
N.S.	1	1.00	0.69	1.19	1.54	1.72	0.00	0.00	0.00
time (sec)	N/A	0.218	0.739	7.085	0.199	2.338	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	182	217	334	189	0	0	0
N.S.	1	1.00	0.83	0.99	1.53	0.86	0.00	0.00	0.00
time (sec)	N/A	0.256	3.114	5.671	0.218	0.464	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	129	147	195	138	0	0	0
N.S.	1	1.00	0.88	1.01	1.34	0.95	0.00	0.00	0.00
time (sec)	N/A	0.152	2.189	1.531	0.209	0.385	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	88	90	86	101	0	70	26927
N.S.	1	1.00	0.91	0.93	0.89	1.04	0.00	0.72	277.60
time (sec)	N/A	0.077	0.859	0.108	0.193	0.314	0.000	0.471	30.609

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	108	3290	0	592	0	1097	0
N.S.	1	1.00	0.85	25.91	0.00	4.66	0.00	8.64	0.00
time (sec)	N/A	0.167	3.856	5.233	0.000	0.364	0.000	1.288	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	171	171	151	8689	0	941	0	0	0
N.S.	1	1.00	0.88	50.81	0.00	5.50	0.00	0.00	0.00
time (sec)	N/A	0.250	1.032	5.541	0.000	0.435	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	234	234	129	8115	0	1307	0	0	0
N.S.	1	1.00	0.55	34.68	0.00	5.59	0.00	0.00	0.00
time (sec)	N/A	0.388	1.311	6.063	0.000	0.557	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	1705	2211	0	1003	0	0	0
N.S.	1	1.00	5.92	7.68	0.00	3.48	0.00	0.00	0.00
time (sec)	N/A	0.536	16.243	10.455	0.000	24.107	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	1315	1611	0	873	0	0	0
N.S.	1	1.00	5.79	7.10	0.00	3.85	0.00	0.00	0.00
time (sec)	N/A	0.357	12.113	8.520	0.000	7.530	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	983	1592	0	879	0	0	0
N.S.	1	1.00	5.89	9.53	0.00	5.26	0.00	0.00	0.00
time (sec)	N/A	0.256	8.398	7.014	0.000	2.094	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	1927	1263	0	881	0	0	0
N.S.	1	1.00	15.42	10.10	0.00	7.05	0.00	0.00	0.00
time (sec)	N/A	0.118	15.750	5.537	0.000	0.711	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	108	136	94	192	0	0	336
N.S.	1	1.00	1.02	1.28	0.89	1.81	0.00	0.00	3.17
time (sec)	N/A	0.124	1.561	5.666	0.197	0.714	0.000	0.000	29.970

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	138	225	216	320	0	0	0
N.S.	1	1.00	0.87	1.42	1.37	2.03	0.00	0.00	0.00
time (sec)	N/A	0.181	4.513	7.221	0.203	2.353	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	173	332	373	460	0	0	0
N.S.	1	1.00	0.77	1.47	1.65	2.04	0.00	0.00	0.00
time (sec)	N/A	0.281	4.934	6.231	0.198	9.338	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD
size	123	0	286	0	0	0	0	0	0
N.S.	1	0.00	2.33	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	4.196	0.000	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	182	182	253	0	0	0	0	0	0
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.215	3.665	0.000	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	178	0	0	0	0	0	0
N.S.	1	1.00	1.52	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.108	2.397	0.000	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	68	0	0	0	0	0	79
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.16
time (sec)	N/A	0.052	0.790	0.000	0.000	0.000	0.000	0.000	19.172

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	1532	0	0	0	0	0	0
N.S.	1	1.00	19.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.088	16.053	0.000	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	266	0	0	0	0	0	0
N.S.	1	1.00	3.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.101	3.224	0.000	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	5878	0	0	0	0	0	0
N.S.	1	1.00	66.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.155	25.843	0.000	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	3781	0	0	0	0	0	0
N.S.	1	1.00	42.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.117	20.190	0.000	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	2137	0	0	0	0	0	0
N.S.	1	1.00	25.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.061	14.980	0.000	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	72	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.087	0.998	0.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	128	132	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.131	1.583	0.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	192	192	149	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.197	1.443	0.000	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	72	97	129	82	0	66	61
N.S.	1	1.00	0.97	1.31	1.74	1.11	0.00	0.89	0.82
time (sec)	N/A	0.053	0.036	0.862	0.187	0.262	0.000	0.303	19.402

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	58	75	81	69	0	53	49
N.S.	1	1.00	1.04	1.34	1.45	1.23	0.00	0.95	0.88
time (sec)	N/A	0.050	0.027	0.550	0.197	0.240	0.000	0.286	19.124

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	42	47	45	56	0	39	33
N.S.	1	1.00	1.11	1.24	1.18	1.47	0.00	1.03	0.87
time (sec)	N/A	0.040	0.018	0.378	0.182	0.249	0.000	0.276	18.012

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	26	17	16	32	0	16	16
N.S.	1	1.00	1.62	1.06	1.00	2.00	0.00	1.00	1.00
time (sec)	N/A	0.018	0.002	0.171	0.197	0.246	0.000	0.261	18.292

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	31	24	26	31	0	45	19
N.S.	1	1.00	1.63	1.26	1.37	1.63	0.00	2.37	1.00
time (sec)	N/A	0.028	0.016	0.152	0.277	0.239	0.000	0.266	18.650

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	36	34	40	75	0	80	31
N.S.	1	1.00	0.97	0.92	1.08	2.03	0.00	2.16	0.84
time (sec)	N/A	0.034	0.020	0.190	0.273	0.243	0.000	0.274	18.372

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	36	44	50	109	0	111	41
N.S.	1	1.00	0.65	0.80	0.91	1.98	0.00	2.02	0.75
time (sec)	N/A	0.047	0.044	0.254	0.280	0.245	0.000	0.291	18.414

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	36	54	60	147	0	139	51
N.S.	1	1.00	0.49	0.74	0.82	2.01	0.00	1.90	0.70
time (sec)	N/A	0.063	0.016	0.324	0.281	0.240	0.000	0.314	19.074

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	137	108	126	114	0	121	102
N.S.	1	1.00	1.40	1.10	1.29	1.16	0.00	1.23	1.04
time (sec)	N/A	0.066	0.037	0.898	0.206	0.247	0.000	0.289	18.120

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	93	85	97	95	0	98	78
N.S.	1	1.00	1.33	1.21	1.39	1.36	0.00	1.40	1.11
time (sec)	N/A	0.055	0.023	0.585	0.192	0.250	0.000	0.285	0.146

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	48	55	58	72	0	60	41
N.S.	1	1.00	1.20	1.38	1.45	1.80	0.00	1.50	1.02
time (sec)	N/A	0.030	0.014	0.342	0.191	0.246	0.000	0.280	18.457

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	35	30	38	40	0	40	22
N.S.	1	1.00	1.46	1.25	1.58	1.67	0.00	1.67	0.92
time (sec)	N/A	0.030	0.041	0.199	0.198	0.248	0.000	0.266	18.342

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	50	31	27	28	0	34	28
N.S.	1	1.00	1.67	1.03	0.90	0.93	0.00	1.13	0.93
time (sec)	N/A	0.053	0.034	0.283	0.204	0.232	0.000	0.275	0.056

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	71	49	43	45	0	57	43
N.S.	1	1.00	1.42	0.98	0.86	0.90	0.00	1.14	0.86
time (sec)	N/A	0.080	0.021	0.431	0.198	0.257	0.000	0.280	18.146

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	81	78	60	74	0	79	56
N.S.	1	1.00	0.93	0.90	0.69	0.85	0.00	0.91	0.64
time (sec)	N/A	0.058	0.237	0.685	0.183	0.253	0.000	0.282	18.159

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	61	58	43	56	0	57	42
N.S.	1	1.00	0.94	0.89	0.66	0.86	0.00	0.88	0.65
time (sec)	N/A	0.049	0.151	0.475	0.191	0.236	0.000	0.274	18.416

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	36	35	34	37	0	34	28
N.S.	1	1.00	0.84	0.81	0.79	0.86	0.00	0.79	0.65
time (sec)	N/A	0.047	0.069	0.366	0.186	0.234	0.000	0.278	17.960

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	31	0	15	17
N.S.	1	1.00	1.00	1.07	1.00	2.07	0.00	1.00	1.13
time (sec)	N/A	0.016	0.001	0.161	0.183	0.246	0.000	0.267	18.417

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	33	24	37	28	51	37	25
N.S.	1	1.00	1.06	0.77	1.19	0.90	1.65	1.19	0.81
time (sec)	N/A	0.033	0.026	0.188	0.271	0.249	2.475	0.281	19.576

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	45	44	73	49	0	73	67
N.S.	1	1.00	0.74	0.72	1.20	0.80	0.00	1.20	1.10
time (sec)	N/A	0.046	0.104	0.333	0.274	0.257	0.000	0.274	18.806

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	68	61	103	68	0	96	91
N.S.	1	1.00	0.76	0.69	1.16	0.76	0.00	1.08	1.02
time (sec)	N/A	0.061	0.108	0.498	0.268	0.265	0.000	0.281	19.157

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	258	180	200	168	0	221	170
N.S.	1	1.00	1.56	1.09	1.21	1.02	0.00	1.34	1.03
time (sec)	N/A	0.165	0.048	1.380	0.194	0.261	0.000	0.302	18.832

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	187	147	166	143	0	183	134
N.S.	1	1.00	1.45	1.14	1.29	1.11	0.00	1.42	1.04
time (sec)	N/A	0.159	0.031	1.027	0.194	0.255	0.000	0.307	18.608

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	109	107	119	116	0	120	86
N.S.	1	1.00	1.20	1.18	1.31	1.27	0.00	1.32	0.95
time (sec)	N/A	0.088	0.026	0.645	0.203	0.258	0.000	0.290	18.690

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	80	69	87	94	0	79	55
N.S.	1	1.00	1.43	1.23	1.55	1.68	0.00	1.41	0.98
time (sec)	N/A	0.078	0.051	0.517	0.184	0.248	0.000	0.294	0.114

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	72	55	63	66	0	70	48
N.S.	1	1.00	1.47	1.12	1.29	1.35	0.00	1.43	0.98
time (sec)	N/A	0.074	0.052	0.418	0.198	0.251	0.000	0.287	17.975

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	106	67	55	59	0	76	44
N.S.	1	1.00	2.00	1.26	1.04	1.11	0.00	1.43	0.83
time (sec)	N/A	0.079	0.040	0.474	0.182	0.265	0.000	0.287	18.545

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	96	134	103	120	0	152	94
N.S.	1	1.00	0.91	1.26	0.97	1.13	0.00	1.43	0.89
time (sec)	N/A	0.112	0.350	1.094	0.200	0.250	0.000	0.328	18.233

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	75	104	81	94	0	114	70
N.S.	1	1.00	0.94	1.30	1.01	1.18	0.00	1.42	0.88
time (sec)	N/A	0.091	0.306	0.839	0.189	0.243	0.000	0.305	18.593

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	48	71	71	69	0	76	44
N.S.	1	1.00	0.91	1.34	1.34	1.30	0.00	1.43	0.83
time (sec)	N/A	0.078	0.208	0.619	0.190	0.246	0.000	0.302	18.245

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	41	46	44	58	0	49	42
N.S.	1	1.00	1.02	1.15	1.10	1.45	0.00	1.22	1.05
time (sec)	N/A	0.040	0.084	0.369	0.188	0.251	0.000	0.268	18.489

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	52	51	53	56	0	53	66
N.S.	1	1.00	1.11	1.09	1.13	1.19	0.00	1.13	1.40
time (sec)	N/A	0.093	0.163	0.421	0.278	0.271	0.000	0.289	18.780

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	58	54	87	62	0	87	76
N.S.	1	1.00	0.72	0.67	1.07	0.77	0.00	1.07	0.94
time (sec)	N/A	0.115	0.301	0.371	0.273	0.263	0.000	0.287	18.618

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	99	82	135	89	0	150	123
N.S.	1	1.00	0.83	0.69	1.13	0.75	0.00	1.26	1.03
time (sec)	N/A	0.173	0.269	0.493	0.273	0.253	0.000	0.296	19.077

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	70	84	83	90	0	91	73
N.S.	1	1.00	0.96	1.15	1.14	1.23	0.00	1.25	1.00
time (sec)	N/A	0.056	1.031	0.704	0.186	0.256	0.000	0.287	18.687

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	106	130	134	130	0	148	119
N.S.	1	1.00	0.95	1.17	1.21	1.17	0.00	1.33	1.07
time (sec)	N/A	0.087	3.596	1.082	0.187	0.246	0.000	0.307	18.709

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	1195	106	124	272	0	113	591
N.S.	1	1.00	13.90	1.23	1.44	3.16	0.00	1.31	6.87
time (sec)	N/A	0.151	4.768	0.788	0.270	0.289	0.000	0.287	0.552

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	49	63	83	157	0	74	456
N.S.	1	1.00	0.89	1.15	1.51	2.85	0.00	1.35	8.29
time (sec)	N/A	0.082	0.092	0.364	0.271	0.271	0.000	0.293	18.735

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	28	50	117	0	38	28
N.S.	1	1.00	1.00	0.78	1.39	3.25	0.00	1.06	0.78
time (sec)	N/A	0.051	0.053	0.215	0.267	0.266	0.000	0.274	0.124

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	45	67	164	0	53	44
N.S.	1	1.00	1.00	0.87	1.29	3.15	0.00	1.02	0.85
time (sec)	N/A	0.082	0.079	0.508	0.269	0.274	0.000	0.281	18.389

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	105	70	88	230	0	85	72
N.S.	1	1.00	1.38	0.92	1.16	3.03	0.00	1.12	0.95
time (sec)	N/A	0.104	0.226	0.987	0.269	0.272	0.000	0.294	18.602

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	136	110	117	305	0	129	111
N.S.	1	1.00	1.26	1.02	1.08	2.82	0.00	1.19	1.03
time (sec)	N/A	0.122	0.583	1.939	0.274	0.291	0.000	0.295	0.137

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	224	70	65	354	0	96	72
N.S.	1	1.00	2.91	0.91	0.84	4.60	0.00	1.25	0.94
time (sec)	N/A	0.113	2.791	0.971	0.273	0.279	0.000	0.293	18.549

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	192	45	45	286	0	66	44
N.S.	1	1.00	3.69	0.87	0.87	5.50	0.00	1.27	0.85
time (sec)	N/A	0.087	1.650	0.532	0.268	0.284	0.000	0.294	18.458

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	28	27	209	0	48	31
N.S.	1	1.00	1.00	0.78	0.75	5.81	0.00	1.33	0.86
time (sec)	N/A	0.079	0.033	0.218	0.270	0.261	0.000	0.290	18.856

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	182	46	44	231	0	65	460
N.S.	1	1.00	4.04	1.02	0.98	5.13	0.00	1.44	10.22
time (sec)	N/A	0.059	0.196	0.169	0.271	0.283	0.000	0.268	20.042

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	67	76	72	272	0	94	373
N.S.	1	1.00	0.89	1.01	0.96	3.63	0.00	1.25	4.97
time (sec)	N/A	0.128	0.537	0.683	0.271	0.302	0.000	0.295	20.228

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	95	116	126	343	0	141	1114
N.S.	1	1.00	0.81	0.99	1.08	2.93	0.00	1.21	9.52
time (sec)	N/A	0.213	3.202	1.371	0.269	0.289	0.000	0.289	20.259

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	133	165	189	424	0	216	1979
N.S.	1	1.00	0.82	1.01	1.16	2.60	0.00	1.33	12.14
time (sec)	N/A	0.305	1.855	2.619	0.269	0.301	0.000	0.290	21.549

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	980	110	146	392	0	126	2039
N.S.	1	1.00	9.61	1.08	1.43	3.84	0.00	1.24	19.99
time (sec)	N/A	0.168	4.729	1.060	0.276	0.312	0.000	0.305	21.326

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	71	68	98	262	0	76	62
N.S.	1	1.00	0.96	0.92	1.32	3.54	0.00	1.03	0.84
time (sec)	N/A	0.085	0.363	0.522	0.265	0.280	0.000	0.307	0.139

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	82	80	111	301	0	91	71
N.S.	1	1.00	0.99	0.96	1.34	3.63	0.00	1.10	0.86
time (sec)	N/A	0.078	0.300	0.549	0.271	0.268	0.000	0.306	19.736

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	89	92	133	391	0	113	94
N.S.	1	1.00	0.88	0.91	1.32	3.87	0.00	1.12	0.93
time (sec)	N/A	0.173	0.451	1.275	0.268	0.285	0.000	0.295	0.247

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	139	120	154	490	0	146	124
N.S.	1	1.00	1.10	0.95	1.22	3.89	0.00	1.16	0.98
time (sec)	N/A	0.196	0.834	3.095	0.277	0.308	0.000	0.300	19.791

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	171	158	183	583	0	188	173
N.S.	1	1.00	1.09	1.01	1.17	3.71	0.00	1.20	1.10
time (sec)	N/A	0.207	1.654	4.722	0.270	0.310	0.000	0.310	0.188

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	248	89	110	516	0	120	113
N.S.	1	1.00	2.48	0.89	1.10	5.16	0.00	1.20	1.13
time (sec)	N/A	0.164	3.164	1.211	0.268	0.305	0.000	0.305	19.570

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	84	76	88	406	0	89	70
N.S.	1	1.00	1.02	0.93	1.07	4.95	0.00	1.09	0.85
time (sec)	N/A	0.102	0.288	0.619	0.277	0.283	0.000	0.301	19.611

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	211	64	71	368	0	83	69
N.S.	1	1.00	2.89	0.88	0.97	5.04	0.00	1.14	0.95
time (sec)	N/A	0.077	2.519	0.474	0.266	0.277	0.000	0.303	19.350

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	240	90	106	435	0	114	2056
N.S.	1	1.00	2.61	0.98	1.15	4.73	0.00	1.24	22.35
time (sec)	N/A	0.103	1.410	0.351	0.273	0.295	0.000	0.286	21.719

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	103	120	175	544	0	192	2401
N.S.	1	1.00	0.73	0.85	1.23	3.83	0.00	1.35	16.91
time (sec)	N/A	0.225	1.233	1.876	0.269	0.319	0.000	0.312	22.474

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	138	160	268	656	0	195	2880
N.S.	1	1.00	0.68	0.79	1.32	3.23	0.00	0.96	14.19
time (sec)	N/A	0.341	1.565	3.492	0.276	0.328	0.000	0.316	22.995

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	499	210	369	789	0	270	3310
N.S.	1	1.00	1.79	0.76	1.33	2.84	0.00	0.97	11.91
time (sec)	N/A	0.404	4.748	6.040	0.278	0.313	0.000	0.313	23.683

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	90	108	179	472	0	117	113
N.S.	1	1.00	0.83	1.00	1.66	4.37	0.00	1.08	1.05
time (sec)	N/A	0.119	0.759	1.378	0.276	0.299	0.000	0.336	0.214

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	163	124	212	544	0	155	129
N.S.	1	1.00	1.30	0.99	1.70	4.35	0.00	1.24	1.03
time (sec)	N/A	0.129	0.701	1.711	0.269	0.297	0.000	0.343	19.600

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	927	142	233	613	0	178	149
N.S.	1	1.00	6.44	0.99	1.62	4.26	0.00	1.24	1.03
time (sec)	N/A	0.137	5.451	1.655	0.269	0.293	0.000	0.344	0.335

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	176	149	253	727	0	197	175
N.S.	1	1.00	1.13	0.96	1.62	4.66	0.00	1.26	1.12
time (sec)	N/A	0.247	1.364	3.144	0.275	0.331	0.000	0.340	19.719

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	194	177	272	856	0	229	256
N.S.	1	1.00	1.07	0.98	1.50	4.73	0.00	1.27	1.41
time (sec)	N/A	0.280	3.248	5.839	0.270	0.324	0.000	0.350	0.351

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	2670	214	303	995	0	271	257
N.S.	1	1.00	12.48	1.00	1.42	4.65	0.00	1.27	1.20
time (sec)	N/A	0.314	7.352	10.268	0.272	0.347	0.000	0.356	0.236

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	125	137	210	722	0	185	149
N.S.	1	1.00	0.88	0.96	1.48	5.08	0.00	1.30	1.05
time (sec)	N/A	0.179	0.827	1.474	0.276	0.317	0.000	0.336	20.045

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	283	118	187	654	0	163	125
N.S.	1	1.00	2.30	0.96	1.52	5.32	0.00	1.33	1.02
time (sec)	N/A	0.123	4.134	1.446	0.276	0.320	0.000	0.343	19.349

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	265	100	156	580	0	124	112
N.S.	1	1.00	2.50	0.94	1.47	5.47	0.00	1.17	1.06
time (sec)	N/A	0.099	4.593	1.580	0.284	0.297	0.000	0.334	19.073

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	332	147	231	819	0	196	3271
N.S.	1	1.00	2.31	1.02	1.60	5.69	0.00	1.36	22.72
time (sec)	N/A	0.198	6.650	1.372	0.279	0.324	0.000	0.298	23.592

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	156	177	338	970	0	228	3708
N.S.	1	1.00	0.78	0.88	1.68	4.83	0.00	1.13	18.45
time (sec)	N/A	0.370	4.487	4.398	0.275	0.330	0.000	0.338	24.104

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	1430	215	464	1129	0	464	4158
N.S.	1	1.00	5.32	0.80	1.72	4.20	0.00	1.72	15.46
time (sec)	N/A	0.459	9.964	6.905	0.284	0.347	0.000	0.340	24.036

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	352	352	1770	267	611	1296	0	350	4594
N.S.	1	1.00	5.03	0.76	1.74	3.68	0.00	0.99	13.05
time (sec)	N/A	0.590	11.122	12.974	0.289	0.395	0.000	0.350	25.065

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	1411	229	401	1323	0	324	4506
N.S.	1	1.00	6.92	1.12	1.97	6.49	0.00	1.59	22.09
time (sec)	N/A	0.399	9.439	3.254	0.288	0.365	0.000	0.318	25.028

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	70	145	81	100	0	217	0
N.S.	1	1.00	0.52	1.08	0.60	0.75	0.00	1.62	0.00
time (sec)	N/A	0.086	2.544	5.155	0.277	0.268	0.000	0.546	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	60	135	62	87	0	182	0
N.S.	1	1.00	0.59	1.34	0.61	0.86	0.00	1.80	0.00
time (sec)	N/A	0.070	0.631	3.366	0.264	0.260	0.000	0.438	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	48	98	40	68	0	137	0
N.S.	1	1.00	0.75	1.53	0.62	1.06	0.00	2.14	0.00
time (sec)	N/A	0.054	0.151	0.764	0.272	0.271	0.000	0.357	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	38	21	53	0	141	0
N.S.	1	1.00	1.00	1.15	0.64	1.61	0.00	4.27	0.00
time (sec)	N/A	0.040	0.051	0.207	0.272	0.256	0.000	0.345	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	40	48	37	56	0	0	0
N.S.	1	1.00	1.25	1.50	1.16	1.75	0.00	0.00	0.00
time (sec)	N/A	0.038	0.083	0.200	0.278	0.256	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	57	87	60	94	0	108	0
N.S.	1	1.00	0.85	1.30	0.90	1.40	0.00	1.61	0.00
time (sec)	N/A	0.058	0.167	0.826	0.279	0.265	0.000	0.427	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	67	107	79	125	0	147	0
N.S.	1	1.00	0.67	1.07	0.79	1.25	0.00	1.47	0.00
time (sec)	N/A	0.070	0.306	0.866	0.281	0.282	0.000	0.612	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	79	117	94	162	0	171	0
N.S.	1	1.00	0.59	0.88	0.71	1.22	0.00	1.29	0.00
time (sec)	N/A	0.087	0.391	0.912	0.278	0.250	0.000	0.545	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	372	372	0	8814	0	887	0	0	0
N.S.	1	1.00	0.00	23.69	0.00	2.38	0.00	0.00	0.00
time (sec)	N/A	0.653	0.000	14.829	0.000	0.172	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	288	288	0	6342	0	782	0	0	0
N.S.	1	1.00	0.00	22.02	0.00	2.72	0.00	0.00	0.00
time (sec)	N/A	0.492	0.000	9.800	0.000	0.147	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	218	218	0	4676	0	603	0	0	0
N.S.	1	1.00	0.00	21.45	0.00	2.77	0.00	0.00	0.00
time (sec)	N/A	0.369	0.000	7.335	0.000	0.119	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	80	80	69	4727	0	0	0	0	0
N.S.	1	1.00	0.86	59.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.118	0.637	3.872	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	246	246	402	6300	0	0	0	0	0
N.S.	1	1.00	1.63	25.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.366	13.397	7.231	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	338	338	0	8763	0	0	0	0	0
N.S.	1	1.00	0.00	25.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.570	0.000	9.747	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	407	1375	317	468	0	0	0
N.S.	1	1.00	2.19	7.39	1.70	2.52	0.00	0.00	0.00
time (sec)	N/A	0.201	15.531	17.255	0.200	1.066	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	380	907	173	390	0	0	0
N.S.	1	1.00	3.11	7.43	1.42	3.20	0.00	0.00	0.00
time (sec)	N/A	0.126	7.729	11.174	0.214	0.425	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	210	537	69	320	0	0	0
N.S.	1	1.00	2.76	7.07	0.91	4.21	0.00	0.00	0.00
time (sec)	N/A	0.097	1.887	6.676	0.196	0.308	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	C	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	284	351	3227	1227	0	0	0
N.S.	1	1.00	3.59	4.44	40.85	15.53	0.00	0.00	0.00
time (sec)	N/A	0.061	0.489	2.109	0.801	0.497	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	136	313	0	499	0	0	0
N.S.	1	1.00	1.66	3.82	0.00	6.09	0.00	0.00	0.00
time (sec)	N/A	0.114	0.774	2.806	0.000	0.421	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	152	587	0	567	0	0	0
N.S.	1	1.00	1.09	4.19	0.00	4.05	0.00	0.00	0.00
time (sec)	N/A	0.142	1.381	5.099	0.000	0.546	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	1902	970	0	641	0	0	0
N.S.	1	1.00	9.70	4.95	0.00	3.27	0.00	0.00	0.00
time (sec)	N/A	0.227	16.882	7.745	0.000	1.169	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	450	450	0	11007	0	981	0	0	0
N.S.	1	1.00	0.00	24.46	0.00	2.18	0.00	0.00	0.00
time (sec)	N/A	0.850	0.000	16.319	0.000	0.201	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	0	8946	0	890	0	0	0
N.S.	1	1.00	0.00	24.11	0.00	2.40	0.00	0.00	0.00
time (sec)	N/A	0.712	0.000	13.873	0.000	0.163	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	0	7055	0	798	0	0	0
N.S.	1	1.00	0.00	24.33	0.00	2.75	0.00	0.00	0.00
time (sec)	N/A	0.524	0.000	9.322	0.000	0.141	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	224	224	0	5360	0	0	0	0	0
N.S.	1	1.00	0.00	23.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.270	0.000	6.353	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	179	6947	0	0	0	0	0
N.S.	1	1.00	0.74	28.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.390	2.739	3.142	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	319	319	356	8848	0	0	0	0	0
N.S.	1	1.00	1.12	27.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.614	18.481	7.028	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	243	243	512	1977	415	566	0	0	0
N.S.	1	1.00	2.11	8.14	1.71	2.33	0.00	0.00	0.00
time (sec)	N/A	0.259	11.216	21.487	0.214	4.133	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	165	165	450	1417	243	470	0	0	0
N.S.	1	1.00	2.73	8.59	1.47	2.85	0.00	0.00	0.00
time (sec)	N/A	0.164	8.974	16.720	0.207	1.074	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	84	951	104	390	0	0	0
N.S.	1	1.00	0.76	8.57	0.94	3.51	0.00	0.00	0.00
time (sec)	N/A	0.123	0.458	11.799	0.196	0.432	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	118	118	527	709	0	1457	0	0	0
N.S.	1	1.00	4.47	6.01	0.00	12.35	0.00	0.00	0.00
time (sec)	N/A	0.115	2.143	7.762	0.000	0.605	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	124	124	466	537	0	1403	0	0	0
N.S.	1	1.00	3.76	4.33	0.00	11.31	0.00	0.00	0.00
time (sec)	N/A	0.158	7.324	9.313	0.000	0.605	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	191	602	0	563	0	0	0
N.S.	1	1.00	1.53	4.82	0.00	4.50	0.00	0.00	0.00
time (sec)	N/A	0.140	1.241	3.575	0.000	0.574	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	165	986	0	647	0	0	0
N.S.	1	1.00	0.85	5.11	0.00	3.35	0.00	0.00	0.00
time (sec)	N/A	0.195	2.222	6.036	0.000	1.191	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	166	166	706	1137	0	1611	0	0	0
N.S.	1	1.00	4.25	6.85	0.00	9.70	0.00	0.00	0.00
time (sec)	N/A	0.212	9.620	21.085	0.000	1.291	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	109	189	0	160	0	0	0
N.S.	1	1.00	2.60	4.50	0.00	3.81	0.00	0.00	0.00
time (sec)	N/A	0.046	0.239	8.050	0.000	0.263	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	C	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	57	120	1391	131	0	0	0
N.S.	1	1.00	2.38	5.00	57.96	5.46	0.00	0.00	0.00
time (sec)	N/A	0.020	0.075	5.975	0.565	0.257	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	330	330	0	6274	0	784	0	0	0
N.S.	1	1.00	0.00	19.01	0.00	2.38	0.00	0.00	0.00
time (sec)	N/A	0.577	0.000	12.320	0.000	0.149	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	170	170	0	4103	0	594	0	0	0
N.S.	1	1.00	0.00	24.14	0.00	3.49	0.00	0.00	0.00
time (sec)	N/A	0.360	0.000	7.891	0.000	0.129	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	69	243	0	303	0	0	0
N.S.	1	1.00	0.86	3.04	0.00	3.79	0.00	0.00	0.00
time (sec)	N/A	0.279	0.882	3.490	0.000	0.102	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	105	105	274	4095	0	0	0	0	0
N.S.	1	1.00	2.61	39.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.144	10.867	6.407	0.000	0.000	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	255	255	0	6273	0	0	0	0	0
N.S.	1	1.00	0.00	24.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.417	0.000	9.019	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	345	345	0	8756	0	0	0	0	0
N.S.	1	1.00	0.00	25.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.594	0.000	11.261	0.000	0.000	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	326	1612	160	396	0	0	0
N.S.	1	1.00	2.38	11.77	1.17	2.89	0.00	0.00	0.00
time (sec)	N/A	0.154	8.552	16.828	0.202	0.435	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	326	1039	74	324	0	0	0
N.S.	1	1.00	4.02	12.83	0.91	4.00	0.00	0.00	0.00
time (sec)	N/A	0.110	6.998	11.293	0.202	0.320	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	39	39	87	241	23	215	0	0	0
N.S.	1	1.00	2.23	6.18	0.59	5.51	0.00	0.00	0.00
time (sec)	N/A	0.089	0.152	7.881	0.195	0.301	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	87	138	992	408	0	0	0
N.S.	1	1.00	2.23	3.54	25.44	10.46	0.00	0.00	0.00
time (sec)	N/A	0.034	0.068	3.019	0.403	0.367	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	126	507	0	502	0	0	0
N.S.	1	1.00	1.45	5.83	0.00	5.77	0.00	0.00	0.00
time (sec)	N/A	0.115	0.279	5.121	0.000	0.401	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	1840	809	0	567	0	0	0
N.S.	1	1.00	12.87	5.66	0.00	3.97	0.00	0.00	0.00
time (sec)	N/A	0.168	16.220	7.145	0.000	0.533	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	1739	1136	0	643	0	0	0
N.S.	1	1.00	8.52	5.57	0.00	3.15	0.00	0.00	0.00
time (sec)	N/A	0.244	15.442	9.682	0.000	1.140	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	289	289	0	10329	0	960	0	0	0
N.S.	1	1.00	0.00	35.74	0.00	3.32	0.00	0.00	0.00
time (sec)	N/A	0.533	0.000	9.398	0.000	0.163	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	150	150	113	2535	0	894	0	0	0
N.S.	1	1.00	0.75	16.90	0.00	5.96	0.00	0.00	0.00
time (sec)	N/A	0.334	3.659	6.144	0.000	0.171	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	229	229	905	2534	0	762	0	0	0
N.S.	1	1.00	3.95	11.07	0.00	3.33	0.00	0.00	0.00
time (sec)	N/A	0.425	18.390	5.982	0.000	0.146	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	240	240	0	6126	0	0	0	0	0
N.S.	1	1.00	0.00	25.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.306	0.000	7.970	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	335	335	0	15843	0	0	0	0	0
N.S.	1	1.00	0.00	47.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.521	0.000	10.670	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	436	436	0	18649	0	0	0	0	0
N.S.	1	1.00	0.00	42.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.772	0.000	14.006	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	259	3210	161	524	0	0	0
N.S.	1	1.00	1.88	23.26	1.17	3.80	0.00	0.00	0.00
time (sec)	N/A	0.176	3.393	12.250	0.203	0.475	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	77	77	405	1513	78	410	0	0	0
N.S.	1	1.00	5.26	19.65	1.01	5.32	0.00	0.00	0.00
time (sec)	N/A	0.111	7.307	8.133	0.189	0.333	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	57	49	30	65	0	0	199
N.S.	1	1.00	1.78	1.53	0.94	2.03	0.00	0.00	6.22
time (sec)	N/A	0.087	0.777	1.876	0.191	0.260	0.000	0.000	20.519

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	168	515	2055	601	0	0	0
N.S.	1	1.00	2.18	6.69	26.69	7.81	0.00	0.00	0.00
time (sec)	N/A	0.053	1.544	2.535	0.542	0.446	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	2059	809	0	699	0	0	0
N.S.	1	1.00	15.72	6.18	0.00	5.34	0.00	0.00	0.00
time (sec)	N/A	0.183	15.386	4.129	0.000	0.632	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	2046	1143	0	811	0	0	0
N.S.	1	1.00	10.55	5.89	0.00	4.18	0.00	0.00	0.00
time (sec)	N/A	0.251	16.178	6.399	0.000	1.479	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	2068	1499	0	941	0	0	0
N.S.	1	1.00	7.63	5.53	0.00	3.47	0.00	0.00	0.00
time (sec)	N/A	0.363	17.722	9.780	0.000	4.392	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	321	321	167	16966	0	1374	0	0	0
N.S.	1	1.00	0.52	52.85	0.00	4.28	0.00	0.00	0.00
time (sec)	N/A	0.570	4.177	6.008	0.000	0.221	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	319	319	1204	12063	0	1244	0	0	0
N.S.	1	1.00	3.77	37.82	0.00	3.90	0.00	0.00	0.00
time (sec)	N/A	0.564	22.179	5.086	0.000	0.207	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	327	327	0	16966	0	1293	0	0	0
N.S.	1	1.00	0.00	51.88	0.00	3.95	0.00	0.00	0.00
time (sec)	N/A	0.584	0.000	6.015	0.000	0.211	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	349	349	0	20403	0	0	0	0	0
N.S.	1	1.00	0.00	58.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.452	0.000	7.614	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	441	441	0	24449	0	0	0	0	0
N.S.	1	1.00	0.00	55.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.707	0.000	9.934	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	559	559	0	31577	0	0	0	0	0
N.S.	1	1.00	0.00	56.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.923	0.000	13.435	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	592	2355	275	688	0	0	0
N.S.	1	1.00	4.45	17.71	2.07	5.17	0.00	0.00	0.00
time (sec)	N/A	0.170	10.351	6.327	0.203	0.505	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	74	75	117	134	0	0	153
N.S.	1	1.00	0.94	0.95	1.48	1.70	0.00	0.00	1.94
time (sec)	N/A	0.101	5.042	3.141	0.184	0.343	0.000	0.000	29.436

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	215	84	61	134	0	0	172
N.S.	1	1.00	3.03	1.18	0.86	1.89	0.00	0.00	2.42
time (sec)	N/A	0.100	6.140	3.115	0.183	0.383	0.000	0.000	30.119

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	1927	1263	0	881	0	0	0
N.S.	1	1.00	15.42	10.10	0.00	7.05	0.00	0.00	0.00
time (sec)	N/A	0.117	6.499	3.763	0.000	0.732	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	1775	2143	0	1023	0	0	0
N.S.	1	1.00	9.49	11.46	0.00	5.47	0.00	0.00	0.00
time (sec)	N/A	0.281	16.736	5.352	0.000	1.784	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	1777	2770	0	1187	0	0	0
N.S.	1	1.00	6.81	10.61	0.00	4.55	0.00	0.00	0.00
time (sec)	N/A	0.403	18.420	8.145	0.000	5.422	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	332	332	1776	3399	0	1337	0	0	0
N.S.	1	1.00	5.35	10.24	0.00	4.03	0.00	0.00	0.00
time (sec)	N/A	0.495	21.312	10.807	0.000	16.210	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	1777	2018	0	1241	0	0	0
N.S.	1	1.00	9.93	11.27	0.00	6.93	0.00	0.00	0.00
time (sec)	N/A	0.226	16.808	5.457	0.000	2.122	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	37	59	388	53	0	0	0
N.S.	1	1.00	2.64	4.21	27.71	3.79	0.00	0.00	0.00
time (sec)	N/A	0.026	0.121	0.868	0.379	0.252	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	F	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD
size	111	0	2195	0	0	0	0	0	0
N.S.	1	0.00	19.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	20.076	0.000	0.000	0.000	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	1989	0	0	0	0	0	0
N.S.	1	1.00	19.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.168	17.989	0.000	0.000	0.000	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	1995	0	0	0	0	0	0
N.S.	1	1.00	19.37	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.146	17.416	0.000	0.000	0.000	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	1983	0	0	0	0	0	0
N.S.	1	1.00	19.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.111	17.082	0.000	0.000	0.000	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	1987	0	0	0	0	0	0
N.S.	1	1.00	19.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.146	17.433	0.000	0.000	0.000	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	1997	0	0	0	0	0	0
N.S.	1	1.00	19.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.164	17.602	0.000	0.000	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	216	216	149	0	0	0	0	0	0
N.S.	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.249	2.162	0.000	0.000	0.000	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	129	126	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.118	2.142	0.000	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.071	1.035	0.000	0.000	0.000	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	2137	0	0	0	0	0	0
N.S.	1	1.00	25.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.067	6.248	0.000	0.000	0.000	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	1914	0	0	0	0	0	0
N.S.	1	1.00	23.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.092	16.549	0.000	0.000	0.000	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	1912	0	0	0	0	0	0
N.S.	1	1.00	23.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.095	16.255	0.000	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	1914	0	0	0	0	0	0
N.S.	1	1.00	23.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.096	16.860	0.000	0.000	0.000	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	55	55	95	69	116	280	52
N.S.	1	1.00	0.76	0.76	1.32	0.96	1.61	3.89	0.72
time (sec)	N/A	0.077	0.185	1.626	0.192	0.268	0.547	1.588	19.692

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	43	45	64	50	80	236	46
N.S.	1	1.00	0.88	0.92	1.31	1.02	1.63	4.82	0.94
time (sec)	N/A	0.054	0.084	0.736	0.190	0.259	0.263	0.658	19.536

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	26	33	37	42	190	32
N.S.	1	1.00	1.00	0.87	1.10	1.23	1.40	6.33	1.07
time (sec)	N/A	0.028	0.020	0.332	0.199	0.244	0.128	0.328	18.982

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	50	24	33	35	0	98	32
N.S.	1	1.00	1.79	0.86	1.18	1.25	0.00	3.50	1.14
time (sec)	N/A	0.056	0.029	0.327	0.194	0.258	0.000	0.276	19.680

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	52	39	29	50	0	146	51
N.S.	1	1.00	1.62	1.22	0.91	1.56	0.00	4.56	1.59
time (sec)	N/A	0.060	0.181	0.578	0.188	0.258	0.000	0.310	19.274

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	73	55	49	83	0	238	61
N.S.	1	1.00	1.43	1.08	0.96	1.63	0.00	4.67	1.20
time (sec)	N/A	0.083	0.029	1.253	0.183	0.259	0.000	0.354	19.878

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	73	57	56	89	66	56	51
N.S.	1	1.00	1.14	0.89	0.88	1.39	1.03	0.88	0.80
time (sec)	N/A	0.071	0.024	2.202	0.268	0.257	1.254	2.182	18.993

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	57	47	45	72	54	45	40
N.S.	1	1.00	1.19	0.98	0.94	1.50	1.12	0.94	0.83
time (sec)	N/A	0.067	0.017	1.098	0.267	0.254	0.831	0.943	19.463

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	41	37	33	53	42	33	29
N.S.	1	1.00	1.28	1.16	1.03	1.66	1.31	1.03	0.91
time (sec)	N/A	0.062	0.013	0.562	0.269	0.254	0.584	0.502	19.434

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	31	0	15	17
N.S.	1	1.00	1.00	1.07	1.00	2.07	0.00	1.00	1.13
time (sec)	N/A	0.017	0.002	0.271	0.184	0.251	0.000	0.275	19.591

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	43	33	25	34	0	53	19
N.S.	1	1.00	2.26	1.74	1.32	1.79	0.00	2.79	1.00
time (sec)	N/A	0.065	0.082	0.566	0.273	0.246	0.000	0.297	19.279

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	51	48	41	76	0	111	35
N.S.	1	1.00	1.55	1.45	1.24	2.30	0.00	3.36	1.06
time (sec)	N/A	0.072	0.017	0.886	0.269	0.252	0.000	0.310	19.443

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	63	52	110	0	170	46
N.S.	1	1.00	1.00	1.24	1.02	2.16	0.00	3.33	0.90
time (sec)	N/A	0.083	0.029	1.891	0.274	0.244	0.000	0.337	19.797

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	126	86	147	99	190	431	124
N.S.	1	1.00	1.26	0.86	1.47	0.99	1.90	4.31	1.24
time (sec)	N/A	0.132	0.531	9.059	0.195	0.274	1.130	1.894	19.519

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	107	76	114	79	128	385	92
N.S.	1	1.00	1.39	0.99	1.48	1.03	1.66	5.00	1.19
time (sec)	N/A	0.108	0.291	4.573	0.191	0.267	0.564	0.876	20.179

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	83	41	67	53	61	334	61
N.S.	1	1.00	1.73	0.85	1.40	1.10	1.27	6.96	1.27
time (sec)	N/A	0.054	0.120	2.223	0.190	0.261	0.277	0.420	19.686

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	84	50	64	79	0	247	58
N.S.	1	1.00	1.58	0.94	1.21	1.49	0.00	4.66	1.09
time (sec)	N/A	0.088	0.246	1.850	0.191	0.260	0.000	0.318	20.391

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	81	64	60	100	0	233	68
N.S.	1	1.00	1.42	1.12	1.05	1.75	0.00	4.09	1.19
time (sec)	N/A	0.105	0.195	2.618	0.188	0.260	0.000	0.329	20.281

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	77	71	61	97	0	332	83
N.S.	1	1.00	1.51	1.39	1.20	1.90	0.00	6.51	1.63
time (sec)	N/A	0.111	0.297	5.402	0.182	0.269	0.000	0.363	20.714

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	275	88	84	137	0	91	126
N.S.	1	1.00	2.89	0.93	0.88	1.44	0.00	0.96	1.33
time (sec)	N/A	0.131	3.708	12.310	0.269	0.269	0.000	2.517	19.692

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	395	78	71	113	0	78	97
N.S.	1	1.00	5.13	1.01	0.92	1.47	0.00	1.01	1.26
time (sec)	N/A	0.108	2.302	6.529	0.264	0.264	0.000	1.197	19.432

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	281	68	58	86	0	65	69
N.S.	1	1.00	4.76	1.15	0.98	1.46	0.00	1.10	1.17
time (sec)	N/A	0.102	1.710	3.204	0.264	0.262	0.000	0.651	19.412

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	41	46	44	58	0	49	42
N.S.	1	1.00	1.02	1.15	1.10	1.45	0.00	1.22	1.05
time (sec)	N/A	0.034	0.136	0.545	0.182	0.254	0.000	0.287	19.572

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	82	66	46	67	0	46	44
N.S.	1	1.00	2.28	1.83	1.28	1.86	0.00	1.28	1.22
time (sec)	N/A	0.091	3.125	1.889	0.266	0.251	0.000	0.323	19.059

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	160	73	59	98	0	176	53
N.S.	1	1.00	3.56	1.62	1.31	2.18	0.00	3.91	1.18
time (sec)	N/A	0.109	1.572	3.911	0.271	0.249	0.000	0.323	19.971

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	256	107	72	136	0	273	68
N.S.	1	1.00	3.94	1.65	1.11	2.09	0.00	4.20	1.05
time (sec)	N/A	0.113	3.887	7.660	0.280	0.271	0.000	0.354	19.656

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	99	67	81	84	0	373	103
N.S.	1	1.00	1.43	0.97	1.17	1.22	0.00	5.41	1.49
time (sec)	N/A	0.134	0.332	1.595	0.188	0.298	0.000	1.518	19.461

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	41	42	50	41	0	222	64
N.S.	1	1.00	0.91	0.93	1.11	0.91	0.00	4.93	1.42
time (sec)	N/A	0.098	0.118	0.798	0.190	0.291	0.000	0.605	20.163

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	26	35	26	21	117	129	63
N.S.	1	1.00	1.13	1.52	1.13	0.91	5.09	5.61	2.74
time (sec)	N/A	0.038	0.213	0.351	0.179	0.250	5.559	0.329	19.831

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	68	50	42	0	166	65
N.S.	1	1.00	0.93	1.48	1.09	0.91	0.00	3.61	1.41
time (sec)	N/A	0.098	0.118	0.648	0.186	0.299	0.000	0.296	20.645

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	100	119	87	126	0	294	98
N.S.	1	1.00	1.35	1.61	1.18	1.70	0.00	3.97	1.32
time (sec)	N/A	0.138	0.239	1.273	0.184	0.343	0.000	0.342	20.762

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	138	180	145	265	0	513	160
N.S.	1	1.00	1.28	1.67	1.34	2.45	0.00	4.75	1.48
time (sec)	N/A	0.175	0.717	2.611	0.195	0.458	0.000	0.397	20.663

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	229	101	95	373	0	126	1109
N.S.	1	1.00	2.76	1.22	1.14	4.49	0.00	1.52	13.36
time (sec)	N/A	0.309	3.953	2.321	0.273	0.306	0.000	2.090	19.875

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	206	72	66	297	0	87	410
N.S.	1	1.00	3.49	1.22	1.12	5.03	0.00	1.47	6.95
time (sec)	N/A	0.186	2.119	1.138	0.266	0.283	0.000	0.868	19.158

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	184	48	45	226	0	66	126
N.S.	1	1.00	4.00	1.04	0.98	4.91	0.00	1.43	2.74
time (sec)	N/A	0.144	0.755	0.632	0.266	0.279	0.000	0.452	19.656

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	182	46	44	231	0	65	460
N.S.	1	1.00	4.04	1.02	0.98	5.13	0.00	1.44	10.22
time (sec)	N/A	0.054	0.262	0.204	0.269	0.284	0.000	0.274	19.369

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	204	68	66	310	0	87	637
N.S.	1	1.00	3.29	1.10	1.06	5.00	0.00	1.40	10.27
time (sec)	N/A	0.196	2.248	0.947	0.266	0.283	0.000	0.312	21.799

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	390	90	106	533	0	134	2644
N.S.	1	1.00	4.53	1.05	1.23	6.20	0.00	1.56	30.74
time (sec)	N/A	0.293	4.688	1.806	0.268	0.308	0.000	0.352	24.728

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	671	118	161	833	0	212	4324
N.S.	1	1.00	5.59	0.98	1.34	6.94	0.00	1.77	36.03
time (sec)	N/A	0.389	4.363	3.663	0.271	0.318	0.000	0.386	25.759

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	109	72	98	118	0	532	170
N.S.	1	1.00	1.42	0.94	1.27	1.53	0.00	6.91	2.21
time (sec)	N/A	0.126	0.751	6.079	0.182	0.318	0.000	1.646	20.344

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	81	50	59	53	0	314	97
N.S.	1	1.00	1.59	0.98	1.16	1.04	0.00	6.16	1.90
time (sec)	N/A	0.107	2.341	3.282	0.195	0.256	0.000	0.720	20.325

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	79	59	57	52	0	384	90
N.S.	1	1.00	1.61	1.20	1.16	1.06	0.00	7.84	1.84
time (sec)	N/A	0.073	0.637	2.325	0.185	0.264	0.000	0.428	19.572

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	112	91	117	138	0	396	106
N.S.	1	1.00	1.35	1.10	1.41	1.66	0.00	4.77	1.28
time (sec)	N/A	0.142	0.569	3.656	0.188	0.377	0.000	0.341	19.758

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	130	141	192	312	0	811	160
N.S.	1	1.00	1.17	1.27	1.73	2.81	0.00	7.31	1.44
time (sec)	N/A	0.188	1.784	7.277	0.204	0.498	0.000	0.421	20.508

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	162	201	279	557	0	913	206
N.S.	1	1.00	1.16	1.44	1.99	3.98	0.00	6.52	1.47
time (sec)	N/A	0.232	2.956	13.267	0.196	0.797	0.000	0.505	21.174

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	286	125	127	514	0	155	765
N.S.	1	1.00	2.40	1.05	1.07	4.32	0.00	1.30	6.43
time (sec)	N/A	0.303	6.801	8.526	0.275	0.296	0.000	2.238	20.049

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	249	85	96	393	0	120	285
N.S.	1	1.00	2.77	0.94	1.07	4.37	0.00	1.33	3.17
time (sec)	N/A	0.193	3.735	4.548	0.291	0.294	0.000	1.017	19.786

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	346	77	75	458	0	94	711
N.S.	1	1.00	4.07	0.91	0.88	5.39	0.00	1.11	8.36
time (sec)	N/A	0.167	8.548	2.397	0.275	0.304	0.000	0.554	20.669

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	240	90	106	435	0	114	2056
N.S.	1	1.00	2.61	0.98	1.15	4.73	0.00	1.24	22.35
time (sec)	N/A	0.098	2.200	0.421	0.277	0.295	0.000	0.288	21.731

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	288	102	163	604	0	175	3146
N.S.	1	1.00	2.38	0.84	1.35	4.99	0.00	1.45	26.00
time (sec)	N/A	0.306	5.343	5.162	0.277	0.305	0.000	0.374	24.793

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	1896	124	235	979	0	212	4987
N.S.	1	1.00	11.85	0.78	1.47	6.12	0.00	1.32	31.17
time (sec)	N/A	0.426	8.329	9.732	0.315	0.320	0.000	0.449	25.728

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	3028	152	319	1505	0	298	6017
N.S.	1	1.00	14.63	0.73	1.54	7.27	0.00	1.44	29.07
time (sec)	N/A	0.515	9.491	18.385	0.286	0.360	0.000	0.516	26.616

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	136	80	112	116	0	531	166
N.S.	1	1.00	1.74	1.03	1.44	1.49	0.00	6.81	2.13
time (sec)	N/A	0.141	3.263	20.494	0.196	0.272	0.000	1.932	19.934

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	131	73	113	111	0	656	153
N.S.	1	1.00	1.62	0.90	1.40	1.37	0.00	8.10	1.89
time (sec)	N/A	0.146	1.118	11.325	0.226	0.273	0.000	0.961	19.468

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	129	81	102	102	0	782	142
N.S.	1	1.00	1.74	1.09	1.38	1.38	0.00	10.57	1.92
time (sec)	N/A	0.091	1.910	11.280	0.207	0.274	0.000	0.577	19.330

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	158	144	243	307	0	767	190
N.S.	1	1.00	1.22	1.11	1.87	2.36	0.00	5.90	1.46
time (sec)	N/A	0.193	1.422	17.523	0.201	0.611	0.000	0.422	20.768

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	176	193	344	584	0	1069	272
N.S.	1	1.00	1.14	1.25	2.23	3.79	0.00	6.94	1.77
time (sec)	N/A	0.244	1.981	30.707	0.214	0.929	0.000	0.558	21.188

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	208	254	454	859	0	1912	327
N.S.	1	1.00	1.08	1.32	2.36	4.47	0.00	9.96	1.70
time (sec)	N/A	0.318	5.525	53.496	0.225	1.684	0.000	0.528	22.487

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	760	134	193	664	0	200	615
N.S.	1	1.00	5.17	0.91	1.31	4.52	0.00	1.36	4.18
time (sec)	N/A	0.318	7.611	27.172	0.286	0.304	0.000	2.543	20.948

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	1473	124	163	763	0	159	1117
N.S.	1	1.00	10.75	0.91	1.19	5.57	0.00	1.16	8.15
time (sec)	N/A	0.282	16.892	15.427	0.278	0.308	0.000	1.247	21.390

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	1473	125	191	860	0	172	2405
N.S.	1	1.00	10.67	0.91	1.38	6.23	0.00	1.25	17.43
time (sec)	N/A	0.275	13.269	11.630	0.275	0.331	0.000	0.743	22.823

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	332	147	231	819	0	196	3271
N.S.	1	1.00	2.31	1.02	1.60	5.69	0.00	1.36	22.72
time (sec)	N/A	0.231	6.516	1.483	0.278	0.321	0.000	0.298	23.807

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	2089	149	311	1060	0	247	4890
N.S.	1	1.00	11.54	0.82	1.72	5.86	0.00	1.36	27.02
time (sec)	N/A	0.427	9.532	23.488	0.283	0.345	0.000	0.488	25.316

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	3340	171	409	1649	0	302	7057
N.S.	1	1.00	14.52	0.74	1.78	7.17	0.00	1.31	30.68
time (sec)	N/A	0.541	9.842	41.582	0.287	0.370	0.000	0.475	25.944

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	976	199	520	2229	0	388	7460
N.S.	1	1.00	3.42	0.70	1.82	7.82	0.00	1.36	26.18
time (sec)	N/A	0.701	12.570	69.090	0.295	0.420	0.000	0.471	27.969

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	139	568	0	456	0	1280	0
N.S.	1	1.00	1.25	5.12	0.00	4.11	0.00	11.53	0.00
time (sec)	N/A	0.178	2.828	12.017	0.000	1.972	0.000	1.820	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	74	338	0	386	0	800	0
N.S.	1	1.00	0.92	4.22	0.00	4.82	0.00	10.00	0.00
time (sec)	N/A	0.128	0.918	7.435	0.000	0.593	0.000	0.760	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	119	58	0	312	0	377	46
N.S.	1	1.00	2.20	1.07	0.00	5.78	0.00	6.98	0.85
time (sec)	N/A	0.092	0.561	0.155	0.000	0.371	0.000	0.463	20.345

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	C	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	540	3317	963	0	402	0
N.S.	1	1.00	1.00	7.71	47.39	13.76	0.00	5.74	0.00
time (sec)	N/A	0.140	0.328	1.350	0.603	0.418	0.000	0.692	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	109	109	527	2756	0	1342	0	574	0
N.S.	1	1.00	4.83	25.28	0.00	12.31	0.00	5.27	0.00
time (sec)	N/A	0.172	6.913	1.388	0.000	0.557	0.000	0.905	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	B	F	B	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	161	161	0	4323	0	1953	0	866	0
N.S.	1	1.00	0.00	26.85	0.00	12.13	0.00	5.38	0.00
time (sec)	N/A	0.289	0.000	1.359	0.000	1.388	0.000	1.299	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	263	1581	0	1775	0	0	0
N.S.	1	1.00	1.20	7.22	0.00	8.11	0.00	0.00	0.00
time (sec)	N/A	0.486	4.130	21.774	0.000	4.224	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	208	1065	0	1621	0	0	0
N.S.	1	1.00	1.26	6.45	0.00	9.82	0.00	0.00	0.00
time (sec)	N/A	0.349	3.055	16.737	0.000	1.312	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	526	661	0	1471	0	0	0
N.S.	1	1.00	4.46	5.60	0.00	12.47	0.00	0.00	0.00
time (sec)	N/A	0.275	4.606	12.297	0.000	0.619	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	C	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	284	351	3227	1227	0	0	0
N.S.	1	1.00	3.59	4.44	40.85	15.53	0.00	0.00	0.00
time (sec)	N/A	0.062	0.548	4.094	0.814	0.487	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	130	224	0	499	0	0	0
N.S.	1	1.00	1.88	3.25	0.00	7.23	0.00	0.00	0.00
time (sec)	N/A	0.213	0.731	3.215	0.000	0.429	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	176	596	0	629	0	0	0
N.S.	1	1.00	1.54	5.23	0.00	5.52	0.00	0.00	0.00
time (sec)	N/A	0.281	0.857	4.712	0.000	0.722	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	178	1016	0	849	0	0	0
N.S.	1	1.00	1.07	6.08	0.00	5.08	0.00	0.00	0.00
time (sec)	N/A	0.382	2.876	5.901	0.000	2.297	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	149	826	0	527	0	2026	0
N.S.	1	1.00	1.10	6.12	0.00	3.90	0.00	15.01	0.00
time (sec)	N/A	0.199	2.404	16.964	0.000	7.519	0.000	3.262	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	99	518	0	443	0	1431	0
N.S.	1	1.00	0.95	4.98	0.00	4.26	0.00	13.76	0.00
time (sec)	N/A	0.154	1.718	14.217	0.000	1.970	0.000	1.671	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	84	81	0	373	0	867	66
N.S.	1	1.00	1.08	1.04	0.00	4.78	0.00	11.12	0.85
time (sec)	N/A	0.099	0.333	0.141	0.000	0.599	0.000	1.000	23.605

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	91	91	506	1330	0	1075	0	718	0
N.S.	1	1.00	5.56	14.62	0.00	11.81	0.00	7.89	0.00
time (sec)	N/A	0.160	6.079	5.571	0.000	0.578	0.000	4.131	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	114	114	622	1569	0	1300	0	0	0
N.S.	1	1.00	5.46	13.76	0.00	11.40	0.00	0.00	0.00
time (sec)	N/A	0.205	6.308	1.263	0.000	0.605	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	159	159	684	4839	0	1801	0	0	0
N.S.	1	1.00	4.30	30.43	0.00	11.33	0.00	0.00	0.00
time (sec)	N/A	0.298	6.905	1.178	0.000	1.612	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	353	2229	0	1973	0	0	0
N.S.	1	1.00	1.22	7.69	0.00	6.80	0.00	0.00	0.00
time (sec)	N/A	0.691	7.880	30.399	0.000	14.978	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	258	1619	0	1777	0	0	0
N.S.	1	1.00	1.21	7.57	0.00	8.30	0.00	0.00	0.00
time (sec)	N/A	0.561	5.287	21.723	0.000	4.149	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	703	1119	0	1627	0	0	0
N.S.	1	1.00	4.23	6.74	0.00	9.80	0.00	0.00	0.00
time (sec)	N/A	0.399	7.206	17.357	0.000	1.269	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	118	118	527	709	0	1457	0	0	0
N.S.	1	1.00	4.47	6.01	0.00	12.35	0.00	0.00	0.00
time (sec)	N/A	0.114	2.095	8.033	0.000	0.623	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	111	111	410	539	0	1446	0	0	0
N.S.	1	1.00	3.69	4.86	0.00	13.03	0.00	0.00	0.00
time (sec)	N/A	0.267	6.497	9.300	0.000	0.650	0.000	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	100	369	0	597	0	0	0
N.S.	1	1.00	0.89	3.29	0.00	5.33	0.00	0.00	0.00
time (sec)	N/A	0.321	0.578	3.742	0.000	0.837	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	139	793	0	767	0	0	0
N.S.	1	1.00	0.84	4.81	0.00	4.65	0.00	0.00	0.00
time (sec)	N/A	0.406	1.497	5.016	0.000	3.056	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	107	303	0	410	0	0	0
N.S.	1	1.00	1.20	3.40	0.00	4.61	0.00	0.00	0.00
time (sec)	N/A	0.148	1.562	8.109	0.000	0.621	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	256	0	328	0	0	0
N.S.	1	1.00	1.00	4.57	0.00	5.86	0.00	0.00	0.00
time (sec)	N/A	0.118	0.604	4.970	0.000	0.371	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	42	0	261	0	0	27
N.S.	1	1.00	1.00	1.27	0.00	7.91	0.00	0.00	0.82
time (sec)	N/A	0.063	0.143	0.225	0.000	0.323	0.000	0.000	20.533

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	294	343	0	1015	0	0	0
N.S.	1	1.00	4.20	4.90	0.00	14.50	0.00	0.00	0.00
time (sec)	N/A	0.131	2.798	1.458	0.000	0.437	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	B	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	116	116	0	1944	0	1550	0	0	0
N.S.	1	1.00	0.00	16.76	0.00	13.36	0.00	0.00	0.00
time (sec)	N/A	0.203	0.000	1.378	0.000	0.618	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	B	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	166	166	0	3596	0	2257	0	0	0
N.S.	1	1.00	0.00	21.66	0.00	13.60	0.00	0.00	0.00
time (sec)	N/A	0.325	0.000	1.546	0.000	1.378	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	230	1930	0	1673	0	0	0
N.S.	1	1.00	1.33	11.16	0.00	9.67	0.00	0.00	0.00
time (sec)	N/A	0.382	3.645	17.678	0.000	1.390	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	196	1334	0	1507	0	0	0
N.S.	1	1.00	1.63	11.12	0.00	12.56	0.00	0.00	0.00
time (sec)	N/A	0.265	2.542	12.662	0.000	0.665	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	80	80	296	344	0	1259	0	0	0
N.S.	1	1.00	3.70	4.30	0.00	15.74	0.00	0.00	0.00
time (sec)	N/A	0.227	2.479	8.970	0.000	0.503	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	87	138	992	408	0	0	0
N.S.	1	1.00	2.23	3.54	25.44	10.46	0.00	0.00	0.00
time (sec)	N/A	0.036	0.117	3.317	0.415	0.372	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	127	507	0	525	0	0	0
N.S.	1	1.00	1.72	6.85	0.00	7.09	0.00	0.00	0.00
time (sec)	N/A	0.240	0.252	4.121	0.000	0.442	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	168	891	0	723	0	0	0
N.S.	1	1.00	1.41	7.49	0.00	6.08	0.00	0.00	0.00
time (sec)	N/A	0.346	1.494	5.279	0.000	0.677	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	199	1279	0	987	0	0	0
N.S.	1	1.00	1.16	7.44	0.00	5.74	0.00	0.00	0.00
time (sec)	N/A	0.424	3.337	6.782	0.000	1.910	0.000	0.000	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	121	562	0	458	0	0	0
N.S.	1	1.00	1.38	6.39	0.00	5.20	0.00	0.00	0.00
time (sec)	N/A	0.174	5.465	5.375	0.000	0.828	0.000	0.000	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	62	2293	2532	417	0	0	0
N.S.	1	1.00	0.98	36.40	40.19	6.62	0.00	0.00	0.00
time (sec)	N/A	0.143	0.829	1.587	0.629	0.385	0.000	0.000	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	382	62	0	392	75	0	49
N.S.	1	1.00	6.70	1.09	0.00	6.88	1.32	0.00	0.86
time (sec)	N/A	0.092	6.657	0.155	0.000	0.370	5.886	0.000	21.308

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	B	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	100	100	0	9462	0	1569	0	0	0
N.S.	1	1.00	0.00	94.62	0.00	15.69	0.00	0.00	0.00
time (sec)	N/A	0.168	0.000	1.806	0.000	0.610	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	153	153	0	13610	0	2347	0	0	0
N.S.	1	1.00	0.00	88.95	0.00	15.34	0.00	0.00	0.00
time (sec)	N/A	0.318	0.000	1.157	0.000	1.485	0.000	0.000	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	213	213	0	17868	0	3501	0	0	0
N.S.	1	1.00	0.00	83.89	0.00	16.44	0.00	0.00	0.00
time (sec)	N/A	0.398	0.000	2.335	0.000	4.800	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	247	2792	0	1895	0	0	0
N.S.	1	1.00	1.44	16.23	0.00	11.02	0.00	0.00	0.00
time (sec)	N/A	0.399	9.405	13.013	0.000	1.765	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	116	116	201	1186	0	1655	0	0	0
N.S.	1	1.00	1.73	10.22	0.00	14.27	0.00	0.00	0.00
time (sec)	N/A	0.275	4.615	10.586	0.000	0.715	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	169	282	2005	548	0	0	0
N.S.	1	1.00	2.38	3.97	28.24	7.72	0.00	0.00	0.00
time (sec)	N/A	0.234	2.612	4.092	0.497	0.434	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	168	515	2055	601	0	0	0
N.S.	1	1.00	2.18	6.69	26.69	7.81	0.00	0.00	0.00
time (sec)	N/A	0.059	1.452	4.431	0.540	0.432	0.000	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	119	119	182	1228	0	741	0	0	0
N.S.	1	1.00	1.53	10.32	0.00	6.23	0.00	0.00	0.00
time (sec)	N/A	0.325	4.462	5.738	0.000	0.731	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	174	174	224	1779	0	1061	0	0	0
N.S.	1	1.00	1.29	10.22	0.00	6.10	0.00	0.00	0.00
time (sec)	N/A	0.414	4.241	6.691	0.000	2.113	0.000	0.000	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	241	241	237	2449	0	1517	0	0	0
N.S.	1	1.00	0.98	10.16	0.00	6.29	0.00	0.00	0.00
time (sec)	N/A	0.535	6.093	8.818	0.000	7.207	0.000	0.000	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	187	6373	0	564	0	0	0
N.S.	1	1.00	1.93	65.70	0.00	5.81	0.00	0.00	0.00
time (sec)	N/A	0.181	8.911	2.938	0.000	0.973	0.000	0.000	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	613	6313	0	522	0	0	0
N.S.	1	1.00	6.89	70.93	0.00	5.87	0.00	0.00	0.00
time (sec)	N/A	0.152	10.448	2.750	0.000	0.838	0.000	0.000	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	A	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	613	86	0	494	100	0	68
N.S.	1	1.00	7.39	1.04	0.00	5.95	1.20	0.00	0.82
time (sec)	N/A	0.103	7.123	0.165	0.000	0.714	9.786	0.000	24.035

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	B	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	137	137	0	43855	0	2279	0	0	0
N.S.	1	1.00	0.00	320.11	0.00	16.64	0.00	0.00	0.00
time (sec)	N/A	0.246	0.000	4.924	0.000	1.483	0.000	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	200	200	0	73511	0	3507	0	0	0
N.S.	1	1.00	0.00	367.56	0.00	17.54	0.00	0.00	0.00
time (sec)	N/A	0.371	0.000	8.714	0.000	4.923	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	268	268	0	77709	0	4751	0	0	0
N.S.	1	1.00	0.00	289.96	0.00	17.73	0.00	0.00	0.00
time (sec)	N/A	0.565	0.000	10.432	0.000	18.704	0.000	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	157	157	316	1500	0	2035	0	0	0
N.S.	1	1.00	2.01	9.55	0.00	12.96	0.00	0.00	0.00
time (sec)	N/A	0.408	11.147	9.863	0.000	2.147	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	409	657	0	661	0	0	0
N.S.	1	1.00	3.41	5.48	0.00	5.51	0.00	0.00	0.00
time (sec)	N/A	0.329	6.554	4.586	0.000	0.952	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	410	1003	0	773	0	0	0
N.S.	1	1.00	3.45	8.43	0.00	6.50	0.00	0.00	0.00
time (sec)	N/A	0.286	4.692	5.600	0.000	0.825	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	1927	1263	0	881	0	0	0
N.S.	1	1.00	15.42	10.10	0.00	7.05	0.00	0.00	0.00
time (sec)	N/A	0.122	6.484	6.398	0.000	0.733	0.000	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	174	174	247	1862	0	1097	0	0	0
N.S.	1	1.00	1.42	10.70	0.00	6.30	0.00	0.00	0.00
time (sec)	N/A	0.446	7.765	7.589	0.000	2.145	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	236	236	234	2612	0	1579	0	0	0
N.S.	1	1.00	0.99	11.07	0.00	6.69	0.00	0.00	0.00
time (sec)	N/A	0.579	11.188	9.646	0.000	7.358	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	315	315	272	3418	0	2059	0	0	0
N.S.	1	1.00	0.86	10.85	0.00	6.54	0.00	0.00	0.00
time (sec)	N/A	0.722	16.993	11.213	0.000	24.643	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	259	0	0	0	0	0	0
N.S.	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.214	5.254	0.000	0.000	0.000	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	94	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.181	0.903	0.000	0.000	0.000	0.000	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	61	0	0	0	0	0	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.109	0.204	0.000	0.000	0.000	0.000	0.000	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.066	0.060	0.000	0.000	0.000	0.000	0.000	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	114	114	115	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.141	1.903	0.000	0.000	0.000	0.000	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	157	157	139	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.198	3.958	0.000	0.000	0.000	0.000	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	2777	0	0	0	0	0	0
N.S.	1	1.00	31.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.158	18.567	0.000	0.000	0.000	0.000	0.000	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	2465	0	0	0	0	0	0
N.S.	1	1.00	28.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.161	17.515	0.000	0.000	0.000	0.000	0.000	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	2137	0	0	0	0	0	0
N.S.	1	1.00	25.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.058	6.254	0.000	0.000	0.000	0.000	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	2469	0	0	0	0	0	0
N.S.	1	1.00	29.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.153	17.480	0.000	0.000	0.000	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	3033	0	0	0	0	0	0
N.S.	1	1.00	34.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.163	18.978	0.000	0.000	0.000	0.000	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	87	70	73	81	119	340	227
N.S.	1	1.00	0.95	0.76	0.79	0.88	1.29	3.70	2.47
time (sec)	N/A	0.078	0.584	3.185	0.181	0.266	0.794	1.627	24.339

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	59	49	51	59	82	271	167
N.S.	1	1.00	0.97	0.80	0.84	0.97	1.34	4.44	2.74
time (sec)	N/A	0.067	0.327	1.561	0.199	0.257	0.380	0.694	24.518

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	26	28	37	42	181	83
N.S.	1	1.00	1.00	0.87	0.93	1.23	1.40	6.03	2.77
time (sec)	N/A	0.031	0.015	0.722	0.190	0.261	0.174	0.352	20.762

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	69	42	45	61	0	87	72
N.S.	1	1.00	1.28	0.78	0.83	1.13	0.00	1.61	1.33
time (sec)	N/A	0.082	0.017	0.636	0.188	0.249	0.000	0.284	20.052

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	114	63	62	99	0	172	86
N.S.	1	1.00	1.58	0.88	0.86	1.38	0.00	2.39	1.19
time (sec)	N/A	0.080	1.176	1.473	0.183	0.253	0.000	0.298	20.234

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	251	233	218	4427	0	0	7402
N.S.	1	1.00	1.15	1.06	1.00	20.21	0.00	0.00	33.80
time (sec)	N/A	0.381	0.892	4.023	0.282	0.995	0.000	0.000	22.955

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	242	113	159	1052	0	0	1620
N.S.	1	1.00	1.46	0.68	0.96	6.34	0.00	0.00	9.76
time (sec)	N/A	0.172	0.644	4.378	0.275	0.893	0.000	0.000	24.316

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	35	21	21	139	0	114
N.S.	1	1.00	1.00	1.52	0.91	0.91	6.04	0.00	4.96
time (sec)	N/A	0.037	0.020	1.704	0.197	0.258	18.047	0.000	20.761

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	215	288	306	6482	0	0	11182
N.S.	1	1.00	0.73	0.98	1.04	21.97	0.00	0.00	37.91
time (sec)	N/A	0.623	0.690	3.434	0.298	1.063	0.000	0.000	23.775

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	393	336	374	488	10746	0	0	58699
N.S.	1	1.00	0.85	0.95	1.24	27.34	0.00	0.00	149.36
time (sec)	N/A	0.771	2.653	8.548	0.277	2.208	0.000	0.000	35.830

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	29	26	29	31
N.S.	1	1.00	1.07	1.00	1.07	1.07	0.96	1.07	1.15
time (sec)	N/A	0.073	5.725	0.832	5.460	0.257	104.479	0.827	21.065

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	226	226	245	0	0	0	0	0	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.657	13.762	0.000	0.000	0.000	0.000	0.000	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	162	0	0	0	0	0	0
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.379	6.427	0.000	0.000	0.000	0.000	0.000	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	59	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.097	0.114	0.000	0.000	0.000	0.000	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	22	25	27
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.96	1.09	1.17
time (sec)	N/A	0.049	4.534	0.602	3.783	0.260	12.664	0.795	20.712

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	27	27	0	27	29
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.16
time (sec)	N/A	0.063	31.508	0.622	9.322	0.257	0.000	1.040	21.969

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	27	27	24	27	29
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.96	1.08	1.16
time (sec)	N/A	0.064	3.319	0.459	5.858	0.258	19.172	1.015	21.034

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	20
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.25
time (sec)	N/A	0.018	1.738	0.479	3.951	0.258	1.725	0.520	20.775

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	27	27	24	27	29
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.96	1.08	1.16
time (sec)	N/A	0.062	2.788	0.602	6.345	0.263	73.546	0.975	21.323

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [255] had the largest ratio of [.5999999999999999978]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	21	0.095
2	A	3	2	1.00	21	0.095
3	A	3	2	1.00	21	0.095
4	A	3	2	1.00	19	0.105
5	A	3	3	1.00	19	0.158
6	A	4	4	1.00	21	0.190
7	A	5	4	1.00	21	0.190
8	A	6	6	1.00	21	0.286
9	A	5	5	1.00	21	0.238
10	A	4	4	1.00	21	0.190
11	A	3	2	1.00	12	0.167
12	A	3	2	1.00	21	0.095
13	A	3	2	1.00	21	0.095
14	A	3	2	1.00	21	0.095
15	A	3	2	1.00	23	0.087
16	A	3	2	1.00	23	0.087
17	A	3	2	1.00	21	0.095
18	A	4	3	1.00	21	0.143
19	A	5	5	1.00	23	0.217
20	A	6	5	1.00	23	0.217
21	A	7	6	1.00	23	0.261
22	A	6	5	1.00	23	0.217
23	A	5	5	1.00	23	0.217
24	A	4	3	1.00	14	0.214

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	3	2	1.00	23	0.087
26	A	3	2	1.00	23	0.087
27	A	3	2	1.00	23	0.087
28	A	4	3	1.00	23	0.130
29	A	4	4	1.00	23	0.174
30	A	3	3	1.00	21	0.143
31	A	4	4	1.00	21	0.190
32	A	5	5	1.00	23	0.217
33	A	6	6	1.00	23	0.261
34	A	7	7	1.00	23	0.304
35	A	6	6	1.00	23	0.261
36	A	5	5	1.00	23	0.217
37	A	3	3	1.00	14	0.214
38	A	3	3	1.00	23	0.130
39	A	4	4	1.00	23	0.174
40	A	4	3	1.00	23	0.130
41	A	6	5	1.00	23	0.217
42	A	5	4	1.00	23	0.174
43	A	4	4	1.00	21	0.190
44	A	5	5	1.00	21	0.238
45	A	6	6	1.00	23	0.261
46	A	7	6	1.00	23	0.261
47	A	8	7	1.00	23	0.304
48	A	7	6	1.00	23	0.261
49	A	6	6	1.00	23	0.261
50	A	5	5	1.00	14	0.357
51	A	4	4	1.00	23	0.174
52	A	5	4	1.00	23	0.174
53	A	6	5	1.00	23	0.217
54	A	6	5	1.00	23	0.217
55	A	6	5	1.00	23	0.217
56	A	5	4	1.00	21	0.190
57	A	6	6	1.00	21	0.286
58	A	7	7	1.00	23	0.304
59	A	8	7	1.00	23	0.304

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	9	7	1.00	23	0.304
61	A	8	6	1.00	23	0.261
62	A	7	6	1.00	23	0.261
63	A	6	6	1.00	14	0.429
64	A	5	4	1.00	23	0.174
65	A	6	5	1.00	23	0.217
66	A	7	6	1.00	23	0.261
67	A	6	6	1.00	25	0.240
68	A	5	5	1.00	25	0.200
69	A	4	4	1.00	23	0.174
70	A	6	6	1.00	23	0.261
71	A	7	7	1.00	25	0.280
72	A	8	8	1.00	25	0.320
73	A	9	8	1.00	25	0.320
74	A	8	8	1.00	25	0.320
75	A	7	7	1.00	25	0.280
76	A	6	6	1.00	16	0.375
77	A	4	4	1.00	25	0.160
78	A	5	5	1.00	25	0.200
79	A	6	6	1.00	25	0.240
80	A	7	7	1.00	25	0.280
81	A	6	6	1.00	25	0.240
82	A	5	5	1.00	23	0.217
83	A	7	7	1.00	23	0.304
84	A	8	8	1.00	25	0.320
85	A	9	9	1.00	25	0.360
86	A	10	10	1.00	25	0.400
87	A	9	9	1.00	25	0.360
88	A	8	8	1.00	25	0.320
89	A	7	7	1.00	16	0.438
90	A	5	5	1.00	25	0.200
91	A	6	6	1.00	25	0.240
92	A	7	7	1.00	25	0.280
93	A	4	4	1.00	25	0.160
94	A	3	3	1.00	25	0.120

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	2	2	1.00	23	0.087
96	A	3	3	1.00	23	0.130
97	A	5	5	1.00	25	0.200
98	A	6	6	1.00	25	0.240
99	A	7	7	1.00	25	0.280
100	A	6	6	1.00	25	0.240
101	A	5	5	1.00	25	0.200
102	A	3	3	1.00	16	0.188
103	A	2	2	1.00	25	0.080
104	A	3	3	1.00	25	0.120
105	A	4	4	1.00	25	0.160
106	A	5	5	1.00	25	0.200
107	A	4	4	1.00	25	0.160
108	A	3	3	1.00	23	0.130
109	A	4	4	1.00	23	0.174
110	A	6	6	1.00	25	0.240
111	A	7	6	1.00	25	0.240
112	A	8	7	1.00	25	0.280
113	A	7	6	1.00	25	0.240
114	A	6	6	1.00	25	0.240
115	A	4	4	1.00	16	0.250
116	A	3	3	1.00	25	0.120
117	A	4	4	1.00	25	0.160
118	A	5	5	1.00	25	0.200
119	A	6	6	1.00	25	0.240
120	A	5	5	1.00	25	0.200
121	A	4	4	1.00	23	0.174
122	A	6	6	1.00	23	0.261
123	A	7	6	1.00	25	0.240
124	A	8	6	1.00	25	0.240
125	A	9	7	1.00	25	0.280
126	A	8	6	1.00	25	0.240
127	A	7	6	1.00	25	0.240
128	A	6	6	1.00	16	0.375
129	A	4	4	1.00	25	0.160

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	5	5	1.00	25	0.200
131	A	6	6	1.00	25	0.240
132	F	0	0	N/A	0.000	N/A
133	A	5	5	1.00	23	0.217
134	A	4	4	1.00	23	0.174
135	A	3	3	1.00	21	0.143
136	A	3	3	1.00	21	0.143
137	A	3	3	1.00	23	0.130
138	A	3	3	1.00	23	0.130
139	A	3	3	1.00	23	0.130
140	A	3	3	1.00	14	0.214
141	A	3	3	1.00	23	0.130
142	A	4	4	1.00	23	0.174
143	A	5	5	1.00	23	0.217
144	A	6	3	1.00	15	0.200
145	A	5	3	1.00	15	0.200
146	A	4	3	1.00	15	0.200
147	A	3	2	1.00	13	0.154
148	A	3	3	1.00	15	0.200
149	A	4	3	1.00	15	0.200
150	A	5	3	1.00	15	0.200
151	A	6	3	1.00	15	0.200
152	A	4	3	1.00	21	0.143
153	A	3	3	1.00	21	0.143
154	A	2	2	1.00	19	0.105
155	A	2	2	1.00	19	0.105
156	A	3	2	1.00	21	0.095
157	A	4	3	1.00	21	0.143
158	A	3	2	1.00	21	0.095
159	A	3	2	1.00	21	0.095
160	A	3	3	1.00	21	0.143
161	A	3	2	1.00	12	0.167
162	A	2	2	1.00	21	0.095
163	A	3	3	1.00	21	0.143
164	A	4	3	1.00	21	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	6	5	1.00	23	0.217
166	A	5	5	1.00	23	0.217
167	A	4	4	1.00	21	0.190
168	A	5	4	1.00	21	0.190
169	A	4	3	1.00	23	0.130
170	A	3	2	1.00	23	0.087
171	A	3	2	1.00	23	0.087
172	A	3	2	1.00	23	0.087
173	A	3	2	1.00	23	0.087
174	A	4	3	1.00	14	0.214
175	A	5	4	1.00	23	0.174
176	A	4	4	1.00	23	0.174
177	A	5	5	1.00	23	0.217
178	A	4	3	1.00	14	0.214
179	A	4	3	1.00	14	0.214
180	A	5	5	1.00	23	0.217
181	A	4	4	1.00	23	0.174
182	A	2	2	1.00	21	0.095
183	A	3	3	1.00	21	0.143
184	A	4	3	1.00	23	0.130
185	A	4	3	1.00	23	0.130
186	A	4	3	1.00	23	0.130
187	A	3	3	1.00	23	0.130
188	A	2	2	1.00	23	0.087
189	A	3	3	1.00	14	0.214
190	A	5	5	1.00	23	0.217
191	A	6	6	1.00	23	0.261
192	A	7	6	1.00	23	0.261
193	A	5	5	1.00	23	0.217
194	A	3	3	1.00	23	0.130
195	A	3	3	1.00	21	0.143
196	A	5	4	1.00	21	0.190
197	A	5	4	1.00	23	0.174
198	A	5	4	1.00	23	0.174
199	A	5	4	1.00	23	0.174

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
200	A	3	3	1.00	23	0.130
201	A	3	3	1.00	23	0.130
202	A	5	5	1.00	14	0.357
203	A	6	6	1.00	23	0.261
204	A	7	6	1.00	23	0.261
205	A	8	6	1.00	23	0.261
206	A	4	3	1.00	23	0.130
207	A	4	4	1.00	23	0.174
208	A	4	4	1.00	21	0.190
209	A	6	5	1.00	21	0.238
210	A	6	5	1.00	23	0.217
211	A	6	5	1.00	23	0.217
212	A	4	4	1.00	23	0.174
213	A	4	4	1.00	23	0.174
214	A	4	3	1.00	23	0.130
215	A	6	6	1.00	14	0.429
216	A	7	6	1.00	23	0.261
217	A	8	6	1.00	23	0.261
218	A	9	6	1.00	23	0.261
219	A	7	6	1.00	14	0.429
220	A	6	4	1.00	17	0.235
221	A	5	4	1.00	17	0.235
222	A	4	4	1.00	17	0.235
223	A	3	3	1.00	17	0.176
224	A	3	3	1.00	17	0.176
225	A	4	4	1.00	17	0.235
226	A	5	4	1.00	17	0.235
227	A	6	4	1.00	17	0.235
228	A	11	10	1.00	25	0.400
229	A	10	10	1.00	25	0.400
230	A	10	10	1.00	23	0.435
231	A	5	5	1.00	23	0.217
232	A	9	9	1.00	25	0.360
233	A	10	10	1.00	25	0.400
234	A	6	6	1.00	25	0.240

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	5	5	1.00	25	0.200
236	A	4	4	1.00	25	0.160
237	A	6	6	1.00	16	0.375
238	A	4	4	1.00	25	0.160
239	A	5	5	1.00	25	0.200
240	A	7	6	1.00	25	0.240
241	A	12	10	1.00	25	0.400
242	A	11	10	1.00	25	0.400
243	A	10	10	1.00	23	0.435
244	A	9	9	1.00	23	0.391
245	A	9	9	1.00	25	0.360
246	A	10	10	1.00	25	0.400
247	A	7	6	1.00	25	0.240
248	A	6	5	1.00	25	0.200
249	A	5	4	1.00	25	0.160
250	A	7	7	1.00	16	0.438
251	A	7	7	1.00	25	0.280
252	A	5	4	1.00	25	0.160
253	A	6	5	1.00	25	0.200
254	A	8	8	1.00	16	0.500
255	A	6	6	1.00	10	0.600
256	A	5	5	1.00	10	0.500
257	A	10	10	1.00	25	0.400
258	A	7	7	1.00	25	0.280
259	A	5	5	1.00	23	0.217
260	A	5	5	1.00	23	0.217
261	A	9	9	1.00	25	0.360
262	A	10	10	1.00	25	0.400
263	A	5	5	1.00	25	0.200
264	A	4	4	1.00	25	0.160
265	A	3	3	1.00	25	0.120
266	A	3	3	1.00	16	0.188
267	A	4	4	1.00	25	0.160
268	A	6	6	1.00	25	0.240
269	A	7	6	1.00	25	0.240

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
270	A	10	10	1.00	25	0.400
271	A	7	7	1.00	25	0.280
272	A	9	9	1.00	23	0.391
273	A	9	9	1.00	23	0.391
274	A	10	10	1.00	25	0.400
275	A	11	10	1.00	25	0.400
276	A	5	5	1.00	25	0.200
277	A	4	4	1.00	25	0.160
278	A	2	2	1.00	25	0.080
279	A	4	4	1.00	16	0.250
280	A	6	6	1.00	25	0.240
281	A	7	6	1.00	25	0.240
282	A	8	6	1.00	25	0.240
283	A	10	10	1.00	25	0.400
284	A	10	10	1.00	25	0.400
285	A	10	10	1.00	23	0.435
286	A	10	10	1.00	23	0.435
287	A	11	11	1.00	25	0.440
288	A	12	11	1.00	25	0.440
289	A	5	5	1.00	25	0.200
290	A	3	3	1.00	25	0.120
291	A	3	3	1.00	25	0.120
292	A	6	6	1.00	16	0.375
293	A	7	6	1.00	25	0.240
294	A	8	6	1.00	25	0.240
295	A	9	6	1.00	25	0.240
296	A	7	6	1.00	16	0.375
297	A	3	3	1.00	10	0.300
298	F	0	0	N/A	0.000	N/A
299	A	5	5	1.00	23	0.217
300	A	5	5	1.00	21	0.238
301	A	5	5	1.00	21	0.238
302	A	5	5	1.00	23	0.217
303	A	5	5	1.00	23	0.217
304	A	5	5	1.00	23	0.217

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	A	4	4	1.00	23	0.174
306	A	3	3	1.00	23	0.130
307	A	3	3	1.00	14	0.214
308	A	3	3	1.00	23	0.130
309	A	3	3	1.00	23	0.130
310	A	3	3	1.00	23	0.130
311	A	4	3	1.00	21	0.143
312	A	4	3	1.00	21	0.143
313	A	3	2	1.00	19	0.105
314	A	4	3	1.00	19	0.158
315	A	4	3	1.00	21	0.143
316	A	4	3	1.00	21	0.143
317	A	4	3	1.00	21	0.143
318	A	4	3	1.00	21	0.143
319	A	4	3	1.00	21	0.143
320	A	3	2	1.00	12	0.167
321	A	4	3	1.00	21	0.143
322	A	4	3	1.00	21	0.143
323	A	4	3	1.00	21	0.143
324	A	4	3	1.00	23	0.130
325	A	4	3	1.00	23	0.130
326	A	4	3	1.00	21	0.143
327	A	4	3	1.00	21	0.143
328	A	4	3	1.00	23	0.130
329	A	4	3	1.00	23	0.130
330	A	4	3	1.00	23	0.130
331	A	4	3	1.00	23	0.130
332	A	4	3	1.00	23	0.130
333	A	4	3	1.00	14	0.214
334	A	4	3	1.00	23	0.130
335	A	4	3	1.00	23	0.130
336	A	4	3	1.00	23	0.130
337	A	4	3	1.00	23	0.130
338	A	4	3	1.00	23	0.130
339	A	2	2	1.00	21	0.095

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
340	A	4	3	1.00	21	0.143
341	A	4	3	1.00	23	0.130
342	A	4	3	1.00	23	0.130
343	A	7	7	1.00	23	0.304
344	A	6	6	1.00	23	0.261
345	A	5	5	1.00	23	0.217
346	A	3	3	1.00	14	0.214
347	A	6	6	1.00	23	0.261
348	A	7	7	1.00	23	0.304
349	A	8	7	1.00	23	0.304
350	A	4	3	1.00	23	0.130
351	A	4	3	1.00	23	0.130
352	A	4	3	1.00	21	0.143
353	A	4	3	1.00	21	0.143
354	A	4	3	1.00	23	0.130
355	A	4	3	1.00	23	0.130
356	A	7	7	1.00	23	0.304
357	A	6	6	1.00	23	0.261
358	A	6	6	1.00	23	0.261
359	A	5	5	1.00	14	0.357
360	A	7	7	1.00	23	0.304
361	A	8	7	1.00	23	0.304
362	A	9	7	1.00	23	0.304
363	A	4	3	1.00	23	0.130
364	A	4	3	1.00	23	0.130
365	A	4	3	1.00	21	0.143
366	A	4	3	1.00	21	0.143
367	A	4	3	1.00	23	0.130
368	A	4	3	1.00	23	0.130
369	A	7	7	1.00	23	0.304
370	A	7	7	1.00	23	0.304
371	A	7	7	1.00	23	0.304
372	A	6	6	1.00	14	0.429
373	A	8	8	1.00	23	0.348
374	A	9	8	1.00	23	0.348

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
375	A	10	8	1.00	23	0.348
376	A	7	6	1.00	25	0.240
377	A	6	6	1.00	25	0.240
378	A	5	5	1.00	23	0.217
379	A	7	5	1.00	23	0.217
380	A	8	6	1.00	25	0.240
381	A	9	7	1.00	25	0.280
382	A	10	9	1.00	25	0.360
383	A	9	9	1.00	25	0.360
384	A	8	8	1.00	25	0.320
385	A	6	6	1.00	16	0.375
386	A	6	6	1.00	25	0.240
387	A	7	7	1.00	25	0.280
388	A	8	7	1.00	25	0.280
389	A	8	6	1.00	25	0.240
390	A	7	6	1.00	25	0.240
391	A	6	5	1.00	23	0.217
392	A	8	6	1.00	23	0.261
393	A	8	6	1.00	25	0.240
394	A	9	7	1.00	25	0.280
395	A	11	9	1.00	25	0.360
396	A	10	9	1.00	25	0.360
397	A	9	9	1.00	25	0.360
398	A	7	7	1.00	16	0.438
399	A	8	8	1.00	25	0.320
400	A	7	7	1.00	25	0.280
401	A	8	7	1.00	25	0.280
402	A	6	5	1.00	25	0.200
403	A	5	5	1.00	25	0.200
404	A	4	4	1.00	23	0.174
405	A	7	5	1.00	23	0.217
406	A	8	6	1.00	25	0.240
407	A	9	7	1.00	25	0.280
408	A	9	9	1.00	25	0.360
409	A	8	8	1.00	25	0.320

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
410	A	7	7	1.00	25	0.280
411	A	3	3	1.00	16	0.188
412	A	6	6	1.00	25	0.240
413	A	7	7	1.00	25	0.280
414	A	8	7	1.00	25	0.280
415	A	6	5	1.00	25	0.200
416	A	5	5	1.00	25	0.200
417	A	5	5	1.00	23	0.217
418	A	8	6	1.00	23	0.261
419	A	9	7	1.00	25	0.280
420	A	10	8	1.00	25	0.320
421	A	9	9	1.00	25	0.360
422	A	8	8	1.00	25	0.320
423	A	5	5	1.00	25	0.200
424	A	4	4	1.00	16	0.250
425	A	7	7	1.00	25	0.280
426	A	8	7	1.00	25	0.280
427	A	9	7	1.00	25	0.280
428	A	6	5	1.00	25	0.200
429	A	6	6	1.00	25	0.240
430	A	6	5	1.00	23	0.217
431	A	9	7	1.00	23	0.304
432	A	10	7	1.00	25	0.280
433	A	11	8	1.00	25	0.320
434	A	9	9	1.00	25	0.360
435	A	7	7	1.00	25	0.280
436	A	7	7	1.00	25	0.280
437	A	6	6	1.00	16	0.375
438	A	8	8	1.00	25	0.320
439	A	9	8	1.00	25	0.320
440	A	10	8	1.00	25	0.320
441	A	4	4	1.00	25	0.160
442	A	5	4	1.00	23	0.174
443	A	4	4	1.00	23	0.174
444	A	3	3	1.00	21	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
445	A	5	5	1.00	21	0.238
446	A	6	6	1.00	23	0.261
447	A	4	4	1.00	23	0.174
448	A	4	4	1.00	23	0.174
449	A	3	3	1.00	14	0.214
450	A	4	4	1.00	23	0.174
451	A	4	4	1.00	23	0.174
452	A	3	2	1.00	21	0.095
453	A	3	2	1.00	21	0.095
454	A	3	2	1.00	19	0.105
455	A	3	2	1.00	19	0.105
456	A	5	4	1.00	21	0.190
457	A	11	10	1.00	23	0.435
458	A	9	9	1.00	23	0.391
459	A	2	2	1.00	21	0.095
460	A	11	10	1.00	21	0.476
461	A	11	10	1.00	23	0.435
462	N/A	0	0	1.00	27	0.000
463	A	15	8	1.00	25	0.320
464	A	11	8	1.00	25	0.320
465	A	5	5	1.00	23	0.217
466	N/A	0	0	1.00	23	0.000
467	N/A	0	0	1.00	25	0.000
468	N/A	0	0	1.00	25	0.000
469	N/A	0	0	1.00	16	0.000
470	N/A	0	0	1.00	25	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (a+b\sec^2(e+fx))\sin^7(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int (a+b\sec^2(e+fx))\sin^7(e+fx)\,dx = -\frac{(a-3b)\cos(e+fx)}{f}+\frac{(a-b)\cos^3(e+fx)}{f} - \frac{(3a-b)\cos^5(e+fx)}{5f} + \frac{a\cos^7(e+fx)}{7f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -(a-3*b)*cos(f*x+e)/f+(a-b)*cos(f*x+e)^3/f-1/5*(3*a-b)*cos(f*x+e)^5/f+1/7*a
*cos(f*x+e)^7/f+b*sec(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4218, 459}

$$\int (a+b\sec^2(e+fx))\sin^7(e+fx)\,dx = -\frac{(3a-b)\cos^5(e+fx)}{5f} + \frac{(a-b)\cos^3(e+fx)}{f}-\frac{(a-3b)\cos(e+fx)}{f} + \frac{a\cos^7(e+fx)}{7f}+\frac{b\sec(e+fx)}{f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^7,x]
```

```
[Out] -(((a - 3*b)*Cos[e + f*x])/f) + ((a - b)*Cos[e + f*x]^3)/f - ((3*a - b)*Cos
[e + f*x]^5)/(5*f) + (a*Cos[e + f*x]^7)/(7*f) + (b*Sec[e + f*x])/f
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^3(b+ax^2)}{x^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \left(a\left(1-\frac{3b}{a}\right)+\frac{b}{x^2}-3(a-b)x^2+(3a-b)x^4-ax^6\right)\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{(a-3b)\cos(e+fx)}{f}+\frac{(a-b)\cos^3(e+fx)}{f} - \frac{(3a-b)\cos^5(e+fx)}{5f}+\frac{a\cos^7(e+fx)}{7f}+\frac{b\sec(e+fx)}{f}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.45

$$\int \left(a+b\sec^2(e+fx)\right)\sin^7(e+fx)\,dx = -\frac{35a\cos(e+fx)}{64f}+\frac{19b\cos(e+fx)}{8f} + \frac{7a\cos(3(e+fx))}{64f}-\frac{3b\cos(3(e+fx))}{16f} - \frac{7a\cos(5(e+fx))}{320f}+\frac{b\cos(5(e+fx))}{80f} + \frac{a\cos(7(e+fx))}{448f}+\frac{b\sec(e+fx)}{f}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^7,x]
```

```
[Out] (-35*a*Cos[e + f*x])/(64*f) + (19*b*Cos[e + f*x])/(8*f) + (7*a*Cos[3*(e + f
*x)])/(64*f) - (3*b*Cos[3*(e + f*x)])/(16*f) - (7*a*Cos[5*(e + f*x)])/(320*
f) + (b*Cos[5*(e + f*x)])/(80*f) + (a*Cos[7*(e + f*x)])/(448*f) + (b*Sec[e
+ f*x])/f
```

Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.18

method	result
parallelrisch	$\frac{(-980a+4900b)\cos(2fx+2e)+(196a-392b)\cos(4fx+4e)+(-44a+28b)\cos(6fx+6e)+5a\cos(8fx+8e)+(-2048a+1433}{4480f\cos(fx+e)}$
derivativedivides	$\frac{-\frac{a\left(\frac{16}{5}+\sin(fx+e)^6+\frac{6\sin(fx+e)^4}{5}+\frac{8\sin(fx+e)^2}{5}\right)\cos(fx+e)}{7}+b\left(\frac{\sin(fx+e)^8}{\cos(fx+e)}+\left(\frac{16}{5}+\sin(fx+e)^6+\frac{6\sin(fx+e)^4}{5}+\frac{8\sin(fx+e}{5}\right.\right.}{f}$
default	$\frac{-\frac{a\left(\frac{16}{5}+\sin(fx+e)^6+\frac{6\sin(fx+e)^4}{5}+\frac{8\sin(fx+e)^2}{5}\right)\cos(fx+e)}{7}+b\left(\frac{\sin(fx+e)^8}{\cos(fx+e)}+\left(\frac{16}{5}+\sin(fx+e)^6+\frac{6\sin(fx+e)^4}{5}+\frac{8\sin(fx+e}{5}\right.\right.}{f}$
parts	$-\frac{a\left(\frac{16}{5}+\sin(fx+e)^6+\frac{6\sin(fx+e)^4}{5}+\frac{8\sin(fx+e)^2}{5}\right)\cos(fx+e)}{7f}+\frac{b\left(\frac{\sin(fx+e)^8}{\cos(fx+e)}+\left(\frac{16}{5}+\sin(fx+e)^6+\frac{6\sin(fx+e)^4}{5}+8\sin\right.\right.}{f}$
norman	$\frac{\frac{32a-224b}{35f}-\frac{32(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{f}+\frac{6(32a-224b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{35f}+\frac{2(32a-224b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{5f}+\frac{2(32a-224b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{5f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^7}$
risch	$\frac{19\,e^{i(fx+e)}b}{16f}-\frac{35\,e^{i(fx+e)}a}{128f}+\frac{19\,e^{-i(fx+e)}b}{16f}-\frac{35\,e^{-i(fx+e)}a}{128f}+\frac{2b\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)}+\frac{a\cos(7fx+7e)}{448f}-\frac{7\cos(5f}{32}$

```
[In] int((a+b*sec(f*x+e)^2)*sin(f*x+e)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4480*((-980*a+4900*b)*cos(2*f*x+2*e)+(196*a-392*b)*cos(4*f*x+4*e)+(-44*a+
28*b)*cos(6*f*x+6*e)+5*a*cos(8*f*x+8*e)+(-2048*a+14336*b)*cos(f*x+e)-1225*a
+9800*b)/f/cos(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int (a+b\sec^2(e+fx))\sin^7(e+fx)\,dx = \frac{5\,a\cos(fx+e)^8-7\,(3\,a-b)\cos(fx+e)^6+35\,(a-b)\cos(fx+e)^4-35\,(a-3\,b)\cos(fx+e)^2+35\,b}{35\,f\cos(fx+e)}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^7,x, algorithm="fricas")
```

```
[Out] 1/35*(5*a*cos(f*x + e)^8 - 7*(3*a - b)*cos(f*x + e)^6 + 35*(a - b)*cos(f*x
+ e)^4 - 35*(a - 3*b)*cos(f*x + e)^2 + 35*b)/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)\sin^7(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**7,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \left(a+b\sec^2(e+fx)\right)\sin^7(e+fx)\,dx = \frac{5\,a\cos\left(fx+e\right)^7-7\left(3\,a-b\right)\cos\left(fx+e\right)^5+35\left(a-b\right)\cos\left(fx+e\right)^3-35\left(a-3\,b\right)\cos\left(fx+e\right)+\frac{35\,b}{\cos(fx+e)}}{35\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^7,x, algorithm="maxima")
[Out] 1/35*(5*a*cos(f*x + e)^7 - 7*(3*a - b)*cos(f*x + e)^5 + 35*(a - b)*cos(f*x
+ e)^3 - 35*(a - 3*b)*cos(f*x + e) + 35*b/cos(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(79) = 158.

Time = 0.32 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.20

$$\int \left(a+b\sec^2(e+fx)\right)\sin^7(e+fx)\,dx = \frac{2\left(\frac{35\,b}{\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1}+\frac{16\,a-77\,b-\frac{112\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{504\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{336\,a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{1337\,b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{560\,a(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{1680}{(}}{\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-1\right)^7}\right)}{35\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^7,x, algorithm="giac")
[Out] 2/35*(35*b/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1) + (16*a - 77*b - 112
*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 504*b*(cos(f*x + e) - 1)/(cos(f*
x + e) + 1) + 336*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 1337*b*(cos
(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 560*a*(cos(f*x + e) - 1)^3/(cos(f*x
 + e) + 1)^3 + 1680*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 1015*b*(c
os(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 280*b*(cos(f*x + e) - 1)^5/(cos(f
*x + e) + 1)^5 - 35*b*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6)/((cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) - 1)^7)/f
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.84

$$\int (a + b\sec^2(e + fx))\sin^7(e + fx)\,dx$$
$$= \frac{\frac{a\cos(e+f\,x)^7}{7} - \cos\left(e + f\,x\right)\,\left(a - 3\,b\right) - \cos\left(e + f\,x\right)^5\,\left(\frac{3\,a}{5} - \frac{b}{5}\right) + \frac{b}{\cos(e+f\,x)} + \cos\left(e + f\,x\right)^3\,\left(a - b\right)}{f}$$

```
[In] int(sin(e + f*x)^7*(a + b/cos(e + f*x)^2),x)
[Out] ((a*cos(e + f*x)^7)/7 - cos(e + f*x)*(a - 3*b) - cos(e + f*x)^5*((3*a)/5 -
b/5) + b/cos(e + f*x) + cos(e + f*x)^3*(a - b))/f
```

3.2 $\int (a+b\sec^2(e+fx))\sin^5(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 66

$$\int (a+b\sec^2(e+fx))\sin^5(e+fx)\,dx=-\frac{(a-2b)\cos(e+fx)}{f}+\frac{(2a-b)\cos^3(e+fx)}{3f}-\frac{a\cos^5(e+fx)}{5f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -(a-2*b)*cos(f*x+e)/f+1/3*(2*a-b)*cos(f*x+e)^3/f-1/5*a*cos(f*x+e)^5/f+b*sec
(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4218, 459}

$$\int (a+b\sec^2(e+fx))\sin^5(e+fx)\,dx=\frac{(2a-b)\cos^3(e+fx)}{3f}-\frac{(a-2b)\cos(e+fx)}{f}-\frac{a\cos^5(e+fx)}{5f}+\frac{b\sec(e+fx)}{f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^5,x]
```

```
[Out] -(((a - 2*b)*Cos[e + f*x])/f) + ((2*a - b)*Cos[e + f*x]^3)/(3*f) - (a*Cos[e
 + f*x]^5)/(5*f) + (b*Sec[e + f*x])/f
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
```

```
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2(b+ax^2)}{x^2}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \left(a\left(1-\frac{2b}{a}\right)+\frac{b}{x^2}-(2a-b)x^2+ax^4\right)\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{(a-2b)\cos(e+fx)}{f}+\frac{(2a-b)\cos^3(e+fx)}{3f}-\frac{a\cos^5(e+fx)}{5f}+\frac{b\sec(e+fx)}{f}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.33

$$\int \left(a+b\sec^2(e+fx)\right)\sin^5(e+fx)\,dx = -\frac{5a\cos(e+fx)}{8f}+\frac{7b\cos(e+fx)}{4f}+\frac{5a\cos(3(e+fx))}{48f}-\frac{b\cos(3(e+fx))}{12f}-\frac{a\cos(5(e+fx))}{80f}+\frac{b\sec(e+fx)}{f}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^5,x]
```

```
[Out] (-5*a*Cos[e + f*x])/(8*f) + (7*b*Cos[e + f*x])/(4*f) + (5*a*Cos[3*(e + f*x)
])/(48*f) - (b*Cos[3*(e + f*x)])/(12*f) - (a*Cos[5*(e + f*x)])/(80*f) + (b*
Sec[e + f*x])/f
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23

method	result
parallelrisch	$\frac{(-125a+400b)\cos(2fx+2e)+(22a-20b)\cos(4fx+4e)-3\cos(6fx+6e)a+(-256a+1280b)\cos(fx+e)-150a+900b}{480f\cos(fx+e)}$
derivativedivides	$\frac{-\frac{a\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)}{5}+b\left(\frac{\sin(fx+e)^6}{\cos(fx+e)}+\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)\right)}{f}$
default	$\frac{-\frac{a\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)}{5}+b\left(\frac{\sin(fx+e)^6}{\cos(fx+e)}+\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)\right)}{f}$
parts	$-\frac{a\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)}{5f}+\frac{b\left(\frac{\sin(fx+e)^6}{\cos(fx+e)}+\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)\right)}{f}$
norman	$\frac{\frac{16a-80b}{15f}-\frac{32(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{3f}+\frac{4(16a-80b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{15f}+\frac{(16a-80b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{3f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^5}$
risch	$-\frac{5\,e^{i(fx+e)}a}{16f}+\frac{7\,e^{i(fx+e)}b}{8f}-\frac{5\,e^{-i(fx+e)}a}{16f}+\frac{7\,e^{-i(fx+e)}b}{8f}+\frac{2b\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)}-\frac{\cos(5fx+5e)a}{80f}+\frac{5\cos(3fx+3}{48f}$

```
[In] int((a+b*sec(f*x+e)^2)*sin(f*x+e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/480*((-125*a+400*b)*cos(2*f*x+2*e)+(22*a-20*b)*cos(4*f*x+4*e)-3*cos(6*f*x
+6*e)*a+(-256*a+1280*b)*cos(f*x+e)-150*a+900*b)/f/cos(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.91

$$\int (a+b\sec^2(e+fx))\sin^5(e+fx)\,dx = -\frac{3\,a\cos(fx+e)^6-5\,(2\,a-b)\cos(fx+e)^4+15\,(a-2\,b)\cos(fx+e)^2-15\,b}{15\,f\cos(fx+e)}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] -1/15*(3*a*cos(f*x + e)^6 - 5*(2*a - b)*cos(f*x + e)^4 + 15*(a - 2*b)*cos(f
*x + e)^2 - 15*b)/(f*cos(f*x + e))
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)\sin^5(e+fx)\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)\sin^5\left(e+fx\right)dx$$

[In] integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**5,x)

[Out] Integral((a + b*sec(e + f*x)**2)*sin(e + f*x)**5, x)

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

$$\int \left(a+b\sec^2(e+fx)\right)\sin^5(e+fx)\,dx$$
$$=-\frac{3\,a\cos\left(fx+e\right)^5-5\left(2\,a-b\right)\cos\left(fx+e\right)^3+15\left(a-2\,b\right)\cos\left(fx+e\right)-\frac{15\,b}{\cos(fx+e)}}{15\,f}$$

[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^5,x, algorithm="maxima")

[Out] -1/15*(3*a*cos(f*x + e)^5 - 5*(2*a - b)*cos(f*x + e)^3 + 15*(a - 2*b)*cos(f*x + e) - 15*b/cos(f*x + e))/f

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(62) = 124.

Time = 0.32 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.98

$$\int \left(a+b\sec^2(e+fx)\right)\sin^5(e+fx)\,dx$$
$$=\frac{2\left(\frac{15\,b}{\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1}+\frac{8\,a-25\,b-\frac{40\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{110\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{80\,a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{160\,b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{90\,b(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}-\frac{15\,b(\cos}{(\cos(f}}{\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-1\right)^5}\right.}{15\,f}$$

[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^5,x, algorithm="giac")

[Out] 2/15*(15*b/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1) + (8*a - 25*b - 40*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 110*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 80*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 160*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 90*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 15*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 1)^5)/f

Mupad [B] (verification not implemented)

Time = 17.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

$$\int \left(a+b\sec^2(e+fx)\right)\sin^5(e+fx)\,dx$$
$$=\frac{\cos\left(e+f\,x\right)^3\,\left(\frac{2\,a}{3}-\frac{b}{3}\right)-\cos\left(e+f\,x\right)\,\left(a-2\,b\right)-\frac{a\cos(e+f\,x)^5}{5}+\frac{b}{\cos(e+f\,x)}}{f}$$

```
[In] int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2),x)
```

```
[Out] (cos(e + f*x)^3*((2*a)/3 - b/3) - cos(e + f*x)*(a - 2*b) - (a*cos(e + f*x)^
5)/5 + b/cos(e + f*x))/f
```

3.3 $\int (a+b\sec^2(e+fx))\sin^3(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 44

$$\int (a+b\sec^2(e+fx))\sin^3(e+fx)\,dx = -\frac{(a-b)\cos(e+fx)}{f} + \frac{a\cos^3(e+fx)}{3f} + \frac{b\sec(e+fx)}{f}$$

```
[Out] -(a-b)*cos(f*x+e)/f+1/3*a*cos(f*x+e)^3/f+b*sec(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4218, 459}

$$\int (a+b\sec^2(e+fx))\sin^3(e+fx)\,dx = -\frac{(a-b)\cos(e+fx)}{f} + \frac{a\cos^3(e+fx)}{3f} + \frac{b\sec(e+fx)}{f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^3,x]
```

```
[Out] -(((a - b)*Cos[e + f*x])/f) + (a*Cos[e + f*x]^3)/(3*f) + (b*Sec[e + f*x])/f
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)(b+ax^2)}{x^2}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \left(a\left(1-\frac{b}{a}\right)+\frac{b}{x^2}-ax^2\right)\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{(a-b)\cos(e+fx)}{f}+\frac{a\cos^3(e+fx)}{3f}+\frac{b\sec(e+fx)}{f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int (a+b\sec^2(e+fx))\sin^3(e+fx)\,dx = -\frac{3a\cos(e+fx)}{4f}+\frac{b\cos(e+fx)}{f}+\frac{a\cos(3(e+fx))}{12f}+\frac{b\sec(e+fx)}{f}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^3,x]
[Out] (-3*a*Cos[e + f*x])/(4*f) + (b*Cos[e + f*x])/f + (a*Cos[3*(e + f*x)])/(12*f
) + (b*Sec[e + f*x])/f
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.41

method	result	size
derivativedivides	$\frac{-\frac{a\left(2+\sin(fx+e)^{2}\right)\cos(fx+e)}{3}+b\left(\frac{\sin(fx+e)^{4}}{\cos(fx+e)}+\left(2+\sin(fx+e)^{2}\right)\cos(fx+e)\right)}{f}$	62
default	$\frac{-\frac{a\left(2+\sin(fx+e)^{2}\right)\cos(fx+e)}{3}+b\left(\frac{\sin(fx+e)^{4}}{\cos(fx+e)}+\left(2+\sin(fx+e)^{2}\right)\cos(fx+e)\right)}{f}$	62
parallelrisch	$\frac{(-8a+12b)\cos(2fx+2e)+\cos(4fx+4e)a+(-16a+48b)\cos(fx+e)-9a+36b}{24f\cos(fx+e)}$	63
parts	$-\frac{a\left(2+\sin(fx+e)^{2}\right)\cos(fx+e)}{3f}+\frac{b\left(\frac{\sin(fx+e)^{4}}{\cos(fx+e)}+\left(2+\sin(fx+e)^{2}\right)\cos(fx+e)\right)}{f}$	64
norman	$\frac{\frac{4a-12b}{3f}-\frac{4(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{f}+\frac{2(4a-12b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{3f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)^{3}}$	87
risch	$-\frac{3\,e^{i(fx+e)}a}{8f}+\frac{e^{i(fx+e)}b}{2f}-\frac{3\,e^{-i(fx+e)}a}{8f}+\frac{e^{-i(fx+e)}b}{2f}+\frac{2b\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)}+\frac{\cos(3fx+3e)a}{12f}$	105

```
[In] int((a+b*sec(f*x+e)^2)*sin(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/3*a*(2+sin(f*x+e)^2)*cos(f*x+e)+b*(sin(f*x+e)^4/cos(f*x+e)+(2+sin(f
*x+e)^2)*cos(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \left(a+b\sec^2(e+fx)\right)\sin^3(e+fx)\,dx=\frac{a\cos\left(fx+e\right)^4-3\left(a-b\right)\cos\left(fx+e\right)^2+3\,b}{3\,f\cos\left(fx+e\right)}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(a*cos(f*x + e)^4 - 3*(a - b)*cos(f*x + e)^2 + 3*b)/(f*cos(f*x + e))
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)\sin^3(e+fx)\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)\sin^3\left(e+fx\right)\,dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*sin(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int \left(a+b\sec^2(e+fx)\right)\sin^3(e+fx)\,dx=\frac{a\cos\left(fx+e\right)^3-3\left(a-b\right)\cos\left(fx+e\right)+\frac{3\,b}{\cos(fx+e)}}{3\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^3,x, algorithm="maxima")
[Out] 1/3*(a*cos(f*x + e)^3 - 3*(a - b)*cos(f*x + e) + 3*b/cos(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

$$\int \left(a+b\sec^2(e+fx)\right)\sin^3(e+fx)\,dx$$
$$=\frac{b}{f\cos\left(fx+e\right)}+\frac{af^5\cos\left(fx+e\right)^3-3\,af^5\cos\left(fx+e\right)+3\,bf^5\cos\left(fx+e\right)}{3\,f^6}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^3,x, algorithm="giac")
[Out] b/(f*cos(f*x + e)) + 1/3*(a*f^5*cos(f*x + e)^3 - 3*a*f^5*cos(f*x + e) + 3*b
*f^5*cos(f*x + e))/f^6
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \left(a+b\sec^2(e+fx)\right)\sin^3(e+fx)\,dx=\frac{\frac{a\cos(e+f\,x)^3}{3}-\cos\left(e+f\,x\right)\,\left(a-b\right)+\frac{b}{\cos(e+f\,x)}}{f}$$

```
[In] int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2),x)
[Out] ((a*cos(e + f*x)^3)/3 - cos(e + f*x)*(a - b) + b/cos(e + f*x))/f
```

3.4 $\int (a+b\sec^2(e+fx))\sin(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 24

$$\int (a+b\sec^2(e+fx))\sin(e+fx)\,dx=-\frac{a\cos(e+fx)}{f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -a*cos(f*x+e)/f+b*sec(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4218, 14}

$$\int (a+b\sec^2(e+fx))\sin(e+fx)\,dx=\frac{b\sec(e+fx)}{f}-\frac{a\cos(e+fx)}{f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x],x]
```

```
[Out] -((a*Cos[e + f*x])/f) + (b*Sec[e + f*x])/f
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
```

```
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{b+ax^2}{x^2}\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \left(a+\frac{b}{x^2}\right)\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{a\cos(e+fx)}{f}+\frac{b\sec(e+fx)}{f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \left(a+b\sec^2(e+fx)\right)\sin(e+fx)\,dx = -\frac{a\cos(e)\cos(fx)}{f}+\frac{b\sec(e+fx)}{f}+\frac{a\sin(e)\sin(fx)}{f}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x],x]
[Out] -((a*Cos[e]*Cos[f*x])/f) + (b*Sec[e + f*x])/f + (a*Sin[e]*Sin[f*x])/f
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{\sec(fx+e)b-\frac{a}{\sec(fx+e)}}{f}$	25
default	$\frac{\sec(fx+e)b-\frac{a}{\sec(fx+e)}}{f}$	25
parts	$-\frac{a\cos(fx+e)}{f}+\frac{b\sec(fx+e)}{f}$	25
parallelrisch	$\frac{-a\cos(2fx+2e)+(-2a+2b)\cos(fx+e)-a+2b}{2f\cos(fx+e)}$	47
risch	$-\frac{a\,e^{3i(fx+e)}+(3a-4b)\cos(fx+e)+i(a-4b)\sin(fx+e)}{2f\left(e^{2i(fx+e)}+1\right)}$	59
norman	$\frac{\frac{2a-2b}{f}-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)}$	63

```
[In] int((a+b*sec(f*x+e)^2)*sin(f*x+e),x,method=_RETURNVERBOSE)
[Out] 1/f*(sec(f*x+e)*b-a/sec(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \left(a+b\sec^2(e+fx)\right)\sin(e+fx)\,dx=-\frac{a\cos\left(fx+e\right)^2-b}{f\cos\left(fx+e\right)}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e),x, algorithm="fricas")
[Out] -(a*cos(f*x + e)^2 - b)/(f*cos(f*x + e))
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)\sin(e+fx)\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)\sin\left(e+fx\right)dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)*sin(f*x+e),x)
[Out] Integral((a + b*sec(e + f*x)**2)*sin(e + f*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \left(a+b\sec^2(e+fx)\right)\sin(e+fx)\,dx=-\frac{a\cos\left(fx+e\right)-\frac{b}{\cos(fx+e)}}{f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e),x, algorithm="maxima")
[Out] -(a*cos(f*x + e) - b/cos(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \left(a+b\sec^2(e+fx)\right)\sin(e+fx)\,dx=-\frac{a\cos\left(fx+e\right)}{f}+\frac{b}{f\cos\left(fx+e\right)}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e),x, algorithm="giac")
[Out] -a*cos(f*x + e)/f + b/(f*cos(f*x + e))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int (a+b\sec^2(e+fx))\sin(e+fx)\,dx=-\frac{a\,\cos(e+f\,x)-\frac{b}{\cos(e+f\,x)}}{f}$$

```
[In] int(sin(e + f*x)*(a + b/cos(e + f*x)^2),x)
[Out] -(a*cos(e + f*x) - b/cos(e + f*x))/f
```

3.5 $\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 27

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{(a+b)\text{arctanh}(\cos(e+fx))}{f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -(a+b)*arctanh(cos(f*x+e))/f+b*sec(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4218, 464, 212}

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{b\sec(e+fx)}{f}-\frac{(a+b)\text{arctanh}(\cos(e+fx))}{f}$$

```
[In] Int[Csc[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(((a + b)*ArcTanh[Cos[e + f*x]])/f) + (b*Sec[e + f*x])/f
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
```

```
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{b+ax^2}{x^2(1-x^2)}\,dx, x, \cos(e+fx)\right)}{f} \\
&= \frac{b\sec(e+fx)}{f} - \frac{(a+b)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(e+fx)\right)}{f} \\
&= -\frac{(a+b)\text{arctanh}(\cos(e+fx))}{f} + \frac{b\sec(e+fx)}{f}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 84 vs. $2(27) = 54$.

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.11

$$\begin{aligned}
\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = &-\frac{a\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f} - \frac{b\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f} \\
&+ \frac{a\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f} \\
&+ \frac{b\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f} + \frac{b\sec(e+fx)}{f}
\end{aligned}$$

```
[In] Integrate[Csc[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((a*Log[Cos[e/2 + (f*x)/2]])/f) - (b*Log[Cos[(e + f*x)/2]])/f + (a*Log[Sin
[e/2 + (f*x)/2]])/f + (b*Log[Sin[(e + f*x)/2]])/f + (b*Sec[e + f*x])/f
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

method	result	size
norman	$-\frac{2b}{f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)}+\frac{(a+b)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}$	40
parallelrisch	$\frac{\cos(fx+e)(a+b)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+b(1+\cos(fx+e))}{f\cos(fx+e)}$	44
derivativedivides	$\frac{a\ln(\csc(fx+e)-\cot(fx+e))+b\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)}{f}$	51
default	$\frac{a\ln(\csc(fx+e)-\cot(fx+e))+b\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)}{f}$	51
risch	$\frac{2b\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)}+\frac{\ln\left(e^{i(fx+e)}-1\right)a}{f}+\frac{\ln\left(e^{i(fx+e)}-1\right)b}{f}-\frac{\ln\left(e^{i(fx+e)}+1\right)a}{f}-\frac{\ln\left(e^{i(fx+e)}+1\right)b}{f}$	100

```
[In] int(csc(f*x+e)*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] -2*b/f/(tan(1/2*f*x+1/2*e)^2-1)+(a+b)/f*ln(tan(1/2*f*x+1/2*e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(27) = 54$.

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.22

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{(a+b)\cos\left(fx+e\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-(a+b)\cos\left(fx+e\right)\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-2\,b}{2\,f\cos\left(fx+e\right)}$$

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
[Out] -1/2*((a + b)*cos(f*x + e)*log(1/2*cos(f*x + e) + 1/2) - (a + b)*cos(f*x +
e)*log(-1/2*cos(f*x + e) + 1/2) - 2*b)/(f*cos(f*x + e))
```

Sympy [F]

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)\csc\left(e+fx\right)dx$$

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2),x)
[Out] Integral((a + b*sec(e + f*x)**2)*csc(e + f*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{(a+b)\log\left(\cos\left(fx+e\right)+1\right)-(a+b)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\,b}{\cos(fx+e)}}{2\,f}$$

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] -1/2*((a + b)*log(cos(f*x + e) + 1) - (a + b)*log(cos(f*x + e) - 1) - 2*b/c
os(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(27) = 54.

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.22

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(a+b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)+\frac{4\,b}{\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1}}{2\,f}$$

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] 1/2*((a + b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) + 4*b/((cos(
f*x + e) - 1)/(cos(f*x + e) + 1) + 1))/f
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b}{f\,\cos\left(e+f\,x\right)}-\frac{\mathrm{atanh}(\cos\left(e+f\,x\right))\,\left(a+b\right)}{f}$$

```
[In] int((a + b/cos(e + f*x)^2)/sin(e + f*x),x)
[Out] b/(f*cos(e + f*x)) - (atanh(cos(e + f*x))*(a + b))/f
```

3.6 $\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{(a+3b)\text{arctanh}(\cos(e+fx))}{2f}-\frac{(a+b)\cot(e+fx)\csc(e+fx)}{2f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -1/2*(a+3*b)*arctanh(cos(f*x+e))/f-1/2*(a+b)*cot(f*x+e)*csc(f*x+e)/f+b*sec(
f*x+e)/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4218, 467, 464, 212}

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{(a+3b)\text{arctanh}(\cos(e+fx))}{2f}-\frac{(a+b)\cot(e+fx)\csc(e+fx)}{2f}+\frac{b\sec(e+fx)}{f}$$

```
[In] Int[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/2*((a + 3*b)*ArcTanh[Cos[e + f*x]])/f - ((a + b)*Cot[e + f*x]*Csc[e + f*
x])/(2*f) + (b*Sec[e + f*x])/f
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{b+ax^2}{x^2(1-x^2)^2}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{(a+b)\cot(e+fx)\csc(e+fx)}{2f}+\frac{\text{Subst}\left(\int \frac{-2b-(a+b)x^2}{x^2(1-x^2)}\,dx, x, \cos(e+fx)\right)}{2f}\\
&= -\frac{(a+b)\cot(e+fx)\csc(e+fx)}{2f}+\frac{b\sec(e+fx)}{f}-\frac{(a+3b)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(e+fx)\right)}{2f}\\
&= -\frac{(a+3b)\text{arctanh}(\cos(e+fx))}{2f}-\frac{(a+b)\cot(e+fx)\csc(e+fx)}{2f}+\frac{b\sec(e+fx)}{f}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 236 vs. 2(53) = 106.

Time = 0.87 (sec) , antiderivative size = 236, normalized size of antiderivative = 4.45

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{a\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f}-\frac{b\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f}-\frac{a\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2f}-\frac{3b\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2f}+\frac{a\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f}+\frac{3b\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f}+\frac{a\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f}+\frac{b\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f}+\frac{b\sin\left(\frac{1}{2}(e+fx)\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}-\frac{b\sin\left(\frac{1}{2}(e+fx)\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/8*(a*Csc[(e + f*x)/2]^2)/f - (b*Csc[(e + f*x)/2]^2)/(8*f) - (a*Log[Cos[(
e + f*x)/2]])/(2*f) - (3*b*Log[Cos[(e + f*x)/2]])/(2*f) + (a*Log[Sin[(e + f
*x)/2]])/(2*f) + (3*b*Log[Sin[(e + f*x)/2]])/(2*f) + (a*Sec[(e + f*x)/2]^2)
/(8*f) + (b*Sec[(e + f*x)/2]^2)/(8*f) + (b*Sin[(e + f*x)/2])/(f*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])) - (b*Sin[(e + f*x)/2])/(f*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2]))
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.70

method	result
derivativedivides	$\frac{a\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)+b\left(-\frac{1}{2\sin(fx+e)^2\cos(fx+e)}+\frac{3}{2\cos(fx+e)}+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)}{f}$
default	$\frac{a\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)+b\left(-\frac{1}{2\sin(fx+e)^2\cos(fx+e)}+\frac{3}{2\cos(fx+e)}+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)}{f}$
norman	$\frac{\frac{a+b}{8f}+\frac{(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{8f}-\frac{(a+9b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{4f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}+\frac{(a+3b)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2f}$
parallelrisch	$\frac{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)(a+3b)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+(a+b)\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^2-2a-18b}{8f\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-8f}$
risch	$\frac{e^{i(fx+e)}\left(a\,e^{4i(fx+e)}+3b\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}-2b\,e^{2i(fx+e)}+a+3b\right)}{f\left(e^{2i(fx+e)}+1\right)\left(e^{2i(fx+e)}-1\right)^2}-\frac{\ln\left(e^{i(fx+e)}+1\right)a}{2f}-\frac{3\ln\left(e^{i(fx+e)}+1\right)b}{2f}+$ ln

```
[In] int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*(-1/2*csc(f*x+e)*cot(f*x+e)+1/2*ln(csc(f*x+e)-cot(f*x+e)))+b*(-1/2/s
in(f*x+e)^2/cos(f*x+e)+3/2/cos(f*x+e)+3/2*ln(csc(f*x+e)-cot(f*x+e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(49) = 98$.

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.34

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{2\,(a+3\,b)\cos\left(fx+e\right)^2-\left((a+3\,b)\cos\left(fx+e\right)^3-(a+3\,b)\cos\left(fx+e\right)\right)\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)+\big((a}{4\left(f\cos\left(fx+e\right)^3-f\cos\left(fx+\right.\right.}$$

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a + 3*b)*cos(f*x + e)^2 - ((a + 3*b)*cos(f*x + e)^3 - (a + 3*b)*cos
(f*x + e))*log(1/2*cos(f*x + e) + 1/2) + ((a + 3*b)*cos(f*x + e)^3 - (a + 3
*b)*cos(f*x + e))*log(-1/2*cos(f*x + e) + 1/2) - 4*b)/(f*cos(f*x + e)^3 - f
*cos(f*x + e))
```

Sympy [F]

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)\csc^3\left(e+fx\right)dx$$

```
[In] integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2),x)
[Out] Integral((a + b*sec(e + f*x)**2)*csc(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.43

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= -\frac{(a+3b)\log\left(\cos\left(fx+e\right)+1\right)-(a+3b)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left((a+3b)\cos(fx+e)^2-2b\right)}{\cos(fx+e)^3-\cos(fx+e)}}{4f}$$

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] -1/4*((a + 3*b)*log(cos(f*x + e) + 1) - (a + 3*b)*log(cos(f*x + e) - 1) - 2
*((a + 3*b)*cos(f*x + e)^2 - 2*b)/(cos(f*x + e)^3 - cos(f*x + e)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(49) = 98.

Time = 0.30 (sec) , antiderivative size = 193, normalized size of antiderivative = 3.64

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= \frac{2(a+3b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)-\frac{a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a+b+\frac{14b(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{3b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+\frac{(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}}{8f}$$

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] 1/8*(2*(a + 3*b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - a*(cos
(f*x + e) - 1)/(cos(f*x + e) + 1) - b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1)
 + (a + b + 14*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - a*(cos(f*x + e) -
1)^2/(cos(f*x + e) + 1)^2 - 3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/
((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + (cos(f*x + e) - 1)^2/(cos(f*x + e)
 + 1)^2))/f
```

Mupad [B] (verification not implemented)

Time = 17.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b-\cos\left(e+f\,x\right)^2\,\left(\frac{a}{2}+\frac{3\,b}{2}\right)}{f\,\left(\cos\left(e+f\,x\right)-\cos\left(e+f\,x\right)^3\right)} - \frac{\mathrm{atanh}(\cos\left(e+f\,x\right))\,\left(\frac{a}{2}+\frac{3\,b}{2}\right)}{f}$$

```
[In] int((a + b/cos(e + f*x)^2)/sin(e + f*x)^3,x)
```

```
[Out] (b - cos(e + f*x)^2*(a/2 + (3*b)/2))/(f*(cos(e + f*x) - cos(e + f*x)^3)) -
(atanh(cos(e + f*x))*(a/2 + (3*b)/2))/f
```

3.7 $\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{3(a+5b)\text{arctanh}(\cos(e+fx))}{8f}-\frac{(3a+7b)\cot(e+fx)\csc(e+fx)}{8f}-\frac{(a+b)\cot(e+fx)\csc^3(e+fx)}{4f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -3/8*(a+5*b)*arctanh(cos(f*x+e))/f-1/8*(3*a+7*b)*cot(f*x+e)*csc(f*x+e)/f-1/
4*(a+b)*cot(f*x+e)*csc(f*x+e)^3/f+b*sec(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4218, 467, 464, 212}

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{3(a+5b)\text{arctanh}(\cos(e+fx))}{8f}-\frac{(a+b)\cot(e+fx)\csc^3(e+fx)}{4f}-\frac{(3a+7b)\cot(e+fx)\csc(e+fx)}{8f}+\frac{b\sec(e+fx)}{f}$$

```
[In] Int[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (-3*(a + 5*b)*ArcTanh[Cos[e + f*x]])/(8*f) - ((3*a + 7*b)*Cot[e + f*x]*Csc[
e + f*x])/(8*f) - ((a + b)*Cot[e + f*x]*Csc[e + f*x]^3)/(4*f) + (b*Sec[e +
f*x])/f
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{b+ax^2}{x^2(1-x^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\cot(e+fx)\csc^3(e+fx)}{4f} + \frac{\text{Subst}\left(\int \frac{-4b-3(a+b)x^2}{x^2(1-x^2)^2}\,dx, x, \cos(e+fx)\right)}{4f}$$

$$= -\frac{(3a+7b)\cot(e+fx)\csc(e+fx)}{8f} - \frac{(a+b)\cot(e+fx)\csc^3(e+fx)}{4f} - \frac{\text{Subst}\left(\int \frac{8b+(3a+7b)x^2}{x^2(1-x^2)}\,dx, x, \cos(e+fx)\right)}{8f}$$

$$= -\frac{(3a+7b)\cot(e+fx)\csc(e+fx)}{8f} - \frac{(a+b)\cot(e+fx)\csc^3(e+fx)}{4f} + \frac{b\sec(e+fx)}{f} - \frac{(3(a+5b))\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(e+fx)\right)}{8f}$$

$$= -\frac{3(a+5b)\text{arctanh}(\cos(e+fx))}{8f} - \frac{(3a+7b)\cot(e+fx)\csc(e+fx)}{8f} - \frac{(a+b)\cot(e+fx)\csc^3(e+fx)}{4f} + \frac{b\sec(e+fx)}{f}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 198 vs. 2(81) = 162.

Time = 6.26 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.44

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$= \frac{-2(3a+7b)\csc^2\left(\frac{1}{2}(e+fx)\right) - (a+b)\csc^4\left(\frac{1}{2}(e+fx)\right) + \frac{2\left(-3(a+13b)+4\cos(e+fx)\left(8b+3(a+5b)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)-\right.\right.}{}}{64f}$$

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (-2*(3*a + 7*b)*Csc[(e + f*x)/2]^2 - (a + b)*Csc[(e + f*x)/2]^4 + (2*(-3*(a
 + 13*b) + 4*Cos[e + f*x]*(8*b + 3*(a + 5*b)*Log[Cos[(e + f*x)/2]] - 3*(a +
 5*b)*Log[Sin[(e + f*x)/2]]))*Sec[(e + f*x)/2]^2 - (a + b)*Sec[(e + f*x)/2]
^4 + (4*(a + 2*b) + (3*a + 7*b)*Cos[e + f*x])*Sec[(e + f*x)/2]^4*Tan[(e + f
*x)/2]^2)/(-1 + Tan[(e + f*x)/2]^2))/(64*f)
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.48

method	result
derivativedivides	$\frac{a\left(\left(-\frac{\csc(fx+e)^3}{4}-\frac{3\csc(fx+e)}{8}\right)\cot(fx+e)+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{8}\right)+b\left(-\frac{1}{4\sin(fx+e)^4\cos(fx+e)}-\frac{5}{8\sin(fx+e)^2\cos(fx+}\right.}{f}$
default	$\frac{a\left(\left(-\frac{\csc(fx+e)^3}{4}-\frac{3\csc(fx+e)}{8}\right)\cot(fx+e)+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{8}\right)+b\left(-\frac{1}{4\sin(fx+e)^4\cos(fx+e)}-\frac{5}{8\sin(fx+e)^2\cos(fx+}\right.}{f}$
parallelrisch	$\frac{24\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)(a+5b)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+(7a+15b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+(a+b)\,c}{64f\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-64f}$
norman	$\frac{\frac{a+b}{64f}+\frac{(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{64f}+\frac{(7a+15b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{64f}+\frac{(7a+15b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{64f}-\frac{(a+10b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{4f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}+\frac{3(a+5b)\ln\left(\tan\left(\frac{fx}{2}\right.\right.}{8f}$
risch	$\frac{e^{i(fx+e)}\left(3a\,e^{8i(fx+e)}+15b\,e^{8i(fx+e)}-8a\,e^{6i(fx+e)}-40b\,e^{6i(fx+e)}-22a\,e^{4i(fx+e)}+18b\,e^{4i(fx+e)}-8a\,e^{2i(fx+e)}-40b\,e^{2i(f}\right.}{4f\left(e^{2i(fx+e)}-1\right)^4\left(e^{2i(fx+e)}+1\right)}$

```
[In] int(csc(f*x+e)^5*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*((-1/4*csc(f*x+e)^3-3/8*csc(f*x+e))*cot(f*x+e)+3/8*ln(csc(f*x+e)-cot
(f*x+e)))+b*(-1/4/sin(f*x+e)^4/cos(f*x+e)-5/8/sin(f*x+e)^2/cos(f*x+e)+15/8/
cos(f*x+e)+15/8*ln(csc(f*x+e)-cot(f*x+e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(75) = 150$.

Time = 0.26 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.20

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= \frac{6\,(a+5\,b)\cos{(fx+e)}^4-10\,(a+5\,b)\cos{(fx+e)}^2-3\,\left((a+5\,b)\cos{(fx+e)}^5-2\,(a+5\,b)\cos{(fx+e)}^3\right.}{1}$$

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/16*(6*(a + 5*b)*cos(f*x + e)^4 - 10*(a + 5*b)*cos(f*x + e)^2 - 3*((a + 5*
b)*cos(f*x + e)^5 - 2*(a + 5*b)*cos(f*x + e)^3 + (a + 5*b)*cos(f*x + e))*lo
g(1/2*cos(f*x + e) + 1/2) + 3*((a + 5*b)*cos(f*x + e)^5 - 2*(a + 5*b)*cos(f
*x + e)^3 + (a + 5*b)*cos(f*x + e))*log(-1/2*cos(f*x + e) + 1/2) + 16*b)/(f
*cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*cos(f*x + e))
```

Sympy [F]

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)\csc^5\left(e+fx\right)dx$$

```
[In] integrate(csc(f*x+e)**5*(a+b*sec(f*x+e)**2),x)
[Out] Integral((a + b*sec(e + f*x)**2)*csc(e + f*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.25

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{3\,(a+5\,b)\log\left(\cos\left(fx+e\right)+1\right)-3\,(a+5\,b)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\,(a+5\,b)\cos(fx+e)^4-5\,(a+5\,b)\cos(fx+e)^2+8\,b\right)}{\cos(fx+e)^5-2\,\cos(fx+e)^3+\cos(fx+e)}}{16\,f}$$

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] -1/16*(3*(a + 5*b)*log(cos(f*x + e) + 1) - 3*(a + 5*b)*log(cos(f*x + e) - 1
) - 2*(3*(a + 5*b)*cos(f*x + e)^4 - 5*(a + 5*b)*cos(f*x + e)^2 + 8*b)/(cos(
f*x + e)^5 - 2*cos(f*x + e)^3 + cos(f*x + e)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(75) = 150.

Time = 0.31 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.23

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{12\,(a+5\,b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)-\frac{\left(a+b-\frac{8\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{16\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{18\,a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{90\,b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)^2}{(\cos(fx+e)-1)^2}-\cdots}{64\,f}$$

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] 1/64*(12*(a + 5*b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - (a +
 b - 8*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 16*b*(cos(f*x + e) - 1)/(c
os(f*x + e) + 1) + 18*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 90*b*(c
os(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(cos(f*x + e)
 - 1)^2 - 8*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 16*b*(cos(f*x + e) -
1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(co
s(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 128*b/((cos(f*x + e) - 1)/(cos(f*x
 + e) + 1) + 1))/f
```

Mupad [B] (verification not implemented)

Time = 17.54 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\left(\frac{3a}{8}+\frac{15b}{8}\right)\cos(e+fx)^4+\left(-\frac{5a}{8}-\frac{25b}{8}\right)\cos(e+fx)^2+b}{f\left(\cos(e+fx)^5-2\cos(e+fx)^3+\cos(e+fx)\right)} - \frac{\operatorname{atanh}(\cos(e+fx))\left(\frac{3a}{8}+\frac{15b}{8}\right)}{f}$$

```
[In] int((a + b/cos(e + f*x)^2)/sin(e + f*x)^5,x)
```

```
[Out] (b + cos(e + f*x)^4*((3*a)/8 + (15*b)/8) - cos(e + f*x)^2*((5*a)/8 + (25*b)
/8))/(f*(cos(e + f*x) - 2*cos(e + f*x)^3 + cos(e + f*x)^5)) - (atanh(cos(e
+ f*x))*((3*a)/8 + (15*b)/8))/f
```

3.8 $\int (a+b\sec^2(e+fx))\sin^6(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int (a+b\sec^2(e+fx))\sin^6(e+fx)\,dx = \frac{5}{16}(a-6b)x-\frac{(11a-18b)\cos(e+fx)\sin(e+fx)}{16f}+\frac{(13a-6b)\cos^3(e+fx)\sin(e+fx)}{24f}-\frac{a\cos^5(e+fx)\sin(e+fx)}{6f}+\frac{b\tan(e+fx)}{f}$$

```
[Out] 5/16*(a-6*b)*x-1/16*(11*a-18*b)*cos(f*x+e)*sin(f*x+e)/f+1/24*(13*a-6*b)*cos
(f*x+e)^3*sin(f*x+e)/f-1/6*a*cos(f*x+e)^5*sin(f*x+e)/f+b*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4217, 466, 1828, 1171, 396, 209}

$$\int (a+b\sec^2(e+fx))\sin^6(e+fx)\,dx = \frac{(13a-6b)\sin(e+fx)\cos^3(e+fx)}{24f}-\frac{(11a-18b)\sin(e+fx)\cos(e+fx)}{16f}+\frac{5}{16}x(a-6b)-\frac{a\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{b\tan(e+fx)}{f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^6,x]
```

```
[Out] (5*(a - 6*b)*x)/16 - ((11*a - 18*b)*Cos[e + f*x]*Sin[e + f*x])/(16*f) + ((1
3*a - 6*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - (a*Cos[e + f*x]^5*Sin[e +
f*x])/(6*f) + (b*Tan[e + f*x])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^6(a+b+bx^2)}{(1+x^2)^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\cos^5(e+fx)\sin(e+fx)}{6f} - \frac{\text{Subst}\left(\int \frac{-a+6ax^2-6ax^4-6bx^6}{(1+x^2)^3}\,dx, x, \tan(e+fx)\right)}{6f}$$

$$= \frac{(13a-6b)\cos^3(e+fx)\sin(e+fx)}{24f} - \frac{a\cos^5(e+fx)\sin(e+fx)}{6f} + \frac{\text{Subst}\left(\int \frac{-3(3a-2b)+24(a-b)x^2+24bx^4}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{24f}$$

$$= -\frac{(11a-18b)\cos(e+fx)\sin(e+fx)}{16f} + \frac{(13a-6b)\cos^3(e+fx)\sin(e+fx)}{24f} - \frac{a\cos^5(e+fx)\sin(e+fx)}{6f} - \frac{\text{Subst}\left(\int \frac{-3(5a-14b)-48bx^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{48f}$$

$$= -\frac{(11a-18b)\cos(e+fx)\sin(e+fx)}{16f} + \frac{(13a-6b)\cos^3(e+fx)\sin(e+fx)}{24f} - \frac{a\cos^5(e+fx)\sin(e+fx)}{6f} + \frac{b\tan(e+fx)}{f} + \frac{(5(a-6b))\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{16f}$$

$$= \frac{5}{16}(a-6b)x - \frac{(11a-18b)\cos(e+fx)\sin(e+fx)}{16f} + \frac{(13a-6b)\cos^3(e+fx)\sin(e+fx)}{24f} - \frac{a\cos^5(e+fx)\sin(e+fx)}{6f} + \frac{b\tan(e+fx)}{f}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\int (a+b\sec^2(e+fx))\sin^6(e+fx)\,dx = \frac{60ae-360be+60afx-360bfx+(-45a+96b)\sin(2(e+fx))+(9a-6b)\sin(4(e+fx))-a\sin(6(e+ \ldots}{192f}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^6,x]
```

```
[Out] (60*a*e - 360*b*e + 60*a*f*x - 360*b*f*x + (-45*a + 96*b)*Sin[2*(e + f*x)]
+ (9*a - 6*b)*Sin[4*(e + f*x)] - a*Sin[6*(e + f*x)] + 192*b*Tan[e + f*x])/(
192*f)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.91

method	result
parallelrisch	$\frac{(-36a+90b)\sin(3fx+3e)+(8a-6b)\sin(5fx+5e)-\sin(7fx+7e)a+120fx(a-6b)\cos(fx+e)-45\sin(fx+e)\left(a-\frac{32b}{3}\right)}{384f\cos(fx+e)}$
derivativedivides	$\frac{a\left(-\frac{\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+b\left(\frac{\sin(fx+e)^7}{\cos(fx+e)}+\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15\sin(\ldots}{8}\right.\right.}{f}$
default	$\frac{a\left(-\frac{\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+b\left(\frac{\sin(fx+e)^7}{\cos(fx+e)}+\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15\sin(\ldots}{8}\right.\right.}{f}$
parts	$\frac{a\left(-\frac{\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)}{f}+\frac{b\left(\frac{\sin(fx+e)^7}{\cos(fx+e)}+\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15 s\ldots}{}\right.\right.}{f}$
risch	$\frac{5ax}{16}-\frac{15xb}{8}+\frac{15ie^{2i(fx+e)}a}{128f}-\frac{ie^{2i(fx+e)}b}{4f}-\frac{15ie^{-2i(fx+e)}a}{128f}+\frac{ie^{-2i(fx+e)}b}{4f}+\frac{2ib}{f\left(e^{2i(fx+e)}+1\right)}-\frac{a\sin(6fx+\ldots}{192f}$
norman	$\left(-\frac{5a}{16}+\frac{15b}{8}\right)x+\left(-\frac{45a}{16}+\frac{135b}{8}\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+\left(-\frac{25a}{16}+\frac{75b}{8}\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\left(-\frac{25a}{16}+\frac{75b}{8}\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+\left(\frac{5a}{16}-\frac{15b}{8}\ldots\right.$

```
[In] int((a+b*sec(f*x+e)^2)*sin(f*x+e)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/384*((-36*a+90*b)*sin(3*f*x+3*e)+(8*a-6*b)*sin(5*f*x+5*e)-sin(7*f*x+7*e)*
a+120*f*x*(a-6*b)*cos(f*x+e)-45*sin(f*x+e)*(a-32/3*b))/f/cos(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$\int \left(a+b\sec^2(e+fx)\right)\sin^6(e+fx)\,dx = \frac{15\,(a-6\,b)fx\cos\left(fx+e\right)-\left(8\,a\cos\left(fx+e\right)^6-2\,(13\,a-6\,b)\cos\left(fx+e\right)^4+3\,(11\,a-18\,b)\cos\left(fx+e\right.}{48\,f\cos\left(fx+e\right)}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] 1/48*(15*(a - 6*b)*f*x*cos(f*x + e) - (8*a*cos(f*x + e)^6 - 2*(13*a - 6*b)*
cos(f*x + e)^4 + 3*(11*a - 18*b)*cos(f*x + e)^2 - 48*b)*sin(f*x + e))/(f*co
s(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)\sin^6(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**6,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.13

$$\int \left(a+b\sec^2(e+fx)\right)\sin^6(e+fx)\,dx = \frac{15\,(fx+e)(a-6\,b)+48\,b\tan\left(fx+e\right)-\frac{3\,(11\,a-18\,b)\tan(fx+e)^5+8\,(5\,a-12\,b)\tan(fx+e)^3+3\,(5\,a-14\,b)\tan(fx+e)}{\tan(fx+e)^6+3\,\tan(fx+e)^4+3\,\tan(fx+e)^2+1}}{48\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] 1/48*(15*(f*x + e)*(a - 6*b) + 48*b*tan(f*x + e) - (3*(11*a - 18*b)*tan(f*x
 + e)^5 + 8*(5*a - 12*b)*tan(f*x + e)^3 + 3*(5*a - 14*b)*tan(f*x + e))/(tan
(f*x + e)^6 + 3*tan(f*x + e)^4 + 3*tan(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.06

$$\int \left(a+b\sec^2(e+fx)\right)\sin^6(e+fx)\,dx$$
$$=\frac{15\,(fx+e)(a-6\,b)+48\,b\tan\left(fx+e\right)-\frac{33\,a\tan(fx+e)^5-54\,b\tan(fx+e)^5+40\,a\tan(fx+e)^3-96\,b\tan(fx+e)^3+15\,a\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^3}}{48\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^6,x, algorithm="giac")
```

```
[Out] 1/48*(15*(f*x + e)*(a - 6*b) + 48*b*tan(f*x + e) - (33*a*tan(f*x + e)^5 - 5
4*b*tan(f*x + e)^5 + 40*a*tan(f*x + e)^3 - 96*b*tan(f*x + e)^3 + 15*a*tan(f
*x + e) - 42*b*tan(f*x + e))/(tan(f*x + e)^2 + 1)^3)/f
```

Mupad [B] (verification not implemented)

Time = 18.47 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.07

$$\int \left(a+b\sec^2(e+fx)\right)\sin^6(e+fx)\,dx$$
$$=x\left(\frac{5\,a}{16}-\frac{15\,b}{8}\right)$$
$$-\frac{\left(\frac{11\,a}{16}-\frac{9\,b}{8}\right)\,\tan(e+f\,x)^5+\left(\frac{5\,a}{6}-2\,b\right)\,\tan(e+f\,x)^3+\left(\frac{5\,a}{16}-\frac{7\,b}{8}\right)\,\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^6+3\tan\left(e+f\,x\right)^4+3\tan\left(e+f\,x\right)^2+1\right)}$$
$$+\frac{b\tan(e+f\,x)}{f}$$

```
[In] int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2),x)
```

```
[Out] x*((5*a)/16 - (15*b)/8) - (tan(e + f*x)^3*((5*a)/6 - 2*b) + tan(e + f*x)^5*
((11*a)/16 - (9*b)/8) + tan(e + f*x)*((5*a)/16 - (7*b)/8))/(f*(3*tan(e + f*
x)^2 + 3*tan(e + f*x)^4 + tan(e + f*x)^6 + 1)) + (b*tan(e + f*x))/f
```

3.9 $\int (a+b\sec^2(e+fx))\sin^4(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int (a+b\sec^2(e+fx))\sin^4(e+fx)\,dx=\frac{3}{8}(a-4b)x-\frac{(5a-4b)\cos(e+fx)\sin(e+fx)}{8f}+\frac{a\cos^3(e+fx)\sin(e+fx)}{4f}+\frac{b\tan(e+fx)}{f}$$

```
[Out] 3/8*(a-4*b)*x-1/8*(5*a-4*b)*cos(f*x+e)*sin(f*x+e)/f+1/4*a*cos(f*x+e)^3*sin(
f*x+e)/f+b*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4217, 466, 1171, 396, 209}

$$\int (a+b\sec^2(e+fx))\sin^4(e+fx)\,dx=-\frac{(5a-4b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}x(a-4b)+\frac{a\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{b\tan(e+fx)}{f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^4,x]
```

```
[Out] (3*(a - 4*b)*x)/8 - ((5*a - 4*b)*Cos[e + f*x]*Sin[e + f*x])/(8*f) + (a*Cos[
e + f*x]^3*Sin[e + f*x])/(4*f) + (b*Tan[e + f*x])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4(a+b+bx^2)}{(1+x^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{a\cos^3(e+fx)\sin(e+fx)}{4f} - \frac{\text{Subst}\left(\int \frac{a-4ax^2-4bx^4}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{4f}$$

$$= -\frac{(5a-4b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a\cos^3(e+fx)\sin(e+fx)}{4f} + \frac{\operatorname{Subst}\left(\int \frac{3a-4b+8bx^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{8f}$$

$$= -\frac{(5a-4b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a\cos^3(e+fx)\sin(e+fx)}{4f} + \frac{b\tan(e+fx)}{f} + \frac{(3(a-4b))\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{8f}$$

$$= \frac{3}{8}(a-4b)x - \frac{(5a-4b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a\cos^3(e+fx)\sin(e+fx)}{4f} + \frac{b\tan(e+fx)}{f}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int (a+b\sec^2(e+fx))\sin^4(e+fx)\,dx = \frac{12(a-4b)(e+fx)-8(a-b)\sin(2(e+fx))+a\sin(4(e+fx))+32b\tan(e+fx)}{32f}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^4,x]
[Out] (12*(a - 4*b)*(e + f*x) - 8*(a - b)*Sin[2*(e + f*x)] + a*Sin[4*(e + f*x)] +
 32*b*Tan[e + f*x])/(32*f)
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

method	result
parallelrisch	$\frac{(-7a+8b)\sin(3fx+3e)+\sin(5fx+5e)a+24fx(a-4b)\cos(fx+e)-8\sin(fx+e)(a-9b)}{64f\cos(fx+e)}$
derivativedivides	$\frac{a\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+b\left(\frac{\sin(fx+e)^5}{\cos(fx+e)}+\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)-\frac{3fx}{2}-\frac{3e}{2}\right)}{f}$
default	$\frac{a\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+b\left(\frac{\sin(fx+e)^5}{\cos(fx+e)}+\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)-\frac{3fx}{2}-\frac{3e}{2}\right)}{f}$
parts	$\frac{a\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)}{f}+\frac{b\left(\frac{\sin(fx+e)^5}{\cos(fx+e)}+\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)-\frac{3fx}{2}-\frac{3e}{2}\right)}{f}$
risch	$\frac{3ax}{8}-\frac{3xb}{2}+\frac{ie^{2i(fx+e)}a}{8f}-\frac{ie^{2i(fx+e)}b}{8f}-\frac{ie^{-2i(fx+e)}a}{8f}+\frac{ie^{-2i(fx+e)}b}{8f}+\frac{2ib}{f\left(e^{2i(fx+e)}+1\right)}+\frac{\sin(4fx+4e)a}{32f}$
norman	$\frac{\left(-\frac{3a}{8}+\frac{3b}{2}\right)x+\left(-\frac{9a}{8}+\frac{9b}{2}\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\left(-\frac{3a}{4}+3b\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+\left(\frac{3a}{4}-3b\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+\left(\frac{3a}{8}-\frac{3b}{2}\right)x\tan\left(\frac{fx}{2}+\dots\right.}{\left(\tan\left(\frac{f\dots}{}\right.\right.}$

```
[In] int((a+b*sec(f*x+e)^2)*sin(f*x+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*((-7*a+8*b)*sin(3*f*x+3*e)+sin(5*f*x+5*e)*a+24*f*x*(a-4*b)*cos(f*x+e)-
8*sin(f*x+e)*(a-9*b))/f/cos(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int \left(a+b\sec^2(e+fx)\right)\sin^4(e+fx)\,dx$$
$$=\frac{3\,(a-4\,b)fx\cos\left(fx+e\right)+\left(2\,a\cos\left(fx+e\right)^4-\left(5\,a-4\,b\right)\cos\left(fx+e\right)^2+8\,b\right)\sin\left(fx+e\right)}{8\,f\cos\left(fx+e\right)}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(3*(a - 4*b)*f*x*cos(f*x + e) + (2*a*cos(f*x + e)^4 - (5*a - 4*b)*cos(f
*x + e)^2 + 8*b)*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)\sin^4(e+fx)\,dx=\int \left(a+b\sec^2{(e+fx)}\right)\sin^4{(e+fx)}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**4,x)
[Out] Integral((a + b*sec(e + f*x)**2)*sin(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17

$$\int \left(a+b\sec^2(e+fx)\right)\sin^4(e+fx)\,dx=\frac{3\,(fx+e)(a-4\,b)+8\,b\tan\left(fx+e\right)-\frac{(5\,a-4\,b)\tan(fx+e)^3+(3\,a-4\,b)\tan(fx+e)}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}}{8\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^4,x, algorithm="maxima")
[Out] 1/8*(3*(f*x + e)*(a - 4*b) + 8*b*tan(f*x + e) - ((5*a - 4*b)*tan(f*x + e)^3
 + (3*a - 4*b)*tan(f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17

$$\int \left(a+b\sec^2(e+fx)\right)\sin^4(e+fx)\,dx=\frac{3\,(fx+e)(a-4\,b)+8\,b\tan\left(fx+e\right)-\frac{5\,a\tan(fx+e)^3-4\,b\tan(fx+e)^3+3\,a\tan(fx+e)-4\,b\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^2}}{8\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^4,x, algorithm="giac")
[Out] 1/8*(3*(f*x + e)*(a - 4*b) + 8*b*tan(f*x + e) - (5*a*tan(f*x + e)^3 - 4*b*t
an(f*x + e)^3 + 3*a*tan(f*x + e) - 4*b*tan(f*x + e))/(tan(f*x + e)^2 + 1)^2
)/f
```

Mupad [B] (verification not implemented)

Time = 17.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.13

$$\int (a+b\sec^2(e+fx))\sin^4(e+fx)\,dx = x\left(\frac{3a}{8}-\frac{3b}{2}\right) - \frac{\left(\frac{5a}{8}-\frac{b}{2}\right)\tan(e+f\,x)^3+\left(\frac{3a}{8}-\frac{b}{2}\right)\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^4+2\tan\left(e+f\,x\right)^2+1\right)} + \frac{b\tan(e+f\,x)}{f}$$

```
[In] int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2),x)
```

```
[Out] x*((3*a)/8 - (3*b)/2) - (tan(e + f*x)^3*((5*a)/8 - b/2) + tan(e + f*x)*((3*
a)/8 - b/2))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1)) + (b*tan(e + f*x))
/f
```

3.10 $\int (a+b\sec^2(e+fx))\sin^2(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 42

$$\int (a+b\sec^2(e+fx))\sin^2(e+fx)\,dx = \frac{1}{2}(a-2b)x - \frac{a\cos(e+fx)\sin(e+fx)}{2f} + \frac{b\tan(e+fx)}{f}$$

```
[Out] 1/2*(a-2*b)*x-1/2*a*cos(f*x+e)*sin(f*x+e)/f+b*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4217, 466, 396, 209}

$$\int (a+b\sec^2(e+fx))\sin^2(e+fx)\,dx = \frac{1}{2}x(a-2b) - \frac{a\sin(e+fx)\cos(e+fx)}{2f} + \frac{b\tan(e+fx)}{f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^2,x]
```

```
[Out] ((a - 2*b)*x)/2 - (a*Cos[e + f*x]*Sin[e + f*x])/(2*f) + (b*Tan[e + f*x])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2(a+b+bx^2)}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{a\cos(e+fx)\sin(e+fx)}{2f} - \frac{\text{Subst}\left(\int \frac{-a-2bx^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{2f}\\
&= -\frac{a\cos(e+fx)\sin(e+fx)}{2f} + \frac{b\tan(e+fx)}{f} + \frac{(a-2b)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{2f}\\
&= \frac{1}{2}(a-2b)x - \frac{a\cos(e+fx)\sin(e+fx)}{2f} + \frac{b\tan(e+fx)}{f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.29

$$\int \left(a+b\sec^2(e+fx)\right)\sin^2(e+fx)\,dx=\frac{a(e+fx)}{2f}-\frac{b\arctan(\tan(e+fx))}{f}-\frac{a\sin(2(e+fx))}{4f}+\frac{b\tan(e+fx)}{f}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^2,x]
[Out] (a*(e + f*x))/(2*f) - (b*ArcTan[Tan[e + f*x]])/f - (a*Sin[2*(e + f*x)])/(4*
f) + (b*Tan[e + f*x])/f
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{a\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b(\tan(fx+e)-fx-e)}{f}$
default	$\frac{a\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b(\tan(fx+e)-fx-e)}{f}$
parts	$\frac{a\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{b(\tan(fx+e)-fx-e)}{f}$
parallelrisch	$\frac{-\sin(3fx+3e)a+4fx(a-2b)\cos(fx+e)-\sin(fx+e)(a-8b)}{8f\cos(fx+e)}$
risch	$\frac{ax}{2}-xb+\frac{i\mathrm{e}^{2i(fx+e)}a}{8f}-\frac{i\mathrm{e}^{-2i(fx+e)}a}{8f}+\frac{2ib}{f\left(\mathrm{e}^{2i(fx+e)}+1\right)}$
norman	$\frac{\left(-\frac{a}{2}+b\right)x+\frac{(a-2b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{(a-2b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}+\left(-\frac{a}{2}+b\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\left(\frac{a}{2}-b\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+\left(\frac{a}{2}-b\right)x\tan\left(\right.}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^2}$

```
[In] int((a+b*sec(f*x+e)^2)*sin(f*x+e)^2,x,method=_RETURNVERBOSE)
[Out] 1/f*(a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+b*(tan(f*x+e)-f*x-e))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

$$\int \left(a+b\sec^2(e+fx)\right)\sin^2(e+fx)\,dx = \frac{(a-2\,b)fx\cos\left(fx+e\right)-\left(a\cos\left(fx+e\right)^2-2\,b\right)\sin\left(fx+e\right)}{2\,f\cos\left(fx+e\right)}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^2,x, algorithm="fricas")
[Out] 1/2*((a - 2*b)*f*x*cos(f*x + e) - (a*cos(f*x + e)^2 - 2*b)*sin(f*x + e))/(f
*cos(f*x + e))
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)\sin^2(e+fx)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)\sin^2\left(e+fx\right)\,dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**2,x)
[Out] Integral((a + b*sec(e + f*x)**2)*sin(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12

$$\int \left(a+b\sec^2(e+fx)\right)\sin^2(e+fx)\,dx = \frac{(fx+e)(a-2\,b)+2\,b\tan\left(fx+e\right)-\frac{a\tan(fx+e)}{\tan(fx+e)^2+1}}{2\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^2,x, algorithm="maxima")
[Out] 1/2*((f*x + e)*(a - 2*b) + 2*b*tan(f*x + e) - a*tan(f*x + e)/(tan(f*x + e)^
2 + 1))/f
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12

$$\int \left(a+b\sec^2(e+fx)\right)\sin^2(e+fx)\,dx=\frac{(fx+e)(a-2\,b)+2\,b\tan\left(fx+e\right)-\frac{a\tan(fx+e)}{\tan(fx+e)^2+1}}{2\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^2,x, algorithm="giac")
```

```
[Out] 1/2*((f*x + e)*(a - 2*b) + 2*b*tan(f*x + e) - a*tan(f*x + e)/(tan(f*x + e)^
2 + 1))/f
```

Mupad [B] (verification not implemented)

Time = 17.63 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \left(a+b\sec^2(e+fx)\right)\sin^2(e+fx)\,dx=\frac{b\tan(e+f\,x)-\frac{a\,\sin(2\,e+2\,f\,x)}{4}+f\,x\,\left(\frac{a}{2}-b\right)}{f}$$

```
[In] int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2),x)
```

```
[Out] (b*tan(e + f*x) - (a*sin(2*e + 2*f*x))/4 + f*x*(a/2 - b))/f
```

3.11 $\int (a + b\sec^2(e + fx))\,dx$

Optimal result

Integrand size = 12, antiderivative size = 15

$$\int (a + b\sec^2(e + fx))\,dx = ax + \frac{b\tan(e + fx)}{f}$$

```
[Out] a*x+b*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3852, 8}

$$\int (a + b\sec^2(e + fx))\,dx = ax + \frac{b\tan(e + fx)}{f}$$

```
[In] Int[a + b*Sec[e + f*x]^2,x]
```

```
[Out] a*x + (b*Tan[e + f*x])/f
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= ax + b\int \sec^2(e+fx)\,dx\\ &= ax - \frac{b\text{Subst}\left(\int 1\,dx, x, -\tan(e+fx)\right)}{f}\\ &= ax + \frac{b\tan(e+fx)}{f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \left(a+b\sec^2(e+fx)\right)\,dx = ax + \frac{b\tan(e+fx)}{f}$$

```
[In] Integrate[a + b*Sec[e + f*x]^2,x]
[Out] a*x + (b*Tan[e + f*x])/f
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$ax+\frac{b\tan(fx+e)}{f}$	16
parts	$ax+\frac{b\tan(fx+e)}{f}$	16
derivativedivides	$\frac{(fx+e)a+b\tan(fx+e)}{f}$	21
risch	$ax+\frac{2ib}{f\left(e^{2i(fx+e)}+1\right)}$	25
parallelrisch	$-\frac{2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}+ax$	35
norman	$\frac{ax\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-ax-\frac{2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}$	51

```
[In] int(a+b*sec(f*x+e)^2,x,method=_RETURNVERBOSE)
[Out] a*x+b*tan(f*x+e)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(15) = 30$.

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \left(a + b\sec^2(e + fx)\right)\,dx = \frac{afx\cos\left(fx + e\right) + b\sin\left(fx + e\right)}{f\cos\left(fx + e\right)}$$

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="fricas")
[Out] (a*f*x*cos(f*x + e) + b*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [F]

$$\int \left(a + b\sec^2(e + fx)\right)\,dx = \int \left(a + b\sec^2\left(e + fx\right)\right)\,dx$$

```
[In] integrate(a+b*sec(f*x+e)**2,x)
[Out] Integral(a + b*sec(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \left(a + b\sec^2(e + fx)\right)\,dx = ax + \frac{b\tan\left(fx + e\right)}{f}$$

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="maxima")
[Out] a*x + b*tan(f*x + e)/f
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \left(a + b\sec^2(e + fx)\right)\,dx = ax + \frac{b\tan\left(fx + e\right)}{f}$$

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="giac")
[Out] a*x + b*tan(f*x + e)/f
```

Mupad [B] (verification not implemented)

Time = 17.71 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \left(a + b\sec^2(e + fx)\right) \, dx = \frac{b\tan(e + f\,x) + a\,f\,x}{f}$$

```
[In] int(a + b/cos(e + f*x)^2,x)
[Out] (b*tan(e + f*x) + a*f*x)/f
```

3.12 $\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 26

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{(a+b)\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}$$

```
[Out] -(a+b)*cot(f*x+e)/f+b*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4217, 14}

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{b\tan(e+fx)}{f}-\frac{(a+b)\cot(e+fx)}{f}$$

```
[In] Int[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(((a + b)*Cot[e + f*x])/f) + (b*Tan[e + f*x])/f
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
```

```
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{a+b+bx^2}{x^2}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(b+\frac{a+b}{x^2}\right)\,dx, x, \tan(e+fx)\right)}{f}\\ &= -\frac{(a+b)\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{a\cot(e+fx)}{f}-\frac{b\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}$$

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
[Out] -((a*Cot[e + f*x])/f) - (b*Cot[e + f*x])/f + (b*Tan[e + f*x])/f
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

method	result	size
derivativedivides	$\frac{-\cot(fx+e)a+b\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)}{f}$	43
default	$\frac{-\cot(fx+e)a+b\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)}{f}$	43
risch	$-\frac{2i\left(a\,e^{2i(fx+e)}+a+2b\right)}{f\left(e^{2i(fx+e)}+1\right)\left(e^{2i(fx+e)}-1\right)}$	49
parallelrisch	$\frac{\left((a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+(-2a-6b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+a+b\right)\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{2f\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-2f}$	68
norman	$\frac{\frac{a+b}{2f}+\frac{(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{2f}-\frac{(a+3b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}$	77

```
[In] int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-cot(f*x+e)*a+b*(1/sin(f*x+e)/cos(f*x+e)-2*cot(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{(a+2\,b)\cos\left(fx+e\right)^2-b}{f\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
[Out] -((a + 2*b)*cos(f*x + e)^2 - b)/(f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F]

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)\csc^2\left(e+fx\right)dx$$

```
[In] integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2),x)
[Out] Integral((a + b*sec(e + f*x)**2)*csc(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{b\tan\left(fx+e\right)-\frac{a+b}{\tan(fx+e)}}{f}$$

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] (b*tan(f*x + e) - (a + b)/tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{b\tan\left(fx+e\right)-\frac{a+b}{\tan(fx+e)}}{f}$$

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] (b*tan(f*x + e) - (a + b)/tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 19.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{b\tan(e+f\,x)}{f}-\frac{a+b}{f\tan\left(e+f\,x\right)}$$

```
[In] int((a + b/cos(e + f*x)^2)/sin(e + f*x)^2,x)
[Out] (b*tan(e + f*x))/f - (a + b)/(f*tan(e + f*x))
```

3.13 $\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 46

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{(a+2b)\cot(e+fx)}{f}-\frac{(a+b)\cot^3(e+fx)}{3f}+\frac{b\tan(e+fx)}{f}$$

```
[Out] -(a+2*b)*cot(f*x+e)/f-1/3*(a+b)*cot(f*x+e)^3/f+b*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4217, 459}

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{(a+b)\cot^3(e+fx)}{3f}-\frac{(a+2b)\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}$$

```
[In] Int[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(((a + 2*b)*Cot[e + f*x])/f) - ((a + b)*Cot[e + f*x]^3)/(3*f) + (b*Tan[e +
 f*x])/f
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)(a+b+bx^2)}{x^4}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \left(b+\frac{a+b}{x^4}+\frac{a+2b}{x^2}\right)\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{(a+2b)\cot(e+fx)}{f}-\frac{(a+b)\cot^3(e+fx)}{3f}+\frac{b\tan(e+fx)}{f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.83

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{2a\cot(e+fx)}{3f}-\frac{5b\cot(e+fx)}{3f}-\frac{a\cot(e+fx)\csc^2(e+fx)}{3f}-\frac{b\cot(e+fx)\csc^2(e+fx)}{3f}+\frac{b\tan(e+fx)}{f}$$

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
[Out] (-2*a*Cot[e + f*x])/(3*f) - (5*b*Cot[e + f*x])/(3*f) - (a*Cot[e + f*x]*Csc[
e + f*x]^2)/(3*f) - (b*Cot[e + f*x]*Csc[e + f*x]^2)/(3*f) + (b*Tan[e + f*x]
)/f
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.59

method	result
derivativedivides	$\frac{a\left(-\frac{2}{3}-\frac{\csc(fx+e)^2}{3}\right)\cot(fx+e)+b\left(-\frac{1}{3\sin(fx+e)^3\cos(fx+e)}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right)}{f}$
default	$\frac{a\left(-\frac{2}{3}-\frac{\csc(fx+e)^2}{3}\right)\cot(fx+e)+b\left(-\frac{1}{3\sin(fx+e)^3\cos(fx+e)}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right)}{f}$
risch	$\frac{4i\left(3a\,\mathrm{e}^{4i(fx+e)}+2a\,\mathrm{e}^{2i(fx+e)}+8b\,\mathrm{e}^{2i(fx+e)}-a-4b\right)}{3f\left(\mathrm{e}^{2i(fx+e)}-1\right)^3\left(\mathrm{e}^{2i(fx+e)}+1\right)}$
parallelrisch	$\frac{\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3\left((a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8+(8a+20b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+(-18a-90b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+(8a+20b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+a+b\right.}{24f\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-24f}$
norman	$\frac{\frac{a+b}{24f}+\frac{(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{24f}-\frac{3(a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{4f}+\frac{(2a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{6f}+\frac{(2a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{6f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}$

```
[In] int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*(-2/3-1/3*csc(f*x+e)^2)*cot(f*x+e)+b*(-1/3/sin(f*x+e)^3/cos(f*x+e)+4
/3/sin(f*x+e)/cos(f*x+e)-8/3*cot(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.43

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{2\,(a+4\,b)\cos\left(fx+e\right)^4-3\,(a+4\,b)\cos\left(fx+e\right)^2+3\,b}{3\,\left(f\cos\left(fx+e\right)^3-f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a + 4*b)*cos(f*x + e)^4 - 3*(a + 4*b)*cos(f*x + e)^2 + 3*b)/((f*co
s(f*x + e)^3 - f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F]

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)\csc^4\left(e+fx\right)dx$$

```
[In] integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2),x)
[Out] Integral((a + b*sec(e + f*x)**2)*csc(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{3\,b\tan\left(fx+e\right)-\frac{3\,(a+2\,b)\tan(fx+e)^2+a+b}{\tan(fx+e)^3}}{3\,f}$$

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] 1/3*(3*b*tan(f*x + e) - (3*(a + 2*b)*tan(f*x + e)^2 + a + b)/tan(f*x + e)^3
)/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{3\,b\tan\left(fx+e\right)-\frac{3\,a\tan(fx+e)^2+6\,b\tan(fx+e)^2+a+b}{\tan(fx+e)^3}}{3\,f}$$

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] 1/3*(3*b*tan(f*x + e) - (3*a*tan(f*x + e)^2 + 6*b*tan(f*x + e)^2 + a + b)/t
an(f*x + e)^3)/f
```

Mupad [B] (verification not implemented)

Time = 19.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{b\tan(e+f\,x)}{f}-\frac{(a+2\,b)\,\tan(e+f\,x)^2+\frac{a}{3}+\frac{b}{3}}{f\tan\left(e+f\,x\right)^3}$$

```
[In] int((a + b/cos(e + f*x)^2)/sin(e + f*x)^4,x)
[Out] (b*tan(e + f*x))/f - (a/3 + b/3 + tan(e + f*x)^2*(a + 2*b))/(f*tan(e + f*x)
^3)
```

3.14 $\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 68

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{(a+3b)\cot(e+fx)}{f}-\frac{(2a+3b)\cot^3(e+fx)}{3f}-\frac{(a+b)\cot^5(e+fx)}{5f}+\frac{b\tan(e+fx)}{f}$$

```
[Out] -(a+3*b)*cot(f*x+e)/f-1/3*(2*a+3*b)*cot(f*x+e)^3/f-1/5*(a+b)*cot(f*x+e)^5/f
+b*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4217, 459}

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{(a+b)\cot^5(e+fx)}{5f}-\frac{(2a+3b)\cot^3(e+fx)}{3f}-\frac{(a+3b)\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}$$

```
[In] Int[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(((a + 3*b)*Cot[e + f*x])/f) - ((2*a + 3*b)*Cot[e + f*x]^3)/(3*f) - ((a +
b)*Cot[e + f*x]^5)/(5*f) + (b*Tan[e + f*x])/f
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
```

```
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2(a+b+bx^2)}{x^6}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(b+\frac{a+b}{x^6}+\frac{2a+3b}{x^4}+\frac{a+3b}{x^2}\right)\,dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{(a+3b)\cot(e+fx)}{f}-\frac{(2a+3b)\cot^3(e+fx)}{3f}-\frac{(a+b)\cot^5(e+fx)}{5f}+\frac{b\tan(e+fx)}{f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.88

$$
\begin{aligned}
\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = &-\frac{8a\cot(e+fx)}{15f}-\frac{11b\cot(e+fx)}{5f}\\
&-\frac{4a\cot(e+fx)\csc^2(e+fx)}{15f}\\
&-\frac{3b\cot(e+fx)\csc^2(e+fx)}{5f}\\
&-\frac{a\cot(e+fx)\csc^4(e+fx)}{5f}\\
&-\frac{b\cot(e+fx)\csc^4(e+fx)}{5f}+\frac{b\tan(e+fx)}{f}
\end{aligned}
$$

```
[In] Integrate[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (-8*a*Cot[e + f*x])/(15*f) - (11*b*Cot[e + f*x])/(5*f) - (4*a*Cot[e + f*x]*
Csc[e + f*x]^2)/(15*f) - (3*b*Cot[e + f*x]*Csc[e + f*x]^2)/(5*f) - (a*Cot[e
 + f*x]*Csc[e + f*x]^4)/(5*f) - (b*Cot[e + f*x]*Csc[e + f*x]^4)/(5*f) + (b*
Tan[e + f*x])/f
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.31

method	result
parallelrisch	$-\frac{\left((a+6b)\cos(2fx+2e)+\frac{4(-a-6b)\cos(4fx+4e)}{5}+\frac{(a+6b)\cos(6fx+6e)}{5}+2a\right)\sec\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}\csc\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{384f\cos(fx+e)}$
risch	$-\frac{16i\left(10a\,e^{6i(fx+e)}+5a\,e^{4i(fx+e)}+30b\,e^{4i(fx+e)}-4a\,e^{2i(fx+e)}-24b\,e^{2i(fx+e)}+a+6b\right)}{15f\left(e^{2i(fx+e)}-1\right)^{5}\left(e^{2i(fx+e)}+1\right)}$
derivativedivides	$\frac{a\left(-\frac{8}{15}-\frac{\csc(fx+e)^{4}}{5}-\frac{4\csc(fx+e)^{2}}{15}\right)\cot(fx+e)+b\left(-\frac{1}{5\sin(fx+e)^{5}\cos(fx+e)}-\frac{2}{5\sin(fx+e)^{3}\cos(fx+e)}+\frac{8}{5\sin(fx+e)\cos(fx+\dots}\right.}{f}$
default	$\frac{a\left(-\frac{8}{15}-\frac{\csc(fx+e)^{4}}{5}-\frac{4\csc(fx+e)^{2}}{15}\right)\cot(fx+e)+b\left(-\frac{1}{5\sin(fx+e)^{5}\cos(fx+e)}-\frac{2}{5\sin(fx+e)^{3}\cos(fx+e)}+\frac{8}{5\sin(fx+e)\cos(fx+\dots}\right.}{f}$
norman	$\frac{\frac{a+b}{160f}+\frac{(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}}{160f}-\frac{5(a+7b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{8f}+\frac{5(5a+21b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{96f}+\frac{5(5a+21b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{96f}+\frac{(11a+21b)\tan\left(\frac{fx}{2}+\dots\right.}{240f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)}$

```
[In] int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/384*((a+6*b)*cos(2*f*x+2*e)+4/5*(-a-6*b)*cos(4*f*x+4*e)+1/5*(a+6*b)*cos(
6*f*x+6*e)+2*a)*sec(1/2*f*x+1/2*e)^5*csc(1/2*f*x+1/2*e)^5/f/cos(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.34

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=-\frac{8\,(a+6\,b)\cos\left(fx+e\right)^{6}-20\,(a+6\,b)\cos\left(fx+e\right)^{4}+15\,(a+6\,b)\cos\left(fx+e\right)^{2}-15\,b}{15\left(f\cos\left(fx+e\right)^{5}-2\,f\cos\left(fx+e\right)^{3}+f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a + 6*b)*cos(f*x + e)^6 - 20*(a + 6*b)*cos(f*x + e)^4 + 15*(a + 6
*b)*cos(f*x + e)^2 - 15*b)/((f*cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*cos(
f*x + e))*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\text{Timed out}$$

```
[In] integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.94

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{15\,b\tan\left(fx+e\right)-\frac{15\,(a+3\,b)\tan(fx+e)^4+5\,(2\,a+3\,b)\tan(fx+e)^2+3\,a+3\,b}{\tan(fx+e)^5}}{15\,f}$$

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] 1/15*(15*b*tan(f*x + e) - (15*(a + 3*b)*tan(f*x + e)^4 + 5*(2*a + 3*b)*tan(
f*x + e)^2 + 3*a + 3*b)/tan(f*x + e)^5)/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{15\,b\tan\left(fx+e\right)-\frac{15\,a\tan(fx+e)^4+45\,b\tan(fx+e)^4+10\,a\tan(fx+e)^2+15\,b\tan(fx+e)^2+3\,a+3\,b}{\tan(fx+e)^5}}{15\,f}$$

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] 1/15*(15*b*tan(f*x + e) - (15*a*tan(f*x + e)^4 + 45*b*tan(f*x + e)^4 + 10*a
*tan(f*x + e)^2 + 15*b*tan(f*x + e)^2 + 3*a + 3*b)/tan(f*x + e)^5)/f
```

Mupad [B] (verification not implemented)

Time = 18.93 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \csc^6(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx = \frac{b\tan(e+f\,x)}{f} - \frac{(a+3\,b)\,\tan(e+f\,x)^4 + \left(\frac{2\,a}{3}+b\right)\,\tan(e+f\,x)^2 + \frac{a}{5} + \frac{b}{5}}{f\tan\left(e+f\,x\right)^5}$$

```
[In] int((a + b/cos(e + f*x)^2)/sin(e + f*x)^6,x)
```

```
[Out] (b*tan(e + f*x))/f - (a/5 + b/5 + tan(e + f*x)^2*((2*a)/3 + b) + tan(e + f*
x)^4*(a + 3*b))/(f*tan(e + f*x)^5)
```

3.15 $\int (a+b\sec^2(e+fx))^2 \sin^5(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 97

$$\int (a+b\sec^2(e+fx))^2\sin^5(e+fx)\,dx = -\frac{(a^2-4ab+b^2)\cos(e+fx)}{f} + \frac{2a(a-b)\cos^3(e+fx)}{3f} - \frac{a^2\cos^5(e+fx)}{5f} + \frac{2(a-b)b\sec(e+fx)}{f} + \frac{b^2\sec^3(e+fx)}{3f}$$

```
[Out] -(a^2-4*a*b+b^2)*cos(f*x+e)/f+2/3*a*(a-b)*cos(f*x+e)^3/f-1/5*a^2*cos(f*x+e)
^5/f+2*(a-b)*b*sec(f*x+e)/f+1/3*b^2*sec(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4218, 459}

$$\int (a+b\sec^2(e+fx))^2\sin^5(e+fx)\,dx = -\frac{(a^2-4ab+b^2)\cos(e+fx)}{f} - \frac{a^2\cos^5(e+fx)}{5f} + \frac{2a(a-b)\cos^3(e+fx)}{3f} + \frac{2b(a-b)\sec(e+fx)}{f} + \frac{b^2\sec^3(e+fx)}{3f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^5,x]
```

```
[Out] -(((a^2 - 4*a*b + b^2)*Cos[e + f*x])/f) + (2*a*(a - b)*Cos[e + f*x]^3)/(3*f
) - (a^2*Cos[e + f*x]^5)/(5*f) + (2*(a - b)*b*Sec[e + f*x])/f + (b^2*Sec[e
+ f*x]^3)/(3*f)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2(b+ax^2)^2}{x^4}\,dx, x, \cos(e+fx)\right)}{f} \\
&= -\frac{\text{Subst}\left(\int \left(a^2\left(1+\frac{b(-4a+b)}{a^2}\right)+\frac{b^2}{x^4}+\frac{2(a-b)b}{x^2}-2a(a-b)x^2+a^2x^4\right)\,dx, x, \cos(e+fx)\right)}{f} \\
&= -\frac{(a^2-4ab+b^2)\cos(e+fx)}{f}+\frac{2a(a-b)\cos^3(e+fx)}{3f} \\
&\quad -\frac{a^2\cos^5(e+fx)}{5f}+\frac{2(a-b)b\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.33 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.22

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^5(e+fx)\,dx =$$
$$-\frac{(425a^2-4400ab+2000b^2+24(22a^2-215ab+120b^2)\cos(2(e+fx))+12(7a^2-60ab+20b^2)\cos(4(e}{1920f}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^5,x]
```

```
[Out] -1/1920*((425*a^2 - 4400*a*b + 2000*b^2 + 24*(22*a^2 - 215*a*b + 120*b^2)*C
os[2*(e + f*x)] + 12*(7*a^2 - 60*a*b + 20*b^2)*Cos[4*(e + f*x)] - 16*a^2*Co
s[6*(e + f*x)] + 40*a*b*Cos[6*(e + f*x)] + 3*a^2*Cos[8*(e + f*x)])*Sec[e +
f*x]^3)/f
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{-\frac{a^2\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)}{5}+2ab\left(\frac{\sin(fx+e)^6}{\cos(fx+e)}+\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)\right)+b^2\left(\frac{\sin(f}{3\cos(}\right.}{f}$
default	$\frac{-\frac{a^2\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)}{5}+2ab\left(\frac{\sin(fx+e)^6}{\cos(fx+e)}+\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)\right)+b^2\left(\frac{\sin(f}{3\cos(}\right.}{f}$
parts	$-\frac{a^2\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)}{5f}+\frac{b^2\left(\frac{\sin(fx+e)^6}{3\cos(fx+e)^3}-\frac{\sin(fx+e)^6}{\cos(fx+e)}-\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(\right.}{f}$
parallelrisch	$\frac{(-256a^2+2560ab-1280b^2)\cos(3fx+3e)+(-528a^2+5160ab-2880b^2)\cos(2fx+2e)+(-84a^2+720ab-240b^2)\cos(4fx+4e}{480f(\cos(3fx+3e)+}$
norman	$\frac{\frac{16a^2-160ab+80b^2}{15f}+\frac{16\left(5a^2-2ab-7b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{3f}+\frac{2\left(16a^2-160ab+80b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{15f}-\frac{2\left(16a^2-160ab+80b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{15f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^5}$
risch	$-\frac{a^2 e^{5i(fx+e)}}{160f}+\frac{5\,e^{3i(fx+e)}a^2}{96f}-\frac{e^{3i(fx+e)}ab}{12f}-\frac{5\,e^{i(fx+e)}a^2}{16f}+\frac{7\,e^{i(fx+e)}ab}{4f}-\frac{e^{i(fx+e)}b^2}{2f}-\frac{5\,e^{-i(fx+e)}a^2}{16f}+$

```
[In] int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/5*a^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+2*a*b*(sin(f*x
+e)^6/cos(f*x+e)+(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e))+b^2*(1/3*s
in(f*x+e)^6/cos(f*x+e)^3-sin(f*x+e)^6/cos(f*x+e)-(8/3+sin(f*x+e)^4+4/3*sin(
f*x+e)^2)*cos(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^5(e+fx)\,dx =$$
$$-\frac{3\,a^2\cos\left(fx+e\right)^8-10\left(a^2-ab\right)\cos\left(fx+e\right)^6+15\left(a^2-4\,ab+b^2\right)\cos\left(fx+e\right)^4-30\left(ab-b^2\right)\cos\left(fx+\right.}{15\,f\cos\left(fx+e\right)^3}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] -1/15*(3*a^2*cos(f*x + e)^8 - 10*(a^2 - a*b)*cos(f*x + e)^6 + 15*(a^2 - 4*a
*b + b^2)*cos(f*x + e)^4 - 30*(a*b - b^2)*cos(f*x + e)^2 - 5*b^2)/(f*cos(f*
x + e)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a + b\sec^2(e + fx)\right)^2 \sin^5(e + fx)\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**5,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

$$\int \left(a + b\sec^2(e + fx)\right)^2 \sin^5(e + fx)\,dx = -\frac{3\,a^2\cos\left(fx+e\right)^5 - 10\left(a^2 - ab\right)\cos\left(fx+e\right)^3 + 15\left(a^2 - 4\,ab + b^2\right)\cos\left(fx+e\right) - \frac{5\left(6\left(ab-b^2\right)\cos(fx+e)^2+\cdots\right.}{\cos(fx+e)^3}}{15\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^5,x, algorithm="maxima")
[Out] -1/15*(3*a^2*cos(f*x + e)^5 - 10*(a^2 - a*b)*cos(f*x + e)^3 + 15*(a^2 - 4*a
*b + b^2)*cos(f*x + e) - 5*(6*(a*b - b^2)*cos(f*x + e)^2 + b^2)/cos(f*x + e
)^3)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. 2(91) = 182.

Time = 0.41 (sec) , antiderivative size = 414, normalized size of antiderivative = 4.27

$$\int \left(a + b\sec^2(e + fx)\right)^2 \sin^5(e + fx)\,dx = \frac{2\left(\frac{5\left(6\,ab-5\,b^2+\frac{12\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{12\,b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{6\,ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{3\,b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right)^3}+\frac{8\,a^2-50\,ab+15\,b^2-\frac{40\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}\cdots}{\cdots}\right.}{\cdots}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^5,x, algorithm="giac")
[Out] 2/15*(5*(6*a*b - 5*b^2 + 12*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 12*
b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 6*a*b*(cos(f*x + e) - 1)^2/(cos
(f*x + e) + 1)^2 - 3*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((cos(f
*x + e) - 1)/(cos(f*x + e) + 1) + 1)^3 + (8*a^2 - 50*a*b + 15*b^2 - 40*a^2*
```

```
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 220*a*b*(cos(f*x + e) - 1)/(cos(f*x
 + e) + 1) - 60*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 80*a^2*(cos(f*x
 + e) - 1)^2/(cos(f*x + e) + 1)^2 - 320*a*b*(cos(f*x + e) - 1)^2/(cos(f*x +
 e) + 1)^2 + 90*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 180*a*b*(co
s(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 60*b^2*(cos(f*x + e) - 1)^3/(cos(f
*x + e) + 1)^3 - 30*a*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 15*b^2*
(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/((cos(f*x + e) - 1)/(cos(f*x + e
) + 1) - 1)^5)/f
```

Mupad [B] (verification not implemented)

Time = 18.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.90

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^5(e+fx)\,dx = \frac{\frac{\frac{b^2}{3}+\cos(e+f\,x)^2\,\left(2\,a\,b-2\,b^2\right)}{\cos(e+f\,x)^3}-\cos\left(e+f\,x\right)\,\left(a^2-4\,a\,b+b^2\right)-\frac{a^2\cos(e+f\,x)^5}{5}+\frac{2\,a\cos(e+f\,x)^3\,(a-b)}{3}}{f}$$

```
[In] int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] ((b^2/3 + cos(e + f*x)^2*(2*a*b - 2*b^2))/cos(e + f*x)^3 - cos(e + f*x)*(a^
2 - 4*a*b + b^2) - (a^2*cos(e + f*x)^5)/5 + (2*a*cos(e + f*x)^3*(a - b))/3)
/f
```

3.16 $\int (a+b\sec^2(e+fx))^2 \sin^3(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int (a+b\sec^2(e+fx))^2 \sin^3(e+fx)\,dx = -\frac{a(a-2b)\cos(e+fx)}{f}+\frac{a^2\cos^3(e+fx)}{3f}+\frac{(2a-b)b\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[Out] -a*(a-2*b)*cos(f*x+e)/f+1/3*a^2*cos(f*x+e)^3/f+(2*a-b)*b*sec(f*x+e)/f+1/3*b
^2*sec(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4218, 459}

$$\int (a+b\sec^2(e+fx))^2 \sin^3(e+fx)\,dx = \frac{a^2\cos^3(e+fx)}{3f}-\frac{a(a-2b)\cos(e+fx)}{f}+\frac{b(2a-b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^3,x]
```

```
[Out] -((a*(a - 2*b)*Cos[e + f*x])/f) + (a^2*Cos[e + f*x]^3)/(3*f) + ((2*a - b)*b
*Sec[e + f*x])/f + (b^2*Sec[e + f*x]^3)/(3*f)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
```

```
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)(b+ax^2)^2}{x^4}\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \left(a(a-2b)+\frac{b^2}{x^4}+\frac{(2a-b)b}{x^2}-a^2x^2\right)\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{a(a-2b)\cos(e+fx)}{f}+\frac{a^2\cos^3(e+fx)}{3f}+\frac{(2a-b)b\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.88 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.15

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^3(e+fx)\,dx = \frac{(-26a^2+168ab-16b^2-3(11a^2-64ab+16b^2)\cos(2(e+fx))-6a(a-4b)\cos(4(e+fx))+a^2\cos(6(e+ }{96f}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^3,x]
```

```
[Out] ((-26*a^2 + 168*a*b - 16*b^2 - 3*(11*a^2 - 64*a*b + 16*b^2)*Cos[2*(e + f*x)
] - 6*a*(a - 4*b)*Cos[4*(e + f*x)] + a^2*Cos[6*(e + f*x)])*Sec[e + f*x]^3)/
(96*f)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\frac{b^2\sec(fx+e)^3}{3}+2ab\sec(fx+e)-b^2\sec(fx+e)+\frac{a^2}{3\sec(fx+e)^3}-\frac{a(a-2b)}{\sec(fx+e)}}{f}$
default	$\frac{\frac{b^2\sec(fx+e)^3}{3}+2ab\sec(fx+e)-b^2\sec(fx+e)+\frac{a^2}{3\sec(fx+e)^3}-\frac{a(a-2b)}{\sec(fx+e)}}{f}$
parts	$-\frac{a^2\left(2+\sin(fx+e)^2\right)\cos(fx+e)}{3f}+\frac{b^2\left(\frac{\sec(fx+e)^3}{3}-\sec(fx+e)\right)}{f}+\frac{2ab\left(\frac{\sin(fx+e)^4}{\cos(fx+e)}+\left(2+\sin(fx+e)^2\right)\cos(fx+e)\right)}{f}$
norman	$\frac{\frac{4a^2-24ab+4b^2}{3f}+\frac{32\left(a^2-b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{3f}-\frac{\left(4a^2+8ab+4b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{f}-\frac{\left(8a^2-16ab+8b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^3}$
parallelrisch	$\frac{(-16a^2+96ab-16b^2)\cos(3fx+3e)+(-33a^2+192ab-48b^2)\cos(2fx+2e)-6a(a-4b)\cos(4fx+4e)+a^2\cos(6fx+6e)+(-}{24f(\cos(3fx+3e)+3\cos(fx+e))}$
risch	$\frac{e^{3i(fx+e)}a^2}{24f}-\frac{3\,e^{i(fx+e)}a^2}{8f}+\frac{e^{i(fx+e)}ab}{f}-\frac{3\,e^{-i(fx+e)}a^2}{8f}+\frac{e^{-i(fx+e)}ab}{f}+\frac{e^{-3i(fx+e)}a^2}{24f}-\frac{2\,e^{i(fx+e)}b(-6a}{}$

```
[In] int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/3*b^2*sec(f*x+e)^3+2*a*b*sec(f*x+e)-b^2*sec(f*x+e)+1/3*a^2/sec(f*x+e
)^3-a*(a-2*b)/sec(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^3(e+fx)\,dx = \frac{a^2\cos\left(fx+e\right)^6-3\left(a^2-2\,ab\right)\cos\left(fx+e\right)^4+3\left(2\,ab-b^2\right)\cos\left(fx+e\right)^2+b^2}{3\,f\cos\left(fx+e\right)^3}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*cos(f*x + e)^6 - 3*(a^2 - 2*a*b)*cos(f*x + e)^4 + 3*(2*a*b - b^2)*
cos(f*x + e)^2 + b^2)/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^3(e+fx)\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^2\sin^3\left(e+fx\right)dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*sin(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^3(e+fx)\,dx=\frac{a^2\cos\left(fx+e\right)^3-3\left(a^2-2\,ab\right)\cos\left(fx+e\right)+\frac{3\left(2\,ab-b^2\right)\cos(fx+e)^2+b^2}{\cos(fx+e)^3}}{3\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(a^2*cos(f*x + e)^3 - 3*(a^2 - 2*a*b)*cos(f*x + e) + (3*(2*a*b - b^2)*c
os(f*x + e)^2 + b^2)/cos(f*x + e)^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^3(e+fx)\,dx=\frac{6\,ab\cos\left(fx+e\right)^2-3\,b^2\cos\left(fx+e\right)^2+b^2}{3\,f\cos\left(fx+e\right)^3}+\frac{a^2f^{11}\cos\left(fx+e\right)^3-3\,a^2f^{11}\cos\left(fx+e\right)+6\,abf^{11}\cos\left(fx+e\right)}{3\,f^{12}}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^3,x, algorithm="giac")
```

```
[Out] 1/3*(6*a*b*cos(f*x + e)^2 - 3*b^2*cos(f*x + e)^2 + b^2)/(f*cos(f*x + e)^3)
+ 1/3*(a^2*f^11*cos(f*x + e)^3 - 3*a^2*f^11*cos(f*x + e) + 6*a*b*f^11*cos(f
*x + e))/f^12
```

Mupad [B] (verification not implemented)

Time = 18.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^3(e+fx)\,dx = \frac{\frac{\frac{b^2}{3}+\cos(e+f\,x)^2\,\left(2\,a\,b-b^2\right)}{\cos(e+f\,x)^3}+\frac{a^2\cos(e+f\,x)^3}{3}-a\,\cos\left(e+f\,x\right)\,\left(a-2\,b\right)}{f}$$

```
[In] int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^2,x)
[Out] ((b^2/3 + cos(e + f*x)^2*(2*a*b - b^2))/cos(e + f*x)^3 + (a^2*cos(e + f*x)^
3)/3 - a*cos(e + f*x)*(a - 2*b))/f
```

3.17 $\int (a+b\sec^2(e+fx))^2 \sin(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 46

$$\int (a+b\sec^2(e+fx))^2 \sin(e+fx)\,dx = -\frac{a^2\cos(e+fx)}{f}+\frac{2ab\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[Out] -a^2*cos(f*x+e)/f+2*a*b*sec(f*x+e)/f+1/3*b^2*sec(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4218, 276}

$$\int (a+b\sec^2(e+fx))^2 \sin(e+fx)\,dx = -\frac{a^2\cos(e+fx)}{f}+\frac{2ab\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x],x]
```

```
[Out] -((a^2*Cos[e + f*x])/f) + (2*a*b*Sec[e + f*x])/f + (b^2*Sec[e + f*x]^3)/(3*
f)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
```

```
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(b+ax^2)^2}{x^4}\,dx, x, \cos(e+fx)\right)}{f} \\
&= -\frac{\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^4}+\frac{2ab}{x^2}\right)\,dx, x, \cos(e+fx)\right)}{f} \\
&= -\frac{a^2\cos(e+fx)}{f}+\frac{2ab\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.63

$$
\int \left(a+b\sec^2(e+fx)\right)^2\sin(e+fx)\,dx
= \frac{4(b+a\cos^2(e+fx))^2\left(b^2+6ab\cos^2(e+fx)-3a^2\cos^4(e+fx)\right)\sec^3(e+fx)}{3f(a+2b+a\cos(2(e+fx)))^2}
$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x],x]
[Out] (4*(b + a*Cos[e + f*x]^2)^2*(b^2 + 6*a*b*Cos[e + f*x]^2 - 3*a^2*Cos[e + f*x
]^4)*Sec[e + f*x]^3)/(3*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{b^2\sec(fx+e)^3}{3}+2ab\sec(fx+e)-\frac{a^2}{\sec(fx+e)}}{f}$
default	$\frac{\frac{b^2\sec(fx+e)^3}{3}+2ab\sec(fx+e)-\frac{a^2}{\sec(fx+e)}}{f}$
parts	$-\frac{a^2\cos(fx+e)}{f}+\frac{2ab\sec(fx+e)}{f}+\frac{b^2\sec(fx+e)^3}{3f}$
risch	$-\frac{e^{i(fx+e)}a^2}{2f}-\frac{e^{-i(fx+e)}a^2}{2f}+\frac{4b(3a\,e^{5i(fx+e)}+6a\,e^{3i(fx+e)}+2b\,e^{3i(fx+e)}+3a\,e^{i(fx+e)})}{3f(e^{2i(fx+e)}+1)^3}$
parallelrisch	$\frac{2(-3a^2+6ab+b^2)\cos(3fx+3e)-12a(a-2b)\cos(2fx+2e)-3a^2\cos(4fx+4e)+6(-3a^2+6ab+b^2)\cos(fx+e)-9a^2+24ab+8}{6f(\cos(3fx+3e)+3\cos(fx+e))}$
norman	$\frac{\frac{6a^2-12ab-2b^2}{3f}+\frac{2(3a^2+2ab-b^2)\tan(\frac{fx}{2}+\frac{e}{2})^4}{f}-\frac{2(a^2+2ab+b^2)\tan(\frac{fx}{2}+\frac{e}{2})^6}{f}-\frac{2(9a^2-6ab+b^2)\tan(\frac{fx}{2}+\frac{e}{2})^2}{3f}}{\left(\tan(\frac{fx}{2}+\frac{e}{2})^2-1\right)^3\left(1+\tan(\frac{fx}{2}+\frac{e}{2})^2\right)}$

```
[In] int((a+b*sec(f*x+e)^2)^2*sin(f*x+e),x,method=_RETURNVERBOSE)
[Out] 1/f*(1/3*b^2*sec(f*x+e)^3+2*a*b*sec(f*x+e)-a^2/sec(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin(e+fx)\,dx=-\frac{3\,a^2\cos\left(fx+e\right)^4-6\,ab\cos\left(fx+e\right)^2-b^2}{3\,f\cos\left(fx+e\right)^3}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e),x, algorithm="fricas")
[Out] -1/3*(3*a^2*cos(f*x + e)^4 - 6*a*b*cos(f*x + e)^2 - b^2)/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin(e+fx)\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^2\sin\left(e+fx\right)dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e),x)
[Out] Integral((a + b*sec(e + f*x)**2)**2*sin(e + f*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin(e+fx)\,dx=-\frac{3\,a^2\cos\left(fx+e\right)-\frac{6\,ab}{\cos(fx+e)}-\frac{b^2}{\cos(fx+e)^3}}{3\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e),x, algorithm="maxima")
[Out] -1/3*(3*a^2*cos(f*x + e) - 6*a*b/cos(f*x + e) - b^2/cos(f*x + e)^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin(e+fx)\,dx=-\frac{a^2\cos\left(fx+e\right)}{f}+\frac{6\,ab\cos\left(fx+e\right)^2+b^2}{3\,f\cos\left(fx+e\right)^3}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e),x, algorithm="giac")
[Out] -a^2*cos(f*x + e)/f + 1/3*(6*a*b*cos(f*x + e)^2 + b^2)/(f*cos(f*x + e)^3)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin(e+fx)\,dx=\frac{\frac{b^2}{3}+2\,a\,b\cos\left(e+f\,x\right)^2}{f\cos\left(e+f\,x\right)^3}-\frac{a^2\,\cos\left(e+f\,x\right)}{f}$$

```
[In] int(sin(e + f*x)*(a + b/cos(e + f*x)^2)^2,x)
[Out] (b^2/3 + 2*a*b*cos(e + f*x)^2)/(f*cos(e + f*x)^3) - (a^2*cos(e + f*x))/f
```

3.18 $\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 52

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=-\frac{(a+b)^2\text{arctanh}(\cos(e+fx))}{f}+\frac{b(2a+b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[Out] -(a+b)^2*arctanh(cos(f*x+e))/f+b*(2*a+b)*sec(f*x+e)/f+1/3*b^2*sec(f*x+e)^3/
f
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4218, 472, 213}

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=-\frac{(a+b)^2\text{arctanh}(\cos(e+fx))}{f}+\frac{b(2a+b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[In] Int[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(((a + b)^2*ArcTanh[Cos[e + f*x]])/f) + (b*(2*a + b)*Sec[e + f*x])/f + (b^
2*Sec[e + f*x]^3)/(3*f)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(b+ax^2)^2}{x^4(1-x^2)}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{x^4}+\frac{b(2a+b)}{x^2}-\frac{(a+b)^2}{-1+x^2}\right)\,dx, x, \cos(e+fx)\right)}{f}\\
&= \frac{b(2a+b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}+\frac{(a+b)^2\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{(a+b)^2\text{arctanh}(\cos(e+fx))}{f}+\frac{b(2a+b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 108 vs. $2(52) = 104$.

Time = 0.91 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.08

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{4\left(b+a\cos^2(e+fx)\right)^2\left(-b^2-3b(2a+b)\cos^2(e+fx)+3(a+b)^2\cos^3(e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)\right.\right.}{3f(a+2b+a\cos(2(e+fx)))^2}$$

```
[In] Integrate[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (-4*(b + a*Cos[e + f*x]^2)^2*(-b^2 - 3*b*(2*a + b)*Cos[e + f*x]^2 + 3*(a +
b)^2*Cos[e + f*x]^3*(Log[Cos[(e + f*x)/2]] - Log[Sin[(e + f*x)/2]]))*Sec[e
+ f*x]^3)/(3*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.81

method	result
derivativedivides	$\frac{a^2\ln(\csc(fx+e)-\cot(fx+e))+2ab\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)+b^2\left(\frac{1}{3\cos(fx+e)^3}+\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)\right.}{f}$
default	$\frac{a^2\ln(\csc(fx+e)-\cot(fx+e))+2ab\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)+b^2\left(\frac{1}{3\cos(fx+e)^3}+\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)\right.}{f}$
norman	$\frac{\frac{(8ab+4b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{f}-\frac{12ab+8b^2}{3f}-\frac{(4ab+4b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}+\frac{(a^2+2ab+b^2)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}$
parallelrisch	$\frac{3\left(\frac{\cos(3fx+3e)}{3}+\cos(fx+e)\right)(a+b)^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+6b\left(\frac{\left(a+\frac{2b}{3}\right)\cos(3fx+3e)}{3}+\frac{(2a+b)\cos(2fx+2e)}{3}+\left(a+\frac{2b}{3}\right)\cos(fx+e)\right.}{f(\cos(3fx+3e)+3\cos(fx+e))}$
risch	$\frac{2b\,e^{i(fx+e)}\left(6a\,e^{4i(fx+e)}+3b\,e^{4i(fx+e)}+12a\,e^{2i(fx+e)}+10b\,e^{2i(fx+e)}+6a+3b\right)}{3f\left(e^{2i(fx+e)}+1\right)^3}+\frac{\ln(e^{i(fx+e)}-1)a^2}{f}+\frac{2\ln(e^{i(fx+e)}-1}{f}$

```
[In] int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*ln(csc(f*x+e)-cot(f*x+e))+2*a*b*(1/cos(f*x+e)+ln(csc(f*x+e)-cot(f*
x+e)))+b^2*(1/3/cos(f*x+e)^3+1/cos(f*x+e)+ln(csc(f*x+e)-cot(f*x+e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(50) = 100.

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.94

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx =$$
$$-\frac{3\,(a^2+2\,ab+b^2)\cos\left(fx+e\right)^3\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)-3\,(a^2+2\,ab+b^2)\cos\left(fx+e\right)^3\log\left(-\frac{1}{2}\,\cos\left(f\right.\right.}{6\,f\cos\left(fx+e\right)^3}$$

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*(a^2 + 2*a*b + b^2)*cos(f*x + e)^3*log(1/2*cos(f*x + e) + 1/2) - 3*
(a^2 + 2*a*b + b^2)*cos(f*x + e)^3*log(-1/2*cos(f*x + e) + 1/2) - 6*(2*a*b
+ b^2)*cos(f*x + e)^2 - 2*b^2)/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^2\csc\left(e+fx\right)dx$$

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2)**2,x)
[Out] Integral((a + b*sec(e + f*x)**2)**2*csc(e + f*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.58

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=$$

$$-\frac{3\left(a^2+2\,ab+b^2\right)\log\left(\cos\left(fx+e\right)+1\right)-3\left(a^2+2\,ab+b^2\right)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\left(2\,ab+b^2\right)\cos(fx+e)^2+b^2\right)}{\cos(fx+e)^3}}{6\,f}$$

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
[Out] -1/6*(3*(a^2 + 2*a*b + b^2)*log(cos(f*x + e) + 1) - 3*(a^2 + 2*a*b + b^2)*l
og(cos(f*x + e) - 1) - 2*(3*(2*a*b + b^2)*cos(f*x + e)^2 + b^2)/cos(f*x + e
)^3)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(50) = 100$.

Time = 0.32 (sec) , antiderivative size = 172, normalized size of antiderivative = 3.31

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$

$$=\frac{3\left(a^2+2\,ab+b^2\right)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)+\frac{8\left(3\,ab+2\,b^2+\frac{6\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{3\,b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{3\,ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{3\,b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right)^3}}{6\,f}$$

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
[Out] 1/6*(3*(a^2 + 2*a*b + b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)
) + 8*(3*a*b + 2*b^2 + 6*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 3*b^2*
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 3*a*b*(cos(f*x + e) - 1)^2/(cos(f*x
 + e) + 1)^2 + 3*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((cos(f*x +
 e) - 1)/(cos(f*x + e) + 1) + 1)^3)/f
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\cos\left(e+f\,x\right)^2\,\left(b^2+2\,a\,b\right)+\frac{b^2}{3}}{f\,\cos\left(e+f\,x\right)^3} - \frac{\mathrm{atanh}(\cos\left(e+f\,x\right))\,\left(a+b\right)^2}{f}$$

```
[In] int((a + b/cos(e + f*x)^2)^2/sin(e + f*x),x)
```

```
[Out] (cos(e + f*x)^2*(2*a*b + b^2) + b^2/3)/(f*cos(e + f*x)^3) - (atanh(cos(e +
f*x))*(a + b)^2)/f
```

3.19 $\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 104

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=-\frac{(a+b)(a+5b)\text{arctanh}(\cos(e+fx))}{2f}-\frac{(3a^2+6ab+5b^2)\cot(e+fx)\csc(e+fx)}{6f}+\frac{b(6a+5b)\sec(e+fx)}{3f}+\frac{b^2\csc^2(e+fx)\sec^3(e+fx)}{3f}$$

```
[Out] -1/2*(a+b)*(a+5*b)*arctanh(cos(f*x+e))/f-1/6*(3*a^2+6*a*b+5*b^2)*cot(f*x+e)
*csc(f*x+e)/f+1/3*b*(6*a+5*b)*sec(f*x+e)/f+1/3*b^2*csc(f*x+e)^2*sec(f*x+e)^
3/f
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4218, 473, 467, 464, 212}

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=-\frac{(3a^2+6ab+5b^2)\cot(e+fx)\csc(e+fx)}{6f}-\frac{(a+b)(a+5b)\text{arctanh}(\cos(e+fx))}{2f}+\frac{b(6a+5b)\sec(e+fx)}{3f}+\frac{b^2\csc^2(e+fx)\sec^3(e+fx)}{3f}$$

```
[In] Int[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/2*((a + b)*(a + 5*b)*ArcTanh[Cos[e + f*x]])/f - ((3*a^2 + 6*a*b + 5*b^2)
*Cot[e + f*x]*Csc[e + f*x])/(6*f) + (b*(6*a + 5*b)*Sec[e + f*x])/(3*f) + (b
^2*Csc[e + f*x]^2*Sec[e + f*x]^3)/(3*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{(b+ax^2)^2}{x^4(1-x^2)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{b^2\csc^2(e+fx)\sec^3(e+fx)}{3f} - \frac{\text{Subst}\left(\int \frac{b(6a+5b)+3a^2x^2}{x^2(1-x^2)^2}\,dx, x, \cos(e+fx)\right)}{3f}$$

$$= -\frac{(3a^2+6ab+5b^2)\cot(e+fx)\csc(e+fx)}{6f} + \frac{b^2\csc^2(e+fx)\sec^3(e+fx)}{3f} + \frac{\text{Subst}\left(\int \frac{-2b(6a+5b)-(3a^2+6ab+5b^2)x^2}{x^2(1-x^2)}\,dx, x, \cos(e+fx)\right)}{6f}$$

$$= -\frac{(3a^2+6ab+5b^2)\cot(e+fx)\csc(e+fx)}{6f} + \frac{b(6a+5b)\sec(e+fx)}{3f} + \frac{b^2\csc^2(e+fx)\sec^3(e+fx)}{3f} - \frac{((a+b)(a+5b))\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(e+fx)\right)}{2f}$$

$$= -\frac{(a+b)(a+5b)\text{arctanh}(\cos(e+fx))}{2f} - \frac{(3a^2+6ab+5b^2)\cot(e+fx)\csc(e+fx)}{6f} + \frac{b(6a+5b)\sec(e+fx)}{3f} + \frac{b^2\csc^2(e+fx)\sec^3(e+fx)}{3f}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1021 vs. $2(104) = 208$.

Time = 8.09 (sec) , antiderivative size = 1021, normalized size of antiderivative = 9.82

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{(-a^2-2ab-b^2)\cos^4(e+fx)\csc^2\left(\frac{e}{2}+\frac{fx}{2}\right)\left(a+b\sec^2(e+fx)\right)^2}{2f(a+2b+a\cos(2e+2fx))^2}$$
$$-\frac{2(a^2+6ab+5b^2)\cos^4(e+fx)\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)\left(a+b\sec^2(e+fx)\right)^2}{f(a+2b+a\cos(2e+2fx))^2}$$
$$+\frac{2(a^2+6ab+5b^2)\cos^4(e+fx)\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)\left(a+b\sec^2(e+fx)\right)^2}{f(a+2b+a\cos(2e+2fx))^2}$$
$$+\frac{2b(12a+13b)\cos^4(e+fx)\sec(e)\left(a+b\sec^2(e+fx)\right)^2}{3f(a+2b+a\cos(2e+2fx))^2}$$
$$+\frac{(a^2+2ab+b^2)\cos^4(e+fx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\left(a+b\sec^2(e+fx)\right)^2}{2f(a+2b+a\cos(2e+2fx))^2}$$
$$+\frac{2b^2\cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\sin\left(\frac{fx}{2}\right)}{3f(a+2b+a\cos(2e+2fx))^2\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)-\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)^3}$$
$$+\frac{\cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\left(b^2\cos\left(\frac{e}{2}\right)+b^2\sin\left(\frac{e}{2}\right)\right)}{3f(a+2b+a\cos(2e+2fx))^2\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)-\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)^2}$$
$$+\frac{2\cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\left(12ab\sin\left(\frac{fx}{2}\right)+13b^2\sin\left(\frac{fx}{2}\right)\right)}{3f(a+2b+a\cos(2e+2fx))^2\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)-\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}$$
$$-\frac{2b^2\cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\sin\left(\frac{fx}{2}\right)}{3f(a+2b+a\cos(2e+2fx))^2\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)^3}$$
$$+\frac{\cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\left(b^2\cos\left(\frac{e}{2}\right)-b^2\sin\left(\frac{e}{2}\right)\right)}{3f(a+2b+a\cos(2e+2fx))^2\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)^2}$$
$$-\frac{2\cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\left(12ab\sin\left(\frac{fx}{2}\right)+13b^2\sin\left(\frac{fx}{2}\right)\right)}{3f(a+2b+a\cos(2e+2fx))^2\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}$$

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((-a^2 - 2*a*b - b^2)*Cos[e + f*x]^4*Csc[e/2 + (f*x)/2]^2*(a + b*Sec[e + f*
x]^2)^2)/(2*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2) - (2*(a^2 + 6*a*b + 5*b^2)*
Cos[e + f*x]^4*Log[Cos[e/2 + (f*x)/2]]*(a + b*Sec[e + f*x]^2)^2)/(f*(a + 2*
b + a*Cos[2*e + 2*f*x])^2) + (2*(a^2 + 6*a*b + 5*b^2)*Cos[e + f*x]^4*Log[Si
n[e/2 + (f*x)/2]]*(a + b*Sec[e + f*x]^2)^2)/(f*(a + 2*b + a*Cos[2*e + 2*f*x
])^2) + (2*b*(12*a + 13*b)*Cos[e + f*x]^4*Sec[e]*(a + b*Sec[e + f*x]^2)^2)/
(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2) + ((a^2 + 2*a*b + b^2)*Cos[e + f*x]^
4*Sec[e/2 + (f*x)/2]^2*(a + b*Sec[e + f*x]^2)^2)/(2*f*(a + 2*b + a*Cos[2*e
+ 2*f*x])^2) + (2*b^2*Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*Sin[(f*x)/2])
/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2*(Cos[e/2] - Sin[e/2])*(Cos[e/2 + (f*
```

```
x)/2] - Sin[e/2 + (f*x)/2])^3) + (Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*(
b^2*Cos[e/2] + b^2*Sin[e/2]))/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2*(Cos[e/
2] - Sin[e/2])*(Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])^2) + (2*Cos[e + f*
x]^4*(a + b*Sec[e + f*x]^2)^2*(12*a*b*Sin[(f*x)/2] + 13*b^2*Sin[(f*x)/2]))/
(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2*(Cos[e/2] - Sin[e/2])*(Cos[e/2 + (f*x
)/2] - Sin[e/2 + (f*x)/2])) - (2*b^2*Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^
2*Sin[(f*x)/2])/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2*(Cos[e/2] + Sin[e/2])
*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^3) + (Cos[e + f*x]^4*(a + b*Sec[
e + f*x]^2)^2*(b^2*Cos[e/2] - b^2*Sin[e/2]))/(3*f*(a + 2*b + a*Cos[2*e + 2*
f*x])^2*(Cos[e/2] + Sin[e/2])*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2)
- (2*Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*(12*a*b*Sin[(f*x)/2] + 13*b^2*
Sin[(f*x)/2]))/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2*(Cos[e/2] + Sin[e/2])*
(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2]))
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.57

method	result
derivativedivides	$\frac{a^2\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)+2ab\left(-\frac{1}{2\sin(fx+e)^2\cos(fx+e)}+\frac{3}{2\cos(fx+e)}+\frac{3\ln(\csc(fx+e)-\cot(fx+}{2}\right.}{f}$
default	$\frac{a^2\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)+2ab\left(-\frac{1}{2\sin(fx+e)^2\cos(fx+e)}+\frac{3}{2\cos(fx+e)}+\frac{3\ln(\csc(fx+e)-\cot(fx+}{2}\right.}{f}$
norman	$\frac{\frac{a^2+2ab+b^2}{8f}+\frac{\left(a^2+2ab+b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{8f}-\frac{\left(7a^2+46ab+55b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{8f}-\frac{\left(9a^2+66ab+65b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{12f}+\frac{\left(11a^2+86ab-\right.}{}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}$
parallelrisch	$\frac{\frac{3\left(\frac{\cos(3fx+3e)}{3}+\cos(fx+e)\right)(a+5b)(a+b)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{3\csc\left(\frac{fx}{2}+\frac{e}{2}\right)^2\left(\left(a^2+\frac{22}{3}ab+\frac{65}{9}b^2\right)\cos(3fx+3e)+\frac{32\left(a^2+2ab+\frac{5}{3}b^2\right)}{3}\right.}{}}{f(\cos(3f}$
risch	$\frac{e^{i(fx+e)}\left(3a^2e^{8i(fx+e)}+18ab\,e^{8i(fx+e)}+15b^2e^{8i(fx+e)}+12a^2e^{6i(fx+e)}+24ab\,e^{6i(fx+e)}+20b^2e^{6i(fx+e)}+18a^2e^{4i(fx+e)}\right.}{3f\left(e^{2i(fx+e)}+1\right)^3\left(e^{2i(fx+\right.}}$

```
[In] int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(-1/2*csc(f*x+e)*cot(f*x+e)+1/2*ln(csc(f*x+e)-cot(f*x+e)))+2*a*b*(
-1/2/sin(f*x+e)^2/cos(f*x+e)+3/2/cos(f*x+e)+3/2*ln(csc(f*x+e)-cot(f*x+e)))+
b^2*(1/3/sin(f*x+e)^2/cos(f*x+e)^3-5/6/sin(f*x+e)^2/cos(f*x+e)+5/2/cos(f*x+
e)+5/2*ln(csc(f*x+e)-cot(f*x+e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(96) = 192$.

Time = 0.26 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.86

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{6\left(a^2+6\,ab+5\,b^2\right)\cos\left(fx+e\right)^4-4\left(6\,ab+5\,b^2\right)\cos\left(fx+e\right)^2-4\,b^2-3\left(\left(a^2+6\,ab+5\,b^2\right)\cos\left(fx+e\right)^5 \cdots\right.}{\cdots}$$

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*(a^2 + 6*a*b + 5*b^2)*cos(f*x + e)^4 - 4*(6*a*b + 5*b^2)*cos(f*x +
e)^2 - 4*b^2 - 3*((a^2 + 6*a*b + 5*b^2)*cos(f*x + e)^5 - (a^2 + 6*a*b + 5*b
^2)*cos(f*x + e)^3)*log(1/2*cos(f*x + e) + 1/2) + 3*((a^2 + 6*a*b + 5*b^2)*
cos(f*x + e)^5 - (a^2 + 6*a*b + 5*b^2)*cos(f*x + e)^3)*log(-1/2*cos(f*x + e
) + 1/2))/(f*cos(f*x + e)^5 - f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^2\csc^3\left(e+fx\right)dx$$

```
[In] integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*csc(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.21

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx =$$
$$-\frac{3\left(a^2+6\,ab+5\,b^2\right)\log\left(\cos\left(fx+e\right)+1\right)-3\left(a^2+6\,ab+5\,b^2\right)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\left(a^2+6\,ab+5\,b^2\right)\cos\cdots\right.}{\cos\cdots}}{12\,f}$$

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/12*(3*(a^2 + 6*a*b + 5*b^2)*log(cos(f*x + e) + 1) - 3*(a^2 + 6*a*b + 5*b
^2)*log(cos(f*x + e) - 1) - 2*(3*(a^2 + 6*a*b + 5*b^2)*cos(f*x + e)^4 - 2*(
6*a*b + 5*b^2)*cos(f*x + e)^2 - 2*b^2)/(cos(f*x + e)^5 - cos(f*x + e)^3))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(96) = 192.

Time = 0.35 (sec) , antiderivative size = 341, normalized size of antiderivative = 3.28

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx =$$

$$-\frac{\frac{3a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{6ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{3b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-6\left(a^2+6ab+5b^2\right)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)-\frac{3\left(a^2+2ab\cdots\right.}{}}{}$$

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/24*(3*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 6*a*b*(cos(f*x + e) -
1)/(cos(f*x + e) + 1) + 3*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 6*(a^
2 + 6*a*b + 5*b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - 3*(a
^2 + 2*a*b + b^2 - 2*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 12*a*b*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) - 10*b^2*(cos(f*x + e) - 1)/(cos(f*x + e
) + 1))*(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - 16*(6*a*b + 7*b^2 + 12*a*b*
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 12*b^2*(cos(f*x + e) - 1)/(cos(f*x
+ e) + 1) + 6*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 9*b^2*(cos(f*
x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((cos(f*x + e) - 1)/(cos(f*x + e) + 1)
+ 1)^3)/f
```

Mupad [B] (verification not implemented)

Time = 18.51 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.92

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\frac{b^2}{3}+\cos(e+f\,x)^2\left(\frac{5\,b^2}{3}+2\,a\,b\right)-\cos(e+f\,x)^4\left(\frac{a^2}{2}+3\,a\,b+\frac{5\,b^2}{2}\right)}{f\,\left(\cos(e+f\,x)^3-\cos(e+f\,x)^5\right)} - \frac{\mathrm{atanh}(\cos(e+f\,x))\,(a+b)\,(a+5\,b)}{2\,f}$$

```
[In] int((a + b/cos(e + f*x)^2)^2/sin(e + f*x)^3,x)
```

```
[Out] (b^2/3 + cos(e + f*x)^2*(2*a*b + (5*b^2)/3) - cos(e + f*x)^4*(3*a*b + a^2/2
 + (5*b^2)/2))/(f*(cos(e + f*x)^3 - cos(e + f*x)^5)) - (atanh(cos(e + f*x))
*(a + b)*(a + 5*b))/(2*f)
```

3.20 $\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 141

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{(3a^2+30ab+35b^2)\operatorname{arctanh}(\cos(e+fx))}{8f} - \frac{(3a+7b)^2\cot(e+fx)\csc(e+fx)}{24f} - \frac{(3a^2+6ab+7b^2)\cot(e+fx)\csc^3(e+fx)}{12f} + \frac{b(6a+7b)\sec(e+fx)}{3f} + \frac{b^2\csc^4(e+fx)\sec^3(e+fx)}{3f}$$

```
[Out] -1/8*(3*a^2+30*a*b+35*b^2)*arctanh(cos(f*x+e))/f-1/24*(3*a+7*b)^2*cot(f*x+e
)*csc(f*x+e)/f-1/12*(3*a^2+6*a*b+7*b^2)*cot(f*x+e)*csc(f*x+e)^3/f+1/3*b*(6*
a+7*b)*sec(f*x+e)/f+1/3*b^2*csc(f*x+e)^4*sec(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used

$= \{4218, 473, 467, 464, 212\}$

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{(3a^2+30ab+35b^2)\operatorname{arctanh}(\cos(e+fx))}{8f} - \frac{(3a^2+6ab+7b^2)\cot(e+fx)\csc^3(e+fx)}{12f} + \frac{b(6a+7b)\sec(e+fx)}{3f} - \frac{(3a+7b)^2\cot(e+fx)\csc(e+fx)}{24f} + \frac{b^2\csc^4(e+fx)\sec^3(e+fx)}{3f}$$

```
[In] Int[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/8*((3*a^2 + 30*a*b + 35*b^2)*ArcTanh[Cos[e + f*x]])/f - ((3*a + 7*b)^2*C
ot[e + f*x]*Csc[e + f*x])/(24*f) - ((3*a^2 + 6*a*b + 7*b^2)*Cot[e + f*x]*Cs
c[e + f*x]^3)/(12*f) + (b*(6*a + 7*b)*Sec[e + f*x])/(3*f) + (b^2*Csc[e + f*
x]^4*Sec[e + f*x]^3)/(3*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(b+ax^2)^2}{x^4(1-x^2)^3}\,dx,x,\cos(e+fx)\right)}{f}\\
&= \frac{b^2\csc^4(e+fx)\sec^3(e+fx)}{3f}-\frac{\text{Subst}\left(\int \frac{b(6a+7b)+3a^2x^2}{x^2(1-x^2)^3}\,dx,x,\cos(e+fx)\right)}{3f}\\
&= -\frac{(3a^2+6ab+7b^2)\cot(e+fx)\csc^3(e+fx)}{12f}+\frac{b^2\csc^4(e+fx)\sec^3(e+fx)}{3f}\\
&\quad+\frac{\text{Subst}\left(\int \frac{-4b(6a+7b)-3(3a^2+6ab+7b^2)x^2}{x^2(1-x^2)^2}\,dx,x,\cos(e+fx)\right)}{12f}\\
&= -\frac{(3a+7b)^2\cot(e+fx)\csc(e+fx)}{24f}-\frac{(3a^2+6ab+7b^2)\cot(e+fx)\csc^3(e+fx)}{12f}\\
&\quad+\frac{b^2\csc^4(e+fx)\sec^3(e+fx)}{3f}-\frac{\text{Subst}\left(\int \frac{8b(6a+7b)+(3a+7b)^2x^2}{x^2(1-x^2)}\,dx,x,\cos(e+fx)\right)}{24f}\\
&= -\frac{(3a+7b)^2\cot(e+fx)\csc(e+fx)}{24f}-\frac{(3a^2+6ab+7b^2)\cot(e+fx)\csc^3(e+fx)}{12f}\\
&\quad+\frac{b(6a+7b)\sec(e+fx)}{3f}+\frac{b^2\csc^4(e+fx)\sec^3(e+fx)}{3f}\\
&\quad-\frac{(3a^2+30ab+35b^2)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\cos(e+fx)\right)}{8f}
\end{aligned}$$

$$= -\frac{(3a^2+30ab+35b^2)\operatorname{arctanh}(\cos(e+fx))}{8f} - \frac{(3a+7b)^2\cot(e+fx)\csc(e+fx)}{24f}$$
$$- \frac{(3a^2+6ab+7b^2)\cot(e+fx)\csc^3(e+fx)}{12f}$$
$$+ \frac{b(6a+7b)\sec(e+fx)}{3f} + \frac{b^2\csc^4(e+fx)\sec^3(e+fx)}{3f}$$

Mathematica [A] (verified)

Time = 9.79 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.55

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx =$$
$$-\frac{(b+a\cos^2(e+fx))^2\left((90a^2+132ab-102b^2+(6a^2+60ab+70b^2)\cos(4(e+fx))-3(3a^2+30ab+3\cdots\right.}{\cdots}$$

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/192*((b + a*Cos[e + f*x]^2)^2*((90*a^2 + 132*a*b - 102*b^2 + (6*a^2 + 60
*a*b + 70*b^2)*Cos[4*(e + f*x)] - 3*(3*a^2 + 30*a*b + 35*b^2)*Cos[6*(e + f*
x)])*Cot[e + f*x]*Csc[e + f*x]^3 + ((105*a^2 + 282*a*b + 329*b^2)*(Cos[e +
f*x] + Cos[3*(e + f*x)])*Csc[e + f*x]^4)/2 + 96*(3*a^2 + 30*a*b + 35*b^2)*C
os[e + f*x]^4*(Log[Cos[(e + f*x)/2]] - Log[Sin[(e + f*x)/2]]))*Sec[e + f*x]
^4)/(f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.50

method	result
derivativedivides	$\frac{a^2\left(\left(-\frac{\csc(fx+e)^3}{4}-\frac{3\csc(fx+e)}{8}\right)\cot(fx+e)+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{8}\right)+2ab\left(-\frac{1}{4\sin(fx+e)^4\cos(fx+e)}-\frac{5}{8\sin(fx+e)^2\cos\cdots}\right.}{\cdots}$
default	$\frac{a^2\left(\left(-\frac{\csc(fx+e)^3}{4}-\frac{3\csc(fx+e)}{8}\right)\cot(fx+e)+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{8}\right)+2ab\left(-\frac{1}{4\sin(fx+e)^4\cos(fx+e)}-\frac{5}{8\sin(fx+e)^2\cos\cdots}\right.}{\cdots}$
norman	$\frac{\frac{a^2+2ab+b^2}{64f}+\frac{(a^2+2ab+b^2)\tan(\frac{fx}{2}+\frac{e}{2})^{14}}{64f}+\frac{(5a^2+26ab+21b^2)\tan(\frac{fx}{2}+\frac{e}{2})^2}{64f}+\frac{(5a^2+26ab+21b^2)\tan(\frac{fx}{2}+\frac{e}{2})^{12}}{64f}-\frac{(3a^2+29ab\cdots}{\cdots}}{\tan(\frac{fx}{2}+\frac{e}{2})^4\left(\tan(\frac{fx}{2}+\frac{e}{2})^2-1\cdots\right.}$
parallelrisch	$\frac{55296\left(\frac{\cos(3fx+3e)}{3}+\cos(fx+e)\right)(a^2+10ab+\frac{35}{3}b^2)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-63\left((3a^2+\frac{218}{7}ab+\frac{121}{3}b^2)\cos(3fx+3e)+(\frac{80}{3}a^2+\cdots\right.}{\cdots}$
risch	$\frac{e^{i(fx+e)}(9a^2e^{12i(fx+e)}+90ab\,e^{12i(fx+e)}+105b^2e^{12i(fx+e)}-6a^2e^{10i(fx+e)}-60ab\,e^{10i(fx+e)}-70b^2e^{10i(fx+e)}-105a^2e^{8i\cdots}}{\cdots}$

```
[In] int(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*((-1/4*csc(f*x+e)^3-3/8*csc(f*x+e))*cot(f*x+e)+3/8*ln(csc(f*x+e)-c
ot(f*x+e)))+2*a*b*(-1/4/sin(f*x+e)^4/cos(f*x+e)-5/8/sin(f*x+e)^2/cos(f*x+e)
+15/8/cos(f*x+e)+15/8*ln(csc(f*x+e)-cot(f*x+e)))+b^2*(-1/4/sin(f*x+e)^4/cos
(f*x+e)^3+7/12/sin(f*x+e)^2/cos(f*x+e)^3-35/24/sin(f*x+e)^2/cos(f*x+e)+35/8
/cos(f*x+e)+35/8*ln(csc(f*x+e)-cot(f*x+e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(131) = 262$.

Time = 0.26 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.03

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{6\left(3a^2+30ab+35b^2\right)\cos\left(fx+e\right)^6-10\left(3a^2+30ab+35b^2\right)\cos\left(fx+e\right)^4+16\left(6ab+7b^2\right)\cos\left(fx+e\right)}{}$$

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(6*(3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^6 - 10*(3*a^2 + 30*a*b + 35*
b^2)*cos(f*x + e)^4 + 16*(6*a*b + 7*b^2)*cos(f*x + e)^2 + 16*b^2 - 3*((3*a^
2 + 30*a*b + 35*b^2)*cos(f*x + e)^7 - 2*(3*a^2 + 30*a*b + 35*b^2)*cos(f*x +
 e)^5 + (3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^3)*log(1/2*cos(f*x + e) + 1/
2) + 3*((3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^7 - 2*(3*a^2 + 30*a*b + 35*b
^2)*cos(f*x + e)^5 + (3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^3)*log(-1/2*cos
(f*x + e) + 1/2))/(f*cos(f*x + e)^7 - 2*f*cos(f*x + e)^5 + f*cos(f*x + e)^3
)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)**5*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.17

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx =$$

$$-\frac{3\,(3\,a^2+30\,ab+35\,b^2)\log\left(\cos\left(fx+e\right)+1\right)-3\,(3\,a^2+30\,ab+35\,b^2)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\,(3\,a^2\right.}{}}{48\,f}$$

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/48*(3*(3*a^2 + 30*a*b + 35*b^2)*log(cos(f*x + e) + 1) - 3*(3*a^2 + 30*a*
b + 35*b^2)*log(cos(f*x + e) - 1) - 2*(3*(3*a^2 + 30*a*b + 35*b^2)*cos(f*x
+ e)^6 - 5*(3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^4 + 8*(6*a*b + 7*b^2)*cos
(f*x + e)^2 + 8*b^2)/(cos(f*x + e)^7 - 2*cos(f*x + e)^5 + cos(f*x + e)^3))/
f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. 2(131) = 262.

Time = 0.38 (sec) , antiderivative size = 493, normalized size of antiderivative = 3.50

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx =$$

$$-\frac{\frac{24\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{96\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{72\,b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{3\,a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{6\,ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{3\,b^2(\cos(fx+e)}{(\cos(fx+e)+}}{}$$

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/192*(24*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 96*a*b*(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) + 72*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 3
*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 6*a*b*(cos(f*x + e) - 1)^2
/(cos(f*x + e) + 1)^2 - 3*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 1
2*(3*a^2 + 30*a*b + 35*b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1
)) + 3*(a^2 + 2*a*b + b^2 - 8*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 3
2*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 24*b^2*(cos(f*x + e) - 1)/(co
s(f*x + e) + 1) + 18*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 180*a*
b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 210*b^2*(cos(f*x + e) - 1)^2/
(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(cos(f*x + e) - 1)^2 - 256*(3*a*
b + 5*b^2 + 6*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 9*b^2*(cos(f*x +
e) - 1)/(cos(f*x + e) + 1) + 3*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^
2 + 6*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((cos(f*x + e) - 1)/(c
os(f*x + e) + 1) + 1)^3)/f
```

Mupad [B] (verification not implemented)

Time = 18.30 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.96

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{\frac{b^2}{3}+\cos\left(e+f\,x\right)^2\left(\frac{7\,b^2}{3}+2\,a\,b\right)+\cos\left(e+f\,x\right)^6\left(\frac{3\,a^2}{8}+\frac{15\,a\,b}{4}+\frac{35\,b^2}{8}\right)-\cos\left(e+f\,x\right)^4\left(\frac{5\,a^2}{8}+\frac{25\,a\,b}{4}+\frac{175\,b^2}{24}\right.}{f\,\left(\cos\left(e+f\,x\right)^7-2\cos\left(e+f\,x\right)^5+\cos\left(e+f\,x\right)^3\right)}$$
$$-\frac{\mathrm{atanh}(\cos\left(e+f\,x\right))\left(\frac{3\,a^2}{8}+\frac{15\,a\,b}{4}+\frac{35\,b^2}{8}\right)}{f}$$

```
[In] int((a + b/cos(e + f*x)^2)^2/sin(e + f*x)^5,x)
```

```
[Out] (b^2/3 + cos(e + f*x)^2*(2*a*b + (7*b^2)/3) + cos(e + f*x)^6*((15*a*b)/4 +
(3*a^2)/8 + (35*b^2)/8) - cos(e + f*x)^4*((25*a*b)/4 + (5*a^2)/8 + (175*b^2
)/24))/(f*(cos(e + f*x)^3 - 2*cos(e + f*x)^5 + cos(e + f*x)^7)) - (atanh(co
s(e + f*x))*((15*a*b)/4 + (3*a^2)/8 + (35*b^2)/8))/f
```

3.21 $\int (a+b\sec^2(e+fx))^2\sin^6(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 148

$$\int (a+b\sec^2(e+fx))^2\sin^6(e+fx)\,dx = \frac{5}{16}(a^2-12ab+8b^2)\,x - \frac{(3a^2-36ab+8b^2)\cos(e+fx)\sin(e+fx)}{16f} + \frac{a(a-12b)\cos^3(e+fx)\sin(e+fx)}{24f} - \frac{(a^2-12ab+12b^2)\tan(e+fx)}{6f} + \frac{a^2\sin^6(e+fx)\tan(e+fx)}{6f} + \frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] 5/16*(a^2-12*a*b+8*b^2)*x-1/16*(3*a^2-36*a*b+8*b^2)*cos(f*x+e)*sin(f*x+e)/f
+1/24*a*(a-12*b)*cos(f*x+e)^3*sin(f*x+e)/f-1/6*(a^2-12*a*b+12*b^2)*tan(f*x+
e)/f+1/6*a^2*sin(f*x+e)^6*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

$= \{4217, 474, 466, 1828, 1167, 209\}$

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^6(e+fx)\,dx$$
$$=-\frac{(a^2-12ab+12b^2)\tan(e+fx)}{6f}-\frac{(3a^2-36ab+8b^2)\sin(e+fx)\cos(e+fx)}{16f}$$
$$+\frac{5}{16}x\left(a^2-12ab+8b^2\right)+\frac{a^2\sin^6(e+fx)\tan(e+fx)}{6f}$$
$$+\frac{a(a-12b)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^6,x]
```

```
[Out] (5*(a^2 - 12*a*b + 8*b^2)*x)/16 - ((3*a^2 - 36*a*b + 8*b^2)*Cos[e + f*x]*Si
n[e + f*x])/(16*f) + (a*(a - 12*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - ((
a^2 - 12*a*b + 12*b^2)*Tan[e + f*x])/(6*f) + (a^2*Sin[e + f*x]^6*Tan[e + f*
x])/(6*f) + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^6(a+b+bx^2)^2}{(1+x^2)^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{a^2\sin^6(e+fx)\tan(e+fx)}{6f} - \frac{\text{Subst}\left(\int \frac{x^6(7a^2-6(a+b)^2-6b^2x^2)}{(1+x^2)^3}\,dx, x, \tan(e+fx)\right)}{6f}$$

$$= \frac{a(a-12b)\cos^3(e+fx)\sin(e+fx)}{24f} + \frac{a^2\sin^6(e+fx)\tan(e+fx)}{6f} + \frac{\text{Subst}\left(\int \frac{-a(a-12b)+4a(a-12b)x^2-4a(a-12b)x^4+24b^2x^6}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{24f}$$

$$= -\frac{(3a^2-36ab+8b^2)\cos(e+fx)\sin(e+fx)}{16f} + \frac{a(a-12b)\cos^3(e+fx)\sin(e+fx)}{24f} + \frac{a^2\sin^6(e+fx)\tan(e+fx)}{6f} - \frac{\text{Subst}\left(\int \frac{-7a^2+84ab-24b^2+8(a^2-12ab+6b^2)x^2-48b^2x^4}{1+x^2}\,dx, x, \tan(e+fx)\right)}{48f}$$

$$= -\frac{(3a^2-36ab+8b^2)\cos(e+fx)\sin(e+fx)}{16f} + \frac{a(a-12b)\cos^3(e+fx)\sin(e+fx)}{24f} + \frac{a^2\sin^6(e+fx)\tan(e+fx)}{6f} - \frac{\text{Subst}\left(\int \left(8(a^2-12ab+12b^2)-48b^2x^2-\frac{15(a^2-12ab+8b^2)}{1+x^2}\right)dx, x, \tan(e+fx)\right)}{48f}$$

$$= -\frac{(3a^2-36ab+8b^2)\cos(e+fx)\sin(e+fx)}{16f} + \frac{a(a-12b)\cos^3(e+fx)\sin(e+fx)}{24f} - \frac{(a^2-12ab+12b^2)\tan(e+fx)}{6f} + \frac{a^2\sin^6(e+fx)\tan(e+fx)}{6f} + \frac{b^2\tan^3(e+fx)}{3f} + \frac{(5(a^2-12ab+8b^2))\,\text{Subst}\left(\int \frac{1}{1+x^2}dx, x, \tan(e+fx)\right)}{16f}$$

$$= \frac{5}{16}(a^2-12ab+8b^2)\,x - \frac{(3a^2-36ab+8b^2)\cos(e+fx)\sin(e+fx)}{16f} + \frac{a(a-12b)\cos^3(e+fx)\sin(e+fx)}{24f} - \frac{(a^2-12ab+12b^2)\tan(e+fx)}{6f} + \frac{a^2\sin^6(e+fx)\tan(e+fx)}{6f} + \frac{b^2\tan^3(e+fx)}{3f}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 499 vs. $2(148) = 296$.

Time = 3.43 (sec) , antiderivative size = 499, normalized size of antiderivative = 3.37

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^6(e+fx)\,dx$$

$$= \frac{\left(b+a\cos^2(e+fx)\right)^2\sec(e)\sec^3(e+fx)\left(360(a^2-12ab+8b^2)\,fx\cos(fx)+360(a^2-12ab+8b^2)\,fx\cos(2\right.}{}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^6,x]
```

```
[Out] ((b + a*Cos[e + f*x]^2)^2*Sec[e]*Sec[e + f*x]^3*(360*(a^2 - 12*a*b + 8*b^2)
*f*x*Cos[f*x] + 360*(a^2 - 12*a*b + 8*b^2)*f*x*Cos[2*e + f*x] + 120*a^2*f*x
*Cos[2*e + 3*f*x] - 1440*a*b*f*x*Cos[2*e + 3*f*x] + 960*b^2*f*x*Cos[2*e + 3
*f*x] + 120*a^2*f*x*Cos[4*e + 3*f*x] - 1440*a*b*f*x*Cos[4*e + 3*f*x] + 960*
b^2*f*x*Cos[4*e + 3*f*x] - 81*a^2*Sin[f*x] + 3444*a*b*Sin[f*x] - 3168*b^2*S
in[f*x] - 81*a^2*Sin[2*e + f*x] - 1164*a*b*Sin[2*e + f*x] + 2208*b^2*Sin[2*
e + f*x] - 109*a^2*Sin[2*e + 3*f*x] + 2076*a*b*Sin[2*e + 3*f*x] - 1936*b^2*
Sin[2*e + 3*f*x] - 109*a^2*Sin[4*e + 3*f*x] + 540*a*b*Sin[4*e + 3*f*x] - 14
4*b^2*Sin[4*e + 3*f*x] - 21*a^2*Sin[4*e + 5*f*x] + 156*a*b*Sin[4*e + 5*f*x]
 - 48*b^2*Sin[4*e + 5*f*x] - 21*a^2*Sin[6*e + 5*f*x] + 156*a*b*Sin[6*e + 5*
f*x] - 48*b^2*Sin[6*e + 5*f*x] + 6*a^2*Sin[6*e + 7*f*x] - 12*a*b*Sin[6*e +
7*f*x] + 6*a^2*Sin[8*e + 7*f*x] - 12*a*b*Sin[8*e + 7*f*x] - a^2*Sin[8*e + 9
*f*x] - a^2*Sin[10*e + 9*f*x]))/(768*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.20

method	result
parallelrisch	$\frac{120fx(a^2-12ab+8b^2)\cos(3fx+3e)+(-109a^2+1308ab-1040b^2)\sin(3fx+3e)+(-21a^2+156ab-48b^2)\sin(5fx+5e)+6}{384f(\cos(3fx+3e)+3}$
derivativedivides	$a^2\left(-\frac{\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+2ab\left(\frac{\sin(fx+e)^7}{\cos(fx+e)}+\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\right.\right.$
default	$a^2\left(-\frac{\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+2ab\left(\frac{\sin(fx+e)^7}{\cos(fx+e)}+\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\right.\right.$
parts	$\frac{a^2\left(-\frac{\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)}{f}+\frac{b^2\left(\frac{\sin(fx+e)^7}{3\cos(fx+e)^3}-\frac{4\sin(fx+e)^7}{3\cos(fx+e)}-\frac{4\left(\sin(fx+e)^5\right.}{}\right.}{}$
risch	$\frac{5a^2x}{16}-\frac{15xab}{4}+\frac{5x\,b^2}{2}-\frac{3ie^{4i(fx+e)}a^2}{128f}+\frac{ie^{4i(fx+e)}ab}{32f}+\frac{15ie^{2i(fx+e)}a^2}{128f}-\frac{ie^{2i(fx+e)}ab}{2f}+\frac{ie^{2i(fx+e)}b^2}{8f}-$

```
[In] int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/384*(120*f*x*(a^2-12*a*b+8*b^2)*cos(3*f*x+3*e)+(-109*a^2+1308*a*b-1040*b^
2)*sin(3*f*x+3*e)+(-21*a^2+156*a*b-48*b^2)*sin(5*f*x+5*e)+6*a*(a-2*b)*sin(7
*f*x+7*e)-a^2*sin(9*f*x+9*e)+360*f*x*(a^2-12*a*b+8*b^2)*cos(f*x+e)-81*(a^2-
380/27*a*b+160/27*b^2)*sin(f*x+e))/f/(cos(3*f*x+3*e)+3*cos(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.89

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^6(e+fx)\,dx$$
$$=\frac{15\,(a^2-12\,ab+8\,b^2)fx\cos{(fx+e)}^3-\left(8\,a^2\cos{(fx+e)}^8-2\,(13\,a^2-12\,ab)\cos{(fx+e)}^6+3\,(11\,a^2-\right.}{48\,f\cos{(fx+e)}^3}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] 1/48*(15*(a^2 - 12*a*b + 8*b^2)*f*x*cos(f*x + e)^3 - (8*a^2*cos(f*x + e)^8
- 2*(13*a^2 - 12*a*b)*cos(f*x + e)^6 + 3*(11*a^2 - 36*a*b + 8*b^2)*cos(f*x
+ e)^4 - 16*(6*a*b - 7*b^2)*cos(f*x + e)^2 - 16*b^2)*sin(f*x + e))/(f*cos(f
*x + e)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^6(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**6,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.11

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^6(e+fx)\,dx$$
$$= \frac{16\,b^2\tan\left(fx+e\right)^3 + 15\left(a^2-12\,ab+8\,b^2\right)(fx+e) + 96\left(ab-b^2\right)\tan\left(fx+e\right) - \frac{3\left(11\,a^2-36\,ab+8\,b^2\right)\tan(fx+e)^5 \cdots}{\tan(fx \cdots}}{48\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] 1/48*(16*b^2*tan(f*x + e)^3 + 15*(a^2 - 12*a*b + 8*b^2)*(f*x + e) + 96*(a*b
 - b^2)*tan(f*x + e) - (3*(11*a^2 - 36*a*b + 8*b^2)*tan(f*x + e)^5 + 8*(5*a
^2 - 24*a*b + 6*b^2)*tan(f*x + e)^3 + 3*(5*a^2 - 28*a*b + 8*b^2)*tan(f*x +
e))/(tan(f*x + e)^6 + 3*tan(f*x + e)^4 + 3*tan(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.24

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^6(e+fx)\,dx$$
$$= \frac{16\,b^2\tan\left(fx+e\right)^3 + 96\,ab\tan\left(fx+e\right) - 96\,b^2\tan\left(fx+e\right) + 15\left(a^2-12\,ab+8\,b^2\right)(fx+e) - \frac{33\,a^2\tan(fx+e \cdots}{\cdots}}{\cdots}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^6,x, algorithm="giac")
```

```
[Out] 1/48*(16*b^2*tan(f*x + e)^3 + 96*a*b*tan(f*x + e) - 96*b^2*tan(f*x + e) + 1
5*(a^2 - 12*a*b + 8*b^2)*(f*x + e) - (33*a^2*tan(f*x + e)^5 - 108*a*b*tan(f
*x + e)^5 + 24*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 - 192*a*b*tan(f*x
 + e)^3 + 48*b^2*tan(f*x + e)^3 + 15*a^2*tan(f*x + e) - 84*a*b*tan(f*x + e)
 + 24*b^2*tan(f*x + e))/(tan(f*x + e)^2 + 1)^3)/f
```

Mupad [B] (verification not implemented)

Time = 19.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.10

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^6(e+fx)\,dx = x\left(\frac{5\,a^2}{16}-\frac{15\,a\,b}{4}+\frac{5\,b^2}{2}\right) - \frac{\left(\frac{11\,a^2}{16}-\frac{9\,a\,b}{4}+\frac{b^2}{2}\right)\,\tan(e+f\,x)^5+\left(\frac{5\,a^2}{6}-4\,a\,b+b^2\right)\,\tan(e+f\,x)^3+\left(\frac{5\,a^2}{16}-\frac{7\,a\,b}{4}+\frac{b^2}{2}\right)\,\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^6+3\tan\left(e+f\,x\right)^4+3\tan\left(e+f\,x\right)^2+1\right)} + \frac{b^2\tan(e+f\,x)^3}{3\,f} - \frac{\tan(e+f\,x)\,\left(4\,b^2-2\,b\,(a+b)\right)}{f}$$

```
[In] int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] x*((5*a^2)/16 - (15*a*b)/4 + (5*b^2)/2) - (tan(e + f*x)*((5*a^2)/16 - (7*a*
b)/4 + b^2/2) + tan(e + f*x)^3*((5*a^2)/6 - 4*a*b + b^2) + tan(e + f*x)^5*(
(11*a^2)/16 - (9*a*b)/4 + b^2/2))/(f*(3*tan(e + f*x)^2 + 3*tan(e + f*x)^4 +
 tan(e + f*x)^6 + 1)) + (b^2*tan(e + f*x)^3)/(3*f) - (tan(e + f*x)*(4*b^2 -
 2*b*(a + b)))/f
```

3.22 $\int (a+b\sec^2(e+fx))^2 \sin^4(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 114

$$\int (a+b\sec^2(e+fx))^2 \sin^4(e+fx)\,dx = \frac{1}{8}(3a^2-24ab+8b^2)\,x - \frac{a(a-8b)\cos(e+fx)\sin(e+fx)}{8f} - \frac{(a^2-8ab+4b^2)\tan(e+fx)}{4f} + \frac{a^2\sin^4(e+fx)\tan(e+fx)}{4f} + \frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] 1/8*(3*a^2-24*a*b+8*b^2)*x-1/8*a*(a-8*b)*cos(f*x+e)*sin(f*x+e)/f-1/4*(a^2-8
*a*b+4*b^2)*tan(f*x+e)/f+1/4*a^2*sin(f*x+e)^4*tan(f*x+e)/f+1/3*b^2*tan(f*x+
e)^3/f
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4217, 474, 466, 1167, 209}

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^4(e+fx)\,dx = -\frac{(a^2-8ab+4b^2)\tan(e+fx)}{4f} + \frac{1}{8}x\left(3a^2-24ab+8b^2\right) + \frac{a^2\sin^4(e+fx)\tan(e+fx)}{4f} - \frac{a(a-8b)\sin(e+fx)\cos(e+fx)}{8f} + \frac{b^2\tan^3(e+fx)}{3f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^4,x]
```

```
[Out] ((3*a^2 - 24*a*b + 8*b^2)*x)/8 - (a*(a - 8*b)*Cos[e + f*x]*Sin[e + f*x])/(8
*f) - ((a^2 - 8*a*b + 4*b^2)*Tan[e + f*x])/(4*f) + (a^2*Sin[e + f*x]^4*Tan[
e + f*x])/(4*f) + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
```

```
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4(a+b+bx^2)^2}{(1+x^2)^3}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{a^2\sin^4(e+fx)\tan(e+fx)}{4f} - \frac{\text{Subst}\left(\int \frac{x^4(5a^2-4(a+b)^2-4b^2x^2)}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{4f}\\
&= -\frac{a(a-8b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a^2\sin^4(e+fx)\tan(e+fx)}{4f}\\
&\quad + \frac{\text{Subst}\left(\int \frac{a(a-8b)-2a(a-8b)x^2+8b^2x^4}{1+x^2}\,dx, x, \tan(e+fx)\right)}{8f}\\
&= -\frac{a(a-8b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a^2\sin^4(e+fx)\tan(e+fx)}{4f}\\
&\quad + \frac{\text{Subst}\left(\int \left(-2(a^2-8ab+4b^2)+8b^2x^2+\frac{3a^2-24ab+8b^2}{1+x^2}\right)dx, x, \tan(e+fx)\right)}{8f}\\
&= -\frac{a(a-8b)\cos(e+fx)\sin(e+fx)}{8f} - \frac{(a^2-8ab+4b^2)\tan(e+fx)}{4f}\\
&\quad + \frac{a^2\sin^4(e+fx)\tan(e+fx)}{4f} + \frac{b^2\tan^3(e+fx)}{3f}\\
&\quad + \frac{(3a^2-24ab+8b^2)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{8f}
\end{aligned}
$$

$$= \frac{1}{8}(3a^2 - 24ab + 8b^2)\,x - \frac{a(a-8b)\cos(e+fx)\sin(e+fx)}{8f} - \frac{(a^2-8ab+4b^2)\tan(e+fx)}{4f} + \frac{a^2\sin^4(e+fx)\tan(e+fx)}{4f} + \frac{b^2\tan^3(e+fx)}{3f}$$

Mathematica [A] (verified)

Time = 2.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.34

$$\int (a + b\sec^2(e+fx))^2 \sin^4(e+fx)\,dx$$
$$= \frac{(b+a\cos^2(e+fx))^2\sec^3(e+fx)\,(32b^2\sec(e)\sin(fx) + 64(3a-2b)b\cos^2(e+fx)\sec(e)\sin(fx) + 3\cos^3}{24f(a+2b+}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^4,x]
```

```
[Out] ((b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^3*(32*b^2*Sec[e]*Sin[f*x] + 64*(3*a
- 2*b)*b*Cos[e + f*x]^2*Sec[e]*Sin[f*x] + 3*Cos[e + f*x]^3*(4*(3*a^2 - 24*a
*b + 8*b^2)*f*x - 8*a*(a - 2*b)*Sin[2*(e + f*x)] + a^2*Sin[4*(e + f*x)]) +
32*b^2*Cos[e + f*x]*Tan[e]))/(24*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(\frac{\sin(fx+e)^5}{\cos(fx+e)}+\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)-\frac{3fx}{2}\right.}{f}$
default	$\frac{a^2\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(\frac{\sin(fx+e)^5}{\cos(fx+e)}+\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)-\frac{3fx}{2}\right.}{f}$
parts	$\frac{a^2\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)}{f}+\frac{b^2\left(\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+fx+e\right)}{f}+\frac{2ab\left(\frac{\sin(fx+e)^5}{\cos(fx+e)}+\right.}{}$
parallelrisch	$\frac{72fx\left(a^2-8ab+\frac{8}{3}b^2\right)\cos(3fx+3e)+\left(-63a^2+528ab-256b^2\right)\sin(3fx+3e)+\left(-15a^2+48ab\right)\sin(5fx+5e)+3a^2\sin(7fx+7}{192f(\cos(3fx+3e)+3\cos(fx+e))}$
risch	$\frac{3a^2x}{8}-3xab+x\,b^2-\frac{i e^{4i(fx+e)}a^2}{64f}+\frac{i e^{2i(fx+e)}a^2}{8f}-\frac{i e^{2i(fx+e)}ab}{4f}-\frac{i e^{-2i(fx+e)}a^2}{8f}+\frac{i e^{-2i(fx+e)}ab}{4f}+$
norman	$\frac{\left(-\frac{3}{8}a^2+3ab-b^2\right)x+\left(-\frac{9}{8}a^2+9ab-3b^2\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8+\left(-\frac{9}{8}a^2+9ab-3b^2\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}+\left(-\frac{3}{8}a^2+3ab-b^2\right)x\tan\left(\frac{f}{2}\right.}{}$

```
[In] int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+2*a*
b*(sin(f*x+e)^5/cos(f*x+e)+(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/2*f*x
-3/2*e)+b^2*(1/3*tan(f*x+e)^3-tan(f*x+e)+f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.94

$$\int (a + b\sec^2(e + fx))^2 \sin^4(e + fx)\, dx = \frac{3\,(3\,a^2 - 24\,ab + 8\,b^2)fx\cos{(fx + e)}^3 + (6\,a^2\cos{(fx + e)}^6 - 3\,(5\,a^2 - 8\,ab)\cos{(fx + e)}^4 + 16\,(3\,ab - 2\,b^2}{24\,f\cos{(fx + e)}^3}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^4,x, algorithm="fricas")
[Out] 1/24*(3*(3*a^2 - 24*a*b + 8*b^2)*f*x*cos(f*x + e)^3 + (6*a^2*cos(f*x + e)^6
 - 3*(5*a^2 - 8*a*b)*cos(f*x + e)^4 + 16*(3*a*b - 2*b^2)*cos(f*x + e)^2 + 8
*b^2)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (a + b\sec^2(e + fx))^2 \sin^4(e + fx)\, dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**4,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.05

$$\int (a + b\sec^2(e + fx))^2 \sin^4(e + fx)\, dx = \frac{8\,b^2\tan{(fx + e)}^3 + 3\,(3\,a^2 - 24\,ab + 8\,b^2)(fx + e) + 24\,(2\,ab - b^2)\tan{(fx + e)} - \frac{3\,((5\,a^2 - 8\,ab)\tan(fx+e)^3 + (3\,a}{\tan(fx+e)^4 + 2\,\tan(f}}{24\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^4,x, algorithm="maxima")
[Out] 1/24*(8*b^2*tan(f*x + e)^3 + 3*(3*a^2 - 24*a*b + 8*b^2)*(f*x + e) + 24*(2*a
*b - b^2)*tan(f*x + e) - 3*((5*a^2 - 8*a*b)*tan(f*x + e)^3 + (3*a^2 - 8*a*b
)*tan(f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^4(e+fx)\,dx$$
$$=\frac{8\,b^2\tan\left(fx+e\right)^3+48\,ab\tan\left(fx+e\right)-24\,b^2\tan\left(fx+e\right)+3\,(3\,a^2-24\,ab+8\,b^2)(fx+e)-\frac{3\left(5\,a^2\tan(fx\right.}{}}{24\,f}$$

[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^4,x, algorithm="giac")

[Out] 1/24*(8*b^2*tan(f*x + e)^3 + 48*a*b*tan(f*x + e) - 24*b^2*tan(f*x + e) + 3*(3*a^2 - 24*a*b + 8*b^2)*(f*x + e) - 3*(5*a^2*tan(f*x + e)^3 - 8*a*b*tan(f*x + e)^3 + 3*a^2*tan(f*x + e) - 8*a*b*tan(f*x + e))/(tan(f*x + e)^2 + 1)^2)/f

Mupad [B] (verification not implemented)

Time = 18.78 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^4(e+fx)\,dx$$
$$=x\left(\frac{3\,a^2}{8}-3\,a\,b+b^2\right)+\frac{\left(a\,b-\frac{5\,a^2}{8}\right)\,\tan(e+f\,x)^3+\left(a\,b-\frac{3\,a^2}{8}\right)\,\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^4+2\tan\left(e+f\,x\right)^2+1\right)}+\frac{b^2\tan(e+f\,x)^3}{3\,f}-\frac{\tan(e+f\,x)\,\left(3\,b^2-2\,b\,(a+b)\right)}{f}$$

[In] int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^2,x)

[Out] x*((3*a^2)/8 - 3*a*b + b^2) + (tan(e + f*x)*(a*b - (3*a^2)/8) + tan(e + f*x)^3*(a*b - (5*a^2)/8))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1)) + (b^2*tan(e + f*x)^3)/(3*f) - (tan(e + f*x)*(3*b^2 - 2*b*(a + b)))/f

3.23 $\int (a+b\sec^2(e+fx))^2\sin^2(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 73

$$\int (a+b\sec^2(e+fx))^2\sin^2(e+fx)\,dx = \frac{1}{2}a(a-4b)x-\frac{a(a-4b)\tan(e+fx)}{2f}+\frac{a^2\sin^2(e+fx)\tan(e+fx)}{2f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] 1/2*a*(a-4*b)*x-1/2*a*(a-4*b)*tan(f*x+e)/f+1/2*a^2*sin(f*x+e)^2*tan(f*x+e)/
f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4217, 474, 470, 327, 209}

$$\int (a+b\sec^2(e+fx))^2\sin^2(e+fx)\,dx = \frac{a^2\sin^2(e+fx)\tan(e+fx)}{2f}-\frac{a(a-4b)\tan(e+fx)}{2f}+\frac{1}{2}ax(a-4b)+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^2,x]
```

```
[Out] (a*(a - 4*b)*x)/2 - (a*(a - 4*b)*Tan[e + f*x])/(2*f) + (a^2*Sin[e + f*x]^2*
Tan[e + f*x])/(2*f) + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2(a+b+bx^2)^2}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{a^2\sin^2(e+fx)\tan(e+fx)}{2f} - \frac{\text{Subst}\left(\int \frac{x^2(3a^2-2(a+b)^2-2b^2x^2)}{1+x^2}\,dx, x, \tan(e+fx)\right)}{2f}$$

$$= \frac{a^2 \sin^2(e+fx)\tan(e+fx)}{2f} + \frac{b^2 \tan^3(e+fx)}{3f} - \frac{(a(a-4b))\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{2f}$$

$$= -\frac{a(a-4b)\tan(e+fx)}{2f} + \frac{a^2 \sin^2(e+fx)\tan(e+fx)}{2f} + \frac{b^2 \tan^3(e+fx)}{3f} + \frac{(a(a-4b))\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{2f}$$

$$= \frac{1}{2}a(a-4b)x - \frac{a(a-4b)\tan(e+fx)}{2f} + \frac{a^2 \sin^2(e+fx)\tan(e+fx)}{2f} + \frac{b^2 \tan^3(e+fx)}{3f}$$

Mathematica [A] (verified)

Time = 2.19 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.73

$$\int \left(a+b\sec^2(e+fx)\right)^2 \sin^2(e+fx)\,dx =$$

$$-\frac{(b+a\cos^2(e+fx))^2 \sec^3(e+fx)\left(-4b^2\sec(e)\sin(fx) - 4(6a-b)b\cos^2(e+fx)\sec(e)\sin(fx) + 3a\cos\right.}{3f(a+2b+a\cos(2(e+fx)))^2}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^2,x]
```

```
[Out] -1/3*((b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^3*(-4*b^2*Sec[e]*Sin[f*x] - 4*(
6*a - b)*b*Cos[e + f*x]^2*Sec[e]*Sin[f*x] + 3*a*Cos[e + f*x]^3*(-2*(a - 4*b
)*f*x + a*Sin[2*(e + f*x)]) - 4*b^2*Cos[e + f*x]*Tan[e]))/(f*(a + 2*b + a*C
os[2*(e + f*x)])^2)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{a^{2}\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+2ab(\tan(fx+e)-fx-e)+\frac{b^{2}\sin(fx+e)^{3}}{3\cos(fx+e)^{3}}}{f}$
default	$\frac{a^{2}\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+2ab(\tan(fx+e)-fx-e)+\frac{b^{2}\sin(fx+e)^{3}}{3\cos(fx+e)^{3}}}{f}$
parts	$\frac{a^{2}\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{b^{2}\sin(fx+e)^{3}}{3f\cos(fx+e)^{3}}+\frac{2ab(\tan(fx+e)-fx-e)}{f}$
risch	$\frac{a^{2}x}{2}-2xab+\frac{i\mathrm{e}^{2i(fx+e)}a^{2}}{8f}-\frac{i\mathrm{e}^{-2i(fx+e)}a^{2}}{8f}-\frac{2ib\left(-6a\,\mathrm{e}^{4i(fx+e)}+3b\,\mathrm{e}^{4i(fx+e)}-12a\,\mathrm{e}^{2i(fx+e)}-6a+b\right)}{3f\left(\mathrm{e}^{2i(fx+e)}+1\right)^{3}}$
parallelrisch	$\frac{12afx(a-4b)\cos(3fx+3e)+\left(-9a^{2}+48ab-8b^{2}\right)\sin(3fx+3e)-3\sin(5fx+5e)a^{2}+36afx(a-4b)\cos(fx+e)-6\sin(fx+e)}{24f(\cos(3fx+3e)+3\cos(fx+e))}$
norman	$\frac{(-\frac{1}{2}a^{2}+2ab)x+(-\frac{1}{2}a^{2}+2ab)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}+(\frac{1}{2}a^{2}-2ab)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+(\frac{1}{2}a^{2}-2ab)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}+(-a^{2}+4ab)x}{}$

```
[In] int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+2*a*b*(tan(f*x+e)-f*x-e
)+1/3*b^2*sin(f*x+e)^3/cos(f*x+e)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.11

$$\int \left(a+b\sec^{2}(e+fx)\right)^{2}\sin^{2}(e+fx)\,dx$$
$$=\frac{3\left(a^{2}-4\,ab\right)fx\cos\left(fx+e\right)^{3}-\left(3\,a^{2}\cos\left(fx+e\right)^{4}-2\left(6\,ab-b^{2}\right)\cos\left(fx+e\right)^{2}-2\,b^{2}\right)\sin\left(fx+e\right)}{6\,f\cos\left(fx+e\right)^{3}}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(a^2 - 4*a*b)*f*x*cos(f*x + e)^3 - (3*a^2*cos(f*x + e)^4 - 2*(6*a*b
- b^2)*cos(f*x + e)^2 - 2*b^2)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^2 \sin^2(e+fx)\,dx = \int \left(a+b\sec^2{(e+fx)}\right)^2 \sin^2{(e+fx)}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**2,x)
[Out] Integral((a + b*sec(e + f*x)**2)**2*sin(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int \left(a+b\sec^2(e+fx)\right)^2 \sin^2(e+fx)\,dx = \frac{2\,b^2\tan\left(fx+e\right)^3+12\,ab\tan\left(fx+e\right)+3\,(a^2-4\,ab)(fx+e)-\frac{3\,a^2\tan(fx+e)}{\tan(fx+e)^2+1}}{6\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^2,x, algorithm="maxima")
[Out] 1/6*(2*b^2*tan(f*x + e)^3 + 12*a*b*tan(f*x + e) + 3*(a^2 - 4*a*b)*(f*x + e)
 - 3*a^2*tan(f*x + e)/(tan(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int \left(a+b\sec^2(e+fx)\right)^2 \sin^2(e+fx)\,dx = \frac{2\,b^2\tan\left(fx+e\right)^3+12\,ab\tan\left(fx+e\right)+3\,(a^2-4\,ab)(fx+e)-\frac{3\,a^2\tan(fx+e)}{\tan(fx+e)^2+1}}{6\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^2,x, algorithm="giac")
[Out] 1/6*(2*b^2*tan(f*x + e)^3 + 12*a*b*tan(f*x + e) + 3*(a^2 - 4*a*b)*(f*x + e)
 - 3*a^2*tan(f*x + e)/(tan(f*x + e)^2 + 1))/f
```

Mupad [B] (verification not implemented)

Time = 19.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.29

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^2(e+fx)\,dx=\frac{b^2\tan(e+f\,x)^3}{3\,f}-\frac{a^2\,\sin\left(2\,e+2\,f\,x\right)}{4\,f}-\frac{\tan(e+f\,x)\,\left(2\,b^2-2\,b\,(a+b)\right)}{f}-\frac{a\,\mathrm{atan}\left(\frac{a\tan(e+f\,x)\,(a-4\,b)}{2\,\left(2\,a\,b-\frac{a^2}{2}\right)}\right)\,(a-4\,b)}{2\,f}$$

```
[In] int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (b^2*tan(e + f*x)^3)/(3*f) - (a^2*sin(2*e + 2*f*x))/(4*f) - (tan(e + f*x)*(
2*b^2 - 2*b*(a + b)))/f - (a*atan((a*tan(e + f*x)*(a - 4*b))/(2*(2*a*b - a^
2/2)))*(a - 4*b))/(2*f)
```

3.24 $\int \left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 40

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx=a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] a^2*x+b*(2*a+b)*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4213, 398, 209}

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx=a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] a^2*x + (b*(2*a + b)*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \left(b(2a+b)+b^2x^2+\frac{a^2}{1+x^2}\right)\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}+\frac{a^2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{3a^2fx+3b(2a+b)\tan(e+fx)+b^2\tan^3(e+fx)}{3f}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2,x]
[Out] (3*a^2*f*x + 3*b*(2*a + b)*Tan[e + f*x] + b^2*Tan[e + f*x]^3)/(3*f)
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

method	result
parts	$a^{2} x -\frac{b^{2} \left(-\frac{2}{3}-\frac{\sec\left(f x +e \right)^{2}}{3}\right) \tan\left(f x +e \right)}{f}+\frac{2 a b \tan\left(f x +e \right)}{f}$
derivativedivides	$\frac{a^{2} \left(f x +e \right)+2 a b \tan\left(f x +e \right)-b^{2} \left(-\frac{2}{3}-\frac{\sec\left(f x +e \right)^{2}}{3}\right) \tan\left(f x +e \right)}{f}$
default	$\frac{a^{2} \left(f x +e \right)+2 a b \tan\left(f x +e \right)-b^{2} \left(-\frac{2}{3}-\frac{\sec\left(f x +e \right)^{2}}{3}\right) \tan\left(f x +e \right)}{f}$
risch	$a^{2} x +\frac{4 i b \left(3 a \,{\mathrm e}^{4 i \left(f x +e \right)}+6 a \,{\mathrm e}^{2 i \left(f x +e \right)}+3 b \,{\mathrm e}^{2 i \left(f x +e \right)}+3 a +b \right)}{3 f \left({\mathrm e}^{2 i \left(f x +e \right)}+1\right)^{3}}$
norman	$\frac{a^{2} x \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{6}-a^{2} x +3 a^{2} x \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-3 a^{2} x \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{4}-\frac{2 b \left(2 a +b \right) \tan\left(\frac{f x}{2}+\frac{e}{2}\right)}{f}-\frac{2 b \left(2 a +b \right) \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{f}+\frac{4 b \left(6 a +b\right.}{}}{\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right)^{3}}$
parallelrisch	$\frac{3 x \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{6} a^{2} f +\left(-12 a b -6 b^{2}\right) \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}-9 x \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{4} a^{2} f +\left(24 a b +4 b^{2}\right) \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}+9 x \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2} a}{3 f \left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)^{3} \left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{3}}$

```
[In] int((a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x-b^2/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+2*a*b/f*tan(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{3\,a^{2} f x \cos\left(f x + e\right)^{3} + \left(2\,{\left(3\,a b + b^{2}\right)} \cos\left(f x + e\right)^{2} + b^{2}\right)\sin\left(f x + e\right)}{3\,f \cos\left(f x + e\right)^{3}}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^2*f*x*cos(f*x + e)^3 + (2*(3*a*b + b^2)*cos(f*x + e)^2 + b^2)*sin(
f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^2\,dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**2,x)
[Out] Integral((a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx=a^2x+\frac{\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)b^2}{3\,f}+\frac{2\,ab\tan\left(fx+e\right)}{f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
[Out] a^2*x + 1/3*(tan(f*x + e)^3 + 3*tan(f*x + e))*b^2/f + 2*a*b*tan(f*x + e)/f
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{b^2\tan\left(fx+e\right)^3+3\left(fx+e\right)a^2+6\,ab\tan\left(fx+e\right)+3\,b^2\tan\left(fx+e\right)}{3\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
[Out] 1/3*(b^2*tan(f*x + e)^3 + 3*(f*x + e)*a^2 + 6*a*b*tan(f*x + e) + 3*b^2*tan(
f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 18.82 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{\frac{b^2\tan(e+f\,x)^3}{3}-\tan(e+f\,x)\,\left(b^2-2\,b\,(a+b)\right)+a^2\,f\,x}{f}$$

```
[In] int((a + b/cos(e + f*x)^2)^2,x)
[Out] ((b^2*tan(e + f*x)^3)/3 - tan(e + f*x)*(b^2 - 2*b*(a + b)) + a^2*f*x)/f
```

3.25 $\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 50

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=-\frac{(a+b)^2\cot(e+fx)}{f}+\frac{2b(a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] -(a+b)^2*cot(f*x+e)/f+2*b*(a+b)*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4217, 276}

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{2b(a+b)\tan(e+fx)}{f}-\frac{(a+b)^2\cot(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[In] Int[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(((a + b)^2*Cot[e + f*x])/f) + (2*b*(a + b)*Tan[e + f*x])/f + (b^2*Tan[e +
 f*x]^3)/(3*f)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^2}{x^2}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(2b(a+b)+\frac{(a+b)^2}{x^2}+b^2x^2\right)\,dx, x, \tan(e+fx)\right)}{f}\\ &= -\frac{(a+b)^2\cot(e+fx)}{f}+\frac{2b(a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 109 vs. $2(50) = 100$.

Time = 2.62 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.18

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{4(b+a\cos^2(e+fx))^2\sec^3(e+fx)\left(b^2\sec(e)\sin(fx)+\cos^2(e+fx)\left(3(a+b)^2\cot(e+fx)\csc(e)+b(6a+\right.\right.}{3f(a+2b+a\cos(2(e+fx)))^2}$$

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (4*(b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^3*(b^2*Sec[e]*Sin[f*x] + Cos[e + f
*x]^2*(3*(a + b)^2*Cot[e + f*x]*Csc[e] + b*(6*a + 5*b)*Sec[e])*Sin[f*x] + b
^2*Cos[e + f*x]*Tan[e]))/(3*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.92

method	result
derivativedivides	$\frac{-a^2\cot(fx+e)+2ab\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)+b^2\left(\frac{1}{3\sin(fx+e)\cos(fx+e)^3}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right.}{f}$
default	$\frac{-a^2\cot(fx+e)+2ab\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)+b^2\left(\frac{1}{3\sin(fx+e)\cos(fx+e)^3}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right.}{f}$
risch	$-\frac{2i\left(3a^2e^{6i(fx+e)}+9a^2e^{4i(fx+e)}+12ab\,e^{4i(fx+e)}+9a^2e^{2i(fx+e)}+24ab\,e^{2i(fx+e)}+16b^2e^{2i(fx+e)}+3a^2+12ab+8b^2\right)}{3f\left(e^{2i(fx+e)}+1\right)^3\left(e^{2i(fx+e)}-1\right)}$
parallelrisch	$\frac{\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\left((a+b)^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8-4(a+3b)(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+(6a^2+28ab+\frac{50}{3}b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-4(a+3b)(a+b)\,\mathrm{ta}\right.}{2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}$
norman	$\frac{\frac{a^2+2ab+b^2}{2f}+\frac{\left(a^2+2ab+b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{2f}-\frac{2\left(a^2+4ab+3b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{f}-\frac{2\left(a^2+4ab+3b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{f}+\frac{\left(9a^2+42ab+25b^2\right)}{3f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}$

```
[In] int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a^2*cot(f*x+e)+2*a*b*(1/sin(f*x+e)/cos(f*x+e)-2*cot(f*x+e))+b^2*(1/3/
sin(f*x+e)/cos(f*x+e)^3+4/3/sin(f*x+e)/cos(f*x+e)-8/3*cot(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.42

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=-\frac{(3\,a^2+12\,ab+8\,b^2)\cos{(fx+e)}^4-2\,(3\,ab+2\,b^2)\cos{(fx+e)}^2-b^2}{3\,f\cos{(fx+e)}^3\sin{(fx+e)}}$$

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*((3*a^2 + 12*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 2*b^2)*cos(f*x +
 e)^2 - b^2)/(f*cos(f*x + e)^3*sin(f*x + e))
```

Sympy [F]

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\int\left(a+b\sec^2{(e+fx)}\right)^2\csc^2{(e+fx)}\,dx$$

```
[In] integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*csc(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

$$\int \csc^2(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = \frac{b^2 \tan(fx+e)^3 + 6\,(ab+b^2)\tan(fx+e) - \frac{3\,(a^2+2\,ab+b^2)}{\tan(fx+e)}}{3\,f}$$

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
[Out] 1/3*(b^2*tan(f*x + e)^3 + 6*(a*b + b^2)*tan(f*x + e) - 3*(a^2 + 2*a*b + b^2
)/tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20

$$\int \csc^2(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = \frac{b^2 \tan(fx+e)^3 + 6\,ab\tan(fx+e) + 6\,b^2\tan(fx+e) - \frac{3\,(a^2+2\,ab+b^2)}{\tan(fx+e)}}{3\,f}$$

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
[Out] 1/3*(b^2*tan(f*x + e)^3 + 6*a*b*tan(f*x + e) + 6*b^2*tan(f*x + e) - 3*(a^2
+ 2*a*b + b^2)/tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 18.62 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12

$$\int \csc^2(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = \frac{b^2 \tan(e+f\,x)^3}{3\,f} - \frac{a^2+2\,a\,b+b^2}{f\tan(e+f\,x)} + \frac{2\,b\tan(e+f\,x)\,(a+b)}{f}$$

```
[In] int((a + b/cos(e + f*x)^2)^2/sin(e + f*x)^2,x)
[Out] (b^2*tan(e + f*x)^3)/(3*f) - (2*a*b + a^2 + b^2)/(f*tan(e + f*x)) + (2*b*ta
n(e + f*x)*(a + b))/f
```

3.26 $\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{(a+b)(a+3b)\cot(e+fx)}{f} - \frac{(a+b)^2\cot^3(e+fx)}{3f} + \frac{b(2a+3b)\tan(e+fx)}{f} + \frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] -(a+b)*(a+3*b)*cot(f*x+e)/f-1/3*(a+b)^2*cot(f*x+e)^3/f+b*(2*a+3*b)*tan(f*x+
e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4217, 459}

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{b(2a+3b)\tan(e+fx)}{f} - \frac{(a+b)^2\cot^3(e+fx)}{3f} - \frac{(a+b)(a+3b)\cot(e+fx)}{f} + \frac{b^2\tan^3(e+fx)}{3f}$$

```
[In] Int[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(((a + b)*(a + 3*b)*Cot[e + f*x])/f) - ((a + b)^2*Cot[e + f*x]^3)/(3*f) +
(b*(2*a + 3*b)*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(1+x^2)(a+b+bx^2)^2}{x^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(b(2a+3b) + \frac{(a+b)^2}{x^4} + \frac{(a+b)(a+3b)}{x^2} + b^2x^2\right)\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)(a+3b)\cot(e+fx)}{f} - \frac{(a+b)^2\cot^3(e+fx)}{3f} + \frac{b(2a+3b)\tan(e+fx)}{f} + \frac{b^2\tan^3(e+fx)}{3f}$$

Mathematica [A] (verified)

Time = 5.18 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.99

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx =$$
$$-\frac{\csc(2e)\csc^3(2(e+fx))\left(8a(a+2b)\sin(2e) - 6(a+2b)^2\sin(2fx) - 3a^2\sin(2(e+fx)) - 6ab\sin(2(e+f\right.}{}$$

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/6*(Csc[2*e]*Csc[2*(e + f*x)]^3*(8*a*(a + 2*b)*Sin[2*e] - 6*(a + 2*b)^2*S
in[2*f*x] - 3*a^2*Sin[2*(e + f*x)] - 6*a*b*Sin[2*(e + f*x)] + a^2*Sin[6*(e
+ f*x)] + 2*a*b*Sin[6*(e + f*x)] + 3*a^2*Sin[4*e + 2*f*x] + a^2*Sin[4*e + 6
*f*x] + 8*a*b*Sin[4*e + 6*f*x] + 8*b^2*Sin[4*e + 6*f*x]))/f
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.71

method	result
risch	$\frac{4i\left(3a^2 e^{8i(fx+e)}+8a^2 e^{6i(fx+e)}+16ab\, e^{6i(fx+e)}+6a^2 e^{4i(fx+e)}+24ab\, e^{4i(fx+e)}+24b^2 e^{4i(fx+e)}-a^2-8ab-8b^2\right)}{3f\left(e^{2i(fx+e)}-1\right)^3\left(e^{2i(fx+e)}+1\right)^3}$
parallelrisch	$-\frac{(9\cos(2fx+2e)a^2+24\cos(2fx+2e)ab+24\cos(2fx+2e)b^2-a^2\cos(6fx+6e)-8ab\cos(6fx+6e)-8b^2\cos(6fx+6e)+8a}{96f(\cos(3fx+3e)+3\cos(fx+e))}$
derivativedivides	$\frac{a^2\left(-\frac{2}{3}-\frac{\csc(fx+e)^2}{3}\right)\cot(fx+e)+2ab\left(-\frac{1}{3\sin(fx+e)^3\cos(fx+e)}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right)+b^2\left(\frac{1}{3\sin(fx+e)^3 c}\right.}{f}$
default	$\frac{a^2\left(-\frac{2}{3}-\frac{\csc(fx+e)^2}{3}\right)\cot(fx+e)+2ab\left(-\frac{1}{3\sin(fx+e)^3\cos(fx+e)}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right)+b^2\left(\frac{1}{3\sin(fx+e)^3 c}\right.}{f}$
norman	$\frac{\frac{a^2+2ab+b^2}{24f}+\frac{(a^2+2ab+b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}}{24f}+\frac{(a^2+6ab+5b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{4f}+\frac{(a^2+6ab+5b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{4f}-\frac{(11a^2+86ab+91b^2}{8f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}$

```
[In] int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/3*I*(3*a^2*exp(8*I*(f*x+e))+8*a^2*exp(6*I*(f*x+e))+16*a*b*exp(6*I*(f*x+e)
)+6*a^2*exp(4*I*(f*x+e))+24*a*b*exp(4*I*(f*x+e))+24*b^2*exp(4*I*(f*x+e))-a^
2-8*a*b-8*b^2)/f/(exp(2*I*(f*x+e))-1)^3/(exp(2*I*(f*x+e))+1)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.33

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{2\,(a^2+8\,ab+8\,b^2)\cos\left(fx+e\right)^6-3\,(a^2+8\,ab+8\,b^2)\cos\left(fx+e\right)^4+6\,(ab+b^2)\cos\left(fx+e\right)^2+b^2}{3\left(f\cos\left(fx+e\right)^5-f\cos\left(fx+e\right)^3\right)\sin\left(fx+e\right)}$$

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^6 - 3*(a^2 + 8*a*b + 8*b^2)*cos(
f*x + e)^4 + 6*(a*b + b^2)*cos(f*x + e)^2 + b^2)/((f*cos(f*x + e)^5 - f*cos
(f*x + e)^3)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\text{Timed out}$$

```
[In] integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{b^2\tan\left(fx+e\right)^3+3\left(2\,ab+3\,b^2\right)\tan\left(fx+e\right)-\frac{3\,\left(a^2+4\,ab+3\,b^2\right)\tan\left(fx+e\right)^2+a^2+2\,ab+b^2}{\tan\left(fx+e\right)^3}}{3\,f}$$

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 3*(2*a*b + 3*b^2)*tan(f*x + e) - (3*(a^2 + 4*a*b
+ 3*b^2)*tan(f*x + e)^2 + a^2 + 2*a*b + b^2)/tan(f*x + e)^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.29

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{b^2\tan\left(fx+e\right)^3+6\,ab\tan\left(fx+e\right)+9\,b^2\tan\left(fx+e\right)-\frac{3\,a^2\tan\left(fx+e\right)^2+12\,ab\tan\left(fx+e\right)^2+9\,b^2\tan\left(fx+e\right)^2+a^2+2\,ab+b^2}{\tan\left(fx+e\right)^3}}{3\,f}$$

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 6*a*b*tan(f*x + e) + 9*b^2*tan(f*x + e) - (3*a^2*
tan(f*x + e)^2 + 12*a*b*tan(f*x + e)^2 + 9*b^2*tan(f*x + e)^2 + a^2 + 2*a*b
 + b^2)/tan(f*x + e)^3)/f
```

Mupad [B] (verification not implemented)

Time = 18.49 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.12

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{b^2\tan(e+f\,x)^3}{3\,f}-\frac{\frac{2\,a\,b}{3}+\tan(e+f\,x)^2\,(a^2+4\,a\,b+3\,b^2)+\frac{a^2}{3}+\frac{b^2}{3}}{f\tan\left(e+f\,x\right)^3}$$
$$+\frac{b\tan(e+f\,x)\,\left(2\,a+3\,b\right)}{f}$$

```
[In] int((a + b/cos(e + f*x)^2)^2/sin(e + f*x)^4,x)
```

```
[Out] (b^2*tan(e + f*x)^3)/(3*f) - ((2*a*b)/3 + tan(e + f*x)^2*(4*a*b + a^2 + 3*b
^2) + a^2/3 + b^2/3)/(f*tan(e + f*x)^3) + (b*tan(e + f*x)*(2*a + 3*b))/f
```

3.27 $\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 103

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=-\frac{(a^2+6ab+6b^2)\cot(e+fx)}{f}-\frac{2(a+b)(a+2b)\cot^3(e+fx)}{3f}-\frac{(a+b)^2\cot^5(e+fx)}{5f}+\frac{2b(a+2b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] -(a^2+6*a*b+6*b^2)*cot(f*x+e)/f-2/3*(a+b)*(a+2*b)*cot(f*x+e)^3/f-1/5*(a+b)^
2*cot(f*x+e)^5/f+2*b*(a+2*b)*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4217, 459}

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=-\frac{(a^2+6ab+6b^2)\cot(e+fx)}{f}+\frac{2b(a+2b)\tan(e+fx)}{f}-\frac{(a+b)^2\cot^5(e+fx)}{5f}-\frac{2(a+b)(a+2b)\cot^3(e+fx)}{3f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[In] Int[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(((a^2 + 6*a*b + 6*b^2)*Cot[e + f*x])/f) - (2*(a + b)*(a + 2*b)*Cot[e + f*
x]^3)/(3*f) - ((a + b)^2*Cot[e + f*x]^5)/(5*f) + (2*b*(a + 2*b)*Tan[e + f*x
])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2(a+b+bx^2)^2}{x^6}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(2b(a+2b) + \frac{(a+b)^2}{x^6} + \frac{2(a+b)(a+2b)}{x^4} + \frac{a^2+6ab+6b^2}{x^2} + b^2x^2\right)dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a^2+6ab+6b^2)\cot(e+fx)}{f} - \frac{2(a+b)(a+2b)\cot^3(e+fx)}{3f} - \frac{(a+b)^2\cot^5(e+fx)}{5f} + \frac{2b(a+2b)\tan(e+fx)}{f} + \frac{b^2\tan^3(e+fx)}{3f}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 353 vs. 2(103) = 206.

Time = 2.67 (sec) , antiderivative size = 353, normalized size of antiderivative = 3.43

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx =$$

$$-\frac{\csc(e)\csc^5(e+fx)\sec(e)\sec^3(e+fx)\left(20a(5a+12b)\sin(2e) - 32(2a^2+9ab+12b^2)\sin(2fx) - 24a^2 \text{s}\right.}{}$$

```
[In] Integrate[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/1920*(Csc[e]*Csc[e + f*x]^5*Sec[e]*Sec[e + f*x]^3*(20*a*(5*a + 12*b)*Sin
[2*e] - 32*(2*a^2 + 9*a*b + 12*b^2)*Sin[2*f*x] - 24*a^2*Sin[2*(e + f*x)] -
108*a*b*Sin[2*(e + f*x)] - 54*b^2*Sin[2*(e + f*x)] + 8*a^2*Sin[4*(e + f*x)]
 + 36*a*b*Sin[4*(e + f*x)] + 18*b^2*Sin[4*(e + f*x)] + 8*a^2*Sin[6*(e + f*x
)] + 36*a*b*Sin[6*(e + f*x)] + 18*b^2*Sin[6*(e + f*x)] - 4*a^2*Sin[8*(e + f
*x)] - 18*a*b*Sin[8*(e + f*x)] - 9*b^2*Sin[8*(e + f*x)] + 8*a^2*Sin[2*(e +
2*f*x)] + 96*a*b*Sin[2*(e + 2*f*x)] + 128*b^2*Sin[2*(e + 2*f*x)] + 40*a^2*S
in[4*e + 2*f*x] + 8*a^2*Sin[4*e + 6*f*x] + 96*a*b*Sin[4*e + 6*f*x] + 128*b^
2*Sin[4*e + 6*f*x] - 4*a^2*Sin[6*e + 8*f*x] - 48*a*b*Sin[6*e + 8*f*x] - 64*
b^2*Sin[6*e + 8*f*x]))/f
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.50

method	result
parallelrisch	$\frac{\left((-13a^2-36ab-48b^2)\cos(2fx+2e)+(a^2+12ab+16b^2)\cos(4fx+4e)+(a^2+12ab+16b^2)\cos(6fx+6e)+(-\frac{1}{2}a^2-6ab-8b^2)\right.}{960f(\cos(3fx+3e)+3\cos(fx+e))}$
derivativedivides	$\frac{a^2\left(-\frac{8}{15}-\frac{\csc(fx+e)^4}{5}-\frac{4\csc(fx+e)^2}{15}\right)\cot(fx+e)+2ab\left(-\frac{1}{5\sin(fx+e)^5\cos(fx+e)}-\frac{2}{5\sin(fx+e)^3\cos(fx+e)}+\frac{8}{5\sin(fx+e)\cos(fx\right.}{}$
default	$\frac{a^2\left(-\frac{8}{15}-\frac{\csc(fx+e)^4}{5}-\frac{4\csc(fx+e)^2}{15}\right)\cot(fx+e)+2ab\left(-\frac{1}{5\sin(fx+e)^5\cos(fx+e)}-\frac{2}{5\sin(fx+e)^3\cos(fx+e)}+\frac{8}{5\sin(fx+e)\cos(fx\right.}{}$
risch	$-\frac{16i\left(10a^2e^{10i(fx+e)}+25a^2e^{8i(fx+e)}+60ab\,e^{8i(fx+e)}+16a^2e^{6i(fx+e)}+72ab\,e^{6i(fx+e)}+96b^2e^{6i(fx+e)}-2a^2e^{4i(fx+e)}-24\right.}{15f\left(e^{2i(fx+e)}-1\right)^5\left(e^{2i(fx+e)}\right.}$

```
[In] int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/960*((-13*a^2-36*a*b-48*b^2)*cos(2*f*x+2*e)+(a^2+12*a*b+16*b^2)*cos(4*f*x
+4*e)+(a^2+12*a*b+16*b^2)*cos(6*f*x+6*e)+(-1/2*a^2-6*a*b-8*b^2)*cos(8*f*x+8
*e)-25/2*a^2-30*a*b)*csc(1/2*f*x+1/2*e)^5*sec(1/2*f*x+1/2*e)^5/f/(cos(3*f*x
+3*e)+3*cos(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.35

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{8\left(a^2+12\,ab+16\,b^2\right)\cos\left(fx+e\right)^8-20\left(a^2+12\,ab+16\,b^2\right)\cos\left(fx+e\right)^6+15\left(a^2+12\,ab+16\,b^2\right)\cos\left(\right.}{15\left(f\cos\left(fx+e\right)^7-2\,f\cos\left(fx+e\right)^5+f\cos\left(fx+e\right)^3\right)\sin\left(f\right.}$$

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a^2 + 12*a*b + 16*b^2)*cos(f*x + e)^8 - 20*(a^2 + 12*a*b + 16*b^2
)*cos(f*x + e)^6 + 15*(a^2 + 12*a*b + 16*b^2)*cos(f*x + e)^4 - 10*(3*a*b +
4*b^2)*cos(f*x + e)^2 - 5*b^2)/((f*cos(f*x + e)^7 - 2*f*cos(f*x + e)^5 + f*
cos(f*x + e)^3)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{5\,b^2\tan\left(fx+e\right)^3 + 30\left(ab+2\,b^2\right)\tan\left(fx+e\right) - \frac{15\left(a^2+6\,ab+6\,b^2\right)\tan(fx+e)^4+10\left(a^2+3\,ab+2\,b^2\right)\tan(fx+e)^2+3\,a^2+6\,ab+}{\tan(fx+e)^5}}{15\,f}$$

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(5*b^2*tan(f*x + e)^3 + 30*(a*b + 2*b^2)*tan(f*x + e) - (15*(a^2 + 6*a
*b + 6*b^2)*tan(f*x + e)^4 + 10*(a^2 + 3*a*b + 2*b^2)*tan(f*x + e)^2 + 3*a^
2 + 6*a*b + 3*b^2)/tan(f*x + e)^5)/f
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.37

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{5\,b^2\tan\left(fx+e\right)^3 + 30\,ab\tan\left(fx+e\right) + 60\,b^2\tan\left(fx+e\right) - \frac{15\,a^2\tan(fx+e)^4+90\,ab\tan(fx+e)^4+90\,b^2\tan(fx+e)^4+10}{}}{15\,f}$$

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(5*b^2*tan(f*x + e)^3 + 30*a*b*tan(f*x + e) + 60*b^2*tan(f*x + e) - (1
5*a^2*tan(f*x + e)^4 + 90*a*b*tan(f*x + e)^4 + 90*b^2*tan(f*x + e)^4 + 10*a
^2*tan(f*x + e)^2 + 30*a*b*tan(f*x + e)^2 + 20*b^2*tan(f*x + e)^2 + 3*a^2 +
 6*a*b + 3*b^2)/tan(f*x + e)^5)/f
```

Mupad [B] (verification not implemented)

Time = 18.75 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.05

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{b^2\tan(e+f\,x)^3}{3\,f}-\frac{\frac{2\,a\,b}{5}+\tan(e+f\,x)^4\,(a^2+6\,a\,b+6\,b^2)+\frac{a^2}{5}+\frac{b^2}{5}+\tan(e+f\,x)^2\left(\frac{2\,a^2}{3}+2\,a\,b+\frac{4\,b^2}{3}\right)}{f\tan\left(e+f\,x\right)^5}+\frac{2\,b\tan(e+f\,x)\,\left(a+2\,b\right)}{f}$$

```
[In] int((a + b/cos(e + f*x)^2)^2/sin(e + f*x)^6,x)
```

```
[Out] (b^2*tan(e + f*x)^3)/(3*f) - ((2*a*b)/5 + tan(e + f*x)^4*(6*a*b + a^2 + 6*b
^2) + a^2/5 + b^2/5 + tan(e + f*x)^2*(2*a*b + (2*a^2)/3 + (4*b^2)/3))/(f*ta
n(e + f*x)^5) + (2*b*tan(e + f*x)*(a + 2*b))/f
```

3.28 $\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 98

$$\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}(a+b)^2\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{7/2}f} - \frac{(a+b)^2\cos(e+fx)}{a^3f} + \frac{(2a+b)\cos^3(e+fx)}{3a^2f} - \frac{\cos^5(e+fx)}{5af}$$

```
[Out] -(a+b)^2*cos(f*x+e)/a^3/f+1/3*(2*a+b)*cos(f*x+e)^3/a^2/f-1/5*cos(f*x+e)^5/a
/f+(a+b)^2*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(7/2)/f
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4218, 472, 211}

$$\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}(a+b)^2\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{7/2}f} - \frac{(a+b)^2\cos(e+fx)}{a^3f} + \frac{(2a+b)\cos^3(e+fx)}{3a^2f} - \frac{\cos^5(e+fx)}{5af}$$

```
[In] Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (Sqrt[b]*(a + b)^2*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(a^(7/2)*f) - ((
a + b)^2*Cos[e + f*x])/(a^3*f) + ((2*a + b)*Cos[e + f*x]^3)/(3*a^2*f) - Cos
[e + f*x]^5/(5*a*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^2(1-x^2)^2}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{(a+b)^2}{a^3} - \frac{(2a+b)x^2}{a^2} + \frac{x^4}{a} + \frac{-a^2b-2ab^2-b^3}{a^3(b+ax^2)}\right)dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{(a+b)^2\cos(e+fx)}{a^3f} + \frac{(2a+b)\cos^3(e+fx)}{3a^2f} - \frac{\cos^5(e+fx)}{5af} + \frac{(b(a+b)^2)\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{a^3f}$$

$$= \frac{\sqrt{b}(a+b)^2\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{7/2}f} - \frac{(a+b)^2\cos(e+fx)}{a^3f} + \frac{(2a+b)\cos^3(e+fx)}{3a^2f} - \frac{\cos^5(e+fx)}{5af}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.85 (sec) , antiderivative size = 425, normalized size of antiderivative = 4.34

$$\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\left(15(5a^3+64a^2b+128ab^2+64b^3)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\,\mathrm{ta}}{}\right.\right.}{}$$

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*(15*(5*a^3 + 64*a^2*b + 128*a*b^2 + 64*b^3)
*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[
(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f
*x)/2]))/Sqrt[b]] + 15*(5*a^3 + 64*a^2*b + 128*a*b^2 + 64*b^3)*ArcTan[((-Sq
rt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Co
s[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt
[b]] - 75*a^3*ArcTan[(Sqrt[a] - Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]] - 75
*a^3*ArcTan[(Sqrt[a] + Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]] - 8*Sqrt[a]*S
qrt[b]*Cos[e + f*x]*(89*a^2 + 220*a*b + 120*b^2 - 4*a*(7*a + 5*b)*Cos[2*(e
+ f*x)] + 3*a^2*Cos[4*(e + f*x)]))*Sec[e + f*x]^2)/(1920*a^(7/2)*Sqrt[b]*f*
(a + b*Sec[e + f*x]^2))
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{-\frac{\frac{\cos(fx+e)^5a^2}{5}-\frac{2a^2\cos(fx+e)^3}{3}-\frac{a\cos(fx+e)^3b}{3}+a^2\cos(fx+e)+2ab\cos(fx+e)+b^2\cos(fx+e)}{a^3}+\frac{b\left(a^2+2ab+b^2\right)\arctan\left(\frac{a\cos(fx}{\sqrt{ab}}\right.}{a^3\sqrt{ab}}}{f}$
default	$\frac{-\frac{\frac{\cos(fx+e)^5a^2}{5}-\frac{2a^2\cos(fx+e)^3}{3}-\frac{a\cos(fx+e)^3b}{3}+a^2\cos(fx+e)+2ab\cos(fx+e)+b^2\cos(fx+e)}{a^3}+\frac{b\left(a^2+2ab+b^2\right)\arctan\left(\frac{a\cos(fx}{\sqrt{ab}}\right.}{a^3\sqrt{ab}}}{f}$
risch	$-\frac{5\,\mathrm{e}^{i(fx+e)}}{16af}-\frac{7\,\mathrm{e}^{i(fx+e)}b}{8a^2f}-\frac{\mathrm{e}^{i(fx+e)}b^2}{2a^3f}-\frac{5\,\mathrm{e}^{-i(fx+e)}}{16af}-\frac{7\,\mathrm{e}^{-i(fx+e)}b}{8a^2f}-\frac{\mathrm{e}^{-i(fx+e)}b^2}{2a^3f}-\frac{i\sqrt{ab}\,\ln\left(\mathrm{e}^{2i(fx+e)}-\right.}{2a}$

```
[In] int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^3*(1/5*cos(f*x+e)^5*a^2-2/3*a^2*cos(f*x+e)^3-1/3*a*cos(f*x+e)^3*b
+a^2*cos(f*x+e)+2*a*b*cos(f*x+e)+b^2*cos(f*x+e))+b*(a^2+2*a*b+b^2)/a^3/(a*b
)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.34

$$\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[-\frac{6\,a^2\cos\left(fx+e\right)^5-10\,(2\,a^2+ab)\cos\left(fx+e\right)^3-15\,(a^2+2\,ab+b^2)\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos(fx+e)^2+2\,a\sqrt{-\frac{b}{a}}\cos(}{a\cos(fx+e)^2+b}\right.}{30\,a^3f}\right.$$
$$-\frac{3\,a^2\cos\left(fx+e\right)^5-5\,(2\,a^2+ab)\cos\left(fx+e\right)^3-15\,(a^2+2\,ab+b^2)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}\cos(fx+e)}{b}\right)+15\,(a^2}{15\,a^3f}$$

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/30*(6*a^2*cos(f*x + e)^5 - 10*(2*a^2 + a*b)*cos(f*x + e)^3 - 15*(a^2 +
2*a*b + b^2)*sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e
) - b)/(a*cos(f*x + e)^2 + b)) + 30*(a^2 + 2*a*b + b^2)*cos(f*x + e))/(a^3*
f), -1/15*(3*a^2*cos(f*x + e)^5 - 5*(2*a^2 + a*b)*cos(f*x + e)^3 - 15*(a^2
+ 2*a*b + b^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) + 15*(a^2 + 2*a
*b + b^2)*cos(f*x + e))/(a^3*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04

$$\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{15\,(a^2b+2\,ab^2+b^3)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} - \frac{3\,a^2\cos(fx+e)^5-5\,(2\,a^2+ab)\cos(fx+e)^3+15\,(a^2+2\,ab+b^2)\cos(fx+e)}{a^3}}{15\,f}$$

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(15*(a^2*b + 2*a*b^2 + b^3)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b
)*a^3) - (3*a^2*cos(f*x + e)^5 - 5*(2*a^2 + a*b)*cos(f*x + e)^3 + 15*(a^2 +
 2*a*b + b^2)*cos(f*x + e))/a^3)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(86) = 172.

Time = 0.31 (sec) , antiderivative size = 349, normalized size of antiderivative = 3.56

$$\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{15\,(a^2b+2\,ab^2+b^3)\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{\sqrt{ab}a^3} - \frac{2\left(8\,a^2+25\,ab+15\,b^2-\frac{40\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{110\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{60\,b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+8\right.}{}}{}$$

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*(a^2*b + 2*a*b^2 + b^3)*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*b)*c
os(f*x + e) + sqrt(a*b)))/(sqrt(a*b)*a^3) - 2*(8*a^2 + 25*a*b + 15*b^2 - 40
*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 110*a*b*(cos(f*x + e) - 1)/(co
s(f*x + e) + 1) - 60*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 80*a^2*(co
s(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 160*a*b*(cos(f*x + e) - 1)^2/(cos(
f*x + e) + 1)^2 + 90*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 90*a*b
*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 60*b^2*(cos(f*x + e) - 1)^3/(c
os(f*x + e) + 1)^3 + 15*a*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 15*
b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/(a^3*((cos(f*x + e) - 1)/(co
s(f*x + e) + 1) - 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 18.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.26

$$\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\cos\left(e+f\,x\right)^3\,\left(\frac{b}{3\,a^2}+\frac{2}{3\,a}\right)}{f}-\frac{\cos\left(e+f\,x\right)^5}{5\,a\,f} -\frac{\cos\left(e+f\,x\right)\,\left(\frac{1}{a}+\frac{b\,\left(\frac{b}{a^2}+\frac{2}{a}\right)}{a}\right)}{f} +\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,\cos(e+f\,x)\,{(a+b)}^2}{a^2\,b+2\,a\,b^2+b^3}\right)\,{\left(a+b\right)}^2}{a^{7/2}\,f}$$

```
[In] int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2),x)
```

```
[Out] (cos(e + f*x)^3*(b/(3*a^2) + 2/(3*a)))/f - cos(e + f*x)^5/(5*a*f) - (cos(e
+ f*x)*(1/a + (b*(b/a^2 + 2/a))/a))/f + (b^(1/2)*atan((a^(1/2)*b^(1/2)*cos(
e + f*x)*(a + b)^2)/(2*a*b^2 + a^2*b + b^3))*(a + b)^2)/(a^(7/2)*f)
```

3.29 $\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 71

$$\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}(a+b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{5/2}f} - \frac{(a+b)\cos(e+fx)}{a^2f} + \frac{\cos^3(e+fx)}{3af}$$

```
[Out] -(a+b)*cos(f*x+e)/a^2/f+1/3*cos(f*x+e)^3/a/f+(a+b)*arctan(cos(f*x+e)*a^(1/2
)/b^(1/2))*b^(1/2)/a^(5/2)/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4218, 470, 327, 211}

$$\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}(a+b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{5/2}f} - \frac{(a+b)\cos(e+fx)}{a^2f} + \frac{\cos^3(e+fx)}{3af}$$

```
[In] Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (Sqrt[b]*(a + b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(a^(5/2)*f) - ((a
+ b)*Cos[e + f*x])/(a^2*f) + Cos[e + f*x]^3/(3*a*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^2(1-x^2)}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{f}\\
&= \frac{\cos^3(e+fx)}{3af} - \frac{(a+b)\text{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{af}\\
&= -\frac{(a+b)\cos(e+fx)}{a^2f} + \frac{\cos^3(e+fx)}{3af} + \frac{(b(a+b))\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{a^2f}\\
&= \frac{\sqrt{b}(a+b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{5/2}f} - \frac{(a+b)\cos(e+fx)}{a^2f} + \frac{\cos^3(e+fx)}{3af}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.21 (sec) , antiderivative size = 376, normalized size of antiderivative = 5.30

$$\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\left(3(a^2+8ab+8b^2)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\right.}{\sqrt{b}}\right.\right.}{}$$

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*(3*(a^2 + 8*a*b + 8*b^2)*ArcTan[((-Sqrt[a]
- I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(
Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] +
 3*(a^2 + 8*a*b + 8*b^2)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I
*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[
e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] - 3*a^2*ArcTan[(Sqrt[a] - Sqrt[a
+ b]*Tan[(e + f*x)/2])/Sqrt[b]] - 3*a^2*ArcTan[(Sqrt[a] + Sqrt[a + b]*Tan[(
e + f*x)/2])/Sqrt[b]] + 4*Sqrt[a]*Sqrt[b]*Cos[e + f*x]*(-5*a - 6*b + a*Cos[
2*(e + f*x)]))*Sec[e + f*x]^2)/(48*a^(5/2)*Sqrt[b]*f*(a + b*Sec[e + f*x]^2)
)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\frac{\frac{a\cos(fx+e)^3}{3}-\cos(fx+e)a-b\cos(fx+e)}{a^2}+\frac{b(a+b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}}{f}$
default	$\frac{\frac{\frac{a\cos(fx+e)^3}{3}-\cos(fx+e)a-b\cos(fx+e)}{a^2}+\frac{b(a+b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}}{f}$
risch	$-\frac{3\,e^{i(fx+e)}}{8af}-\frac{e^{i(fx+e)}b}{2a^2f}-\frac{3\,e^{-i(fx+e)}}{8af}-\frac{e^{-i(fx+e)}b}{2a^2f}+\frac{i\sqrt{ab}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{ab}\,e^{i(fx+e)}}{a}+1\right)}{2a^2f}+\frac{i\sqrt{ab}\,\ln\left(e\right.}{}$

```
[In] int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^2*(1/3*a*cos(f*x+e)^3-cos(f*x+e)*a-b*cos(f*x+e))+b*(a+b)/a^2/(a*b)
^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.17

$$\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[\frac{2\,a\cos\left(fx+e\right)^3+3\,(a+b)\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos(fx+e)^2+2\,a\sqrt{-\frac{b}{a}}\cos(fx+e)-b}{a\cos(fx+e)^2+b}\right)-6\,(a+b)\cos\left(fx+e\right)}{6\,a^2f},\ \frac{a\cos\left(fx\right.}{}\right.$$

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*a*cos(f*x + e)^3 + 3*(a + b)*sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*
a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) - 6*(a + b)*cos(f*x
+ e))/(a^2*f), 1/3*(a*cos(f*x + e)^3 + 3*(a + b)*sqrt(b/a)*arctan(a*sqrt(b/
a)*cos(f*x + e)/b) - 3*(a + b)*cos(f*x + e))/(a^2*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{3\,(ab+b^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^2}+\frac{a\cos(fx+e)^3-3\,(a+b)\cos(fx+e)}{a^2}}{3\,f}$$

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(a*b + b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^2) + (a*co
s(f*x + e)^3 - 3*(a + b)*cos(f*x + e))/a^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20

$$\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(ab+b^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^2f} + \frac{a^2f^5\cos(fx+e)^3 - 3\,a^2f^5\cos(fx+e) - 3\,abf^5\cos(fx+e)}{3\,a^3f^6}$$

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] (a*b + b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^2*f) + 1/3*(a^2*f
^5*cos(f*x + e)^3 - 3*a^2*f^5*cos(f*x + e) - 3*a*b*f^5*cos(f*x + e))/(a^3*f
^6)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\cos(e+f\,x)^3}{3\,a\,f} - \frac{\cos(e+f\,x)\,\left(\frac{b}{a^2}+\frac{1}{a}\right)}{f} + \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,\cos(e+f\,x)\,(a+b)}{b^2+a\,b}\right)\,(a+b)}{a^{5/2}\,f}$$

```
[In] int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2),x)
```

```
[Out] cos(e + f*x)^3/(3*a*f) - (cos(e + f*x)*(b/a^2 + 1/a))/f + (b^(1/2)*atan((a^
(1/2)*b^(1/2)*cos(e + f*x)*(a + b))/(a*b + b^2))*(a + b))/(a^(5/2)*f)
```

3.30 $\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 47

$$\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx=\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{3/2}f}-\frac{\cos(e+fx)}{af}$$

```
[Out] -cos(f*x+e)/a/f+arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(3/2)/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4218, 327, 211}

$$\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx=\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{3/2}f}-\frac{\cos(e+fx)}{af}$$

```
[In] Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(a^(3/2)*f) - Cos[e + f*x]
/(a*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)}{af} + \frac{b\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{af}$$

$$= \frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{3/2}f} - \frac{\cos(e+fx)}{af}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.71 (sec) , antiderivative size = 329, normalized size of antiderivative = 7.00

$$\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\left((a+4b)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)+(a+\right.}{}$$

```
[In] Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (((a + 4*b)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*
Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e]
)^2]*Tan[(f*x)/2]))/Sqrt[b]] + (a + 4*b)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*
Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a
 + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] - a*ArcTan[(Sqrt[
a] - Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]] - a*ArcTan[(Sqrt[a] + Sqrt[a +
b]*Tan[(e + f*x)/2])/Sqrt[b]] - 4*Sqrt[a]*Sqrt[b]*Cos[e + f*x])*(a + 2*b +
a*Cos[2*(e + f*x)]))/(8*a^(3/2)*Sqrt[b]*f*(b + a*Cos[e + f*x]^2))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

method	result	si
derivativedivides	$\frac{-\frac{1}{a\sec(fx+e)}-\frac{b\arctan\left(\frac{\sec(fx+e)b}{\sqrt{ab}}\right)}{a\sqrt{ab}}}{f}$	4
default	$\frac{-\frac{1}{a\sec(fx+e)}-\frac{b\arctan\left(\frac{\sec(fx+e)b}{\sqrt{ab}}\right)}{a\sqrt{ab}}}{f}$	4
risch	$-\frac{e^{i(fx+e)}}{2af}-\frac{e^{-i(fx+e)}}{2af}-\frac{i\sqrt{ab}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{ab}\,e^{i(fx+e)}}{a}+1\right)}{2a^2f}+\frac{i\sqrt{ab}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{ab}\,e^{i(fx+e)}}{a}+1\right)}{2a^2f}$	1

```
[In] int(sin(f*x+e)/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(-1/a/sec(f*x+e)-b/a/(a*b)^(1/2)*arctan(sec(f*x+e)*b/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.51

$$\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx = \left[\frac{\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos(fx+e)^2+2\,a\sqrt{-\frac{b}{a}}\cos(fx+e)-b}{a\cos(fx+e)^2+b}\right)-2\cos(fx+e)}{2\,af}, \frac{\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}\cos(fx+e)}{b}\right)-\cos(fx+e)}{af}\right]$$

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
[Out] [1/2*(sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/
(a*cos(f*x + e)^2 + b)) - 2*cos(f*x + e))/(a*f), (sqrt(b/a)*arctan(a*sqrt(b
/a)*cos(f*x + e)/b) - cos(f*x + e))/(a*f)]
```

Sympy [F]

$$\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2),x)
[Out] Integral(sin(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{\cos(fx+e)}{a}}{f}$$

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] (b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a) - cos(f*x + e)/a)/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}af} - \frac{\cos(fx+e)}{af}$$

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a*f) - cos(f*x + e)/(a*f)
```

Mupad [B] (verification not implemented)

Time = 17.89 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\cos(e+f\,x)}{\sqrt{b}}\right)}{a^{3/2}\,f} - \frac{\cos(e+f\,x)}{a\,f}$$

```
[In] int(sin(e + f*x)/(a + b/cos(e + f*x)^2),x)
[Out] (b^(1/2)*atan((a^(1/2)*cos(e + f*x))/b^(1/2)))/(a^(3/2)*f) - cos(e + f*x)/(
a*f)
```

3.31 $\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 55

$$\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)f} - \frac{\operatorname{arctanh}(\cos(e+fx))}{(a+b)f}$$

```
[Out] -arctanh(cos(f*x+e))/(a+b)/f+arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/(a+
b)/f/a^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4218, 492, 212, 211}

$$\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}f(a+b)} - \frac{\operatorname{arctanh}(\cos(e+fx))}{f(a+b)}$$

```
[In] Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(Sqrt[a]*(a + b)*f) - ArcT
anh[Cos[e + f*x]]/((a + b)*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{f} \\
&= -\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(e+fx)\right)}{(a+b)f} + \frac{b\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{(a+b)f} \\
&= \frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)f} - \frac{\text{arctanh}(\cos(e+fx))}{(a+b)f}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.24 (sec) , antiderivative size = 239, normalized size of antiderivative = 4.35

$$
\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx
$$

$$
= \frac{\frac{\sqrt{b}\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)}{\sqrt{a}} + \frac{\sqrt{b}\arctan\left(\frac{\left(-\sqrt{a}+i\sqrt{a+b}\sqrt{(\cos(e}{}\right.}{}}{(a+b}
$$

```
[In] Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((Sqrt[b]*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Si
n[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^
2]*Tan[(f*x)/2]))/Sqrt[b]])/Sqrt[a] + (Sqrt[b]*ArcTan[((-Sqrt[a] + I*Sqrt[a
 + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] +
Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/Sqrt[a] -
Log[Cos[(e + f*x)/2]] + Log[Sin[(e + f*x)/2]])/((a + b)*f)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{\frac{\ln(-1+\cos(fx+e))}{2a+2b}+\frac{b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}-\frac{\ln(1+\cos(fx+e))}{2a+2b}}{f}$
default	$\frac{\frac{\ln(-1+\cos(fx+e))}{2a+2b}+\frac{b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}-\frac{\ln(1+\cos(fx+e))}{2a+2b}}{f}$
risch	$-\frac{\ln(e^{i(fx+e)}+1)}{f(a+b)}+\frac{\ln(e^{i(fx+e)}-1)}{f(a+b)}+\frac{i\sqrt{ab}\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{ab}\,e^{i(fx+e)}}{a}+1\right)}{2a(a+b)f}-\frac{i\sqrt{ab}\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{ab}\,e^{i(fx+}}{a}\right.}{2a(a+b)f}$

```
[In] int(csc(f*x+e)/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(2*a+2*b)*ln(-1+cos(f*x+e))+b/(a+b)/(a*b)^(1/2)*arctan(a*cos(f*x+e)/
(a*b)^(1/2))-1/(2*a+2*b)*ln(1+cos(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.84

$$\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx = \left[\frac{\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos(fx+e)^2+2a\sqrt{-\frac{b}{a}}\cos(fx+e)-b}{a\cos(fx+e)^2+b}\right)-\log\left(\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)+\log\left(-\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)}{2(a+b)f}, \frac{2\sqrt{}}{}\right.$$

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/
(a*cos(f*x + e)^2 + b)) - log(1/2*cos(f*x + e) + 1/2) + log(-1/2*cos(f*x +
e) + 1/2))/((a + b)*f), 1/2*(2*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b)
 - log(1/2*cos(f*x + e) + 1/2) + log(-1/2*cos(f*x + e) + 1/2))/((a + b)*f)]
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2),x)
[Out] Integral(csc(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16

$$\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{2\,b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}(a+b)} - \frac{\log(\cos(fx+e)+1)}{a+b} + \frac{\log(\cos(fx+e)-1)}{a+b}}{2\,f}$$

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] 1/2*(2*b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*(a + b)) - log(cos(f*x
 + e) + 1)/(a + b) + log(cos(f*x + e) - 1)/(a + b))/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.55

$$\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{2\,b\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{\sqrt{ab}(a+b)} - \frac{\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a+b}}{2\,f}$$

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] -1/2*(2*b*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*b)*cos(f*x + e) + sqrt(a*b))
)/(sqrt(a*b)*(a + b)) - log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(
a + b))/f
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.24

$$\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\operatorname{atanh}\left(\frac{\cos(e+f\,x)\,\left(2\,a^3+2\,a\,b^2\right)-\frac{\cos(e+f\,x)\,\left(8\,a^5+8\,a^4\,b-8\,a^3\,b^2-8\,a^2\,b^3\right)}{4\,{(a+b)}^2}}{2\,a\,b\,(a+b)}\right)}{f\,\left(a+b\right)} - \frac{\operatorname{atanh}\left(\frac{\cos(e+f\,x)\,\sqrt{-a\,b}}{b}\right)\,\sqrt{-a\,b}}{f\,\left(a^2+b\,a\right)}$$

```
[In] int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] - atanh((cos(e + f*x)*(2*a*b^2 + 2*a^3) - (cos(e + f*x)*(8*a^4*b + 8*a^5 -
8*a^2*b^3 - 8*a^3*b^2))/(4*(a + b)^2))/(2*a*b*(a + b)))/(f*(a + b)) - (atan
h((cos(e + f*x)*(-a*b)^(1/2))/b)*(-a*b)^(1/2))/(f*(a*b + a^2))
```

3.32 $\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx=\frac{\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{(a+b)^2f}-\frac{(a-b)\text{arctanh}(\cos(e+fx))}{2(a+b)^2f}-\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f}$$

```
[Out] -1/2*(a-b)*arctanh(cos(f*x+e))/(a+b)^2/f-1/2*cot(f*x+e)*csc(f*x+e)/(a+b)/f+
arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*a^(1/2)*b^(1/2)/(a+b)^2/f
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4218, 482, 536, 212, 211}

$$\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx=\frac{\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{f(a+b)^2}-\frac{(a-b)\text{arctanh}(\cos(e+fx))}{2f(a+b)^2}-\frac{\cot(e+fx)\csc(e+fx)}{2f(a+b)}$$

```
[In] Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/((a + b)^2*f) - ((
a - b)*ArcTanh[Cos[e + f*x]])/(2*(a + b)^2*f) - (Cot[e + f*x]*Csc[e + f*x])
/(2*(a + b)*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f} + \frac{\text{Subst}\left(\int \frac{b-ax^2}{(1-x^2)(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{2(a+b)f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f} - \frac{(a-b)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(e+fx)\right)}{2(a+b)^2f} + \frac{(ab)\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{(a+b)^2f}$$

$$= \frac{\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{(a+b)^2f} - \frac{(a-b)\text{arctanh}(\cos(e+fx))}{2(a+b)^2f} - \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.92 (sec) , antiderivative size = 371, normalized size of antiderivative = 4.31

$$\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left(-8\sqrt{a}\sqrt{b}\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{\cdots}\right.}{\sqrt{b}}\right.\right.}{}$$

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/16*((a + 2*b + a*Cos[2*(e + f*x)])*(-8*Sqrt[a]*Sqrt[b]*ArcTan[((-Sqrt[a]
 - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*
(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]
- 8*Sqrt[a]*Sqrt[b]*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[
e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] -
I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] + a*Csc[(e + f*x)/2]^2 + b*Csc[(e + f*
x)/2]^2 + 4*a*Log[Cos[(e + f*x)/2]] - 4*b*Log[Cos[(e + f*x)/2]] - 4*a*Log[S
in[(e + f*x)/2]] + 4*b*Log[Sin[(e + f*x)/2]] - a*Sec[(e + f*x)/2]^2 - b*Sec
[(e + f*x)/2]^2)*Sec[e + f*x]^2)/((a + b)^2*f*(a + b*Sec[e + f*x]^2))
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{\frac{1}{(4a+4b)(-1+\cos(fx+e))}+\frac{(a-b)\ln(-1+\cos(fx+e))}{4(a+b)^2}+\frac{ab\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a+b)^2\sqrt{ab}}+\frac{1}{(4a+4b)(1+\cos(fx+e))}+\frac{(-a+b)\ln(1+\cos(fx+e)}{4(a+b)^2}}{f}$
default	$\frac{\frac{1}{(4a+4b)(-1+\cos(fx+e))}+\frac{(a-b)\ln(-1+\cos(fx+e))}{4(a+b)^2}+\frac{ab\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a+b)^2\sqrt{ab}}+\frac{1}{(4a+4b)(1+\cos(fx+e))}+\frac{(-a+b)\ln(1+\cos(fx+e)}{4(a+b)^2}}{f}$
risch	$\frac{e^{3i(fx+e)}+e^{i(fx+e)}}{f(a+b)\left(e^{2i(fx+e)}-1\right)^2}-\frac{\ln\left(e^{i(fx+e)}+1\right)a}{2f(a^2+2ab+b^2)}+\frac{\ln\left(e^{i(fx+e)}+1\right)b}{2f(a^2+2ab+b^2)}+\frac{\ln\left(e^{i(fx+e)}-1\right)a}{2f(a^2+2ab+b^2)}-\frac{\ln\left(e^{i(fx+e)}-1\right)b}{2f(a^2+2ab+b^2)}-\frac{i\sqrt{ab}}{}$

```
[In] int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(4*a+4*b)/(-1+cos(f*x+e))+1/4*(a-b)/(a+b)^2*ln(-1+cos(f*x+e))+a*b/(a
+b)^2/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))+1/(4*a+4*b)/(1+cos(f*x+e
```

```
))+1/4/(a+b)^2*(-a+b)*ln(1+cos(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 327, normalized size of antiderivative = 3.80

$$\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[\frac{2\sqrt{-ab}\left(\cos\left(fx+e\right)^2-1\right)\log\left(-\frac{a\cos(fx+e)^2+2\sqrt{-ab}\cos(fx+e)-b}{a\cos(fx+e)^2+b}\right)+2\left(a+b\right)\cos\left(fx+e\right)-\left(\left(a-b\right)\cos\left(fx\right.\right.}{4\left(\left(a^2+2\,ab+b^2\right)f\cos\left(fx+\right.\right.}\right.$$

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
[Out] [1/4*(2*sqrt(-a*b)*(cos(f*x + e)^2 - 1)*log(-(a*cos(f*x + e)^2 + 2*sqrt(-a*
b)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 2*(a + b)*cos(f*x + e) - ((a
 - b)*cos(f*x + e)^2 - a + b)*log(1/2*cos(f*x + e) + 1/2) + ((a - b)*cos(f*
x + e)^2 - a + b)*log(-1/2*cos(f*x + e) + 1/2))/((a^2 + 2*a*b + b^2)*f*cos(
f*x + e)^2 - (a^2 + 2*a*b + b^2)*f), 1/4*(4*sqrt(a*b)*(cos(f*x + e)^2 - 1)*
arctan(sqrt(a*b)*cos(f*x + e)/b) + 2*(a + b)*cos(f*x + e) - ((a - b)*cos(f*
x + e)^2 - a + b)*log(1/2*cos(f*x + e) + 1/2) + ((a - b)*cos(f*x + e)^2 - a
 + b)*log(-1/2*cos(f*x + e) + 1/2))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 -
 (a^2 + 2*a*b + b^2)*f)]
```

Sympy [F]

$$\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\csc^3\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

```
[In] integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
[Out] Integral(csc(e + f*x)**3/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.49

$$\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\frac{4\,ab\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a^2+2\,ab+b^2)\sqrt{ab}}-\frac{(a-b)\log(\cos(fx+e)+1)}{a^2+2\,ab+b^2}+\frac{(a-b)\log(\cos(fx+e)-1)}{a^2+2\,ab+b^2}+\frac{2\,\cos(fx+e)}{(a+b)\cos(fx+e)^2-a-b}}{4\,f}$$

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*a*b*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b))
 - (a - b)*log(cos(f*x + e) + 1)/(a^2 + 2*a*b + b^2) + (a - b)*log(cos(f*x
+ e) - 1)/(a^2 + 2*a*b + b^2) + 2*cos(f*x + e)/((a + b)*cos(f*x + e)^2 - a
- b))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(74) = 148$.

Time = 0.32 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.43

$$\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{8\,ab\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{(a^2+2\,ab+b^2)\sqrt{ab}} - \frac{2\,(a-b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^2+2\,ab+b^2} - \frac{\left(a+b-\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)(\cos(fx+e)+1)}{(a^2+2\,ab+b^2)(\cos(fx+e)-1)} + \frac{}{(a+}}{8\,f}$$

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/8*(8*a*b*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*b)*cos(f*x + e) + sqrt(a*b
)))/((a^2 + 2*a*b + b^2)*sqrt(a*b)) - 2*(a - b)*log(abs(-cos(f*x + e) + 1)/
abs(cos(f*x + e) + 1))/(a^2 + 2*a*b + b^2) - (a + b - 2*a*(cos(f*x + e) - 1
)/(cos(f*x + e) + 1) + 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))*(cos(f*x
+ e) + 1)/((a^2 + 2*a*b + b^2)*(cos(f*x + e) - 1)) + (cos(f*x + e) - 1)/((a
 + b)*(cos(f*x + e) + 1)))/f
```

Mupad [B] (verification not implemented)

Time = 18.89 (sec) , antiderivative size = 392, normalized size of antiderivative = 4.56

$$\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{2\,a\,\cos(e+f\,x)+2\,b\,\cos(e+f\,x)-a\,\ln(\cos(e+f\,x)-1)+a\,\ln(\cos(e+f\,x)+1)+b\,\ln(\cos(e+}{}$$

```
[In] int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] -(2*a*cos(e + f*x) + 2*b*cos(e + f*x) - atan((a^3*cos(e + f*x)*(-a*b)^(1/2)
*1i + a*b^2*cos(e + f*x)*(-a*b)^(1/2)*1i + a^2*b*cos(e + f*x)*(-a*b)^(1/2)*
2i)/(a*b^3 + a^3*b + 2*a^2*b^2))*(-a*b)^(1/2)*4i - a*log(cos(e + f*x) - 1)
+ a*log(cos(e + f*x) + 1) + b*log(cos(e + f*x) - 1) - b*log(cos(e + f*x) +
1) + cos(e + f*x)^2*atan((a^3*cos(e + f*x)*(-a*b)^(1/2)*1i + a*b^2*cos(e +
f*x)*(-a*b)^(1/2)*1i + a^2*b*cos(e + f*x)*(-a*b)^(1/2)*2i)/(a*b^3 + a^3*b +
```

```
2*a^2*b^2))*(-a*b)^(1/2)*4i + a*log(cos(e + f*x) - 1)*cos(e + f*x)^2 - a*l
og(cos(e + f*x) + 1)*cos(e + f*x)^2 - b*log(cos(e + f*x) - 1)*cos(e + f*x)^
2 + b*log(cos(e + f*x) + 1)*cos(e + f*x)^2)/(4*a^2*f + 4*b^2*f - 4*a^2*f*co
s(e + f*x)^2 - 4*b^2*f*cos(e + f*x)^2 + 8*a*b*f - 8*a*b*f*cos(e + f*x)^2)
```

3.33 $\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 129

$$\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{a^{3/2}\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{(a+b)^3 f} - \frac{(3a^2-6ab-b^2)\,\text{arctanh}(\cos(e+fx))}{8(a+b)^3 f} - \frac{(3a-b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2 f} - \frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b)f}$$

```
[Out] -1/8*(3*a^2-6*a*b-b^2)*arctanh(cos(f*x+e))/(a+b)^3/f-1/8*(3*a-b)*cot(f*x+e)
*csc(f*x+e)/(a+b)^2/f-1/4*cot(f*x+e)*csc(f*x+e)^3/(a+b)/f+a^(3/2)*arctan(co
s(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/(a+b)^3/f
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4218, 482, 541, 536, 212, 211}

$$\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{a^{3/2}\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{f(a+b)^3} - \frac{(3a^2-6ab-b^2)\,\text{arctanh}(\cos(e+fx))}{8f(a+b)^3} - \frac{\cot(e+fx)\csc^3(e+fx)}{4f(a+b)} - \frac{(3a-b)\cot(e+fx)\csc(e+fx)}{8f(a+b)^2}$$

```
[In] Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/((a + b)^3*f) - ((
3*a^2 - 6*a*b - b^2)*ArcTanh[Cos[e + f*x]])/(8*(a + b)^3*f) - ((3*a - b)*Co
t[e + f*x]*Csc[e + f*x])/(8*(a + b)^2*f) - (Cot[e + f*x]*Csc[e + f*x]^3)/(4
*(a + b)*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
```

```
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)^3(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b)f} + \frac{\text{Subst}\left(\int \frac{b-3ax^2}{(1-x^2)^2(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{4(a+b)f}$$

$$= -\frac{(3a-b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f} - \frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b)f} + \frac{\text{Subst}\left(\int \frac{b(5a+b)-a(3a-b)x^2}{(1-x^2)(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{8(a+b)^2f}$$

$$= -\frac{(3a-b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f} - \frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b)f} + \frac{(a^2b)\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{(a+b)^3f} - \frac{(3a^2-6ab-b^2)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(e+fx)\right)}{8(a+b)^3f}$$

$$= \frac{a^{3/2}\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{(a+b)^3f} - \frac{(3a^2-6ab-b^2)\,\text{arctanh}(\cos(e+fx))}{8(a+b)^3f} - \frac{(3a-b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f} - \frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b)f}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.21 (sec) , antiderivative size = 549, normalized size of antiderivative = 4.26

$$\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left(-64a^{3/2}\sqrt{b}\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+\right.}{\sqrt{b}}\right.\right.}{}$$

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/128*((a + 2*b + a*Cos[2*(e + f*x)])*(-64*a^(3/2)*Sqrt[b]*ArcTan[((-Sqrt[
a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e
]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]
] - 64*a^(3/2)*Sqrt[b]*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*S
in[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e]
 - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] + 6*a^2*Csc[(e + f*x)/2]^2 + 4*a*b*
Csc[(e + f*x)/2]^2 - 2*b^2*Csc[(e + f*x)/2]^2 + a^2*Csc[(e + f*x)/2]^4 + 2*
a*b*Csc[(e + f*x)/2]^4 + b^2*Csc[(e + f*x)/2]^4 + 24*a^2*Log[Cos[(e + f*x)/
2]] - 48*a*b*Log[Cos[(e + f*x)/2]] - 8*b^2*Log[Cos[(e + f*x)/2]] - 24*a^2*L
og[Sin[(e + f*x)/2]] + 48*a*b*Log[Sin[(e + f*x)/2]] + 8*b^2*Log[Sin[(e + f*
x)/2]] - 6*a^2*Sec[(e + f*x)/2]^2 - 4*a*b*Sec[(e + f*x)/2]^2 + 2*b^2*Sec[(e
 + f*x)/2]^2 - a^2*Sec[(e + f*x)/2]^4 - 2*a*b*Sec[(e + f*x)/2]^4 - b^2*Sec[
(e + f*x)/2]^4)*Sec[e + f*x]^2)/((a + b)^3*f*(a + b*Sec[e + f*x]^2))
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{-\frac{1}{2(8a+8b)(-1+\cos(fx+e))^2}-\frac{-3a+b}{16(a+b)^2(-1+\cos(fx+e))}+\frac{(3a^2-6ab-b^2)\ln(-1+\cos(fx+e))}{16(a+b)^3}+\frac{a^2 b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a+b)^3\sqrt{ab}}+\frac{\cdots}{2(8a+8\cdots}}{f}$
default	$\frac{-\frac{1}{2(8a+8b)(-1+\cos(fx+e))^2}-\frac{-3a+b}{16(a+b)^2(-1+\cos(fx+e))}+\frac{(3a^2-6ab-b^2)\ln(-1+\cos(fx+e))}{16(a+b)^3}+\frac{a^2 b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a+b)^3\sqrt{ab}}+\frac{\cdots}{2(8a+8\cdots}}{f}$
risch	$\frac{3a\,e^{7i(fx+e)}-b\,e^{7i(fx+e)}-11a\,e^{5i(fx+e)}-7b\,e^{5i(fx+e)}-11a\,e^{3i(fx+e)}-7b\,e^{3i(fx+e)}+3a\,e^{i(fx+e)}-b\,e^{i(fx+e)}}{4f(a+b)^2\left(e^{2i(fx+e)}-1\right)^4}-\frac{3\ln(e^{\cdots}}{8f(a^3+3\cdots}$

```
[In] int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/2/(8*a+8*b)/(-1+cos(f*x+e))^2-1/16*(-3*a+b)/(a+b)^2/(-1+cos(f*x+e))
+1/16*(3*a^2-6*a*b-b^2)/(a+b)^3*ln(-1+cos(f*x+e))+a^2*b/(a+b)^3/(a*b)^(1/2)
*arctan(a*cos(f*x+e)/(a*b)^(1/2))+1/2/(8*a+8*b)/(1+cos(f*x+e))^2-1/16*(-3*a
+b)/(a+b)^2/(1+cos(f*x+e))+1/16/(a+b)^3*(-3*a^2+6*a*b+b^2)*ln(1+cos(f*x+e))
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(115) = 230.

Time = 0.32 (sec) , antiderivative size = 693, normalized size of antiderivative = 5.37

$$\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$=\left[\frac{2\left(3a^2+2ab-b^2\right)\cos(fx+e)^3+8\left(a\cos(fx+e)^4-2a\cos(fx+e)^2+a\right)\sqrt{-ab}\log\left(-\frac{a\cos(fx+e)^2+2\sqrt{\cdots}}{a\cos(fx\cdots}\right.}{\cdots}\right.$$

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^3 + 8*(a*cos(f*x + e)^4 - 2*a*c
os(f*x + e)^2 + a)*sqrt(-a*b)*log(-(a*cos(f*x + e)^2 + 2*sqrt(-a*b)*cos(f*x
 + e) - b)/(a*cos(f*x + e)^2 + b)) - 2*(5*a^2 + 6*a*b + b^2)*cos(f*x + e) -
 ((3*a^2 - 6*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a^2 - 6*a*b - b^2)*cos(f*x +
e)^2 + 3*a^2 - 6*a*b - b^2)*log(1/2*cos(f*x + e) + 1/2) + ((3*a^2 - 6*a*b -
 b^2)*cos(f*x + e)^4 - 2*(3*a^2 - 6*a*b - b^2)*cos(f*x + e)^2 + 3*a^2 - 6*a
*b - b^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*
cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^2 + (a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*f), 1/16*(2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^3
 + 16*(a*cos(f*x + e)^4 - 2*a*cos(f*x + e)^2 + a)*sqrt(a*b)*arctan(sqrt(a*b
)*cos(f*x + e)/b) - 2*(5*a^2 + 6*a*b + b^2)*cos(f*x + e) - ((3*a^2 - 6*a*b
- b^2)*cos(f*x + e)^4 - 2*(3*a^2 - 6*a*b - b^2)*cos(f*x + e)^2 + 3*a^2 - 6*
a*b - b^2)*log(1/2*cos(f*x + e) + 1/2) + ((3*a^2 - 6*a*b - b^2)*cos(f*x + e
)^4 - 2*(3*a^2 - 6*a*b - b^2)*cos(f*x + e)^2 + 3*a^2 - 6*a*b - b^2)*log(-1/
2*cos(f*x + e) + 1/2))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^4 -
2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*f)]
```

Sympy [F]

$$\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

```
[In] integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(csc(e + f*x)**5/(a + b*sec(e + f*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(115) = 230$.

Time = 0.26 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.79

$$\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{16\,a^2b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{ab}} - \frac{(3\,a^2-6\,ab-b^2)\log(\cos(fx+e)+1)}{a^3+3\,a^2b+3\,ab^2+b^3} + \frac{(3\,a^2-6\,ab-b^2)\log(\cos(fx+e)-1)}{a^3+3\,a^2b+3\,ab^2+b^3} + \frac{2\left((3\,a-b)\cos(fx+e)\right.}{(a^2+2\,ab+b^2)\cos(fx+e)^4-2\,(a^2+}}{16\,f}$$

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/16*(16*a^2*b*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*sqrt(a*b)) - (3*a^2 - 6*a*b - b^2)*log(cos(f*x + e) + 1)/(a^3 + 3*a^2
```

```
*b + 3*a*b^2 + b^3) + (3*a^2 - 6*a*b - b^2)*log(cos(f*x + e) - 1)/(a^3 + 3*
a^2*b + 3*a*b^2 + b^3) + 2*((3*a - b)*cos(f*x + e)^3 - (5*a + b)*cos(f*x +
e))/((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e
)^2 + a^2 + 2*a*b + b^2))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(115) = 230$.

Time = 0.33 (sec) , antiderivative size = 389, normalized size of antiderivative = 3.02

$$\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{64\,a^2b\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{ab}} - \frac{4\,(3\,a^2-6\,ab-b^2)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^3+3\,a^2b+3\,ab^2+b^3} + \frac{\frac{8\,a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} - \frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{a^2+2\,ab+b^2} +}{64\,f}$$

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/64*(64*a^2*b*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*b)*cos(f*x + e) + sqrt
(a*b)))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)) - 4*(3*a^2 - 6*a*b - b^
2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(a^3 + 3*a^2*b + 3*a*b
^2 + b^3) + (8*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - a*(cos(f*x + e) -
1)^2/(cos(f*x + e) + 1)^2 - b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a
^2 + 2*a*b + b^2) + (a^2 + 2*a*b + b^2 - 8*a^2*(cos(f*x + e) - 1)/(cos(f*x
+ e) + 1) - 8*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 18*a^2*(cos(f*x +
 e) - 1)^2/(cos(f*x + e) + 1)^2 - 36*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e)
 + 1)^2 - 6*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) +
1)^2/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(cos(f*x + e) - 1)^2))/f
```

Mupad [B] (verification not implemented)

Time = 22.21 (sec) , antiderivative size = 870, normalized size of antiderivative = 6.74

$$\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{3\,a^2\cos(e+f\,x)^3 - b^2\,\cos(e+f\,x) - 5\,a^2\,\cos(e+f\,x) - b^2\cos(e+f\,x)^3 - 3\,a^2\,\mathrm{atanh}(\cos(e+f\,x)) + b^2}{}$$

```
[In] int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] (atan((a^5*cos(e + f*x)*(-a^3*b)^(1/2)*9i + a^2*b^3*cos(e + f*x)*(-a^3*b)^(
1/2)*12i + a^3*b^2*cos(e + f*x)*(-a^3*b)^(1/2)*30i + a*b^4*cos(e + f*x)*(-a
^3*b)^(1/2)*1i + a^4*b*cos(e + f*x)*(-a^3*b)^(1/2)*28i)/(9*a^6*b + a^2*b^5
```

```
+ 12*a^3*b^4 + 30*a^4*b^3 + 28*a^5*b^2))*(-a^3*b)^(1/2)*8i - 5*a^2*cos(e +
f*x) - b^2*cos(e + f*x) + 3*a^2*cos(e + f*x)^3 - b^2*cos(e + f*x)^3 - 3*a^2
*atanh(cos(e + f*x)) + b^2*atanh(cos(e + f*x)) - atan((a^5*cos(e + f*x)*(-a
^3*b)^(1/2)*9i + a^2*b^3*cos(e + f*x)*(-a^3*b)^(1/2)*12i + a^3*b^2*cos(e +
f*x)*(-a^3*b)^(1/2)*30i + a*b^4*cos(e + f*x)*(-a^3*b)^(1/2)*1i + a^4*b*cos(
e + f*x)*(-a^3*b)^(1/2)*28i)/(9*a^6*b + a^2*b^5 + 12*a^3*b^4 + 30*a^4*b^3 +
 28*a^5*b^2))*cos(e + f*x)^2*(-a^3*b)^(1/2)*16i + atan((a^5*cos(e + f*x)*(-
a^3*b)^(1/2)*9i + a^2*b^3*cos(e + f*x)*(-a^3*b)^(1/2)*12i + a^3*b^2*cos(e +
 f*x)*(-a^3*b)^(1/2)*30i + a*b^4*cos(e + f*x)*(-a^3*b)^(1/2)*1i + a^4*b*cos
(e + f*x)*(-a^3*b)^(1/2)*28i)/(9*a^6*b + a^2*b^5 + 12*a^3*b^4 + 30*a^4*b^3
+ 28*a^5*b^2))*cos(e + f*x)^4*(-a^3*b)^(1/2)*8i - 6*a*b*cos(e + f*x) + 6*a^
2*cos(e + f*x)^2*atanh(cos(e + f*x)) - 3*a^2*cos(e + f*x)^4*atanh(cos(e + f
*x)) - 2*b^2*cos(e + f*x)^2*atanh(cos(e + f*x)) + b^2*cos(e + f*x)^4*atanh(
cos(e + f*x)) + 2*a*b*cos(e + f*x)^3 + 6*a*b*atanh(cos(e + f*x)) - 12*a*b*c
os(e + f*x)^2*atanh(cos(e + f*x)) + 6*a*b*cos(e + f*x)^4*atanh(cos(e + f*x)
))/(8*a^3*f + 8*b^3*f - 16*a^3*f*cos(e + f*x)^2 + 8*a^3*f*cos(e + f*x)^4 -
16*b^3*f*cos(e + f*x)^2 + 8*b^3*f*cos(e + f*x)^4 + 24*a*b^2*f + 24*a^2*b*f
- 48*a*b^2*f*cos(e + f*x)^2 - 48*a^2*b*f*cos(e + f*x)^2 + 24*a*b^2*f*cos(e
+ f*x)^4 + 24*a^2*b*f*cos(e + f*x)^4)
```

3.34 $\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 166

$$\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(5a^3+30a^2b+40ab^2+16b^3)\,x}{16a^4} - \frac{\sqrt{b}(a+b)^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^4f} - \frac{(11a^2+18ab+8b^2)\cos(e+fx)\sin(e+fx)}{16a^3f} + \frac{(3a+2b)\cos^3(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af}$$

```
[Out] 1/16*(5*a^3+30*a^2*b+40*a*b^2+16*b^3)*x/a^4-1/16*(11*a^2+18*a*b+8*b^2)*cos(
f*x+e)*sin(f*x+e)/a^3/f+1/8*(3*a+2*b)*cos(f*x+e)^3*sin(f*x+e)/a^2/f+1/6*cos
(f*x+e)^3*sin(f*x+e)^3/a/f-(a+b)^(5/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2
))*b^(1/2)/a^4/f
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used

= {4217, 481, 592, 541, 536, 209, 211}

$$\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\sqrt{b}(a+b)^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^4f} + \frac{(3a+2b)\sin(e+fx)\cos^3(e+fx)}{8a^2f} - \frac{(11a^2+18ab+8b^2)\sin(e+fx)\cos(e+fx)}{16a^3f} + \frac{x(5a^3+30a^2b+40ab^2+16b^3)}{16a^4} + \frac{\sin^3(e+fx)\cos^3(e+fx)}{6af}$$

```
[In] Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((5*a^3 + 30*a^2*b + 40*a*b^2 + 16*b^3)*x)/(16*a^4) - (Sqrt[b]*(a + b)^(5/2
)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a^4*f) - ((11*a^2 + 18*a*b +
 8*b^2)*Cos[e + f*x]*Sin[e + f*x])/(16*a^3*f) + ((3*a + 2*b)*Cos[e + f*x]^3
*Sin[e + f*x])/(8*a^2*f) + (Cos[e + f*x]^3*Sin[e + f*x]^3)/(6*a*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^4(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af} - \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)-3(2a+b)x^2)}{(1+x^2)^3(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{6af}$$

$$= \frac{(3a+2b)\cos^3(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af} - \frac{\text{Subst}\left(\int \frac{3(a+b)(3a+2b)-3(8a^2+13ab+6b^2)x^2}{(1+x^2)^2(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{24a^2f}$$

$$= -\frac{(11a^2+18ab+8b^2)\cos(e+fx)\sin(e+fx)}{16a^3f} + \frac{(3a+2b)\cos^3(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af} + \frac{\text{Subst}\left(\int \frac{3(a+b)(a+2b)(5a+4b)-3b(11a^2+18ab+8b^2)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{48a^3f}$$

$$= -\frac{(11a^2+18ab+8b^2)\cos(e+fx)\sin(e+fx)}{16a^3f} + \frac{(3a+2b)\cos^3(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af} - \frac{(b(a+b)^3)\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{a^4f} + \frac{(5a^3+30a^2b+40ab^2+16b^3)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{16a^4f}$$

$$= \frac{(5a^3+30a^2b+40ab^2+16b^3)\,x}{16a^4} - \frac{\sqrt{b}(a+b)^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^4f} - \frac{(11a^2+18ab+8b^2)\cos(e+fx)\sin(e+fx)}{16a^3f} + \frac{(3a+2b)\cos^3(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.57 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.15

$$\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(3\sqrt{b}(9a^4+136a^3b+384a^2b^2+384ab^3+128b^4)\arctan\left(\frac{\sec(fx)(\cdots}{\cdots}\right)\cdots\right)}{\cdots}$$

```
[In] Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(3*Sqrt[b]*(9*a^4 + 136*a^3*
b + 384*a^2*b^2 + 384*a*b^3 + 128*b^4)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2
*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos
[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[b*(Cos[e] - I*Sin[e])^4
]*(3*a^3*(9*a + 8*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]] + 2*Sqrt[b]
*Sqrt[a + b]*(-12*a^3*e + 60*a^3*f*x + 360*a^2*b*f*x + 480*a*b^2*f*x + 192*
b^3*f*x - 3*a*(15*a^2 + 32*a*b + 16*b^2)*Sin[2*(e + f*x)] + 3*a^2*(3*a + 2*
b)*Sin[4*(e + f*x)] - a^3*Sin[6*(e + f*x)]))))/(768*a^4*Sqrt[b]*Sqrt[a + b]
*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{-\frac{b(a+b)^3 \arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^4\sqrt{(a+b)b}}+\frac{\frac{\left(-\frac{9}{8}a^2b-\frac{1}{2}a\,b^2-\frac{11}{16}a^3\right)\tan(fx+e)^5+\left(-2a^2b-a\,b^2-\frac{5}{6}a^3\right)\tan(fx+e)^3+\left(-\frac{5}{16}a^3-\frac{7}{8}a^2b-\frac{1}{2}a\,b^2\right)}{\left(1+\tan(fx+e)^2\right)^3}}{a^4}}{f}$
default	$\frac{-\frac{b(a+b)^3 \arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^4\sqrt{(a+b)b}}+\frac{\frac{\left(-\frac{9}{8}a^2b-\frac{1}{2}a\,b^2-\frac{11}{16}a^3\right)\tan(fx+e)^5+\left(-2a^2b-a\,b^2-\frac{5}{6}a^3\right)\tan(fx+e)^3+\left(-\frac{5}{16}a^3-\frac{7}{8}a^2b-\frac{1}{2}a\,b^2\right)}{\left(1+\tan(fx+e)^2\right)^3}}{a^4}}{f}$
risch	$\frac{5x}{16a}+\frac{15xb}{8a^2}+\frac{5x\,b^2}{2a^3}+\frac{x\,b^3}{a^4}+\frac{15i\mathrm{e}^{2i(fx+e)}}{128af}+\frac{i\mathrm{e}^{2i(fx+e)}b}{4a^2f}-\frac{i\mathrm{e}^{-2i(fx+e)}b^2}{8a^3f}-\frac{i\mathrm{e}^{-2i(fx+e)}b}{4a^2f}-\frac{15i\mathrm{e}^{-2i(fx+e)}}{128af}$

```
[In] int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-b/a^4*(a+b)^3/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))+1/
a^4*(((-9/8*a^2*b-1/2*a*b^2-11/16*a^3)*tan(f*x+e)^5+(-2*a^2*b-a*b^2-5/6*a^3
)*tan(f*x+e)^3+(-5/16*a^3-7/8*a^2*b-1/2*a*b^2)*tan(f*x+e))/(1+tan(f*x+e)^2)
^3+1/16*(5*a^3+30*a^2*b+40*a*b^2+16*b^3)*arctan(tan(f*x+e))))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.58

$$\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{3\left(5a^3+30a^2b+40ab^2+16b^3\right)fx+12\left(a^2+2ab+b^2\right)\sqrt{-ab-b^2}\log\left(\frac{\left(a^2+8ab+8b^2\right)\cos(fx+e)^4-2\left(3ab+4b^2\right)\cdots}{a^2\cdots}\right.}{\cdots}\right.$$

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(5*a^3 + 30*a^2*b + 40*a*b^2 + 16*b^3)*f*x + 12*(a^2 + 2*a*b + b^2
)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4
*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-
a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 +
 b^2)) - (8*a^3*cos(f*x + e)^5 - 2*(13*a^3 + 6*a^2*b)*cos(f*x + e)^3 + 3*(1
1*a^3 + 18*a^2*b + 8*a*b^2)*cos(f*x + e))*sin(f*x + e))/(a^4*f), 1/48*(3*(5
*a^3 + 30*a^2*b + 40*a*b^2 + 16*b^3)*f*x + 24*(a^2 + 2*a*b + b^2)*sqrt(a*b
+ b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x +
 e)*sin(f*x + e))) - (8*a^3*cos(f*x + e)^5 - 2*(13*a^3 + 6*a^2*b)*cos(f*x +
```

```
 e)^3 + 3*(11*a^3 + 18*a^2*b + 8*a*b^2)*cos(f*x + e))*sin(f*x + e))/(a^4*f)
]
```

Sympy [F]

$$\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

```
[In] integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
[Out] Integral(sin(e + f*x)**6/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.26

$$\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx =$$

$$-\frac{\frac{3\,(11\,a^2+18\,ab+8\,b^2)\tan(fx+e)^5+8\,(5\,a^2+12\,ab+6\,b^2)\tan(fx+e)^3+3\,(5\,a^2+14\,ab+8\,b^2)\tan(fx+e)}{a^3\tan(fx+e)^6+3\,a^3\tan(fx+e)^4+3\,a^3\tan(fx+e)^2+a^3} - \frac{3\,(5\,a^3+30\,a^2b+40\,ab^2+16\,b^3)(fx+}{a^4}}{48\,f}$$

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] -1/48*((3*(11*a^2 + 18*a*b + 8*b^2)*tan(f*x + e)^5 + 8*(5*a^2 + 12*a*b + 6*
b^2)*tan(f*x + e)^3 + 3*(5*a^2 + 14*a*b + 8*b^2)*tan(f*x + e))/(a^3*tan(f*x
 + e)^6 + 3*a^3*tan(f*x + e)^4 + 3*a^3*tan(f*x + e)^2 + a^3) - 3*(5*a^3 + 3
0*a^2*b + 40*a*b^2 + 16*b^3)*(f*x + e)/a^4 + 48*(a^3*b + 3*a^2*b^2 + 3*a*b^
3 + b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^4))/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.43

$$\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{\frac{3\,(5\,a^3+30\,a^2b+40\,ab^2+16\,b^3)(fx+e)}{a^4} - \frac{48\,(a^3b+3\,a^2b^2+3\,ab^3+b^4)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,a^4} - \frac{33\,a^2\tan(fx+e)^5+54\,ab}{}}{48}$$

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/48*(3*(5*a^3 + 30*a^2*b + 40*a*b^2 + 16*b^3)*(f*x + e)/a^4 - 48*(a^3*b +
3*a^2*b^2 + 3*a*b^3 + b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*
tan(f*x + e)/sqrt(a*b + b^2)))/(sqrt(a*b + b^2)*a^4) - (33*a^2*tan(f*x + e)
^5 + 54*a*b*tan(f*x + e)^5 + 24*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3
+ 96*a*b*tan(f*x + e)^3 + 48*b^2*tan(f*x + e)^3 + 15*a^2*tan(f*x + e) + 42*
a*b*tan(f*x + e) + 24*b^2*tan(f*x + e))/((tan(f*x + e)^2 + 1)^3*a^3))/f
```

Mupad [B] (verification not implemented)

Time = 19.65 (sec) , antiderivative size = 1448, normalized size of antiderivative = 8.72

$$\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

```
[In] int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2),x)
```

```
[Out] (atanh((25*b^3*tan(e + f*x)*(- 5*a*b^5 - a^5*b - b^6 - 10*a^2*b^4 - 10*a^3*
b^3 - 5*a^4*b^2)^(1/2))/(128*((227*a*b^5)/128 + (217*b^6)/128 + (119*a^2*b^
4)/128 + (25*a^3*b^3)/128 + (13*b^7)/(16*a) + (5*b^8)/(32*a^2))) + (11*b^4*
tan(e + f*x)*(- 5*a*b^5 - a^5*b - b^6 - 10*a^2*b^4 - 10*a^3*b^3 - 5*a^4*b^2
)^(1/2))/(32*((217*a*b^6)/128 + (13*b^7)/16 + (227*a^2*b^5)/128 + (119*a^3*
b^4)/128 + (25*a^4*b^3)/128 + (5*b^8)/(32*a))) + (5*b^5*tan(e + f*x)*(- 5*a
*b^5 - a^5*b - b^6 - 10*a^2*b^4 - 10*a^3*b^3 - 5*a^4*b^2)^(1/2))/(32*((13*a
*b^7)/16 + (5*b^8)/32 + (217*a^2*b^6)/128 + (227*a^3*b^5)/128 + (119*a^4*b^
4)/128 + (25*a^5*b^3)/128)))*(-b*(a + b)^5)^(1/2))/(a^4*f) - (atan((((((((51
2*a^8*b^5 + 1408*a^9*b^4 + 1216*a^10*b^3 + 320*a^11*b^2)/(256*a^9) - (tan(e
 + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(a*b^2*40i + a^2*b*30i + a^3*5i + b^3
*16i))/(4096*a^10))*(a*b^2*40i + a^2*b*30i + a^3*5i + b^3*16i))/(32*a^4) -
(tan(e + f*x)*(2816*a*b^8 + 512*b^9 + 6400*a^2*b^7 + 7680*a^3*b^6 + 5140*a^
4*b^5 + 1836*a^5*b^4 + 281*a^6*b^3))/(128*a^6))*(a*b^2*40i + a^2*b*30i + a^
3*5i + b^3*16i)*1i)/(32*a^4) - (((((512*a^8*b^5 + 1408*a^9*b^4 + 1216*a^10*
b^3 + 320*a^11*b^2)/(256*a^9) + (tan(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)
*(a*b^2*40i + a^2*b*30i + a^3*5i + b^3*16i))/(4096*a^10))*(a*b^2*40i + a^2*
b*30i + a^3*5i + b^3*16i))/(32*a^4) + (tan(e + f*x)*(2816*a*b^8 + 512*b^9 +
 6400*a^2*b^7 + 7680*a^3*b^6 + 5140*a^4*b^5 + 1836*a^5*b^4 + 281*a^6*b^3))/
(128*a^6))*(a*b^2*40i + a^2*b*30i + a^3*5i + b^3*16i)*1i)/(32*a^4))/((992*a
*b^10 + 128*b^11 + 3344*a^2*b^9 + 6380*a^3*b^8 + 7496*a^4*b^7 + 5511*a^5*b^
6 + 2445*a^6*b^5 + 585*a^7*b^4 + 55*a^8*b^3)/(128*a^9) + (((((512*a^8*b^5 +
 1408*a^9*b^4 + 1216*a^10*b^3 + 320*a^11*b^2)/(256*a^9) - (tan(e + f*x)*(20
48*a^8*b^3 + 1024*a^9*b^2)*(a*b^2*40i + a^2*b*30i + a^3*5i + b^3*16i))/(409
6*a^10))*(a*b^2*40i + a^2*b*30i + a^3*5i + b^3*16i))/(32*a^4) - (tan(e + f*
x)*(2816*a*b^8 + 512*b^9 + 6400*a^2*b^7 + 7680*a^3*b^6 + 5140*a^4*b^5 + 183
6*a^5*b^4 + 281*a^6*b^3))/(128*a^6))*(a*b^2*40i + a^2*b*30i + a^3*5i + b^3*
16i))/(32*a^4) + (((((512*a^8*b^5 + 1408*a^9*b^4 + 1216*a^10*b^3 + 320*a^11
```

```
*b^2)/(256*a^9) + (tan(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(a*b^2*40i +
a^2*b*30i + a^3*5i + b^3*16i))/(4096*a^10))*(a*b^2*40i + a^2*b*30i + a^3*5i
 + b^3*16i))/(32*a^4) + (tan(e + f*x)*(2816*a*b^8 + 512*b^9 + 6400*a^2*b^7
+ 7680*a^3*b^6 + 5140*a^4*b^5 + 1836*a^5*b^4 + 281*a^6*b^3))/(128*a^6))*(a*
b^2*40i + a^2*b*30i + a^3*5i + b^3*16i))/(32*a^4)))*(a*b^2*40i + a^2*b*30i
+ a^3*5i + b^3*16i)*1i)/(16*a^4*f) - ((tan(e + f*x)*(14*a*b + 5*a^2 + 8*b^2
))/(16*a^3) + (tan(e + f*x)^3*(12*a*b + 5*a^2 + 6*b^2))/(6*a^3) + (tan(e +
f*x)^5*(18*a*b + 11*a^2 + 8*b^2))/(16*a^3))/(f*(3*tan(e + f*x)^2 + 3*tan(e
+ f*x)^4 + tan(e + f*x)^6 + 1))
```

3.35 $\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(3a^2+12ab+8b^2)\,x}{8a^3} - \frac{\sqrt{b}(a+b)^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^3f} - \frac{(5a+4b)\cos(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af}$$

```
[Out] 1/8*(3*a^2+12*a*b+8*b^2)*x/a^3-1/8*(5*a+4*b)*cos(f*x+e)*sin(f*x+e)/a^2/f+1/
4*cos(f*x+e)^3*sin(f*x+e)/a/f-(a+b)^(3/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(
1/2))*b^(1/2)/a^3/f
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4217, 481, 541, 536, 209, 211}

$$\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\sqrt{b}(a+b)^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^3f} - \frac{(5a+4b)\sin(e+fx)\cos(e+fx)}{8a^2f} + \frac{x(3a^2+12ab+8b^2)}{8a^3} + \frac{\sin(e+fx)\cos^3(e+fx)}{4af}$$

```
[In] Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((3*a^2 + 12*a*b + 8*b^2)*x)/(8*a^3) - (Sqrt[b]*(a + b)^(3/2)*ArcTan[(Sqrt[
b]*Tan[e + f*x])/Sqrt[a + b]])/(a^3*f) - ((5*a + 4*b)*Cos[e + f*x]*Sin[e +
f*x])/(8*a^2*f) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
```

```
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^3(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\cos^3(e+fx)\sin(e+fx)}{4af} - \frac{\text{Subst}\left(\int \frac{a+b+(b-4(a+b))x^2}{(1+x^2)^2(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{4af}\\
&= -\frac{(5a+4b)\cos(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af}\\
&\quad + \frac{\text{Subst}\left(\int \frac{(a+b)(3a+4b)-b(5a+4b)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{8a^2f}\\
&= -\frac{(5a+4b)\cos(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af}\\
&\quad - \frac{(b(a+b)^2)\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{a^3f}\\
&\quad + \frac{(3a^2+12ab+8b^2)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{8a^3f}\\
&= \frac{(3a^2+12ab+8b^2)\,x}{8a^3} - \frac{\sqrt{b}(a+b)^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^3f}\\
&\quad - \frac{(5a+4b)\cos(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.38 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.59

$$\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(\sqrt{b}(3a^3+34a^2b+64ab^2+32b^3)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((}{2\sqrt{a+b}\sqrt{b(\cos}\right.\right.}{}$$

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[b]*(3*a^3 + 34*a^2*b +
 64*a*b^2 + 32*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*S
```

```
in[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])
]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[b*(Cos[e] - I*Sin[e])^4]*(a^2*(3*a + 2*b)*
ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]] + Sqrt[b]*Sqrt[a + b]*(-2*a^2*e
+ 12*a^2*f*x + 48*a*b*f*x + 32*b^2*f*x - 8*a*(a + b)*Sin[2*(e + f*x)] + a^2
*Sin[4*(e + f*x)]))))/(64*a^3*Sqrt[b]*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)*
Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{-\frac{(a+b)^2 b \arctan\left(\frac{b \tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^3\sqrt{(a+b)b}}+\frac{\frac{\left(-\frac{1}{2}ab-\frac{5}{8}a^2\right)\tan(fx+e)^3+\left(-\frac{3}{8}a^2-\frac{1}{2}ab\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{\left(3a^2+12ab+8b^2\right)\arctan(\tan(fx+e))}{8}}{a^3}}{f}$
default	$\frac{-\frac{(a+b)^2 b \arctan\left(\frac{b \tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^3\sqrt{(a+b)b}}+\frac{\frac{\left(-\frac{1}{2}ab-\frac{5}{8}a^2\right)\tan(fx+e)^3+\left(-\frac{3}{8}a^2-\frac{1}{2}ab\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{\left(3a^2+12ab+8b^2\right)\arctan(\tan(fx+e))}{8}}{a^3}}{f}$
risch	$\frac{3x}{8a}+\frac{3xb}{2a^2}+\frac{x\,b^2}{a^3}+\frac{i e^{2i(fx+e)}}{8af}+\frac{i e^{2i(fx+e)} b}{8a^2 f}-\frac{i e^{-2i(fx+e)}}{8af}-\frac{i e^{-2i(fx+e)} b}{8a^2 f}+\frac{\sqrt{-ab-b^2}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{}}{}\right.}{2f\,a^2}$

```
[In] int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-(a+b)^2*b/a^3/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))+1/
a^3*(((-1/2*a*b-5/8*a^2)*tan(f*x+e)^3+(-3/8*a^2-1/2*a*b)*tan(f*x+e))/(1+tan
(f*x+e)^2)^2+1/8*(3*a^2+12*a*b+8*b^2)*arctan(tan(f*x+e))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.84

$$\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \left[\frac{(3a^2+12ab+8b^2)fx+2\sqrt{-ab-b^2}(a+b)\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4-2(3ab+4b^2)\cos(fx+e)^2+4\left((a+2b)\cos(\right.}{a^2\cos(fx+e)^4+2ab\cos(fx+e)}\right.}{8a^3 f}\right.$$

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/8*((3*a^2 + 12*a*b + 8*b^2)*f*x + 2*sqrt(-a*b - b^2)*(a + b)*log(((a^2 +
 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a +
 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)
```

```
/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + (2*a^2*cos(f*x + e)^3
 - (5*a^2 + 4*a*b)*cos(f*x + e))*sin(f*x + e))/(a^3*f), 1/8*((3*a^2 + 12*a*
b + 8*b^2)*f*x + 4*sqrt(a*b + b^2)*(a + b)*arctan(1/2*((a + 2*b)*cos(f*x +
e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) + (2*a^2*cos(f*x + e
)^3 - (5*a^2 + 4*a*b)*cos(f*x + e))*sin(f*x + e))/(a^3*f)]
```

Sympy [F]

$$\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

```
[In] integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
[Out] Integral(sin(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.17

$$\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{(5a+4b)\tan(fx+e)^3+(3a+4b)\tan(fx+e)}{a^2\tan(fx+e)^4+2a^2\tan(fx+e)^2+a^2} - \frac{(3a^2+12ab+8b^2)(fx+e)}{a^3} + \frac{8(a^2b+2ab^2+b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^3}}{8f}$$

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] -1/8*(((5*a + 4*b)*tan(f*x + e)^3 + (3*a + 4*b)*tan(f*x + e))/(a^2*tan(f*x
+ e)^4 + 2*a^2*tan(f*x + e)^2 + a^2) - (3*a^2 + 12*a*b + 8*b^2)*(f*x + e)/a
^3 + 8*(a^2*b + 2*a*b^2 + b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt
((a + b)*b)*a^3))/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.30

$$\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{(3a^2+12ab+8b^2)(fx+e)}{a^3} - \frac{8(a^2b+2ab^2+b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}a^3} - \frac{5a\tan(fx+e)^3+4b\tan(fx+e)^3+3a\tan(fx+e}{\left(\tan(fx+e)^2+1\right)^2a^2}}{8f}$$

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/8*((3*a^2 + 12*a*b + 8*b^2)*(f*x + e)/a^3 - 8*(a^2*b + 2*a*b^2 + b^3)*(pi
*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))
/(sqrt(a*b + b^2)*a^3) - (5*a*tan(f*x + e)^3 + 4*b*tan(f*x + e)^3 + 3*a*tan
(f*x + e) + 4*b*tan(f*x + e))/((tan(f*x + e)^2 + 1)^2*a^2))/f
```

Mupad [B] (verification not implemented)

Time = 18.78 (sec) , antiderivative size = 494, normalized size of antiderivative = 4.22

$$\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\operatorname{atanh}\left(\frac{9\,b^3\tan(e+f\,x)\,\sqrt{-a^3\,b-3\,a^2\,b^2-3\,a\,b^3-b^4}}{32\,\left(\frac{13\,a\,b^4}{16}+\frac{25\,b^5}{32}+\frac{9\,a^2\,b^3}{32}+\frac{b^6}{4\,a}\right)}+\frac{b^4\tan(e+f\,x)\,\sqrt{-a^3\,b-3\,a^2\,b^2-3\,a\,b^3-b^4}}{4\,\left(\frac{9\,a^3\,b^3}{32}+\frac{13\,a^2\,b^4}{16}+\frac{25\,a\,b^5}{32}+\frac{b^6}{4}\right)}\right)\sqrt{-b\,(a+b)^3}}{a^3\,f} - \frac{\frac{\tan(e+f\,x)\,(3\,a+4\,b)}{8\,a^2}+\frac{\tan(e+f\,x)^3\,(5\,a+4\,b)}{8\,a^2}}{f\,\left(\tan\left(e+f\,x\right)^4+2\tan\left(e+f\,x\right)^2+1\right)} - \frac{\operatorname{atan}\left(\frac{159\,b^3\tan(e+f\,x)}{256\,\left(\frac{27\,a\,b^2}{256}+\frac{159\,b^3}{256}+\frac{75\,b^4}{64\,a}+\frac{29\,b^5}{32\,a^2}+\frac{b^6}{4\,a^3}\right)}+\frac{75\,b^4\tan(e+f\,x)}{64\,\left(\frac{159\,a\,b^3}{256}+\frac{75\,b^4}{64}+\frac{27\,a^2\,b^2}{256}+\frac{29\,b^5}{32\,a}+\frac{b^6}{4\,a^2}\right)}+\frac{29\,b^5\tan(e+f\,x)}{32\,\left(\frac{75\,a\,b^4}{64}+\frac{29\,b^5}{32}+\frac{159\,a^2\,b^3}{256}+\frac{27\,a^3\,b^2}{256}+\cdots\right)}\cdots\right)}{8\,a^3\cdots}$$

```
[In] int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2),x)
```

```
[Out] (atanh((9*b^3*tan(e + f*x)*(- 3*a*b^3 - a^3*b - b^4 - 3*a^2*b^2)^(1/2))/(32
*((13*a*b^4)/16 + (25*b^5)/32 + (9*a^2*b^3)/32 + b^6/(4*a))) + (b^4*tan(e +
 f*x)*(- 3*a*b^3 - a^3*b - b^4 - 3*a^2*b^2)^(1/2))/(4*((25*a*b^5)/32 + b^6/
4 + (13*a^2*b^4)/16 + (9*a^3*b^3)/32)))*(-b*(a + b)^3)^(1/2))/(a^3*f) - ((t
an(e + f*x)*(3*a + 4*b))/(8*a^2) + (tan(e + f*x)^3*(5*a + 4*b))/(8*a^2))/(f
*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1)) - (atan((159*b^3*tan(e + f*x))/(2
56*((27*a*b^2)/256 + (159*b^3)/256 + (75*b^4)/(64*a) + (29*b^5)/(32*a^2) +
b^6/(4*a^3))) + (75*b^4*tan(e + f*x))/(64*((159*a*b^3)/256 + (75*b^4)/64 +
(27*a^2*b^2)/256 + (29*b^5)/(32*a) + b^6/(4*a^2))) + (29*b^5*tan(e + f*x))/
(32*((75*a*b^4)/64 + (29*b^5)/32 + (159*a^2*b^3)/256 + (27*a^3*b^2)/256 + b
^6/(4*a))) + (b^6*tan(e + f*x))/(4*((29*a*b^5)/32 + b^6/4 + (75*a^2*b^4)/64
 + (159*a^3*b^3)/256 + (27*a^4*b^2)/256)) + (27*b^2*tan(e + f*x))/(256*((27
*b^2)/256 + (159*b^3)/(256*a) + (75*b^4)/(64*a^2) + (29*b^5)/(32*a^3) + b^6
/(4*a^4))))*(a*b*12i + a^2*3i + b^2*8i)*1i)/(8*a^3*f)
```

3.36 $\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a+2b)x}{2a^2} - \frac{\sqrt{b}\sqrt{a+b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^2f} - \frac{\cos(e+fx)\sin(e+fx)}{2af}$$

```
[Out] 1/2*(a+2*b)*x/a^2-1/2*cos(f*x+e)*sin(f*x+e)/a/f-arctan(b^(1/2)*tan(f*x+e)/(
a+b)^(1/2))*b^(1/2)*(a+b)^(1/2)/a^2/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4217, 482, 536, 209, 211}

$$\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\sqrt{b}\sqrt{a+b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^2f} + \frac{x(a+2b)}{2a^2} - \frac{\sin(e+fx)\cos(e+fx)}{2af}$$

```
[In] Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b)*x)/(2*a^2) - (Sqrt[b]*Sqrt[a + b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/
Sqrt[a + b]])/(a^2*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*a*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{\cos(e+fx)\sin(e+fx)}{2af} + \frac{\text{Subst}\left(\int \frac{a+b-bx^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{2af}\\
&= -\frac{\cos(e+fx)\sin(e+fx)}{2af} - \frac{(b(a+b))\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{a^2f}\\
&\quad + \frac{(a+2b)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{2a^2f}
\end{aligned}
$$

$$
= \frac{(a+2b)x}{2a^2} - \frac{\sqrt{b}\sqrt{a+b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^2 f} - \frac{\cos(e+fx)\sin(e+fx)}{2af}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 245, normalized size of antiderivative = 3.22

$$
\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx
$$

$$
= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(\frac{\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}f} - \frac{-4(a+2b)x-\frac{\left(a^2+8ab+8b^2\right)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(\cdots}{2\sqrt{a+b}\sqrt{b(\cdots}}\right.}{\sqrt{a+b}f\sqrt{b(\cos\cdots}}}{}\right.}{16\left(a+b\sec^2(e+fx)\right)}
$$

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(ArcTan[(Sqrt[b]*Tan[e + f*x
])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*f) - (-4*(a + 2*b)*x - ((a^2 + 8*a*b +
 8*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a
*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e]
- I*Sin[2*e]))/(Sqrt[a + b]*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (2*a*Cos[2*f
*x]*Sin[2*e])/f + (2*a*Cos[2*e]*Sin[2*f*x])/f)/a^2))/(16*(a + b*Sec[e + f*x
]^2))
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{-\frac{(a+b)b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^2\sqrt{(a+b)b}}+\frac{-\frac{a\tan(fx+e)}{2\left(1+\tan(fx+e)^2\right)}+\frac{(a+2b)\arctan(\tan(fx+e))}{2}}{a^2}}{f}$
default	$\frac{-\frac{(a+b)b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^2\sqrt{(a+b)b}}+\frac{-\frac{a\tan(fx+e)}{2\left(1+\tan(fx+e)^2\right)}+\frac{(a+2b)\arctan(\tan(fx+e))}{2}}{a^2}}{f}$
risch	$\frac{x}{2a}+\frac{xb}{a^2}+\frac{i\mathrm{e}^{2i(fx+e)}}{8af}-\frac{i\mathrm{e}^{-2i(fx+e)}}{8af}-\frac{\sqrt{-ab-b^2}\,\ln\left(\mathrm{e}^{2i(fx+e)}-\frac{2i\sqrt{-ab-b^2}-a-2b}{a}\right)}{2f\,a^2}+\frac{\sqrt{-ab-b^2}\,\ln\left(\mathrm{e}^{2i(fx+e)}+\cdots\right.}{2f\,a^2}$

```
[In] int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-(a+b)*b/a^2/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))+1/a^
2*(-1/2*a*tan(f*x+e)/(1+tan(f*x+e)^2)+1/2*(a+2*b)*arctan(tan(f*x+e))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 257, normalized size of antiderivative = 3.38

$$\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \left[\frac{2\,(a+2\,b)fx - 2\,a\cos\left(fx+e\right)\sin\left(fx+e\right) + \sqrt{-ab-b^2}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2}{a^2\cos(fx+e)}\right.}{4\,a^2f}\right.$$

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a + 2*b)*f*x - 2*a*cos(f*x + e)*sin(f*x + e) + sqrt(-a*b - b^2)*lo
g(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2
+ 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x +
e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(a^2*f), 1/2*
((a + 2*b)*f*x - a*cos(f*x + e)*sin(f*x + e) + sqrt(a*b + b^2)*arctan(1/2*(
(a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))))
/(a^2*f)]
```

Sympy [F]

$$\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\sin^2\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

```
[In] integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sin(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01

$$\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{(fx+e)(a+2\,b)}{a^2} - \frac{\tan(fx+e)}{a\tan(fx+e)^2+a} - \frac{2\,(ab+b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^2}}{2\,f}$$

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*((f*x + e)*(a + 2*b)/a^2 - tan(f*x + e)/(a*tan(f*x + e)^2 + a) - 2*(a*b
 + b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^2))/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.21

$$\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{(fx+e)(a+2\,b)}{a^2} - \frac{2\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)\sqrt{ab+b^2}}{a^2} - \frac{\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)a}}{2\,f}$$

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*((f*x + e)*(a + 2*b)/a^2 - 2*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arc
tan(b*tan(f*x + e)/sqrt(a*b + b^2)))*sqrt(a*b + b^2)/a^2 - tan(f*x + e)/((t
an(f*x + e)^2 + 1)*a))/f
```

Mupad [B] (verification not implemented)

Time = 18.39 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.46

$$\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\mathrm{atanh}\left(\frac{\sin(e+f\,x)\,\sqrt{-b^2-a\,b}}{a\,\cos(e+f\,x)+b\,\cos(e+f\,x)}\right)\sqrt{-b^2-a\,b} - a\left(\frac{\sin(2\,e+2\,f\,x)}{4} - \frac{\mathrm{atan}\left(\frac{\sin(e+f\,x)}{\cos(e+f\,x)}\right)}{2}\right) + b\,\mathrm{atan}\left(\frac{\sin(e+f\,x)}{\cos(e+f\,x)}\right)}{a^2\,f}$$

```
[In] int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2),x)
```

```
[Out] (atanh((sin(e + f*x)*(- a*b - b^2)^(1/2))/(a*cos(e + f*x) + b*cos(e + f*x))
)*(- a*b - b^2)^(1/2) - a*(sin(2*e + 2*f*x)/4 - atan(sin(e + f*x)/cos(e + f
*x))/2) + b*atan(sin(e + f*x)/cos(e + f*x)))/(a^2*f)
```

3.37 $\int \frac{1}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 45

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx=\frac{x}{a}+\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{a\sqrt{a+b}f}$$

```
[Out] x/a+arctan(cot(f*x+e)*(a+b)^(1/2)/b^(1/2))*b^(1/2)/a/f/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4212, 3260, 211}

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx=\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{af\sqrt{a+b}}+\frac{x}{a}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-1),x]
```

```
[Out] x/a + (Sqrt[b]*ArcTan[(Sqrt[a + b]*Cot[e + f*x])/Sqrt[b]])/(a*Sqrt[a + b]*f
)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
```

```
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 4212

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/a, x
] - Dist[b/a, Int[1/(b + a*Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f}, x
] && NeQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x}{a} - \frac{b\int \frac{1}{b+a\cos^2(e+fx)}\,dx}{a} \\
&= \frac{x}{a} + \frac{b\text{Subst}\left(\int \frac{1}{b+(a+b)x^2}\,dx, x, \cot(e+fx)\right)}{af} \\
&= \frac{x}{a} + \frac{\sqrt{b}\arctan\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{a\sqrt{a+b}f}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.04

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(\sqrt{a+b}fx\sqrt{b(\cos(e)-i\sin(e))^4}+b\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))}{2\sqrt{a+b}\sqrt{}}\right.\right.}{2a\sqrt{a+b}f\left(a+b\sec^2(e+fx)\right)\sqrt{b(\cos(e)-i\sin(e))^4}}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-1),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[a + b]*f*x*Sqrt[b*(Cos
[e] - I*Sin[e])^4] + b*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b
)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^
4])]*(Cos[2*e] - I*Sin[2*e])))/(2*a*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)*Sq
rt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$\frac{-\frac{b \arctan\left(\frac{b \tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}+\frac{\arctan(\tan(fx+e))}{a}}{f}$	46
default	$\frac{-\frac{b \arctan\left(\frac{b \tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}+\frac{\arctan(\tan(fx+e))}{a}}{f}$	46
risch	$\frac{x}{a}+\frac{\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{2(a+b)fa}-\frac{\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-(a+b)b}-a-2b}{a}\right)}{2(a+b)fa}$	114

```
[In] int(1/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(-b/a/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))+1/a*arctan(t
an(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 231, normalized size of antiderivative = 5.13

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \left[\frac{4\,fx+\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-}}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right.}{4\,af}\right.$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
[Out] [1/4*(4*f*x + sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3
- (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f
*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(a*f), 1/2*(2*f*x + sqrt(b/(a + b
))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e
)*sin(f*x + e))))/(a*f)]
```

Sympy [F]

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \int \frac{1}{a+b\sec^2{(e+fx)}}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(1/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a} - \frac{fx+e}{a}}{f}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -(b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a) - (f*x + e)/
a)/f
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{\sqrt{ab+b^2}a} - \frac{fx+e}{a}}{f}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b
^2)))*b/(sqrt(a*b + b^2)*a) - (f*x + e)/a)/f
```

Mupad [B] (verification not implemented)

Time = 18.48 (sec) , antiderivative size = 460, normalized size of antiderivative = 10.22

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx=\frac{x}{a}-\frac{\operatorname{atan}\left(\frac{\frac{\left(2\,b^3\,\tan(e+f\,x)-\frac{\left(2\,a^2\,b^2-\frac{\tan(e+f\,x)\,\left(8\,a^3\,b^2+16\,a^2\,b^3\right)\,\sqrt{-b\,(a+b)}}{4\,\left(a^2+b\,a\right)}\right)\,\sqrt{-b\,(a+b)}}{2\,\left(a^2+b\,a\right)}\right)\,\sqrt{-b\,(a+b)}\,1\mathrm{i}}{a^2+b\,a}+\frac{\left(2\,b^3\,\tan(e+f\,x)+\frac{\left(2\,a^2\,b^2+\frac{\tan(e+f\,x)\,\left(8\right.}{}\right.}{}\right.}{}}{\frac{\left(2\,b^3\,\tan(e+f\,x)-\frac{\left(2\,a^2\,b^2-\frac{\tan(e+f\,x)\,\left(8\,a^3\,b^2+16\,a^2\,b^3\right)\,\sqrt{-b\,(a+b)}}{4\,\left(a^2+b\,a\right)}\right)\,\sqrt{-b\,(a+b)}}{2\,\left(a^2+b\,a\right)}\right)\,\sqrt{-b\,(a+b)}}{a^2+b\,a}-\frac{\left(2\,b^3\,\tan(e+f\,x)+\frac{\left(2\,a^2\,b^2+\frac{\tan(e+f\,x)\,\left(8\right.}{}\right.}{}\right.}{}}\right.}{f\,\left(a^2+b\,a\right)}$$

```
[In] int(1/(a + b/cos(e + f*x)^2),x)
```

```
[Out] x/a - (atan((((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^2*b^3
 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(
a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(a*b + a^2) + ((2*b^3*tan(e + f*x) + ((
2*a^2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(
a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(a
*b + a^2))/(((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^2*b^3
+ 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a
*b + a^2)))*(-b*(a + b))^(1/2))/(a*b + a^2) - ((2*b^3*tan(e + f*x) + ((2*a^
2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b
+ a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2))/(a*b + a^
2)))*(-b*(a + b))^(1/2)*1i)/(f*(a*b + a^2))
```

3.38 $\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}f} - \frac{\cot(e+fx)}{(a+b)f}$$

```
[Out] -cot(f*x+e)/(a+b)/f-arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/(a+b)^(3
/2)/f
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4217, 331, 211}

$$\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{f(a+b)^{3/2}} - \frac{\cot(e+fx)}{f(a+b)}$$

```
[In] Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/((a + b)^(3/2)*f)) -
 Cot[e + f*x]/((a + b)*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)}{(a+b)f} - \frac{b\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{(a+b)f}$$

$$= -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}f} - \frac{\cot(e+fx)}{(a+b)f}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 189, normalized size of antiderivative = 3.50

$$\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(b\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2 \cdots}{2(a+b)^{3/2}f\left(a+b\sec^2(e+fx)\right)\sqrt{b(\cos(e \cdots}}$$

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(b*ArcTan[(Sec[f*x]*(Cos[2*e
] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*
Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a + b]*Csc[e
]*Csc[e + f*x]*Sqrt[b*(Cos[e] - I*Sin[e])^4]*Sin[f*x]))/(2*(a + b)^(3/2)*f*
(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{-\frac{1}{(a+b)\tan(fx+e)}-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)\sqrt{(a+b)b}}}{f}$
default	$\frac{-\frac{1}{(a+b)\tan(fx+e)}-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)\sqrt{(a+b)b}}}{f}$
risch	$-\frac{2i}{f(a+b)\left(e^{2i(fx+e)}-1\right)}-\frac{\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-(a+b)b}-a-2b}{a}\right)}{2(a+b)^2 f}+\frac{\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+}{a}\right.}{2(a+b)^2 f}$

```
[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/(a+b)/tan(f*x+e)-b/(a+b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)
*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 271, normalized size of antiderivative = 5.02

$$\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$=\left[\frac{\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\,\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-\frac{b}{a+b}}\sin(fx}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right.}{4\,(a+b)f\sin\left(fx+e\right)}\right.$$

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b
 + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b +
 b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^
4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) - 4*cos(f*x + e))/((a + b)*f*
sin(f*x + e)), 1/2*(sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 -
b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) - 2*cos(f*x
+ e))/((a + b)*f*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\csc^2{(e+fx)}}{a+b\sec^2{(e+fx)}}\,dx$$

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
[Out] Integral(csc(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}(a+b)}+\frac{1}{(a+b)\tan(fx+e)}}{f}$$

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] -(b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*(a + b)) + 1/((
a + b)*tan(f*x + e)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.31

$$\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{\sqrt{ab+b^2}(a+b)}+\frac{1}{(a+b)\tan(fx+e)}}{f}$$

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] -((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b
^2)))*b/(sqrt(a*b + b^2)*(a + b)) + 1/((a + b)*tan(f*x + e)))/f
```

Mupad [B] (verification not implemented)

Time = 18.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\cot(e+f\,x)}{f\,\left(a+b\right)} - \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(e+f\,x)}{\sqrt{a+b}}\right)}{f\,{\left(a+b\right)}^{3/2}}$$

```
[In] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] - cot(e + f*x)/(f*(a + b)) - (b^(1/2)*atan((b^(1/2)*tan(e + f*x))/(a + b)^(
1/2)))/(f*(a + b)^(3/2))
```

3.39 $\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx=-\frac{a\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}f}-\frac{a\cot(e+fx)}{(a+b)^2f}-\frac{\cot^3(e+fx)}{3(a+b)f}$$

```
[Out] -a*cot(f*x+e)/(a+b)^2/f-1/3*cot(f*x+e)^3/(a+b)/f-a*arctan(b^(1/2)*tan(f*x+e
)/(a+b)^(1/2))*b^(1/2)/(a+b)^(5/2)/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4217, 464, 331, 211}

$$\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx=-\frac{a\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{f(a+b)^{5/2}}-\frac{\cot^3(e+fx)}{3f(a+b)}-\frac{a\cot(e+fx)}{f(a+b)^2}$$

```
[In] Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((a*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/((a + b)^(5/2)*f))
 - (a*Cot[e + f*x])/((a + b)^2*f) - Cot[e + f*x]^3/(3*(a + b)*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{\cot^3(e+fx)}{3(a+b)f} + \frac{a\text{Subst}\left(\int \frac{1}{x^2(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{(a+b)f}\\
&= -\frac{a\cot(e+fx)}{(a+b)^2f} - \frac{\cot^3(e+fx)}{3(a+b)f} - \frac{(ab)\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{(a+b)^2f}\\
&= -\frac{a\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}f} - \frac{a\cot(e+fx)}{(a+b)^2f} - \frac{\cot^3(e+fx)}{3(a+b)f}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.86 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.97

$$\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(3ab\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\mathrm{co}\right.}{6(a+b)^{5/2}f\,(a}$$

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(3*a*b*ArcTan[(Sec[f*x]*(Cos
[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a +
 b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + (Sqrt[a + b]*
Csc[e]*Csc[e + f*x]^3*Sqrt[b*(Cos[e] - I*Sin[e])^4]*(6*a*Sin[f*x] - 3*b*Sin
[2*e + f*x] + (-2*a + b)*Sin[2*e + 3*f*x]))/4))/(6*(a + b)^(5/2)*f*(a + b*S
ec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{-\frac{ab\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)^2\sqrt{(a+b)b}}-\frac{1}{3(a+b)\tan(fx+e)^3}-\frac{a}{(a+b)^2\tan(fx+e)}}{f}$
default	$\frac{-\frac{ab\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)^2\sqrt{(a+b)b}}-\frac{1}{3(a+b)\tan(fx+e)^3}-\frac{a}{(a+b)^2\tan(fx+e)}}{f}$
risch	$\frac{2i\left(3b\,e^{4i(fx+e)}+6a\,e^{2i(fx+e)}-2a+b\right)}{3f(a+b)^2\left(e^{2i(fx+e)}-1\right)^3}+\frac{\sqrt{-(a+b)b}\,a\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{2(a+b)^3f}-\frac{\sqrt{-(a+b)b}\,a\ln\left(e^{2i(fx+}\right.}{2(a+}$

```
[In] int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a*b/(a+b)^2/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))-1/3/
(a+b)/tan(f*x+e)^3-a/(a+b)^2/tan(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(66) = 132$.

Time = 0.30 (sec) , antiderivative size = 397, normalized size of antiderivative = 5.22

$$\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[-\frac{4\,(2\,a-b)\cos\left(fx+e\right)^3 - 3\left(a\cos\left(fx+e\right)^2-a\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2}{a^2\cos(}\right.}{12\left((a^2+2\,ab+b^2)f\cos\left(fx+e\right)^2-(a^2\right.} \right.$$
$$-\frac{2\,(2\,a-b)\cos\left(fx+e\right)^3-3\left(a\cos\left(fx+e\right)^2-a\right)\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)\sin\left(fx+e\right)-}{6\left((a^2+2\,ab+b^2)f\cos\left(fx+e\right)^2-(a^2+2\,ab+b^2)f\right)\sin\left(fx+e\right)}$$

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/12*(4*(2*a - b)*cos(f*x + e)^3 - 3*(a*cos(f*x + e)^2 - a)*sqrt(-b/(a +
b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
 e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))
*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x +
 e)^2 + b^2))*sin(f*x + e) - 12*a*cos(f*x + e))/(((a^2 + 2*a*b + b^2)*f*cos
(f*x + e)^2 - (a^2 + 2*a*b + b^2)*f)*sin(f*x + e)), -1/6*(2*(2*a - b)*cos(f
*x + e)^3 - 3*(a*cos(f*x + e)^2 - a)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*
cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x
+ e) - 6*a*cos(f*x + e))/(((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 - (a^2 + 2*
a*b + b^2)*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\csc^4\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

```
[In] integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(csc(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{3\,ab\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^2+2\,ab+b^2)\sqrt{(a+b)b}} + \frac{3\,a\tan(fx+e)^2+a+b}{(a^2+2\,ab+b^2)\tan(fx+e)^3}}{3\,f}$$

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*a*b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^2 + 2*a*b + b^2)*sqr
t((a + b)*b)) + (3*a*tan(f*x + e)^2 + a + b)/((a^2 + 2*a*b + b^2)*tan(f*x +
 e)^3))/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.36

$$\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)ab}{(a^2+2\,ab+b^2)\sqrt{ab+b^2}} + \frac{3\,a\tan(fx+e)^2+a+b}{(a^2+2\,ab+b^2)\tan(fx+e)^3}}{3\,f}$$

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b + b^2)))*a*b/((a^2 + 2*a*b + b^2)*sqrt(a*b + b^2)) + (3*a*tan(f*x + e)^2
 + a + b)/((a^2 + 2*a*b + b^2)*tan(f*x + e)^3))/f
```

Mupad [B] (verification not implemented)

Time = 18.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{1}{3\,(a+b)}+\frac{a\tan(e+f\,x)^2}{(a+b)^2}}{f\tan\left(e+f\,x\right)^3} - \frac{a\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)\,\left(a^2+2\,a\,b+b^2\right)}{(a+b)^{5/2}}\right)}{f\,\left(a+b\right)^{5/2}}$$

```
[In] int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] - (1/(3*(a + b)) + (a*tan(e + f*x)^2)/(a + b)^2)/(f*tan(e + f*x)^3) - (a*b^
(1/2)*atan((b^(1/2)*tan(e + f*x)*(2*a*b + a^2 + b^2))/(a + b)^(5/2)))/(f*(a
 + b)^(5/2))
```

3.40 $\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 105

$$\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{a^2\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{7/2}f} - \frac{a^2\cot(e+fx)}{(a+b)^3f} - \frac{(2a+b)\cot^3(e+fx)}{3(a+b)^2f} - \frac{\cot^5(e+fx)}{5(a+b)f}$$

```
[Out] -a^2*cot(f*x+e)/(a+b)^3/f-1/3*(2*a+b)*cot(f*x+e)^3/(a+b)^2/f-1/5*cot(f*x+e)
^5/(a+b)/f-a^2*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/(a+b)^(7/2)/f
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4217, 472, 211}

$$\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{a^2\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{f(a+b)^{7/2}} - \frac{a^2\cot(e+fx)}{f(a+b)^3} - \frac{\cot^5(e+fx)}{5f(a+b)} - \frac{(2a+b)\cot^3(e+fx)}{3f(a+b)^2}$$

```
[In] Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((a^2*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/((a + b)^(7/2)*f
)) - (a^2*Cot[e + f*x])/((a + b)^3*f) - ((2*a + b)*Cot[e + f*x]^3)/(3*(a +
b)^2*f) - Cot[e + f*x]^5/(5*(a + b)*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^6(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)x^6}+\frac{2a+b}{(a+b)^2x^4}+\frac{a^2}{(a+b)^3x^2}-\frac{a^2b}{(a+b)^3(a+b+bx^2)}\right)dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a^2\cot(e+fx)}{(a+b)^3f}-\frac{(2a+b)\cot^3(e+fx)}{3(a+b)^2f}-\frac{\cot^5(e+fx)}{5(a+b)f} - \frac{(a^2b)\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{(a+b)^3f}$$

$$= -\frac{a^2\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{7/2}f}-\frac{a^2\cot(e+fx)}{(a+b)^3f}-\frac{(2a+b)\cot^3(e+fx)}{3(a+b)^2f}-\frac{\cot^5(e+fx)}{5(a+b)f}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.78 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.03

$$\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(240a^2b\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)\right.}{}$$

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(240*a^2*b*ArcTan[(Sec[f*x]*
(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt
[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a +
b]*Csc[e]*Csc[e + f*x]^5*Sqrt[b*(Cos[e] - I*Sin[e])^4]*(10*(8*a^2 + b^2)*Si
n[f*x] - 30*b*(3*a + b)*Sin[2*e + f*x] - 40*a^2*Sin[2*e + 3*f*x] + 30*a*b*S
in[2*e + 3*f*x] + 10*b^2*Sin[2*e + 3*f*x] + 15*a*b*Sin[4*e + 3*f*x] + 8*a^2
*Sin[4*e + 5*f*x] - 9*a*b*Sin[4*e + 5*f*x] - 2*b^2*Sin[4*e + 5*f*x])))/(480
*(a + b)^(7/2)*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{-\frac{a^2 b \arctan\left(\frac{b \tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)^3 \sqrt{(a+b)b}}-\frac{1}{5(a+b)\tan(fx+e)^5}-\frac{a^2}{(a+b)^3 \tan(fx+e)}-\frac{2a+b}{3(a+b)^2 \tan(fx+e)^3}}{f}$
default	$\frac{-\frac{a^2 b \arctan\left(\frac{b \tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)^3 \sqrt{(a+b)b}}-\frac{1}{5(a+b)\tan(fx+e)^5}-\frac{a^2}{(a+b)^3 \tan(fx+e)}-\frac{2a+b}{3(a+b)^2 \tan(fx+e)^3}}{f}$
risch	$\frac{2i\left(15ab\,e^{8i(fx+e)}-90ab\,e^{6i(fx+e)}-30b^2 e^{6i(fx+e)}-80a^2 e^{4i(fx+e)}-10b^2 e^{4i(fx+e)}+40a^2 e^{2i(fx+e)}-30ab\,e^{2i(fx+e)}-10b^2 e\right.}{15f(a+b)^3\left(e^{2i(fx+e)}-1\right)^5}$

```
[In] int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a^2*b/(a+b)^3/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))-1/
5/(a+b)/tan(f*x+e)^5-a^2/(a+b)^3/tan(f*x+e)-1/3*(2*a+b)/(a+b)^2/tan(f*x+e)^
3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(93) = 186$.

Time = 0.29 (sec) , antiderivative size = 587, normalized size of antiderivative = 5.59

$$\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \left[-\frac{4\,(8\,a^2-9\,ab-2\,b^2)\cos{(fx+e)}^5-20\,(4\,a^2-3\,ab-b^2)\cos{(fx+e)}^3-15\,\left(a^2\cos{(fx+e)}^4-2\,a^2\,\mathrm{co}\right.}{60\,\left((a^3+3\,a^2b+3\,ab^2+b^3)f\cos\right.} \right.$$

$$-\frac{2\,(8\,a^2-9\,ab-2\,b^2)\cos{(fx+e)}^5-10\,(4\,a^2-3\,ab-b^2)\cos{(fx+e)}^3-15\,\left(a^2\cos{(fx+e)}^4-2\,a^2\,\mathrm{co}\right.}{30\,\left((a^3+3\,a^2b+3\,ab^2+b^3)f\cos{(fx+e)}^4-2\,(a^3+3\,a^2b+3\,ab^2+b^3\right.}$$

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(4*(8*a^2 - 9*a*b - 2*b^2)*cos(f*x + e)^5 - 20*(4*a^2 - 3*a*b - b^2)
*cos(f*x + e)^3 - 15*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt
(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*
cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(
f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*
cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*a^2*cos(f*x + e))/(((a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*f*cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*co
s(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f)*sin(f*x + e)), -1/30*(2*(
8*a^2 - 9*a*b - 2*b^2)*cos(f*x + e)^5 - 10*(4*a^2 - 3*a*b - b^2)*cos(f*x +
e)^3 - 15*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt(b/(a + b))
*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*
sin(f*x + e)))*sin(f*x + e) + 30*a^2*cos(f*x + e))/(((a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*f*cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x +
e)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\csc^6{(e+fx)}}{a+b\sec^2{(e+fx)}}\,dx$$

```
[In] integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(csc(e + f*x)**6/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.30

$$\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{15\,a^2b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{(a+b)b}}+\frac{15\,a^2\tan(fx+e)^4+5\,(2\,a^2+3\,ab+b^2)\tan(fx+e)^2+3\,a^2+6\,ab+3\,b^2}{(a^3+3\,a^2b+3\,ab^2+b^3)\tan(fx+e)^5}}{15\,f}$$

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*a^2*b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*sqrt((a + b)*b)) + (15*a^2*tan(f*x + e)^4 + 5*(2*a^2 + 3*a*b +
 b^2)*tan(f*x + e)^2 + 3*a^2 + 6*a*b + 3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b
^3)*tan(f*x + e)^5))/f
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.65

$$\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)a^2b}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{ab+b^2}}+\frac{15\,a^2\tan(fx+e)^4+10\,a^2\tan(fx+e)^2+15\,ab\tan(fx+e)^2+5\,b^2\tan(fx+e)^2+3\,a^2+6\,ab-}{(a^3+3\,a^2b+3\,ab^2+b^3)\tan(fx+e)^5}}{15\,f}$$

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt
(a*b + b^2)))*a^2*b/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b + b^2)) + (15
*a^2*tan(f*x + e)^4 + 10*a^2*tan(f*x + e)^2 + 15*a*b*tan(f*x + e)^2 + 5*b^2
*tan(f*x + e)^2 + 3*a^2 + 6*a*b + 3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*t
an(f*x + e)^5))/f
```

Mupad [B] (verification not implemented)

Time = 19.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.07

$$\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{1}{5\,(a+b)}+\frac{\tan(e+f\,x)^2\,(2\,a+b)}{3\,(a+b)^2}+\frac{a^2\tan(e+f\,x)^4}{(a+b)^3}}{f\tan\left(e+f\,x\right)^5} - \frac{a^2\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{(a+b)^{7/2}}\right)}{f\,(a+b)^{7/2}}$$

```
[In] int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] - (1/(5*(a + b)) + (tan(e + f*x)^2*(2*a + b))/(3*(a + b)^2) + (a^2*tan(e +
f*x)^4)/(a + b)^3)/(f*tan(e + f*x)^5) - (a^2*b^(1/2)*atan((b^(1/2)*tan(e +
f*x)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(a + b)^(7/2)))/(f*(a + b)^(7/2))
```

3.41 $\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 161

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\sqrt{b}(a+b)(3a+7b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{9/2}f} - \frac{(a+b)(3a+7b)\cos(e+fx)}{2a^4f} + \frac{(a+b)(3a+7b)\cos^3(e+fx)}{6a^3bf} - \frac{\cos^5(e+fx)}{5a^2f} - \frac{(a+b)^2\cos^5(e+fx)}{2a^2bf\,(b+a\cos^2(e+fx))}$$

```
[Out] -1/2*(a+b)*(3*a+7*b)*cos(f*x+e)/a^4/f+1/6*(a+b)*(3*a+7*b)*cos(f*x+e)^3/a^3/
b/f-1/5*cos(f*x+e)^5/a^2/f-1/2*(a+b)^2*cos(f*x+e)^5/a^2/b/f/(b+a*cos(f*x+e)
^2)+1/2*(a+b)*(3*a+7*b)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(9/2)/
f
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used

= {4218, 474, 470, 308, 211}

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\sqrt{b}(a+b)(3a+7b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{9/2}f} - \frac{(a+b)(3a+7b)\cos(e+fx)}{2a^4f} + \frac{(a+b)(3a+7b)\cos^3(e+fx)}{6a^3bf} - \frac{(a+b)^2\cos^5(e+fx)}{2a^2bf\,(a\cos^2(e+fx)+b)} - \frac{\cos^5(e+fx)}{5a^2f}$$

```
[In] Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (Sqrt[b]*(a + b)*(3*a + 7*b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*a^(
9/2)*f) - ((a + b)*(3*a + 7*b)*Cos[e + f*x])/(2*a^4*f) + ((a + b)*(3*a + 7*
b)*Cos[e + f*x]^3)/(6*a^3*b*f) - Cos[e + f*x]^5/(5*a^2*f) - ((a + b)^2*Cos[
e + f*x]^5)/(2*a^2*b*f*(b + a*Cos[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^4(1-x^2)^2}{(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{(a+b)^2\cos^5(e+fx)}{2a^2bf\,(b+a\cos^2(e+fx))} + \frac{\text{Subst}\left(\int \frac{x^4(-2a^2+5(a+b)^2-2abx^2)}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{2a^2bf}$$

$$= -\frac{\cos^5(e+fx)}{5a^2f} - \frac{(a+b)^2\cos^5(e+fx)}{2a^2bf\,(b+a\cos^2(e+fx))} + \frac{((a+b)(3a+7b))\text{Subst}\left(\int \frac{x^4}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{2a^2bf}$$

$$= -\frac{\cos^5(e+fx)}{5a^2f} - \frac{(a+b)^2\cos^5(e+fx)}{2a^2bf\,(b+a\cos^2(e+fx))} + \frac{((a+b)(3a+7b))\text{Subst}\left(\int \left(-\frac{b}{a^2}+\frac{x^2}{a}+\frac{b^2}{a^2(b+ax^2)}\right)dx, x, \cos(e+fx)\right)}{2a^2bf}$$

$$= -\frac{(a+b)(3a+7b)\cos(e+fx)}{2a^4f} + \frac{(a+b)(3a+7b)\cos^3(e+fx)}{6a^3bf} - \frac{\cos^5(e+fx)}{5a^2f} - \frac{(a+b)^2\cos^5(e+fx)}{2a^2bf\,(b+a\cos^2(e+fx))} + \frac{(b(a+b)(3a+7b))\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{2a^4f}$$

$$= \frac{\sqrt{b}(a+b)(3a+7b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{9/2}f} - \frac{(a+b)(3a+7b)\cos(e+fx)}{2a^4f} + \frac{(a+b)(3a+7b)\cos^3(e+fx)}{6a^3bf} - \frac{\cos^5(e+fx)}{5a^2f} - \frac{(a+b)^2\cos^5(e+fx)}{2a^2bf\,(b+a\cos^2(e+fx))}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.66 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.82

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \frac{\frac{15(3a^4+384a^2b^2+1280ab^3+896b^4)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)}{b^{3/2}}+}{}$$

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((15*(3*a^4 + 384*a^2*b^2 + 1280*a*b^3 + 896*b^4)*ArcTan[((-Sqrt[a] - I*Sqr
t[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a]
 - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/2)
 + (15*(3*a^4 + 384*a^2*b^2 + 1280*a*b^3 + 896*b^4)*ArcTan[((-Sqrt[a] + I*S
qrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[
a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/
2) - (45*a^4*ArcTan[(Sqrt[a] - Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(3
/2) - (45*a^4*ArcTan[(Sqrt[a] + Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(
3/2) - (16*Sqrt[a]*Cos[e + f*x]*(150*a^3 + 1436*a^2*b + 2960*a*b^2 + 1680*b
^3 + a*(125*a^2 + 688*a*b + 560*b^2)*Cos[2*(e + f*x)] - 2*a^2*(11*a + 14*b)
*Cos[4*(e + f*x)] + 3*a^3*Cos[6*(e + f*x)]))/(a + 2*b + a*Cos[2*(e + f*x)])
)/(3840*a^(9/2)*f)
```

Maple [A] (verified)

Time = 4.61 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{-\frac{\frac{\cos(fx+e)^5a^2}{5}-\frac{2a^2\cos(fx+e)^3}{3}-\frac{2a\cos(fx+e)^3b}{3}+a^2\cos(fx+e)+4ab\cos(fx+e)+3b^2\cos(fx+e)}{a^4}+\frac{b\left(\frac{\left(-\frac{1}{2}a^2-ab-\frac{1}{2}b^2\right)\cos(fx+}{b+a\cos(fx+e)^2}\right.}{}}{f}$
default	$\frac{-\frac{\frac{\cos(fx+e)^5a^2}{5}-\frac{2a^2\cos(fx+e)^3}{3}-\frac{2a\cos(fx+e)^3b}{3}+a^2\cos(fx+e)+4ab\cos(fx+e)+3b^2\cos(fx+e)}{a^4}+\frac{b\left(\frac{\left(-\frac{1}{2}a^2-ab-\frac{1}{2}b^2\right)\cos(fx+}{b+a\cos(fx+e)^2}\right.}{}}{f}$
risch	$\frac{5\,e^{3i(fx+e)}}{96a^2f}+\frac{e^{3i(fx+e)}b}{12a^3f}-\frac{5\,e^{i(fx+e)}}{16f\,a^2}-\frac{7\,e^{i(fx+e)}b}{4f\,a^3}-\frac{3\,e^{i(fx+e)}b^2}{2f\,a^4}-\frac{5\,e^{-i(fx+e)}}{16f\,a^2}-\frac{7\,e^{-i(fx+e)}b}{4f\,a^3}-\frac{3\,e^{-i(f}}{2f}$

```
[In] int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^4*(1/5*cos(f*x+e)^5*a^2-2/3*a^2*cos(f*x+e)^3-2/3*a*cos(f*x+e)^3*b
+a^2*cos(f*x+e)+4*a*b*cos(f*x+e)+3*b^2*cos(f*x+e))+b/a^4*((-1/2*a^2-a*b-1/2
*b^2)*cos(f*x+e)/(b+a*cos(f*x+e)^2)+1/2*(3*a^2+10*a*b+7*b^2)/(a*b)^(1/2)*ar
ctan(a*cos(f*x+e)/(a*b)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.52

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \left[-\frac{12\,a^3\cos\left(fx+e\right)^7 - 4\,(10\,a^3+7\,a^2b)\cos\left(fx+e\right)^5 + 20\,(3\,a^3+10\,a^2b+7\,ab^2)\cos\left(fx+e\right)^3 - 15\,(3\,a^2}{}\right.$$

$$-\frac{6\,a^3\cos\left(fx+e\right)^7 - 2\,(10\,a^3+7\,a^2b)\cos\left(fx+e\right)^5 + 10\,(3\,a^3+10\,a^2b+7\,ab^2)\cos\left(fx+e\right)^3 - 15\,(3\,a^2b}{30\,(a^5}$$

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(12*a^3*cos(f*x + e)^7 - 4*(10*a^3 + 7*a^2*b)*cos(f*x + e)^5 + 20*(3
*a^3 + 10*a^2*b + 7*a*b^2)*cos(f*x + e)^3 - 15*(3*a^2*b + 10*a*b^2 + 7*b^3
+ (3*a^3 + 10*a^2*b + 7*a*b^2)*cos(f*x + e)^2)*sqrt(-b/a)*log(-(a*cos(f*x +
 e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 30*(3*a^
2*b + 10*a*b^2 + 7*b^3)*cos(f*x + e))/(a^5*f*cos(f*x + e)^2 + a^4*b*f), -1/
30*(6*a^3*cos(f*x + e)^7 - 2*(10*a^3 + 7*a^2*b)*cos(f*x + e)^5 + 10*(3*a^3
+ 10*a^2*b + 7*a*b^2)*cos(f*x + e)^3 - 15*(3*a^2*b + 10*a*b^2 + 7*b^3 + (3*
a^3 + 10*a^2*b + 7*a*b^2)*cos(f*x + e)^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(
f*x + e)/b) + 15*(3*a^2*b + 10*a*b^2 + 7*b^3)*cos(f*x + e))/(a^5*f*cos(f*x
+ e)^2 + a^4*b*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.92

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{15\left(a^2b+2\,ab^2+b^3\right)\cos(fx+e)}{a^5\cos(fx+e)^2+a^4b} - \frac{15\left(3\,a^2b+10\,ab^2+7\,b^3\right)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^4} + \frac{2\left(3\,a^2\cos(fx+e)^5-10\left(a^2+ab\right)\cos(fx+e)^3+15\left(a^2+4\,a\right.\right.}{a^4}}{30\,f}$$

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/30*(15*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)/(a^5*cos(f*x + e)^2 + a^4*b)
 - 15*(3*a^2*b + 10*a*b^2 + 7*b^3)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a
*b)*a^4) + 2*(3*a^2*cos(f*x + e)^5 - 10*(a^2 + a*b)*cos(f*x + e)^3 + 15*(a^
2 + 4*a*b + 3*b^2)*cos(f*x + e))/a^4)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(143) = 286$.

Time = 0.36 (sec) , antiderivative size = 509, normalized size of antiderivative = 3.16

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{15\left(3\,a^2b+10\,ab^2+7\,b^3\right)\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{\sqrt{ab}a^4} + \frac{30\left(a^2b+2\,ab^2+b^3+\frac{a^2b(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{b^3(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)}{\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)a^4}}{}$$

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/30*(15*(3*a^2*b + 10*a*b^2 + 7*b^3)*arctan(-(a*cos(f*x + e) - b)/(sqrt(a
*b)*cos(f*x + e) + sqrt(a*b)))/(sqrt(a*b)*a^4) + 30*(a^2*b + 2*a*b^2 + b^3
+ a^2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - b^3*(cos(f*x + e) - 1)/(cos
(f*x + e) + 1))/((a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(
cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e)
 + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*a^4) - 4*(8*a^2 + 50
*a*b + 45*b^2 - 40*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 220*a*b*(cos
(f*x + e) - 1)/(cos(f*x + e) + 1) - 180*b^2*(cos(f*x + e) - 1)/(cos(f*x + e
) + 1) + 80*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 320*a*b*(cos(f*
x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 270*b^2*(cos(f*x + e) - 1)^2/(cos(f*x
+ e) + 1)^2 - 180*a*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 180*b^2*(
cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 30*a*b*(cos(f*x + e) - 1)^4/(cos
(f*x + e) + 1)^4 + 45*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/(a^4*(
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.21

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\cos(e+f\,x)^3\,\left(\frac{2\,b}{3\,a^3}+\frac{2}{3\,a^2}\right)}{f}-\frac{\cos(e+f\,x)^5}{5\,a^2\,f}-\frac{\cos(e+f\,x)\,\left(\frac{1}{a^2}-\frac{b^2}{a^4}+\frac{2\,b\,\left(\frac{2\,b}{a^3}+\frac{2}{a^2}\right)}{a}\right)}{f}-\frac{\cos(e+f\,x)\,\left(\frac{a^2\,b}{2}+a\,b^2+\frac{b^3}{2}\right)}{f\,\left(a^5\cos(e+f\,x)^2+b\,a^4\right)}+\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,\cos(e+f\,x)\,(a+b)\,(3\,a+7\,b)}{3\,a^2\,b+10\,a\,b^2+7\,b^3}\right)\,(a+b)\,(3\,a+7\,b)}{2\,a^{9/2}\,f}$$

```
[In] int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (cos(e + f*x)^3*((2*b)/(3*a^3) + 2/(3*a^2)))/f - cos(e + f*x)^5/(5*a^2*f) -
 (cos(e + f*x)*(1/a^2 - b^2/a^4 + (2*b*((2*b)/a^3 + 2/a^2))/a))/f - (cos(e
+ f*x)*(a*b^2 + (a^2*b)/2 + b^3/2))/(f*(a^4*b + a^5*cos(e + f*x)^2)) + (b^(
1/2)*atan((a^(1/2)*b^(1/2)*cos(e + f*x)*(a + b)*(3*a + 7*b))/(10*a*b^2 + 3*
a^2*b + 7*b^3))*(a + b)*(3*a + 7*b))/(2*a^(9/2)*f)
```

3.42 $\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 114

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\sqrt{b}(3a+5b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{7/2}f} - \frac{(a+2b)\cos(e+fx)}{a^3f} + \frac{\cos^3(e+fx)}{3a^2f} - \frac{b(a+b)\cos(e+fx)}{2a^3f\,(b+a\cos^2(e+fx))}$$

```
[Out] -(a+2*b)*cos(f*x+e)/a^3/f+1/3*cos(f*x+e)^3/a^2/f-1/2*b*(a+b)*cos(f*x+e)/a^3
/f/(b+a*cos(f*x+e)^2)+1/2*(3*a+5*b)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1
/2)/a^(7/2)/f
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4218, 466, 1167, 211}

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\sqrt{b}(3a+5b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{7/2}f} - \frac{b(a+b)\cos(e+fx)}{2a^3f\,(a\cos^2(e+fx)+b)} - \frac{(a+2b)\cos(e+fx)}{a^3f} + \frac{\cos^3(e+fx)}{3a^2f}$$

```
[In] Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (Sqrt[b]*(3*a + 5*b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*a^(7/2)*f)
- ((a + 2*b)*Cos[e + f*x])/(a^3*f) + Cos[e + f*x]^3/(3*a^2*f) - (b*(a + b)*
Cos[e + f*x])/(2*a^3*f*(b + a*Cos[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^4(1-x^2)}{(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{b(a+b)\cos(e+fx)}{2a^3f\,(b+a\cos^2(e+fx))} + \frac{\text{Subst}\left(\int \frac{b(a+b)-2a(a+b)x^2+2a^2x^4}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{2a^3f}\\
&= -\frac{b(a+b)\cos(e+fx)}{2a^3f\,(b+a\cos^2(e+fx))} + \frac{\text{Subst}\left(\int \left(-2(a+2b)+2ax^2+\frac{3ab+5b^2}{b+ax^2}\right)dx, x, \cos(e+fx)\right)}{2a^3f}\\
&= -\frac{(a+2b)\cos(e+fx)}{a^3f} + \frac{\cos^3(e+fx)}{3a^2f} - \frac{b(a+b)\cos(e+fx)}{2a^3f\,(b+a\cos^2(e+fx))}\\
&\quad + \frac{(b(3a+5b))\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{2a^3f}
\end{aligned}$$

$$= \frac{\sqrt{b}(3a+5b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{7/2}f} - \frac{(a+2b)\cos(e+fx)}{a^3f} + \frac{\cos^3(e+fx)}{3a^2f} - \frac{b(a+b)\cos(e+fx)}{2a^3f\left(b+a\cos^2(e+fx)\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.91 (sec) , antiderivative size = 403, normalized size of antiderivative = 3.54

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{\frac{3(3a^3+192ab^2+320b^3)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)}{b^{3/2}} + \frac{3(3a^3+192a}{}}{}$$

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((3*(3*a^3 + 192*a*b^2 + 320*b^3)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(C
os[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*S
qrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/2) + (3*(3*a^3 + 1
92*a*b^2 + 320*b^3)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[
e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] -
I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/2) - (9*a^3*ArcTan[(Sqrt[a] - Sq
rt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(3/2) - (9*a^3*ArcTan[(Sqrt[a] + Sq
rt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(3/2) - (32*Sqrt[a]*Cos[e + f*x]*(9
*a^2 + 56*a*b + 60*b^2 + 4*a*(2*a + 5*b)*Cos[2*(e + f*x)] - a^2*Cos[4*(e +
f*x)]))/(a + 2*b + a*Cos[2*(e + f*x)]))/(384*a^(7/2)*f)
```

Maple [A] (verified)

Time = 2.46 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\frac{\frac{a\cos(fx+e)^3}{3}-\cos(fx+e)a-2b\cos(fx+e)}{a^3}+\frac{b\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\cos(fx+e)}{b+a\cos(fx+e)^2}+\frac{(3a+5b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}}{f}$
default	$\frac{\frac{\frac{a\cos(fx+e)^3}{3}-\cos(fx+e)a-2b\cos(fx+e)}{a^3}+\frac{b\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\cos(fx+e)}{b+a\cos(fx+e)^2}+\frac{(3a+5b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}}{f}$
risch	$\frac{e^{3i(fx+e)}}{24a^2f}-\frac{3\,e^{i(fx+e)}}{8f\,a^2}-\frac{e^{i(fx+e)}b}{f\,a^3}-\frac{3\,e^{-i(fx+e)}}{8f\,a^2}-\frac{e^{-i(fx+e)}b}{f\,a^3}+\frac{e^{-3i(fx+e)}}{24a^2f}-\frac{(a+b)b(e^{3i(fx+e)}+e}{a^3f(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}}$

```
[In] int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^3*(1/3*a*cos(f*x+e)^3-cos(f*x+e)*a-2*b*cos(f*x+e))+b/a^3*((-1/2*a-
1/2*b)*cos(f*x+e)/(b+a*cos(f*x+e)^2)+1/2*(3*a+5*b)/(a*b)^(1/2)*arctan(a*cos
(f*x+e)/(a*b)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.61

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$=\left[\frac{4\,a^2\cos\left(fx+e\right)^5-4\left(3\,a^2+5\,ab\right)\cos\left(fx+e\right)^3+3\left(\left(3\,a^2+5\,ab\right)\cos\left(fx+e\right)^2+3\,ab+5\,b^2\right)\sqrt{-\frac{b}{a}}\log\left(\right.}{12\left(a^4f\cos\left(fx+e\right)^2+a^3bf\right)}\right.$$

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*a^2*cos(f*x + e)^5 - 4*(3*a^2 + 5*a*b)*cos(f*x + e)^3 + 3*((3*a^2
+ 5*a*b)*cos(f*x + e)^2 + 3*a*b + 5*b^2)*sqrt(-b/a)*log(-(a*cos(f*x + e)^2
+ 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) - 6*(3*a*b + 5*b
^2)*cos(f*x + e))/(a^4*f*cos(f*x + e)^2 + a^3*b*f), 1/6*(2*a^2*cos(f*x + e)
^5 - 2*(3*a^2 + 5*a*b)*cos(f*x + e)^3 + 3*((3*a^2 + 5*a*b)*cos(f*x + e)^2 +
 3*a*b + 5*b^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) - 3*(3*a*b + 5
*b^2)*cos(f*x + e))/(a^4*f*cos(f*x + e)^2 + a^3*b*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.91

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{3\,(ab+b^2)\cos(fx+e)}{a^4\cos(fx+e)^2+a^3b} - \frac{3\,(3\,ab+5\,b^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} - \frac{2\left(a\cos(fx+e)^3-3\,(a+2\,b)\cos(fx+e)\right)}{a^3}}{6\,f}$$

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
[Out] -1/6*(3*(a*b + b^2)*cos(f*x + e)/(a^4*cos(f*x + e)^2 + a^3*b) - 3*(3*a*b +
5*b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2*(a*cos(f*x + e)
^3 - 3*(a + 2*b)*cos(f*x + e))/a^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.19

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{(3\,ab+5\,b^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3f} - \frac{\frac{ab\cos(fx+e)}{f} + \frac{b^2\cos(fx+e)}{f}}{2\left(a\cos\left(fx+e\right)^2+b\right)a^3} + \frac{a^4f^{11}\cos\left(fx+e\right)^3 - 3\,a^4f^{11}\cos\left(fx+e\right) - 6\,a^3bf^{11}\cos\left(fx+e\right)}{3\,a^6f^{12}}$$

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
[Out] 1/2*(3*a*b + 5*b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^3*f) - 1/
2*(a*b*cos(f*x + e)/f + b^2*cos(f*x + e)/f)/((a*cos(f*x + e)^2 + b)*a^3) +
1/3*(a^4*f^11*cos(f*x + e)^3 - 3*a^4*f^11*cos(f*x + e) - 6*a^3*b*f^11*cos(f
*x + e))/(a^6*f^12)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.14

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\cos\left(e+f\,x\right)^3}{3\,a^2\,f}-\frac{\cos\left(e+f\,x\right)\,\left(\frac{2\,b}{a^3}+\frac{1}{a^2}\right)}{f}-\frac{\cos\left(e+f\,x\right)\,\left(\frac{b^2}{2}+\frac{a\,b}{2}\right)}{f\,\left(a^4\cos\left(e+f\,x\right)^2+b\,a^3\right)}+\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,\cos(e+f\,x)\,(3\,a+5\,b)}{5\,b^2+3\,a\,b}\right)\,(3\,a+5\,b)}{2\,a^{7/2}\,f}$$

```
[In] int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] cos(e + f*x)^3/(3*a^2*f) - (cos(e + f*x)*((2*b)/a^3 + 1/a^2))/f - (cos(e +
f*x)*((a*b)/2 + b^2/2))/(f*(a^3*b + a^4*cos(e + f*x)^2)) + (b^(1/2)*atan((a
^(1/2)*b^(1/2)*cos(e + f*x)*(3*a + 5*b))/(3*a*b + 5*b^2))*(3*a + 5*b))/(2*a
^(7/2)*f)
```

3.43 $\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 84

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{3\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{5/2}f} - \frac{3\cos(e+fx)}{2a^2f} + \frac{\cos^3(e+fx)}{2af\,(b+a\cos^2(e+fx))}$$

```
[Out] -3/2*cos(f*x+e)/a^2/f+1/2*cos(f*x+e)^3/a/f/(b+a*cos(f*x+e)^2)+3/2*arctan(co
s(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(5/2)/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4218, 294, 327, 211}

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{3\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{5/2}f} - \frac{3\cos(e+fx)}{2a^2f} + \frac{\cos^3(e+fx)}{2af\,(a\cos^2(e+fx)+b)}$$

```
[In] Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (3*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*a^(5/2)*f) - (3*Cos[e
 + f*x])/(2*a^2*f) + Cos[e + f*x]^3/(2*a*f*(b + a*Cos[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^4}{(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)}{2af\,(b+a\cos^2(e+fx))} - \frac{3\text{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{2af}$$

$$= -\frac{3\cos(e+fx)}{2a^2f} + \frac{\cos^3(e+fx)}{2af\,(b+a\cos^2(e+fx))} + \frac{(3b)\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{2a^2f}$$

$$= \frac{3\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{5/2}f} - \frac{3\cos(e+fx)}{2a^2f} + \frac{\cos^3(e+fx)}{2af\,(b+a\cos^2(e+fx))}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.58 (sec) , antiderivative size = 393, normalized size of antiderivative = 4.68

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))^2 \left(\frac{(a^2+24b^2)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right.}{\sqrt{b}}\right.}{b^{3/2}}\right.}{}$$

```
[In] Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^2*(((a^2 + 24*b^2)*ArcTan[((-Sqrt[a] - I*Sq
rt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a
] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/2
) + ((a^2 + 24*b^2)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[
e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] -
I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/2) - (a^2*ArcTan[(Sqrt[a] - Sqrt
[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(3/2) - (a^2*ArcTan[(Sqrt[a] + Sqrt[a
 + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(3/2) - (16*Sqrt[a]*Cos[e + f*x]*(a + 3
*b + a*Cos[2*(e + f*x)]))/(a + 2*b + a*Cos[2*(e + f*x)]))*Sec[e + f*x]^4)/(
64*a^(5/2)*f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\sec(fx+e)}{2a+2b\sec(fx+e)^2}+\frac{3\arctan\left(\frac{\sec(fx+e)b}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2}-\frac{1}{a^2\sec(fx+e)}}{f}$
default	$\frac{-\frac{b\left(\frac{\sec(fx+e)}{2a+2b\sec(fx+e)^2}+\frac{3\arctan\left(\frac{\sec(fx+e)b}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2}-\frac{1}{a^2\sec(fx+e)}}{f}$
risch	$-\frac{e^{i(fx+e)}}{2f\,a^2}-\frac{e^{-i(fx+e)}}{2f\,a^2}-\frac{b\left(e^{3i(fx+e)}+e^{i(fx+e)}\right)}{a^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}-\frac{3i\sqrt{ab}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{ab}\,e^{i(fx+e)}}{a}\right.}{4a^3f}$

```
[In] int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-b/a^2*(1/2*sec(f*x+e)/(a+b*sec(f*x+e)^2)+3/2/(a*b)^(1/2)*arctan(sec(f
*x+e)*b/(a*b)^(1/2)))-1/a^2/sec(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.39

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \left[-\frac{4\,a\cos\left(fx+e\right)^3-3\left(a\cos\left(fx+e\right)^2+b\right)\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos(fx+e)^2+2\,a\sqrt{-\frac{b}{a}}\cos(fx+e)-b}{a\cos(fx+e)^2+b}\right)+6\,b\cos\left(fx+e\right)}{4\left(a^3f\cos\left(fx+e\right)^2+a^2bf\right)},\right.$$
$$\left.-\frac{2\,a\cos\left(fx+e\right)^3-3\left(a\cos\left(fx+e\right)^2+b\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}\cos(fx+e)}{b}\right)+3\,b\cos\left(fx+e\right)}{2\left(a^3f\cos\left(fx+e\right)^2+a^2bf\right)}\right]$$

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*cos(f*x + e)^3 - 3*(a*cos(f*x + e)^2 + b)*sqrt(-b/a)*log(-(a*cos
(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 6*
b*cos(f*x + e))/(a^3*f*cos(f*x + e)^2 + a^2*b*f), -1/2*(2*a*cos(f*x + e)^3
- 3*(a*cos(f*x + e)^2 + b)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) + 3
*b*cos(f*x + e))/(a^3*f*cos(f*x + e)^2 + a^2*b*f)]
```

Sympy [F]

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\sin\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(sin(e + f*x)/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{b\cos(fx+e)}{a^3\cos(fx+e)^2+a^2b} - \frac{3\,b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{2\,\cos(fx+e)}{a^2}}{2\,f}$$

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*cos(f*x + e)/(a^3*cos(f*x + e)^2 + a^2*b) - 3*b*arctan(a*cos(f*x +
e)/sqrt(a*b))/(sqrt(a*b)*a^2) + 2*cos(f*x + e)/a^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{3\,b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2f} - \frac{\cos\left(fx+e\right)}{a^2f} - \frac{b\cos\left(fx+e\right)}{2\,\left(a\cos\left(fx+e\right)^2+b\right)a^2f}$$

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 3/2*b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^2*f) - cos(f*x + e)/(a^
2*f) - 1/2*b*cos(f*x + e)/((a*cos(f*x + e)^2 + b)*a^2*f)
```

Mupad [B] (verification not implemented)

Time = 18.82 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\cos(e+f\,x)}{\sqrt{b}}\right)}{2\,a^{5/2}\,f} - \frac{b\,\cos\left(e+f\,x\right)}{2\,f\,\left(a^3\cos\left(e+f\,x\right)^2+b\,a^2\right)} - \frac{\cos\left(e+f\,x\right)}{a^2\,f}$$

```
[In] int(sin(e + f*x)/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (3*b^(1/2)*atan((a^(1/2)*cos(e + f*x))/b^(1/2)))/(2*a^(5/2)*f) - (b*cos(e +
 f*x))/(2*f*(a^2*b + a^3*cos(e + f*x)^2)) - cos(e + f*x)/(a^2*f)
```

3.44 $\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 99

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{3/2}(a+b)^2f} - \frac{\operatorname{arctanh}(\cos(e+fx))}{(a+b)^2f} - \frac{b\cos(e+fx)}{2a(a+b)f\,(b+a\cos^2(e+fx))}$$

```
[Out] -arctanh(cos(f*x+e))/(a+b)^2/f-1/2*b*cos(f*x+e)/a/(a+b)/f/(b+a*cos(f*x+e)^2
)+1/2*(3*a+b)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(3/2)/(a+b)^2/f
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4218, 481, 536, 212, 211}

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{3/2}f(a+b)^2} - \frac{\operatorname{arctanh}(\cos(e+fx))}{f(a+b)^2} - \frac{b\cos(e+fx)}{2af(a+b)\,(a\cos^2(e+fx)+b)}$$

```
[In] Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*a^(3/2)*(a +
b)^2*f) - ArcTanh[Cos[e + f*x]]/((a + b)^2*f) - (b*Cos[e + f*x])/(2*a*(a +
b)*f*(b + a*Cos[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{b\cos(e+fx)}{2a(a+b)f\left(b+a\cos^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{b+(-2a-b)x^2}{(1-x^2)(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{2a(a+b)f}$$

$$= -\frac{b\cos(e+fx)}{2a(a+b)f\left(b+a\cos^2(e+fx)\right)} - \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(e+fx)\right)}{(a+b)^2 f} + \frac{(b(3a+b))\operatorname{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{2a(a+b)^2 f}$$

$$= \frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{3/2}(a+b)^2 f} - \frac{\operatorname{arctanh}(\cos(e+fx))}{(a+b)^2 f} - \frac{b\cos(e+fx)}{2a(a+b)f\left(b+a\cos^2(e+fx)\right)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.63 (sec) , antiderivative size = 384, normalized size of antiderivative = 3.88

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(-\frac{2b(a+b)}{a} + \frac{\sqrt{b}(3a+b)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e\cdots}{\sqrt{b}}\right)\cdots}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*((-2*b*(a + b))/a + (Sqrt[b]
*(3*a + b)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*S
in[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])
^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x])/a^
(3/2) + (Sqrt[b]*(3*a + b)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] -
 I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Co
s[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])*
Sec[e + f*x])/a^(3/2) - 2*(a + 2*b + a*Cos[2*(e + f*x)])*Log[Cos[(e + f*x)/
2]]*Sec[e + f*x] + 2*(a + 2*b + a*Cos[2*(e + f*x)])*Log[Sin[(e + f*x)/2]]*S
ec[e + f*x]))/(8*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{\frac{\ln(-1+\cos(fx+e))}{2(a+b)^2}+\frac{b\left(-\frac{(a+b)\cos(fx+e)}{2a\left(b+a\cos(fx+e)^2\right)}+\frac{(3a+b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a+b)^2}-\frac{\ln(1+\cos(fx+e))}{2(a+b)^2}}{f}$
default	$\frac{\frac{\ln(-1+\cos(fx+e))}{2(a+b)^2}+\frac{b\left(-\frac{(a+b)\cos(fx+e)}{2a\left(b+a\cos(fx+e)^2\right)}+\frac{(3a+b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a+b)^2}-\frac{\ln(1+\cos(fx+e))}{2(a+b)^2}}{f}$
risch	$-\frac{b(\mathrm{e}^{3i(fx+e)}+\mathrm{e}^{i(fx+e)})}{af(a+b)(a\,\mathrm{e}^{4i(fx+e)}+2a\,\mathrm{e}^{2i(fx+e)}+4b\,\mathrm{e}^{2i(fx+e)}+a)}+\frac{\ln(\mathrm{e}^{i(fx+e)}-1)}{f(a^2+2ab+b^2)}-\frac{\ln(\mathrm{e}^{i(fx+e)}+1)}{f(a^2+2ab+b^2)}+\frac{3i\sqrt{ab}\,\ln\left(\mathrm{e}^{2i(fx+e}\right.}{4a(}$

```
[In] int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2/(a+b)^2*ln(-1+cos(f*x+e))+b/(a+b)^2*(-1/2*(a+b)/a*cos(f*x+e)/(b+a*
cos(f*x+e)^2)+1/2*(3*a+b)/a/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2)))-1
/2/(a+b)^2*ln(1+cos(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(87) = 174.

Time = 0.32 (sec) , antiderivative size = 390, normalized size of antiderivative = 3.94

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \left[\frac{\left((3\,a^2+ab)\cos\left(fx+e\right)^2+3\,ab+b^2\right)\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos(fx+e)^2+2\,a\sqrt{-\frac{b}{a}}\cos(fx+e)-b}{a\cos(fx+e)^2+b}\right)-2\left(ab+b^2\right)\cos\left(fx-\right.}{4\left((a^4+2\,a^3b+a^2b^2)f\cos\left(\right.\right.}\right.$$

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(((3*a^2 + a*b)*cos(f*x + e)^2 + 3*a*b + b^2)*sqrt(-b/a)*log(-(a*cos(f
*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) - 2*(a
*b + b^2)*cos(f*x + e) - 2*(a^2*cos(f*x + e)^2 + a*b)*log(1/2*cos(f*x + e)
+ 1/2) + 2*(a^2*cos(f*x + e)^2 + a*b)*log(-1/2*cos(f*x + e) + 1/2))/((a^4 +
 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b + 2*a^2*b^2 + a*b^3)*f), 1/2*
(((3*a^2 + a*b)*cos(f*x + e)^2 + 3*a*b + b^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*
cos(f*x + e)/b) - (a*b + b^2)*cos(f*x + e) - (a^2*cos(f*x + e)^2 + a*b)*log
(1/2*cos(f*x + e) + 1/2) + (a^2*cos(f*x + e)^2 + a*b)*log(-1/2*cos(f*x + e)
 + 1/2))/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b + 2*a^2*b^2 +
 a*b^3)*f)]
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx=\int \frac{\csc\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(csc(e + f*x)/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.39

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{b\cos(fx+e)}{a^2b+ab^2+(a^3+a^2b)\cos(fx+e)^2}-\frac{(3\,ab+b^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a^3+2\,a^2b+ab^2)\sqrt{ab}}+\frac{\log(\cos(fx+e)+1)}{a^2+2\,ab+b^2}-\frac{\log(\cos(fx+e)-1)}{a^2+2\,ab+b^2}}{2\,f}$$

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*cos(f*x + e)/(a^2*b + a*b^2 + (a^3 + a^2*b)*cos(f*x + e)^2) - (3*a*
b + b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b
)) + log(cos(f*x + e) + 1)/(a^2 + 2*a*b + b^2) - log(cos(f*x + e) - 1)/(a^2
 + 2*a*b + b^2))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(87) = 174$.

Time = 0.33 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.75

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{(3\,ab+b^2)\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{(a^3+2\,a^2b+ab^2)\sqrt{ab}}-\frac{\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^2+2\,ab+b^2}+\frac{2\left(ab+b^2+\frac{ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)}{(a^3+2\,a^2b+ab^2)\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)}{(\cos(fx+e)}\right.}}{2\,f}$$

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((3*a*b + b^2)*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*b)*cos(f*x + e) +
sqrt(a*b)))/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b)) - log(abs(-cos(f*x + e) + 1
```

```
)/abs(cos(f*x + e) + 1))/(a^2 + 2*a*b + b^2) + 2*(a*b + b^2 + a*b*(cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) - b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/(
(a^3 + 2*a^2*b + a*b^2)*(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1)
- 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f
*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)))/f
```

Mupad [B] (verification not implemented)

Time = 20.18 (sec) , antiderivative size = 2188, normalized size of antiderivative = 22.10

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] (atan((((3*a + b)*(-a^3*b)^(1/2)*((cos(e + f*x)*(6*a*b^3 + 4*a^4 + b^4 + 9*
a^2*b^2))/(2*(a*b^2 + 2*a^2*b + a^3)) + ((3*a + b)*(-a^3*b)^(1/2)*((2*a^6*b
 + 2*a^2*b^5 + 8*a^3*b^4 + 12*a^4*b^3 + 8*a^5*b^2)/(a*b^3 + 3*a^3*b + a^4 +
 3*a^2*b^2) - (cos(e + f*x)*(3*a + b)*(-a^3*b)^(1/2)*(48*a^7*b + 16*a^8 - 1
6*a^3*b^5 - 48*a^4*b^4 - 32*a^5*b^3 + 32*a^6*b^2))/(8*(a*b^2 + 2*a^2*b + a^
3)*(2*a^4*b + a^5 + a^3*b^2))))/(4*(2*a^4*b + a^5 + a^3*b^2)))*1i)/(4*(2*a^
4*b + a^5 + a^3*b^2)) + ((3*a + b)*(-a^3*b)^(1/2)*((cos(e + f*x)*(6*a*b^3 +
 4*a^4 + b^4 + 9*a^2*b^2))/(2*(a*b^2 + 2*a^2*b + a^3)) - ((3*a + b)*(-a^3*b
)^(1/2)*((2*a^6*b + 2*a^2*b^5 + 8*a^3*b^4 + 12*a^4*b^3 + 8*a^5*b^2)/(a*b^3
+ 3*a^3*b + a^4 + 3*a^2*b^2) + (cos(e + f*x)*(3*a + b)*(-a^3*b)^(1/2)*(48*a
^7*b + 16*a^8 - 16*a^3*b^5 - 48*a^4*b^4 - 32*a^5*b^3 + 32*a^6*b^2))/(8*(a*b
^2 + 2*a^2*b + a^3)*(2*a^4*b + a^5 + a^3*b^2))))/(4*(2*a^4*b + a^5 + a^3*b^
2)))*1i)/(4*(2*a^4*b + a^5 + a^3*b^2)))/(((5*a*b^2)/2 + 3*a^2*b + b^3/2)/(a
*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2) - ((3*a + b)*(-a^3*b)^(1/2)*((cos(e + f*x
)*(6*a*b^3 + 4*a^4 + b^4 + 9*a^2*b^2))/(2*(a*b^2 + 2*a^2*b + a^3)) + ((3*a
+ b)*(-a^3*b)^(1/2)*((2*a^6*b + 2*a^2*b^5 + 8*a^3*b^4 + 12*a^4*b^3 + 8*a^5*
b^2)/(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2) - (cos(e + f*x)*(3*a + b)*(-a^3*b)
^(1/2)*(48*a^7*b + 16*a^8 - 16*a^3*b^5 - 48*a^4*b^4 - 32*a^5*b^3 + 32*a^6*b
^2))/(8*(a*b^2 + 2*a^2*b + a^3)*(2*a^4*b + a^5 + a^3*b^2))))/(4*(2*a^4*b +
a^5 + a^3*b^2))))/(4*(2*a^4*b + a^5 + a^3*b^2)) + ((3*a + b)*(-a^3*b)^(1/2)
*((cos(e + f*x)*(6*a*b^3 + 4*a^4 + b^4 + 9*a^2*b^2))/(2*(a*b^2 + 2*a^2*b +
a^3)) - ((3*a + b)*(-a^3*b)^(1/2)*((2*a^6*b + 2*a^2*b^5 + 8*a^3*b^4 + 12*a^
4*b^3 + 8*a^5*b^2)/(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2) + (cos(e + f*x)*(3*a
 + b)*(-a^3*b)^(1/2)*(48*a^7*b + 16*a^8 - 16*a^3*b^5 - 48*a^4*b^4 - 32*a^5*
b^3 + 32*a^6*b^2))/(8*(a*b^2 + 2*a^2*b + a^3)*(2*a^4*b + a^5 + a^3*b^2))))/
(4*(2*a^4*b + a^5 + a^3*b^2))))/(4*(2*a^4*b + a^5 + a^3*b^2))))*(3*a + b)*(
-a^3*b)^(1/2)*1i)/(2*f*(2*a^4*b + a^5 + a^3*b^2)) - (atan((((((2*a^6*b + 2*
a^2*b^5 + 8*a^3*b^4 + 12*a^4*b^3 + 8*a^5*b^2)/(2*(a*b^3 + 3*a^3*b + a^4 + 3
*a^2*b^2)) - (cos(e + f*x)*(48*a^7*b + 16*a^8 - 16*a^3*b^5 - 48*a^4*b^4 - 3
2*a^5*b^3 + 32*a^6*b^2))/(8*(a + b)^2*(a*b^2 + 2*a^2*b + a^3)))*1i)/(2*(a +
```

```
b)^2) + (cos(e + f*x)*(6*a*b^3 + 4*a^4 + b^4 + 9*a^2*b^2)*1i)/(4*(a*b^2 +
2*a^2*b + a^3)))/(a + b)^2 - ((((2*a^6*b + 2*a^2*b^5 + 8*a^3*b^4 + 12*a^4*b
^3 + 8*a^5*b^2)/(2*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)) + (cos(e + f*x)*(48
*a^7*b + 16*a^8 - 16*a^3*b^5 - 48*a^4*b^4 - 32*a^5*b^3 + 32*a^6*b^2))/(8*(a
 + b)^2*(a*b^2 + 2*a^2*b + a^3)))*1i)/(2*(a + b)^2) - (cos(e + f*x)*(6*a*b^
3 + 4*a^4 + b^4 + 9*a^2*b^2)*1i)/(4*(a*b^2 + 2*a^2*b + a^3)))/(a + b)^2)/((
((2*a^6*b + 2*a^2*b^5 + 8*a^3*b^4 + 12*a^4*b^3 + 8*a^5*b^2)/(2*(a*b^3 + 3*a
^3*b + a^4 + 3*a^2*b^2)) - (cos(e + f*x)*(48*a^7*b + 16*a^8 - 16*a^3*b^5 -
48*a^4*b^4 - 32*a^5*b^3 + 32*a^6*b^2))/(8*(a + b)^2*(a*b^2 + 2*a^2*b + a^3)
))/(2*(a + b)^2) + (cos(e + f*x)*(6*a*b^3 + 4*a^4 + b^4 + 9*a^2*b^2))/(4*(a
*b^2 + 2*a^2*b + a^3)))/(a + b)^2 - ((5*a*b^2)/2 + 3*a^2*b + b^3/2)/(a*b^3
+ 3*a^3*b + a^4 + 3*a^2*b^2) + (((2*a^6*b + 2*a^2*b^5 + 8*a^3*b^4 + 12*a^4*
b^3 + 8*a^5*b^2)/(2*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)) + (cos(e + f*x)*(4
8*a^7*b + 16*a^8 - 16*a^3*b^5 - 48*a^4*b^4 - 32*a^5*b^3 + 32*a^6*b^2))/(8*(
a + b)^2*(a*b^2 + 2*a^2*b + a^3)))/(2*(a + b)^2) - (cos(e + f*x)*(6*a*b^3 +
 4*a^4 + b^4 + 9*a^2*b^2))/(4*(a*b^2 + 2*a^2*b + a^3)))/(a + b)^2))*1i)/(f*
(a + b)^2) - (b*cos(e + f*x))/(2*a*f*(a + b)*(b + a*cos(e + f*x)^2))
```

3.45 $\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{(3a-b)\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2\sqrt{a}(a+b)^3f} - \frac{(a-3b)\operatorname{arctanh}(\cos(e+fx))}{2(a+b)^3f} + \frac{(a-b)\cos(e+fx)}{2(a+b)^2f\,(b+a\cos^2(e+fx))} - \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\,(b+a\cos^2(e+fx))}$$

```
[Out] -1/2*(a-3*b)*arctanh(cos(f*x+e))/(a+b)^3/f+1/2*(a-b)*cos(f*x+e)/(a+b)^2/f/(
b+a*cos(f*x+e)^2)-1/2*cot(f*x+e)*csc(f*x+e)/(a+b)/f/(b+a*cos(f*x+e)^2)+1/2*
(3*a-b)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/(a+b)^3/f/a^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used

$= \{4218, 481, 541, 536, 212, 211\}$

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\sqrt{b}(3a-b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2\sqrt{a}f(a+b)^3} - \frac{(a-3b)\text{arctanh}(\cos(e+fx))}{2f(a+b)^3} + \frac{(a-b)\cos(e+fx)}{2f(a+b)^2\,(a\cos^2(e+fx)+b)} - \frac{\cot(e+fx)\csc(e+fx)}{2f(a+b)\,(a\cos^2(e+fx)+b)}$$

```
[In] Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*Sqrt[a]*(a +
b)^3*f) - ((a - 3*b)*ArcTanh[Cos[e + f*x]])/(2*(a + b)^3*f) + ((a - b)*Cos[
e + f*x])/(2*(a + b)^2*f*(b + a*Cos[e + f*x]^2)) - (Cot[e + f*x]*Csc[e + f*
x])/(2*(a + b)*f*(b + a*Cos[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\,(b+a\cos^2(e+fx))} + \frac{\text{Subst}\left(\int \frac{b+(-a+2b)x^2}{(1-x^2)(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{2(a+b)f}\\
&= \frac{(a-b)\cos(e+fx)}{2(a+b)^2f\,(b+a\cos^2(e+fx))} - \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\,(b+a\cos^2(e+fx))}\\
&\quad - \frac{\text{Subst}\left(\int \frac{-4b^2+2(a-b)bx^2}{(1-x^2)(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{4b(a+b)^2f}\\
&= \frac{(a-b)\cos(e+fx)}{2(a+b)^2f\,(b+a\cos^2(e+fx))} - \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\,(b+a\cos^2(e+fx))}\\
&\quad - \frac{(a-3b)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(e+fx)\right)}{2(a+b)^3f}\\
&\quad + \frac{((3a-b)b)\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{2(a+b)^3f}\\
&= \frac{(3a-b)\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2\sqrt{a}(a+b)^3f} - \frac{(a-3b)\text{arctanh}(\cos(e+fx))}{2(a+b)^3f}\\
&\quad + \frac{(a-b)\cos(e+fx)}{2(a+b)^2f\,(b+a\cos^2(e+fx))} - \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\,(b+a\cos^2(e+fx))}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.26 (sec) , antiderivative size = 468, normalized size of antiderivative = 3.18

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(-8b(a+b)-\frac{4\sqrt{b}(-3a+b)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right.}{}\right.}{}\right.}{}$$

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(-8*b*(a + b) - (4*Sqrt[b]*(
-3*a + b)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Si
n[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^
2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x])/Sqr
t[a] - (4*Sqrt[b]*(-3*a + b)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e]
 - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(
Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)]
)*Sec[e + f*x])/Sqrt[a] - (a + b)*(a + 2*b + a*Cos[2*(e + f*x)])*Csc[(e + f
*x)/2]^2*Sec[e + f*x] - 4*(a - 3*b)*(a + 2*b + a*Cos[2*(e + f*x)])*Log[Cos[
(e + f*x)/2]]*Sec[e + f*x] + 4*(a - 3*b)*(a + 2*b + a*Cos[2*(e + f*x)])*Log
[Sin[(e + f*x)/2]]*Sec[e + f*x] + (a + b)*(a + 2*b + a*Cos[2*(e + f*x)])*Se
c[(e + f*x)/2]^2*Sec[e + f*x]))/(32*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{1}{4(a+b)^2(-1+\cos(fx+e))}+\frac{(a-3b)\ln(-1+\cos(fx+e))}{4(a+b)^3}+\frac{b\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\cos(fx+e)}{b+a\cos(fx+e)^2}+\frac{(3a-b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a+b)^3}+\frac{1}{4(a+b)^2(1+\cos(f}}{f}$
default	$\frac{\frac{1}{4(a+b)^2(-1+\cos(fx+e))}+\frac{(a-3b)\ln(-1+\cos(fx+e))}{4(a+b)^3}+\frac{b\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\cos(fx+e)}{b+a\cos(fx+e)^2}+\frac{(3a-b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a+b)^3}+\frac{1}{4(a+b)^2(1+\cos(f}}{f}$
risch	$\frac{a\,e^{7i(fx+e)}-b\,e^{7i(fx+e)}+3a\,e^{5i(fx+e)}+5b\,e^{5i(fx+e)}+3a\,e^{3i(fx+e)}+5b\,e^{3i(fx+e)}+a\,e^{i(fx+e)}-b\,e^{i(fx+e)}}{f(a+b)^2\left(e^{2i(fx+e)}-1\right)^2\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{\ln\left(e^{i(fx+}\right.}{2f(a^3+3a^2b+}$

```
[In] int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/4/(a+b)^2/(-1+cos(f*x+e))+1/4*(a-3*b)/(a+b)^3*ln(-1+cos(f*x+e))+b/(a
+b)^3*((-1/2*a-1/2*b)*cos(f*x+e)/(b+a*cos(f*x+e)^2)+1/2*(3*a-b)/(a*b)^(1/2)
*arctan(a*cos(f*x+e)/(a*b)^(1/2)))+1/4/(a+b)^2/(1+cos(f*x+e))+1/4/(a+b)^3*(
-a+3*b)*ln(1+cos(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(131) = 262$.

Time = 0.33 (sec) , antiderivative size = 698, normalized size of antiderivative = 4.75

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \left[\frac{2\,(a^2-b^2)\cos{(fx+e)}^3 - \left((3\,a^2-ab)\cos{(fx+e)}^4 - (3\,a^2-4\,ab+b^2)\cos{(fx+e)}^2 - 3\,ab+b^2\right)\sqrt{-}\right.$$

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2 - b^2)*cos(f*x + e)^3 - ((3*a^2 - a*b)*cos(f*x + e)^4 - (3*a^2
 - 4*a*b + b^2)*cos(f*x + e)^2 - 3*a*b + b^2)*sqrt(-b/a)*log((a*cos(f*x + e
)^2 - 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 4*(a*b + b
^2)*cos(f*x + e) - ((a^2 - 3*a*b)*cos(f*x + e)^4 - (a^2 - 4*a*b + 3*b^2)*co
s(f*x + e)^2 - a*b + 3*b^2)*log(1/2*cos(f*x + e) + 1/2) + ((a^2 - 3*a*b)*co
s(f*x + e)^4 - (a^2 - 4*a*b + 3*b^2)*cos(f*x + e)^2 - a*b + 3*b^2)*log(-1/2
*cos(f*x + e) + 1/2))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4
 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*f*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 +
3*a*b^3 + b^4)*f), 1/4*(2*(a^2 - b^2)*cos(f*x + e)^3 + 2*((3*a^2 - a*b)*cos
(f*x + e)^4 - (3*a^2 - 4*a*b + b^2)*cos(f*x + e)^2 - 3*a*b + b^2)*sqrt(b/a)
*arctan(a*sqrt(b/a)*cos(f*x + e)/b) + 4*(a*b + b^2)*cos(f*x + e) - ((a^2 -
3*a*b)*cos(f*x + e)^4 - (a^2 - 4*a*b + 3*b^2)*cos(f*x + e)^2 - a*b + 3*b^2)
*log(1/2*cos(f*x + e) + 1/2) + ((a^2 - 3*a*b)*cos(f*x + e)^4 - (a^2 - 4*a*b
 + 3*b^2)*cos(f*x + e)^2 - a*b + 3*b^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^4
 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2*a^3*b - 2*a*b^3
 - b^4)*f*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f)]
```

Sympy [F]

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

```
[In] integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(csc(e + f*x)**3/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.57

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{(a-3b)\log(\cos(fx+e)+1)}{a^3+3a^2b+3ab^2+b^3} - \frac{(a-3b)\log(\cos(fx+e)-1)}{a^3+3a^2b+3ab^2+b^3} - \frac{2(3ab-b^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a^3+3a^2b+3ab^2+b^3)\sqrt{ab}} - \frac{2\left((a-b)\cos(fx+e)^3+2\right.}{(a^3+2a^2b+ab^2)\cos(fx+e)^4-a^2b-2ab^2-b}}{4f}$$

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*((a - 3*b)*log(cos(f*x + e) + 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (a
- 3*b)*log(cos(f*x + e) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(3*a*b - b
^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(
a*b)) - 2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))/((a^3 + 2*a^2*b + a*b
^2)*cos(f*x + e)^4 - a^2*b - 2*a*b^2 - b^3 - (a^3 + a^2*b - a*b^2 - b^3)*co
s(f*x + e)^2))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(131) = 262.

Time = 0.37 (sec) , antiderivative size = 550, normalized size of antiderivative = 3.74

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{6(a-3b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^3+3a^2b+3ab^2+b^3} - \frac{12(3ab-b^2)\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{(a^3+3a^2b+3ab^2+b^3)\sqrt{ab}} + \frac{3a^2+6ab+3b^2+\frac{4a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{20ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{24b^2(}{\cos}}{(a^3+3a^2b+3ab^2+b^3)\left(\frac{a(\cos(}{\cos(}\right.}}{}$$

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(6*(a - 3*b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(a^3 +
3*a^2*b + 3*a*b^2 + b^3) - 12*(3*a*b - b^2)*arctan(-(a*cos(f*x + e) - b)/(s
qrt(a*b)*cos(f*x + e) + sqrt(a*b)))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a
*b)) + (3*a^2 + 6*a*b + 3*b^2 + 4*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1)
 - 20*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 24*b^2*(cos(f*x + e) - 1)
/(cos(f*x + e) + 1) - a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 2*a*b
*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 15*b^2*(cos(f*x + e) - 1)^2/(c
os(f*x + e) + 1)^2 - 2*a^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 4*a*
b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 6*b^2*(cos(f*x + e) - 1)^3/(c
os(f*x + e) + 1)^3)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(a*(cos(f*x + e) - 1)/
(cos(f*x + e) + 1) + b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2*a*(cos(f*x
 + e) - 1)^2/(cos(f*x + e) + 1)^2 - 2*b*(cos(f*x + e) - 1)^2/(cos(f*x + e)
+ 1)^2 + a*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + b*(cos(f*x + e) - 1)
^3/(cos(f*x + e) + 1)^3)) - 3*(cos(f*x + e) - 1)/((a^2 + 2*a*b + b^2)*(cos(
f*x + e) + 1)))/f
```

Mupad [B] (verification not implemented)

Time = 18.84 (sec) , antiderivative size = 1845, normalized size of antiderivative = 12.55

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] - ((cos(e + f*x)^3*(a - b))/(2*(2*a*b + a^2 + b^2)) + (b*cos(e + f*x))/(2*a
*b + a^2 + b^2))/(f*(b - a*cos(e + f*x)^4 + cos(e + f*x)^2*(a - b))) - (log
(cos(e + f*x) - 1)*(b/(a + b)^3 - 1/(4*(a + b)^2)))/f - (log(cos(e + f*x) +
 1)*(a - 3*b))/(4*f*(a + b)^3) - (atan((((-a*b)^(1/2)*((cos(e + f*x)*(a*b^4
 - 6*a^4*b + a^5 - 6*a^2*b^3 + 18*a^3*b^2))/(2*(4*a*b^3 + 4*a^3*b + a^4 + b
^4 + 6*a^2*b^2)) + ((-a*b)^(1/2)*((4*a^8*b + 4*a^2*b^7 + 24*a^3*b^6 + 60*a^
4*b^5 + 80*a^5*b^4 + 60*a^6*b^3 + 24*a^7*b^2)/(6*a*b^5 + 6*a^5*b + a^6 + b^
6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2) - (cos(e + f*x)*(-a*b)^(1/2)*(3*a
 - b)*(80*a^8*b + 16*a^9 - 16*a^2*b^7 - 80*a^3*b^6 - 144*a^4*b^5 - 80*a^5*b
^4 + 80*a^6*b^3 + 144*a^7*b^2))/(8*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)*(4*a
*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)))*(3*a - b))/(4*(a*b^3 + 3*a^3*b +
a^4 + 3*a^2*b^2)))*(3*a - b)*1i)/(4*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)) +
((-a*b)^(1/2)*((cos(e + f*x)*(a*b^4 - 6*a^4*b + a^5 - 6*a^2*b^3 + 18*a^3*b^
2))/(2*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)) - ((-a*b)^(1/2)*((4*a^8
*b + 4*a^2*b^7 + 24*a^3*b^6 + 60*a^4*b^5 + 80*a^5*b^4 + 60*a^6*b^3 + 24*a^7
*b^2)/(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2
) + (cos(e + f*x)*(-a*b)^(1/2)*(3*a - b)*(80*a^8*b + 16*a^9 - 16*a^2*b^7 -
80*a^3*b^6 - 144*a^4*b^5 - 80*a^5*b^4 + 80*a^6*b^3 + 144*a^7*b^2))/(8*(a*b^
3 + 3*a^3*b + a^4 + 3*a^2*b^2)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2))
)*(3*a - b))/(4*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)))*(3*a - b)*1i)/(4*(a*b
```

```
^3 + 3*a^3*b + a^4 + 3*a^2*b^2)))/(((3*a*b^4)/4 - (3*a^4*b)/4 - (13*a^2*b^3
)/4 + (13*a^3*b^2)/4)/(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*
b^3 + 15*a^4*b^2) + ((-a*b)^(1/2)*((cos(e + f*x)*(a*b^4 - 6*a^4*b + a^5 - 6
*a^2*b^3 + 18*a^3*b^2))/(2*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)) + (
(-a*b)^(1/2)*((4*a^8*b + 4*a^2*b^7 + 24*a^3*b^6 + 60*a^4*b^5 + 80*a^5*b^4 +
 60*a^6*b^3 + 24*a^7*b^2)/(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*
a^3*b^3 + 15*a^4*b^2) - (cos(e + f*x)*(-a*b)^(1/2)*(3*a - b)*(80*a^8*b + 16
*a^9 - 16*a^2*b^7 - 80*a^3*b^6 - 144*a^4*b^5 - 80*a^5*b^4 + 80*a^6*b^3 + 14
4*a^7*b^2))/(8*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)*(4*a*b^3 + 4*a^3*b + a^4
 + b^4 + 6*a^2*b^2)))*(3*a - b))/(4*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)))*(
3*a - b))/(4*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)) - ((-a*b)^(1/2)*((cos(e +
 f*x)*(a*b^4 - 6*a^4*b + a^5 - 6*a^2*b^3 + 18*a^3*b^2))/(2*(4*a*b^3 + 4*a^3
*b + a^4 + b^4 + 6*a^2*b^2)) - ((-a*b)^(1/2)*((4*a^8*b + 4*a^2*b^7 + 24*a^3
*b^6 + 60*a^4*b^5 + 80*a^5*b^4 + 60*a^6*b^3 + 24*a^7*b^2)/(6*a*b^5 + 6*a^5*
b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2) + (cos(e + f*x)*(-a*b
)^(1/2)*(3*a - b)*(80*a^8*b + 16*a^9 - 16*a^2*b^7 - 80*a^3*b^6 - 144*a^4*b^
5 - 80*a^5*b^4 + 80*a^6*b^3 + 144*a^7*b^2))/(8*(a*b^3 + 3*a^3*b + a^4 + 3*a
^2*b^2)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)))*(3*a - b))/(4*(a*b^3
+ 3*a^3*b + a^4 + 3*a^2*b^2)))*(3*a - b))/(4*(a*b^3 + 3*a^3*b + a^4 + 3*a^2
*b^2))))*(-a*b)^(1/2)*(3*a - b)*1i)/(2*f*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2
))
```

3.46 $\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 197

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{3\sqrt{a}(a-b)\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2(a+b)^4 f} - \frac{3(a^2-6ab+b^2)\,\text{arctanh}(\cos(e+fx))}{8(a+b)^4 f} + \frac{3a(a-3b)\cos(e+fx)}{8(a+b)^3 f\,(b+a\cos^2(e+fx))} - \frac{(a-5b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2 f\,(b+a\cos^2(e+fx))} - \frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b) f\,(b+a\cos^2(e+fx))}$$

```
[Out] -3/8*(a^2-6*a*b+b^2)*arctanh(cos(f*x+e))/(a+b)^4/f+3/8*a*(a-3*b)*cos(f*x+e)
/(a+b)^3/f/(b+a*cos(f*x+e)^2)-1/8*(a-5*b)*cot(f*x+e)*csc(f*x+e)/(a+b)^2/f/(
b+a*cos(f*x+e)^2)-1/4*cot(f*x+e)*csc(f*x+e)^3/(a+b)/f/(b+a*cos(f*x+e)^2)+3/
2*(a-b)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*a^(1/2)*b^(1/2)/(a+b)^4/f
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4218, 481, 541, 536, 212, 211}

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{3(a^2-6ab+b^2)\,\text{arctanh}(\cos(e+fx))}{8f(a+b)^4} + \frac{3\sqrt{a}\sqrt{b}(a-b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2f(a+b)^4} + \frac{3a(a-3b)\cos(e+fx)}{8f(a+b)^3\,(a\cos^2(e+fx)+b)} - \frac{\cot(e+fx)\csc^3(e+fx)}{4f(a+b)\,(a\cos^2(e+fx)+b)} - \frac{(a-5b)\cot(e+fx)\csc(e+fx)}{8f(a+b)^2\,(a\cos^2(e+fx)+b)}$$

```
[In] Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (3*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*(a +
b)^4*f) - (3*(a^2 - 6*a*b + b^2)*ArcTanh[Cos[e + f*x]])/(8*(a + b)^4*f) + (
3*a*(a - 3*b)*Cos[e + f*x])/(8*(a + b)^3*f*(b + a*Cos[e + f*x]^2)) - ((a -
5*b)*Cot[e + f*x]*Csc[e + f*x])/(8*(a + b)^2*f*(b + a*Cos[e + f*x]^2)) - (C
ot[e + f*x]*Csc[e + f*x]^3)/(4*(a + b)*f*(b + a*Cos[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
```

```
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^3(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{f} \\
&= -\frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b)f\left(b+a\cos^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{b+(-a+4b)x^2}{(1-x^2)^2(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{4(a+b)f} \\
&= -\frac{(a-5b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(b+a\cos^2(e+fx)\right)} - \frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b)f\left(b+a\cos^2(e+fx)\right)} \\
&\quad + \frac{\text{Subst}\left(\int \frac{3(a-b)b-3a(a-5b)x^2}{(1-x^2)(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{8(a+b)^2f} \\
&= \frac{3a(a-3b)\cos(e+fx)}{8(a+b)^3f\left(b+a\cos^2(e+fx)\right)} - \frac{(a-5b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(b+a\cos^2(e+fx)\right)} \\
&\quad - \frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b)f\left(b+a\cos^2(e+fx)\right)} \\
&\quad - \frac{\text{Subst}\left(\int \frac{-6(3a-b)b^2+6a(a-3b)bx^2}{(1-x^2)(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{16b(a+b)^3f}
\end{aligned}
$$

$$= \frac{3a(a-3b)\cos(e+fx)}{8(a+b)^3f\left(b+a\cos^2(e+fx)\right)} - \frac{(a-5b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(b+a\cos^2(e+fx)\right)} - \frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b)f\left(b+a\cos^2(e+fx)\right)} + \frac{(3a(a-b)b)\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx,x,\cos(e+fx)\right)}{2(a+b)^4f} - \frac{\left(3\left(a^2-6ab+b^2\right)\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\cos(e+fx)\right)}{8(a+b)^4f}$$

$$= \frac{3\sqrt{a}(a-b)\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2(a+b)^4f} - \frac{3\left(a^2-6ab+b^2\right)\text{arctanh}(\cos(e+fx))}{8(a+b)^4f} + \frac{3a(a-3b)\cos(e+fx)}{8(a+b)^3f\left(b+a\cos^2(e+fx)\right)} - \frac{(a-5b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(b+a\cos^2(e+fx)\right)} - \frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b)f\left(b+a\cos^2(e+fx)\right)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.64 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.28

$$\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\left(96\sqrt{a}(a-b)\sqrt{b}\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+\right.}{\sqrt{b}}\right.\right.}{}$$

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*(96*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan[((-Sqrt[
a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e
]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]
]*(a + 2*b + a*Cos[2*(e + f*x)]) + 96*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan[((-Sqr
t[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos
[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[
b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - 2*(a + b)*(11*a^2 + 43*a*b - 4*b^2 + 4
*(2*a^2 - 5*a*b + 5*b^2)*Cos[2*(e + f*x)] - 3*a*(a - 3*b)*Cos[4*(e + f*x)])
*Cot[e + f*x]*Csc[e + f*x]^3 - 24*(a^2 - 6*a*b + b^2)*(a + 2*b + a*Cos[2*(e
 + f*x)])*Log[Cos[(e + f*x)/2]] + 24*(a^2 - 6*a*b + b^2)*(a + 2*b + a*Cos[2
*(e + f*x)])*Log[Sin[(e + f*x)/2]])*Sec[e + f*x]^4)/(256*(a + b)^4*f*(a + b
*Sec[e + f*x]^2)^2)
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{-\frac{1}{16(a+b)^2(-1+\cos(fx+e))^2}-\frac{-3a+5b}{16(a+b)^3(-1+\cos(fx+e))}+\frac{(3a^2-18ab+3b^2)\ln(-1+\cos(fx+e))}{16(a+b)^4}+\frac{ab\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\cos(fx+e)}{b+a\cos(fx+e)^2}+\frac{3(a-}{}\right.}{(a+b)}}{f}$
default	$\frac{-\frac{1}{16(a+b)^2(-1+\cos(fx+e))^2}-\frac{-3a+5b}{16(a+b)^3(-1+\cos(fx+e))}+\frac{(3a^2-18ab+3b^2)\ln(-1+\cos(fx+e))}{16(a+b)^4}+\frac{ab\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\cos(fx+e)}{b+a\cos(fx+e)^2}+\frac{3(a-}{}\right.}{(a+b)}}{f}$
risch	$-\frac{-3a^2 e^{11i(fx+e)}+9ab\,e^{11i(fx+e)}+5a^2 e^{9i(fx+e)}-11ab\,e^{9i(fx+e)}+20b^2 e^{9i(fx+e)}+30a^2 e^{7i(fx+e)}+66ab\,e^{7i(fx+e)}+12b}{4f(a+b)^3\left(e^{2i(fx+e)}-1\right)^4(}$

```
[In] int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/16/(a+b)^2/(-1+cos(f*x+e))^2-1/16*(-3*a+5*b)/(a+b)^3/(-1+cos(f*x+e)
)+1/16/(a+b)^4*(3*a^2-18*a*b+3*b^2)*ln(-1+cos(f*x+e))+a*b/(a+b)^4*((-1/2*a-
1/2*b)*cos(f*x+e)/(b+a*cos(f*x+e)^2)+3/2*(a-b)/(a*b)^(1/2)*arctan(a*cos(f*x
+e)/(a*b)^(1/2)))+1/16/(a+b)^2/(1+cos(f*x+e))^2-1/16*(-3*a+5*b)/(a+b)^3/(1+
cos(f*x+e))+1/16/(a+b)^4*(-3*a^2+18*a*b-3*b^2)*ln(1+cos(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(179) = 358$.

Time = 0.37 (sec) , antiderivative size = 1202, normalized size of antiderivative = 6.10

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/16*(6*(a^3 - 2*a^2*b - 3*a*b^2)*cos(f*x + e)^5 - 2*(5*a^3 - 9*a^2*b - 9*
a*b^2 + 5*b^3)*cos(f*x + e)^3 - 12*((a^2 - a*b)*cos(f*x + e)^6 - (2*a^2 - 3
*a*b + b^2)*cos(f*x + e)^4 + (a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^2 + a*b - b
^2)*sqrt(-a*b)*log((a*cos(f*x + e)^2 - 2*sqrt(-a*b)*cos(f*x + e) - b)/(a*co
s(f*x + e)^2 + b)) - 6*(3*a^2*b + 2*a*b^2 - b^3)*cos(f*x + e) - 3*((a^3 - 6
*a^2*b + a*b^2)*cos(f*x + e)^6 - (2*a^3 - 13*a^2*b + 8*a*b^2 - b^3)*cos(f*x
 + e)^4 + a^2*b - 6*a*b^2 + b^3 + (a^3 - 8*a^2*b + 13*a*b^2 - 2*b^3)*cos(f*
x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 3*((a^3 - 6*a^2*b + a*b^2)*cos(f*x
+ e)^6 - (2*a^3 - 13*a^2*b + 8*a*b^2 - b^3)*cos(f*x + e)^4 + a^2*b - 6*a*b^
2 + b^3 + (a^3 - 8*a^2*b + 13*a*b^2 - 2*b^3)*cos(f*x + e)^2)*log(-1/2*cos(f
*x + e) + 1/2))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*f*cos(f*x
```

```
+ e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*f*cos(f*
x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*f*cos(
f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f), 1/16*(6*(a
^3 - 2*a^2*b - 3*a*b^2)*cos(f*x + e)^5 - 2*(5*a^3 - 9*a^2*b - 9*a*b^2 + 5*b
^3)*cos(f*x + e)^3 + 24*((a^2 - a*b)*cos(f*x + e)^6 - (2*a^2 - 3*a*b + b^2)
*cos(f*x + e)^4 + (a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^2 + a*b - b^2)*sqrt(a*
b)*arctan(sqrt(a*b)*cos(f*x + e)/b) - 6*(3*a^2*b + 2*a*b^2 - b^3)*cos(f*x +
 e) - 3*((a^3 - 6*a^2*b + a*b^2)*cos(f*x + e)^6 - (2*a^3 - 13*a^2*b + 8*a*b
^2 - b^3)*cos(f*x + e)^4 + a^2*b - 6*a*b^2 + b^3 + (a^3 - 8*a^2*b + 13*a*b^
2 - 2*b^3)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 3*((a^3 - 6*a^2*b
+ a*b^2)*cos(f*x + e)^6 - (2*a^3 - 13*a^2*b + 8*a*b^2 - b^3)*cos(f*x + e)^4
 + a^2*b - 6*a*b^2 + b^3 + (a^3 - 8*a^2*b + 13*a*b^2 - 2*b^3)*cos(f*x + e)^
2)*log(-1/2*cos(f*x + e) + 1/2))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 +
a*b^4)*f*cos(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^
4 - b^5)*f*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^
4 - 2*b^5)*f*cos(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^
5)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(179) = 358$.

Time = 0.27 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.87

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{3\,(a^2-6\,ab+b^2)\log(\cos(fx+e)+1)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4} - \frac{3\,(a^2-6\,ab+b^2)\log(\cos(fx+e)-1)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4} - \frac{24\,(a^2b-ab^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\sqrt{ab}} - \frac{\cdots}{(a^4+3\,a^3b+3\,a^2b^2+ab^3)\cdots}}{16\,f}$$

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(3*(a^2 - 6*a*b + b^2)*log(cos(f*x + e) + 1)/(a^4 + 4*a^3*b + 6*a^2*b
^2 + 4*a*b^3 + b^4) - 3*(a^2 - 6*a*b + b^2)*log(cos(f*x + e) - 1)/(a^4 + 4*
a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 24*(a^2*b - a*b^2)*arctan(a*cos(f*x +
```

```
e)/sqrt(a*b))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt(a*b)) - 2*(
3*(a^2 - 3*a*b)*cos(f*x + e)^5 - (5*a^2 - 14*a*b + 5*b^2)*cos(f*x + e)^3 -
3*(3*a*b - b^2)*cos(f*x + e))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x
+ e)^6 - (2*a^4 + 5*a^3*b + 3*a^2*b^2 - a*b^3 - b^4)*cos(f*x + e)^4 + a^3*b
 + 3*a^2*b^2 + 3*a*b^3 + b^4 + (a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 - 2*b^4)*
cos(f*x + e)^2))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 669 vs. $2(179) = 358$.

Time = 0.41 (sec) , antiderivative size = 669, normalized size of antiderivative = 3.40

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \frac{\frac{12\,(a^2-6\,ab+b^2)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4} - \frac{96\,(a^2b-ab^2)\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\sqrt{ab}} - \frac{\frac{8\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{8\,b^2(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{a^2(\cos(fx+e)}{(\cos(fx+e)+}}{a^4+4\,a^3b+6\,a^2b^2}}{}$$

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/64*(12*(a^2 - 6*a*b + b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) +
1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 96*(a^2*b - a*b^2)*arctan
(-(a*cos(f*x + e) - b)/(sqrt(a*b)*cos(f*x + e) + sqrt(a*b)))/((a^4 + 4*a^3*
b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt(a*b)) - (8*a^2*(cos(f*x + e) - 1)/(cos(
f*x + e) + 1) - 8*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - a^2*(cos(f*x
+ e) - 1)^2/(cos(f*x + e) + 1)^2 - 2*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e)
 + 1)^2 - b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^4 + 4*a^3*b + 6
*a^2*b^2 + 4*a*b^3 + b^4) - (a^2 + 2*a*b + b^2 - 8*a^2*(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) + 8*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 18*a^2*(c
os(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 108*a*b*(cos(f*x + e) - 1)^2/(cos
(f*x + e) + 1)^2 + 18*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f
*x + e) + 1)^2/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(cos(f*x + e) -
 1)^2) - 64*(a^2*b + a*b^2 + a^2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) -
a*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/((a^4 + 4*a^3*b + 6*a^2*b^2 +
4*a*b^3 + b^4)*(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2))/f
```

Mupad [B] (verification not implemented)

Time = 23.16 (sec) , antiderivative size = 4338, normalized size of antiderivative = 22.02

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] (atan((((-a*b)^(1/2)*((cos(e + f*x)*(9*a^7 - 108*a^6*b + 153*a^3*b^4 - 396*
a^4*b^3 + 486*a^5*b^2))/(32*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 2
0*a^3*b^3 + 15*a^4*b^2)) + (3*(-a*b)^(1/2)*(a - b)*(((9*a^11*b)/2 - (3*a^2*
b^10)/2 - (15*a^3*b^9)/2 - 6*a^4*b^8 + 42*a^5*b^7 + 147*a^6*b^6 + 231*a^7*b
^5 + 210*a^8*b^4 + 114*a^9*b^3 + (69*a^10*b^2)/2)/(9*a*b^8 + 9*a^8*b + a^9
+ b^9 + 36*a^2*b^7 + 84*a^3*b^6 + 126*a^4*b^5 + 126*a^5*b^4 + 84*a^6*b^3 +
36*a^7*b^2) - (3*cos(e + f*x)*(-a*b)^(1/2)*(a - b)*(1792*a^10*b + 256*a^11
- 256*a^2*b^9 - 1792*a^3*b^8 - 5120*a^4*b^7 - 7168*a^5*b^6 - 3584*a^6*b^5 +
 3584*a^7*b^4 + 7168*a^8*b^3 + 5120*a^9*b^2))/(128*(4*a*b^3 + 4*a^3*b + a^4
 + b^4 + 6*a^2*b^2)*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^
3 + 15*a^4*b^2))))/(4*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)))*(a - b)
*3i)/(4*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)) + ((-a*b)^(1/2)*((cos(
e + f*x)*(9*a^7 - 108*a^6*b + 153*a^3*b^4 - 396*a^4*b^3 + 486*a^5*b^2))/(32
*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)) -
(3*(-a*b)^(1/2)*(a - b)*(((9*a^11*b)/2 - (3*a^2*b^10)/2 - (15*a^3*b^9)/2 -
6*a^4*b^8 + 42*a^5*b^7 + 147*a^6*b^6 + 231*a^7*b^5 + 210*a^8*b^4 + 114*a^9*
b^3 + (69*a^10*b^2)/2)/(9*a*b^8 + 9*a^8*b + a^9 + b^9 + 36*a^2*b^7 + 84*a^3
*b^6 + 126*a^4*b^5 + 126*a^5*b^4 + 84*a^6*b^3 + 36*a^7*b^2) + (3*cos(e + f*
x)*(-a*b)^(1/2)*(a - b)*(1792*a^10*b + 256*a^11 - 256*a^2*b^9 - 1792*a^3*b^
8 - 5120*a^4*b^7 - 7168*a^5*b^6 - 3584*a^6*b^5 + 3584*a^7*b^4 + 7168*a^8*b^
3 + 5120*a^9*b^2))/(128*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)*(6*a*b^
5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2))))/(4*(4*a*
b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)))*(a - b)*3i)/(4*(4*a*b^3 + 4*a^3*b
+ a^4 + b^4 + 6*a^2*b^2)))/(((27*a^7*b)/64 + (81*a^3*b^5)/64 - (297*a^4*b^4
)/32 + (189*a^5*b^3)/16 - (135*a^6*b^2)/32)/(9*a*b^8 + 9*a^8*b + a^9 + b^9
+ 36*a^2*b^7 + 84*a^3*b^6 + 126*a^4*b^5 + 126*a^5*b^4 + 84*a^6*b^3 + 36*a^7
*b^2) - (3*(-a*b)^(1/2)*((cos(e + f*x)*(9*a^7 - 108*a^6*b + 153*a^3*b^4 - 3
96*a^4*b^3 + 486*a^5*b^2))/(32*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4
+ 20*a^3*b^3 + 15*a^4*b^2)) + (3*(-a*b)^(1/2)*(a - b)*(((9*a^11*b)/2 - (3*a
^2*b^10)/2 - (15*a^3*b^9)/2 - 6*a^4*b^8 + 42*a^5*b^7 + 147*a^6*b^6 + 231*a^
7*b^5 + 210*a^8*b^4 + 114*a^9*b^3 + (69*a^10*b^2)/2)/(9*a*b^8 + 9*a^8*b + a
^9 + b^9 + 36*a^2*b^7 + 84*a^3*b^6 + 126*a^4*b^5 + 126*a^5*b^4 + 84*a^6*b^3
 + 36*a^7*b^2) - (3*cos(e + f*x)*(-a*b)^(1/2)*(a - b)*(1792*a^10*b + 256*a^
11 - 256*a^2*b^9 - 1792*a^3*b^8 - 5120*a^4*b^7 - 7168*a^5*b^6 - 3584*a^6*b^
5 + 3584*a^7*b^4 + 7168*a^8*b^3 + 5120*a^9*b^2))/(128*(4*a*b^3 + 4*a^3*b +
a^4 + b^4 + 6*a^2*b^2)*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3
```

*b^3 + 15*a^4*b^2)))))/(4*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)))*(a - b))/(4*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)) + (3*(-a*b)^(1/2)*((cos(e + f*x)*(9*a^7 - 108*a^6*b + 153*a^3*b^4 - 396*a^4*b^3 + 486*a^5*b^2))/(32*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)) - (3*(-a*b)^(1/2)*(a - b)*(((9*a^11*b)/2 - (3*a^2*b^10)/2 - (15*a^3*b^9)/2 - 6*a^4*b^8 + 42*a^5*b^7 + 147*a^6*b^6 + 231*a^7*b^5 + 210*a^8*b^4 + 114*a^9*b^3 + (69*a^10*b^2)/2)/(9*a*b^8 + 9*a^8*b + a^9 + b^9 + 36*a^2*b^7 + 84*a^3*b^6 + 126*a^4*b^5 + 126*a^5*b^4 + 84*a^6*b^3 + 36*a^7*b^2) + (3*cos(e + f*x)*(-a*b)^(1/2)*(a - b)*(1792*a^10*b + 256*a^11 - 256*a^2*b^9 - 1792*a^3*b^8 - 5120*a^4*b^7 - 7168*a^5*b^6 - 3584*a^6*b^5 + 3584*a^7*b^4 + 7168*a^8*b^3 + 5120*a^9*b^2))/(128*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2))))/(4*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)))*(a - b))/(4*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2))))*(-a*b)^(1/2)*(a - b)*3i)/(2*f*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)) - ((3*cos(e + f*x)^5*(3*a*b - a^2))/(8*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) + (3*b*cos(e + f*x)*(3*a - b))/(8*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) + (cos(e + f*x)^3*(5*a^2 - 14*a*b + 5*b^2))/(8*(a + b)*(2*a*b + a^2 + b^2)))/(f*(b - cos(e + f*x)^4*(2*a - b) + a*cos(e + f*x)^6 + cos(e + f*x)^2*(a - 2*b))) - (atan((((((9*a^11*b)/2 - (3*a^2*b^10)/2 - (15*a^3*b^9)/2 - 6*a^4*b^8 + 42*a^5*b^7 + 147*a^6*b^6 + 231*a^7*b^5 + 210*a^8*b^4 + 114*a^9*b^3 + (69*a^10*b^2)/2)/(9*a*b^8 + 9*a^8*b + a^9 + b^9 + 36*a^2*b^7 + 84*a^3*b^6 + 126*a^4*b^5 + 126*a^5*b^4 + 84*a^6*b^3 + 36*a^7*b^2) - (cos(e + f*x)*(3/(16*(a + b)^2) - (3*b)/(2*(a + b)^3) + (3*b^2)/(2*(a + b)^4))*(1792*a^10*b + 256*a^11 - 256*a^2*b^9 - 1792*a^3*b^8 - 5120*a^4*b^7 - 7168*a^5*b^6 - 3584*a^6*b^5 + 3584*a^7*b^4 + 7168*a^8*b^3 + 5120*a^9*b^2))/(32*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)))*(3/(16*(a + b)^2) - (3*b)/(2*(a + b)^3) + (3*b^2)/(2*(a + b)^4)) + (cos(e + f*x)*(9*a^7 - 108*a^6*b + 153*a^3*b^4 - 396*a^4*b^3 + 486*a^5*b^2))/(32*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)))*(3/(16*(a + b)^2) - (3*b)/(2*(a + b)^3) + (3*b^2)/(2*(a + b)^4))*1i - ((((9*a^11*b)/2 - (3*a^2*b^10)/2 - (15*a^3*b^9)/2 - 6*a^4*b^8 + 42*a^5*b^7 + 147*a^6*b^6 + 231*a^7*b^5 + 210*a^8*b^4 + 114*a^9*b^3 + (69*a^10*b^2)/2)/(9*a*b^8 + 9*a^8*b + a^9 + b^9 + 36*a^2*b^7 + 84*a^3*b^6 + 126*a^4*b^5 + 126*a^5*b^4 + 84*a^6*b^3 + 36*a^7*b^2) + (cos(e + f*x)*(3/(16*(a + b)^2) - (3*b)/(2*(a + b)^3) + (3*b^2)/(2*(a + b)^4))*(1792*a^10*b + 256*a^11 - 256*a^2*b^9 - 1792*a^3*b^8 - 5120*a^4*b^7 - 7168*a^5*b^6 - 3584*a^6*b^5 + 3584*a^7*b^4 + 7168*a^8*b^3 + 5120*a^9*b^2))/(32*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)))*(3/(16*(a + b)^2) - (3*b)/(2*(a + b)^3) + (3*b^2)/(2*(a + b)^4)) - (cos(e + f*x)*(9*a^7 - 108*a^6*b + 153*a^3*b^4 - 396*a^4*b^3 + 486*a^5*b^2))/(32*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)))*(3/(16*(a + b)^2) - (3*b)/(2*(a + b)^3) + (3*b^2)/(2*(a + b)^4))*1i)/((((((9*a^11*b)/2 - (3*a^2*b^10)/2 - (15*a^3*b^9)/2 - 6*a^4*b^8 + 42*a^5*b^7 + 147*a^6*b^6 + 231*a^7*b^5 + 210*a^8*b^4 + 114*a^9*b^3 + (69*a^10*b^2)/2)/(9*a*b^8 + 9*a^8*b + a^9 + b^9 + 36*a^2*b^7 + 84*a^3*b^6 + 126*a^4*b^5 + 126*a^5*b^4 + 84*a^6*b^3 + 36*a^7*b^2) - (cos(e + f*x)*(3

```
/(16*(a + b)^2) - (3*b)/(2*(a + b)^3) + (3*b^2)/(2*(a + b)^4))*(1792*a^10*b
 + 256*a^11 - 256*a^2*b^9 - 1792*a^3*b^8 - 5120*a^4*b^7 - 7168*a^5*b^6 - 35
84*a^6*b^5 + 3584*a^7*b^4 + 7168*a^8*b^3 + 5120*a^9*b^2))/(32*(6*a*b^5 + 6*
a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)))*(3/(16*(a + b)^
2) - (3*b)/(2*(a + b)^3) + (3*b^2)/(2*(a + b)^4)) + (cos(e + f*x)*(9*a^7 -
108*a^6*b + 153*a^3*b^4 - 396*a^4*b^3 + 486*a^5*b^2))/(32*(6*a*b^5 + 6*a^5*
b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)))*(3/(16*(a + b)^2) -
 (3*b)/(2*(a + b)^3) + (3*b^2)/(2*(a + b)^4)) - ((27*a^7*b)/64 + (81*a^3*b^
5)/64 - (297*a^4*b^4)/32 + (189*a^5*b^3)/16 - (135*a^6*b^2)/32)/(9*a*b^8 +
9*a^8*b + a^9 + b^9 + 36*a^2*b^7 + 84*a^3*b^6 + 126*a^4*b^5 + 126*a^5*b^4 +
 84*a^6*b^3 + 36*a^7*b^2) + ((((9*a^11*b)/2 - (3*a^2*b^10)/2 - (15*a^3*b^9)
/2 - 6*a^4*b^8 + 42*a^5*b^7 + 147*a^6*b^6 + 231*a^7*b^5 + 210*a^8*b^4 + 114
*a^9*b^3 + (69*a^10*b^2)/2)/(9*a*b^8 + 9*a^8*b + a^9 + b^9 + 36*a^2*b^7 + 8
4*a^3*b^6 + 126*a^4*b^5 + 126*a^5*b^4 + 84*a^6*b^3 + 36*a^7*b^2) + (cos(e +
 f*x)*(3/(16*(a + b)^2) - (3*b)/(2*(a + b)^3) + (3*b^2)/(2*(a + b)^4))*(179
2*a^10*b + 256*a^11 - 256*a^2*b^9 - 1792*a^3*b^8 - 5120*a^4*b^7 - 7168*a^5*
b^6 - 3584*a^6*b^5 + 3584*a^7*b^4 + 7168*a^8*b^3 + 5120*a^9*b^2))/(32*(6*a*
b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)))*(3/(16*
(a + b)^2) - (3*b)/(2*(a + b)^3) + (3*b^2)/(2*(a + b)^4)) - (cos(e + f*x)*(
9*a^7 - 108*a^6*b + 153*a^3*b^4 - 396*a^4*b^3 + 486*a^5*b^2))/(32*(6*a*b^5
+ 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)))*(3/(16*(a +
 b)^2) - (3*b)/(2*(a + b)^3) + (3*b^2)/(2*(a + b)^4))))*(3i/(8*(a + b)^2) -
 (b*3i)/(a + b)^3 + (b^2*3i)/(a + b)^4))/f
```

3.47 $\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 267

$$\begin{aligned}\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx =& \frac{(5a^3+60a^2b+120ab^2+64b^3)\,x}{16a^5} \\ &- \frac{\sqrt{b}(a+b)^{3/2}(3a+8b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^5f} \\ &- \frac{(33a^2+82ab+48b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\,(a+b+b\tan^2(e+fx))} \\ &+ \frac{(9a+8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\,(a+b+b\tan^2(e+fx))} \\ &+ \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\,(a+b+b\tan^2(e+fx))} \\ &- \frac{b(19a^2+52ab+32b^2)\tan(e+fx)}{16a^4f\,(a+b+b\tan^2(e+fx))}\end{aligned}$$

```
[Out] 1/16*(5*a^3+60*a^2*b+120*a*b^2+64*b^3)*x/a^5-1/2*(a+b)^(3/2)*(3*a+8*b)*arct
an(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/a^5/f-1/48*(33*a^2+82*a*b+48*b^2
)*cos(f*x+e)*sin(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)+1/24*(9*a+8*b)*cos(f*x+e
)^3*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)+1/6*cos(f*x+e)^3*sin(f*x+e)^3/a/f
/(a+b+b*tan(f*x+e)^2)-1/16*b*(19*a^2+52*a*b+32*b^2)*tan(f*x+e)/a^4/f/(a+b+b
*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4217, 481, 592, 541, 536, 209, 211}

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\sqrt{b}(a+b)^{3/2}(3a+8b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^5f} + \frac{(9a+8b)\sin(e+fx)\cos^3(e+fx)}{24a^2f\,(a+b\tan^2(e+fx)+b)} - \frac{b(19a^2+52ab+32b^2)\tan(e+fx)}{16a^4f\,(a+b\tan^2(e+fx)+b)} - \frac{(33a^2+82ab+48b^2)\sin(e+fx)\cos(e+fx)}{48a^3f\,(a+b\tan^2(e+fx)+b)} + \frac{x(5a^3+60a^2b+120ab^2+64b^3)}{16a^5} + \frac{\sin^3(e+fx)\cos^3(e+fx)}{6af\,(a+b\tan^2(e+fx)+b)}$$

```
[In] Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((5*a^3 + 60*a^2*b + 120*a*b^2 + 64*b^3)*x)/(16*a^5) - (Sqrt[b]*(a + b)^(3/
2)*(3*a + 8*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*a^5*f) - ((33
*a^2 + 82*a*b + 48*b^2)*Cos[e + f*x]*Sin[e + f*x])/(48*a^3*f*(a + b + b*Tan
[e + f*x]^2)) + ((9*a + 8*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*a^2*f*(a + b
+ b*Tan[e + f*x]^2)) + (Cos[e + f*x]^3*Sin[e + f*x]^3)/(6*a*f*(a + b + b*Ta
n[e + f*x]^2)) - (b*(19*a^2 + 52*a*b + 32*b^2)*Tan[e + f*x])/(16*a^4*f*(a +
 b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
```

```
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^4(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)} - \frac{\operatorname{Subst}\left(\int \frac{x^2(3(a+b)+(b-6(a+b))x^2)}{(1+x^2)^3(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{6af}$$

$$= \frac{(9a+8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)} - \frac{\operatorname{Subst}\left(\int \frac{(a+b)(9a+8b)+(-24a^2-65ab-40b^2)x^2}{(1+x^2)^2(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{24a^2f}$$

$$= -\frac{(33a^2+82ab+48b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(9a+8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{3(a+b)(5a^2+22ab+16b^2)-3b(33a^2+82ab+48b^2)x^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{48a^3f}$$

$$= -\frac{(33a^2+82ab+48b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(9a+8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)} - \frac{b(19a^2+52ab+32b^2)\tan(e+fx)}{16a^4f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{6(a+b)^2(5a^2+36ab+32b^2)-6b(a+b)(19a^2+52ab+32b^2)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{96a^4(a+b)f}$$

$$= -\frac{(33a^2+82ab+48b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(9a+8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)} - \frac{b(19a^2+52ab+32b^2)\tan(e+fx)}{16a^4f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\left(b(a+b)^2(3a+8b)\right)\operatorname{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{2a^5f} + \frac{(5a^3+60a^2b+120ab^2+64b^3)\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{16a^5f}$$

$$= \frac{(5a^3+60a^2b+120ab^2+64b^3)\,x}{16a^5} - \frac{\sqrt{b}(a+b)^{3/2}(3a+8b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^5f}$$
$$- \frac{(33a^2+82ab+48b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\,(a+b+b\tan^2(e+fx))}$$
$$+ \frac{(9a+8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\,(a+b+b\tan^2(e+fx))} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\,(a+b+b\tan^2(e+fx))}$$
$$- \frac{b(19a^2+52ab+32b^2)\tan(e+fx)}{16a^4f\,(a+b+b\tan^2(e+fx))}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 19.91 (sec) , antiderivative size = 2468, normalized size of antiderivative = 9.24

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/512*((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*(16*x + ((-a^3 + 6*
a^2*b + 24*a*b^2 + 16*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a +
 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[
e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin
[e])^4]) + ((a^2 + 8*a*b + 8*b^2)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(
a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])
)))/(a^2*(a + b*Sec[e + f*x]^2)^2) + (3*(a + 2*b + a*Cos[2*e + 2*f*x])^2*Se
c[e + f*x]^4*(-64*(a + 2*b)*x + ((a^4 - 16*a^3*b - 144*a^2*b^2 - 256*a*b^3
- 128*b^4)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x])
+ a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*
e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (16*a
*Cos[2*f*x]*Sin[2*e])/f + (16*a*Cos[2*e]*Sin[2*f*x])/f - ((a^3 + 18*a^2*b +
 48*a*b^2 + 32*b^3)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a + b)*f*(a +
2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(4096*a^3*
(a + b*Sec[e + f*x]^2)^2) + (3*(a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x
]^4*(((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2) -
 (a*Sqrt[b]*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)]))))/(2
048*b^(3/2)*f*(a + b*Sec[e + f*x]^2)^2) - ((a + 2*b + a*Cos[2*e + 2*f*x])^2
*Sec[e + f*x]^4*(-((a*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(
3/2)) + (Sqrt[b]*(a + 2*b)*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e
 + f*x)]))))/(2048*b^(3/2)*f*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[
2*e + 2*f*x])^2*Sec[e + f*x]^4*(-(((a^5 - 30*a^4*b - 480*a^3*b^2 - 1600*a^2
*b^3 - 1920*a*b^4 - 768*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a
 + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Si
```

```
n[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])
^4])) + (Sec[2*e]*(32*b*(5*a^4 + 39*a^3*b + 106*a^2*b^2 + 120*a*b^3 + 48*b^
4)*f*x*Cos[2*e] + 16*a*b*(5*a^3 + 29*a^2*b + 48*a*b^2 + 24*b^3)*f*x*Cos[2*f
*x] + 80*a^4*b*f*x*Cos[4*e + 2*f*x] + 464*a^3*b^2*f*x*Cos[4*e + 2*f*x] + 76
8*a^2*b^3*f*x*Cos[4*e + 2*f*x] + 384*a*b^4*f*x*Cos[4*e + 2*f*x] + a^5*Sin[2
*e] + 34*a^4*b*Sin[2*e] + 224*a^3*b^2*Sin[2*e] + 576*a^2*b^3*Sin[2*e] + 640
*a*b^4*Sin[2*e] + 256*b^5*Sin[2*e] - a^5*Sin[2*f*x] - 62*a^4*b*Sin[2*f*x] -
 318*a^3*b^2*Sin[2*f*x] - 512*a^2*b^3*Sin[2*f*x] - 256*a*b^4*Sin[2*f*x] - 1
2*a^4*b*Sin[2*(e + 2*f*x)] - 36*a^3*b^2*Sin[2*(e + 2*f*x)] - 24*a^2*b^3*Sin
[2*(e + 2*f*x)] - 30*a^4*b*Sin[4*e + 2*f*x] - 158*a^3*b^2*Sin[4*e + 2*f*x]
- 256*a^2*b^3*Sin[4*e + 2*f*x] - 128*a*b^4*Sin[4*e + 2*f*x] - 12*a^4*b*Sin[
6*e + 4*f*x] - 36*a^3*b^2*Sin[6*e + 4*f*x] - 24*a^2*b^3*Sin[6*e + 4*f*x] +
2*a^4*b*Sin[4*e + 6*f*x] + 2*a^3*b^2*Sin[4*e + 6*f*x] + 2*a^4*b*Sin[8*e + 6
*f*x] + 2*a^3*b^2*Sin[8*e + 6*f*x]))/(a + 2*b + a*Cos[2*(e + f*x)])))/(2048
*a^4*b*(a + b)*f*(a + b*Sec[e + f*x]^2)^2) - ((a + 2*b + a*Cos[2*e + 2*f*x]
)^2*Sec[e + f*x]^4*((3*(a^6 - 48*a^5*b - 1200*a^4*b^2 - 6400*a^3*b^3 - 1344
0*a^2*b^4 - 12288*a*b^5 - 4096*b^6)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e]
)*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e]
 - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*
Sin[e])^4]) + (Sec[2*e]*(-192*b*(a + 2*b)^2*(5*a^3 + 37*a^2*b + 64*a*b^2 +
32*b^3)*f*x*Cos[2*e] - 96*a*b*(5*a^4 + 47*a^3*b + 138*a^2*b^2 + 160*a*b^3 +
 64*b^4)*f*x*Cos[2*f*x] - 480*a^5*b*f*x*Cos[4*e + 2*f*x] - 4512*a^4*b^2*f*x
*Cos[4*e + 2*f*x] - 13248*a^3*b^3*f*x*Cos[4*e + 2*f*x] - 15360*a^2*b^4*f*x*
Cos[4*e + 2*f*x] - 6144*a*b^5*f*x*Cos[4*e + 2*f*x] - 3*a^6*Sin[2*e] - 156*a
^5*b*Sin[2*e] - 1500*a^4*b^2*Sin[2*e] - 5760*a^3*b^3*Sin[2*e] - 10560*a^2*b
^4*Sin[2*e] - 9216*a*b^5*Sin[2*e] - 3072*b^6*Sin[2*e] + 3*a^6*Sin[2*f*x] +
366*a^5*b*Sin[2*f*x] + 3000*a^4*b^2*Sin[2*f*x] + 8400*a^3*b^3*Sin[2*f*x] +
9600*a^2*b^4*Sin[2*f*x] + 3840*a*b^5*Sin[2*f*x] + 76*a^5*b*Sin[2*(e + 2*f*x
)] + 460*a^4*b^2*Sin[2*(e + 2*f*x)] + 768*a^3*b^3*Sin[2*(e + 2*f*x)] + 384*
a^2*b^4*Sin[2*(e + 2*f*x)] + 216*a^5*b*Sin[4*e + 2*f*x] + 1800*a^4*b^2*Sin[
4*e + 2*f*x] + 5040*a^3*b^3*Sin[4*e + 2*f*x] + 5760*a^2*b^4*Sin[4*e + 2*f*x
] + 2304*a*b^5*Sin[4*e + 2*f*x] + 76*a^5*b*Sin[6*e + 4*f*x] + 460*a^4*b^2*S
in[6*e + 4*f*x] + 768*a^3*b^3*Sin[6*e + 4*f*x] + 384*a^2*b^4*Sin[6*e + 4*f*
x] - 16*a^5*b*Sin[4*e + 6*f*x] - 48*a^4*b^2*Sin[4*e + 6*f*x] - 32*a^3*b^3*S
in[4*e + 6*f*x] - 16*a^5*b*Sin[8*e + 6*f*x] - 48*a^4*b^2*Sin[8*e + 6*f*x] -
 32*a^3*b^3*Sin[8*e + 6*f*x] + 4*a^5*b*Sin[6*e + 8*f*x] + 4*a^4*b^2*Sin[6*e
 + 8*f*x] + 4*a^5*b*Sin[10*e + 8*f*x] + 4*a^4*b^2*Sin[10*e + 8*f*x]))/(a +
2*b + a*Cos[2*(e + f*x)])))/(12288*a^5*b*(a + b)*f*(a + b*Sec[e + f*x]^2)^2
)
```

Maple [A] (verified)

Time = 5.73 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{-\frac{(a+b)^2 b\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(3a+8b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^5}+\frac{\left(-\frac{9}{4}a^2b-\frac{3}{2}ab^2-\frac{11}{16}a^3\right)\tan(fx+e)^5+\left(-4a^2b-3ab^2-\frac{5}{6}a^3\right.}{\left(1+\tan(fx+e\right.}}{f}$
default	$\frac{-\frac{(a+b)^2 b\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(3a+8b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^5}+\frac{\left(-\frac{9}{4}a^2b-\frac{3}{2}ab^2-\frac{11}{16}a^3\right)\tan(fx+e)^5+\left(-4a^2b-3ab^2-\frac{5}{6}a^3\right.}{\left(1+\tan(fx+e\right.}}{f}$
risch	$\frac{5x}{16a^2}+\frac{15xb}{4a^3}+\frac{15x\,b^2}{2a^4}+\frac{4x\,b^3}{a^5}-\frac{3i e^{4i(fx+e)}}{128a^2f}-\frac{i e^{-2i(fx+e)}b}{2a^3f}-\frac{15i e^{-2i(fx+e)}}{128a^2f}+\frac{3i e^{-4i(fx+e)}}{128a^2f}+\frac{i e^{-4i(fx+}}{32a^3f}$

```
[In] int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-(a+b)^2*b/a^5*(1/2*a*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a+8*b)/((
a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^5*(((-9/4*a^2*b-3/2
*a*b^2-11/16*a^3)*tan(f*x+e)^5+(-4*a^2*b-3*a*b^2-5/6*a^3)*tan(f*x+e)^3+(-5/
16*a^3-7/4*a^2*b-3/2*a*b^2)*tan(f*x+e))/(1+tan(f*x+e)^2)^3+1/16*(5*a^3+60*a
^2*b+120*a*b^2+64*b^3)*arctan(tan(f*x+e))))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 674, normalized size of antiderivative = 2.52

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \left[\frac{3\,(5\,a^4+60\,a^3b+120\,a^2b^2+64\,ab^3)fx\cos{(fx+e)}^2+3\,(5\,a^3b+60\,a^2b^2+120\,ab^3+64\,b^4)fx+6\,(3\,a^2b\right.}{}$$

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(5*a^4 + 60*a^3*b + 120*a^2*b^2 + 64*a*b^3)*f*x*cos(f*x + e)^2 + 3
*(5*a^3*b + 60*a^2*b^2 + 120*a*b^3 + 64*b^4)*f*x + 6*(3*a^2*b + 11*a*b^2 +
8*b^3 + (3*a^3 + 11*a^2*b + 8*a*b^2)*cos(f*x + e)^2)*sqrt(-a*b - b^2)*log((
(a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4
*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e)
+ b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - (8*a^4*cos(f*x
+ e)^7 - 2*(13*a^4 + 8*a^3*b)*cos(f*x + e)^5 + (33*a^4 + 82*a^3*b + 48*a^2*
```

```
b^2)*cos(f*x + e)^3 + 3*(19*a^3*b + 52*a^2*b^2 + 32*a*b^3)*cos(f*x + e))*si
n(f*x + e))/(a^6*f*cos(f*x + e)^2 + a^5*b*f), 1/48*(3*(5*a^4 + 60*a^3*b + 1
20*a^2*b^2 + 64*a*b^3)*f*x*cos(f*x + e)^2 + 3*(5*a^3*b + 60*a^2*b^2 + 120*a
*b^3 + 64*b^4)*f*x + 12*(3*a^2*b + 11*a*b^2 + 8*b^3 + (3*a^3 + 11*a^2*b + 8
*a*b^2)*cos(f*x + e)^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^
2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) - (8*a^4*cos(f*x + e)^7
 - 2*(13*a^4 + 8*a^3*b)*cos(f*x + e)^5 + (33*a^4 + 82*a^3*b + 48*a^2*b^2)*c
os(f*x + e)^3 + 3*(19*a^3*b + 52*a^2*b^2 + 32*a*b^3)*cos(f*x + e))*sin(f*x
+ e))/(a^6*f*cos(f*x + e)^2 + a^5*b*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.13

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx=$$

$$-\frac{\frac{3\left(19\,a^2b+52\,ab^2+32\,b^3\right)\tan(fx+e)^7+\left(33\,a^3+253\,a^2b+516\,ab^2+288\,b^3\right)\tan(fx+e)^5+\left(40\,a^3+319\,a^2b+564\,ab^2+288\,b^3\right)\tan(fx+e)^3+3\left(5\,a^3+\cdots\right.}{a^4b\tan(fx+e)^8+\left(a^5+4\,a^4b\right)\tan(fx+e)^6+a^5+a^4b+3\left(a^5+2\,a^4b\right)\tan(fx+e)^4+\left(3\,a^5+4\,a^4b\right)\tan(fx+e)^2}\cdots}{48\,f}$$

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/48*((3*(19*a^2*b + 52*a*b^2 + 32*b^3)*tan(f*x + e)^7 + (33*a^3 + 253*a^2
*b + 516*a*b^2 + 288*b^3)*tan(f*x + e)^5 + (40*a^3 + 319*a^2*b + 564*a*b^2
+ 288*b^3)*tan(f*x + e)^3 + 3*(5*a^3 + 41*a^2*b + 68*a*b^2 + 32*b^3)*tan(f*
x + e))/(a^4*b*tan(f*x + e)^8 + (a^5 + 4*a^4*b)*tan(f*x + e)^6 + a^5 + a^4*
b + 3*(a^5 + 2*a^4*b)*tan(f*x + e)^4 + (3*a^5 + 4*a^4*b)*tan(f*x + e)^2) -
3*(5*a^3 + 60*a^2*b + 120*a*b^2 + 64*b^3)*(f*x + e)/a^5 + 24*(3*a^3*b + 14*
a^2*b^2 + 19*a*b^3 + 8*b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a
 + b)*b)*a^5))/f
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.10

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \frac{\frac{3\,(5\,a^3+60\,a^2b+120\,ab^2+64\,b^3)(fx+e)}{a^5} - \frac{24\,(3\,a^3b+14\,a^2b^2+19\,ab^3+8\,b^4)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,a^5} - \frac{24\,(a^2b\tan(fx+e}{(b\mathrm{t}}}{}$$

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/48*(3*(5*a^3 + 60*a^2*b + 120*a*b^2 + 64*b^3)*(f*x + e)/a^5 - 24*(3*a^3*b
 + 14*a^2*b^2 + 19*a*b^3 + 8*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + ar
ctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/(sqrt(a*b + b^2)*a^5) - 24*(a^2*b*tan
(f*x + e) + 2*a*b^2*tan(f*x + e) + b^3*tan(f*x + e))/((b*tan(f*x + e)^2 + a
 + b)*a^4) - (33*a^2*tan(f*x + e)^5 + 108*a*b*tan(f*x + e)^5 + 72*b^2*tan(f
*x + e)^5 + 40*a^2*tan(f*x + e)^3 + 192*a*b*tan(f*x + e)^3 + 144*b^2*tan(f*
x + e)^3 + 15*a^2*tan(f*x + e) + 84*a*b*tan(f*x + e) + 72*b^2*tan(f*x + e))
/((tan(f*x + e)^2 + 1)^3*a^4))/f
```

Mupad [B] (verification not implemented)

Time = 20.40 (sec) , antiderivative size = 1461, normalized size of antiderivative = 5.47

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (atanh((75*b^3*tan(e + f*x)*(- 3*a*b^3 - a^3*b - b^4 - 3*a^2*b^2)^(1/2))/(2
56*((211*a*b^4)/128 + (811*b^5)/256 + (75*a^2*b^3)/256 + (41*b^6)/(16*a) +
(3*b^7)/(4*a^2))) + (17*b^4*tan(e + f*x)*(- 3*a*b^3 - a^3*b - b^4 - 3*a^2*b
^2)^(1/2))/(16*((811*a*b^5)/256 + (41*b^6)/16 + (211*a^2*b^4)/128 + (75*a^3
*b^3)/256 + (3*b^7)/(4*a))) + (3*b^5*tan(e + f*x)*(- 3*a*b^3 - a^3*b - b^4
- 3*a^2*b^2)^(1/2))/(4*((41*a*b^6)/16 + (3*b^7)/4 + (811*a^2*b^5)/256 + (21
1*a^3*b^4)/128 + (75*a^4*b^3)/256)))*(-b*(a + b)^3)^(1/2)*(3*a + 8*b))/(2*a
^5*f) - (atan((((((((8*a^10*b^5 + 17*a^11*b^4 + (41*a^12*b^3)/4 + (5*a^13*b^
2)/4)/a^12 - (tan(e + f*x)*(2048*a^10*b^3 + 1024*a^11*b^2)*(a*b^2*120i + a^
2*b*60i + a^3*5i + b^3*64i))/(4096*a^13))*(a*b^2*120i + a^2*b*60i + a^3*5i
+ b^3*64i))/(32*a^5) - (tan(e + f*x)*(34816*a*b^8 + 8192*b^9 + 59520*a^2*b^
7 + 52160*a^3*b^6 + 24640*a^4*b^5 + 5976*a^5*b^4 + 601*a^6*b^3))/(128*a^8))
*(a*b^2*120i + a^2*b*60i + a^3*5i + b^3*64i)*1i)/(32*a^5) - ((((8*a^10*b^5
```

```
 + 17*a^11*b^4 + (41*a^12*b^3)/4 + (5*a^13*b^2)/4)/a^12 + (tan(e + f*x)*(20
48*a^10*b^3 + 1024*a^11*b^2)*(a*b^2*120i + a^2*b*60i + a^3*5i + b^3*64i))/(
4096*a^13))*(a*b^2*120i + a^2*b*60i + a^3*5i + b^3*64i))/(32*a^5) + (tan(e
+ f*x)*(34816*a*b^8 + 8192*b^9 + 59520*a^2*b^7 + 52160*a^3*b^6 + 24640*a^4*
b^5 + 5976*a^5*b^4 + 601*a^6*b^3))/(128*a^8))*(a*b^2*120i + a^2*b*60i + a^3
*5i + b^3*64i)*1i)/(32*a^5))/((376*a*b^10 + 64*b^11 + 937*a^2*b^9 + (10285*
a^3*b^8)/8 + (33701*a^4*b^7)/32 + (8333*a^5*b^6)/16 + (38085*a^6*b^5)/256 +
 (2765*a^7*b^4)/128 + (285*a^8*b^3)/256)/a^12 + (((((8*a^10*b^5 + 17*a^11*b
^4 + (41*a^12*b^3)/4 + (5*a^13*b^2)/4)/a^12 - (tan(e + f*x)*(2048*a^10*b^3
+ 1024*a^11*b^2)*(a*b^2*120i + a^2*b*60i + a^3*5i + b^3*64i))/(4096*a^13))*
(a*b^2*120i + a^2*b*60i + a^3*5i + b^3*64i))/(32*a^5) - (tan(e + f*x)*(3481
6*a*b^8 + 8192*b^9 + 59520*a^2*b^7 + 52160*a^3*b^6 + 24640*a^4*b^5 + 5976*a
^5*b^4 + 601*a^6*b^3))/(128*a^8))*(a*b^2*120i + a^2*b*60i + a^3*5i + b^3*64
i))/(32*a^5) + (((((8*a^10*b^5 + 17*a^11*b^4 + (41*a^12*b^3)/4 + (5*a^13*b^
2)/4)/a^12 + (tan(e + f*x)*(2048*a^10*b^3 + 1024*a^11*b^2)*(a*b^2*120i + a^
2*b*60i + a^3*5i + b^3*64i))/(4096*a^13))*(a*b^2*120i + a^2*b*60i + a^3*5i
+ b^3*64i))/(32*a^5) + (tan(e + f*x)*(34816*a*b^8 + 8192*b^9 + 59520*a^2*b^
7 + 52160*a^3*b^6 + 24640*a^4*b^5 + 5976*a^5*b^4 + 601*a^6*b^3))/(128*a^8))
*(a*b^2*120i + a^2*b*60i + a^3*5i + b^3*64i))/(32*a^5)))*(a*b^2*120i + a^2*
b*60i + a^3*5i + b^3*64i)*1i)/(16*a^5*f) - ((tan(e + f*x)*(68*a*b^2 + 41*a^
2*b + 5*a^3 + 32*b^3))/(16*a^4) + (tan(e + f*x)^5*(516*a*b^2 + 253*a^2*b +
33*a^3 + 288*b^3))/(48*a^4) + (tan(e + f*x)^3*(564*a*b^2 + 319*a^2*b + 40*a
^3 + 288*b^3))/(48*a^4) + (b*tan(e + f*x)^7*(52*a*b + 19*a^2 + 32*b^2))/(16
*a^4))/(f*(a + b + tan(e + f*x)^2*(3*a + 4*b) + tan(e + f*x)^4*(3*a + 6*b)
+ b*tan(e + f*x)^8 + tan(e + f*x)^6*(a + 4*b)))
```

3.48 $\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 191

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{3(a^2+8ab+8b^2)\,x}{8a^4} - \frac{3\sqrt{b}\sqrt{a+b}(a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^4f} - \frac{(5a+6b)\cos(e+fx)\sin(e+fx)}{8a^2f\,(a+b+b\tan^2(e+fx))} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))} - \frac{3b(3a+4b)\tan(e+fx)}{8a^3f\,(a+b+b\tan^2(e+fx))}$$

```
[Out] 3/8*(a^2+8*a*b+8*b^2)*x/a^4-3/2*(a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/
2))*b^(1/2)*(a+b)^(1/2)/a^4/f-1/8*(5*a+6*b)*cos(f*x+e)*sin(f*x+e)/a^2/f/(a+
b+b*tan(f*x+e)^2)+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)-3/8*
b*(3*a+4*b)*tan(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used

$= \{4217, 481, 541, 536, 209, 211\}$

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{3\sqrt{b}\sqrt{a+b}(a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^4f} - \frac{3b(3a+4b)\tan(e+fx)}{8a^3f\,(a+b\tan^2(e+fx)+b)} - \frac{(5a+6b)\sin(e+fx)\cos(e+fx)}{8a^2f\,(a+b\tan^2(e+fx)+b)} + \frac{3x(a^2+8ab+8b^2)}{8a^4} + \frac{\sin(e+fx)\cos^3(e+fx)}{4af\,(a+b\tan^2(e+fx)+b)}$$

```
[In] Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (3*(a^2 + 8*a*b + 8*b^2)*x)/(8*a^4) - (3*Sqrt[b]*Sqrt[a + b]*(a + 2*b)*ArcT
an[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*a^4*f) - ((5*a + 6*b)*Cos[e + f*
x]*Sin[e + f*x])/(8*a^2*f*(a + b + b*Tan[e + f*x]^2)) + (Cos[e + f*x]^3*Sin
[e + f*x])/(4*a*f*(a + b + b*Tan[e + f*x]^2)) - (3*b*(3*a + 4*b)*Tan[e + f*
x])/(8*a^3*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^3(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{a+b+(-4a-5b)x^2}{(1+x^2)^2(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= -\frac{(5a+6b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{3(a+b)(a+2b)-3b(5a+6b)x^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{8a^2f}$$

$$= -\frac{(5a+6b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)} - \frac{3b(3a+4b)\tan(e+fx)}{8a^3f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{6(a+b)^2(a+4b)-6b(a+b)(3a+4b)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{16a^3(a+b)f}$$

$$= -\frac{(5a+6b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)} - \frac{3b(3a+4b)\tan(e+fx)}{8a^3f\left(a+b+b\tan^2(e+fx)\right)} - \frac{(3b(a+b)(a+2b))\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{2a^4f} + \frac{(3(a^2+8ab+8b^2))\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{8a^4f}$$

$$= \frac{3(a^2+8ab+8b^2)\,x}{8a^4} - \frac{3\sqrt{b}\sqrt{a+b}(a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^4f} - \frac{(5a+6b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)} - \frac{3b(3a+4b)\tan(e+fx)}{8a^3f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.56 (sec) , antiderivative size = 1105, normalized size of antiderivative = 5.79

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx =$$

$$-\frac{(a+2b+a\cos(2e+2fx))^2\sec^4(e+fx)\left(16x+\frac{(-a^3+6a^2b+24ab^2+16b^3)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))}}\right.}{b(a+b)^{3/2}f\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{256a^2\left(a+b\sec^2(e+fx)\right)}$$

$$+\frac{3(a+2b+a\cos(2e+2fx))^2\sec^4(e+fx)\left(\frac{(a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{a\sqrt{b}\sin(2(e+fx))}{(a+b)(a+2b+a\cos(2(e+fx)))}\right)}{1024b^{3/2}f\left(a+b\sec^2(e+fx)\right)^2}$$

$$+\frac{(a+2b+a\cos(2e+2fx))^2\sec^4(e+fx)\left(-\frac{(a^5-30a^4b-480a^3b^2-1600a^2b^3-1920ab^4-768b^5)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e}{2\sqrt{a+b}}\right.}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{}$$

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/256*((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*(16*x + ((-a^3 + 6*
a^2*b + 24*a*b^2 + 16*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a +
 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[
e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin
```

```
[e])^4]) + ((a^2 + 8*a*b + 8*b^2)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(
a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])
)))/(a^2*(a + b*Sec[e + f*x]^2)^2) + (3*(a + 2*b + a*Cos[2*e + 2*f*x])^2*Se
c[e + f*x]^4*(((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b
)^(3/2) - (a*Sqrt[b]*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x
)]))))/(1024*b^(3/2)*f*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[2*e +
2*f*x])^2*Sec[e + f*x]^4*(-(((a^5 - 30*a^4*b - 480*a^3*b^2 - 1600*a^2*b^3 -
 1920*a*b^4 - 768*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b
)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^
4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]))
+ (Sec[2*e]*(32*b*(5*a^4 + 39*a^3*b + 106*a^2*b^2 + 120*a*b^3 + 48*b^4)*f*x
*Cos[2*e] + 16*a*b*(5*a^3 + 29*a^2*b + 48*a*b^2 + 24*b^3)*f*x*Cos[2*f*x] +
80*a^4*b*f*x*Cos[4*e + 2*f*x] + 464*a^3*b^2*f*x*Cos[4*e + 2*f*x] + 768*a^2*
b^3*f*x*Cos[4*e + 2*f*x] + 384*a*b^4*f*x*Cos[4*e + 2*f*x] + a^5*Sin[2*e] +
34*a^4*b*Sin[2*e] + 224*a^3*b^2*Sin[2*e] + 576*a^2*b^3*Sin[2*e] + 640*a*b^4
*Sin[2*e] + 256*b^5*Sin[2*e] - a^5*Sin[2*f*x] - 62*a^4*b*Sin[2*f*x] - 318*a
^3*b^2*Sin[2*f*x] - 512*a^2*b^3*Sin[2*f*x] - 256*a*b^4*Sin[2*f*x] - 12*a^4*
b*Sin[2*(e + 2*f*x)] - 36*a^3*b^2*Sin[2*(e + 2*f*x)] - 24*a^2*b^3*Sin[2*(e
+ 2*f*x)] - 30*a^4*b*Sin[4*e + 2*f*x] - 158*a^3*b^2*Sin[4*e + 2*f*x] - 256*
a^2*b^3*Sin[4*e + 2*f*x] - 128*a*b^4*Sin[4*e + 2*f*x] - 12*a^4*b*Sin[6*e +
4*f*x] - 36*a^3*b^2*Sin[6*e + 4*f*x] - 24*a^2*b^3*Sin[6*e + 4*f*x] + 2*a^4*
b*Sin[4*e + 6*f*x] + 2*a^3*b^2*Sin[4*e + 6*f*x] + 2*a^4*b*Sin[8*e + 6*f*x]
+ 2*a^3*b^2*Sin[8*e + 6*f*x]))/(a + 2*b + a*Cos[2*(e + f*x)])))/(1024*a^4*b
*(a + b)*f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] (verified)

Time = 3.17 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{-\frac{(a+b)b\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{3(a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^4}+\frac{\frac{\left(-ab-\frac{5}{8}a^2\right)\tan(fx+e)^3+\left(-\frac{3}{8}a^2-ab\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{3(a^2+8}{}}{a^4}}{f}$
default	$\frac{-\frac{(a+b)b\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{3(a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^4}+\frac{\frac{\left(-ab-\frac{5}{8}a^2\right)\tan(fx+e)^3+\left(-\frac{3}{8}a^2-ab\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{3(a^2+8}{}}{a^4}}{f}$
risch	$\frac{3x}{8a^2}+\frac{3xb}{a^3}+\frac{3x\,b^2}{a^4}-\frac{i\,e^{4i(fx+e)}}{64a^2f}+\frac{i\,e^{2i(fx+e)}}{8a^2f}+\frac{i\,e^{2i(fx+e)}b}{4a^3f}-\frac{i\,e^{-2i(fx+e)}}{8a^2f}-\frac{i\,e^{-2i(fx+e)}b}{4a^3f}+\frac{i\,e^{-4i(fx+e)}}{64a^2f}$

```
[In] int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-(a+b)*b/a^4*(1/2*a*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+3/2*(a+2*b)/((a+b)
*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^4*(((-a*b-5/8*a^2)*tan(
```

```
f*x+e)^3+(-3/8*a^2-a*b)*tan(f*x+e))/(1+tan(f*x+e)^2)^2+3/8*(a^2+8*a*b+8*b^2
)*arctan(tan(f*x+e))))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 522, normalized size of antiderivative = 2.73

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \left[\frac{3\,(a^3+8\,a^2b+8\,ab^2)fx\cos{(fx+e)}^2+3\,(a^2b+8\,ab^2+8\,b^3)fx+3\,\left((a^2+2\,ab)\cos{(fx+e)}^2+ab+2\,b^2\right.}{}\right.$$

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(3*(a^3 + 8*a^2*b + 8*a*b^2)*f*x*cos(f*x + e)^2 + 3*(a^2*b + 8*a*b^2 +
 8*b^3)*f*x + 3*((a^2 + 2*a*b)*cos(f*x + e)^2 + a*b + 2*b^2)*sqrt(-a*b - b^
2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f
*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + (2*a^3*
cos(f*x + e)^5 - (5*a^3 + 6*a^2*b)*cos(f*x + e)^3 - 3*(3*a^2*b + 4*a*b^2)*c
os(f*x + e))*sin(f*x + e))/(a^5*f*cos(f*x + e)^2 + a^4*b*f), 1/8*(3*(a^3 +
8*a^2*b + 8*a*b^2)*f*x*cos(f*x + e)^2 + 3*(a^2*b + 8*a*b^2 + 8*b^3)*f*x + 6
*((a^2 + 2*a*b)*cos(f*x + e)^2 + a*b + 2*b^2)*sqrt(a*b + b^2)*arctan(1/2*((
a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) +
 (2*a^3*cos(f*x + e)^5 - (5*a^3 + 6*a^2*b)*cos(f*x + e)^3 - 3*(3*a^2*b + 4*
a*b^2)*cos(f*x + e))*sin(f*x + e))/(a^5*f*cos(f*x + e)^2 + a^4*b*f)]
```

Sympy [F]

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx=\int \frac{\sin^4{(e+fx)}}{(a+b\sec^2{(e+fx)})^2}\,dx$$

```
[In] integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(sin(e + f*x)**4/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.07

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx =$$

$$-\frac{\frac{3\,(3\,ab+4\,b^2)\tan(fx+e)^5+(5\,a^2+24\,ab+24\,b^2)\tan(fx+e)^3+3\,(a^2+5\,ab+4\,b^2)\tan(fx+e)}{a^3b\tan(fx+e)^6+(a^4+3\,a^3b)\tan(fx+e)^4+a^4+a^3b+(2\,a^4+3\,a^3b)\tan(fx+e)^2}-\frac{3\,(a^2+8\,ab+8\,b^2)(fx+e)}{a^4}+\frac{12\,(a^2b+3\,ab^2+2}{\sqrt{}}}{8\,f}$$

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/8*((3*(3*a*b + 4*b^2)*tan(f*x + e)^5 + (5*a^2 + 24*a*b + 24*b^2)*tan(f*x
 + e)^3 + 3*(a^2 + 5*a*b + 4*b^2)*tan(f*x + e))/(a^3*b*tan(f*x + e)^6 + (a^
4 + 3*a^3*b)*tan(f*x + e)^4 + a^4 + a^3*b + (2*a^4 + 3*a^3*b)*tan(f*x + e)^
2) - 3*(a^2 + 8*a*b + 8*b^2)*(f*x + e)/a^4 + 12*(a^2*b + 3*a*b^2 + 2*b^3)*a
rctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^4))/f
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.01

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$=\frac{\frac{3\,(a^2+8\,ab+8\,b^2)(fx+e)}{a^4}-\frac{12\,(a^2b+3\,ab^2+2\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}a^4}-\frac{4\,(ab\tan(fx+e)+b^2\tan(fx+e))}{\left(b\tan(fx+e)^2+a+b\right)a^3}-\frac{5\,a\tan}{}}{8\,f}$$

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^2 + 8*a*b + 8*b^2)*(f*x + e)/a^4 - 12*(a^2*b + 3*a*b^2 + 2*b^3)*(
pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)
))/(sqrt(a*b + b^2)*a^4) - 4*(a*b*tan(f*x + e) + b^2*tan(f*x + e))/((b*tan(
f*x + e)^2 + a + b)*a^3) - (5*a*tan(f*x + e)^3 + 8*b*tan(f*x + e)^3 + 3*a*t
an(f*x + e) + 8*b*tan(f*x + e))/((tan(f*x + e)^2 + 1)^2*a^3))/f
```

Mupad [B] (verification not implemented)

Time = 19.42 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.28

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \frac{3\,\mathrm{atanh}\left(\frac{27\,b^3\tan(e+f\,x)\,\sqrt{-b^2-a\,b}}{64\,\left(\frac{27\,a\,b^3}{64}+\frac{81\,b^4}{64}+\frac{27\,b^5}{32\,a}\right)}+\frac{27\,b^4\tan(e+f\,x)\,\sqrt{-b^2-a\,b}}{32\,\left(\frac{27\,a^2\,b^3}{64}+\frac{81\,a\,b^4}{64}+\frac{27\,b^5}{32}\right)}\right)\,(a+2\,b)\,\sqrt{-b\,(a+b)}}{2\,a^4\,f}$$

$$-\frac{\frac{3\tan(e+f\,x)^5\,(4\,b^2+3\,a\,b)}{8\,a^3}+\frac{\tan(e+f\,x)^3\,(5\,a^2+24\,a\,b+24\,b^2)}{8\,a^3}+\frac{3\tan(e+f\,x)\,(a^2+5\,a\,b+4\,b^2)}{8\,a^3}}{f\,\left(b\tan\left(e+f\,x\right)^6+(a+3\,b)\,\tan\left(e+f\,x\right)^4+(2\,a+3\,b)\,\tan\left(e+f\,x\right)^2+a+b\right)}$$

$$-\frac{\mathrm{atan}\left(\frac{27\,b^2\tan(e+f\,x)}{256\,\left(\frac{27\,b^2}{256}+\frac{243\,b^3}{256\,a}+\frac{27\,b^4}{16\,a^2}+\frac{27\,b^5}{32\,a^3}\right)}+\frac{243\,b^3\tan(e+f\,x)}{256\,\left(\frac{27\,a\,b^2}{256}+\frac{243\,b^3}{256}+\frac{27\,b^4}{16\,a}+\frac{27\,b^5}{32\,a^2}\right)}+\frac{27\,b^4\tan(e+f\,x)}{16\,\left(\frac{243\,a\,b^3}{256}+\frac{27\,b^4}{16}+\frac{27\,a^2\,b^2}{256}+\frac{27\,b^5}{32\,a}\right)}+\frac{27\cdots}{32\,\left(\frac{27\,a^3\,b^2}{256}+\cdots\right.}\cdots}{8\,a^4\,f}$$

```
[In] int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (3*atanh((27*b^3*tan(e + f*x)*(- a*b - b^2)^(1/2))/(64*((27*a*b^3)/64 + (81
*b^4)/64 + (27*b^5)/(32*a))) + (27*b^4*tan(e + f*x)*(- a*b - b^2)^(1/2))/(3
2*((81*a*b^4)/64 + (27*b^5)/32 + (27*a^2*b^3)/64)))*(a + 2*b)*(-b*(a + b))^
(1/2))/(2*a^4*f) - (atan((27*b^2*tan(e + f*x))/(256*((27*b^2)/256 + (243*b^
3)/(256*a) + (27*b^4)/(16*a^2) + (27*b^5)/(32*a^3))) + (243*b^3*tan(e + f*x
))/(256*((27*a*b^2)/256 + (243*b^3)/256 + (27*b^4)/(16*a) + (27*b^5)/(32*a^
2))) + (27*b^4*tan(e + f*x))/(16*((243*a*b^3)/256 + (27*b^4)/16 + (27*a^2*b
^2)/256 + (27*b^5)/(32*a))) + (27*b^5*tan(e + f*x))/(32*((27*a*b^4)/16 + (2
7*b^5)/32 + (243*a^2*b^3)/256 + (27*a^3*b^2)/256)))*(a*b*8i + a^2*1i + b^2*
8i)*3i)/(8*a^4*f) - ((3*tan(e + f*x)^5*(3*a*b + 4*b^2))/(8*a^3) + (tan(e +
f*x)^3*(24*a*b + 5*a^2 + 24*b^2))/(8*a^3) + (3*tan(e + f*x)*(5*a*b + a^2 +
4*b^2))/(8*a^3))/(f*(a + b + tan(e + f*x)^2*(2*a + 3*b) + b*tan(e + f*x)^6
+ tan(e + f*x)^4*(a + 3*b)))
```

3.49 $\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 130

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^2} dx = \frac{(a+4b)x}{2a^3} - \frac{\sqrt{b}(3a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^3\sqrt{a+b}f} - \frac{\cos(e+fx)\sin(e+fx)}{2af(a+b+b\tan^2(e+fx))} - \frac{b\tan(e+fx)}{a^2f(a+b+b\tan^2(e+fx))}$$

```
[Out] 1/2*(a+4*b)*x/a^3-1/2*(3*a+4*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1
/2)/a^3/f/(a+b)^(1/2)-1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)-b*
tan(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4217, 482, 541, 536, 209, 211}

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^2} dx = -\frac{\sqrt{b}(3a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^3f\sqrt{a+b}} + \frac{x(a+4b)}{2a^3} - \frac{b\tan(e+fx)}{a^2f(a+b\tan^2(e+fx)+b)} - \frac{\sin(e+fx)\cos(e+fx)}{2af(a+b\tan^2(e+fx)+b)}$$

```
[In] Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 4*b)*x)/(2*a^3) - (Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/
Sqrt[a + b]])/(2*a^3*Sqrt[a + b]*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*(a
```

```
 + b + b*Tan[e + f*x]^2)) - (b*Tan[e + f*x])/(a^2*f*(a + b + b*Tan[e + f*x]
^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{a+b-3bx^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\tan(e+fx)}{a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2(a+b)(a+2b)-4b(a+b)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{4a^2(a+b)f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\tan(e+fx)}{a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(a+4b)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{2a^3f} - \frac{(b(3a+4b))\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{2a^3f}$$

$$= \frac{(a+4b)x}{2a^3} - \frac{\sqrt{b}(3a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^3\sqrt{a+b}f} - \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\tan(e+fx)}{a^2f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.73 (sec) , antiderivative size = 699, normalized size of antiderivative = 5.38

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))^2\sec^4(e+fx)\left(-\frac{2\left(16x+\frac{\left(-a^3+6a^2b+24ab^2+16b^3\right)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx)}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{b(a+b)^{3/2}f\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{}\right.}{}$$

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x]^4*((-2*(16*x + ((-a^3 + 6*a^
2*b + 24*a*b^2 + 16*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2
*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e]
)^4])]*(Cos[2*e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e
])^4]) + ((a^2 + 8*a*b + 8*b^2)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a
+ b)*f*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])))
)/a^2 - (-64*(a + 2*b)*x + ((a^4 - 16*a^3*b - 144*a^2*b^2 - 256*a*b^3 - 128
*b^4)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*S
in[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] -
I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (16*a*Cos[
2*f*x]*Sin[2*e])/f + (16*a*Cos[2*e]*Sin[2*f*x])/f - ((a^3 + 18*a^2*b + 48*a
*b^2 + 32*b^3)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a + b)*f*(a + 2*b +
 a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])))/a^3 + (2*(((a +
2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2) - (a*Sqrt[b]
*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)]))))/(b^(3/2)*f) +
 (-((a*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2)) + (Sqrt[b
]*(a + 2*b)*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])))/(b^
(3/2)*f)))/(256*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(3a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^3}+\frac{-\frac{a\tan(fx+e)}{2\left(1+\tan(fx+e)^2\right)}+\frac{(a+4b)\arctan(\tan(fx+e))}{2}}{a^3}}{f}$
default	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(3a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^3}+\frac{-\frac{a\tan(fx+e)}{2\left(1+\tan(fx+e)^2\right)}+\frac{(a+4b)\arctan(\tan(fx+e))}{2}}{a^3}}{f}$
risch	$\frac{x}{2a^2}+\frac{2xb}{a^3}+\frac{i\mathrm{e}^{2i(fx+e)}}{8a^2f}-\frac{i\mathrm{e}^{-2i(fx+e)}}{8a^2f}-\frac{ib\left(a\,\mathrm{e}^{2i(fx+e)}+2b\,\mathrm{e}^{2i(fx+e)}+a\right)}{a^3f\left(a\,\mathrm{e}^{4i(fx+e)}+2a\,\mathrm{e}^{2i(fx+e)}+4b\,\mathrm{e}^{2i(fx+e)}+a\right)}-\frac{3\sqrt{-(a+b)b}\,\ln\left(\mathrm{e}^{2i(\cdots}{4(\cdots}$

```
[In] int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^3*b*(1/2*a*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a+4*b)/((a+b)*b
)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^3*(-1/2*a*tan(f*x+e)/(1+t
an(f*x+e)^2)+1/2*(a+4*b)*arctan(tan(f*x+e))))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 441, normalized size of antiderivative = 3.39

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \left[\frac{4\,(a^2+4\,ab)fx\cos{(fx+e)}^2+4\,(ab+4\,b^2)fx+\left((3\,a^2+4\,ab)\cos{(fx+e)}^2+3\,ab+4\,b^2\right)\sqrt{-\frac{b}{a+b}}\log\left(\right.}{}\right.$$

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a^2 + 4*a*b)*f*x*cos(f*x + e)^2 + 4*(a*b + 4*b^2)*f*x + ((3*a^2 +
4*a*b)*cos(f*x + e)^2 + 3*a*b + 4*b^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b +
 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b
 + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f
*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 4*(a^2*
cos(f*x + e)^3 + 2*a*b*cos(f*x + e))*sin(f*x + e))/(a^4*f*cos(f*x + e)^2 +
a^3*b*f), 1/4*(2*(a^2 + 4*a*b)*f*x*cos(f*x + e)^2 + 2*(a*b + 4*b^2)*f*x + (
(3*a^2 + 4*a*b)*cos(f*x + e)^2 + 3*a*b + 4*b^2)*sqrt(b/(a + b))*arctan(1/2*
((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)
)) - 2*(a^2*cos(f*x + e)^3 + 2*a*b*cos(f*x + e))*sin(f*x + e))/(a^4*f*cos(f
*x + e)^2 + a^3*b*f)]
```

Sympy [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\sin^2{(e+fx)}}{(a+b\sec^2{(e+fx)})^2}\,dx$$

```
[In] integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(sin(e + f*x)**2/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.97

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{2\,b\tan(fx+e)^3+(a+2\,b)\tan(fx+e)}{a^2b\tan(fx+e)^4+a^3+a^2b+(a^3+2\,a^2b)\tan(fx+e)^2}-\frac{(fx+e)(a+4\,b)}{a^3}+\frac{(3\,ab+4\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^3}}{2\,f}$$

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((2*b*tan(f*x + e)^3 + (a + 2*b)*tan(f*x + e))/(a^2*b*tan(f*x + e)^4 +
 a^3 + a^2*b + (a^3 + 2*a^2*b)*tan(f*x + e)^2) - (f*x + e)*(a + 4*b)/a^3 +
(3*a*b + 4*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^3
))/f
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.15

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{(fx+e)(a+4\,b)}{a^3}-\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3\,ab+4\,b^2)}{\sqrt{ab+b^2}a^3}-\frac{2\,b\tan(fx+e)^3+a\tan(fx+e)+2\,b\tan(fx+e)}{\left(b\tan(fx+e)^4+a\tan(fx+e)^2+2\,b\tan(fx+e)^2+a+b\right)a^2}}{2\,f}$$

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((f*x + e)*(a + 4*b)/a^3 - (pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arcta
n(b*tan(f*x + e)/sqrt(a*b + b^2)))*(3*a*b + 4*b^2)/(sqrt(a*b + b^2)*a^3) -
(2*b*tan(f*x + e)^3 + a*tan(f*x + e) + 2*b*tan(f*x + e))/((b*tan(f*x + e)^4
 + a*tan(f*x + e)^2 + 2*b*tan(f*x + e)^2 + a + b)*a^2))/f
```

Mupad [B] (verification not implemented)

Time = 18.50 (sec) , antiderivative size = 816, normalized size of antiderivative = 6.28

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{\tan(e+f\,x)\,(a+2\,b)}{2\,a^2}+\frac{b\tan(e+f\,x)^3}{a^2}}{f\,\left(b\tan\left(e+f\,x\right)^4+(a+2\,b)\,\tan\left(e+f\,x\right)^2+a+b\right)}$$

$$-\frac{\mathrm{atan}\left(\frac{b^2\tan(e+f\,x)}{4\left(\frac{b^2}{4}+\frac{b^3}{a}\right)}+\frac{b^3\tan(e+f\,x)}{b^3+\frac{a\,b^2}{4}}\right)\,(a\,1\mathrm{i}+b\,4\mathrm{i})\,1\mathrm{i}}{2\,a^3\,f}$$

$$+\frac{\mathrm{atan}\left(\frac{\left(\frac{\tan(e+f\,x)\left(5\,a^2\,b^3+16\,a\,b^4+16\,b^5\right)}{a^4}+\frac{\left(\frac{2\,a^7\,b^2+4\,a^6\,b^3}{a^6}+\frac{\tan(e+f\,x)\left(8\,a^7\,b^2+16\,a^6\,b^3\right)\left(\frac{3\,a}{4}+b\right)\sqrt{-b\,(a+b)}}{a^4\left(a^4+b\,a^3\right)}\right)\left(\frac{3\,a}{4}+b\right)\sqrt{-b\,(a+b)}}{a^4+b\,a^3}\right)\left(\cdots\right.}{a^4+b\,a^3} \Bigg/ \left(\frac{\frac{3\,a^2\,b^3}{2}+8\,a\,b^4+8\,b^5}{a^6}+\frac{\left(\frac{\tan(e+f\,x)\left(5\,a^2\,b^3+16\,a\,b^4+16\,b^5\right)}{a^4}+\frac{\left(\frac{2\,a^7\,b^2+4\,a^6\,b^3}{a^6}+\frac{\tan(e+f\,x)\left(8\,a^7\,b^2+16\,a^6\,b^3\right)\left(\frac{3\,a}{4}+b\right)\sqrt{-b\,(a+b)}}{a^4\left(a^4+b\,a^3\right)}\right)\left(\frac{3\,a}{4}+b\right)\cdots}{a^4+b\,a^3}\right.\cdots}{a^4+b\,a^3}\right.\cdots}{\cdots}$$

```
[In] int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (atan(((((tan(e + f*x)*(16*a*b^4 + 16*b^5 + 5*a^2*b^3))/a^4 + (((4*a^6*b^3
+ 2*a^7*b^2)/a^6 + (tan(e + f*x)*(16*a^6*b^3 + 8*a^7*b^2)*((3*a)/4 + b)*(-b
*(a + b))^(1/2))/(a^4*(a^3*b + a^4)))*((3*a)/4 + b)*(-b*(a + b))^(1/2))/(a^
3*b + a^4))*((3*a)/4 + b)*(-b*(a + b))^(1/2)*1i)/(a^3*b + a^4) + (((tan(e +
 f*x)*(16*a*b^4 + 16*b^5 + 5*a^2*b^3))/a^4 - (((4*a^6*b^3 + 2*a^7*b^2)/a^6
- (tan(e + f*x)*(16*a^6*b^3 + 8*a^7*b^2)*((3*a)/4 + b)*(-b*(a + b))^(1/2))/
(a^4*(a^3*b + a^4)))*((3*a)/4 + b)*(-b*(a + b))^(1/2))/(a^3*b + a^4))*((3*a
)/4 + b)*(-b*(a + b))^(1/2)*1i)/(a^3*b + a^4))/((8*a*b^4 + 8*b^5 + (3*a^2*b
^3)/2)/a^6 + (((tan(e + f*x)*(16*a*b^4 + 16*b^5 + 5*a^2*b^3))/a^4 + (((4*a^
6*b^3 + 2*a^7*b^2)/a^6 + (tan(e + f*x)*(16*a^6*b^3 + 8*a^7*b^2)*((3*a)/4 +
b)*(-b*(a + b))^(1/2))/(a^4*(a^3*b + a^4)))*((3*a)/4 + b)*(-b*(a + b))^(1/2
))/(a^3*b + a^4))*((3*a)/4 + b)*(-b*(a + b))^(1/2))/(a^3*b + a^4) - (((tan(
e + f*x)*(16*a*b^4 + 16*b^5 + 5*a^2*b^3))/a^4 - (((4*a^6*b^3 + 2*a^7*b^2)/a
^6 - (tan(e + f*x)*(16*a^6*b^3 + 8*a^7*b^2)*((3*a)/4 + b)*(-b*(a + b))^(1/2
))/(a^4*(a^3*b + a^4)))*((3*a)/4 + b)*(-b*(a + b))^(1/2))/(a^3*b + a^4))*((
3*a)/4 + b)*(-b*(a + b))^(1/2))/(a^3*b + a^4)))*((3*a)/4 + b)*(-b*(a + b))^
(1/2)*2i)/(f*(a^3*b + a^4)) - (atan((b^2*tan(e + f*x))/(4*(b^2/4 + b^3/a))
+ (b^3*tan(e + f*x))/((a*b^2)/4 + b^3))*(a*1i + b*4i)*1i)/(2*a^3*f) - ((tan
(e + f*x)*(a + 2*b))/(2*a^2) + (b*tan(e + f*x)^3)/a^2)/(f*(a + b + b*tan(e
+ f*x)^4 + tan(e + f*x)^2*(a + 2*b)))
```

3.50 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 92

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx=\frac{x}{a^2}-\frac{\sqrt{b}(3a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}f}-\frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

```
[Out] x/a^2-1/2*(3*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/a^2/(a+b
)^(3/2)/f-1/2*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4213, 425, 536, 209, 211}

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx=-\frac{\sqrt{b}(3a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{3/2}}+\frac{x}{a^2}-\frac{b\tan(e+fx)}{2af(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-2),x]
```

```
[Out] x/a^2 - (Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2
*a^2*(a + b)^(3/2)*f) - (b*Tan[e + f*x])/(2*a*(a + b)*f*(a + b + b*Tan[e +
f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a+b-bx^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{2a(a+b)f} \\
&= -\frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^2f} \\
&\quad - \frac{(b(3a+2b))\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{2a^2(a+b)f}
\end{aligned}$$

$$= \frac{x}{a^2} - \frac{\sqrt{b}(3a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}f} - \frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.68 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.61

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(2x(a+2b+a\cos(2(e+fx)))+\frac{b(3a+2b)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))}{2\sqrt{a+b}\sqrt{b}}\right.}{}\right.}{8a^2\left(a+b\sec^2(e+f\right.}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(2*x*(a + 2*b + a*Cos[2*(e +
 f*x)]) + (b*(3*a + 2*b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2
*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e]
)^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/((a + b)^(3/
2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (b*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]
))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*a^2*(a + b*Sec[e +
f*x]^2)^2)
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(3a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^2}+\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
default	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(3a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^2}+\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
risch	$\frac{x}{a^2}-\frac{ib\left(a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a\right)}{a^2(a+b)f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{3\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{4(a+b)^2fa}+$

```
[In] int(1/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-b/a^2*(1/2*a/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a+2*b)/(a+b
)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^2*arctan(tan(f*
x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(80) = 160.

Time = 0.31 (sec) , antiderivative size = 435, normalized size of antiderivative = 4.73

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \left[\frac{8\,(a^2+ab)fx\cos{(fx+e)}^2-4\,ab\cos{(fx+e)}\sin{(fx+e)}+8\,(ab+b^2)fx+\left((3\,a^2+2\,ab)\cos{(fx+e)}\right.}{8\left((a^4+a\right.}\right.$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 4*a*b*cos(f*x + e)*sin(f*x + e) +
8*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt(-
b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*co
s(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*
x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*co
s(f*x + e)^2 + b^2)))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*b^2)*f
), 1/4*(4*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 2*a*b*cos(f*x + e)*sin(f*x + e)
+ 4*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt
(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*co
s(f*x + e)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*b
^2)*f)]
```

Sympy [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{1}{(a+b\sec^2{(e+fx)})^2}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{b\tan(fx+e)}{a^3+2a^2b+ab^2+(a^2b+ab^2)\tan(fx+e)^2}+\frac{(3ab+2b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^3+a^2b)\sqrt{(a+b)b}}-\frac{2(fx+e)}{a^2}}{2f}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*tan(f*x + e)/(a^3 + 2*a^2*b + a*b^2 + (a^2*b + a*b^2)*tan(f*x + e)^
2) + (3*a*b + 2*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + a^2*b)*
sqrt((a + b)*b)) - 2*(f*x + e)/a^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.24

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3ab+2b^2)}{(a^3+a^2b)\sqrt{ab+b^2}}+\frac{b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)(a^2+ab)}-\frac{2(fx+e)}{a^2}}{2f}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
 + b^2)))*(3*a*b + 2*b^2)/((a^3 + a^2*b)*sqrt(a*b + b^2)) + b*tan(f*x + e)/
((b*tan(f*x + e)^2 + a + b)*(a^2 + a*b)) - 2*(f*x + e)/a^2)/f
```

Mupad [B] (verification not implemented)

Time = 20.41 (sec) , antiderivative size = 2056, normalized size of antiderivative = 22.35

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] atan((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^
2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8
*a^2*(2*a^3*b + a^4 + a^2*b^2)))/(2*a^2) + (tan(e + f*x)*(20*a*b^4 + 8*b^5
+ 13*a^2*b^3))/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 - ((((2*a^4*b^4 + 6*a^5*b
^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^2)) + (tan(e + f*x)*(32*a^4*b
^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2
)))/(2*a^2) - (tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(4*(2*a^3*b +
a^4 + a^2*b^2)))/a^2)/((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^
4*b + a^5 + a^3*b^2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3
 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) + (tan(e + f
*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2
+ (((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^2))
 + (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^
2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) - (tan(e + f*x)*(20*a*b^4 + 8*b^5
 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 + ((3*a*b^3)/2 + b^4)
/(2*a^4*b + a^5 + a^3*b^2)))/(a^2*f) + (atan(((((tan(e + f*x)*(20*a*b^4 + 8
*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((-b*(a + b)^3)^(1/2)*(
(2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a^3*b^2) - (tan(e + f*
x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 +
 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3
*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(-b*(a + b)
^3)^(1/2)*(3*a + 2*b)*1i)/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)) + (((ta
n(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) +
 ((-b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5
+ a^3*b^2) + (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 8
0*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b
 + a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 +
3*a^3*b^2)))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*1i)/(4*(3*a^4*b + a^5 + a^2*b
^3 + 3*a^3*b^2)))/(((3*a*b^3)/2 + b^4)/(2*a^4*b + a^5 + a^3*b^2) - (((tan(e
 + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((
-b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a
^3*b^2) - (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a
^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b +
a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a
^3*b^2)))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3
*a^3*b^2)) + (((tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b +
 a^4 + a^2*b^2)) + ((-b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^
2)/(2*a^4*b + a^5 + a^3*b^2) + (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*
b)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 +
 a^2*b^2)*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b
+ a^5 + a^2*b^3 + 3*a^3*b^2)))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b))/(4*(3*a^4*
b + a^5 + a^2*b^3 + 3*a^3*b^2))))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*1i)/(2*f
*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)) - (b*tan(e + f*x))/(2*a*f*(a + b)*(
a + b + b*tan(e + f*x)^2))
```

3.51 $\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}f} - \frac{3\cot(e+fx)}{2(a+b)^2 f} + \frac{\cot(e+fx)}{2(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

```
[Out] -3/2*cot(f*x+e)/(a+b)^2/f-3/2*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2
)/(a+b)^(5/2)/f+1/2*cot(f*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4217, 296, 331, 211}

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2f(a+b)^{5/2}} - \frac{3\cot(e+fx)}{2f(a+b)^2} + \frac{\cot(e+fx)}{2f(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

```
[In] Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*(a + b)^(5/2)*f)
 - (3*Cot[e + f*x])/(2*(a + b)^2*f) + Cot[e + f*x]/(2*(a + b)*f*(a + b + b*
Tan[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\cot(e+fx)}{2(a+b)f\,(a+b+b\tan^2(e+fx))} + \frac{3\text{Subst}\left(\int \frac{1}{x^2(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{2(a+b)f}\\
&= -\frac{3\cot(e+fx)}{2(a+b)^2f} + \frac{\cot(e+fx)}{2(a+b)f\,(a+b+b\tan^2(e+fx))}\\
&\quad - \frac{(3b)\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{2(a+b)^2f}\\
&= -\frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}f} - \frac{3\cot(e+fx)}{2(a+b)^2f} + \frac{\cot(e+fx)}{2(a+b)f\,(a+b+b\tan^2(e+fx))}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.54 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.66

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(\frac{3b\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a\cos(2(e+\ldots}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{8(a+b)^2f\,(a+b\ldots}$$

[In] Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]

[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((3*b*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + 2*(a + 2*b + a*Cos[2*(e + f*x)])*Csc[e]*Csc[e + f*x]*Sin[f*x] + (b*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(a*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^2)

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^2}-\frac{1}{(a+b)^2\tan(fx+e)}}{f}$
default	$\frac{-\frac{b\left(\frac{\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^2}-\frac{1}{(a+b)^2\tan(fx+e)}}{f}$
risch	$-\frac{i\left(2a^2e^{4i(fx+e)}+ab\,e^{4i(fx+e)}+2b^2e^{4i(fx+e)}+4a^2e^{2i(fx+e)}+8ab\,e^{2i(fx+e)}-2b^2e^{2i(fx+e)}+2a^2-ab\right)}{a(a+b)^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)\left(e^{2i(fx+e)}-1\right)}-\frac{3\sqrt{-(a+b)b}\,\ln\ldots}{\ldots}$

[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/f*(-b/(a+b)^2*(1/2*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+3/2/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))-1/(a+b)^2/tan(f*x+e))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(77) = 154$.

Time = 0.31 (sec) , antiderivative size = 407, normalized size of antiderivative = 4.47

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \left[-\frac{4\,(2\,a-b)\cos\left(fx+e\right)^3 - 3\left(a\cos\left(fx+e\right)^2+b\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)}{a^2\cos}\right.}{8\left((a^3+2\,a^2b+ab^2)f\cos\left(fx+e\right)^2+(a\right.}\right.$$
$$-\frac{2\,(2\,a-b)\cos\left(fx+e\right)^3-3\left(a\cos\left(fx+e\right)^2+b\right)\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)\sin\left(fx+e\right)}{4\left((a^3+2\,a^2b+ab^2)f\cos\left(fx+e\right)^2+(a^2b+2\,ab^2+b^3)f\right)\sin\left(fx+e\right)}$$

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(2*a - b)*cos(f*x + e)^3 - 3*(a*cos(f*x + e)^2 + b)*sqrt(-b/(a + b
))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*
sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x +
e)^2 + b^2))*sin(f*x + e) + 12*b*cos(f*x + e))/(((a^3 + 2*a^2*b + a*b^2)*f*
cos(f*x + e)^2 + (a^2*b + 2*a*b^2 + b^3)*f)*sin(f*x + e)), -1/4*(2*(2*a - b
)*cos(f*x + e)^3 - 3*(a*cos(f*x + e)^2 + b)*sqrt(b/(a + b))*arctan(1/2*((a
+ 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*s
in(f*x + e) + 6*b*cos(f*x + e))/(((a^3 + 2*a^2*b + a*b^2)*f*cos(f*x + e)^2
+ (a^2*b + 2*a*b^2 + b^3)*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\csc^2\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(csc(e + f*x)**2/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.29

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{3\,b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^2+2\,ab+b^2)\sqrt{(a+b)b}}+\frac{3\,b\tan(fx+e)^2+2\,a+2\,b}{(a^2b+2\,ab^2+b^3)\tan(fx+e)^3+(a^3+3\,a^2b+3\,ab^2+b^3)\tan(fx+e)}}{2\,f}$$

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^2 + 2*a*b + b^2)*sqrt(
(a + b)*b)) + (3*b*tan(f*x + e)^2 + 2*a + 2*b)/((a^2*b + 2*a*b^2 + b^3)*tan
(f*x + e)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(f*x + e)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.40

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{(a^2+2\,ab+b^2)\sqrt{ab+b^2}}+\frac{3\,b\tan(fx+e)^2+2\,a+2\,b}{\left(b\tan(fx+e)^3+a\tan(fx+e)+b\tan(fx+e)\right)(a^2+2\,ab+b^2)}}{2\,f}$$

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b + b^2)))*b/((a^2 + 2*a*b + b^2)*sqrt(a*b + b^2)) + (3*b*tan(f*x + e)^2 +
 2*a + 2*b)/((b*tan(f*x + e)^3 + a*tan(f*x + e) + b*tan(f*x + e))*(a^2 + 2*
a*b + b^2)))/f
```

Mupad [B] (verification not implemented)

Time = 18.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{1}{a+b}+\frac{3\,b\tan(e+f\,x)^2}{2\,(a+b)^2}}{f\,\left(b\tan\left(e+f\,x\right)^3+\left(a+b\right)\,\tan\left(e+f\,x\right)\right)} - \frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)\,\left(a^2+2\,a\,b+b^2\right)}{(a+b)^{5/2}}\right)}{2\,f\,\left(a+b\right)^{5/2}}$$

```
[In] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] - (1/(a + b) + (3*b*tan(e + f*x)^2)/(2*(a + b)^2))/(f*(b*tan(e + f*x)^3 + t
an(e + f*x)*(a + b))) - (3*b^(1/2)*atan((b^(1/2)*tan(e + f*x)*(2*a*b + a^2
+ b^2))/(a + b)^(5/2)))/(2*f*(a + b)^(5/2))
```

3.52 $\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 123

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2} dx = -\frac{(3a-2b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2(a+b)^{7/2}f} - \frac{(a-b)\cot(e+fx)}{(a+b)^3f} - \frac{\cot^3(e+fx)}{3(a+b)^2f} - \frac{ab\tan(e+fx)}{2(a+b)^3f\left(a+b+b\tan^2(e+fx)\right)}$$

```
[Out] -(a-b)*cot(f*x+e)/(a+b)^3/f-1/3*cot(f*x+e)^3/(a+b)^2/f-1/2*(3*a-2*b)*arctan
(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/(a+b)^(7/2)/f-1/2*a*b*tan(f*x+e)/(
a+b)^3/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4217, 467, 1275, 211}

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2} dx = -\frac{\sqrt{b}(3a-2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2f(a+b)^{7/2}} - \frac{ab\tan(e+fx)}{2f(a+b)^3\left(a+b\tan^2(e+fx)+b\right)} - \frac{\cot^3(e+fx)}{3f(a+b)^2} - \frac{(a-b)\cot(e+fx)}{f(a+b)^3}$$

```
[In] Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/2*((3*a - 2*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/((a +
 b)^(7/2)*f) - ((a - b)*Cot[e + f*x])/((a + b)^3*f) - Cot[e + f*x]^3/(3*(a
+ b)^2*f) - (a*b*Tan[e + f*x])/(2*(a + b)^3*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{ab\tan(e+fx)}{2(a+b)^3f\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\text{Subst}\left(\int \frac{-\frac{2}{b(a+b)}-\frac{2ax^2}{b(a+b)^2}+\frac{ax^4}{(a+b)^3}}{x^4(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{2f}$$

$$= -\frac{ab\tan(e+fx)}{2(a+b)^3f\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\text{Subst}\left(\int \left(-\frac{2}{b(a+b)^2x^4}-\frac{2(a-b)}{b(a+b)^3x^2}+\frac{3a-2b}{(a+b)^3(a+b+bx^2)}\right)dx, x, \tan(e+fx)\right)}{2f}$$

$$= -\frac{(a-b)\cot(e+fx)}{(a+b)^3 f} - \frac{\cot^3(e+fx)}{3(a+b)^2 f} - \frac{ab\tan(e+fx)}{2(a+b)^3 f\,(a+b+b\tan^2(e+fx))} - \frac{((3a-2b)b)\mathrm{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{2(a+b)^3 f}$$

$$= -\frac{(3a-2b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2(a+b)^{7/2} f} - \frac{(a-b)\cot(e+fx)}{(a+b)^3 f} - \frac{\cot^3(e+fx)}{3(a+b)^2 f} - \frac{ab\tan(e+fx)}{2(a+b)^3 f\,(a+b+b\tan^2(e+fx))}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.80 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.46

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(-2(a+b)(a+2b+a\cos(2(e+fx)))\cot(e)\csc^2(e+fx) + \frac{3(3a}{}\right.}{}$$

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(-2*(a + b)*(a + 2*b + a*Cos
[2*(e + f*x)])*Cot[e]*Csc[e + f*x]^2 + (3*(3*a - 2*b)*b*ArcTan[(Sec[f*x]*(C
os[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a
 + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2
*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + 4*(a - 2*b
)*(a + 2*b + a*Cos[2*(e + f*x)])*Csc[e]*Csc[e + f*x]*Sin[f*x] + 2*(a + b)*(
a + 2*b + a*Cos[2*(e + f*x)])*Csc[e]*Csc[e + f*x]^3*Sin[f*x] - 3*a*b*Sec[2*
e]*Sin[2*f*x] + 3*b*(a + 2*b)*Tan[2*e]))/(24*(a + b)^3*f*(a + b*Sec[e + f*x
]^2)^2)
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(3a-2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^3}-\frac{1}{3(a+b)^2\tan(fx+e)^3}-\frac{a-b}{(a+b)^3\tan(fx+e)}}{f}$
default	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(3a-2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^3}-\frac{1}{3(a+b)^2\tan(fx+e)^3}-\frac{a-b}{(a+b)^3\tan(fx+e)}}{f}$
risch	$\frac{i(9ab\,e^{8i(fx+e)}-6b^2e^{8i(fx+e)}+12a^2e^{6i(fx+e)}+18ab\,e^{6i(fx+e)}+66b^2e^{6i(fx+e)}+20a^2e^{4i(fx+e)}+44ab\,e^{4i(fx+e)}-66b^2e^{4i}}{3f(a+b)^3\left(e^{2i(fx+e)}-1\right)^3\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(f}\right.}$

```
[In] int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/(a+b)^3*b*(1/2*a*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a-2*b)/((a+
b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))-1/3/(a+b)^2/tan(f*x+e)^3-
(a-b)/(a+b)^3/tan(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(109) = 218$.

Time = 0.31 (sec) , antiderivative size = 663, normalized size of antiderivative = 5.39

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \left[-\frac{4\left(4a^2-11ab\right)\cos\left(fx+e\right)^5-8\left(3a^2-8ab+4b^2\right)\cos\left(fx+e\right)^3+3\left(\left(3a^2-2ab\right)\cos\left(fx+e\right)^4-(3\right.}{24\left(\left(a^4+3a^3b+\right.\right.}\right.$$

$$-\frac{2\left(4a^2-11ab\right)\cos\left(fx+e\right)^5-4\left(3a^2-8ab+4b^2\right)\cos\left(fx+e\right)^3-3\left(\left(3a^2-2ab\right)\cos\left(fx+e\right)^4-(3\right.}{12\left(\left(a^4+3a^3b+3a^2b^2+ab^3\right)f\cos\left(fx+e\right)^4-(a^4+\right.}$$

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/24*(4*(4*a^2 - 11*a*b)*cos(f*x + e)^5 - 8*(3*a^2 - 8*a*b + 4*b^2)*cos(f
*x + e)^3 + 3*((3*a^2 - 2*a*b)*cos(f*x + e)^4 - (3*a^2 - 5*a*b + 2*b^2)*cos
(f*x + e)^2 - 3*a*b + 2*b^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*co
s(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*
cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) +
b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) - 12*(
3*a*b - 2*b^2)*cos(f*x + e))/(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*
```

```
x + e)^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*f*cos(f*x + e)^2 - (a^3*b + 3*a^
2*b^2 + 3*a*b^3 + b^4)*f)*sin(f*x + e)), -1/12*(2*(4*a^2 - 11*a*b)*cos(f*x
+ e)^5 - 4*(3*a^2 - 8*a*b + 4*b^2)*cos(f*x + e)^3 - 3*((3*a^2 - 2*a*b)*cos(
f*x + e)^4 - (3*a^2 - 5*a*b + 2*b^2)*cos(f*x + e)^2 - 3*a*b + 2*b^2)*sqrt(b
/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(
f*x + e)*sin(f*x + e)))*sin(f*x + e) - 6*(3*a*b - 2*b^2)*cos(f*x + e))/(((a
^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2*a^3*b - 2*a*b
^3 - b^4)*f*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f)*sin(f*x
 + e))]
```

Sympy [F]

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

```
[In] integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(csc(e + f*x)**4/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.57

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{3\,(3\,ab-2\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{(a+b)b}}+\frac{3\,(3\,ab-2\,b^2)\tan(fx+e)^4+2\,(3\,a^2+ab-2\,b^2)\tan(fx+e)^2+2\,a^2+4\,ab+2\,b^2}{(a^3b+3\,a^2b^2+3\,ab^3+b^4)\tan(fx+e)^5+(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\tan(fx+e)^3}}{6\,f}$$

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(3*a*b - 2*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*sqrt((a + b)*b)) + (3*(3*a*b - 2*b^2)*tan(f*x + e)^4 +
 2*(3*a^2 + a*b - 2*b^2)*tan(f*x + e)^2 + 2*a^2 + 4*a*b + 2*b^2)/((a^3*b +
3*a^2*b^2 + 3*a*b^3 + b^4)*tan(f*x + e)^5 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*
a*b^3 + b^4)*tan(f*x + e)^3))/f
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.50

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{3\,ab\tan(fx+e)}{(a^3+3\,a^2b+3\,ab^2+b^3)\left(b\tan(fx+e)^2+a+b\right)} + \frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3\,ab-2\,b^2)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{ab+b^2}} + \frac{2\left(3\,a\tan(fx+e)^2-3\,b\tan(f\right.}{(a^3+3\,a^2b+3\,ab^2+b^3)\tan}}{6\,f}$$

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*a*b*tan(f*x + e)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(b*tan(f*x + e)^2
 + a + b)) + 3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)
/sqrt(a*b + b^2)))*(3*a*b - 2*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*
b + b^2)) + 2*(3*a*tan(f*x + e)^2 - 3*b*tan(f*x + e)^2 + a + b)/((a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*tan(f*x + e)^3))/f
```

Mupad [B] (verification not implemented)

Time = 19.58 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.15

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{1}{3\,(a+b)}+\frac{\tan(e+f\,x)^2\,(3\,a-2\,b)}{3\,(a+b)^2}+\frac{b\tan(e+f\,x)^4\,(3\,a-2\,b)}{2\,(a+b)^3}}{f\,\left(b\tan\left(e+f\,x\right)^5+\left(a+b\right)\,\tan\left(e+f\,x\right)^3\right)} - \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{(a+b)^{7/2}}\right)\,\left(3\,a-2\,b\right)}{2\,f\,\left(a+b\right)^{7/2}}$$

```
[In] int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] - (1/(3*(a + b)) + (tan(e + f*x)^2*(3*a - 2*b))/(3*(a + b)^2) + (b*tan(e +
f*x)^4*(3*a - 2*b))/(2*(a + b)^3))/(f*(tan(e + f*x)^3*(a + b) + b*tan(e + f
*x)^5)) - (b^(1/2)*atan((b^(1/2)*tan(e + f*x)*(3*a*b^2 + 3*a^2*b + a^3 + b^
3))/(a + b)^(7/2))*(3*a - 2*b))/(2*f*(a + b)^(7/2))
```

3.53 $\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 188

$$\begin{aligned}\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = &-\frac{a(3a-4b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2(a+b)^{9/2}f}\\ &-\frac{(5a^2-10ab-b^2)\cot(e+fx)}{5(a+b)^4f}-\frac{(10a+3b)\cot^3(e+fx)}{15(a+b)^3f}\\ &-\frac{\cot^5(e+fx)}{5(a+b)f\,(a+b+b\tan^2(e+fx))}\\ &-\frac{b(5a^2+2b^2)\tan(e+fx)}{10(a+b)^4f\,(a+b+b\tan^2(e+fx))}\end{aligned}$$

```
[Out] -1/5*(5*a^2-10*a*b-b^2)*cot(f*x+e)/(a+b)^4/f-1/15*(10*a+3*b)*cot(f*x+e)^3/(
a+b)^3/f-1/2*a*(3*a-4*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/(a+
b)^(9/2)/f-1/5*cot(f*x+e)^5/(a+b)/f/(a+b+b*tan(f*x+e)^2)-1/10*b*(5*a^2+2*b^
2)*tan(f*x+e)/(a+b)^4/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used

$= \{4217, 473, 467, 1275, 211\}$

$$\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{b(5a^2+2b^2)\tan(e+fx)}{10f(a+b)^4\left(a+b\tan^2(e+fx)+b\right)} - \frac{(5a^2-10ab-b^2)\cot(e+fx)}{5f(a+b)^4} - \frac{a\sqrt{b}(3a-4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2f(a+b)^{9/2}} - \frac{(10a+3b)\cot^3(e+fx)}{15f(a+b)^3} - \frac{\cot^5(e+fx)}{5f(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

```
[In] Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/2*(a*(3*a - 4*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/((a
 + b)^(9/2)*f) - ((5*a^2 - 10*a*b - b^2)*Cot[e + f*x])/(5*(a + b)^4*f) - ((
10*a + 3*b)*Cot[e + f*x]^3)/(15*(a + b)^3*f) - Cot[e + f*x]^5/(5*(a + b)*f*
(a + b + b*Tan[e + f*x]^2)) - (b*(5*a^2 + 2*b^2)*Tan[e + f*x])/(10*(a + b)^
4*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^6(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\,(a+b+b\tan^2(e+fx))} + \frac{\text{Subst}\left(\int \frac{10a+3b+5(a+b)x^2}{x^4(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{5(a+b)f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\,(a+b+b\tan^2(e+fx))} - \frac{b(5a^2+2b^2)\tan(e+fx)}{10(a+b)^4f\,(a+b+b\tan^2(e+fx))} - \frac{b\text{Subst}\left(\int \frac{-\frac{2(10a+3b)}{b(a+b)}-\frac{2\left(5a^2+2b^2\right)x^2}{b(a+b)^2}+\frac{\left(5a^2+2b^2\right)x^4}{(a+b)^3}}{x^4(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{10(a+b)f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\,(a+b+b\tan^2(e+fx))} - \frac{b(5a^2+2b^2)\tan(e+fx)}{10(a+b)^4f\,(a+b+b\tan^2(e+fx))} - \frac{b\text{Subst}\left(\int \left(-\frac{2(10a+3b)}{b(a+b)^2x^4}+\frac{2(-5a^2+10ab+b^2)}{b(a+b)^3x^2}+\frac{5a(3a-4b)}{(a+b)^3(a+b+bx^2)}\right)dx, x, \tan(e+fx)\right)}{10(a+b)f}$$

$$= -\frac{(5a^2-10ab-b^2)\cot(e+fx)}{5(a+b)^4f} - \frac{(10a+3b)\cot^3(e+fx)}{15(a+b)^3f} - \frac{\cot^5(e+fx)}{5(a+b)f\,(a+b+b\tan^2(e+fx))} - \frac{b(5a^2+2b^2)\tan(e+fx)}{10(a+b)^4f\,(a+b+b\tan^2(e+fx))} - \frac{(a(3a-4b)b)\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{2(a+b)^4f}$$

$$= -\frac{a(3a-4b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2(a+b)^{9/2}f} - \frac{(5a^2-10ab-b^2)\cot(e+fx)}{5(a+b)^4f} - \frac{(10a+3b)\cot^3(e+fx)}{15(a+b)^3f} - \frac{\cot^5(e+fx)}{5(a+b)f\,(a+b+b\tan^2(e+fx))} - \frac{b(5a^2+2b^2)\tan(e+fx)}{10(a+b)^4f\,(a+b+b\tan^2(e+fx))}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.24 (sec) , antiderivative size = 777, normalized size of antiderivative = 4.13

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(\frac{960a(3a-4b)b\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{}$$

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((960*a*(3*a - 4*b)*b*ArcTan
[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x
]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e +
f*x)])*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])
 - Csc[e]*Csc[e + f*x]^5*Sec[2*e]*(10*a*(16*a^2 + 34*a*b + 123*b^2)*Sin[f*x
] - a*(16*a^2 - 223*a*b + 1336*b^2)*Sin[3*f*x] + 240*a^3*Sin[2*e - f*x] + 6
40*a^2*b*Sin[2*e - f*x] - 1460*a*b^2*Sin[2*e - f*x] + 240*b^3*Sin[2*e - f*x
] - 240*a^3*Sin[2*e + f*x] - 715*a^2*b*Sin[2*e + f*x] + 860*a*b^2*Sin[2*e +
 f*x] - 240*b^3*Sin[2*e + f*x] + 160*a^3*Sin[4*e + f*x] + 415*a^2*b*Sin[4*e
 + f*x] + 1830*a*b^2*Sin[4*e + f*x] + 165*a^2*b*Sin[2*e + 3*f*x] - 30*a*b^2
*Sin[2*e + 3*f*x] + 120*b^3*Sin[2*e + 3*f*x] - 16*a^3*Sin[4*e + 3*f*x] + 20
8*a^2*b*Sin[4*e + 3*f*x] - 1036*a*b^2*Sin[4*e + 3*f*x] + 180*a^2*b*Sin[6*e
+ 3*f*x] - 330*a*b^2*Sin[6*e + 3*f*x] + 120*b^3*Sin[6*e + 3*f*x] + 48*a^3*S
in[2*e + 5*f*x] - 268*a^2*b*Sin[2*e + 5*f*x] + 290*a*b^2*Sin[2*e + 5*f*x] -
 24*b^3*Sin[2*e + 5*f*x] + 48*a^3*Sin[6*e + 5*f*x] - 223*a^2*b*Sin[6*e + 5*
f*x] + 230*a*b^2*Sin[6*e + 5*f*x] - 24*b^3*Sin[6*e + 5*f*x] - 45*a^2*b*Sin[
8*e + 5*f*x] + 60*a*b^2*Sin[8*e + 5*f*x] - 16*a^3*Sin[4*e + 7*f*x] + 83*a^2
*b*Sin[4*e + 7*f*x] - 6*a*b^2*Sin[4*e + 7*f*x] - 15*a^2*b*Sin[6*e + 7*f*x]
- 16*a^3*Sin[8*e + 7*f*x] + 68*a^2*b*Sin[8*e + 7*f*x] - 6*a*b^2*Sin[8*e + 7
*f*x])))/(7680*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.66

method	result
derivativedivides	$\frac{-\frac{ba\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(3a-4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^4}-\frac{1}{5(a+b)^2\tan(fx+e)^5}-\frac{a(a-2b)}{(a+b)^4\tan(fx+e)}-\frac{2a}{3(a+b)^3\tan(fx+e)^3}}{f}$
default	$\frac{-\frac{ba\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(3a-4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^4}-\frac{1}{5(a+b)^2\tan(fx+e)^5}-\frac{a(a-2b)}{(a+b)^4\tan(fx+e)}-\frac{2a}{3(a+b)^3\tan(fx+e)^3}}{f}$
risch	$-\frac{i\left(-45a^2b\,e^{12i(fx+e)}+60a\,b^2e^{12i(fx+e)}+180a^2b\,e^{10i(fx+e)}-330a\,b^2e^{10i(fx+e)}+120b^3e^{10i(fx+e)}+160a^3e^{8i(fx+e)}+415\right.}{}$

[In] int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/f*(-b*a/(a+b)^4*(1/2*a*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a-4*b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))-1/5/(a+b)^2/tan(f*x+e)^5-a*(a-2*b)/(a+b)^4/tan(f*x+e)-2/3*a/(a+b)^3/tan(f*x+e)^3)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. 2(170) = 340.

Time = 0.32 (sec) , antiderivative size = 987, normalized size of antiderivative = 5.25

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")

[Out] [-1/120*(4*(16*a^3 - 83*a^2*b + 6*a*b^2)*cos(f*x + e)^7 - 4*(40*a^3 - 201*a^2*b + 68*a*b^2 - 6*b^3)*cos(f*x + e)^5 + 20*(6*a^3 - 29*a^2*b + 28*a*b^2)*cos(f*x + e)^3 + 15*((3*a^3 - 4*a^2*b)*cos(f*x + e)^6 - (6*a^3 - 11*a^2*b + 4*a*b^2)*cos(f*x + e)^4 + 3*a^2*b - 4*a*b^2 + (3*a^3 - 10*a^2*b + 8*a*b^2)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*(3*a^2*b - 4*a*b^2)*cos(f*x + e))/(((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*f*cos(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*f*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*f*cos(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f)*sin(f*x + e)), -1/60*(2*(16*a^3 - 83*a^2*b + 6*a*b^2)*cos(f*x + e)^7 - 2*(40*a^3

```
 - 201*a^2*b + 68*a*b^2 - 6*b^3)*cos(f*x + e)^5 + 10*(6*a^3 - 29*a^2*b + 28
*a*b^2)*cos(f*x + e)^3 - 15*((3*a^3 - 4*a^2*b)*cos(f*x + e)^6 - (6*a^3 - 11
*a^2*b + 4*a*b^2)*cos(f*x + e)^4 + 3*a^2*b - 4*a*b^2 + (3*a^3 - 10*a^2*b +
8*a*b^2)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)
^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 30*(3
*a^2*b - 4*a*b^2)*cos(f*x + e))/(((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 +
a*b^4)*f*cos(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^
4 - b^5)*f*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^
4 - 2*b^5)*f*cos(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^
5)*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.43

$$\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx=-\frac{\frac{15\,(3\,a^2b-4\,ab^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\sqrt{(a+b)b}}+\frac{15\,(3\,a^2b-4\,ab^2)\tan(fx+e)^6+10\,(3\,a^3-a^2b-4\,ab^2)\tan(fx+e)^4+6\,a^3+18\,a^2b+18\,ab^2+6\,b^3+2\,(1}{(a^4b+4\,a^3b^2+6\,a^2b^3+4\,ab^4+b^5)\tan(fx+e)^7+(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,a}}{30\,f}$$

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/30*(15*(3*a^2*b - 4*a*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4
+ 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt((a + b)*b)) + (15*(3*a^2*b - 4*
a*b^2)*tan(f*x + e)^6 + 10*(3*a^3 - a^2*b - 4*a*b^2)*tan(f*x + e)^4 + 6*a^3
 + 18*a^2*b + 18*a*b^2 + 6*b^3 + 2*(10*a^3 + 23*a^2*b + 16*a*b^2 + 3*b^3)*t
an(f*x + e)^2)/((a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*tan(f*x + e
)^7 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*tan(f*x + e
)^5))/f
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.35

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{15\,a^2b\tan(fx+e)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\left(b\tan(fx+e)^2+a+b\right)} + \frac{15\,(3\,a^2b-4\,ab^2)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\sqrt{ab+b^2}} + \frac{2\left(15\,a^2\tan(fx+e)\right.}{}}{30\,f}$$

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/30*(15*a^2*b*tan(f*x + e)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(
b*tan(f*x + e)^2 + a + b)) + 15*(3*a^2*b - 4*a*b^2)*(pi*floor((f*x + e)/pi
+ 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^4 + 4*a^3*b + 6
*a^2*b^2 + 4*a*b^3 + b^4)*sqrt(a*b + b^2)) + 2*(15*a^2*tan(f*x + e)^4 - 30*
a*b*tan(f*x + e)^4 + 10*a^2*tan(f*x + e)^2 + 10*a*b*tan(f*x + e)^2 + 3*a^2
+ 6*a*b + 3*b^2)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*tan(f*x + e)^
5))/f
```

Mupad [B] (verification not implemented)

Time = 20.34 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.05

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{a\,\sqrt{b}\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\tan(e+f\,x)\,(3\,a-4\,b)\,(a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4)}{(a+b)^{9/2}\,(4\,a\,b-3\,a^2)}\right)\,(3\,a-4\,b)}{2\,f\,(a+b)^{9/2}} - \frac{\frac{1}{5\,(a+b)} - \frac{\tan(e+f\,x)^4\,(4\,a\,b-3\,a^2)}{3\,(a+b)^3} + \frac{\tan(e+f\,x)^2\,(10\,a+3\,b)}{15\,(a+b)^2} - \frac{b\tan(e+f\,x)^6\,(4\,a\,b-3\,a^2)}{2\,(a+b)^4}}{f\,\left(b\tan\left(e+f\,x\right)^7+(a+b)\,\tan\left(e+f\,x\right)^5\right)}$$

```
[In] int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] (a*b^(1/2)*atan((a*b^(1/2)*tan(e + f*x)*(3*a - 4*b)*(4*a*b^3 + 4*a^3*b + a^
4 + b^4 + 6*a^2*b^2))/((a + b)^(9/2)*(4*a*b - 3*a^2)))*(3*a - 4*b))/(2*f*(a
 + b)^(9/2)) - (1/(5*(a + b)) - (tan(e + f*x)^4*(4*a*b - 3*a^2))/(3*(a + b)
^3) + (tan(e + f*x)^2*(10*a + 3*b))/(15*(a + b)^2) - (b*tan(e + f*x)^6*(4*a
*b - 3*a^2))/(2*(a + b)^4))/(f*(tan(e + f*x)^5*(a + b) + b*tan(e + f*x)^7))
```

3.54 $\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 214

$$\begin{aligned}\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx &= \frac{\sqrt{b}(15a^2+70ab+63b^2)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{11/2}f} \\ &\quad - \frac{(3a^2+14ab+13b^2)\cos(e+fx)}{2a^5f} \\ &\quad + \frac{(a+3b)(3a+5b)\cos^3(e+fx)}{12a^4bf} \\ &\quad - \frac{\cos^5(e+fx)}{5a^3f} - \frac{(a+b)^2\cos^7(e+fx)}{4a^2bf\,(b+a\cos^2(e+fx))^2} \\ &\quad - \frac{b(a+b)(3a+11b)\cos(e+fx)}{8a^5f\,(b+a\cos^2(e+fx))}\end{aligned}$$

```
[Out] -1/2*(3*a^2+14*a*b+13*b^2)*cos(f*x+e)/a^5/f+1/12*(a+3*b)*(3*a+5*b)*cos(f*x+
e)^3/a^4/b/f-1/5*cos(f*x+e)^5/a^3/f-1/4*(a+b)^2*cos(f*x+e)^7/a^2/b/f/(b+a*c
os(f*x+e)^2)^2-1/8*b*(a+b)*(3*a+11*b)*cos(f*x+e)/a^5/f/(b+a*cos(f*x+e)^2)+1
/8*(15*a^2+70*a*b+63*b^2)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(11/
2)/f
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4218, 474, 466, 1824, 211}

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{b(a+b)(3a+11b)\cos(e+fx)}{8a^5f\left(a\cos^2(e+fx)+b\right)} + \frac{(a+3b)(3a+5b)\cos^3(e+fx)}{12a^4bf} - \frac{\cos^5(e+fx)}{5a^3f} - \frac{(a+b)^2\cos^7(e+fx)}{4a^2bf\left(a\cos^2(e+fx)+b\right)^2} + \frac{\sqrt{b}(15a^2+70ab+63b^2)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{11/2}f} - \frac{(3a^2+14ab+13b^2)\cos(e+fx)}{2a^5f}$$

```
[In] Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (Sqrt[b]*(15*a^2 + 70*a*b + 63*b^2)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])
/(8*a^(11/2)*f) - ((3*a^2 + 14*a*b + 13*b^2)*Cos[e + f*x])/(2*a^5*f) + ((a
+ 3*b)*(3*a + 5*b)*Cos[e + f*x]^3)/(12*a^4*b*f) - Cos[e + f*x]^5/(5*a^3*f)
- ((a + b)^2*Cos[e + f*x]^7)/(4*a^2*b*f*(b + a*Cos[e + f*x]^2)^2) - (b*(a +
 b)*(3*a + 11*b)*Cos[e + f*x])/(8*a^5*f*(b + a*Cos[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
```

```
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^6(1-x^2)^2}{(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{(a+b)^2\cos^7(e+fx)}{4a^2bf\left(b+a\cos^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{x^6(-4a^2+7(a+b)^2-4abx^2)}{(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{4a^2bf}$$

$$= -\frac{(a+b)^2\cos^7(e+fx)}{4a^2bf\left(b+a\cos^2(e+fx)\right)^2} - \frac{b(a+b)(3a+11b)\cos(e+fx)}{8a^5f\left(b+a\cos^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-ab^2(a+b)(3a+11b)+2a^2b(a+b)(3a+11b)x^2-2a^3(a+b)(3a+11b)x^4+8a^4bx^6}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{8a^6bf}$$

$$= -\frac{(a+b)^2\cos^7(e+fx)}{4a^2bf\left(b+a\cos^2(e+fx)\right)^2} - \frac{b(a+b)(3a+11b)\cos(e+fx)}{8a^5f\left(b+a\cos^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \left(4ab(3a^2+14ab+13b^2)-2a^2(a+3b)(3a+5b)x^2+8a^3bx^4+\frac{-15a^3b^2-70a^2b^3-63ab^4}{b+ax^2}\right)\,d}{8a^6bf}$$

$$= -\frac{(3a^2+14ab+13b^2)\cos(e+fx)}{2a^5f} + \frac{(a+3b)(3a+5b)\cos^3(e+fx)}{12a^4bf} - \frac{\cos^5(e+fx)}{5a^3f} - \frac{(a+b)^2\cos^7(e+fx)}{4a^2bf\left(b+a\cos^2(e+fx)\right)^2} - \frac{b(a+b)(3a+11b)\cos(e+fx)}{8a^5f\left(b+a\cos^2(e+fx)\right)} + \frac{(b(15a^2+70ab+63b^2))\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{8a^5f}$$

$$= \frac{\sqrt{b}(15a^2+70ab+63b^2)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{11/2}f} - \frac{(3a^2+14ab+13b^2)\cos(e+fx)}{2a^5f}$$
$$+ \frac{(a+3b)(3a+5b)\cos^3(e+fx)}{12a^4bf} - \frac{\cos^5(e+fx)}{5a^3f}$$
$$- \frac{(a+b)^2\cos^7(e+fx)}{4a^2bf\left(b+a\cos^2(e+fx)\right)^2} - \frac{b(a+b)(3a+11b)\cos(e+fx)}{8a^5f\left(b+a\cos^2(e+fx)\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.36 (sec) , antiderivative size = 1641, normalized size of antiderivative = 7.67

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(-900a^{11/2}b^{3/2}\cos(e+fx) - 109000a^{9/2}b^{5/2}\cos(e+fx) - 93600\right.}{}$$

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(-900*a^(11/2)*b^(3/2)*Cos[e
 + f*x] - 109000*a^(9/2)*b^(5/2)*Cos[e + f*x] - 936000*a^(7/2)*b^(7/2)*Cos[
e + f*x] - 2803072*a^(5/2)*b^(9/2)*Cos[e + f*x] - 3763200*a^(3/2)*b^(11/2)*
Cos[e + f*x] - 1935360*Sqrt[a]*b^(13/2)*Cos[e + f*x] - 900*a^(11/2)*b^(3/2)
*Cos[e + f*x]*Cos[2*(e + f*x)] + 900*a^(9/2)*b^(3/2)*Cos[e + f*x]*(a + 2*b
+ a*Cos[2*(e + f*x)]) + 24000*a^(7/2)*b^(5/2)*Cos[e + f*x]*(a + 2*b + a*Cos
[2*(e + f*x)]) + 43200*a^(5/2)*b^(7/2)*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e +
 f*x)]) + 225*a^5*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e]
)^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*
Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 11520
0*a^2*b^3*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Si
n[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^
2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 537600*a*b^4*
ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(
f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*
x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 483840*b^5*ArcTan[((-Sq
rt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Co
s[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt
[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 225*a^5*ArcTan[((-Sqrt[a] + I*Sqrt[
a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] +
 Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b +
 a*Cos[2*(e + f*x)])^2 + 115200*a^2*b^3*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*S
qrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a
```

```
+ b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2
*(e + f*x)])^2 + 537600*a*b^4*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e
] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[
(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)
])^2 + 483840*b^5*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e]
)^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*
Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 - 225*a
^5*ArcTan[(Sqrt[a] - Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]]*(a + 2*b + a*Co
s[2*(e + f*x)])^2 - 225*a^5*ArcTan[(Sqrt[a] + Sqrt[a + b]*Tan[(e + f*x)/2])
/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 19200*a^(5/2)*b^(5/2)*Cos[e]*C
os[f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^2 - 20352*a^(9/2)*b^(5/2)*Cos[e + f*
x]*Cos[4*(e + f*x)] - 115712*a^(7/2)*b^(7/2)*Cos[e + f*x]*Cos[4*(e + f*x)]
- 129024*a^(5/2)*b^(9/2)*Cos[e + f*x]*Cos[4*(e + f*x)] + 2048*a^(9/2)*b^(5/
2)*Cos[e + f*x]*Cos[6*(e + f*x)] + 4608*a^(7/2)*b^(7/2)*Cos[e + f*x]*Cos[6*
(e + f*x)] - 384*a^(9/2)*b^(5/2)*Cos[e + f*x]*Cos[8*(e + f*x)] - 19200*a^(5
/2)*b^(5/2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sin[e]*Sin[f*x] - 32496*a^(9/2
)*b^(5/2)*Csc[e + f*x]*Sin[4*(e + f*x)] - 252080*a^(7/2)*b^(7/2)*Csc[e + f*
x]*Sin[4*(e + f*x)] - 577024*a^(5/2)*b^(9/2)*Csc[e + f*x]*Sin[4*(e + f*x)]
- 403200*a^(3/2)*b^(11/2)*Csc[e + f*x]*Sin[4*(e + f*x)]))/(491520*a^(11/2)*
b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A] (verified)

Time = 10.37 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{-\frac{\frac{\cos(fx+e)^5 a^2}{5}-\frac{2a^2\cos(fx+e)^3}{3}-a\cos(fx+e)^3 b+a^2\cos(fx+e)+6ab\cos(fx+e)+6b^2\cos(fx+e)}{a^5}+\frac{b\left(\frac{\left(-\frac{9}{8}a^3-\frac{13}{4}a^2 b-\frac{17}{8}a\,b^2\right)\cos \ldots}{\left(b \ldots\right.}\right.}{\ldots}}{f}$
default	$\frac{-\frac{\frac{\cos(fx+e)^5 a^2}{5}-\frac{2a^2\cos(fx+e)^3}{3}-a\cos(fx+e)^3 b+a^2\cos(fx+e)+6ab\cos(fx+e)+6b^2\cos(fx+e)}{a^5}+\frac{b\left(\frac{\left(-\frac{9}{8}a^3-\frac{13}{4}a^2 b-\frac{17}{8}a\,b^2\right)\cos \ldots}{\left(b \ldots\right.}\right.}{\ldots}}{f}$
risch	$-\frac{e^{5i(fx+e)}}{160a^3 f}+\frac{5\,e^{3i(fx+e)}}{96a^3 f}+\frac{e^{3i(fx+e)}b}{8a^4 f}-\frac{5\,e^{i(fx+e)}}{16a^3 f}-\frac{21\,e^{i(fx+e)}b}{8a^4 f}-\frac{3\,e^{i(fx+e)}b^2}{a^5 f}-\frac{5\,e^{-i(fx+e)}}{16a^3 f}-\frac{21\,e^{-i(\ldots}}{8a\ldots}$

```
[In] int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^5*(1/5*cos(f*x+e)^5*a^2-2/3*a^2*cos(f*x+e)^3-a*cos(f*x+e)^3*b+a^2
*cos(f*x+e)+6*a*b*cos(f*x+e)+6*b^2*cos(f*x+e))+b/a^5*(((-9/8*a^3-13/4*a^2*b
-17/8*a*b^2)*cos(f*x+e)^3-1/8*b*(7*a^2+22*a*b+15*b^2)*cos(f*x+e))/(b+a*cos(
f*x+e)^2)^2+1/8*(15*a^2+70*a*b+63*b^2)/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b
)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 579, normalized size of antiderivative = 2.71

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \left[-\frac{48\,a^4\cos{(fx+e)}^9 - 16\,(10\,a^4 + 9\,a^3b)\cos{(fx+e)}^7 + 16\,(15\,a^4 + 70\,a^3b + 63\,a^2b^2)\cos{(fx+e)}^5 + 50\,(\ldots}{\ldots} \right.$$

$$-\frac{24\,a^4\cos{(fx+e)}^9 - 8\,(10\,a^4 + 9\,a^3b)\cos{(fx+e)}^7 + 8\,(15\,a^4 + 70\,a^3b + 63\,a^2b^2)\cos{(fx+e)}^5 + 25\,(15\ldots}{\ldots}$$

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/240*(48*a^4*cos(f*x + e)^9 - 16*(10*a^4 + 9*a^3*b)*cos(f*x + e)^7 + 16*
(15*a^4 + 70*a^3*b + 63*a^2*b^2)*cos(f*x + e)^5 + 50*(15*a^3*b + 70*a^2*b^2
 + 63*a*b^3)*cos(f*x + e)^3 - 15*((15*a^4 + 70*a^3*b + 63*a^2*b^2)*cos(f*x
+ e)^4 + 15*a^2*b^2 + 70*a*b^3 + 63*b^4 + 2*(15*a^3*b + 70*a^2*b^2 + 63*a*b
^3)*cos(f*x + e)^2)*sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(
f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 30*(15*a^2*b^2 + 70*a*b^3 + 63*b^4)
*cos(f*x + e))/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*f*cos(f*x + e)^2 + a^5*b^2*f
), -1/120*(24*a^4*cos(f*x + e)^9 - 8*(10*a^4 + 9*a^3*b)*cos(f*x + e)^7 + 8*
(15*a^4 + 70*a^3*b + 63*a^2*b^2)*cos(f*x + e)^5 + 25*(15*a^3*b + 70*a^2*b^2
 + 63*a*b^3)*cos(f*x + e)^3 - 15*((15*a^4 + 70*a^3*b + 63*a^2*b^2)*cos(f*x
+ e)^4 + 15*a^2*b^2 + 70*a*b^3 + 63*b^4 + 2*(15*a^3*b + 70*a^2*b^2 + 63*a*b
^3)*cos(f*x + e)^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) + 15*(15*a
^2*b^2 + 70*a*b^3 + 63*b^4)*cos(f*x + e))/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*f
*cos(f*x + e)^2 + a^5*b^2*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.95

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{15\left((9\,a^3b+26\,a^2b^2+17\,ab^3)\cos(fx+e)^3+(7\,a^2b^2+22\,ab^3+15\,b^4)\cos(fx+e)\right)}{a^7\cos(fx+e)^4+2\,a^6b\cos(fx+e)^2+a^5b^2} - \frac{15\,(15\,a^2b+70\,ab^2+63\,b^3)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^5} + \frac{8\,\left(3\,a^2 \cdots\right.}{\cdots}}{120\,f}$$

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/120*(15*((9*a^3*b + 26*a^2*b^2 + 17*a*b^3)*cos(f*x + e)^3 + (7*a^2*b^2 +
 22*a*b^3 + 15*b^4)*cos(f*x + e))/(a^7*cos(f*x + e)^4 + 2*a^6*b*cos(f*x + e
)^2 + a^5*b^2) - 15*(15*a^2*b + 70*a*b^2 + 63*b^3)*arctan(a*cos(f*x + e)/sq
rt(a*b))/(sqrt(a*b)*a^5) + 8*(3*a^2*cos(f*x + e)^5 - 5*(2*a^2 + 3*a*b)*cos(
f*x + e)^3 + 15*(a^2 + 6*a*b + 6*b^2)*cos(f*x + e))/a^5)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 781 vs. 2(194) = 388.

Time = 0.47 (sec) , antiderivative size = 781, normalized size of antiderivative = 3.65

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/120*(15*(15*a^2*b + 70*a*b^2 + 63*b^3)*arctan(-(a*cos(f*x + e) - b)/(sqr
t(a*b)*cos(f*x + e) + sqrt(a*b)))/(sqrt(a*b)*a^5) + 30*(9*a^3*b + 33*a^2*b^
2 + 39*a*b^3 + 15*b^4 + 27*a^3*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 49
*a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 23*a*b^3*(cos(f*x + e) - 1
)/(cos(f*x + e) + 1) - 45*b^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 27*a^
3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 27*a^2*b^2*(cos(f*x + e) -
1)^2/(cos(f*x + e) + 1)^2 - 3*a*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)
^2 + 45*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 9*a^3*b*(cos(f*x +
e) - 1)^3/(cos(f*x + e) + 1)^3 + 11*a^2*b^2*(cos(f*x + e) - 1)^3/(cos(f*x +
 e) + 1)^3 - 13*a*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 15*b^4*(c
os(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3)/((a + b + 2*a*(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x
+ e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1
)^2)^2*a^5) - 16*(8*a^2 + 75*a*b + 90*b^2 - 40*a^2*(cos(f*x + e) - 1)/(cos(
f*x + e) + 1) - 330*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 360*b^2*(co
```

```
s(f*x + e) - 1)/(cos(f*x + e) + 1) + 80*a^2*(cos(f*x + e) - 1)^2/(cos(f*x +
 e) + 1)^2 + 480*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 540*b^2*(c
os(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 270*a*b*(cos(f*x + e) - 1)^3/(cos
(f*x + e) + 1)^3 - 360*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 45*a
*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 90*b^2*(cos(f*x + e) - 1)^4/
(cos(f*x + e) + 1)^4)/(a^5*((cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 1)^5))/
f
```

Mupad [B] (verification not implemented)

Time = 17.90 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.19

$$
\begin{aligned}
&\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx \\
&= \frac{\cos(e+f\,x)^3\,\left(\frac{b}{a^4}+\frac{2}{3\,a^3}\right)}{f} \\
&\quad - \frac{\left(\frac{9\,a^3\,b}{8}+\frac{13\,a^2\,b^2}{4}+\frac{17\,a\,b^3}{8}\right)\,\cos(e+f\,x)^3+\left(\frac{7\,a^2\,b^2}{8}+\frac{11\,a\,b^3}{4}+\frac{15\,b^4}{8}\right)\,\cos(e+f\,x)}{f\,\left(a^7\cos(e+f\,x)^4+2\,a^6\,b\cos(e+f\,x)^2+a^5\,b^2\right)} \\
&\quad - \frac{\cos(e+f\,x)^5}{5\,a^3\,f} - \frac{\cos(e+f\,x)\,\left(\frac{1}{a^3}-\frac{3\,b^2}{a^5}+\frac{3\,b\,\left(\frac{3\,b}{a^4}+\frac{2}{a^3}\right)}{a}\right)}{f} \\
&\quad + \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,\cos(e+f\,x)\,(15\,a^2+70\,a\,b+63\,b^2)}{15\,a^2\,b+70\,a\,b^2+63\,b^3}\right)\,(15\,a^2+70\,a\,b+63\,b^2)}{8\,a^{11/2}\,f}
\end{aligned}
$$

```
[In] int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] (cos(e + f*x)^3*(b/a^4 + 2/(3*a^3)))/f - (cos(e + f*x)^3*((17*a*b^3)/8 + (9
*a^3*b)/8 + (13*a^2*b^2)/4) + cos(e + f*x)*((11*a*b^3)/4 + (15*b^4)/8 + (7*
a^2*b^2)/8))/(f*(a^5*b^2 + a^7*cos(e + f*x)^4 + 2*a^6*b*cos(e + f*x)^2)) -
cos(e + f*x)^5/(5*a^3*f) - (cos(e + f*x)*(1/a^3 - (3*b^2)/a^5 + (3*b*((3*b)
/a^4 + 2/a^3))/a))/f + (b^(1/2)*atan((a^(1/2)*b^(1/2)*cos(e + f*x)*(70*a*b
+ 15*a^2 + 63*b^2))/(70*a*b^2 + 15*a^2*b + 63*b^3))*(70*a*b + 15*a^2 + 63*b
^2))/(8*a^(11/2)*f)
```

3.55 $\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 154

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^3} dx = \frac{5\sqrt{b}(3a+7b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{9/2}f} - \frac{(a+3b)\cos(e+fx)}{a^4f} + \frac{\cos^3(e+fx)}{3a^3f} + \frac{b^2(a+b)\cos(e+fx)}{4a^4f(b+a\cos^2(e+fx))^2} - \frac{b(9a+13b)\cos(e+fx)}{8a^4f(b+a\cos^2(e+fx))}$$

```
[Out] -(a+3*b)*cos(f*x+e)/a^4/f+1/3*cos(f*x+e)^3/a^3/f+1/4*b^2*(a+b)*cos(f*x+e)/a
^4/f/(b+a*cos(f*x+e)^2)^2-1/8*b*(9*a+13*b)*cos(f*x+e)/a^4/f/(b+a*cos(f*x+e)
^2)+5/8*(3*a+7*b)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(9/2)/f
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4218, 466, 1828, 1167, 211}

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^3} dx = \frac{5\sqrt{b}(3a+7b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{9/2}f} + \frac{b^2(a+b)\cos(e+fx)}{4a^4f(a\cos^2(e+fx)+b)^2} - \frac{b(9a+13b)\cos(e+fx)}{8a^4f(a\cos^2(e+fx)+b)} - \frac{(a+3b)\cos(e+fx)}{a^4f} + \frac{\cos^3(e+fx)}{3a^3f}$$

```
[In] Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (5*Sqrt[b]*(3*a + 7*b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(8*a^(9/2)*f
) - ((a + 3*b)*Cos[e + f*x])/(a^4*f) + Cos[e + f*x]^3/(3*a^3*f) + (b^2*(a +
 b)*Cos[e + f*x])/(4*a^4*f*(b + a*Cos[e + f*x]^2)^2) - (b*(9*a + 13*b)*Cos[
e + f*x])/(8*a^4*f*(b + a*Cos[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^6(1-x^2)}{(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{b^2(a+b)\cos(e+fx)}{4a^4f\,(b+a\cos^2(e+fx))^2} + \frac{\text{Subst}\left(\int \frac{-b^2(a+b)+4ab(a+b)x^2-4a^2(a+b)x^4+4a^3x^6}{(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{4a^4f}$$

$$= \frac{b^2(a+b)\cos(e+fx)}{4a^4f\,(b+a\cos^2(e+fx))^2} - \frac{b(9a+13b)\cos(e+fx)}{8a^4f\,(b+a\cos^2(e+fx))} - \frac{\text{Subst}\left(\int \frac{-b^2(7a+11b)+8ab(a+2b)x^2-8a^2bx^4}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{8a^4bf}$$

$$= \frac{b^2(a+b)\cos(e+fx)}{4a^4f\,(b+a\cos^2(e+fx))^2} - \frac{b(9a+13b)\cos(e+fx)}{8a^4f\,(b+a\cos^2(e+fx))} - \frac{\text{Subst}\left(\int \left(8b(a+3b)-8abx^2-\frac{5(3ab^2+7b^3)}{b+ax^2}\right)dx, x, \cos(e+fx)\right)}{8a^4bf}$$

$$= -\frac{(a+3b)\cos(e+fx)}{a^4f} + \frac{\cos^3(e+fx)}{3a^3f} + \frac{b^2(a+b)\cos(e+fx)}{4a^4f\,(b+a\cos^2(e+fx))^2} - \frac{b(9a+13b)\cos(e+fx)}{8a^4f\,(b+a\cos^2(e+fx))} + \frac{(5b(3a+7b))\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{8a^4f}$$

$$= \frac{5\sqrt{b}(3a+7b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{9/2}f} - \frac{(a+3b)\cos(e+fx)}{a^4f} + \frac{\cos^3(e+fx)}{3a^3f} + \frac{b^2(a+b)\cos(e+fx)}{4a^4f\,(b+a\cos^2(e+fx))^2} - \frac{b(9a+13b)\cos(e+fx)}{8a^4f\,(b+a\cos^2(e+fx))}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.74 (sec) , antiderivative size = 1153, normalized size of antiderivative = 7.49

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))^3\sec^6(e+fx)\left(3(9a^4+1920ab^3+4480b^4)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))}\right.}{}\right.\right.}{}$$

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*(3*(9*a^4 + 1920*a*b^3 + 4
480*b^4)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin
[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2
]*Tan[(f*x)/2]))/Sqrt[b]] - (-27*a^(11/2)*Sqrt[b]*Cos[e + f*x] + 162*a^(9/2
)*b^(3/2)*Cos[e + f*x] + 10816*a^(7/2)*b^(5/2)*Cos[e + f*x] + 51552*a^(5/2)
*b^(7/2)*Cos[e + f*x] + 87424*a^(3/2)*b^(9/2)*Cos[e + f*x] + 53760*Sqrt[a]*
b^(11/2)*Cos[e + f*x] - 27*a^(11/2)*Sqrt[b]*Cos[e + f*x]*Cos[2*(e + f*x)] +
 47936*a^(5/2)*b^(7/2)*Cos[e + f*x]*Cos[2*(e + f*x)] + 44800*a^(3/2)*b^(9/2
)*Cos[e + f*x]*Cos[2*(e + f*x)] + 27*a^(9/2)*Sqrt[b]*Cos[e + f*x]*(a + 2*b
+ a*Cos[2*(e + f*x)]) - 216*a^(7/2)*b^(3/2)*Cos[e + f*x]*(a + 2*b + a*Cos[2
*(e + f*x)]) - 3600*a^(5/2)*b^(5/2)*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*
x)]) - 5184*a^(3/2)*b^(7/2)*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)]) - 2
7*a^4*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]
*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*T
an[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 - 5760*a*b^3*ArcTan
[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2
] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2])
)/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 - 13440*b^4*ArcTan[((-Sqrt[a] +
 I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(S
qrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a
 + 2*b + a*Cos[2*(e + f*x)])^2 + 27*a^4*ArcTan[(Sqrt[a] - Sqrt[a + b]*Tan[(
e + f*x)/2])/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 27*a^4*ArcTan[(Sqr
t[a] + Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)]
)^2 - 2304*a^(3/2)*b^(5/2)*Cos[e]*Cos[f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^2
 + 1920*a^(7/2)*b^(5/2)*Cos[e + f*x]*Cos[4*(e + f*x)] + 3584*a^(5/2)*b^(7/2
)*Cos[e + f*x]*Cos[4*(e + f*x)] - 128*a^(7/2)*b^(5/2)*Cos[e + f*x]*Cos[6*(e
 + f*x)] + 2304*a^(3/2)*b^(5/2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sin[e]*Sin
[f*x] + 54*a^(9/2)*b^(3/2)*Csc[e + f*x]*Sin[4*(e + f*x)] + 3108*a^(7/2)*b^(
5/2)*Csc[e + f*x]*Sin[4*(e + f*x)])/(a + 2*b + a*Cos[2*(e + f*x)])^2))/(245
76*a^(9/2)*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A] (verified)

Time = 5.74 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{\frac{\frac{a\cos(fx+e)^3}{3}-\cos(fx+e)a-3b\cos(fx+e)}{a^4}+\frac{b\left(\frac{\left(-\frac{9}{8}a^2-\frac{13}{8}ab\right)\cos(fx+e)^3-\frac{b(7a+11b)\cos(fx+e)}{8}}{\left(b+a\cos(fx+e)^2\right)^2}+\frac{5(3a+7b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right.}{8\sqrt{ab}}\right.}{a^4}}{f}$
default	$\frac{\frac{\frac{a\cos(fx+e)^3}{3}-\cos(fx+e)a-3b\cos(fx+e)}{a^4}+\frac{b\left(\frac{\left(-\frac{9}{8}a^2-\frac{13}{8}ab\right)\cos(fx+e)^3-\frac{b(7a+11b)\cos(fx+e)}{8}}{\left(b+a\cos(fx+e)^2\right)^2}+\frac{5(3a+7b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right.}{8\sqrt{ab}}\right.}{a^4}}{f}$
risch	$\frac{e^{3i(fx+e)}}{24a^3f}-\frac{3\,e^{i(fx+e)}}{8a^3f}-\frac{3\,e^{i(fx+e)}b}{2a^4f}-\frac{3\,e^{-i(fx+e)}}{8a^3f}-\frac{3\,e^{-i(fx+e)}b}{2a^4f}+\frac{e^{-3i(fx+e)}}{24a^3f}-\frac{b\left(9a^2e^{7i(fx+e)}+13ab\,e^{7i\ldots}\right.}{}$

```
[In] int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^4*(1/3*a*cos(f*x+e)^3-cos(f*x+e)*a-3*b*cos(f*x+e))+b/a^4*(((-9/8*a
^2-13/8*a*b)*cos(f*x+e)^3-1/8*b*(7*a+11*b)*cos(f*x+e))/(b+a*cos(f*x+e)^2)^2
+5/8*(3*a+7*b)/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.85

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$
$$=\left[\frac{16\,a^3\cos\left(fx+e\right)^7-16\left(3\,a^3+7\,a^2b\right)\cos\left(fx+e\right)^5-50\left(3\,a^2b+7\,ab^2\right)\cos\left(fx+e\right)^3+15\left(\left(3\,a^3+7\,a^2b\ldots\right.\right.}{48\left(a^6f\,\mathrm{co}\ldots\right.}\right.$$

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*a^3*cos(f*x + e)^7 - 16*(3*a^3 + 7*a^2*b)*cos(f*x + e)^5 - 50*(3*
a^2*b + 7*a*b^2)*cos(f*x + e)^3 + 15*((3*a^3 + 7*a^2*b)*cos(f*x + e)^4 + 3*
a*b^2 + 7*b^3 + 2*(3*a^2*b + 7*a*b^2)*cos(f*x + e)^2)*sqrt(-b/a)*log(-(a*co
s(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) - 3
0*(3*a*b^2 + 7*b^3)*cos(f*x + e))/(a^6*f*cos(f*x + e)^4 + 2*a^5*b*f*cos(f*x
 + e)^2 + a^4*b^2*f), 1/24*(8*a^3*cos(f*x + e)^7 - 8*(3*a^3 + 7*a^2*b)*cos(
f*x + e)^5 - 25*(3*a^2*b + 7*a*b^2)*cos(f*x + e)^3 + 15*((3*a^3 + 7*a^2*b)*
cos(f*x + e)^4 + 3*a*b^2 + 7*b^3 + 2*(3*a^2*b + 7*a*b^2)*cos(f*x + e)^2)*sq
rt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) - 15*(3*a*b^2 + 7*b^3)*cos(f*x +
 e))/(a^6*f*cos(f*x + e)^4 + 2*a^5*b*f*cos(f*x + e)^2 + a^4*b^2*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.97

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{3\left((9a^2b+13ab^2)\cos(fx+e)^3+(7ab^2+11b^3)\cos(fx+e)\right)}{a^6\cos(fx+e)^4+2a^5b\cos(fx+e)^2+a^4b^2} - \frac{15(3ab+7b^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^4} - \frac{8\left(a\cos(fx+e)^3-3(a+3b)\cos(fx+e)\right)}{a^4}}{24f}$$

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
[Out] -1/24*(3*((9*a^2*b + 13*a*b^2)*cos(f*x + e)^3 + (7*a*b^2 + 11*b^3)*cos(f*x
+ e))/(a^6*cos(f*x + e)^4 + 2*a^5*b*cos(f*x + e)^2 + a^4*b^2) - 15*(3*a*b +
 7*b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^4) - 8*(a*cos(f*x + e
)^3 - 3*(a + 3*b)*cos(f*x + e))/a^4)/f
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.13

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{5(3ab+7b^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{8\sqrt{ab}a^4f} - \frac{\frac{9a^2b\cos(fx+e)^3}{f}+\frac{13ab^2\cos(fx+e)^3}{f}+\frac{7ab^2\cos(fx+e)}{f}+\frac{11b^3\cos(fx+e)}{f}}{8\left(a\cos(fx+e)^2+b\right)^2a^4} + \frac{a^6f^{17}\cos(fx+e)^3-3a^6f^{17}\cos(fx+e)-9a^5bf^{17}\cos(fx+e)}{3a^9f^{18}}$$

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 5/8*(3*a*b + 7*b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^4*f) - 1/
8*(9*a^2*b*cos(f*x + e)^3/f + 13*a*b^2*cos(f*x + e)^3/f + 7*a*b^2*cos(f*x +
 e)/f + 11*b^3*cos(f*x + e)/f)/((a*cos(f*x + e)^2 + b)^2*a^4) + 1/3*(a^6*f^
17*cos(f*x + e)^3 - 3*a^6*f^17*cos(f*x + e) - 9*a^5*b*f^17*cos(f*x + e))/(a
^9*f^18)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.12

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\cos(e+f\,x)^3}{3\,a^3\,f} - \frac{\left(\frac{9\,a^2\,b}{8}+\frac{13\,a\,b^2}{8}\right)\,\cos(e+f\,x)^3+\left(\frac{11\,b^3}{8}+\frac{7\,a\,b^2}{8}\right)\,\cos(e+f\,x)}{f\,\left(a^6\cos(e+f\,x)^4+2\,a^5\,b\cos(e+f\,x)^2+a^4\,b^2\right)} - \frac{\cos(e+f\,x)\,\left(\frac{3\,b}{a^4}+\frac{1}{a^3}\right)}{f} + \frac{5\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,\cos(e+f\,x)\,(3\,a+7\,b)}{7\,b^2+3\,a\,b}\right)\,(3\,a+7\,b)}{8\,a^{9/2}\,f}$$

```
[In] int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] cos(e + f*x)^3/(3*a^3*f) - (cos(e + f*x)^3*((13*a*b^2)/8 + (9*a^2*b)/8) + c
os(e + f*x)*((7*a*b^2)/8 + (11*b^3)/8))/(f*(a^4*b^2 + a^6*cos(e + f*x)^4 +
2*a^5*b*cos(e + f*x)^2)) - (cos(e + f*x)*((3*b)/a^4 + 1/a^3))/f + (5*b^(1/2
)*atan((a^(1/2)*b^(1/2)*cos(e + f*x)*(3*a + 7*b))/(3*a*b + 7*b^2))*(3*a + 7
*b))/(8*a^(9/2)*f)
```

3.56 $\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^3} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 116

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^3} \, dx = \frac{15\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{7/2}f} - \frac{15\cos(e+fx)}{8a^3f} + \frac{\cos^5(e+fx)}{4af\left(b+a\cos^2(e+fx)\right)^2} + \frac{5\cos^3(e+fx)}{8a^2f\left(b+a\cos^2(e+fx)\right)}$$

```
[Out] -15/8*cos(f*x+e)/a^3/f+1/4*cos(f*x+e)^5/a/f/(b+a*cos(f*x+e)^2)^2+5/8*cos(f*
x+e)^3/a^2/f/(b+a*cos(f*x+e)^2)+15/8*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(
1/2)/a^(7/2)/f
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4218, 294, 327, 211}

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^3} \, dx = \frac{15\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{7/2}f} - \frac{15\cos(e+fx)}{8a^3f} + \frac{5\cos^3(e+fx)}{8a^2f\left(a\cos^2(e+fx)+b\right)} + \frac{\cos^5(e+fx)}{4af\left(a\cos^2(e+fx)+b\right)^2}$$

```
[In] Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (15*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(8*a^(7/2)*f) - (15*Cos
[e + f*x])/(8*a^3*f) + Cos[e + f*x]^5/(4*a*f*(b + a*Cos[e + f*x]^2)^2) + (5
*Cos[e + f*x]^3)/(8*a^2*f*(b + a*Cos[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^6}{(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)}{4af\left(b+a\cos^2(e+fx)\right)^2} - \frac{5\text{Subst}\left(\int \frac{x^4}{(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{4af}$$

$$= \frac{\cos^5(e+fx)}{4af\left(b+a\cos^2(e+fx)\right)^2} + \frac{5\cos^3(e+fx)}{8a^2f\left(b+a\cos^2(e+fx)\right)} - \frac{15\text{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{8a^2f}$$

$$= -\frac{15\cos(e+fx)}{8a^3f} + \frac{\cos^5(e+fx)}{4af\left(b+a\cos^2(e+fx)\right)^2} + \frac{5\cos^3(e+fx)}{8a^2f\left(b+a\cos^2(e+fx)\right)} + \frac{(15b)\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{8a^3f}$$

$$= \frac{15\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{7/2}f} - \frac{15\cos(e+fx)}{8a^3f} + \frac{\cos^5(e+fx)}{4af\left(b+a\cos^2(e+fx)\right)^2} + \frac{5\cos^3(e+fx)}{8a^2f\left(b+a\cos^2(e+fx)\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.52 (sec) , antiderivative size = 656, normalized size of antiderivative = 5.66

$$\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))^3\sec^6(e+fx)\left(15(a^3+64b^3)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+}{\sqrt{b}}\right.\right.}{}$$

```
[In] Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*(15*(a^3 + 64*b^3)*ArcTan[
((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2]
 + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))
/Sqrt[b]] + 15*(a^3 + 64*b^3)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e
] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[
(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] + (Sqrt[a]*(24*a^4*Sqrt[b]*C
os[e + f*x] - 24*a^3*b^(3/2)*Cos[e + f*x] - 144*a^2*b^(5/2)*Cos[e + f*x] +
512*b^(9/2)*Cos[e + f*x] - 72*a^3*b^(3/2)*Cos[e + f*x]*Cos[2*(e + f*x)] - 2
4*a^3*Sqrt[b]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)]) + 72*a^2*b^(3/2)*
Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)]) - 1152*b^(7/2)*Cos[e + f*x]*(a
+ 2*b + a*Cos[2*(e + f*x)]) - 15*a^(5/2)*ArcTan[(Sqrt[a] - Sqrt[a + b]*Tan[
(e + f*x)/2])/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 - 15*a^(5/2)*ArcTan
[(Sqrt[a] + Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]]*(a + 2*b + a*Cos[2*(e +
f*x)])^2 - 512*b^(5/2)*Cos[e]*Cos[f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 5
12*b^(5/2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sin[e]*Sin[f*x] + 6*a^4*Sqrt[b]
*Csc[e + f*x]*Sin[4*(e + f*x)]))/(a + 2*b + a*Cos[2*(e + f*x)])^2))/(4096*a
^(7/2)*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A] (verified)

Time = 2.95 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\frac{7b\sec(fx+e)^3}{8}+\frac{9\sec(fx+e)a}{8}}{\left(a+b\sec(fx+e)^2\right)^2}+\frac{15\arctan\left(\frac{\sec(fx+e)b}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^3}-\frac{1}{a^3\sec(fx+e)}}{f}$
default	$\frac{-\frac{b\left(\frac{\frac{7b\sec(fx+e)^3}{8}+\frac{9\sec(fx+e)a}{8}}{\left(a+b\sec(fx+e)^2\right)^2}+\frac{15\arctan\left(\frac{\sec(fx+e)b}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^3}-\frac{1}{a^3\sec(fx+e)}}{f}$
risch	$-\frac{e^{i(fx+e)}}{2a^3f}-\frac{e^{-i(fx+e)}}{2a^3f}-\frac{b\left(9a\,e^{7i(fx+e)}+27a\,e^{5i(fx+e)}+28b\,e^{5i(fx+e)}+27a\,e^{3i(fx+e)}+28b\,e^{3i(fx+e)}+9a\,e^{i(fx+e)}\right)}{4a^3\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2f}$

[In] int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)

[Out] 1/f*(-1/a^3*b*((7/8*b*sec(f*x+e)^3+9/8*sec(f*x+e)*a)/(a+b*sec(f*x+e)^2)^2+15/8/(a*b)^(1/2)*arctan(sec(f*x+e)*b/(a*b)^(1/2)))-1/a^3/sec(f*x+e))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.58

$$\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \left[-\frac{16\,a^2\cos\left(fx+e\right)^5+50\,ab\cos\left(fx+e\right)^3+30\,b^2\cos\left(fx+e\right)-15\left(a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)\right.}{16\left(a^5f\cos\left(fx+e\right)^4+2\,a^4bf\cos\left(fx+e\right)^2+a^3b^2f\right.}\right.$$

$$-\frac{8\,a^2\cos\left(fx+e\right)^5+25\,ab\cos\left(fx+e\right)^3+15\,b^2\cos\left(fx+e\right)-15\left(a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2\right.}{8\left(a^5f\cos\left(fx+e\right)^4+2\,a^4bf\cos\left(fx+e\right)^2+a^3b^2f\right)}$$

[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")

[Out] [-1/16*(16*a^2*cos(f*x + e)^5 + 50*a*b*cos(f*x + e)^3 + 30*b^2*cos(f*x + e) - 15*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)))/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f), -1/8*(8*a^2*cos(f*x + e)^5 + 25*a*b*cos(f*x + e)^3 + 15*b^2*cos(f*x + e) - 15*(a^2*cos(

```
f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f
*x + e)/b))/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.89

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{9\,ab\cos(fx+e)^3+7\,b^2\cos(fx+e)}{a^5\cos(fx+e)^4+2\,a^4b\cos(fx+e)^2+a^3b^2} - \frac{15\,b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} + \frac{8\cos(fx+e)}{a^3}}{8\,f}$$

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
[Out] -1/8*((9*a*b*cos(f*x + e)^3 + 7*b^2*cos(f*x + e))/(a^5*cos(f*x + e)^4 + 2*a
^4*b*cos(f*x + e)^2 + a^3*b^2) - 15*b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqr
t(a*b)*a^3) + 8*cos(f*x + e)/a^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.79

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{15\,b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3f} - \frac{\cos(fx+e)}{a^3f} - \frac{\frac{9\,ab\cos(fx+e)^3}{f} + \frac{7\,b^2\cos(fx+e)}{f}}{8\,\left(a\cos(fx+e)^2+b\right)^2a^3}$$

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
[Out] 15/8*b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^3*f) - cos(f*x + e)/(a
^3*f) - 1/8*(9*a*b*cos(f*x + e)^3/f + 7*b^2*cos(f*x + e)/f)/((a*cos(f*x + e
)^2 + b)^2*a^3)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.91

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{15\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\cos(e+f\,x)}{\sqrt{b}}\right)}{8\,a^{7/2}\,f} - \frac{\frac{7\,b^2\,\cos(e+f\,x)}{8}+\frac{9\,a\,b\cos(e+f\,x)^3}{8}}{f\,\left(a^5\cos\left(e+f\,x\right)^4+2\,a^4\,b\cos\left(e+f\,x\right)^2+a^3\,b^2\right)} - \frac{\cos\left(e+f\,x\right)}{a^3\,f}$$

```
[In] int(sin(e + f*x)/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] (15*b^(1/2)*atan((a^(1/2)*cos(e + f*x))/b^(1/2)))/(8*a^(7/2)*f) - ((7*b^2*c
os(e + f*x))/8 + (9*a*b*cos(e + f*x)^3)/8)/(f*(a^3*b^2 + a^5*cos(e + f*x)^4
 + 2*a^4*b*cos(e + f*x)^2)) - cos(e + f*x)/(a^3*f)
```

3.57 $\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 154

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\sqrt{b}(15a^2+10ab+3b^2)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{5/2}(a+b)^3f} - \frac{\operatorname{arctanh}(\cos(e+fx))}{(a+b)^3f} - \frac{b\cos^3(e+fx)}{4a(a+b)f\,(b+a\cos^2(e+fx))^2} - \frac{b(7a+3b)\cos(e+fx)}{8a^2(a+b)^2f\,(b+a\cos^2(e+fx))}$$

```
[Out] -arctanh(cos(f*x+e))/(a+b)^3/f-1/4*b*cos(f*x+e)^3/a/(a+b)/f/(b+a*cos(f*x+e)
^2)^2-1/8*b*(7*a+3*b)*cos(f*x+e)/a^2/(a+b)^2/f/(b+a*cos(f*x+e)^2)+1/8*(15*a
^2+10*a*b+3*b^2)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(5/2)/(a+b)^3
/f
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4218, 481, 592, 536, 212, 211}

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{b(7a+3b)\cos(e+fx)}{8a^2f(a+b)^2\,(a\cos^2(e+fx)+b)} + \frac{\sqrt{b}(15a^2+10ab+3b^2)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{5/2}f(a+b)^3} - \frac{\operatorname{arctanh}(\cos(e+fx))}{f(a+b)^3} - \frac{b\cos^3(e+fx)}{4af(a+b)\,(a\cos^2(e+fx)+b)^2}$$

```
[In] Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/
(8*a^(5/2)*(a + b)^3*f) - ArcTanh[Cos[e + f*x]]/((a + b)^3*f) - (b*Cos[e +
f*x]^3)/(4*a*(a + b)*f*(b + a*Cos[e + f*x]^2)^2) - (b*(7*a + 3*b)*Cos[e + f
*x])/(8*a^2*(a + b)^2*f*(b + a*Cos[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{b\cos^3(e+fx)}{4a(a+b)f\left(b+a\cos^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{x^2(3b+(-4a-3b)x^2)}{(1-x^2)(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{4a(a+b)f}$$

$$= -\frac{b\cos^3(e+fx)}{4a(a+b)f\left(b+a\cos^2(e+fx)\right)^2} - \frac{b(7a+3b)\cos(e+fx)}{8a^2(a+b)^2f\left(b+a\cos^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{b(7a+3b)+(-8a^2-7ab-3b^2)x^2}{(1-x^2)(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{8a^2(a+b)^2f}$$

$$= -\frac{b\cos^3(e+fx)}{4a(a+b)f\left(b+a\cos^2(e+fx)\right)^2} - \frac{b(7a+3b)\cos(e+fx)}{8a^2(a+b)^2f\left(b+a\cos^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(e+fx)\right)}{(a+b)^3f} + \frac{\left(b(15a^2+10ab+3b^2)\right)\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{8a^2(a+b)^3f}$$

$$= \frac{\sqrt{b}(15a^2+10ab+3b^2)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{5/2}(a+b)^3f} - \frac{\text{arctanh}(\cos(e+fx))}{(a+b)^3f} - \frac{b\cos^3(e+fx)}{4a(a+b)f\left(b+a\cos^2(e+fx)\right)^2} - \frac{b(7a+3b)\cos(e+fx)}{8a^2(a+b)^2f\left(b+a\cos^2(e+fx)\right)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.22 (sec) , antiderivative size = 447, normalized size of antiderivative = 2.90

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^5(e+fx)\left(\frac{8b^2(a+b)^2}{a^2} - \frac{2b(a+b)(9a+5b)(a+2b+a\cos(2(e+fx)))}{a^2} + \frac{\sqrt{b}(15a^2+10ab+3b^2)\,\text{arcta}}{}\right.}{}$$

```
[In] Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^5*((8*b^2*(a + b)^2)/a^2 - (2*
b*(a + b)*(9*a + 5*b)*(a + 2*b + a*Cos[2*(e + f*x)]))/a^2 + (Sqrt[b]*(15*a^
2 + 10*a*b + 3*b^2)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[
e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] -
I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e
 + f*x])/a^(5/2) + (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[((-Sqrt[a] + I
*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqr
t[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a +
 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x])/a^(5/2) - 8*(a + 2*b + a*Cos[2*(
e + f*x)])^2*Log[Cos[(e + f*x)/2]]*Sec[e + f*x] + 8*(a + 2*b + a*Cos[2*(e +
 f*x)])^2*Log[Sin[(e + f*x)/2]]*Sec[e + f*x]))/(64*(a + b)^3*f*(a + b*Sec[e
 + f*x]^2)^3)
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{\ln(-1+\cos(fx+e))}{2(a+b)^3}+\frac{b\left(\frac{-\frac{\left(9a^2+14ab+5b^2\right)\cos(fx+e)^3}{8a}-\frac{b\left(7a^2+10ab+3b^2\right)\cos(fx+e)}{8a^2}}{\left(b+a\cos(fx+e)^2\right)^2}+\frac{\left(15a^2+10ab+3b^2\right)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}} \cdots\right)}{(a+b)^3}}{f}$
default	$\frac{\frac{\ln(-1+\cos(fx+e))}{2(a+b)^3}+\frac{b\left(\frac{-\frac{\left(9a^2+14ab+5b^2\right)\cos(fx+e)^3}{8a}-\frac{b\left(7a^2+10ab+3b^2\right)\cos(fx+e)}{8a^2}}{\left(b+a\cos(fx+e)^2\right)^2}+\frac{\left(15a^2+10ab+3b^2\right)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}} \cdots\right)}{(a+b)^3}}{f}$
risch	$-\frac{b\left(9a^2e^{7i(fx+e)}+5ab\,e^{7i(fx+e)}+27a^2e^{5i(fx+e)}+43ab\,e^{5i(fx+e)}+12b^2e^{5i(fx+e)}+27a^2e^{3i(fx+e)}+43ab\,e^{3i(fx+e)}+12b^2 \cdots\right)}{4a^2(a+b)^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2}$

```
[In] int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2/(a+b)^3*ln(-1+cos(f*x+e))+b/(a+b)^3*((-1/8*(9*a^2+14*a*b+5*b^2)/a*
cos(f*x+e)^3-1/8*b*(7*a^2+10*a*b+3*b^2)/a^2*cos(f*x+e))/(b+a*cos(f*x+e)^2)^
2+1/8*(15*a^2+10*a*b+3*b^2)/a^2/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2)
))-1/2/(a+b)^3*ln(1+cos(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(140) = 280.

Time = 0.40 (sec) , antiderivative size = 779, normalized size of antiderivative = 5.06

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \left[-\frac{2\,(9\,a^3b + 14\,a^2b^2 + 5\,ab^3)\cos{(fx+e)}^3 - \left((15\,a^4 + 10\,a^3b + 3\,a^2b^2)\cos{(fx+e)}^4 + 15\,a^2b^2 + 10\,ab^3 + 3\right.}{}\right.$$

$$-\frac{(9\,a^3b + 14\,a^2b^2 + 5\,ab^3)\cos{(fx+e)}^3 - \left((15\,a^4 + 10\,a^3b + 3\,a^2b^2)\cos{(fx+e)}^4 + 15\,a^2b^2 + 10\,ab^3 + 3\,b\right.}{}$$

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^3 - ((15*a^4 + 10*a
^3*b + 3*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 10*a*b^3 + 3*b^4 + 2*(15*a^
3*b + 10*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^2)*sqrt(-b/a)*log(-(a*cos(f*x + e)
^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 2*(7*a^2*b^
2 + 10*a*b^3 + 3*b^4)*cos(f*x + e) + 8*(a^4*cos(f*x + e)^4 + 2*a^3*b*cos(f*
x + e)^2 + a^2*b^2)*log(1/2*cos(f*x + e) + 1/2) - 8*(a^4*cos(f*x + e)^4 + 2
*a^3*b*cos(f*x + e)^2 + a^2*b^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^7 + 3*a^
6*b + 3*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^4 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*
b^3 + a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^
5)*f), -1/8*((9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^3 - ((15*a^4 + 1
0*a^3*b + 3*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 10*a*b^3 + 3*b^4 + 2*(15
*a^3*b + 10*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^2)*sqrt(b/a)*arctan(a*sqrt(b/a)
*cos(f*x + e)/b) + (7*a^2*b^2 + 10*a*b^3 + 3*b^4)*cos(f*x + e) + 4*(a^4*cos
(f*x + e)^4 + 2*a^3*b*cos(f*x + e)^2 + a^2*b^2)*log(1/2*cos(f*x + e) + 1/2)
 - 4*(a^4*cos(f*x + e)^4 + 2*a^3*b*cos(f*x + e)^2 + a^2*b^2)*log(-1/2*cos(f
*x + e) + 1/2))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^4 + 2
*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 + 3*
a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.69

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{(15\,a^2b+10\,ab^2+3\,b^3)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a^5+3\,a^4b+3\,a^3b^2+a^2b^3)\sqrt{ab}} - \frac{(9\,a^2b+5\,ab^2)\cos(fx+e)^3+(7\,ab^2+3\,b^3)\cos(fx+e)}{a^4b^2+2\,a^3b^3+a^2b^4+(a^6+2\,a^5b+a^4b^2)\cos(fx+e)^4+2\,(a^5b+2\,a^4b^2+a^3b^3)\cos(fx+e)^2} - \frac{4\log(\cdots}{a^3+3\,\cdots}}{8\,f}$$

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
[Out] 1/8*((15*a^2*b + 10*a*b^2 + 3*b^3)*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^5 +
 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b)) - ((9*a^2*b + 5*a*b^2)*cos(f*x +
 e)^3 + (7*a*b^2 + 3*b^3)*cos(f*x + e))/(a^4*b^2 + 2*a^3*b^3 + a^2*b^4 + (a
^6 + 2*a^5*b + a^4*b^2)*cos(f*x + e)^4 + 2*(a^5*b + 2*a^4*b^2 + a^3*b^3)*co
s(f*x + e)^2) - 4*log(cos(f*x + e) + 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 4
*log(cos(f*x + e) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(140) = 280$.

Time = 0.41 (sec) , antiderivative size = 592, normalized size of antiderivative = 3.84

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{(15\,a^2b+10\,ab^2+3\,b^3)\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{(a^5+3\,a^4b+3\,a^3b^2+a^2b^3)\sqrt{ab}} - \frac{4\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^3+3\,a^2b+3\,ab^2+b^3} + \frac{2\left(9\,a^3b+21\,a^2b^2+15\,ab^3+3\,b^4+\frac{27\,a^3b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\cdots\right.}{\cdots}}{\cdots}$$

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b + 10*a*b^2 + 3*b^3)*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*b)
*cos(f*x + e) + sqrt(a*b)))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b
)) - 4*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(a^3 + 3*a^2*b + 3
*a*b^2 + b^3) + 2*(9*a^3*b + 21*a^2*b^2 + 15*a*b^3 + 3*b^4 + 27*a^3*b*(cos(
f*x + e) - 1)/(cos(f*x + e) + 1) + 13*a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x +
 e) + 1) - 23*a*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 9*b^4*(cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) + 27*a^3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e)
+ 1)^2 - 9*a^2*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 21*a*b^3*(co
s(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 9*b^4*(cos(f*x + e) - 1)^2/(cos(f*
x + e) + 1)^2 + 9*a^3*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - a^2*b^2
*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 13*a*b^3*(cos(f*x + e) - 1)^3/
(cos(f*x + e) + 1)^3 - 3*b^4*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3)/((a
^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*
x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) -
 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)^2
))/f
```

Mupad [B] (verification not implemented)

Time = 21.90 (sec) , antiderivative size = 3557, normalized size of antiderivative = 23.10

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] (atan((((-a^5*b)^(1/2)*((cos(e + f*x)*(60*a*b^5 + 64*a^6 + 9*b^6 + 190*a^2*
b^4 + 300*a^3*b^3 + 225*a^4*b^2))/(32*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3
+ 6*a^5*b^2)) + (((224*a^10*b + 96*a^3*b^8 + 800*a^4*b^7 + 2784*a^5*b^6 + 5
280*a^6*b^5 + 5920*a^7*b^4 + 3936*a^8*b^3 + 1440*a^9*b^2)/(64*(6*a^8*b + a^
9 + a^3*b^6 + 6*a^4*b^5 + 15*a^5*b^4 + 20*a^6*b^3 + 15*a^7*b^2)) - (cos(e +
 f*x)*(-a^5*b)^(1/2)*(10*a*b + 15*a^2 + 3*b^2)*(1280*a^11*b + 256*a^12 - 25
6*a^5*b^7 - 1280*a^6*b^6 - 2304*a^7*b^5 - 1280*a^8*b^4 + 1280*a^9*b^3 + 230
4*a^10*b^2))/(512*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)*(4*a^6*b + a^7 + a^
3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*(-a^5*b)^(1/2)*(10*a*b + 15*a^2 + 3*b^2))/
(16*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)))*(10*a*b + 15*a^2 + 3*b^2)*1i)/(
16*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)) + ((-a^5*b)^(1/2)*((cos(e + f*x)*
(60*a*b^5 + 64*a^6 + 9*b^6 + 190*a^2*b^4 + 300*a^3*b^3 + 225*a^4*b^2))/(32*
(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)) - (((224*a^10*b + 96*a^3
*b^8 + 800*a^4*b^7 + 2784*a^5*b^6 + 5280*a^6*b^5 + 5920*a^7*b^4 + 3936*a^8*
b^3 + 1440*a^9*b^2)/(64*(6*a^8*b + a^9 + a^3*b^6 + 6*a^4*b^5 + 15*a^5*b^4 +
 20*a^6*b^3 + 15*a^7*b^2)) + (cos(e + f*x)*(-a^5*b)^(1/2)*(10*a*b + 15*a^2
+ 3*b^2)*(1280*a^11*b + 256*a^12 - 256*a^5*b^7 - 1280*a^6*b^6 - 2304*a^7*b^
5 - 1280*a^8*b^4 + 1280*a^9*b^3 + 2304*a^10*b^2))/(512*(3*a^7*b + a^8 + a^5
*b^3 + 3*a^6*b^2)*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*(-a^5
```

```
*b)^(1/2)*(10*a*b + 15*a^2 + 3*b^2))/(16*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b
^2)))*(10*a*b + 15*a^2 + 3*b^2)*1i)/(16*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^
2)))/((51*a*b^4 + 120*a^4*b + 9*b^5 + 139*a^2*b^3 + 185*a^3*b^2)/(32*(6*a^8
*b + a^9 + a^3*b^6 + 6*a^4*b^5 + 15*a^5*b^4 + 20*a^6*b^3 + 15*a^7*b^2)) - (
(-a^5*b)^(1/2)*((cos(e + f*x)*(60*a*b^5 + 64*a^6 + 9*b^6 + 190*a^2*b^4 + 30
0*a^3*b^3 + 225*a^4*b^2))/(32*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*
b^2)) + (((224*a^10*b + 96*a^3*b^8 + 800*a^4*b^7 + 2784*a^5*b^6 + 5280*a^6*
b^5 + 5920*a^7*b^4 + 3936*a^8*b^3 + 1440*a^9*b^2)/(64*(6*a^8*b + a^9 + a^3*
b^6 + 6*a^4*b^5 + 15*a^5*b^4 + 20*a^6*b^3 + 15*a^7*b^2)) - (cos(e + f*x)*(-
a^5*b)^(1/2)*(10*a*b + 15*a^2 + 3*b^2)*(1280*a^11*b + 256*a^12 - 256*a^5*b^
7 - 1280*a^6*b^6 - 2304*a^7*b^5 - 1280*a^8*b^4 + 1280*a^9*b^3 + 2304*a^10*b
^2))/(512*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)*(4*a^6*b + a^7 + a^3*b^4 +
4*a^4*b^3 + 6*a^5*b^2)))*(-a^5*b)^(1/2)*(10*a*b + 15*a^2 + 3*b^2))/(16*(3*a
^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)))*(10*a*b + 15*a^2 + 3*b^2))/(16*(3*a^7*b
 + a^8 + a^5*b^3 + 3*a^6*b^2)) + ((-a^5*b)^(1/2)*((cos(e + f*x)*(60*a*b^5 +
 64*a^6 + 9*b^6 + 190*a^2*b^4 + 300*a^3*b^3 + 225*a^4*b^2))/(32*(4*a^6*b +
a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)) - (((224*a^10*b + 96*a^3*b^8 + 800*
a^4*b^7 + 2784*a^5*b^6 + 5280*a^6*b^5 + 5920*a^7*b^4 + 3936*a^8*b^3 + 1440*
a^9*b^2)/(64*(6*a^8*b + a^9 + a^3*b^6 + 6*a^4*b^5 + 15*a^5*b^4 + 20*a^6*b^3
 + 15*a^7*b^2)) + (cos(e + f*x)*(-a^5*b)^(1/2)*(10*a*b + 15*a^2 + 3*b^2)*(1
280*a^11*b + 256*a^12 - 256*a^5*b^7 - 1280*a^6*b^6 - 2304*a^7*b^5 - 1280*a^
8*b^4 + 1280*a^9*b^3 + 2304*a^10*b^2))/(512*(3*a^7*b + a^8 + a^5*b^3 + 3*a^
6*b^2)*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*(-a^5*b)^(1/2)*(
10*a*b + 15*a^2 + 3*b^2))/(16*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)))*(10*a
*b + 15*a^2 + 3*b^2))/(16*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2))))*(-a^5*b)
^(1/2)*(10*a*b + 15*a^2 + 3*b^2)*1i)/(8*f*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*
b^2)) - ((cos(e + f*x)^3*(9*a*b + 5*b^2))/(8*a*(2*a*b + a^2 + b^2)) + (b*co
s(e + f*x)*(7*a*b + 3*b^2))/(8*a^2*(2*a*b + a^2 + b^2)))/(f*(b^2 + a^2*cos(
e + f*x)^4 + 2*a*b*cos(e + f*x)^2)) - (atan((((((224*a^10*b + 96*a^3*b^8 +
800*a^4*b^7 + 2784*a^5*b^6 + 5280*a^6*b^5 + 5920*a^7*b^4 + 3936*a^8*b^3 + 1
440*a^9*b^2)/(64*(6*a^8*b + a^9 + a^3*b^6 + 6*a^4*b^5 + 15*a^5*b^4 + 20*a^6
*b^3 + 15*a^7*b^2)) - (cos(e + f*x)*(1280*a^11*b + 256*a^12 - 256*a^5*b^7 -
 1280*a^6*b^6 - 2304*a^7*b^5 - 1280*a^8*b^4 + 1280*a^9*b^3 + 2304*a^10*b^2)
)/(64*(a + b)^3*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))/(2*(a +
 b)^3) + (cos(e + f*x)*(60*a*b^5 + 64*a^6 + 9*b^6 + 190*a^2*b^4 + 300*a^3*b
^3 + 225*a^4*b^2))/(32*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*
1i)/(2*(a + b)^3) - ((((224*a^10*b + 96*a^3*b^8 + 800*a^4*b^7 + 2784*a^5*b^
6 + 5280*a^6*b^5 + 5920*a^7*b^4 + 3936*a^8*b^3 + 1440*a^9*b^2)/(64*(6*a^8*b
 + a^9 + a^3*b^6 + 6*a^4*b^5 + 15*a^5*b^4 + 20*a^6*b^3 + 15*a^7*b^2)) + (co
s(e + f*x)*(1280*a^11*b + 256*a^12 - 256*a^5*b^7 - 1280*a^6*b^6 - 2304*a^7*
b^5 - 1280*a^8*b^4 + 1280*a^9*b^3 + 2304*a^10*b^2))/(64*(a + b)^3*(4*a^6*b
+ a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))/(2*(a + b)^3) - (cos(e + f*x)*(6
0*a*b^5 + 64*a^6 + 9*b^6 + 190*a^2*b^4 + 300*a^3*b^3 + 225*a^4*b^2))/(32*(4
*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*1i)/(2*(a + b)^3))/((((22
4*a^10*b + 96*a^3*b^8 + 800*a^4*b^7 + 2784*a^5*b^6 + 5280*a^6*b^5 + 5920*a^
```

```
7*b^4 + 3936*a^8*b^3 + 1440*a^9*b^2)/(64*(6*a^8*b + a^9 + a^3*b^6 + 6*a^4*b
^5 + 15*a^5*b^4 + 20*a^6*b^3 + 15*a^7*b^2)) - (cos(e + f*x)*(1280*a^11*b +
256*a^12 - 256*a^5*b^7 - 1280*a^6*b^6 - 2304*a^7*b^5 - 1280*a^8*b^4 + 1280*
a^9*b^3 + 2304*a^10*b^2))/(64*(a + b)^3*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^
3 + 6*a^5*b^2)))/(2*(a + b)^3) + (cos(e + f*x)*(60*a*b^5 + 64*a^6 + 9*b^6 +
 190*a^2*b^4 + 300*a^3*b^3 + 225*a^4*b^2))/(32*(4*a^6*b + a^7 + a^3*b^4 + 4
*a^4*b^3 + 6*a^5*b^2)))/(2*(a + b)^3) - (51*a*b^4 + 120*a^4*b + 9*b^5 + 139
*a^2*b^3 + 185*a^3*b^2)/(32*(6*a^8*b + a^9 + a^3*b^6 + 6*a^4*b^5 + 15*a^5*b
^4 + 20*a^6*b^3 + 15*a^7*b^2)) + (((224*a^10*b + 96*a^3*b^8 + 800*a^4*b^7 +
 2784*a^5*b^6 + 5280*a^6*b^5 + 5920*a^7*b^4 + 3936*a^8*b^3 + 1440*a^9*b^2)/
(64*(6*a^8*b + a^9 + a^3*b^6 + 6*a^4*b^5 + 15*a^5*b^4 + 20*a^6*b^3 + 15*a^7
*b^2)) + (cos(e + f*x)*(1280*a^11*b + 256*a^12 - 256*a^5*b^7 - 1280*a^6*b^6
 - 2304*a^7*b^5 - 1280*a^8*b^4 + 1280*a^9*b^3 + 2304*a^10*b^2))/(64*(a + b)
^3*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))/(2*(a + b)^3) - (cos
(e + f*x)*(60*a*b^5 + 64*a^6 + 9*b^6 + 190*a^2*b^4 + 300*a^3*b^3 + 225*a^4*
b^2))/(32*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))/(2*(a + b)^3)
))*1i)/(f*(a + b)^3)
```

3.58 $\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 213

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3} dx = \frac{\sqrt{b}(15a^2-10ab-b^2)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{3/2}(a+b)^4f} - \frac{(a-5b)\text{arctanh}(\cos(e+fx))}{2(a+b)^4f} - \frac{(2a-b)b\cos(e+fx)}{4a(a+b)^2f\left(b+a\cos^2(e+fx)\right)^2} + \frac{(4a^2-9ab-b^2)\cos(e+fx)}{8a(a+b)^3f\left(b+a\cos^2(e+fx)\right)} - \frac{\cos(e+fx)\cot^2(e+fx)}{2(a+b)f\left(b+a\cos^2(e+fx)\right)^2}$$

```
[Out] -1/2*(a-5*b)*arctanh(cos(f*x+e))/(a+b)^4/f-1/4*(2*a-b)*b*cos(f*x+e)/a/(a+b)
^2/f/(b+a*cos(f*x+e)^2)^2+1/8*(4*a^2-9*a*b-b^2)*cos(f*x+e)/a/(a+b)^3/f/(b+a
*cos(f*x+e)^2)-1/2*cos(f*x+e)*cot(f*x+e)^2/(a+b)/f/(b+a*cos(f*x+e)^2)^2+1/8
*(15*a^2-10*a*b-b^2)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(3/2)/(a+
b)^4/f
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4218, 481, 592, 541, 536, 212, 211}

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{(4a^2-9ab-b^2)\cos(e+fx)}{8af(a+b)^3\,(a\cos^2(e+fx)+b)} + \frac{\sqrt{b}(15a^2-10ab-b^2)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{3/2}f(a+b)^4} - \frac{(a-5b)\text{arctanh}(\cos(e+fx))}{2f(a+b)^4} - \frac{b(2a-b)\cos(e+fx)}{4af(a+b)^2\,(a\cos^2(e+fx)+b)^2} - \frac{\cos(e+fx)\cot^2(e+fx)}{2f(a+b)\,(a\cos^2(e+fx)+b)^2}$$

```
[In] Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (Sqrt[b]*(15*a^2 - 10*a*b - b^2)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(8
*a^(3/2)*(a + b)^4*f) - ((a - 5*b)*ArcTanh[Cos[e + f*x]])/(2*(a + b)^4*f) -
 ((2*a - b)*b*Cos[e + f*x])/(4*a*(a + b)^2*f*(b + a*Cos[e + f*x]^2)^2) + ((
4*a^2 - 9*a*b - b^2)*Cos[e + f*x])/(8*a*(a + b)^3*f*(b + a*Cos[e + f*x]^2))
 - (Cos[e + f*x]*Cot[e + f*x]^2)/(2*(a + b)*f*(b + a*Cos[e + f*x]^2)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
```

```
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^2(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\cot^2(e+fx)}{2(a+b)f\left(b+a\cos^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{x^2(3b+(-a+2b)x^2)}{(1-x^2)(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{2(a+b)f}$$

$$= -\frac{(2a-b)b\cos(e+fx)}{4a(a+b)^2f\left(b+a\cos^2(e+fx)\right)^2} - \frac{\cos(e+fx)\cot^2(e+fx)}{2(a+b)f\left(b+a\cos^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{2(2a-b)b-2(2a^2-8ab-b^2)x^2}{(1-x^2)(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{8a(a+b)^2f}$$

$$= -\frac{(2a-b)b\cos(e+fx)}{4a(a+b)^2f\left(b+a\cos^2(e+fx)\right)^2} + \frac{(4a^2-9ab-b^2)\cos(e+fx)}{8a(a+b)^3f\left(b+a\cos^2(e+fx)\right)} - \frac{\cos(e+fx)\cot^2(e+fx)}{2(a+b)f\left(b+a\cos^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-2(11a-b)b^2+2b(4a^2-9ab-b^2)x^2}{(1-x^2)(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{16ab(a+b)^3f}$$

$$= -\frac{(2a-b)b\cos(e+fx)}{4a(a+b)^2f\left(b+a\cos^2(e+fx)\right)^2} + \frac{(4a^2-9ab-b^2)\cos(e+fx)}{8a(a+b)^3f\left(b+a\cos^2(e+fx)\right)} - \frac{\cos(e+fx)\cot^2(e+fx)}{2(a+b)f\left(b+a\cos^2(e+fx)\right)^2} - \frac{(a-5b)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(e+fx)\right)}{2(a+b)^4f} + \frac{(b(15a^2-10ab-b^2))\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{8a(a+b)^4f}$$

$$= \frac{\sqrt{b}(15a^2-10ab-b^2)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{3/2}(a+b)^4f} - \frac{(a-5b)\text{arctanh}(\cos(e+fx))}{2(a+b)^4f} - \frac{(2a-b)b\cos(e+fx)}{4a(a+b)^2f\left(b+a\cos^2(e+fx)\right)^2} + \frac{(4a^2-9ab-b^2)\cos(e+fx)}{8a(a+b)^3f\left(b+a\cos^2(e+fx)\right)} - \frac{\cos(e+fx)\cot^2(e+fx)}{2(a+b)f\left(b+a\cos^2(e+fx)\right)^2}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.36 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.50

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^5(e+fx)\left(\frac{8b^2(a+b)^2}{a} - \frac{2b(a+b)(9a+b)(a+2b+a\cos(2(e+fx)))}{a} - \frac{\sqrt{b}(-15a^2+10ab+b^2)\,\text{arcta}}{}\right.}{}$$

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^5*((8*b^2*(a + b)^2)/a - (2*b*(a + b)*(9*a + b)*(a + 2*b + a*Cos[2*(e + f*x)]))/a - (Sqrt[b]*(-15*a^2 + 10*a*b + b^2)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x])/a^(3/2) - (Sqrt[b]*(-15*a^2 + 10*a*b + b^2)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x])/a^(3/2) - (a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Csc[(e + f*x)/2]^2*Sec[e + f*x] - 4*(a - 5*b)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Log[Cos[(e + f*x)/2]]*Sec[e + f*x] + 4*(a - 5*b)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Log[Sin[(e + f*x)/2]]*Sec[e + f*x] + (a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[(e + f*x)/2]^2*Sec[e + f*x]))/(64*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^3)

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{\frac{1}{4(a+b)^3(-1+\cos(fx+e))}+\frac{(a-5b)\ln(-1+\cos(fx+e))}{4(a+b)^4}+\frac{b\left(\frac{\left(-\frac{9}{8}a^2-\frac{5}{4}ab-\frac{1}{8}b^2\right)\cos(fx+e)^3-\frac{b\left(7a^2+6ab-b^2\right)\cos(fx+e)}{8a}}{\left(b+a\cos(fx+e)^2\right)^2}+\frac{(15a^2-\cdots}{}\right.}{(a+b)^4}}{f}$
default	$\frac{\frac{1}{4(a+b)^3(-1+\cos(fx+e))}+\frac{(a-5b)\ln(-1+\cos(fx+e))}{4(a+b)^4}+\frac{b\left(\frac{\left(-\frac{9}{8}a^2-\frac{5}{4}ab-\frac{1}{8}b^2\right)\cos(fx+e)^3-\frac{b\left(7a^2+6ab-b^2\right)\cos(fx+e)}{8a}}{\left(b+a\cos(fx+e)^2\right)^2}+\frac{(15a^2-\cdots}{}\right.}{(a+b)^4}}{f}$
risch	$-\frac{-4a^3e^{11i(fx+e)}+9a^2b\,e^{11i(fx+e)}+a\,b^2e^{11i(fx+e)}-20a^3e^{9i(fx+e)}-23a^2b\,e^{9i(fx+e)}+29a\,b^2e^{9i(fx+e)}-4b^3e^{9i(fx+e)}-\cdots}{}$

[In] int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)

[Out] 1/f*(1/4/(a+b)^3/(-1+cos(f*x+e))+1/4*(a-5*b)/(a+b)^4*ln(-1+cos(f*x+e))+b/(a+b)^4*(((-9/8*a^2-5/4*a*b-1/8*b^2)*cos(f*x+e)^3-1/8*b*(7*a^2+6*a*b-b^2)/a*cos(f*x+e))/(b+a*cos(f*x+e)^2)^2+1/8*(15*a^2-10*a*b-b^2)/a/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2)))+1/4/(a+b)^3/(1+cos(f*x+e))+1/4/(a+b)^4*(-a+5*b)*ln(1+cos(f*x+e)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 649 vs. 2(195) = 390.

Time = 0.43 (sec) , antiderivative size = 1332, normalized size of antiderivative = 6.25

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(4*a^4 - 5*a^3*b - 10*a^2*b^2 - a*b^3)*cos(f*x + e)^5 + 2*(17*a^3*
b + 11*a^2*b^2 - 5*a*b^3 + b^4)*cos(f*x + e)^3 - ((15*a^4 - 10*a^3*b - a^2*
b^2)*cos(f*x + e)^6 - (15*a^4 - 40*a^3*b + 19*a^2*b^2 + 2*a*b^3)*cos(f*x +
e)^4 - 15*a^2*b^2 + 10*a*b^3 + b^4 - (30*a^3*b - 35*a^2*b^2 + 8*a*b^3 + b^4
)*cos(f*x + e)^2)*sqrt(-b/a)*log((a*cos(f*x + e)^2 - 2*a*sqrt(-b/a)*cos(f*x
 + e) - b)/(a*cos(f*x + e)^2 + b)) + 2*(11*a^2*b^2 + 10*a*b^3 - b^4)*cos(f*
x + e) - 4*((a^4 - 5*a^3*b)*cos(f*x + e)^6 - (a^4 - 7*a^3*b + 10*a^2*b^2)*c
os(f*x + e)^4 - a^2*b^2 + 5*a*b^3 - (2*a^3*b - 11*a^2*b^2 + 5*a*b^3)*cos(f*
x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 4*((a^4 - 5*a^3*b)*cos(f*x + e)^6 -
 (a^4 - 7*a^3*b + 10*a^2*b^2)*cos(f*x + e)^4 - a^2*b^2 + 5*a*b^3 - (2*a^3*b
 - 11*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^
7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (a^7 + 2*
a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^4 - (
2*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - a*b^6)*f*cos(f*x
+ e)^2 - (a^5*b^2 + 4*a^4*b^3 + 6*a^3*b^4 + 4*a^2*b^5 + a*b^6)*f), 1/8*((4*
a^4 - 5*a^3*b - 10*a^2*b^2 - a*b^3)*cos(f*x + e)^5 + (17*a^3*b + 11*a^2*b^2
 - 5*a*b^3 + b^4)*cos(f*x + e)^3 + ((15*a^4 - 10*a^3*b - a^2*b^2)*cos(f*x +
 e)^6 - (15*a^4 - 40*a^3*b + 19*a^2*b^2 + 2*a*b^3)*cos(f*x + e)^4 - 15*a^2*
b^2 + 10*a*b^3 + b^4 - (30*a^3*b - 35*a^2*b^2 + 8*a*b^3 + b^4)*cos(f*x + e)
^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) + (11*a^2*b^2 + 10*a*b^3 -
 b^4)*cos(f*x + e) - 2*((a^4 - 5*a^3*b)*cos(f*x + e)^6 - (a^4 - 7*a^3*b + 1
0*a^2*b^2)*cos(f*x + e)^4 - a^2*b^2 + 5*a*b^3 - (2*a^3*b - 11*a^2*b^2 + 5*a
*b^3)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 2*((a^4 - 5*a^3*b)*cos(
f*x + e)^6 - (a^4 - 7*a^3*b + 10*a^2*b^2)*cos(f*x + e)^4 - a^2*b^2 + 5*a*b^
3 - (2*a^3*b - 11*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e)
+ 1/2))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6
 - (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*
x + e)^4 - (2*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - a*b^6
)*f*cos(f*x + e)^2 - (a^5*b^2 + 4*a^4*b^3 + 6*a^3*b^4 + 4*a^2*b^5 + a*b^6)*
f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(195) = 390$.

Time = 0.27 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.87

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{2(a-5b)\log(\cos(fx+e)+1)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4} - \frac{2(a-5b)\log(\cos(fx+e)-1)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4} - \frac{(15a^2b-10ab^2-b^3)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4)\sqrt{ab}} - \frac{\cdots}{(a^6+3a^5b+3a^4b^2+a^3b^3)\cos(\cdots}}{8f}$$

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
[Out] -1/8*(2*(a - 5*b)*log(cos(f*x + e) + 1)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^
3 + b^4) - 2*(a - 5*b)*log(cos(f*x + e) - 1)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4
*a*b^3 + b^4) - (15*a^2*b - 10*a*b^2 - b^3)*arctan(a*cos(f*x + e)/sqrt(a*b)
)/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*sqrt(a*b)) - ((4*a^3 - 9
*a^2*b - a*b^2)*cos(f*x + e)^5 + (17*a^2*b - 6*a*b^2 + b^3)*cos(f*x + e)^3
+ (11*a*b^2 - b^3)*cos(f*x + e))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cos
(f*x + e)^6 - a^4*b^2 - 3*a^3*b^3 - 3*a^2*b^4 - a*b^5 - (a^6 + a^5*b - 3*a^
4*b^2 - 5*a^3*b^3 - 2*a^2*b^4)*cos(f*x + e)^4 - (2*a^5*b + 5*a^4*b^2 + 3*a^
3*b^3 - a^2*b^4 - a*b^5)*cos(f*x + e)^2))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 750 vs. $2(195) = 390$.

Time = 0.52 (sec) , antiderivative size = 750, normalized size of antiderivative = 3.52

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{2(a-5b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4} - \frac{(15a^2b-10ab^2-b^3)\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{(a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4)\sqrt{ab}} + \frac{\left(a+b-\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{10b(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)(\cos(fx\cdots}{(a^4+4a^3b+6a^2b^2+4ab^3+b^4)(\cos(fx+e)-1)}\cdots}{\cdots}$$

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(2*(a - 5*b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(a^4 + 4
*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - (15*a^2*b - 10*a*b^2 - b^3)*arctan(-(
a*cos(f*x + e) - b)/(sqrt(a*b)*cos(f*x + e) + sqrt(a*b)))/((a^5 + 4*a^4*b +
 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*sqrt(a*b)) + (a + b - 2*a*(cos(f*x + e) - 1
)/(cos(f*x + e) + 1) + 10*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))*(cos(f*x
 + e) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(cos(f*x + e) - 1))
 - (cos(f*x + e) - 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(cos(f*x + e) + 1))
- 2*(9*a^3*b + 17*a^2*b^2 + 7*a*b^3 - b^4 + 27*a^3*b*(cos(f*x + e) - 1)/(co
s(f*x + e) + 1) + a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 23*a*b^3*
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 3*b^4*(cos(f*x + e) - 1)/(cos(f*x +
 e) + 1) + 27*a^3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 21*a^2*b^2*
(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 29*a*b^3*(cos(f*x + e) - 1)^2/(
cos(f*x + e) + 1)^2 - 3*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 9*a
^3*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 5*a^2*b^2*(cos(f*x + e) -
1)^3/(cos(f*x + e) + 1)^3 - 13*a*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1
)^3 + b^4*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3)/((a^5 + 4*a^4*b + 6*a^
3*b^2 + 4*a^2*b^3 + a*b^4)*(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) +
1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(co
s(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)^2))/f
```

Mupad [B] (verification not implemented)

Time = 19.87 (sec) , antiderivative size = 2728, normalized size of antiderivative = 12.81

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] - ((cos(e + f*x)^3*(17*a^2*b - 6*a*b^2 + b^3))/(8*(a*b^3 + 3*a^3*b + a^4 +
3*a^2*b^2)) - (cos(e + f*x)^5*(9*a*b - 4*a^2 + b^2))/(8*(3*a*b^2 + 3*a^2*b
+ a^3 + b^3)) + (b^2*cos(e + f*x)*(11*a - b))/(8*(a*b^3 + 3*a^3*b + a^4 + 3
*a^2*b^2)))/(f*(b^2 - cos(e + f*x)^4*(2*a*b - a^2) + cos(e + f*x)^2*(2*a*b
- b^2) - a^2*cos(e + f*x)^6)) - (log(cos(e + f*x) - 1)*((3*b)/(2*(a + b)^4)
 - 1/(4*(a + b)^3)))/f - (log(cos(e + f*x) + 1)*(a - 5*b))/(4*f*(a + b)^4)
- (atan(((((cos(e + f*x)*(20*a*b^5 - 160*a^5*b + 16*a^6 + b^6 + 70*a^2*b^4
- 300*a^3*b^3 + 625*a^4*b^2))/(32*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a
^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)) + ((-a^3*b)^(1/2)*(((11*a^11*b)/2 - (a^2
*b^10)/2 + (3*a^3*b^9)/2 + 30*a^4*b^8 + 126*a^5*b^7 + 273*a^6*b^6 + 357*a^7
*b^5 + 294*a^8*b^4 + 150*a^9*b^3 + (87*a^10*b^2)/2)/(a*b^9 + 9*a^9*b + a^10
 + 9*a^2*b^8 + 36*a^3*b^7 + 84*a^4*b^6 + 126*a^5*b^5 + 126*a^6*b^4 + 84*a^7
*b^3 + 36*a^8*b^2) - (cos(e + f*x)*(-a^3*b)^(1/2)*(10*a*b - 15*a^2 + b^2)*(
1792*a^11*b + 256*a^12 - 256*a^3*b^9 - 1792*a^4*b^8 - 5120*a^5*b^7 - 7168*a
```

^6*b^6 - 3584*a^7*b^5 + 3584*a^8*b^4 + 7168*a^9*b^3 + 5120*a^10*b^2))/(512*
(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)*(a*b^6 + 6*a^6*b + a^7 +
6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)))*(10*a*b - 15*a^2 + b^2)
)/(16*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*(-a^3*b)^(1/2)*(1
0*a*b - 15*a^2 + b^2)*1i)/(16*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*
b^2)) + (((cos(e + f*x)*(20*a*b^5 - 160*a^5*b + 16*a^6 + b^6 + 70*a^2*b^4 -
 300*a^3*b^3 + 625*a^4*b^2))/(32*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^
3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)) - ((-a^3*b)^(1/2)*(((11*a^11*b)/2 - (a^2*
b^10)/2 + (3*a^3*b^9)/2 + 30*a^4*b^8 + 126*a^5*b^7 + 273*a^6*b^6 + 357*a^7*
b^5 + 294*a^8*b^4 + 150*a^9*b^3 + (87*a^10*b^2)/2)/(a*b^9 + 9*a^9*b + a^10
+ 9*a^2*b^8 + 36*a^3*b^7 + 84*a^4*b^6 + 126*a^5*b^5 + 126*a^6*b^4 + 84*a^7*
b^3 + 36*a^8*b^2) + (cos(e + f*x)*(-a^3*b)^(1/2)*(10*a*b - 15*a^2 + b^2)*(1
792*a^11*b + 256*a^12 - 256*a^3*b^9 - 1792*a^4*b^8 - 5120*a^5*b^7 - 7168*a^
6*b^6 - 3584*a^7*b^5 + 3584*a^8*b^4 + 7168*a^9*b^3 + 5120*a^10*b^2))/(512*(
4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)*(a*b^6 + 6*a^6*b + a^7 + 6
*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)))*(10*a*b - 15*a^2 + b^2))
/(16*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*(-a^3*b)^(1/2)*(10
*a*b - 15*a^2 + b^2)*1i)/(16*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b
^2)))/(((47*a*b^5)/32 - (15*a^5*b)/16 + (5*b^6)/64 + (21*a^2*b^4)/4 - (473*
a^3*b^3)/32 + (475*a^4*b^2)/64)/(a*b^9 + 9*a^9*b + a^10 + 9*a^2*b^8 + 36*a^
3*b^7 + 84*a^4*b^6 + 126*a^5*b^5 + 126*a^6*b^4 + 84*a^7*b^3 + 36*a^8*b^2) +
 (((cos(e + f*x)*(20*a*b^5 - 160*a^5*b + 16*a^6 + b^6 + 70*a^2*b^4 - 300*a^
3*b^3 + 625*a^4*b^2))/(32*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 +
 20*a^4*b^3 + 15*a^5*b^2)) + ((-a^3*b)^(1/2)*(((11*a^11*b)/2 - (a^2*b^10)/2
 + (3*a^3*b^9)/2 + 30*a^4*b^8 + 126*a^5*b^7 + 273*a^6*b^6 + 357*a^7*b^5 + 2
94*a^8*b^4 + 150*a^9*b^3 + (87*a^10*b^2)/2)/(a*b^9 + 9*a^9*b + a^10 + 9*a^2
*b^8 + 36*a^3*b^7 + 84*a^4*b^6 + 126*a^5*b^5 + 126*a^6*b^4 + 84*a^7*b^3 + 3
6*a^8*b^2) - (cos(e + f*x)*(-a^3*b)^(1/2)*(10*a*b - 15*a^2 + b^2)*(1792*a^1
1*b + 256*a^12 - 256*a^3*b^9 - 1792*a^4*b^8 - 5120*a^5*b^7 - 7168*a^6*b^6 -
 3584*a^7*b^5 + 3584*a^8*b^4 + 7168*a^9*b^3 + 5120*a^10*b^2))/(512*(4*a^6*b
 + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^
5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)))*(10*a*b - 15*a^2 + b^2))/(16*(4
*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*(-a^3*b)^(1/2)*(10*a*b -
15*a^2 + b^2))/(16*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)) - (((
cos(e + f*x)*(20*a*b^5 - 160*a^5*b + 16*a^6 + b^6 + 70*a^2*b^4 - 300*a^3*b^
3 + 625*a^4*b^2))/(32*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*
a^4*b^3 + 15*a^5*b^2)) - ((-a^3*b)^(1/2)*(((11*a^11*b)/2 - (a^2*b^10)/2 + (
3*a^3*b^9)/2 + 30*a^4*b^8 + 126*a^5*b^7 + 273*a^6*b^6 + 357*a^7*b^5 + 294*a
^8*b^4 + 150*a^9*b^3 + (87*a^10*b^2)/2)/(a*b^9 + 9*a^9*b + a^10 + 9*a^2*b^8
 + 36*a^3*b^7 + 84*a^4*b^6 + 126*a^5*b^5 + 126*a^6*b^4 + 84*a^7*b^3 + 36*a^
8*b^2) + (cos(e + f*x)*(-a^3*b)^(1/2)*(10*a*b - 15*a^2 + b^2)*(1792*a^11*b
+ 256*a^12 - 256*a^3*b^9 - 1792*a^4*b^8 - 5120*a^5*b^7 - 7168*a^6*b^6 - 358
4*a^7*b^5 + 3584*a^8*b^4 + 7168*a^9*b^3 + 5120*a^10*b^2))/(512*(4*a^6*b + a
^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 +
15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)))*(10*a*b - 15*a^2 + b^2))/(16*(4*a^6

```
*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*(-a^3*b)^(1/2)*(10*a*b - 15*a
^2 + b^2))/(16*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2))))*(-a^3*b
)^(1/2)*(10*a*b - 15*a^2 + b^2)*1i)/(8*f*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b
^3 + 6*a^5*b^2))
```

3.59 $\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 257

$$\begin{aligned}\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx &= \frac{3\sqrt{b}(5a^2-10ab+b^2)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8\sqrt{a}(a+b)^5f} \\ &- \frac{3(a^2-10ab+5b^2)\,\mathrm{arctanh}(\cos(e+fx))}{8(a+b)^5f} \\ &+ \frac{(a^2-9ab+2b^2)\cos(e+fx)}{8(a+b)^3f\left(b+a\cos^2(e+fx)\right)^2} \\ &+ \frac{3(a^2-6ab+b^2)\cos(e+fx)}{8(a+b)^4f\left(b+a\cos^2(e+fx)\right)} \\ &- \frac{(a-7b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(b+a\cos^2(e+fx)\right)^2} \\ &- \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(b+a\cos^2(e+fx)\right)^2}\end{aligned}$$

```
[Out] -3/8*(a^2-10*a*b+5*b^2)*arctanh(cos(f*x+e))/(a+b)^5/f+1/8*(a^2-9*a*b+2*b^2)
*cos(f*x+e)/(a+b)^3/f/(b+a*cos(f*x+e)^2)^2+3/8*(a^2-6*a*b+b^2)*cos(f*x+e)/(
a+b)^4/f/(b+a*cos(f*x+e)^2)-1/8*(a-7*b)*cot(f*x+e)*csc(f*x+e)/(a+b)^2/f/(b+
a*cos(f*x+e)^2)^2-1/4*cot(f*x+e)^3*csc(f*x+e)/(a+b)/f/(b+a*cos(f*x+e)^2)^2+
3/8*(5*a^2-10*a*b+b^2)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/(a+b)^5/f
/a^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4218, 481, 592, 541, 536, 212, 211}

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{3\sqrt{b}(5a^2-10ab+b^2)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8\sqrt{a}f(a+b)^5} - \frac{3(a^2-10ab+5b^2)\operatorname{arctanh}(\cos(e+fx))}{8f(a+b)^5} + \frac{3(a^2-6ab+b^2)\cos(e+fx)}{8f(a+b)^4(a\cos^2(e+fx)+b)} + \frac{(a^2-9ab+2b^2)\cos(e+fx)}{8f(a+b)^3(a\cos^2(e+fx)+b)^2} - \frac{\cot^3(e+fx)\csc(e+fx)}{4f(a+b)(a\cos^2(e+fx)+b)^2} - \frac{(a-7b)\cot(e+fx)\csc(e+fx)}{8f(a+b)^2(a\cos^2(e+fx)+b)^2}$$

```
[In] Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (3*Sqrt[b]*(5*a^2 - 10*a*b + b^2)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(
8*Sqrt[a]*(a + b)^5*f) - (3*(a^2 - 10*a*b + 5*b^2)*ArcTanh[Cos[e + f*x]])/(
8*(a + b)^5*f) + ((a^2 - 9*a*b + 2*b^2)*Cos[e + f*x])/(8*(a + b)^3*f*(b + a
*Cos[e + f*x]^2)^2) + (3*(a^2 - 6*a*b + b^2)*Cos[e + f*x])/(8*(a + b)^4*f*(
b + a*Cos[e + f*x]^2)) - ((a - 7*b)*Cot[e + f*x]*Csc[e + f*x])/(8*(a + b)^2
*f*(b + a*Cos[e + f*x]^2)^2) - (Cot[e + f*x]^3*Csc[e + f*x])/(4*(a + b)*f*(
b + a*Cos[e + f*x]^2)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
```

```
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(b+a\cos^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{x^2(3b+(-a+4b)x^2)}{(1-x^2)^2(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{4(a+b)f}$$

$$= -\frac{(a-7b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(b+a\cos^2(e+fx)\right)^2} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(b+a\cos^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{(a-7b)b+(-3a^2+29ab-8b^2)x^2}{(1-x^2)(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{8(a+b)^2f}$$

$$= \frac{(a^2-9ab+2b^2)\cos(e+fx)}{8(a+b)^3f\left(b+a\cos^2(e+fx)\right)^2} - \frac{(a-7b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(b+a\cos^2(e+fx)\right)^2} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(b+a\cos^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-12(a-3b)b^2+12b(a^2-9ab+2b^2)x^2}{(1-x^2)(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{32b(a+b)^3f}$$

$$= \frac{(a^2-9ab+2b^2)\cos(e+fx)}{8(a+b)^3f\left(b+a\cos^2(e+fx)\right)^2} + \frac{3(a^2-6ab+b^2)\cos(e+fx)}{8(a+b)^4f\left(b+a\cos^2(e+fx)\right)} - \frac{(a-7b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(b+a\cos^2(e+fx)\right)^2} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(b+a\cos^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{96(a-b)b^3-24b^2(a^2-6ab+b^2)x^2}{(1-x^2)(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{64b^2(a+b)^4f}$$

$$= \frac{(a^2-9ab+2b^2)\cos(e+fx)}{8(a+b)^3f\left(b+a\cos^2(e+fx)\right)^2} + \frac{3(a^2-6ab+b^2)\cos(e+fx)}{8(a+b)^4f\left(b+a\cos^2(e+fx)\right)} - \frac{(a-7b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(b+a\cos^2(e+fx)\right)^2} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(b+a\cos^2(e+fx)\right)^2} + \frac{(3b(5a^2-10ab+b^2))\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{8(a+b)^5f} - \frac{(3(a^2-10ab+5b^2))\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(e+fx)\right)}{8(a+b)^5f}$$

$$= \frac{3\sqrt{b}(5a^2-10ab+b^2)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8\sqrt{a}(a+b)^5f} - \frac{3(a^2-10ab+5b^2)\operatorname{arctanh}(\cos(e+fx))}{8(a+b)^5f} + \frac{(a^2-9ab+2b^2)\cos(e+fx)}{8(a+b)^3f\left(b+a\cos^2(e+fx)\right)^2} + \frac{3(a^2-6ab+b^2)\cos(e+fx)}{8(a+b)^4f\left(b+a\cos^2(e+fx)\right)} - \frac{(a-7b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(b+a\cos^2(e+fx)\right)^2} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(b+a\cos^2(e+fx)\right)^2}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.32 (sec) , antiderivative size = 549, normalized size of antiderivative = 2.14

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\left(\frac{48\sqrt{b}(5a^2-10ab+b^2)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e)\cdots}\right.}{\sqrt{b}}\right.}{\sqrt{a}}\right.}{\cdots}$$

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*((48*Sqrt[b]*(5*a^2 - 10*a*b + b^2)*ArcTan[
((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2]
 + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))
/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2)/Sqrt[a] + (48*Sqrt[b]*(5*a^2 -
10*a*b + b^2)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]
)*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[
e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2)/Sqrt[a] -
2*(a + b)*(30*a^3 + 112*a^2*b + 182*a*b^2 - 140*b^3 + (35*a^3 + 78*a^2*b -
93*a*b^2 + 224*b^3)*Cos[2*(e + f*x)] + 2*(a^3 - 8*a^2*b + 53*a*b^2 - 10*b^3
)*Cos[4*(e + f*x)] - 3*a^3*Cos[6*(e + f*x)] + 18*a^2*b*Cos[6*(e + f*x)] - 3
*a*b^2*Cos[6*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^3 - 48*(a^2 - 10*a*b + 5
*b^2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Log[Cos[(e + f*x)/2]] + 48*(a^2 - 10
*a*b + 5*b^2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Log[Sin[(e + f*x)/2]])*Sec[e
 + f*x]^6)/(1024*(a + b)^5*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.01

method	result
derivativedivides	$-\frac{1}{16(a+b)^3(-1+\cos(fx+e))^2}-\frac{-3a+9b}{16(a+b)^4(-1+\cos(fx+e))}+\frac{(3a^2-30ab+15b^2)\ln(-1+\cos(fx+e))}{16(a+b)^5}+\frac{b\left(\frac{\left(-\frac{9}{8}a^3-\frac{3}{4}a^2b+\frac{3}{8}ab^2\right)\cos\cdots}{(b+\cdots}\right.}{\cdots}$
default	$-\frac{1}{16(a+b)^3(-1+\cos(fx+e))^2}-\frac{-3a+9b}{16(a+b)^4(-1+\cos(fx+e))}+\frac{(3a^2-30ab+15b^2)\ln(-1+\cos(fx+e))}{16(a+b)^5}+\frac{b\left(\frac{\left(-\frac{9}{8}a^3-\frac{3}{4}a^2b+\frac{3}{8}ab^2\right)\cos\cdots}{(b+\cdots}\right.}{\cdots}$
risch	Expression too large to display

```
[In] int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/16/(a+b)^3/(-1+cos(f*x+e))^2-1/16*(-3*a+9*b)/(a+b)^4/(-1+cos(f*x+e)
)+1/16/(a+b)^5*(3*a^2-30*a*b+15*b^2)*ln(-1+cos(f*x+e))+b/(a+b)^5*(((-9/8*a^
3-3/4*a^2*b+3/8*a*b^2)*cos(f*x+e)^3-1/8*b*(7*a^2+2*a*b-5*b^2)*cos(f*x+e))/(
b+a*cos(f*x+e)^2)^2+3/8*(5*a^2-10*a*b+b^2)/(a*b)^(1/2)*arctan(a*cos(f*x+e)/
(a*b)^(1/2)))+1/16/(a+b)^3/(1+cos(f*x+e))^2-1/16*(-3*a+9*b)/(a+b)^4/(1+cos(
f*x+e))+1/16/(a+b)^5*(-3*a^2+30*a*b-15*b^2)*ln(1+cos(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 901 vs. $2(237) = 474$.

Time = 0.47 (sec) , antiderivative size = 1833, normalized size of antiderivative = 7.13

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*(a^4 - 5*a^3*b - 5*a^2*b^2 + a*b^3)*cos(f*x + e)^7 - 2*(5*a^4 - 26
*a^3*b + 26*a*b^3 - 5*b^4)*cos(f*x + e)^5 - 2*(19*a^3*b - 15*a^2*b^2 - 15*a
*b^3 + 19*b^4)*cos(f*x + e)^3 + 3*((5*a^4 - 10*a^3*b + a^2*b^2)*cos(f*x + e
)^8 - 2*(5*a^4 - 15*a^3*b + 11*a^2*b^2 - a*b^3)*cos(f*x + e)^6 + (5*a^4 - 3
0*a^3*b + 46*a^2*b^2 - 14*a*b^3 + b^4)*cos(f*x + e)^4 + 5*a^2*b^2 - 10*a*b^
3 + b^4 + 2*(5*a^3*b - 15*a^2*b^2 + 11*a*b^3 - b^4)*cos(f*x + e)^2)*sqrt(-b
/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x +
e)^2 + b)) - 24*(a^2*b^2 - b^4)*cos(f*x + e) - 3*((a^4 - 10*a^3*b + 5*a^2*b
^2)*cos(f*x + e)^8 - 2*(a^4 - 11*a^3*b + 15*a^2*b^2 - 5*a*b^3)*cos(f*x + e)
^6 + (a^4 - 14*a^3*b + 46*a^2*b^2 - 30*a*b^3 + 5*b^4)*cos(f*x + e)^4 + a^2*
b^2 - 10*a*b^3 + 5*b^4 + 2*(a^3*b - 11*a^2*b^2 + 15*a*b^3 - 5*b^4)*cos(f*x
+ e)^2)*log(1/2*cos(f*x + e) + 1/2) + 3*((a^4 - 10*a^3*b + 5*a^2*b^2)*cos(f
*x + e)^8 - 2*(a^4 - 11*a^3*b + 15*a^2*b^2 - 5*a*b^3)*cos(f*x + e)^6 + (a^4
 - 14*a^3*b + 46*a^2*b^2 - 30*a*b^3 + 5*b^4)*cos(f*x + e)^4 + a^2*b^2 - 10*
a*b^3 + 5*b^4 + 2*(a^3*b - 11*a^2*b^2 + 15*a*b^3 - 5*b^4)*cos(f*x + e)^2)*l
og(-1/2*cos(f*x + e) + 1/2))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*
a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*
b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a
^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*cos(f*x + e)^4 + 2*(a^6*b
+ 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*cos(f*x + e)^2 + (a^
5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f), 1/16*(6*(a
^4 - 5*a^3*b - 5*a^2*b^2 + a*b^3)*cos(f*x + e)^7 - 2*(5*a^4 - 26*a^3*b + 26
*a*b^3 - 5*b^4)*cos(f*x + e)^5 - 2*(19*a^3*b - 15*a^2*b^2 - 15*a*b^3 + 19*b
^4)*cos(f*x + e)^3 + 6*((5*a^4 - 10*a^3*b + a^2*b^2)*cos(f*x + e)^8 - 2*(5*
a^4 - 15*a^3*b + 11*a^2*b^2 - a*b^3)*cos(f*x + e)^6 + (5*a^4 - 30*a^3*b + 4
6*a^2*b^2 - 14*a*b^3 + b^4)*cos(f*x + e)^4 + 5*a^2*b^2 - 10*a*b^3 + b^4 + 2
```

```
*(5*a^3*b - 15*a^2*b^2 + 11*a*b^3 - b^4)*cos(f*x + e)^2)*sqrt(b/a)*arctan(a
*sqrt(b/a)*cos(f*x + e)/b) - 24*(a^2*b^2 - b^4)*cos(f*x + e) - 3*((a^4 - 10
*a^3*b + 5*a^2*b^2)*cos(f*x + e)^8 - 2*(a^4 - 11*a^3*b + 15*a^2*b^2 - 5*a*b
^3)*cos(f*x + e)^6 + (a^4 - 14*a^3*b + 46*a^2*b^2 - 30*a*b^3 + 5*b^4)*cos(f
*x + e)^4 + a^2*b^2 - 10*a*b^3 + 5*b^4 + 2*(a^3*b - 11*a^2*b^2 + 15*a*b^3 -
 5*b^4)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 3*((a^4 - 10*a^3*b +
5*a^2*b^2)*cos(f*x + e)^8 - 2*(a^4 - 11*a^3*b + 15*a^2*b^2 - 5*a*b^3)*cos(f
*x + e)^6 + (a^4 - 14*a^3*b + 46*a^2*b^2 - 30*a*b^3 + 5*b^4)*cos(f*x + e)^4
 + a^2*b^2 - 10*a*b^3 + 5*b^4 + 2*(a^3*b - 11*a^2*b^2 + 15*a*b^3 - 5*b^4)*c
os(f*x + e)^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^7 + 5*a^6*b + 10*a^5*b^2 +
 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b + 5*
a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 + (a^7 + a^6*b -
9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*cos(f*x +
e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*cos(
f*x + e)^2 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7
)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(237) = 474$.

Time = 0.28 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.05

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx =$$

$$-\frac{\frac{3\,(a^2-10\,ab+5\,b^2)\log(\cos(fx+e)+1)}{a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5} - \frac{3\,(a^2-10\,ab+5\,b^2)\log(\cos(fx+e)-1)}{a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5} - \frac{6\,(5\,a^2b-10\,ab^2+b^3)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5)\sqrt{ab}} - \frac{\cdots}{(a^6+4\cdots}}{\cdots}$$

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
[Out] -1/16*(3*(a^2 - 10*a*b + 5*b^2)*log(cos(f*x + e) + 1)/(a^5 + 5*a^4*b + 10*a
^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - 3*(a^2 - 10*a*b + 5*b^2)*log(cos(f*x
 + e) - 1)/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - 6*(5
*a^2*b - 10*a*b^2 + b^3)*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^5 + 5*a^4*b +
```

```
10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*sqrt(a*b)) - 2*(3*(a^3 - 6*a^2*b
+ a*b^2)*cos(f*x + e)^7 - (5*a^3 - 31*a^2*b + 31*a*b^2 - 5*b^3)*cos(f*x + e
)^5 - (19*a^2*b - 34*a*b^2 + 19*b^3)*cos(f*x + e)^3 - 12*(a*b^2 - b^3)*cos(
f*x + e))/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^8
 - 2*(a^6 + 3*a^5*b + 2*a^4*b^2 - 2*a^3*b^3 - 3*a^2*b^4 - a*b^5)*cos(f*x +
e)^6 + a^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6 + (a^6 - 9*a^4*b^2 -
 16*a^3*b^3 - 9*a^2*b^4 + b^6)*cos(f*x + e)^4 + 2*(a^5*b + 3*a^4*b^2 + 2*a^
3*b^3 - 2*a^2*b^4 - 3*a*b^5 - b^6)*cos(f*x + e)^2))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1316 vs. $2(237) = 474$.

Time = 0.46 (sec) , antiderivative size = 1316, normalized size of antiderivative = 5.12

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/64*(12*(a^2 - 10*a*b + 5*b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e)
 + 1))/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - 24*(5*a^
2*b - 10*a*b^2 + b^3)*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*b)*cos(f*x + e)
+ sqrt(a*b)))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*sq
rt(a*b)) - (8*a^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 24*a*b^2*(cos(f*x
 + e) - 1)/(cos(f*x + e) + 1) - 16*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1
) - a^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 3*a^2*b*(cos(f*x + e) -
 1)^2/(cos(f*x + e) + 1)^2 - 3*a*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1
)^2 - b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^6 + 6*a^5*b + 15*a^
4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6) - (a^4 + 4*a^3*b + 6*a^2*b
^2 + 4*a*b^3 + b^4 - 4*a^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 24*a^2*b
^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 32*a*b^3*(cos(f*x + e) - 1)/(cos
(f*x + e) + 1) + 12*b^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 20*a^4*(cos
(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 136*a^3*b*(cos(f*x + e) - 1)^2/(cos
(f*x + e) + 1)^2 + 224*a^2*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 -
40*a*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 108*b^4*(cos(f*x + e)
- 1)^2/(cos(f*x + e) + 1)^2 - 20*a^4*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1
)^3 + 280*a^3*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 64*a^2*b^2*(cos
(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 152*a*b^3*(cos(f*x + e) - 1)^3/(cos
(f*x + e) + 1)^3 + 212*b^4*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 5*a^
4*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 84*a^3*b*(cos(f*x + e) - 1)^4
/(cos(f*x + e) + 1)^4 + 30*a^2*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^
4 + 84*a*b^3*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 - 123*b^4*(cos(f*x +
 e) - 1)^4/(cos(f*x + e) + 1)^4 + 16*a^4*(cos(f*x + e) - 1)^5/(cos(f*x + e)
 + 1)^5 - 104*a^3*b*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 - 24*a^2*b^2*
```

```
(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 + 72*a*b^3*(cos(f*x + e) - 1)^5/(
cos(f*x + e) + 1)^5 - 24*b^4*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 + 6*
a^4*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6 - 48*a^3*b*(cos(f*x + e) - 1)
^6/(cos(f*x + e) + 1)^6 - 84*a^2*b^2*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1
)^6 + 30*b^4*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6)/((a^5 + 5*a^4*b + 1
0*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*(a*(cos(f*x + e) - 1)/(cos(f*x + e)
 + 1) + b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2*a*(cos(f*x + e) - 1)^2/
(cos(f*x + e) + 1)^2 - 2*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + a*(c
os(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + b*(cos(f*x + e) - 1)^3/(cos(f*x +
 e) + 1)^3)^2))/f
```

Mupad [B] (verification not implemented)

Time = 23.78 (sec) , antiderivative size = 5613, normalized size of antiderivative = 21.84

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] (atan(((((cos(e + f*x)*(9*a*b^6 - 180*a^6*b + 9*a^7 - 180*a^2*b^5 + 1215*a^
3*b^4 - 1800*a^4*b^3 + 1215*a^5*b^2))/(32*(8*a*b^7 + 8*a^7*b + a^8 + b^8 +
28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2)) + (3*(-a*b
)^(1/2)*((6*a^13*b - 6*a^2*b^12 - 54*a^3*b^11 - 210*a^4*b^10 - 450*a^5*b^9
- 540*a^6*b^8 - 252*a^7*b^7 + 252*a^8*b^6 + 540*a^9*b^5 + 450*a^10*b^4 + 21
0*a^11*b^3 + 54*a^12*b^2)/(12*a*b^11 + 12*a^11*b + a^12 + b^12 + 66*a^2*b^1
0 + 220*a^3*b^9 + 495*a^4*b^8 + 792*a^5*b^7 + 924*a^6*b^6 + 792*a^7*b^5 + 4
95*a^8*b^4 + 220*a^9*b^3 + 66*a^10*b^2) - (3*cos(e + f*x)*(-a*b)^(1/2)*(5*a
^2 - 10*a*b + b^2)*(2304*a^12*b + 256*a^13 - 256*a^2*b^11 - 2304*a^3*b^10 -
 8960*a^4*b^9 - 19200*a^5*b^8 - 23040*a^6*b^7 - 10752*a^7*b^6 + 10752*a^8*b
^5 + 23040*a^9*b^4 + 19200*a^10*b^3 + 8960*a^11*b^2))/(512*(a*b^5 + 5*a^5*b
 + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^4*b^2)*(8*a*b^7 + 8*a^7*b + a^8 + b^
8 + 28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2)))*(5*a^
2 - 10*a*b + b^2))/(16*(a*b^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10
*a^4*b^2)))*(-a*b)^(1/2)*(5*a^2 - 10*a*b + b^2)*3i)/(16*(a*b^5 + 5*a^5*b +
a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^4*b^2)) + (((cos(e + f*x)*(9*a*b^6 - 18
0*a^6*b + 9*a^7 - 180*a^2*b^5 + 1215*a^3*b^4 - 1800*a^4*b^3 + 1215*a^5*b^2)
)/(32*(8*a*b^7 + 8*a^7*b + a^8 + b^8 + 28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4
 + 56*a^5*b^3 + 28*a^6*b^2)) - (3*(-a*b)^(1/2)*((6*a^13*b - 6*a^2*b^12 - 54
*a^3*b^11 - 210*a^4*b^10 - 450*a^5*b^9 - 540*a^6*b^8 - 252*a^7*b^7 + 252*a^
8*b^6 + 540*a^9*b^5 + 450*a^10*b^4 + 210*a^11*b^3 + 54*a^12*b^2)/(12*a*b^11
 + 12*a^11*b + a^12 + b^12 + 66*a^2*b^10 + 220*a^3*b^9 + 495*a^4*b^8 + 792*
a^5*b^7 + 924*a^6*b^6 + 792*a^7*b^5 + 495*a^8*b^4 + 220*a^9*b^3 + 66*a^10*b
^2) + (3*cos(e + f*x)*(-a*b)^(1/2)*(5*a^2 - 10*a*b + b^2)*(2304*a^12*b + 25
6*a^13 - 256*a^2*b^11 - 2304*a^3*b^10 - 8960*a^4*b^9 - 19200*a^5*b^8 - 2304
```

0*a^6*b^7 - 10752*a^7*b^6 + 10752*a^8*b^5 + 23040*a^9*b^4 + 19200*a^10*b^3 + 8960*a^11*b^2))/(512*(a*b^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^4*b^2)*(8*a*b^7 + 8*a^7*b + a^8 + b^8 + 28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2)))*(5*a^2 - 10*a*b + b^2))/(16*(a*b^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^4*b^2)))*(-a*b)^(1/2)*(5*a^2 - 10*a*b + b^2)*3i)/(16*(a*b^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^4*b^2)))/(((135*a*b^7)/256 + (135*a^7*b)/256 - (1215*a^2*b^6)/128 + (13257*a^3*b^5)/256 - (5913*a^4*b^4)/64 + (13257*a^5*b^3)/256 - (1215*a^6*b^2)/128)/(12*a*b^11 + 12*a^11*b + a^12 + b^12 + 66*a^2*b^10 + 220*a^3*b^9 + 495*a^4*b^8 + 792*a^5*b^7 + 924*a^6*b^6 + 792*a^7*b^5 + 495*a^8*b^4 + 220*a^9*b^3 + 66*a^10*b^2) - (3*((cos(e + f*x)*(9*a*b^6 - 180*a^6*b + 9*a^7 - 180*a^2*b^5 + 1215*a^3*b^4 - 1800*a^4*b^3 + 1215*a^5*b^2))/(32*(8*a*b^7 + 8*a^7*b + a^8 + b^8 + 28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2)) + (3*(-a*b)^(1/2)*((6*a^13*b - 6*a^2*b^12 - 54*a^3*b^11 - 210*a^4*b^10 - 450*a^5*b^9 - 540*a^6*b^8 - 252*a^7*b^7 + 252*a^8*b^6 + 540*a^9*b^5 + 450*a^10*b^4 + 210*a^11*b^3 + 54*a^12*b^2)/(12*a*b^11 + 12*a^11*b + a^12 + b^12 + 66*a^2*b^10 + 220*a^3*b^9 + 495*a^4*b^8 + 792*a^5*b^7 + 924*a^6*b^6 + 792*a^7*b^5 + 495*a^8*b^4 + 220*a^9*b^3 + 66*a^10*b^2) - (3*cos(e + f*x)*(-a*b)^(1/2)*(5*a^2 - 10*a*b + b^2)*(2304*a^12*b + 256*a^13 - 256*a^2*b^11 - 2304*a^3*b^10 - 8960*a^4*b^9 - 19200*a^5*b^8 - 23040*a^6*b^7 - 10752*a^7*b^6 + 10752*a^8*b^5 + 23040*a^9*b^4 + 19200*a^10*b^3 + 8960*a^11*b^2))/(512*(a*b^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^4*b^2)*(8*a*b^7 + 8*a^7*b + a^8 + b^8 + 28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2)))*(5*a^2 - 10*a*b + b^2))/(16*(a*b^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^4*b^2)))*(-a*b)^(1/2)*(5*a^2 - 10*a*b + b^2))/(16*(a*b^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^4*b^2)) + (3*((cos(e + f*x)*(9*a*b^6 - 180*a^6*b + 9*a^7 - 180*a^2*b^5 + 1215*a^3*b^4 - 1800*a^4*b^3 + 1215*a^5*b^2))/(32*(8*a*b^7 + 8*a^7*b + a^8 + b^8 + 28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2)) - (3*(-a*b)^(1/2)*((6*a^13*b - 6*a^2*b^12 - 54*a^3*b^11 - 210*a^4*b^10 - 450*a^5*b^9 - 540*a^6*b^8 - 252*a^7*b^7 + 252*a^8*b^6 + 540*a^9*b^5 + 450*a^10*b^4 + 210*a^11*b^3 + 54*a^12*b^2)/(12*a*b^11 + 12*a^11*b + a^12 + b^12 + 66*a^2*b^10 + 220*a^3*b^9 + 495*a^4*b^8 + 792*a^5*b^7 + 924*a^6*b^6 + 792*a^7*b^5 + 495*a^8*b^4 + 220*a^9*b^3 + 66*a^10*b^2) + (3*cos(e + f*x)*(-a*b)^(1/2)*(5*a^2 - 10*a*b + b^2)*(2304*a^12*b + 256*a^13 - 256*a^2*b^11 - 2304*a^3*b^10 - 8960*a^4*b^9 - 19200*a^5*b^8 - 23040*a^6*b^7 - 10752*a^7*b^6 + 10752*a^8*b^5 + 23040*a^9*b^4 + 19200*a^10*b^3 + 8960*a^11*b^2))/(512*(a*b^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^4*b^2)*(8*a*b^7 + 8*a^7*b + a^8 + b^8 + 28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2)))*(5*a^2 - 10*a*b + b^2))/(16*(a*b^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^4*b^2)))*(-a*b)^(1/2) *(5*a^2 - 10*a*b + b^2))/(16*(a*b^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^4*b^2))))*(-a*b)^(1/2)*(5*a^2 - 10*a*b + b^2)*3i)/(8*f*(a*b^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^4*b^2)) - (atan(((((6*a^13*b - 6*a^2*b^12 - 54*a^3*b^11 - 210*a^4*b^10 - 450*a^5*b^9 - 540*a^6*b^8 - 252*a^7*b^7 + 252*a^8*b^6 + 540*a^9*b^5 + 450*a^10*b^4 + 210*a^11*b^3 + 54*a^12*

b^2)/(12*a*b^11 + 12*a^11*b + a^12 + b^12 + 66*a^2*b^10 + 220*a^3*b^9 + 495
*a^4*b^8 + 792*a^5*b^7 + 924*a^6*b^6 + 792*a^7*b^5 + 495*a^8*b^4 + 220*a^9*
b^3 + 66*a^10*b^2) - (cos(e + f*x)*(3/(16*(a + b)^3) - (9*b)/(4*(a + b)^4)
+ (3*b^2)/(a + b)^5)*(2304*a^12*b + 256*a^13 - 256*a^2*b^11 - 2304*a^3*b^10
 - 8960*a^4*b^9 - 19200*a^5*b^8 - 23040*a^6*b^7 - 10752*a^7*b^6 + 10752*a^8
*b^5 + 23040*a^9*b^4 + 19200*a^10*b^3 + 8960*a^11*b^2))/(32*(8*a*b^7 + 8*a^
7*b + a^8 + b^8 + 28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^
6*b^2)))*(3/(16*(a + b)^3) - (9*b)/(4*(a + b)^4) + (3*b^2)/(a + b)^5) + (co
s(e + f*x)*(9*a*b^6 - 180*a^6*b + 9*a^7 - 180*a^2*b^5 + 1215*a^3*b^4 - 1800
*a^4*b^3 + 1215*a^5*b^2))/(32*(8*a*b^7 + 8*a^7*b + a^8 + b^8 + 28*a^2*b^6 +
 56*a^3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2)))*(3/(16*(a + b)^3) - (
9*b)/(4*(a + b)^4) + (3*b^2)/(a + b)^5)*1i - (((6*a^13*b - 6*a^2*b^12 - 54*
a^3*b^11 - 210*a^4*b^10 - 450*a^5*b^9 - 540*a^6*b^8 - 252*a^7*b^7 + 252*a^8
*b^6 + 540*a^9*b^5 + 450*a^10*b^4 + 210*a^11*b^3 + 54*a^12*b^2)/(12*a*b^11
+ 12*a^11*b + a^12 + b^12 + 66*a^2*b^10 + 220*a^3*b^9 + 495*a^4*b^8 + 792*a
^5*b^7 + 924*a^6*b^6 + 792*a^7*b^5 + 495*a^8*b^4 + 220*a^9*b^3 + 66*a^10*b^
2) + (cos(e + f*x)*(3/(16*(a + b)^3) - (9*b)/(4*(a + b)^4) + (3*b^2)/(a + b
)^5)*(2304*a^12*b + 256*a^13 - 256*a^2*b^11 - 2304*a^3*b^10 - 8960*a^4*b^9
- 19200*a^5*b^8 - 23040*a^6*b^7 - 10752*a^7*b^6 + 10752*a^8*b^5 + 23040*a^9
*b^4 + 19200*a^10*b^3 + 8960*a^11*b^2))/(32*(8*a*b^7 + 8*a^7*b + a^8 + b^8
+ 28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2)))*(3/(16*
(a + b)^3) - (9*b)/(4*(a + b)^4) + (3*b^2)/(a + b)^5) - (cos(e + f*x)*(9*a*
b^6 - 180*a^6*b + 9*a^7 - 180*a^2*b^5 + 1215*a^3*b^4 - 1800*a^4*b^3 + 1215*
a^5*b^2))/(32*(8*a*b^7 + 8*a^7*b + a^8 + b^8 + 28*a^2*b^6 + 56*a^3*b^5 + 70
*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2)))*(3/(16*(a + b)^3) - (9*b)/(4*(a + b)^
4) + (3*b^2)/(a + b)^5)*1i)/((((6*a^13*b - 6*a^2*b^12 - 54*a^3*b^11 - 210*a
^4*b^10 - 450*a^5*b^9 - 540*a^6*b^8 - 252*a^7*b^7 + 252*a^8*b^6 + 540*a^9*b
^5 + 450*a^10*b^4 + 210*a^11*b^3 + 54*a^12*b^2)/(12*a*b^11 + 12*a^11*b + a^
12 + b^12 + 66*a^2*b^10 + 220*a^3*b^9 + 495*a^4*b^8 + 792*a^5*b^7 + 924*a^6
*b^6 + 792*a^7*b^5 + 495*a^8*b^4 + 220*a^9*b^3 + 66*a^10*b^2) - (cos(e + f*
x)*(3/(16*(a + b)^3) - (9*b)/(4*(a + b)^4) + (3*b^2)/(a + b)^5)*(2304*a^12*
b + 256*a^13 - 256*a^2*b^11 - 2304*a^3*b^10 - 8960*a^4*b^9 - 19200*a^5*b^8
- 23040*a^6*b^7 - 10752*a^7*b^6 + 10752*a^8*b^5 + 23040*a^9*b^4 + 19200*a^1
0*b^3 + 8960*a^11*b^2))/(32*(8*a*b^7 + 8*a^7*b + a^8 + b^8 + 28*a^2*b^6 + 5
6*a^3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2)))*(3/(16*(a + b)^3) - (9*
b)/(4*(a + b)^4) + (3*b^2)/(a + b)^5) + (cos(e + f*x)*(9*a*b^6 - 180*a^6*b
+ 9*a^7 - 180*a^2*b^5 + 1215*a^3*b^4 - 1800*a^4*b^3 + 1215*a^5*b^2))/(32*(8
*a*b^7 + 8*a^7*b + a^8 + b^8 + 28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4 + 56*a^
5*b^3 + 28*a^6*b^2)))*(3/(16*(a + b)^3) - (9*b)/(4*(a + b)^4) + (3*b^2)/(a
+ b)^5) - ((135*a*b^7)/256 + (135*a^7*b)/256 - (1215*a^2*b^6)/128 + (13257*
a^3*b^5)/256 - (5913*a^4*b^4)/64 + (13257*a^5*b^3)/256 - (1215*a^6*b^2)/128
)/(12*a*b^11 + 12*a^11*b + a^12 + b^12 + 66*a^2*b^10 + 220*a^3*b^9 + 495*a^
4*b^8 + 792*a^5*b^7 + 924*a^6*b^6 + 792*a^7*b^5 + 495*a^8*b^4 + 220*a^9*b^3
 + 66*a^10*b^2) + (((6*a^13*b - 6*a^2*b^12 - 54*a^3*b^11 - 210*a^4*b^10 - 4
50*a^5*b^9 - 540*a^6*b^8 - 252*a^7*b^7 + 252*a^8*b^6 + 540*a^9*b^5 + 450*a^

```
10*b^4 + 210*a^11*b^3 + 54*a^12*b^2)/(12*a*b^11 + 12*a^11*b + a^12 + b^12 +
 66*a^2*b^10 + 220*a^3*b^9 + 495*a^4*b^8 + 792*a^5*b^7 + 924*a^6*b^6 + 792*
a^7*b^5 + 495*a^8*b^4 + 220*a^9*b^3 + 66*a^10*b^2) + (cos(e + f*x)*(3/(16*(
a + b)^3) - (9*b)/(4*(a + b)^4) + (3*b^2)/(a + b)^5)*(2304*a^12*b + 256*a^1
3 - 256*a^2*b^11 - 2304*a^3*b^10 - 8960*a^4*b^9 - 19200*a^5*b^8 - 23040*a^6
*b^7 - 10752*a^7*b^6 + 10752*a^8*b^5 + 23040*a^9*b^4 + 19200*a^10*b^3 + 896
0*a^11*b^2))/(32*(8*a*b^7 + 8*a^7*b + a^8 + b^8 + 28*a^2*b^6 + 56*a^3*b^5 +
 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2)))*(3/(16*(a + b)^3) - (9*b)/(4*(a +
b)^4) + (3*b^2)/(a + b)^5) - (cos(e + f*x)*(9*a*b^6 - 180*a^6*b + 9*a^7 - 1
80*a^2*b^5 + 1215*a^3*b^4 - 1800*a^4*b^3 + 1215*a^5*b^2))/(32*(8*a*b^7 + 8*
a^7*b + a^8 + b^8 + 28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*
a^6*b^2)))*(3/(16*(a + b)^3) - (9*b)/(4*(a + b)^4) + (3*b^2)/(a + b)^5)))*(
3i/(8*(a + b)^3) - (b*9i)/(2*(a + b)^4) + (b^2*6i)/(a + b)^5))/f - ((cos(e
+ f*x)^3*(19*a^2*b - 34*a*b^2 + 19*b^3))/(8*(4*a*b^3 + 4*a^3*b + a^4 + b^4
+ 6*a^2*b^2)) + (cos(e + f*x)^5*(31*a*b^2 - 31*a^2*b + 5*a^3 - 5*b^3))/(8*(
4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)) + (3*b^2*cos(e + f*x)*(a - b))/
(2*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)) - (3*a*cos(e + f*x)^7*(a^2
- 6*a*b + b^2))/(8*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)))/(f*(cos(e
+ f*x)^4*(a^2 - 4*a*b + b^2) + b^2 + cos(e + f*x)^6*(2*a*b - 2*a^2) + cos(e
 + f*x)^2*(2*a*b - 2*b^2) + a^2*cos(e + f*x)^8))
```

3.60 $\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 314

$$\begin{aligned}\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx &= \frac{5(a+2b)\,(a^2+16ab+16b^2)\,x}{16a^6} \\ &- \frac{5\sqrt{b}\sqrt{a+b}(a+4b)(3a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^6f} \\ &- \frac{(33a^2+110ab+80b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\,(a+b+b\tan^2(e+fx))^2} \\ &+ \frac{(9a+10b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\,(a+b+b\tan^2(e+fx))^2} \\ &+ \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\,(a+b+b\tan^2(e+fx))^2} \\ &- \frac{5b(9a^2+32ab+24b^2)\tan(e+fx)}{48a^4f\,(a+b+b\tan^2(e+fx))^2} \\ &- \frac{5b(5a^2+20ab+16b^2)\tan(e+fx)}{16a^5f\,(a+b+b\tan^2(e+fx))}\end{aligned}$$

```
[Out] 5/16*(a+2*b)*(a^2+16*a*b+16*b^2)*x/a^6-5/8*(a+4*b)*(3*a+4*b)*arctan(b^(1/2)
*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)*(a+b)^(1/2)/a^6/f-1/48*(33*a^2+110*a*b+80*
b^2)*cos(f*x+e)*sin(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)^2+1/24*(9*a+10*b)*cos
(f*x+e)^3*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2+1/6*cos(f*x+e)^3*sin(f*x+
e)^3/a/f/(a+b+b*tan(f*x+e)^2)^2-5/48*b*(9*a^2+32*a*b+24*b^2)*tan(f*x+e)/a^4
/f/(a+b+b*tan(f*x+e)^2)^2-5/16*b*(5*a^2+20*a*b+16*b^2)*tan(f*x+e)/a^5/f/(a+
b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4217, 481, 592, 541, 536, 209, 211}

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{5\sqrt{b}\sqrt{a+b}(a+4b)(3a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^6f} + \frac{(9a+10b)\sin(e+fx)\cos^3(e+fx)}{24a^2f\left(a+b\tan^2(e+fx)+b\right)^2} + \frac{5x(a+2b)\left(a^2+16ab+16b^2\right)}{16a^6} - \frac{5b(5a^2+20ab+16b^2)\tan(e+fx)}{16a^5f\left(a+b\tan^2(e+fx)+b\right)} - \frac{5b(9a^2+32ab+24b^2)\tan(e+fx)}{48a^4f\left(a+b\tan^2(e+fx)+b\right)^2} - \frac{(33a^2+110ab+80b^2)\sin(e+fx)\cos(e+fx)}{48a^3f\left(a+b\tan^2(e+fx)+b\right)^2} + \frac{\sin^3(e+fx)\cos^3(e+fx)}{6af\left(a+b\tan^2(e+fx)+b\right)^2}$$

```
[In] Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (5*(a + 2*b)*(a^2 + 16*a*b + 16*b^2)*x)/(16*a^6) - (5*Sqrt[b]*Sqrt[a + b]*(
a + 4*b)*(3*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*a^6*f)
- ((33*a^2 + 110*a*b + 80*b^2)*Cos[e + f*x]*Sin[e + f*x])/(48*a^3*f*(a + b
+ b*Tan[e + f*x]^2)^2) + ((9*a + 10*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*a^2
*f*(a + b + b*Tan[e + f*x]^2)^2) + (Cos[e + f*x]^3*Sin[e + f*x]^3)/(6*a*f*(
a + b + b*Tan[e + f*x]^2)^2) - (5*b*(9*a^2 + 32*a*b + 24*b^2)*Tan[e + f*x])
/(48*a^4*f*(a + b + b*Tan[e + f*x]^2)^2) - (5*b*(5*a^2 + 20*a*b + 16*b^2)*T
an[e + f*x])/(16*a^5*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^4(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\,(a+b+b\tan^2(e+fx))^2} - \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)+(-6a-7b)x^2)}{(1+x^2)^3(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{6af}$$

$$= \frac{(9a+10b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\,(a+b+b\tan^2(e+fx))^2} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\,(a+b+b\tan^2(e+fx))^2} - \frac{\text{Subst}\left(\int \frac{(a+b)(9a+10b)+(-24a^2-91ab-70b^2)x^2}{(1+x^2)^2(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{24a^2f}$$

$$= -\frac{(33a^2+110ab+80b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\,(a+b+b\tan^2(e+fx))^2} + \frac{(9a+10b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\,(a+b+b\tan^2(e+fx))^2} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\,(a+b+b\tan^2(e+fx))^2} + \frac{\text{Subst}\left(\int \frac{5(a+b)(3a^2+18ab+16b^2)-5b(33a^2+110ab+80b^2)x^2}{(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{48a^3f}$$

$$= -\frac{(33a^2+110ab+80b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\,(a+b+b\tan^2(e+fx))^2} + \frac{(9a+10b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\,(a+b+b\tan^2(e+fx))^2} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\,(a+b+b\tan^2(e+fx))^2} - \frac{5b(9a^2+32ab+24b^2)\tan(e+fx)}{48a^4f\,(a+b+b\tan^2(e+fx))^2} + \frac{\text{Subst}\left(\int \frac{60(a+b)^2(a^2+8ab+8b^2)-60b(a+b)(9a^2+32ab+24b^2)x^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{192a^4(a+b)f}$$

$$= -\frac{(33a^2+110ab+80b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\,(a+b+b\tan^2(e+fx))^2} + \frac{(9a+10b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\,(a+b+b\tan^2(e+fx))^2} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\,(a+b+b\tan^2(e+fx))^2} - \frac{5b(9a^2+32ab+24b^2)\tan(e+fx)}{48a^4f\,(a+b+b\tan^2(e+fx))^2} - \frac{5b(5a^2+20ab+16b^2)\tan(e+fx)}{16a^5f\,(a+b+b\tan^2(e+fx))} + \frac{\text{Subst}\left(\int \frac{120(a+b)^3(a^2+12ab+16b^2)-120b(a+b)^2(5a^2+20ab+16b^2)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{384a^5(a+b)^2f}$$

$$
\begin{aligned}
&= -\frac{(33a^2+110ab+80b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)^2} \\
&\quad + \frac{(9a+10b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^2} \\
&\quad - \frac{5b(9a^2+32ab+24b^2)\tan(e+fx)}{48a^4f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{5b(5a^2+20ab+16b^2)\tan(e+fx)}{16a^5f\left(a+b+b\tan^2(e+fx)\right)} \\
&\quad - \frac{(5b(a+b)(a+4b)(3a+4b))\mathrm{Subst}\left(\int\frac{1}{a+b+bx^2}\,dx,x,\tan(e+fx)\right)}{8a^6f} \\
&\quad + \frac{(5(a+2b)\left(a^2+16ab+16b^2\right))\,\mathrm{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\tan(e+fx)\right)}{16a^6f}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{5(a+2b)\left(a^2+16ab+16b^2\right)x}{16a^6} - \frac{5\sqrt{b}\sqrt{a+b}(a+4b)(3a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^6f} \\
&\quad - \frac{(33a^2+110ab+80b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)^2} \\
&\quad + \frac{(9a+10b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^2} \\
&\quad - \frac{5b(9a^2+32ab+24b^2)\tan(e+fx)}{48a^4f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{5b(5a^2+20ab+16b^2)\tan(e+fx)}{16a^5f\left(a+b+b\tan^2(e+fx)\right)}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 17.42 (sec) , antiderivative size = 1639, normalized size of antiderivative = 5.22

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{5(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(\frac{(3a^2+8ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{a\sqrt{b}(3a^2+16ab+16b^2+3a(a+2b)\cos(2(e+}{(a+b)^2(a+2b+a\cos(2(e+f}\right.}{65536b^{5/2}f\,(a+b\sec^2(e+fx))^3}$$
$$- \frac{15(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(-\frac{6a^2\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{}$$
$$+ \frac{3(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(-1536(a+2b)x - \frac{3(a^6-8a^5b+120a^4b^2+1280a^3b^3+3200a^2b^4+3072ab^5+1}{}\right.}{}$$
$$- \frac{(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(-6144(7a^3+54a^2b+120ab^2+80b^3)\,x - \frac{3(3a^8-64a^7b+2240a^6b^2+}{}\right.}{}$$

```
[In] Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (5*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((3*a^2 + 8*a*b + 8*b^2
)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*(3
*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/
((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(65536*b^(5/2)*f*(a + b*Sec[
e + f*x]^2)^3) - (15*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-6*a
^2*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[
2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*S
in[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (a*Sec[2*e]*((-9*a^
4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^3 + 64*b^4)*Sin[2*f*x] + a*(-3*a^3 + 2*
a^2*b + 24*a*b^2 + 16*b^3)*Sin[2*(e + 2*f*x)] + (3*a^4 - 64*a^2*b^2 - 128*a
*b^3 - 64*b^4)*Sin[4*e + 2*f*x]) + (9*a^5 + 18*a^4*b - 64*a^3*b^2 - 256*a^2
*b^3 - 320*a*b^4 - 128*b^5)*Tan[2*e])/(a^2*(a + 2*b + a*Cos[2*(e + f*x)])^2
)))/(262144*b^2*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3) + (3*(a + 2*b + a*Cos
[2*e + 2*f*x])^3*Sec[e + f*x]^6*(-1536*(a + 2*b)*x - (3*(a^6 - 8*a^5*b + 12
0*a^4*b^2 + 1280*a^3*b^3 + 3200*a^2*b^4 + 3072*a*b^5 + 1024*b^6)*ArcTan[(Se
c[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/
(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(b^
2*(a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (4*(a^4 + 32*a^3*b + 160
*a^2*b^2 + 256*a*b^3 + 128*b^4)*Sec[2*e]*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]
))/(b*(a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)])^2) + (256*a*Sin[2*(e + f*x)]
```

```
)/f + (a*(-3*a^5 + 26*a^4*b + 736*a^3*b^2 + 2624*a^2*b^3 + 3200*a*b^4 + 128
0*b^5)*Sec[2*e]*Sin[2*f*x] + (3*a^6 - 24*a^5*b - 920*a^4*b^2 - 4864*a^3*b^3
 - 10112*a^2*b^4 - 9216*a*b^5 - 3072*b^6)*Tan[2*e])/(b^2*(a + b)^2*f*(a + 2
*b + a*Cos[2*(e + f*x)]))))/(65536*a^4*(a + b*Sec[e + f*x]^2)^3) - ((a + 2*
b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(-6144*(7*a^3 + 54*a^2*b + 120*a*b
^2 + 80*b^3)*x - (3*(3*a^8 - 64*a^7*b + 2240*a^6*b^2 + 53760*a^5*b^3 + 3136
00*a^4*b^4 + 802816*a^3*b^5 + 1032192*a^2*b^6 + 655360*a*b^7 + 163840*b^8)*
ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e
 + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[
2*e]))/(b^2*(a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (12*(a^6 + 72*
a^5*b + 840*a^4*b^2 + 3584*a^3*b^3 + 6912*a^2*b^4 + 6144*a*b^5 + 2048*b^6)*
Sec[2*e]*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a + b)*f*(a + 2*b + a*Cos
[2*(e + f*x)])^2) + (1152*a*(7*a^2 + 32*a*b + 32*b^2)*((-I)*Cos[2*(e + f*x)
] + Sin[2*(e + f*x)]))/f + (1152*a*(7*a^2 + 32*a*b + 32*b^2)*(I*Cos[2*(e +
f*x)] + Sin[2*(e + f*x)]))/f + (192*a^2*(a + 2*b)*((-6*I)*Cos[4*(e + f*x)]
 - 6*Sin[4*(e + f*x)]))/f + ((1152*I)*a^2*(a + 2*b)*(Cos[4*(e + f*x)] + I*Si
n[4*(e + f*x)]))/f + (256*a^3*Sin[6*(e + f*x)])/f + (3*(3*a*(-a^7 + 22*a^6*
b + 1352*a^5*b^2 + 11312*a^4*b^3 + 37120*a^3*b^4 + 57856*a^2*b^5 + 43008*a*
b^6 + 12288*b^7)*Sec[2*e]*Sin[2*f*x] + (3*a^8 - 64*a^7*b - 4480*a^6*b^2 - 4
5696*a^5*b^3 - 196928*a^4*b^4 - 438272*a^3*b^5 - 528384*a^2*b^6 - 327680*a*
b^7 - 81920*b^8)*Tan[2*e]))/(b^2*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f*x)])
)))/(393216*a^6*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A] (verified)

Time = 11.70 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{\frac{\left(-\frac{27}{8}a^2b-3ab^2-\frac{11}{16}a^3\right)\tan(fx+e)^5+\left(-6a^2b-6ab^2-\frac{5}{6}a^3\right)\tan(fx+e)^3+\left(-\frac{5}{16}a^3-\frac{21}{8}a^2b-3ab^2\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^3}+\frac{5\left(a^3+18a^2b+48a\cdots\right.}{\cdots}}{a^6}$
default	$\frac{\frac{\left(-\frac{27}{8}a^2b-3ab^2-\frac{11}{16}a^3\right)\tan(fx+e)^5+\left(-6a^2b-6ab^2-\frac{5}{6}a^3\right)\tan(fx+e)^3+\left(-\frac{5}{16}a^3-\frac{21}{8}a^2b-3ab^2\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^3}+\frac{5\left(a^3+18a^2b+48a\cdots\right.}{\cdots}}{a^6}$
risch	$\frac{5x}{16a^3}+\frac{45xb}{8a^4}+\frac{15xb^2}{a^5}+\frac{10xb^3}{a^6}+\frac{15ie^{2i(fx+e)}}{128a^3f}+\frac{3ie^{-4i(fx+e)}b}{64a^4f}+\frac{3ie^{2i(fx+e)}b}{4a^4f}+\frac{3ie^{2i(fx+e)}b^2}{4a^5f}-\frac{ie^{-6i(f\cdots}}{384a\cdots}$

```
[In] int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^6*(((-27/8*a^2*b-3*a*b^2-11/16*a^3)*tan(f*x+e)^5+(-6*a^2*b-6*a*b^2
-5/6*a^3)*tan(f*x+e)^3+(-5/16*a^3-21/8*a^2*b-3*a*b^2)*tan(f*x+e))/(1+tan(f*
x+e)^2)^3+5/16*(a^3+18*a^2*b+48*a*b^2+32*b^3)*arctan(tan(f*x+e)))-(a+b)*b/a
^6*(((7/8*a^2*b+2*a*b^2)*tan(f*x+e)^3+1/8*a*(9*a^2+25*a*b+16*b^2)*tan(f*x+e
```

```
))/(a+b+b*tan(f*x+e)^2)^2+5/8*(3*a^2+16*a*b+16*b^2)/((a+b)*b)^(1/2)*arctan(
b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 930, normalized size of antiderivative = 2.96

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \left[\frac{30\,(a^5+18\,a^4b+48\,a^3b^2+32\,a^2b^3)fx\cos{(fx+e)}^4+60\,(a^4b+18\,a^3b^2+48\,a^2b^3+32\,ab^4)fx\cos{(fx+e}}{}\right.$$

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/96*(30*(a^5 + 18*a^4*b + 48*a^3*b^2 + 32*a^2*b^3)*f*x*cos(f*x + e)^4 + 6
0*(a^4*b + 18*a^3*b^2 + 48*a^2*b^3 + 32*a*b^4)*f*x*cos(f*x + e)^2 + 30*(a^3
*b^2 + 18*a^2*b^3 + 48*a*b^4 + 32*b^5)*f*x + 15*((3*a^4 + 16*a^3*b + 16*a^2
*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 16*a*b^3 + 16*b^4 + 2*(3*a^3*b + 16*a^2*
b^2 + 16*a*b^3)*cos(f*x + e)^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)
*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x +
 e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x +
 e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 2*(8*a^5*cos(f*x + e)^9 - 2*(13*a^5
+ 10*a^4*b)*cos(f*x + e)^7 + (33*a^5 + 110*a^4*b + 80*a^3*b^2)*cos(f*x + e)
^5 + 20*(6*a^4*b + 23*a^3*b^2 + 18*a^2*b^3)*cos(f*x + e)^3 + 15*(5*a^3*b^2
+ 20*a^2*b^3 + 16*a*b^4)*cos(f*x + e))*sin(f*x + e))/(a^8*f*cos(f*x + e)^4
+ 2*a^7*b*f*cos(f*x + e)^2 + a^6*b^2*f), 1/48*(15*(a^5 + 18*a^4*b + 48*a^3*
b^2 + 32*a^2*b^3)*f*x*cos(f*x + e)^4 + 30*(a^4*b + 18*a^3*b^2 + 48*a^2*b^3
+ 32*a*b^4)*f*x*cos(f*x + e)^2 + 15*(a^3*b^2 + 18*a^2*b^3 + 48*a*b^4 + 32*b
^5)*f*x + 15*((3*a^4 + 16*a^3*b + 16*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 +
16*a*b^3 + 16*b^4 + 2*(3*a^3*b + 16*a^2*b^2 + 16*a*b^3)*cos(f*x + e)^2)*sqr
t(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos
(f*x + e)*sin(f*x + e))) - (8*a^5*cos(f*x + e)^9 - 2*(13*a^5 + 10*a^4*b)*co
s(f*x + e)^7 + (33*a^5 + 110*a^4*b + 80*a^3*b^2)*cos(f*x + e)^5 + 20*(6*a^4
*b + 23*a^3*b^2 + 18*a^2*b^3)*cos(f*x + e)^3 + 15*(5*a^3*b^2 + 20*a^2*b^3 +
 16*a*b^4)*cos(f*x + e))*sin(f*x + e))/(a^8*f*cos(f*x + e)^4 + 2*a^7*b*f*co
s(f*x + e)^2 + a^6*b^2*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.33

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{15\,(5\,a^2b^2+20\,ab^3+16\,b^4)\tan(fx+e)^9+40\,(3\,a^3b+19\,a^2b^2+39\,ab^3+24\,b^4)\tan(fx+e)^7+(33\,a^4+470\,a^3b+1910\,a^2b^2+2880\,ab^3+1440\,b^4)\tan(f}{a^5b^2\tan(fx+e)^{10}+(2\,a^6b+5\,a^5b^2)\tan(fx+e)^8+a^7+2\,a^6b+a^5b^2+(a^7+8\,a^6b+10\,a^5b^2)\tan(fx+e)^6-}$$

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/48*((15*(5*a^2*b^2 + 20*a*b^3 + 16*b^4)*tan(f*x + e)^9 + 40*(3*a^3*b + 1
9*a^2*b^2 + 39*a*b^3 + 24*b^4)*tan(f*x + e)^7 + (33*a^4 + 470*a^3*b + 1910*
a^2*b^2 + 2880*a*b^3 + 1440*b^4)*tan(f*x + e)^5 + 40*(a^4 + 14*a^3*b + 46*a
^2*b^2 + 57*a*b^3 + 24*b^4)*tan(f*x + e)^3 + 15*(a^4 + 14*a^3*b + 41*a^2*b^
2 + 44*a*b^3 + 16*b^4)*tan(f*x + e))/(a^5*b^2*tan(f*x + e)^10 + (2*a^6*b +
5*a^5*b^2)*tan(f*x + e)^8 + a^7 + 2*a^6*b + a^5*b^2 + (a^7 + 8*a^6*b + 10*a
^5*b^2)*tan(f*x + e)^6 + (3*a^7 + 12*a^6*b + 10*a^5*b^2)*tan(f*x + e)^4 + (
3*a^7 + 8*a^6*b + 5*a^5*b^2)*tan(f*x + e)^2) - 15*(a^3 + 18*a^2*b + 48*a*b^
2 + 32*b^3)*(f*x + e)/a^6 + 30*(3*a^3*b + 19*a^2*b^2 + 32*a*b^3 + 16*b^4)*a
rctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^6))/f
```

Giac [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.12

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\frac{15\,(a^3+18\,a^2b+48\,ab^2+32\,b^3)(fx+e)}{a^6}-\frac{30\,(3\,a^3b+19\,a^2b^2+32\,ab^3+16\,b^4)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,a^6}-\frac{6\left(7\,a^2b^2\tan(fx+\right.}{}}{}$$

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/48*(15*(a^3 + 18*a^2*b + 48*a*b^2 + 32*b^3)*(f*x + e)/a^6 - 30*(3*a^3*b +
 19*a^2*b^2 + 32*a*b^3 + 16*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arc
tan(b*tan(f*x + e)/sqrt(a*b + b^2)))/(sqrt(a*b + b^2)*a^6) - 6*(7*a^2*b^2*t
an(f*x + e)^3 + 23*a*b^3*tan(f*x + e)^3 + 16*b^4*tan(f*x + e)^3 + 9*a^3*b*t
an(f*x + e) + 34*a^2*b^2*tan(f*x + e) + 41*a*b^3*tan(f*x + e) + 16*b^4*tan(
f*x + e))/((b*tan(f*x + e)^2 + a + b)^2*a^5) - (33*a^2*tan(f*x + e)^5 + 162
*a*b*tan(f*x + e)^5 + 144*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 + 288*
a*b*tan(f*x + e)^3 + 288*b^2*tan(f*x + e)^3 + 15*a^2*tan(f*x + e) + 126*a*b
*tan(f*x + e) + 144*b^2*tan(f*x + e))/((tan(f*x + e)^2 + 1)^3*a^5))/f
```

Mupad [B] (verification not implemented)

Time = 22.33 (sec) , antiderivative size = 2117, normalized size of antiderivative = 6.74

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] (5*atan(((5*((tan(e + f*x)*(179200*a*b^8 + 51200*b^9 + 249600*a^2*b^7 + 176
000*a^3*b^6 + 65800*a^4*b^5 + 12300*a^5*b^4 + 925*a^6*b^3))/(128*a^10) - ((
(20*a^12*b^5 + 35*a^13*b^4 + (65*a^14*b^3)/4 + (5*a^15*b^2)/4)/a^15 - (tan(
e + f*x)*(2048*a^12*b^3 + 1024*a^13*b^2)*(a + 2*b)*(16*a*b + a^2 + 16*b^2)*
5i)/(4096*a^16))*(a + 2*b)*(16*a*b + a^2 + 16*b^2)*5i)/(32*a^6))*(a + 2*b)*
(16*a*b + a^2 + 16*b^2))/(32*a^6) + (5*((tan(e + f*x)*(179200*a*b^8 + 51200
*b^9 + 249600*a^2*b^7 + 176000*a^3*b^6 + 65800*a^4*b^5 + 12300*a^5*b^4 + 92
5*a^6*b^3))/(128*a^10) + (((20*a^12*b^5 + 35*a^13*b^4 + (65*a^14*b^3)/4 + (
5*a^15*b^2)/4)/a^15 + (tan(e + f*x)*(2048*a^12*b^3 + 1024*a^13*b^2)*(a + 2*
b)*(16*a*b + a^2 + 16*b^2)*5i)/(4096*a^16))*(a + 2*b)*(16*a*b + a^2 + 16*b^
2)*5i)/(32*a^6))*(a + 2*b)*(16*a*b + a^2 + 16*b^2))/(32*a^6))/((4750*a*b^10
 + 1000*b^11 + (18875*a^2*b^9)/2 + (40625*a^3*b^8)/4 + (204875*a^4*b^7)/32
+ (305125*a^5*b^6)/128 + (256125*a^6*b^5)/512 + (53125*a^7*b^4)/1024 + (187
5*a^8*b^3)/1024)/a^15 - (((tan(e + f*x)*(179200*a*b^8 + 51200*b^9 + 249600*
a^2*b^7 + 176000*a^3*b^6 + 65800*a^4*b^5 + 12300*a^5*b^4 + 925*a^6*b^3))/(1
28*a^10) - (((20*a^12*b^5 + 35*a^13*b^4 + (65*a^14*b^3)/4 + (5*a^15*b^2)/4)
/a^15 - (tan(e + f*x)*(2048*a^12*b^3 + 1024*a^13*b^2)*(a + 2*b)*(16*a*b + a
^2 + 16*b^2)*5i)/(4096*a^16))*(a + 2*b)*(16*a*b + a^2 + 16*b^2)*5i)/(32*a^6
))*(a + 2*b)*(16*a*b + a^2 + 16*b^2)*5i)/(32*a^6) + (((tan(e + f*x)*(179200
*a*b^8 + 51200*b^9 + 249600*a^2*b^7 + 176000*a^3*b^6 + 65800*a^4*b^5 + 1230
0*a^5*b^4 + 925*a^6*b^3))/(128*a^10) + (((20*a^12*b^5 + 35*a^13*b^4 + (65*a
^14*b^3)/4 + (5*a^15*b^2)/4)/a^15 + (tan(e + f*x)*(2048*a^12*b^3 + 1024*a^1
3*b^2)*(a + 2*b)*(16*a*b + a^2 + 16*b^2)*5i)/(4096*a^16))*(a + 2*b)*(16*a*b
 + a^2 + 16*b^2)*5i)/(32*a^6))*(a + 2*b)*(16*a*b + a^2 + 16*b^2)*5i)/(32*a^
```

6)))*(a + 2*b)*(16*a*b + a^2 + 16*b^2))/(16*a^6*f) - ((5*tan(e + f*x)^3*(57
*a*b^3 + 14*a^3*b + a^4 + 24*b^4 + 46*a^2*b^2))/(6*a^5) + (5*tan(e + f*x)^7
*(39*a*b^3 + 3*a^3*b + 24*b^4 + 19*a^2*b^2))/(6*a^5) + (tan(e + f*x)^5*(288
0*a*b^3 + 470*a^3*b + 33*a^4 + 1440*b^4 + 1910*a^2*b^2))/(48*a^5) + (5*tan(
e + f*x)*(44*a*b^3 + 14*a^3*b + a^4 + 16*b^4 + 41*a^2*b^2))/(16*a^5) + (5*b
*tan(e + f*x)^9*(20*a*b^2 + 5*a^2*b + 16*b^3))/(16*a^5))/(f*(2*a*b + tan(e
+ f*x)^6*(8*a*b + a^2 + 10*b^2) + a^2 + b^2 + tan(e + f*x)^8*(2*a*b + 5*b^2
) + b^2*tan(e + f*x)^10 + tan(e + f*x)^2*(8*a*b + 3*a^2 + 5*b^2) + tan(e +
f*x)^4*(12*a*b + 3*a^2 + 10*b^2))) + (atan((((a + 4*b)*(3*a + 4*b)*(-b*(a +
b))^(1/2)*((tan(e + f*x)*(179200*a*b^8 + 51200*b^9 + 249600*a^2*b^7 + 1760
00*a^3*b^6 + 65800*a^4*b^5 + 12300*a^5*b^4 + 925*a^6*b^3))/(128*a^10) - (5*
(a + 4*b)*((20*a^12*b^5 + 35*a^13*b^4 + (65*a^14*b^3)/4 + (5*a^15*b^2)/4)/a
^15 - (5*tan(e + f*x)*(2048*a^12*b^3 + 1024*a^13*b^2)*(a + 4*b)*(3*a + 4*b)
(-b(a + b))^(1/2))/(2048*a^16))*(3*a + 4*b)*(-b*(a + b))^(1/2))/(16*a^6))
*5i)/(16*a^6) + ((a + 4*b)*(3*a + 4*b)*(-b*(a + b))^(1/2)*((tan(e + f*x)*(1
79200*a*b^8 + 51200*b^9 + 249600*a^2*b^7 + 176000*a^3*b^6 + 65800*a^4*b^5 +
12300*a^5*b^4 + 925*a^6*b^3))/(128*a^10) + (5*(a + 4*b)*((20*a^12*b^5 + 35
*a^13*b^4 + (65*a^14*b^3)/4 + (5*a^15*b^2)/4)/a^15 + (5*tan(e + f*x)*(2048*
a^12*b^3 + 1024*a^13*b^2)*(a + 4*b)*(3*a + 4*b)*(-b*(a + b))^(1/2))/(2048*a
^16))*(3*a + 4*b)*(-b*(a + b))^(1/2))/(16*a^6))*5i)/(16*a^6))/((4750*a*b^10
+ 1000*b^11 + (18875*a^2*b^9)/2 + (40625*a^3*b^8)/4 + (204875*a^4*b^7)/32
+ (305125*a^5*b^6)/128 + (256125*a^6*b^5)/512 + (53125*a^7*b^4)/1024 + (187
5*a^8*b^3)/1024)/a^15 - (5*(a + 4*b)*(3*a + 4*b)*(-b*(a + b))^(1/2)*((tan(e
+ f*x)*(179200*a*b^8 + 51200*b^9 + 249600*a^2*b^7 + 176000*a^3*b^6 + 65800
*a^4*b^5 + 12300*a^5*b^4 + 925*a^6*b^3))/(128*a^10) - (5*(a + 4*b)*((20*a^1
2*b^5 + 35*a^13*b^4 + (65*a^14*b^3)/4 + (5*a^15*b^2)/4)/a^15 - (5*tan(e + f
x)(2048*a^12*b^3 + 1024*a^13*b^2)*(a + 4*b)*(3*a + 4*b)*(-b*(a + b))^(1/2
))/(2048*a^16))*(3*a + 4*b)*(-b*(a + b))^(1/2))/(16*a^6)))/(16*a^6) + (5*(a
+ 4*b)*(3*a + 4*b)*(-b*(a + b))^(1/2)*((tan(e + f*x)*(179200*a*b^8 + 51200
*b^9 + 249600*a^2*b^7 + 176000*a^3*b^6 + 65800*a^4*b^5 + 12300*a^5*b^4 + 92
5*a^6*b^3))/(128*a^10) + (5*(a + 4*b)*((20*a^12*b^5 + 35*a^13*b^4 + (65*a^1
4*b^3)/4 + (5*a^15*b^2)/4)/a^15 + (5*tan(e + f*x)*(2048*a^12*b^3 + 1024*a^1
3*b^2)*(a + 4*b)*(3*a + 4*b)*(-b*(a + b))^(1/2))/(2048*a^16))*(3*a + 4*b)*(
-b*(a + b))^(1/2))/(16*a^6)))/(16*a^6)))*(a + 4*b)*(3*a + 4*b)*(-b*(a + b))
^(1/2)*5i)/(8*a^6*f)

3.61 $\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 238

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{3(a^2+12ab+16b^2)\,x}{8a^5} - \frac{3\sqrt{b}(5a^2+20ab+16b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^5\sqrt{a+b}f} - \frac{(5a+8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+12b)\tan(e+fx)}{8a^3f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{3b(a+2b)\tan(e+fx)}{2a^4f\left(a+b+b\tan^2(e+fx)\right)}$$

```
[Out] 3/8*(a^2+12*a*b+16*b^2)*x/a^5-3/8*(5*a^2+20*a*b+16*b^2)*arctan(b^(1/2)*tan(
f*x+e)/(a+b)^(1/2))*b^(1/2)/a^5/f/(a+b)^(1/2)-1/8*(5*a+8*b)*cos(f*x+e)*sin(
f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*
tan(f*x+e)^2)^2-1/8*b*(7*a+12*b)*tan(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)^2-3/
2*b*(a+2*b)*tan(f*x+e)/a^4/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4217, 481, 541, 536, 209, 211}

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{3b(a+2b)\tan(e+fx)}{2a^4f\,(a+b\tan^2(e+fx)+b)} - \frac{b(7a+12b)\tan(e+fx)}{8a^3f\,(a+b\tan^2(e+fx)+b)^2} - \frac{(5a+8b)\sin(e+fx)\cos(e+fx)}{8a^2f\,(a+b\tan^2(e+fx)+b)^2} - \frac{3\sqrt{b}(5a^2+20ab+16b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^5f\sqrt{a+b}} + \frac{3x(a^2+12ab+16b^2)}{8a^5} + \frac{\sin(e+fx)\cos^3(e+fx)}{4af\,(a+b\tan^2(e+fx)+b)^2}$$

```
[In] Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (3*(a^2 + 12*a*b + 16*b^2)*x)/(8*a^5) - (3*Sqrt[b]*(5*a^2 + 20*a*b + 16*b^2
)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*a^5*Sqrt[a + b]*f) - ((5*a
 + 8*b)*Cos[e + f*x]*Sin[e + f*x])/(8*a^2*f*(a + b + b*Tan[e + f*x]^2)^2) +
 (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f*(a + b + b*Tan[e + f*x]^2)^2) - (b*(7
*a + 12*b)*Tan[e + f*x])/(8*a^3*f*(a + b + b*Tan[e + f*x]^2)^2) - (3*b*(a +
 2*b)*Tan[e + f*x])/(2*a^4*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
```

```
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^3(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))^2} - \frac{\text{Subst}\left(\int \frac{a+b+(-4a-7b)x^2}{(1+x^2)^2(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{4af}\\
&= -\frac{(5a+8b)\cos(e+fx)\sin(e+fx)}{8a^2f\,(a+b+b\tan^2(e+fx))^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))^2}\\
&\quad + \frac{\text{Subst}\left(\int \frac{(a+b)(3a+8b)-5b(5a+8b)x^2}{(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{8a^2f}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{(5a+8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} \\
&\quad + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+12b)\tan(e+fx)}{8a^3f\left(a+b+b\tan^2(e+fx)\right)^2} \\
&\quad + \frac{\text{Subst}\left(\int \frac{12(a+b)^2(a+4b)-12b(a+b)(7a+12b)x^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{32a^3(a+b)f}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{(5a+8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} \\
&\quad - \frac{b(7a+12b)\tan(e+fx)}{8a^3f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{3b(a+2b)\tan(e+fx)}{2a^4f\left(a+b+b\tan^2(e+fx)\right)} \\
&\quad + \frac{\text{Subst}\left(\int \frac{24(a+b)^2(a^2+8ab+8b^2)-96b(a+b)^2(a+2b)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{64a^4(a+b)^2f}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{(5a+8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} \\
&\quad - \frac{b(7a+12b)\tan(e+fx)}{8a^3f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{3b(a+2b)\tan(e+fx)}{2a^4f\left(a+b+b\tan^2(e+fx)\right)} \\
&\quad + \frac{(3(a^2+12ab+16b^2))\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{8a^5f} \\
&\quad - \frac{(3b(5a^2+20ab+16b^2))\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{8a^5f}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3(a^2+12ab+16b^2)\,x}{8a^5} - \frac{3\sqrt{b}(5a^2+20ab+16b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^5\sqrt{a+b}f} \\
&\quad - \frac{(5a+8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} \\
&\quad - \frac{b(7a+12b)\tan(e+fx)}{8a^3f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{3b(a+2b)\tan(e+fx)}{2a^4f\left(a+b+b\tan^2(e+fx)\right)}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 21.95 (sec) , antiderivative size = 2469, normalized size of antiderivative = 10.37

$$
\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (3*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((3*a^2 + 8*a*b + 8*b^2
)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*(3
*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/
((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(16384*b^(5/2)*f*(a + b*Sec[
e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-3*a*(a
 + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + (Sqrt[b
]*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*Cos[2*(
e + f*x)])*Sin[2*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))
/(16384*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3) - (3*(a + 2*b + a*Cos[2*e + 2*f
*x])^3*Sec[e + f*x]^6*((2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 64
0*a*b^4 + 256*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Si
n[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]
*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (Se
c[2*e]*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*f*x*Cos[2*e] + 512*a*b^2*
(a + b)^2*(a + 2*b)*f*x*Cos[2*f*x] + 128*a^4*b^2*f*x*Cos[2*(e + 2*f*x)] + 2
56*a^3*b^3*f*x*Cos[2*(e + 2*f*x)] + 128*a^2*b^4*f*x*Cos[2*(e + 2*f*x)] + 51
2*a^4*b^2*f*x*Cos[4*e + 2*f*x] + 2048*a^3*b^3*f*x*Cos[4*e + 2*f*x] + 2560*a
^2*b^4*f*x*Cos[4*e + 2*f*x] + 1024*a*b^5*f*x*Cos[4*e + 2*f*x] + 128*a^4*b^2
*f*x*Cos[6*e + 4*f*x] + 256*a^3*b^3*f*x*Cos[6*e + 4*f*x] + 128*a^2*b^4*f*x*
Cos[6*e + 4*f*x] - 9*a^6*Sin[2*e] + 12*a^5*b*Sin[2*e] + 684*a^4*b^2*Sin[2*e
] + 2880*a^3*b^3*Sin[2*e] + 5280*a^2*b^4*Sin[2*e] + 4608*a*b^5*Sin[2*e] + 1
536*b^6*Sin[2*e] + 9*a^6*Sin[2*f*x] - 14*a^5*b*Sin[2*f*x] - 608*a^4*b^2*Sin
[2*f*x] - 2112*a^3*b^3*Sin[2*f*x] - 2560*a^2*b^4*Sin[2*f*x] - 1024*a*b^5*Si
n[2*f*x] + 3*a^6*Sin[2*(e + 2*f*x)] - 12*a^5*b*Sin[2*(e + 2*f*x)] - 204*a^4
*b^2*Sin[2*(e + 2*f*x)] - 384*a^3*b^3*Sin[2*(e + 2*f*x)] - 192*a^2*b^4*Sin[
2*(e + 2*f*x)] - 3*a^6*Sin[4*e + 2*f*x] + 10*a^5*b*Sin[4*e + 2*f*x] + 304*a
^4*b^2*Sin[4*e + 2*f*x] + 1056*a^3*b^3*Sin[4*e + 2*f*x] + 1280*a^2*b^4*Sin[
4*e + 2*f*x] + 512*a*b^5*Sin[4*e + 2*f*x]))/(a + 2*b + a*Cos[2*(e + f*x)])^
2))/(65536*a^3*b^2*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3) - ((a + 2*b + a*Co
s[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-6*a^2*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Si
n[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(
Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e
] - I*Sin[e])^4]) + (a*Sec[2*e]*((-9*a^4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^
3 + 64*b^4)*Sin[2*f*x] + a*(-3*a^3 + 2*a^2*b + 24*a*b^2 + 16*b^3)*Sin[2*(e
+ 2*f*x)] + (3*a^4 - 64*a^2*b^2 - 128*a*b^3 - 64*b^4)*Sin[4*e + 2*f*x]) + (
9*a^5 + 18*a^4*b - 64*a^3*b^2 - 256*a^2*b^3 - 320*a*b^4 - 128*b^5)*Tan[2*e]
)/(a^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(8192*b^2*(a + b)^2*f*(a + b*Sec
[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(-1536*(
a + 2*b)*x - (3*(a^6 - 8*a^5*b + 120*a^4*b^2 + 1280*a^3*b^3 + 3200*a^2*b^4
+ 3072*a*b^5 + 1024*b^6)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2
*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e]
)^4])]*(Cos[2*e] - I*Sin[2*e]))/(b^2*(a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin
[e])^4]) + (4*(a^4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4)*Sec[2*e]
*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a + b)*f*(a + 2*b + a*Cos[2*(e +
f*x)])^2) + (256*a*Sin[2*(e + f*x)])/f + (a*(-3*a^5 + 26*a^4*b + 736*a^3*b^
```

```
2 + 2624*a^2*b^3 + 3200*a*b^4 + 1280*b^5)*Sec[2*e]*Sin[2*f*x] + (3*a^6 - 24
*a^5*b - 920*a^4*b^2 - 4864*a^3*b^3 - 10112*a^2*b^4 - 9216*a*b^5 - 3072*b^6
)*Tan[2*e])/(b^2*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f*x)]))))/(16384*a^4*(
a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6
*(768*(7*a^2 + 32*a*b + 32*b^2)*x + (3*(a^7 - 14*a^6*b + 336*a^5*b^2 + 5600
*a^4*b^3 + 22400*a^3*b^4 + 37632*a^2*b^5 + 28672*a*b^6 + 8192*b^7)*ArcTan[(
Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x])
)/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(
b^2*(a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) - (4*(a^5 + 50*a^4*b + 4
00*a^3*b^2 + 1120*a^2*b^3 + 1280*a*b^4 + 512*b^5)*Sec[2*e]*((a + 2*b)*Sin[2
*e] - a*Sin[2*f*x]))/(b*(a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)])^2) - ((768
*I)*a*(a + 2*b)*(Cos[2*(e + f*x)] - I*Sin[2*(e + f*x)]))/f + ((768*I)*a*(a
+ 2*b)*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]))/f + (128*a^2*Sin[4*(e + f*x
)])/f + (a*(3*a^6 - 44*a^5*b - 1900*a^4*b^2 - 10880*a^3*b^3 - 23360*a^2*b^4
 - 21504*a*b^5 - 7168*b^6)*Sec[2*e]*Sin[2*f*x] + (-3*a^7 + 42*a^6*b + 2192*
a^5*b^2 + 16480*a^4*b^3 + 51200*a^3*b^4 + 77824*a^2*b^5 + 57344*a*b^6 + 163
84*b^7)*Tan[2*e])/(b^2*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f*x)]))))/(32768
*a^5*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A] (verified)

Time = 6.37 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{\frac{\frac{\left(-\frac{3}{2}ab-\frac{5}{8}a^2\right)\tan(fx+e)^3+\left(-\frac{3}{8}a^2-\frac{3}{2}ab\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{3\left(a^2+12ab+16b^2\right)\arctan(\tan(fx+e))}{8}}{a^5}-\frac{b\left(\frac{\left(\frac{7}{8}a^2b+\frac{3}{2}ab^2\right)\tan(fx+e)^3+\frac{3}{\cdots}}{\left(a+b+b\tan(\cdots\right.}\right.}{\cdots}}{f}$
default	$\frac{\frac{\frac{\left(-\frac{3}{2}ab-\frac{5}{8}a^2\right)\tan(fx+e)^3+\left(-\frac{3}{8}a^2-\frac{3}{2}ab\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{3\left(a^2+12ab+16b^2\right)\arctan(\tan(fx+e))}{8}}{a^5}-\frac{b\left(\frac{\left(\frac{7}{8}a^2b+\frac{3}{2}ab^2\right)\tan(fx+e)^3+\frac{3}{\cdots}}{\left(a+b+b\tan(\cdots\right.}\right.}{\cdots}}{f}$
risch	$\frac{3x}{8a^3}+\frac{9xb}{2a^4}+\frac{6x\,b^2}{a^5}-\frac{ib\left(9a^3e^{6i(fx+e)}+36a^2b\,e^{6i(fx+e)}+32a\,b^2e^{6i(fx+e)}+27a^3e^{4i(fx+e)}+114a^2b\,e^{4i(fx+e)}+184a\cdots\right.}{4a^5f\left(a\,e^{4i(fx+e)}+2a\,e^{2i\cdots}\right.}$

```
[In] int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^5*(((-3/2*a*b-5/8*a^2)*tan(f*x+e)^3+(-3/8*a^2-3/2*a*b)*tan(f*x+e))
/(1+tan(f*x+e)^2)^2+3/8*(a^2+12*a*b+16*b^2)*arctan(tan(f*x+e)))-b/a^5*(((7/
8*a^2*b+3/2*a*b^2)*tan(f*x+e)^3+3/8*a*(3*a^2+7*a*b+4*b^2)*tan(f*x+e))/(a+b+
b*tan(f*x+e)^2)^2+3/8*(5*a^2+20*a*b+16*b^2)/((a+b)*b)^(1/2)*arctan(b*tan(f*
x+e)/((a+b)*b)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 803, normalized size of antiderivative = 3.37

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \left[\frac{12\,(a^4+12\,a^3b+16\,a^2b^2)fx\cos{(fx+e)}^4+24\,(a^3b+12\,a^2b^2+16\,ab^3)fx\cos{(fx+e)}^2+12\,(a^2b^2+12\,a\right.$$

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(12*(a^4 + 12*a^3*b + 16*a^2*b^2)*f*x*cos(f*x + e)^4 + 24*(a^3*b + 12
*a^2*b^2 + 16*a*b^3)*f*x*cos(f*x + e)^2 + 12*(a^2*b^2 + 12*a*b^3 + 16*b^4)*
f*x + 3*((5*a^4 + 20*a^3*b + 16*a^2*b^2)*cos(f*x + e)^4 + 5*a^2*b^2 + 20*a*
b^3 + 16*b^4 + 2*(5*a^3*b + 20*a^2*b^2 + 16*a*b^3)*cos(f*x + e)^2)*sqrt(-b/
(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(
f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x
+ e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(
f*x + e)^2 + b^2)) + 4*(2*a^4*cos(f*x + e)^7 - (5*a^4 + 8*a^3*b)*cos(f*x +
e)^5 - (19*a^3*b + 36*a^2*b^2)*cos(f*x + e)^3 - 12*(a^2*b^2 + 2*a*b^3)*cos(
f*x + e))*sin(f*x + e))/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*f*cos(f*x + e)^2 +
a^5*b^2*f), 1/16*(6*(a^4 + 12*a^3*b + 16*a^2*b^2)*f*x*cos(f*x + e)^4 + 12*(
a^3*b + 12*a^2*b^2 + 16*a*b^3)*f*x*cos(f*x + e)^2 + 6*(a^2*b^2 + 12*a*b^3 +
 16*b^4)*f*x + 3*((5*a^4 + 20*a^3*b + 16*a^2*b^2)*cos(f*x + e)^4 + 5*a^2*b^
2 + 20*a*b^3 + 16*b^4 + 2*(5*a^3*b + 20*a^2*b^2 + 16*a*b^3)*cos(f*x + e)^2)
*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/
(b*cos(f*x + e)*sin(f*x + e))) + 2*(2*a^4*cos(f*x + e)^7 - (5*a^4 + 8*a^3*b
)*cos(f*x + e)^5 - (19*a^3*b + 36*a^2*b^2)*cos(f*x + e)^3 - 12*(a^2*b^2 + 2
*a*b^3)*cos(f*x + e))*sin(f*x + e))/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*f*cos(f
*x + e)^2 + a^5*b^2*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.26

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx =$$

$$-\frac{\frac{12\,(ab^2+2\,b^3)\tan(fx+e)^7+(19\,a^2b+72\,ab^2+72\,b^3)\tan(fx+e)^5+(5\,a^3+46\,a^2b+108\,ab^2+72\,b^3)\tan(fx+e)^3+3\,(a^3+9\,a^2b+16\,ab^2+8\,b^3)\tan(}{a^4b^2\tan(fx+e)^8+2\,(a^5b+2\,a^4b^2)\tan(fx+e)^6+a^6+2\,a^5b+a^4b^2+(a^6+6\,a^5b+6\,a^4b^2)\tan(fx+e)^4+2\,(a^6+3\,a^5b+2\,a^4b^2)\tan(fx+e)^2}}{8\,f}$$

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((12*(a*b^2 + 2*b^3)*tan(f*x + e)^7 + (19*a^2*b + 72*a*b^2 + 72*b^3)*t
an(f*x + e)^5 + (5*a^3 + 46*a^2*b + 108*a*b^2 + 72*b^3)*tan(f*x + e)^3 + 3*
(a^3 + 9*a^2*b + 16*a*b^2 + 8*b^3)*tan(f*x + e))/(a^4*b^2*tan(f*x + e)^8 +
2*(a^5*b + 2*a^4*b^2)*tan(f*x + e)^6 + a^6 + 2*a^5*b + a^4*b^2 + (a^6 + 6*a
^5*b + 6*a^4*b^2)*tan(f*x + e)^4 + 2*(a^6 + 3*a^5*b + 2*a^4*b^2)*tan(f*x +
e)^2) - 3*(a^2 + 12*a*b + 16*b^2)*(f*x + e)/a^5 + 3*(5*a^2*b + 20*a*b^2 + 1
6*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^5))/f
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.29

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \frac{\frac{3\,(a^2+12\,ab+16\,b^2)(fx+e)}{a^5}-\frac{3\,(5\,a^2b+20\,ab^2+16\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,a^5}-\frac{12\,ab^2\tan(fx+e)^7+24\,b^3\tan(fx+e)^7}{}}{}$$

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^2 + 12*a*b + 16*b^2)*(f*x + e)/a^5 - 3*(5*a^2*b + 20*a*b^2 + 16*b
^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b +
 b^2)))/(sqrt(a*b + b^2)*a^5) - (12*a*b^2*tan(f*x + e)^7 + 24*b^3*tan(f*x +
 e)^7 + 19*a^2*b*tan(f*x + e)^5 + 72*a*b^2*tan(f*x + e)^5 + 72*b^3*tan(f*x
+ e)^5 + 5*a^3*tan(f*x + e)^3 + 46*a^2*b*tan(f*x + e)^3 + 108*a*b^2*tan(f*x
 + e)^3 + 72*b^3*tan(f*x + e)^3 + 3*a^3*tan(f*x + e) + 27*a^2*b*tan(f*x + e
) + 48*a*b^2*tan(f*x + e) + 24*b^3*tan(f*x + e))/((b*tan(f*x + e)^4 + a*tan
(f*x + e)^2 + 2*b*tan(f*x + e)^2 + a + b)^2*a^4))/f
```

Mupad [B] (verification not implemented)

Time = 21.11 (sec) , antiderivative size = 1317, normalized size of antiderivative = 5.53

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] (atan(((((tan(e + f*x)*(4608*a*b^6 + 2304*b^7 + 3312*a^2*b^5 + 1008*a^3*b^4
 + 117*a^4*b^3))/(16*a^8) - (3*((12*a^10*b^4 + 12*a^11*b^3 + (3*a^12*b^2)/2
)/a^12 - (3*tan(e + f*x)*(256*a^10*b^3 + 128*a^11*b^2)*(-b*(a + b))^(1/2)*(
20*a*b + 5*a^2 + 16*b^2))/(256*a^8*(a^5*b + a^6)))*(-b*(a + b))^(1/2)*(20*a
*b + 5*a^2 + 16*b^2))/(16*(a^5*b + a^6)))*(-b*(a + b))^(1/2)*(20*a*b + 5*a^
2 + 16*b^2)*3i)/(16*(a^5*b + a^6)) + (((tan(e + f*x)*(4608*a*b^6 + 2304*b^7
 + 3312*a^2*b^5 + 1008*a^3*b^4 + 117*a^4*b^3))/(16*a^8) + (3*((12*a^10*b^4
+ 12*a^11*b^3 + (3*a^12*b^2)/2)/a^12 + (3*tan(e + f*x)*(256*a^10*b^3 + 128*
a^11*b^2)*(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(256*a^8*(a^5*b + a
^6)))*(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(16*(a^5*b + a^6)))*(-b
*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2)*3i)/(16*(a^5*b + a^6)))/((540*a*b
^7 + 216*b^8 + (999*a^2*b^6)/2 + (837*a^3*b^5)/4 + (1215*a^4*b^4)/32 + (135
*a^5*b^3)/64)/a^12 - (3*((tan(e + f*x)*(4608*a*b^6 + 2304*b^7 + 3312*a^2*b^
5 + 1008*a^3*b^4 + 117*a^4*b^3))/(16*a^8) - (3*((12*a^10*b^4 + 12*a^11*b^3
+ (3*a^12*b^2)/2)/a^12 - (3*tan(e + f*x)*(256*a^10*b^3 + 128*a^11*b^2)*(-b*
(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(256*a^8*(a^5*b + a^6)))*(-b*(a +
 b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(16*(a^5*b + a^6)))*(-b*(a + b))^(1/2
)*(20*a*b + 5*a^2 + 16*b^2))/(16*(a^5*b + a^6)) + (3*((tan(e + f*x)*(4608*a
*b^6 + 2304*b^7 + 3312*a^2*b^5 + 1008*a^3*b^4 + 117*a^4*b^3))/(16*a^8) + (3
*((12*a^10*b^4 + 12*a^11*b^3 + (3*a^12*b^2)/2)/a^12 + (3*tan(e + f*x)*(256*
a^10*b^3 + 128*a^11*b^2)*(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(256
*a^8*(a^5*b + a^6)))*(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(16*(a^5
*b + a^6)))*(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(16*(a^5*b + a^6)
)))*(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2)*3i)/(8*f*(a^5*b + a^6)) -
(atan((27*b^2*tan(e + f*x))/(256*((27*b^2)/256 + (81*b^3)/(64*a) + (27*b^4)
/(16*a^2))) + (81*b^3*tan(e + f*x))/(64*((27*a*b^2)/256 + (81*b^3)/64 + (27
*b^4)/(16*a))) + (27*b^4*tan(e + f*x))/(16*((81*a*b^3)/64 + (27*b^4)/16 + (
27*a^2*b^2)/256)))*(a*b*12i + a^2*1i + b^2*16i)*3i)/(8*a^5*f) - ((tan(e + f
*x)^5*(72*a*b^2 + 19*a^2*b + 72*b^3))/(8*a^4) + (tan(e + f*x)^3*(108*a*b^2
+ 46*a^2*b + 5*a^3 + 72*b^3))/(8*a^4) + (3*tan(e + f*x)*(16*a*b^2 + 9*a^2*b
 + a^3 + 8*b^3))/(8*a^4) + (3*b*tan(e + f*x)^7*(a*b + 2*b^2))/(2*a^4))/(f*(
2*a*b + tan(e + f*x)^4*(6*a*b + a^2 + 6*b^2) + a^2 + b^2 + tan(e + f*x)^6*(
2*a*b + 4*b^2) + b^2*tan(e + f*x)^8 + tan(e + f*x)^2*(6*a*b + 2*a^2 + 4*b^2
)))
```

3.62 $\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 184

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{(a+6b)x}{2a^4} - \frac{\sqrt{b}(15a^2+40ab+24b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{3/2}f} - \frac{\cos(e+fx)\sin(e+fx)}{2af\,(a+b+b\tan^2(e+fx))^2} - \frac{3b\tan(e+fx)}{4a^2f\,(a+b+b\tan^2(e+fx))^2} - \frac{b(11a+12b)\tan(e+fx)}{8a^3(a+b)f\,(a+b+b\tan^2(e+fx))}$$

```
[Out] 1/2*(a+6*b)*x/a^4-1/8*(15*a^2+40*a*b+24*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b
)^(1/2))*b^(1/2)/a^4/(a+b)^(3/2)/f-1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan
(f*x+e)^2)^2-3/4*b*tan(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2-1/8*b*(11*a+12*b
)*tan(f*x+e)/a^3/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

$= \{4217, 482, 541, 536, 209, 211\}$

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{x(a+6b)}{2a^4} - \frac{b(11a+12b)\tan(e+fx)}{8a^3f(a+b)\left(a+b\tan^2(e+fx)+b\right)} - \frac{3b\tan(e+fx)}{4a^2f\left(a+b\tan^2(e+fx)+b\right)^2} - \frac{\sqrt{b}(15a^2+40ab+24b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^4f(a+b)^{3/2}} - \frac{\sin(e+fx)\cos(e+fx)}{2af\left(a+b\tan^2(e+fx)+b\right)^2}$$

```
[In] Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 6*b)*x)/(2*a^4) - (Sqrt[b]*(15*a^2 + 40*a*b + 24*b^2)*ArcTan[(Sqrt[b]
*Tan[e + f*x])/Sqrt[a + b]])/(8*a^4*(a + b)^(3/2)*f) - (Cos[e + f*x]*Sin[e
+ f*x])/(2*a*f*(a + b + b*Tan[e + f*x]^2)^2) - (3*b*Tan[e + f*x])/(4*a^2*f*
(a + b + b*Tan[e + f*x]^2)^2) - (b*(11*a + 12*b)*Tan[e + f*x])/(8*a^3*(a +
b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2}+\frac{\text{Subst}\left(\int \frac{a+b-5bx^2}{(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{2af}\\
&= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2}-\frac{3b\tan(e+fx)}{4a^2f\left(a+b+b\tan^2(e+fx)\right)^2}\\
&\quad+\frac{\text{Subst}\left(\int \frac{2(a+b)(2a+3b)-18b(a+b)x^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{8a^2(a+b)f}\\
&= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2}-\frac{3b\tan(e+fx)}{4a^2f\left(a+b+b\tan^2(e+fx)\right)^2}\\
&\quad-\frac{b(11a+12b)\tan(e+fx)}{8a^3(a+b)f\left(a+b+b\tan^2(e+fx)\right)}\\
&\quad+\frac{\text{Subst}\left(\int \frac{2(a+b)(4a^2+17ab+12b^2)-2b(a+b)(11a+12b)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{16a^3(a+b)^2f}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{3b\tan(e+fx)}{4a^2f\left(a+b+b\tan^2(e+fx)\right)^2} \\
&\quad - \frac{b(11a+12b)\tan(e+fx)}{8a^3(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(a+6b)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{2a^4f} \\
&\quad - \frac{\left(b(15a^2+40ab+24b^2)\right)\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{8a^4(a+b)f}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(a+6b)x}{2a^4} - \frac{\sqrt{b}(15a^2+40ab+24b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{3/2}f} \\
&\quad - \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{3b\tan(e+fx)}{4a^2f\left(a+b+b\tan^2(e+fx)\right)^2} \\
&\quad - \frac{b(11a+12b)\tan(e+fx)}{8a^3(a+b)f\left(a+b+b\tan^2(e+fx)\right)}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 15.36 (sec) , antiderivative size = 1915, normalized size of antiderivative = 10.41

$$
\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx
$$

$$
\begin{aligned}
&= \frac{5(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(\frac{(3a^2+8ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{a\sqrt{b}(3a^2+16ab+16b^2+3a(a+2b)\cos(2(e+\cdots}{(a+b)^2(a+2b+a\cos(2(e+f\cdots}\right.}{8192b^{5/2}f\left(a+b\sec^2(e+fx)\right)^3} \\
&\quad + \frac{(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(-\frac{3a(a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + \frac{\sqrt{b}(3a^3+14a^2b+24ab^2+16b^3+a(3a^2+4ab+\cdots}{(a+b)^2(a+2b+a\cos(2\cdots}\right.}{2048b^{5/2}f\left(a+b\sec^2(e+fx)\right)^3} \\
&\quad - \frac{(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(\frac{2(3a^5-10a^4b+80a^3b^2+480a^2b^3+640ab^4+256b^5)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-\cdots}{2\sqrt{a+b}\sqrt{b(\cdots}}\right.}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{\cdots} \\
&\quad - \frac{(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(-\frac{6a^2\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-i\,\mathrm{s}\cdots}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{\cdots} \\
&\quad - \frac{(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(-1536(a+2b)x - \frac{3(a^6-8a^5b+120a^4b^2+1280a^3b^3+3200a^2b^4+3072ab^5+10\cdots}{\cdots b\cdots}\right.}{\cdots}
\end{aligned}
$$

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]

[Out] (5*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((3*a^2 + 8*a*b + 8*b^2
)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*(3
*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/
((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(8192*b^(5/2)*f*(a + b*Sec[e
 + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-3*a*(a
+ 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + (Sqrt[b]
*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*Cos[2*(e
 + f*x)])*Sin[2*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/
(2048*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3) - ((a + 2*b + a*Cos[2*e + 2*f*x])
^3*Sec[e + f*x]^6*((2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*
b^4 + 256*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*
x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Co
s[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (Sec[2*
e]*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*f*x*Cos[2*e] + 512*a*b^2*(a +
 b)^2*(a + 2*b)*f*x*Cos[2*f*x] + 128*a^4*b^2*f*x*Cos[2*(e + 2*f*x)] + 256*a
^3*b^3*f*x*Cos[2*(e + 2*f*x)] + 128*a^2*b^4*f*x*Cos[2*(e + 2*f*x)] + 512*a^
4*b^2*f*x*Cos[4*e + 2*f*x] + 2048*a^3*b^3*f*x*Cos[4*e + 2*f*x] + 2560*a^2*b
^4*f*x*Cos[4*e + 2*f*x] + 1024*a*b^5*f*x*Cos[4*e + 2*f*x] + 128*a^4*b^2*f*x
*Cos[6*e + 4*f*x] + 256*a^3*b^3*f*x*Cos[6*e + 4*f*x] + 128*a^2*b^4*f*x*Cos[
6*e + 4*f*x] - 9*a^6*Sin[2*e] + 12*a^5*b*Sin[2*e] + 684*a^4*b^2*Sin[2*e] +
2880*a^3*b^3*Sin[2*e] + 5280*a^2*b^4*Sin[2*e] + 4608*a*b^5*Sin[2*e] + 1536*
b^6*Sin[2*e] + 9*a^6*Sin[2*f*x] - 14*a^5*b*Sin[2*f*x] - 608*a^4*b^2*Sin[2*f
*x] - 2112*a^3*b^3*Sin[2*f*x] - 2560*a^2*b^4*Sin[2*f*x] - 1024*a*b^5*Sin[2*
f*x] + 3*a^6*Sin[2*(e + 2*f*x)] - 12*a^5*b*Sin[2*(e + 2*f*x)] - 204*a^4*b^2
*Sin[2*(e + 2*f*x)] - 384*a^3*b^3*Sin[2*(e + 2*f*x)] - 192*a^2*b^4*Sin[2*(e
 + 2*f*x)] - 3*a^6*Sin[4*e + 2*f*x] + 10*a^5*b*Sin[4*e + 2*f*x] + 304*a^4*b
^2*Sin[4*e + 2*f*x] + 1056*a^3*b^3*Sin[4*e + 2*f*x] + 1280*a^2*b^4*Sin[4*e
+ 2*f*x] + 512*a*b^5*Sin[4*e + 2*f*x]))/(a + 2*b + a*Cos[2*(e + f*x)])^2))/
(4096*a^3*b^2*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3) - ((a + 2*b + a*Cos[2*e
 + 2*f*x])^3*Sec[e + f*x]^6*((-6*a^2*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e
])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e
] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I
*Sin[e])^4]) + (a*Sec[2*e]*((-9*a^4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^3 + 6
4*b^4)*Sin[2*f*x] + a*(-3*a^3 + 2*a^2*b + 24*a*b^2 + 16*b^3)*Sin[2*(e + 2*f
*x)] + (3*a^4 - 64*a^2*b^2 - 128*a*b^3 - 64*b^4)*Sin[4*e + 2*f*x]) + (9*a^5
 + 18*a^4*b - 64*a^3*b^2 - 256*a^2*b^3 - 320*a*b^4 - 128*b^5)*Tan[2*e])/(a^
2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(4096*b^2*(a + b)^2*f*(a + b*Sec[e +
f*x]^2)^3) - ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(-1536*(a + 2
*b)*x - (3*(a^6 - 8*a^5*b + 120*a^4*b^2 + 1280*a^3*b^3 + 3200*a^2*b^4 + 307
2*a*b^5 + 1024*b^6)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*S
in[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])
]*(Cos[2*e] - I*Sin[2*e]))/(b^2*(a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^
4]) + (4*(a^4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4)*Sec[2*e]*((a
```

```
+ 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)]
)^2) + (256*a*Sin[2*(e + f*x)])/f + (a*(-3*a^5 + 26*a^4*b + 736*a^3*b^2 + 2
624*a^2*b^3 + 3200*a*b^4 + 1280*b^5)*Sec[2*e]*Sin[2*f*x] + (3*a^6 - 24*a^5*
b - 920*a^4*b^2 - 4864*a^3*b^3 - 10112*a^2*b^4 - 9216*a*b^5 - 3072*b^6)*Tan
[2*e])/(b^2*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f*x)]))))/(8192*a^4*(a + b*
Sec[e + f*x]^2)^3)
```

Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{\frac{-\frac{a\tan(fx+e)}{2\left(1+\tan(fx+e)^2\right)}+\frac{(a+6b)\arctan(\tan(fx+e))}{2}}{a^4}-\frac{b\left(\frac{\frac{ab(7a+8b)\tan(fx+e)^3}{8a+8b}+\frac{a(9a+8b)\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2+40ab+24b^2\right)\arctan\left(\frac{b\,t}{\sqrt{}}\right.}{8(a+b)\sqrt{(a+b)b}}\right.}{a^4}}{f}$
default	$\frac{\frac{-\frac{a\tan(fx+e)}{2\left(1+\tan(fx+e)^2\right)}+\frac{(a+6b)\arctan(\tan(fx+e))}{2}}{a^4}-\frac{b\left(\frac{\frac{ab(7a+8b)\tan(fx+e)^3}{8a+8b}+\frac{a(9a+8b)\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2+40ab+24b^2\right)\arctan\left(\frac{b\,t}{\sqrt{}}\right.}{8(a+b)\sqrt{(a+b)b}}\right.}{a^4}}{f}$
risch	$\frac{x}{2a^3}+\frac{3xb}{a^4}+\frac{ie^{2i(fx+e)}}{8a^3f}-\frac{ie^{-2i(fx+e)}}{8a^3f}-\frac{ib\left(9a^3e^{6i(fx+e)}+32a^2b\,e^{6i(fx+e)}+24a\,b^2e^{6i(fx+e)}+27a^3e^{4i(fx+e)}+102a\right.}{4a^4(a+b}$

```
[In] int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^4*(-1/2*a*tan(f*x+e)/(1+tan(f*x+e)^2)+1/2*(a+6*b)*arctan(tan(f*x+e
)))-1/a^4*b*((1/8*a*b*(7*a+8*b)/(a+b)*tan(f*x+e)^3+1/8*a*(9*a+8*b)*tan(f*x+
e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(15*a^2+40*a*b+24*b^2)/(a+b)/((a+b)*b)^(1/2)
*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(166) = 332.

Time = 0.33 (sec) , antiderivative size = 815, normalized size of antiderivative = 4.43

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$=\left[\frac{16\left(a^4+7a^3b+6a^2b^2\right)fx\cos\left(fx+e\right)^4+32\left(a^3b+7a^2b^2+6ab^3\right)fx\cos\left(fx+e\right)^2+16\left(a^2b^2+7ab^3+6\right.}{}\right.$$

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(16*(a^4 + 7*a^3*b + 6*a^2*b^2)*f*x*cos(f*x + e)^4 + 32*(a^3*b + 7*a^
2*b^2 + 6*a*b^3)*f*x*cos(f*x + e)^2 + 16*(a^2*b^2 + 7*a*b^3 + 6*b^4)*f*x +
((15*a^4 + 40*a^3*b + 24*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 40*a*b^3 +
24*b^4 + 2*(15*a^3*b + 40*a^2*b^2 + 24*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(a +
b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
 e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))
*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x +
 e)^2 + b^2)) - 4*(4*(a^4 + a^3*b)*cos(f*x + e)^5 + (17*a^3*b + 18*a^2*b^2)
*cos(f*x + e)^3 + (11*a^2*b^2 + 12*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^7
 + a^6*b)*f*cos(f*x + e)^4 + 2*(a^6*b + a^5*b^2)*f*cos(f*x + e)^2 + (a^5*b^
2 + a^4*b^3)*f), 1/16*(8*(a^4 + 7*a^3*b + 6*a^2*b^2)*f*x*cos(f*x + e)^4 + 1
6*(a^3*b + 7*a^2*b^2 + 6*a*b^3)*f*x*cos(f*x + e)^2 + 8*(a^2*b^2 + 7*a*b^3 +
 6*b^4)*f*x + ((15*a^4 + 40*a^3*b + 24*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2
 + 40*a*b^3 + 24*b^4 + 2*(15*a^3*b + 40*a^2*b^2 + 24*a*b^3)*cos(f*x + e)^2)
*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/
(b*cos(f*x + e)*sin(f*x + e))) - 2*(4*(a^4 + a^3*b)*cos(f*x + e)^5 + (17*a^
3*b + 18*a^2*b^2)*cos(f*x + e)^3 + (11*a^2*b^2 + 12*a*b^3)*cos(f*x + e))*si
n(f*x + e))/((a^7 + a^6*b)*f*cos(f*x + e)^4 + 2*(a^6*b + a^5*b^2)*f*cos(f*x
 + e)^2 + (a^5*b^2 + a^4*b^3)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.48

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\frac{\left(15\,a^2b+40\,ab^2+24\,b^3\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\left(a^5+a^4b\right)\sqrt{(a+b)b}}+\frac{\left(11\,ab^2+12\,b^3\right)\tan(fx+e)^5+\left(17\,a^2b+40\,ab^2+24\,b^3\right)\tan(fx+e)^3+\left(4\,a^3+21\,a^2b+\right.}{\left(a^4b^2+a^3b^3\right)\tan(fx+e)^6+a^6+3\,a^5b+3\,a^4b^2+a^3b^3+\left(2\,a^5b+5\,a^4b^2+3\,a^3b^3\right)\tan(fx+e)^4+\left(a\right.}}{8\,f}$$

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((15*a^2*b + 40*a*b^2 + 24*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))
/((a^5 + a^4*b)*sqrt((a + b)*b)) + ((11*a*b^2 + 12*b^3)*tan(f*x + e)^5 + (1
7*a^2*b + 40*a*b^2 + 24*b^3)*tan(f*x + e)^3 + (4*a^3 + 21*a^2*b + 29*a*b^2
+ 12*b^3)*tan(f*x + e))/((a^4*b^2 + a^3*b^3)*tan(f*x + e)^6 + a^6 + 3*a^5*b
 + 3*a^4*b^2 + a^3*b^3 + (2*a^5*b + 5*a^4*b^2 + 3*a^3*b^3)*tan(f*x + e)^4 +
 (a^6 + 5*a^5*b + 7*a^4*b^2 + 3*a^3*b^3)*tan(f*x + e)^2) - 4*(f*x + e)*(a +
 6*b)/a^4)/f
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.13

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\frac{(15\,a^2b+40\,ab^2+24\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^5+a^4b)\sqrt{ab+b^2}}+\frac{7\,ab^2\tan(fx+e)^3+8\,b^3\tan(fx+e)^3+9\,a^2b\tan(fx+e)+17\,ab^2\tan(fx}{(a^4+a^3b)\left(b\tan(fx+e)^2+a+b\right)^2}}{8\,f}$$

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b + 40*a*b^2 + 24*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
 arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + a^4*b)*sqrt(a*b + b^2)) +
(7*a*b^2*tan(f*x + e)^3 + 8*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x + e) + 17*
a*b^2*tan(f*x + e) + 8*b^3*tan(f*x + e))/((a^4 + a^3*b)*(b*tan(f*x + e)^2 +
 a + b)^2) - 4*(f*x + e)*(a + 6*b)/a^4 + 4*tan(f*x + e)/((tan(f*x + e)^2 +
1)*a^3))/f
```

Mupad [B] (verification not implemented)

Time = 22.32 (sec) , antiderivative size = 2628, normalized size of antiderivative = 14.28

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

```
[In] int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] (atan(((((tan(e + f*x)*(3264*a*b^6 + 1152*b^7 + 3296*a^2*b^5 + 1424*a^3*b^4
 + 241*a^4*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) - (((6*a^8*b^5 + (29*a^9*b^
4)/2 + (21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11 + a^9*b^2) - (tan(e +
 f*x)*(a*1i + b*6i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*
b^2))/(128*a^4*(2*a^7*b + a^8 + a^6*b^2)))*(a*1i + b*6i))/(4*a^4))*(a*1i +
b*6i)*1i)/(4*a^4) + (((tan(e + f*x)*(3264*a*b^6 + 1152*b^7 + 3296*a^2*b^5 +
```

1424*a^3*b^4 + 241*a^4*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) + (((6*a^8*b^5 + (29*a^9*b^4)/2 + (21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11 + a^9*b^2) + (tan(e + f*x)*(a*1i + b*6i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2))/(128*a^4*(2*a^7*b + a^8 + a^6*b^2)))*(a*1i + b*6i))/(4*a^4))*(a*1i + b*6i)*1i)/(4*a^4))/(((297*a*b^6)/4 + 27*b^7 + (279*a^2*b^5)/4 + (805*a^3*b^4)/32 + (165*a^4*b^3)/64)/(2*a^10*b + a^11 + a^9*b^2) - (((tan(e + f*x)*(3264*a*b^6 + 1152*b^7 + 3296*a^2*b^5 + 1424*a^3*b^4 + 241*a^4*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) - (((6*a^8*b^5 + (29*a^9*b^4)/2 + (21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11 + a^9*b^2) - (tan(e + f*x)*(a*1i + b*6i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2))/(128*a^4*(2*a^7*b + a^8 + a^6*b^2)))*(a*1i + b*6i))/(4*a^4))*(a*1i + b*6i))/(4*a^4) + (((tan(e + f*x)*(3264*a*b^6 + 1152*b^7 + 3296*a^2*b^5 + 1424*a^3*b^4 + 241*a^4*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) + (((6*a^8*b^5 + (29*a^9*b^4)/2 + (21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11 + a^9*b^2) + (tan(e + f*x)*(a*1i + b*6i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2))/(128*a^4*(2*a^7*b + a^8 + a^6*b^2)))*(a*1i + b*6i))/(4*a^4))*(a*1i + b*6i))/(4*a^4)))*(a*1i + b*6i)*1i)/(2*a^4*f) - ((tan(e + f*x)*(17*a*b + 4*a^2 + 12*b^2))/(8*a^3) + (tan(e + f*x)^5*(11*a*b^2 + 12*b^3))/(8*a^3*(a + b)) + (b*tan(e + f*x)^3*(40*a*b + 17*a^2 + 24*b^2))/(8*a^3*(a + b)))/(f*(2*a*b + tan(e + f*x)^2*(4*a*b + a^2 + 3*b^2) + a^2 + b^2 + tan(e + f*x)^4*(2*a*b + 3*b^2) + b^2*tan(e + f*x)^6)) + (atan((((-b*(a + b)^3)^(1/2)*((tan(e + f*x)*(3264*a*b^6 + 1152*b^7 + 3296*a^2*b^5 + 1424*a^3*b^4 + 241*a^4*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) - ((-b*(a + b)^3)^(1/2)*((6*a^8*b^5 + (29*a^9*b^4)/2 + (21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11 + a^9*b^2) - (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(40*a*b + 15*a^2 + 24*b^2)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2))/(512*(2*a^7*b + a^8 + a^6*b^2)*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(40*a*b + 15*a^2 + 24*b^2))/(16*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(40*a*b + 15*a^2 + 24*b^2)*1i)/(16*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)) + ((-b*(a + b)^3)^(1/2)*((tan(e + f*x)*(3264*a*b^6 + 1152*b^7 + 3296*a^2*b^5 + 1424*a^3*b^4 + 241*a^4*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) + ((-b*(a + b)^3)^(1/2)*((6*a^8*b^5 + (29*a^9*b^4)/2 + (21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11 + a^9*b^2) + (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(40*a*b + 15*a^2 + 24*b^2)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2))/(512*(2*a^7*b + a^8 + a^6*b^2)*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(40*a*b + 15*a^2 + 24*b^2))/(16*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(40*a*b + 15*a^2 + 24*b^2)*1i)/(16*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))/(((297*a*b^6)/4 + 27*b^7 + (279*a^2*b^5)/4 + (805*a^3*b^4)/32 + (165*a^4*b^3)/64)/(2*a^10*b + a^11 + a^9*b^2) - ((-b*(a + b)^3)^(1/2)*((tan(e + f*x)*(3264*a*b^6 + 1152*b^7 + 3296*a^2*b^5 + 1424*a^3*b^4 + 241*a^4*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) - ((-b*(a + b)^3)^(1/2)*((6*a^8*b^5 + (29*a^9*b^4)/2 + (21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11 + a^9*b^2) - (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(40*a*b + 15*a^2 + 24*b^2)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2))/(512*(2*a^7*b + a^8 + a^6*b^2)*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(40*a*b + 15*a^2 + 24*b^2))/(16*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(40*a*b + 15*a^2 +

```
 24*b^2))/(16*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)) + ((-b*(a + b)^3)^(1/2
)*((tan(e + f*x)*(3264*a*b^6 + 1152*b^7 + 3296*a^2*b^5 + 1424*a^3*b^4 + 241
*a^4*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) + ((-b*(a + b)^3)^(1/2)*((6*a^8*b
^5 + (29*a^9*b^4)/2 + (21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11 + a^9*
b^2) + (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(40*a*b + 15*a^2 + 24*b^2)*(512*a
^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2))/(512*(2*a^7*b + a^8
+ a^6*b^2)*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(40*a*b + 15*a^2 + 24*b^
2))/(16*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(40*a*b + 15*a^2 + 24*b^2))
/(16*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2))))*(-b*(a + b)^3)^(1/2)*(40*a*b
+ 15*a^2 + 24*b^2)*1i)/(8*f*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2))
```

3.63 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 144

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{x}{a^3} - \frac{\sqrt{b}(15a^2+20ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}f} - \frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

```
[Out] x/a^3-1/8*(15*a^2+20*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1
/2)/a^3/(a+b)^(5/2)/f-1/4*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/8
*b*(7*a+4*b)*tan(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4213, 425, 541, 536, 209, 211}

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{x}{a^3} - \frac{b(7a+4b)\tan(e+fx)}{8a^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)} - \frac{\sqrt{b}(15a^2+20ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{5/2}} - \frac{b\tan(e+fx)}{4af(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-3),x]
```

```
[Out] x/a^3 - (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sq
rt[a + b]])/(8*a^3*(a + b)^(5/2)*f) - (b*Tan[e + f*x])/(4*a*(a + b)*f*(a +
b + b*Tan[e + f*x]^2)^2) - (b*(7*a + 4*b)*Tan[e + f*x])/(8*a^2*(a + b)^2*f*
(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
```

```
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a+b-3bx^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4a(a+b)f}$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{8a^2+9ab+4b^2-b(7a+4b)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{8a^2(a+b)^2f}$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^3f} - \frac{\left(b(15a^2+20ab+8b^2)\right)\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{8a^3(a+b)^2f}$$

$$= \frac{x}{a^3} - \frac{\sqrt{b}(15a^2+20ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}f} - \frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.39 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.31

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(8x(a+2b+a\cos(2(e+fx)))^2 + \frac{b(15a^2+20ab+8b^2)\arctan\left(\frac{\sec(fx)(\cos}{\ldots}\right.}{\ldots}\right.}{\ldots}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-3),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(8*x*(a + 2*b + a*Cos[2*(e +
 f*x)])^2 + (b*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin
[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(C
os[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2*(Cos[2*e] - I*Sin[2
*e]))/((a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) - (4*b^2*((a + 2*b)*S
in[2*e] - a*Sin[2*f*x]))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])) +
(b*(a + 2*b + a*Cos[2*(e + f*x)])*((9*a^2 + 28*a*b + 16*b^2)*Sin[2*e] - 3*a
*(3*a + 2*b)*Sin[2*f*x]))/((a + b)^2*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))
))/(64*a^3*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a^3}-\frac{b\left(\frac{\frac{ab(7a+4b)\tan(fx+e)^3}{8a^2+16ab+8b^2}+\frac{(9a+4b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}\right)}{a^3}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{a^3}-\frac{b\left(\frac{\frac{ab(7a+4b)\tan(fx+e)^3}{8a^2+16ab+8b^2}+\frac{(9a+4b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}\right)}{a^3}}{f}$
risch	$\frac{x}{a^3}-\frac{ib\left(9a^3\mathrm{e}^{6i(fx+e)}+28a^2b\,\mathrm{e}^{6i(fx+e)}+16a\,b^2\mathrm{e}^{6i(fx+e)}+27a^3\mathrm{e}^{4i(fx+e)}+90a^2b\,\mathrm{e}^{4i(fx+e)}+120a\,b^2\mathrm{e}^{4i(fx+e)}+48b^3\mathrm{e}^{4i(f}\right.}{4a^3(a+b)^2f\left(a\,\mathrm{e}^{4i(fx+e)}+2a\,\mathrm{e}^{2i(fx+e)}+4b\,\mathrm{e}^{2i(fx+e}\right.}$

```
[In] int(1/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^3*arctan(tan(f*x+e))-b/a^3*((1/8*a*b*(7*a+4*b)/(a^2+2*a*b+b^2)*tan
(f*x+e)^3+1/8*(9*a+4*b)*a/(a+b)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(15*
a^2+20*a*b+8*b^2)/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b
)*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(130) = 260$.

Time = 0.33 (sec) , antiderivative size = 819, normalized size of antiderivative = 5.69

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \left[\frac{32\left(a^4+2a^3b+a^2b^2\right)fx\cos(fx+e)^4+64\left(a^3b+2a^2b^2+ab^3\right)fx\cos(fx+e)^2+32\left(a^2b^2+2ab^3+b^4\right)f}{}\right.$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 64*(a^3*b + 2*a^2*
b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 32*(a^2*b^2 + 2*a*b^3 + b^4)*f*x + ((15*a
^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 + 8*b^4 +
 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log((
(a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4
*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/
(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b
^2)) - 4*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 + 4*a*b^3)*co
s(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 + 2*(
a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^3 + a^3*
b^4)*f), 1/16*(16*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 32*(a^3*b
+ 2*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 16*(a^2*b^2 + 2*a*b^3 + b^4)*f*x
+ ((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 +
 8*b^4 + 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(b/(a + b)
)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)
*sin(f*x + e))) - 2*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 +
4*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x +
 e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4
*b^3 + a^3*b^4)*f)]
```

Sympy [F]

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx=\int \frac{1}{\left(a+b\sec^2\left(e+fx\right)\right)^3}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.60

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx=-\frac{\frac{\left(15\,a^2b+20\,ab^2+8\,b^3\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\left(a^5+2\,a^4b+a^3b^2\right)\sqrt{(a+b)b}}+\frac{\left(7\,ab^2+4\,b^3\right)\tan(fx+e)^3+\left(9\,a^2b+13\,ab^2+4\,b^3\right)\tan(fx+e)}{a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4+\left(a^4b^2+2\,a^3b^3+a^2b^4\right)\tan(fx+e)^4+2\left(a^5b+3\,a^4b^2+3\,a^3b^3+a\right)}}{8\,f}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/
((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)) + ((7*a*b^2 + 4*b^3)*tan(f*x +
e)^3 + (9*a^2*b + 13*a*b^2 + 4*b^3)*tan(f*x + e))/(a^6 + 4*a^5*b + 6*a^4*b^
2 + 4*a^3*b^3 + a^2*b^4 + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*tan(f*x + e)^4 +
2*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*tan(f*x + e)^2) - 8*(f*x + e)/a
^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.36

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{(15\,a^2b+20\,ab^2+8\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^5+2\,a^4b+a^3b^2)\sqrt{ab+b^2}}+\frac{7\,ab^2\tan(fx+e)^3+4\,b^3\tan(fx+e)^3+9\,a^2b\tan(fx+e)+13\,ab^2\tan(fx+}{(a^4+2\,a^3b+a^2b^2)\left(b\tan(fx+e)^2+a+b\right)^2}}{8\,f}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(a*b
 + b^2)) + (7*a*b^2*tan(f*x + e)^3 + 4*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x
 + e) + 13*a*b^2*tan(f*x + e) + 4*b^3*tan(f*x + e))/((a^4 + 2*a^3*b + a^2*b
^2)*(b*tan(f*x + e)^2 + a + b)^2) - 8*(f*x + e)/a^3)/f
```

Mupad [B] (verification not implemented)

Time = 22.74 (sec) , antiderivative size = 3271, normalized size of antiderivative = 22.72

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] int(1/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] atan((((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10
*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e +
 f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10
*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6
*b^2)))/(2*a^3) + (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a
^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2
)))/a^3 - ((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*
a^10*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) + (tan
(e + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*
```

a^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6
*a^6*b^2)))/(2*a^3) - (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 8
56*a^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6
*b^2)))/a^3)/(((17*a*b^5)/4 + b^6 + (25*a^2*b^4)/4 + (105*a^3*b^3)/32)/(4*a
^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (((((2*a^6*b^6 + (17*a^7*b
^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)*1i)/(2*(4*a^9*b + a^10 +
a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e + f*x)*(512*a^6*b^7 + 2304*a^7*b
^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(128*a^3*
(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(2*a^3) + (tan(e +
f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3)*1i)/(
64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/a^3 + (((((2*a^6*b^6
 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)*1i)/(2*(4*a^9
*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) + (tan(e + f*x)*(512*a^6*b^7
+ 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2
))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(2*a^3)
 - (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^
4*b^3)*1i)/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/a^3))/(a
^3*f) - ((tan(e + f*x)^3*(7*a*b^2 + 4*b^3))/(8*a^2*(a + b)^2) + (tan(e + f*
x)*(9*a*b + 4*b^2))/(8*a^2*(a + b)))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)^2
*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4)) + (atan(((((tan(e + f*x)*(576*a*b^6
 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(32*(4*a^7*b + a^8
+ a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) - (((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^
8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3
+ 6*a^8*b^2) - (tan(e + f*x)*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)
*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3
+ 256*a^11*b^2))/(512*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)*(5*
a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^
5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^
4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^
2)*1i)/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2))
 + (((tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*
a^4*b^3))/(32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) + (((2*a^6
*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)/(4*a^9*b
+ a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (tan(e + f*x)*(-b*(a + b)^5)^(1
/2)*(20*a*b + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 +
3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(512*(4*a^7*b + a^8 + a^4*b^4
 + 4*a^5*b^3 + 6*a^6*b^2)*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3
 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7
*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^
(1/2)*(20*a*b + 15*a^2 + 8*b^2)*1i)/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^
4 + 10*a^5*b^3 + 10*a^6*b^2)))/(((17*a*b^5)/4 + b^6 + (25*a^2*b^4)/4 + (105
*a^3*b^3)/32)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) - (((tan(e
 + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(
32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) - (((2*a^6*b^6 + (17*
a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)/(4*a^9*b + a^10 + a^

```
6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) - (tan(e + f*x)*(-b*(a + b)^5)^(1/2)*(20*a*b
 + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^
4 + 1536*a^10*b^3 + 256*a^11*b^2))/(512*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^
3 + 6*a^6*b^2)*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b
^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 +
a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a
*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3
 + 10*a^6*b^2)) + (((tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856
*a^3*b^4 + 289*a^4*b^3))/(32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b
^2)) + (((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10
*b^2)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (tan(e + f*x)*(-
b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 +
4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(512*(4*a^7*b
+ a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b
^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b
^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*
(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^
5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2))))*(-b*(a + b)^5)^(1/2)*(20*a*b +
15*a^2 + 8*b^2)*1i)/(8*f*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3
+ 10*a^6*b^2))
```

3.64 $\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^3} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 124

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^3} \, dx = -\frac{15\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}f} - \frac{15\cot(e+fx)}{8(a+b)^3 f} + \frac{\cot(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{5\cot(e+fx)}{8(a+b)^2 f\left(a+b+b\tan^2(e+fx)\right)}$$

```
[Out] -15/8*cot(f*x+e)/(a+b)^3/f-15/8*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1
/2)/(a+b)^(7/2)/f+1/4*cot(f*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2+5/8*cot(f*x
+e)/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4217, 296, 331, 211}

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^3} \, dx = -\frac{15\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8f(a+b)^{7/2}} - \frac{15\cot(e+fx)}{8f(a+b)^3} + \frac{5\cot(e+fx)}{8f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)} + \frac{\cot(e+fx)}{4f(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}$$

```
[In] Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*(a + b)^(7/2)*f
) - (15*Cot[e + f*x])/(8*(a + b)^3*f) + Cot[e + f*x]/(4*(a + b)*f*(a + b +
b*Tan[e + f*x]^2)^2) + (5*Cot[e + f*x])/(8*(a + b)^2*f*(a + b + b*Tan[e + f
*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cot(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{5\text{Subst}\left(\int \frac{1}{x^2(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4(a+b)f}$$

$$= \frac{\cot(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{5\cot(e+fx)}{8(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{15\text{Subst}\left(\int \frac{1}{x^2(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{8(a+b)^2f}$$

$$= -\frac{15\cot(e+fx)}{8(a+b)^3f} + \frac{\cot(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{5\cot(e+fx)}{8(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} - \frac{(15b)\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{8(a+b)^3f}$$

$$= -\frac{15\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}f} - \frac{15\cot(e+fx)}{8(a+b)^3f} + \frac{\cot(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{5\cot(e+fx)}{8(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.88 (sec) , antiderivative size = 749, normalized size of antiderivative = 6.04

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(\frac{120b\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a\cos(2}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{}$$

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*((120*b*ArcTan[(Sec[f*x]*(Co
s[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a
+ b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2*(Cos[
2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (Csc[e]*C
sc[e + f*x]*Sec[2*e]*((-32*a^4 - 64*a^3*b + 22*a^2*b^2 + 80*a*b^3 + 16*b^4)
*Sin[f*x] + 2*a*(16*a^3 + 23*a^2*b - 27*a*b^2 - 4*b^3)*Sin[3*f*x] - 48*a^4*
Sin[2*e - f*x] - 128*a^3*b*Sin[2*e - f*x] - 106*a^2*b^2*Sin[2*e - f*x] + 80
*a*b^3*Sin[2*e - f*x] + 16*b^4*Sin[2*e - f*x] + 48*a^4*Sin[2*e + f*x] + 146
*a^3*b*Sin[2*e + f*x] + 182*a^2*b^2*Sin[2*e + f*x] + 80*a*b^3*Sin[2*e + f*x
] + 16*b^4*Sin[2*e + f*x] - 32*a^4*Sin[4*e + f*x] - 82*a^3*b*Sin[4*e + f*x]
 - 54*a^2*b^2*Sin[4*e + f*x] - 80*a*b^3*Sin[4*e + f*x] - 16*b^4*Sin[4*e + f
*x] - 8*a^4*Sin[2*e + 3*f*x] + 18*a^3*b*Sin[2*e + 3*f*x] + 54*a^2*b^2*Sin[2
*e + 3*f*x] + 8*a*b^3*Sin[2*e + 3*f*x] + 32*a^4*Sin[4*e + 3*f*x] + 73*a^3*b
```

```
*Sin[4*e + 3*f*x] + 24*a^2*b^2*Sin[4*e + 3*f*x] + 8*a*b^3*Sin[4*e + 3*f*x]
- 8*a^4*Sin[6*e + 3*f*x] - 9*a^3*b*Sin[6*e + 3*f*x] - 24*a^2*b^2*Sin[6*e +
3*f*x] - 8*a*b^3*Sin[6*e + 3*f*x] + 8*a^4*Sin[2*e + 5*f*x] - 9*a^3*b*Sin[2*
e + 5*f*x] - 2*a^2*b^2*Sin[2*e + 5*f*x] + 9*a^3*b*Sin[4*e + 5*f*x] + 2*a^2*
b^2*Sin[4*e + 5*f*x] + 8*a^4*Sin[6*e + 5*f*x]))/a^2))/(512*(a + b)^3*f*(a +
 b*Sec[e + f*x]^2)^3)
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\frac{7b\tan(fx+e)^3}{8}+\left(\frac{9a}{8}+\frac{9b}{8}\right)\tan(fx+e)}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{15\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}}\right)}{(a+b)^3}-\frac{1}{(a+b)^3\tan(fx+e)}}{f}$
default	$\frac{-\frac{b\left(\frac{\frac{7b\tan(fx+e)^3}{8}+\left(\frac{9a}{8}+\frac{9b}{8}\right)\tan(fx+e)}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{15\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}}\right)}{(a+b)^3}-\frac{1}{(a+b)^3\tan(fx+e)}}{f}$
risch	$-\frac{i\left(8a^4e^{8i(fx+e)}+9a^3b\,e^{8i(fx+e)}+24a^2b^2e^{8i(fx+e)}+8a\,b^3e^{8i(fx+e)}+32a^4e^{6i(fx+e)}+82a^3b\,e^{6i(fx+e)}+54a^2b^2e^{6i(fx+e)}+\right.}{}$

```
[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
[Out] 1/f*(-b/(a+b)^3*((7/8*b*tan(f*x+e)^3+(9/8*a+9/8*b)*tan(f*x+e))/(a+b+b*tan(f
*x+e)^2)^2+15/8/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))-1/(a+
b)^3/tan(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(108) = 216.

Time = 0.31 (sec) , antiderivative size = 615, normalized size of antiderivative = 4.96

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$
$$=\left[-\frac{4\left(8a^2-9ab-2b^2\right)\cos(fx+e)^5+20\left(5ab-b^2\right)\cos(fx+e)^3-15\left(a^2\cos(fx+e)^4+2ab\cos(fx+\right.}{32\left(\left(a^5+3a^4b+3a^3b^2+a^2b^3\right)f\cos(fx+e\right.}\right.$$
$$-\frac{2\left(8a^2-9ab-2b^2\right)\cos(fx+e)^5+10\left(5ab-b^2\right)\cos(fx+e)^3-15\left(a^2\cos(fx+e)^4+2ab\cos(fx+\right.}{16\left(\left(a^5+3a^4b+3a^3b^2+a^2b^3\right)f\cos(fx+e)^4+2\left(a^4b+3a^3b^2+3a^2b^3+ab^4\right)f\right.}$$

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
[Out] [-1/32*(4*(8*a^2 - 9*a*b - 2*b^2)*cos(f*x + e)^5 + 20*(5*a*b - b^2)*cos(f*x
 + e)^3 - 15*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(-b/(a +
 b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x
+ e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e)
)*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x
+ e)^2 + b^2))*sin(f*x + e) + 60*b^2*cos(f*x + e))/(((a^5 + 3*a^4*b + 3*a^3
*b^2 + a^2*b^3)*f*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4
)*f*cos(f*x + e)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*f)*sin(f*x + e))
, -1/16*(2*(8*a^2 - 9*a*b - 2*b^2)*cos(f*x + e)^5 + 10*(5*a*b - b^2)*cos(f*
x + e)^3 - 15*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(b/(a +
 b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x +
 e)*sin(f*x + e)))*sin(f*x + e) + 30*b^2*cos(f*x + e))/(((a^5 + 3*a^4*b + 3
*a^3*b^2 + a^2*b^3)*f*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a
*b^4)*f*cos(f*x + e)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*f)*sin(f*x +
 e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(108) = 216$.

Time = 0.27 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.77

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{15\,b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{(a+b)b}} + \frac{15\,b^2\tan(fx+e)^4+25\,(ab+b^2)\tan(fx+e)^2+8\,a^2+16\,ab+8\,b^2}{(a^3b^2+3\,a^2b^3+3\,ab^4+b^5)\tan(fx+e)^5+2\,(a^4b+4\,a^3b^2+6\,a^2b^3+4\,ab^4+b^5)\tan(fx+e)^3+(a^5+5\,a^4b+10\,a^3b}}{8\,f}$$

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
[Out] -1/8*(15*b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*sqrt((a + b)*b)) + (15*b^2*tan(f*x + e)^4 + 25*(a*b + b^2)*tan(f*x
+ e)^2 + 8*a^2 + 16*a*b + 8*b^2)/((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*tan
(f*x + e)^5 + 2*(a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*tan(f*x + e
)^3 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*tan(f*x + e
)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.43

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{(a^3+3a^2b+3ab^2+b^3)\sqrt{ab+b^2}}+\frac{7b^2\tan(fx+e)^3+9ab\tan(fx+e)+9b^2\tan(fx+e)}{(a^3+3a^2b+3ab^2+b^3)\left(b\tan(fx+e)^2+a+b\right)^2}+\frac{8}{(a^3+3a^2b+3ab^2+b^3)\tan(fx+e)}}{8f}$$

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(
a*b + b^2)))*b/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b + b^2)) + (7*b^2*t
an(f*x + e)^3 + 9*a*b*tan(f*x + e) + 9*b^2*tan(f*x + e))/((a^3 + 3*a^2*b +
3*a*b^2 + b^3)*(b*tan(f*x + e)^2 + a + b)^2) + 8/((a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*tan(f*x + e)))/f
```

Mupad [B] (verification not implemented)

Time = 19.49 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.18

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{1}{a+b}+\frac{25\,b\tan(e+f\,x)^2}{8\,(a+b)^2}+\frac{15\,b^2\tan(e+f\,x)^4}{8\,(a+b)^3}}{f\,\left(\tan\left(e+f\,x\right)^3\,\left(2\,b^2+2\,a\,b\right)+\tan\left(e+f\,x\right)\,\left(a^2+2\,a\,b+b^2\right)+b^2\tan\left(e+f\,x\right)^5\right)} - \frac{15\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{(a+b)^{7/2}}\right)}{8\,f\,(a+b)^{7/2}}$$

```
[In] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] - (1/(a + b) + (25*b*tan(e + f*x)^2)/(8*(a + b)^2) + (15*b^2*tan(e + f*x)^4
)/(8*(a + b)^3))/(f*(tan(e + f*x)^3*(2*a*b + 2*b^2) + tan(e + f*x)*(2*a*b +
 a^2 + b^2) + b^2*tan(e + f*x)^5)) - (15*b^(1/2)*atan((b^(1/2)*tan(e + f*x)
*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(a + b)^(7/2)))/(8*f*(a + b)^(7/2))
```

3.65 $\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 164

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{5(3a-4b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8(a+b)^{9/2}f} - \frac{(a-2b)\cot(e+fx)}{(a+b)^4f} - \frac{\cot^3(e+fx)}{3(a+b)^3f} - \frac{ab\tan(e+fx)}{4(a+b)^3f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{(7a-4b)b\tan(e+fx)}{8(a+b)^4f\left(a+b+b\tan^2(e+fx)\right)}$$

```
[Out] -(a-2*b)*cot(f*x+e)/(a+b)^4/f-1/3*cot(f*x+e)^3/(a+b)^3/f-5/8*(3*a-4*b)*arct
an(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/(a+b)^(9/2)/f-1/4*a*b*tan(f*x+e)
/(a+b)^3/f/(a+b+b*tan(f*x+e)^2)^2-1/8*(7*a-4*b)*b*tan(f*x+e)/(a+b)^4/f/(a+b
+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4217, 467, 1273, 1275, 211}

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{5\sqrt{b}(3a-4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8f(a+b)^{9/2}} - \frac{b(7a-4b)\tan(e+fx)}{8f(a+b)^4\left(a+b\tan^2(e+fx)+b\right)} - \frac{ab\tan(e+fx)}{4f(a+b)^3\left(a+b\tan^2(e+fx)+b\right)^2} - \frac{\cot^3(e+fx)}{3f(a+b)^3} - \frac{(a-2b)\cot(e+fx)}{f(a+b)^4}$$

```
[In] Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (-5*(3*a - 4*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*(a +
 b)^(9/2)*f) - ((a - 2*b)*Cot[e + f*x])/((a + b)^4*f) - Cot[e + f*x]^3/(3*(
a + b)^3*f) - (a*b*Tan[e + f*x])/(4*(a + b)^3*f*(a + b + b*Tan[e + f*x]^2)^
2) - ((7*a - 4*b)*b*Tan[e + f*x])/(8*(a + b)^4*f*(a + b + b*Tan[e + f*x]^2)
)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
```

```
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{ab\tan(e+fx)}{4(a+b)^3 f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b\text{Subst}\left(\int \frac{-\frac{4}{b(a+b)}-\frac{4ax^2}{b(a+b)^2}+\frac{3ax^4}{(a+b)^3}}{x^4(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4f}$$

$$= -\frac{ab\tan(e+fx)}{4(a+b)^3 f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{(7a-4b)b\tan(e+fx)}{8(a+b)^4 f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-8b(a+b)-8(a-b)bx^2+\frac{(7a-4b)b^2x^4}{a+b}}{x^4(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{8b(a+b)^3 f}$$

$$= -\frac{ab\tan(e+fx)}{4(a+b)^3 f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{(7a-4b)b\tan(e+fx)}{8(a+b)^4 f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \left(-\frac{8b}{x^4}+\frac{8b(-a+2b)}{(a+b)x^2}+\frac{5(3a-4b)b^2}{(a+b)(a+b+bx^2)}\right)dx, x, \tan(e+fx)\right)}{8b(a+b)^3 f}$$

$$= -\frac{(a-2b)\cot(e+fx)}{(a+b)^4 f} - \frac{\cot^3(e+fx)}{3(a+b)^3 f} - \frac{ab\tan(e+fx)}{4(a+b)^3 f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{(7a-4b)b\tan(e+fx)}{8(a+b)^4 f\left(a+b+b\tan^2(e+fx)\right)} - \frac{(5(3a-4b)b)\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{8(a+b)^4 f}$$

$$= -\frac{5(3a-4b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8(a+b)^{9/2}f} - \frac{(a-2b)\cot(e+fx)}{(a+b)^4f} - \frac{\cot^3(e+fx)}{3(a+b)^3f} - \frac{ab\tan(e+fx)}{4(a+b)^3f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{(7a-4b)b\tan(e+fx)}{8(a+b)^4f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.03 (sec) , antiderivative size = 994, normalized size of antiderivative = 6.06

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(\frac{480(3a-4b)b\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{}$$

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*((480*(3*a - 4*b)*b*ArcTan[(
Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x])
)/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*
x)])^2*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])
 - (Csc[e]*Csc[e + f*x]^3*Sec[2*e]*(4*(44*a^4 + 122*a^3*b + 63*a^2*b^2 + 12
6*a*b^3 + 36*b^4)*Sin[f*x] + (-96*a^4 - 71*a^3*b + 344*a^2*b^2 - 1208*a*b^3
 + 48*b^4)*Sin[3*f*x] + 224*a^4*Sin[2*e - f*x] + 576*a^3*b*Sin[2*e - f*x] +
 124*a^2*b^2*Sin[2*e - f*x] - 2184*a*b^3*Sin[2*e - f*x] + 144*b^4*Sin[2*e -
 f*x] - 224*a^4*Sin[2*e + f*x] - 657*a^3*b*Sin[2*e + f*x] - 538*a^2*b^2*Sin
[2*e + f*x] + 984*a*b^3*Sin[2*e + f*x] + 144*b^4*Sin[2*e + f*x] + 176*a^4*S
in[4*e + f*x] + 569*a^3*b*Sin[4*e + f*x] + 666*a^2*b^2*Sin[4*e + f*x] + 170
4*a*b^3*Sin[4*e + f*x] - 144*b^4*Sin[4*e + f*x] + 48*a^4*Sin[2*e + 3*f*x] +
 111*a^3*b*Sin[2*e + 3*f*x] + 360*a^2*b^2*Sin[2*e + 3*f*x] + 312*a*b^3*Sin[
2*e + 3*f*x] - 48*b^4*Sin[2*e + 3*f*x] - 96*a^4*Sin[4*e + 3*f*x] - 152*a^3*
b*Sin[4*e + 3*f*x] + 146*a^2*b^2*Sin[4*e + 3*f*x] - 728*a*b^3*Sin[4*e + 3*f
*x] - 48*b^4*Sin[4*e + 3*f*x] + 48*a^4*Sin[6*e + 3*f*x] + 192*a^3*b*Sin[6*e
 + 3*f*x] + 558*a^2*b^2*Sin[6*e + 3*f*x] - 168*a*b^3*Sin[6*e + 3*f*x] + 48*
b^4*Sin[6*e + 3*f*x] + 16*a^4*Sin[2*e + 5*f*x] - 598*a^2*b^2*Sin[2*e + 5*f*
x] + 48*a*b^3*Sin[2*e + 5*f*x] + 72*a^3*b*Sin[4*e + 5*f*x] + 150*a^2*b^2*Si
n[4*e + 5*f*x] - 48*a*b^3*Sin[4*e + 5*f*x] + 16*a^4*Sin[6*e + 5*f*x] + 27*a
^3*b*Sin[6*e + 5*f*x] - 388*a^2*b^2*Sin[6*e + 5*f*x] + 45*a^3*b*Sin[8*e + 5
*f*x] - 60*a^2*b^2*Sin[8*e + 5*f*x] + 16*a^4*Sin[4*e + 7*f*x] - 83*a^3*b*Si
n[4*e + 7*f*x] + 6*a^2*b^2*Sin[4*e + 7*f*x] + 27*a^3*b*Sin[6*e + 7*f*x] - 6
*a^2*b^2*Sin[6*e + 7*f*x] + 16*a^4*Sin[8*e + 7*f*x] - 56*a^3*b*Sin[8*e + 7*
f*x]))/a))/(6144*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\left(\frac{7}{8}ab-\frac{1}{2}b^2\right)\tan(fx+e)^3+\left(\frac{9}{8}a^2+\frac{5}{8}ab-\frac{1}{2}b^2\right)\tan(fx+e)}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{5(3a-4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}}\right)}{(a+b)^4}-\frac{1}{3(a+b)^3\tan(fx+e)^3}-\frac{a-}{(a+b)^4\text{t}}}{f}$
default	$\frac{-\frac{b\left(\frac{\left(\frac{7}{8}ab-\frac{1}{2}b^2\right)\tan(fx+e)^3+\left(\frac{9}{8}a^2+\frac{5}{8}ab-\frac{1}{2}b^2\right)\tan(fx+e)}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{5(3a-4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}}\right)}{(a+b)^4}-\frac{1}{3(a+b)^3\tan(fx+e)^3}-\frac{a-}{(a+b)^4\text{t}}}{f}$
risch	$-\frac{i\left(-569a^3b\,e^{8i(fx+e)}-666a^2b^2e^{8i(fx+e)}-1704a\,b^3e^{8i(fx+e)}-176a^4e^{8i(fx+e)}-144b^4e^{6i(fx+e)}-576a^3b\,e^{6i(fx+e)}-12\right.}{}$

```
[In] int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/(a+b)^4*b*(((7/8*a*b-1/2*b^2)*tan(f*x+e)^3+(9/8*a^2+5/8*a*b-1/2*b^2
)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+5/8*(3*a-4*b)/((a+b)*b)^(1/2)*arctan(b
*tan(f*x+e)/((a+b)*b)^(1/2)))-1/3/(a+b)^3/tan(f*x+e)^3-(a-2*b)/(a+b)^4/tan(
f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(148) = 296$.

Time = 0.33 (sec) , antiderivative size = 1009, normalized size of antiderivative = 6.15

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/96*(4*(16*a^3 - 83*a^2*b + 6*a*b^2)*cos(f*x + e)^7 - 4*(24*a^3 - 134*a^
2*b + 145*a*b^2 - 12*b^3)*cos(f*x + e)^5 - 20*(15*a^2*b - 32*a*b^2 + 16*b^3
)*cos(f*x + e)^3 + 15*((3*a^3 - 4*a^2*b)*cos(f*x + e)^6 - (3*a^3 - 10*a^2*b
 + 8*a*b^2)*cos(f*x + e)^4 - 3*a*b^2 + 4*b^3 - (6*a^2*b - 11*a*b^2 + 4*b^3)
*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4
 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)
^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*co
s(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) - 60*(3*a*b^2 - 4*
b^3)*cos(f*x + e))/(((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*f*co
s(f*x + e)^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5
)*f*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5
 - b^6)*f*cos(f*x + e)^2 - (a^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6
```

```
)*f)*sin(f*x + e)), -1/48*(2*(16*a^3 - 83*a^2*b + 6*a*b^2)*cos(f*x + e)^7 -
 2*(24*a^3 - 134*a^2*b + 145*a*b^2 - 12*b^3)*cos(f*x + e)^5 - 10*(15*a^2*b
- 32*a*b^2 + 16*b^3)*cos(f*x + e)^3 - 15*((3*a^3 - 4*a^2*b)*cos(f*x + e)^6
- (3*a^3 - 10*a^2*b + 8*a*b^2)*cos(f*x + e)^4 - 3*a*b^2 + 4*b^3 - (6*a^2*b
- 11*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*c
os(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x +
 e) - 30*(3*a*b^2 - 4*b^3)*cos(f*x + e))/(((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a
^3*b^3 + a^2*b^4)*f*cos(f*x + e)^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a^3*b^3
 - 7*a^2*b^4 - 2*a*b^5)*f*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 + 8*a^3*b^3
 + 2*a^2*b^4 - 2*a*b^5 - b^6)*f*cos(f*x + e)^2 - (a^4*b^2 + 4*a^3*b^3 + 6*a
^2*b^4 + 4*a*b^5 + b^6)*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(148) = 296$.

Time = 0.27 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.97

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{15\,(3\,ab-4\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\sqrt{(a+b)b}} + \frac{15\,(3\,ab^2-4\,b^3)\tan(fx+e)^6+25\,(3\,a^2b-ab^2-4\,b^3)\tan(fx+e)^4+8\,a^3+24\,a^2b+24}{(a^4b^2+4\,a^3b^3+6\,a^2b^4+4\,ab^5+b^6)\tan(fx+e)^7+2\,(a^5b+5\,a^4b^2+10\,a^3b^3+10\,a^2b^4+5\,ab^5+b^6)\tan(fx}}{24\,f}$$

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/24*(15*(3*a*b - 4*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4 + 4*
a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt((a + b)*b)) + (15*(3*a*b^2 - 4*b^3)
*tan(f*x + e)^6 + 25*(3*a^2*b - a*b^2 - 4*b^3)*tan(f*x + e)^4 + 8*a^3 + 24*
a^2*b + 24*a*b^2 + 8*b^3 + 8*(3*a^3 + 2*a^2*b - 5*a*b^2 - 4*b^3)*tan(f*x +
e)^2)/((a^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6)*tan(f*x + e)^7 + 2
*(a^5*b + 5*a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*tan(f*x + e)
^5 + (a^6 + 6*a^5*b + 15*a^4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6)
*tan(f*x + e)^3))/f
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.61

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3ab-4b^2)}{(a^4+4a^3b+6a^2b^2+4ab^3+b^4)\sqrt{ab+b^2}}+\frac{3\left(7ab^2\tan(fx+e)^3-4b^3\tan(fx+e)^3+9a^2b\tan(fx+e)+5ab^2\tan(fx+e)-\right.}{(a^4+4a^3b+6a^2b^2+4ab^3+b^4)\left(b\tan(fx+e)^2+a+b\right)^2}}{24f}$$

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/24*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt
(a*b + b^2)))*(3*a*b - 4*b^2)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*
sqrt(a*b + b^2)) + 3*(7*a*b^2*tan(f*x + e)^3 - 4*b^3*tan(f*x + e)^3 + 9*a^2
*b*tan(f*x + e) + 5*a*b^2*tan(f*x + e) - 4*b^3*tan(f*x + e))/((a^4 + 4*a^3*
b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(b*tan(f*x + e)^2 + a + b)^2) + 8*(3*a*tan(f
*x + e)^2 - 6*b*tan(f*x + e)^2 + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b
^3 + b^4)*tan(f*x + e)^3))/f
```

Mupad [B] (verification not implemented)

Time = 20.64 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.26

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{1}{3(a+b)}+\frac{25\tan(e+fx)^4(3ab-4b^2)}{24(a+b)^3}+\frac{\tan(e+fx)^2(3a-4b)}{3(a+b)^2}+\frac{5\tan(e+fx)^6(3ab^2-4b^3)}{8(a+b)^4}}{f\left(\tan(e+fx)^3(a^2+2ab+b^2)+\tan(e+fx)^5(2b^2+2ab)+b^2\tan(e+fx)^7\right)} - \frac{5\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+fx)(a^4+4a^3b+6a^2b^2+4ab^3+b^4)}{(a+b)^{9/2}}\right)(3a-4b)}{8f(a+b)^{9/2}}$$

```
[In] int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] - (1/(3*(a + b)) + (25*tan(e + f*x)^4*(3*a*b - 4*b^2))/(24*(a + b)^3) + (ta
n(e + f*x)^2*(3*a - 4*b))/(3*(a + b)^2) + (5*tan(e + f*x)^6*(3*a*b^2 - 4*b^
3))/(8*(a + b)^4))/(f*(tan(e + f*x)^3*(2*a*b + a^2 + b^2) + tan(e + f*x)^5*
(2*a*b + 2*b^2) + b^2*tan(e + f*x)^7)) - (5*b^(1/2)*atan((b^(1/2)*tan(e + f
*x)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2))/(a + b)^(9/2))*(3*a - 4*b)
)/(8*f*(a + b)^(9/2))
```

3.66 $\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 242

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\sqrt{b}(15a^2-40ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8(a+b)^{11/2}f} - \frac{(5a^2-20ab+2b^2)\cot(e+fx)}{5(a+b)^5f} - \frac{(10a+b)\cot^3(e+fx)}{15(a+b)^4f} - \frac{\cot^5(e+fx)}{5(a+b)f\,(a+b+b\tan^2(e+fx))^2} - \frac{b(5a^2+4b^2)\tan(e+fx)}{20(a+b)^4f\,(a+b+b\tan^2(e+fx))^2} - \frac{b(35a^2-40ab+24b^2)\tan(e+fx)}{40(a+b)^5f\,(a+b+b\tan^2(e+fx))}$$

```
[Out] -1/5*(5*a^2-20*a*b+2*b^2)*cot(f*x+e)/(a+b)^5/f-1/15*(10*a+b)*cot(f*x+e)^3/(
a+b)^4/f-1/8*(15*a^2-40*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b
^(1/2)/(a+b)^(11/2)/f-1/5*cot(f*x+e)^5/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/20*
b*(5*a^2+4*b^2)*tan(f*x+e)/(a+b)^4/f/(a+b+b*tan(f*x+e)^2)^2-1/40*b*(35*a^2-
40*a*b+24*b^2)*tan(f*x+e)/(a+b)^5/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4217, 473, 467, 1273, 1275, 211}

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\sqrt{b}(15a^2-40ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8f(a+b)^{11/2}} - \frac{b(35a^2-40ab+24b^2)\tan(e+fx)}{40f(a+b)^5\,(a+b\tan^2(e+fx)+b)} - \frac{b(5a^2+4b^2)\tan(e+fx)}{20f(a+b)^4\,(a+b\tan^2(e+fx)+b)^2} - \frac{(5a^2-20ab+2b^2)\cot(e+fx)}{5f(a+b)^5} - \frac{(10a+b)\cot^3(e+fx)}{15f(a+b)^4} - \frac{\cot^5(e+fx)}{5f(a+b)\,(a+b\tan^2(e+fx)+b)^2}$$

```
[In] Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -1/8*(Sqrt[b]*(15*a^2 - 40*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[
a + b]])/((a + b)^(11/2)*f) - ((5*a^2 - 20*a*b + 2*b^2)*Cot[e + f*x])/(5*(a
 + b)^5*f) - ((10*a + b)*Cot[e + f*x]^3)/(15*(a + b)^4*f) - Cot[e + f*x]^5/
(5*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) - (b*(5*a^2 + 4*b^2)*Tan[e + f*x
])/(20*(a + b)^4*f*(a + b + b*Tan[e + f*x]^2)^2) - (b*(35*a^2 - 40*a*b + 24
*b^2)*Tan[e + f*x])/(40*(a + b)^5*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
```

```
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^6(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{10a+b+5(a+b)x^2}{x^4(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{5(a+b)f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(5a^2+4b^2)\tan(e+fx)}{20(a+b)^4f\left(a+b+b\tan^2(e+fx)\right)^2}$$

$$- \frac{b\text{Subst}\left(\int \frac{-\frac{4(10a+b)}{b(a+b)} - \frac{4(5a^2+4b^2)x^2}{b(a+b)^2} + \frac{3(5a^2+4b^2)x^4}{(a+b)^3}}{x^4(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{20(a+b)f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(5a^2+4b^2)\tan(e+fx)}{20(a+b)^4 f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(35a^2-40ab+24b^2)\tan(e+fx)}{40(a+b)^5 f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-8b(a+b)(10a+b)-8b(5a^2-10ab+3b^2)x^2+\frac{b^2(35a^2-40ab+24b^2)x^4}{a+b}}{x^4(a+b+bx^2)}dx, x, \tan(e+fx)\right)}{40b(a+b)^4 f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(5a^2+4b^2)\tan(e+fx)}{20(a+b)^4 f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(35a^2-40ab+24b^2)\tan(e+fx)}{40(a+b)^5 f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \left(-\frac{8b(10a+b)}{x^4} - \frac{8b(5a^2-20ab+2b^2)}{(a+b)x^2} + \frac{5b^2(15a^2-40ab+8b^2)}{(a+b)(a+b+bx^2)}\right)dx, x, \tan(e+fx)\right)}{40b(a+b)^4 f}$$

$$= -\frac{(5a^2-20ab+2b^2)\cot(e+fx)}{5(a+b)^5 f} - \frac{(10a+b)\cot^3(e+fx)}{15(a+b)^4 f} - \frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(5a^2+4b^2)\tan(e+fx)}{20(a+b)^4 f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(35a^2-40ab+24b^2)\tan(e+fx)}{40(a+b)^5 f\left(a+b+b\tan^2(e+fx)\right)} - \frac{(b(15a^2-40ab+8b^2))\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}dx, x, \tan(e+fx)\right)}{8(a+b)^5 f}$$

$$= -\frac{\sqrt{b}(15a^2-40ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8(a+b)^{11/2}f} - \frac{(5a^2-20ab+2b^2)\cot(e+fx)}{5(a+b)^5 f} - \frac{(10a+b)\cot^3(e+fx)}{15(a+b)^4 f} - \frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(5a^2+4b^2)\tan(e+fx)}{20(a+b)^4 f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(35a^2-40ab+24b^2)\tan(e+fx)}{40(a+b)^5 f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.57 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.98

$$\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(-8(4a-11b)(a+b)(a+2b+a\cos(2(e+fx)))^2\cot(e)\csc^2(e\right.}{}$$

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(-8*(4*a - 11*b)*(a + b)*(a
+ 2*b + a*Cos[2*(e + f*x)])^2*Cot[e]*Csc[e + f*x]^2 - 24*(a + b)^2*(a + 2*b
 + a*Cos[2*(e + f*x)])^2*Cot[e]*Csc[e + f*x]^4 + (15*b*(15*a^2 - 40*a*b + 8
*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*S
in[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a
*Cos[2*(e + f*x)])^2*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] -
 I*Sin[e])^4]) + 8*(8*a^2 - 59*a*b + 23*b^2)*(a + 2*b + a*Cos[2*(e + f*x)])
^2*Csc[e]*Csc[e + f*x]*Sin[f*x] + 8*(4*a - 11*b)*(a + b)*(a + 2*b + a*Cos[2
*(e + f*x)])^2*Csc[e]*Csc[e + f*x]^3*Sin[f*x] + 24*(a + b)^2*(a + 2*b + a*C
os[2*(e + f*x)])^2*Csc[e]*Csc[e + f*x]^5*Sin[f*x] - 60*b^2*(a + b)*Sec[2*e]
*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]) + 15*b*(a + 2*b + a*Cos[2*(e + f*x)])*
Sec[2*e]*((9*a^2 + 16*a*b - 8*b^2)*Sin[2*e] + 3*a*(-3*a + 2*b)*Sin[2*f*x]))
)/(960*(a + b)^5*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A] (verified)

Time = 2.55 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\left(\frac{7}{8}a^2 b-a\,b^2\right)\tan(fx+e)^3+\frac{a\left(9a^2+ab-8b^2\right)\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2-40ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}}\right)}{(a+b)^5}-\frac{1}{5(a+b)^3\tan(fx+e)^5}-3}{f}$
default	$\frac{-\frac{b\left(\frac{\left(\frac{7}{8}a^2 b-a\,b^2\right)\tan(fx+e)^3+\frac{a\left(9a^2+ab-8b^2\right)\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2-40ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}}\right)}{(a+b)^5}-\frac{1}{5(a+b)^3\tan(fx+e)^5}-3}{f}$
risch	Expression too large to display

```
[In] int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-b/(a+b)^5*(((7/8*a^2*b-a*b^2)*tan(f*x+e)^3+1/8*a*(9*a^2+a*b-8*b^2)*ta
n(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(15*a^2-40*a*b+8*b^2)/((a+b)*b)^(1/2)*
arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))-1/5/(a+b)^3/tan(f*x+e)^5-1/3*(2*a-b)/
(a+b)^4/tan(f*x+e)^3-(a^2-4*a*b+b^2)/(a+b)^5/tan(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 668 vs. $2(222) = 444$.

Time = 0.36 (sec) , antiderivative size = 1423, normalized size of antiderivative = 5.88

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/480*(4*(64*a^4 - 607*a^3*b + 274*a^2*b^2)*cos(f*x + e)^9 - 4*(160*a^4 -
 1533*a^3*b + 1599*a^2*b^2 - 488*a*b^3)*cos(f*x + e)^7 + 4*(120*a^4 - 1205*
a^3*b + 2769*a^2*b^2 - 1392*a*b^3 + 184*b^4)*cos(f*x + e)^5 + 20*(75*a^3*b
- 305*a^2*b^2 + 320*a*b^3 - 56*b^4)*cos(f*x + e)^3 - 15*((15*a^4 - 40*a^3*b
 + 8*a^2*b^2)*cos(f*x + e)^8 - 2*(15*a^4 - 55*a^3*b + 48*a^2*b^2 - 8*a*b^3)
*cos(f*x + e)^6 + (15*a^4 - 100*a^3*b + 183*a^2*b^2 - 72*a*b^3 + 8*b^4)*cos
(f*x + e)^4 + 15*a^2*b^2 - 40*a*b^3 + 8*b^4 + 2*(15*a^3*b - 55*a^2*b^2 + 48
*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)
*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^
2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e)
 + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 6
0*(15*a^2*b^2 - 40*a*b^3 + 8*b^4)*cos(f*x + e))/(((a^7 + 5*a^6*b + 10*a^5*b
^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b
+ 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 + (a^7 + a^6*
b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*cos(f*
x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*
cos(f*x + e)^2 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 +
 b^7)*f)*sin(f*x + e)), -1/240*(2*(64*a^4 - 607*a^3*b + 274*a^2*b^2)*cos(f*
x + e)^9 - 2*(160*a^4 - 1533*a^3*b + 1599*a^2*b^2 - 488*a*b^3)*cos(f*x + e)
^7 + 2*(120*a^4 - 1205*a^3*b + 2769*a^2*b^2 - 1392*a*b^3 + 184*b^4)*cos(f*x
 + e)^5 + 10*(75*a^3*b - 305*a^2*b^2 + 320*a*b^3 - 56*b^4)*cos(f*x + e)^3 -
 15*((15*a^4 - 40*a^3*b + 8*a^2*b^2)*cos(f*x + e)^8 - 2*(15*a^4 - 55*a^3*b
+ 48*a^2*b^2 - 8*a*b^3)*cos(f*x + e)^6 + (15*a^4 - 100*a^3*b + 183*a^2*b^2
- 72*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 15*a^2*b^2 - 40*a*b^3 + 8*b^4 + 2*(15*
a^3*b - 55*a^2*b^2 + 48*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqrt(b/(a + b))*arct
an(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f
*x + e)))*sin(f*x + e) + 30*(15*a^2*b^2 - 40*a*b^3 + 8*b^4)*cos(f*x + e))/(
((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x
+ e)^8 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*co
s(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b
^5 + a*b^6 + b^7)*f*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a
^2*b^5 - 4*a*b^6 - b^7)*f*cos(f*x + e)^2 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^
4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.79

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{15\,(15\,a^2b-40\,ab^2+8\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5)\sqrt{(a+b)b}} + \frac{15\,(15\,a^2b^2-40\,ab^3+8\,b^4)\tan(fx+e)^8+25\,(15\,a^3b-25\,a^2b^2-32\,ab^3+8\,b^4)\tan(fx+e)^6+8}{(a^5b^2+5\,a^4b^3+10\,a^3b^4+10\,a^2b^5+5\,ab^6+b^7)\tan(fx+e)^9+2\,(a^6b+6\,a^5b^2}}{120}$$

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/120*(15*(15*a^2*b - 40*a*b^2 + 8*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)
*b))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*sqrt((a + b
)*b)) + (15*(15*a^2*b^2 - 40*a*b^3 + 8*b^4)*tan(f*x + e)^8 + 25*(15*a^3*b -
 25*a^2*b^2 - 32*a*b^3 + 8*b^4)*tan(f*x + e)^6 + 8*(15*a^4 - 10*a^3*b - 57*
a^2*b^2 - 24*a*b^3 + 8*b^4)*tan(f*x + e)^4 + 24*a^4 + 96*a^3*b + 144*a^2*b^
2 + 96*a*b^3 + 24*b^4 + 8*(10*a^4 + 31*a^3*b + 33*a^2*b^2 + 13*a*b^3 + b^4)
*tan(f*x + e)^2)/((a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6
+ b^7)*tan(f*x + e)^9 + 2*(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*b^4 + 15
*a^2*b^5 + 6*a*b^6 + b^7)*tan(f*x + e)^7 + (a^7 + 7*a^6*b + 21*a^5*b^2 + 35
*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*tan(f*x + e)^5))/f
```

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.52

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{15\,(15\,a^2b-40\,ab^2+8\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5)\sqrt{ab+b^2}} + \frac{15\left(7\,a^2b^2\tan(fx+e)^3-8\,ab^3\tan(fx+e)^3+9\,a^3b\tan(fx+e)+a^2b^2\,\mathrm{t}\right.}{(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5)\left(b\tan(fx+\right.}}{}$$

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/120*(15*(15*a^2*b - 40*a*b^2 + 8*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(
b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 5*a^4*b + 10*a^3*b^2 +
 10*a^2*b^3 + 5*a*b^4 + b^5)*sqrt(a*b + b^2)) + 15*(7*a^2*b^2*tan(f*x + e)^
3 - 8*a*b^3*tan(f*x + e)^3 + 9*a^3*b*tan(f*x + e) + a^2*b^2*tan(f*x + e) -
8*a*b^3*tan(f*x + e))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 +
 b^5)*(b*tan(f*x + e)^2 + a + b)^2) + 8*(15*a^2*tan(f*x + e)^4 - 60*a*b*tan
(f*x + e)^4 + 15*b^2*tan(f*x + e)^4 + 10*a^2*tan(f*x + e)^2 + 5*a*b*tan(f*x
 + e)^2 - 5*b^2*tan(f*x + e)^2 + 3*a^2 + 6*a*b + 3*b^2)/((a^5 + 5*a^4*b + 1
0*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*tan(f*x + e)^5))/f
```

Mupad [B] (verification not implemented)

Time = 21.39 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.10

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{1}{5\,(a+b)}+\frac{\tan(e+f\,x)^2\,(10\,a+b)}{15\,(a+b)^2}+\frac{5\tan(e+f\,x)^6\,(15\,a^2\,b-40\,a\,b^2+8\,b^3)}{24\,(a+b)^4}+\frac{\tan(e+f\,x)^4\,(15\,a^2-40\,a\,b+8\,b^2)}{15\,(a+b)^3}+\frac{\tan(e+f\,x)^8\,(15\,a^2\,b^2-}{8\,(a+b)^5}}{f\,\left(\tan\left(e+f\,x\right)^5\,\left(a^2+2\,a\,b+b^2\right)+\tan\left(e+f\,x\right)^7\,\left(2\,b^2+2\,a\,b\right)+b^2\tan\left(e+f\,x\right)^9\right)} - \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)\,\left(a^5+5\,a^4\,b+10\,a^3\,b^2+10\,a^2\,b^3+5\,a\,b^4+b^5\right)}{(a+b)^{11/2}}\right)\,\left(15\,a^2-40\,a\,b+8\,b^2\right)}{8\,f\,(a+b)^{11/2}}$$

```
[In] int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] - (1/(5*(a + b)) + (tan(e + f*x)^2*(10*a + b))/(15*(a + b)^2) + (5*tan(e +
f*x)^6*(15*a^2*b - 40*a*b^2 + 8*b^3))/(24*(a + b)^4) + (tan(e + f*x)^4*(15*
a^2 - 40*a*b + 8*b^2))/(15*(a + b)^3) + (tan(e + f*x)^8*(8*b^4 - 40*a*b^3 +
 15*a^2*b^2))/(8*(a + b)^5))/(f*(tan(e + f*x)^5*(2*a*b + a^2 + b^2) + tan(e
 + f*x)^7*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^9)) - (b^(1/2)*atan((b^(1/2)*t
an(e + f*x)*(5*a*b^4 + 5*a^4*b + a^5 + b^5 + 10*a^2*b^3 + 10*a^3*b^2))/(a +
 b)^(11/2))*(15*a^2 - 40*a*b + 8*b^2))/(8*f*(a + b)^(11/2))
```

3.67 $\int \sqrt{a+b\sec^2(e+fx)}\sin^5(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 139

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^5(e+fx)\,dx = \frac{\sqrt{b}\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{2(5a+b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{15a^2f} - \frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{5af}$$

```
[Out] 2/15*(5*a+b)*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2)/a^2/f-1/5*cos(f*x+e)^5*(
a+b*sec(f*x+e)^2)^(3/2)/a/f+arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(
1/2))*b^(1/2)/f-cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used

= {4219, 473, 462, 283, 223, 212}

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^5(e+fx)\,dx = \frac{2(5a+b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{15a^2f} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{5af} - \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}$$

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^5,x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - (C
os[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/f + (2*(5*a + b)*Cos[e + f*x]^3*(a
+ b*Sec[e + f*x]^2)^(3/2))/(15*a^2*f) - (Cos[e + f*x]^5*(a + b*Sec[e + f*x]
^2)^(3/2))/(5*a*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(-1+x^2)^2\sqrt{a+bx^2}}{x^6}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)\,(a+b\sec^2(e+fx))^{3/2}}{5af} + \frac{\text{Subst}\left(\int \frac{(-2(5a+b)+5ax^2)\sqrt{a+bx^2}}{x^4}\,dx, x, \sec(e+fx)\right)}{5af}$$

$$= \frac{2(5a+b)\cos^3(e+fx)\,(a+b\sec^2(e+fx))^{3/2}}{15a^2f} - \frac{\cos^5(e+fx)\,(a+b\sec^2(e+fx))^{3/2}}{5af} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{2(5a+b)\cos^3(e+fx)\,(a+b\sec^2(e+fx))^{3/2}}{15a^2f} - \frac{\cos^5(e+fx)\,(a+b\sec^2(e+fx))^{3/2}}{5af} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{2(5a+b)\cos^3(e+fx)\,(a+b\sec^2(e+fx))^{3/2}}{15a^2f} - \frac{\cos^5(e+fx)\,(a+b\sec^2(e+fx))^{3/2}}{5af} + \frac{b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}$$

$$= \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{2(5a+b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{15a^2f} - \frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{5af}$$

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.09

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^5(e+fx)\,dx = -\frac{\cos(e+fx)\left(-2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b+a\cos^2(e+fx)}}{\sqrt{b}}\right)+2\sqrt{b+a\cos^2(e+fx)}-\frac{2(2a+b)\left(b+a\cos^2(e+fx)\right)^{3/2}}{3a^2}+\frac{2(b+a}{}\right.}{\sqrt{2}f\sqrt{a+2b+a\cos(2e+2fx)}}$$

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^5,x]
```

```
[Out] -((Cos[e + f*x]*(-2*Sqrt[b]*ArcTanh[Sqrt[b + a*Cos[e + f*x]^2]/Sqrt[b]] + 2
*Sqrt[b + a*Cos[e + f*x]^2] - (2*(2*a + b)*(b + a*Cos[e + f*x]^2)^(3/2))/(3
*a^2) + (2*(b + a*Cos[e + f*x]^2)^(5/2))/(5*a^2))*Sqrt[a + b*Sec[e + f*x]^2
])/(Sqrt[2]*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. 2(123) = 246.

Time = 3.31 (sec) , antiderivative size = 572, normalized size of antiderivative = 4.12

method	result
default	$\frac{\left(-3\cos(fx+e)^5\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,a^2-3\cos(fx+e)^4\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,a^2+10\cos(fx+e)^3\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,a^2-\cos(fx+e)^3\sqrt{\frac{b+a\,\mathrm{co}}{(1+\mathrm{cos}}}\right.}{}$

```
[In] int(sin(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15/f/a^2*(-3*cos(f*x+e)^5*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2
-3*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+10*cos(f*x+
e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2-cos(f*x+e)^3*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b+10*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*a^2-cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*a*b+15*b^(1/2)*ln(-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)-4*
```

```
sec(f*x+e)*b)*a^2-15*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*a^2+10*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b+2*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*cos(f*x+e)-15*a^2*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)+10*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*a*b+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2)*(a+b*sec(f*x+e)^2)^
(1/2)*cos(f*x+e)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.12

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^5(e+fx)\,dx$$
$$=\left[\frac{15\,a^2\sqrt{b}\log\left(\frac{a\cos(fx+e)^2+2\,\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2\,b}{\cos(fx+e)^2}\right)-2\left(3\,a^2\cos(fx+e)^5-(10\,a^2-ab)\cos(fx+e)\right.}{30\,a^2f}\right.$$
$$-\frac{15\,a^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{b}\right)+\left(3\,a^2\cos(fx+e)^5-(10\,a^2-ab)\cos(fx+e)^3+(15\,a\right.}{15\,a^2f}$$

```
[In] integrate(sin(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*a^2*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) - 2*(3*a^2*cos
(f*x + e)^5 - (10*a^2 - a*b)*cos(f*x + e)^3 + (15*a^2 - 10*a*b - 2*b^2)*cos
(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^2*f), -1/15*(15*
a^2*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*co
s(f*x + e)/b) + (3*a^2*cos(f*x + e)^5 - (10*a^2 - a*b)*cos(f*x + e)^3 + (15
*a^2 - 10*a*b - 2*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2))/(a^2*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^5(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**5*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.23

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^5(e+fx)\,dx$$
$$= \frac{\frac{20\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3}{a} - 30\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e) - 15\sqrt{b}\log\left(\frac{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)-\sqrt{b}}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)+\sqrt{b}}\right) - \frac{2\left(3 \cdots\right.}{\cdots}}{30\,f}$$

```
[In] integrate(sin(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/30*(20*(a + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3/a - 30*sqrt(a + b/cos(
f*x + e)^2)*cos(f*x + e) - 15*sqrt(b)*log((sqrt(a + b/cos(f*x + e)^2)*cos(f
*x + e) - sqrt(b))/(sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b))) - 2
*(3*(a + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 5*(a + b/cos(f*x + e)^2)^
(3/2)*b*cos(f*x + e)^3)/a^2)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1281 vs. 2(123) = 246.

Time = 0.65 (sec) , antiderivative size = 1281, normalized size of antiderivative = 9.22

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^5(e+fx)\,dx = \text{Too large to display}$$

```
[In] integrate(sin(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/15*(15*b*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/
2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - sqrt(a + b))/sqrt(-b))/sqrt(-b) - 2*(
15*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*
tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2
```

```
*e)^2 + a + b))^9*b + 165*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2
+ 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^8*sqrt(a + b)*b - 20*(sqrt(a + b)*ta
n(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*
e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^7*
(16*a^2 + 5*a*b - 27*b^2) + 20*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a
*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*
e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^6*(32*a^2 - 83*a*b + 33*b^2)*sq
rt(a + b) + 2*(416*a^3 + 625*a^2*b - 1230*a*b^2 - 15*b^3)*(sqrt(a + b)*tan(
1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)
^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^5 -
10*(256*a^3 - 391*a^2*b + 90*a*b^2 + 81*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2
*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(
1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^4*sqrt(a + b) + 2
0*(16*a^4 - 161*a^3*b + 157*a^2*b^2 + 45*a*b^3 - 33*b^4)*(sqrt(a + b)*tan(1
/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^
4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^3 + 2
0*(160*a^4 - 159*a^3*b - 49*a^2*b^2 + 75*a*b^3 - 3*b^4)*(sqrt(a + b)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4
 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2*sqrt
(a + b) - 5*(576*a^5 - 667*a^4*b - 76*a^3*b^2 + 286*a^2*b^3 - 140*a*b^4 - 2
7*b^5)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4
+ b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x +
 1/2*e)^2 + a + b)) + (768*a^5 - 1539*a^4*b + 1028*a^3*b^2 - 450*a^2*b^3 +
100*a*b^4 + 45*b^5)*sqrt(a + b))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1
/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 + 2*(sqrt(a + b)*tan(1/2*f
*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 -
2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a +
b) - 3*a + b)^5)*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^5(e+fx)\,dx=\int \sin\left(e+f\,x\right)^5\,\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}\,dx$$

```
[In] int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.68 $\int \sqrt{a+b\sec^2(e+fx)}\sin^3(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 100

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^3(e+fx)\,dx = \frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af}$$

```
[Out] 1/3*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2)/a/f+arctanh(sec(f*x+e)*b^(1/2)/(a
+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4219, 462, 283, 223, 212}

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^3(e+fx)\,dx = \frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} + \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af} - \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}$$

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^3,x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - (C
os[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/f + (Cos[e + f*x]^3*(a + b*Sec[e +
f*x]^2)^(3/2))/(3*a*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(-1+x^2)\sqrt{a+bx^2}}{x^4}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af} + \frac{b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}$$

$$= \frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.20

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^3(e+fx)\,dx = \frac{\sqrt{2}\cos(e+fx)\left(3a\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b+a\cos^2(e+fx)}}{\sqrt{b}}\right) + \sqrt{b+a\cos^2(e+fx)}(-3a+b+a\cos^2(e+fx))\right)\sqrt{a-}}{3af\sqrt{a+2b+a\cos(2(e+fx))}}$$

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^3,x]
[Out] (Sqrt[2]*Cos[e + f*x]*(3*a*Sqrt[b]*ArcTanh[Sqrt[b + a*Cos[e + f*x]^2]/Sqrt[
b]] + Sqrt[b + a*Cos[e + f*x]^2]*(-3*a + b + a*Cos[e + f*x]^2))*Sqrt[a + b*
Sec[e + f*x]^2])/(3*a*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(88) = 176.

Time = 0.38 (sec) , antiderivative size = 340, normalized size of antiderivative = 3.40

method	result
default	$\frac{\left(\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)^3 a+\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)^2 a+3\ln\left(-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sec(fx+e\right.\right.}{3fa}$

```
[In] int(sin(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f/a*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^3*a+((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^2*a+3*ln(-4*b^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*sec(f*x+e)-4*sec(f*x+e)*b)*b^(1/2)*a-3*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*cos(f*x+e)*b-3*a*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b)*(a+b*sec(f*x+e)^2)^(1/2)*cos(f*x+e)/
(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.32

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^3(e+fx)\,dx$$

$$= \left[\frac{3\,a\sqrt{b}\log\left(\frac{a\cos(fx+e)^2+2\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2\,b}{\cos(fx+e)^2}\right)+2\left(a\cos(fx+e)^3-(3\,a-b)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{6\,af}, -\frac{3\,a\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{b}\right)-\left(a\cos(fx+e)^3-(3\,a-b)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{3\,af}\right]$$

```
[In] integrate(sin(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*a*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*(a*cos(f*x + e
)^3 - (3*a - b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/
(a*f), -1/3*(3*a*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*cos(f*x + e)/b) - (a*cos(f*x + e)^3 - (3*a - b)*cos(f*x + e))*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^3(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**3*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.16

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^3(e+fx)\,dx = \frac{\frac{2\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3}{a} - 6\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e) - 3\sqrt{b}\log\left(\frac{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)-\sqrt{b}}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)+\sqrt{b}}\right)}{6f}$$

```
[In] integrate(sin(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] 1/6*(2*(a + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3/a - 6*sqrt(a + b/cos(f*x
 + e)^2)*cos(f*x + e) - 3*sqrt(b)*log((sqrt(a + b/cos(f*x + e)^2)*cos(f*x +
 e) - sqrt(b))/(sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b))))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 805 vs. 2(88) = 176.

Time = 0.47 (sec) , antiderivative size = 805, normalized size of antiderivative = 8.05

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^3(e+fx)\,dx = \text{Too large to display}$$

```
[In] integrate(sin(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] -2/3*(3*b*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*
f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*
b*tan(1/2*f*x + 1/2*e)^2 + a + b) - sqrt(a + b))/sqrt(-b))/sqrt(-b) - 2*(3*
(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan
(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)
^2 + a + b))^5*b - 3*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f
*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b
```

```
*tan(1/2*f*x + 1/2*e)^2 + a + b))^4*(4*a - 3*b)*sqrt(a + b) + 2*(sqrt(a + b
)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x +
1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)
)^3*(8*a^2 - 9*a*b + 3*b^2) + 6*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2
*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2*(4*a^2 - a*b - b^2)*sqrt(a +
 b) - 3*(16*a^3 - 5*a^2*b - 2*a*b^2 + 3*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2
*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(
1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)) + (20*a^3 - 19*a^
2*b + 6*a*b^2 - 3*b^3)*sqrt(a + b))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 -
sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x
+ 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 + 2*(sqrt(a + b)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4
 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a
 + b) - 3*a + b)^3)*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^3(e+fx)\,dx=\int \sin(e+f\,x)^3\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,dx$$

```
[In] int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.69 $\int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 66

$$\int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx = \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}$$

```
[Out] arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-cos(f*x+e)*(
a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4219, 283, 223, 212}

$$\int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx = \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}$$

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - (C
os[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/f
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2}\,dx, x, \sec(e+fx)\right)}{f} \\
&= -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f} \\
&= -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} \\
&= \frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.50

$$\int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx = -\frac{\sqrt{2}\cos(e+fx)\left(-\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b+a\cos^2(e+fx)}}{\sqrt{b}}\right)+\sqrt{b+a\cos^2(e+fx)}\right)\sqrt{a+b\sec^2(e+fx)}}{f\sqrt{a+2b+a\cos(2e+2fx)}}$$

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x],x]
```

```
[Out] -((Sqrt[2]*Cos[e + f*x]*(-(Sqrt[b]*ArcTanh[Sqrt[b + a*Cos[e + f*x]^2]/Sqrt[
b]]) + Sqrt[b + a*Cos[e + f*x]^2])*Sqrt[a + b*Sec[e + f*x]^2])/(f*Sqrt[a +
2*b + a*Cos[2*e + 2*f*x]]))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.41

method	result	size
derivativedivides	$-\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}{fa\sec(fx+e)}+\frac{b\sec(fx+e)\sqrt{a+b\sec(fx+e)^2}}{fa}+\frac{\sqrt{b}\,\ln\left(\sqrt{b}\,\sec(fx+e)+\sqrt{a+b\sec(fx+e)^2}\right)}{f}$	93
default	$-\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}{fa\sec(fx+e)}+\frac{b\sec(fx+e)\sqrt{a+b\sec(fx+e)^2}}{fa}+\frac{\sqrt{b}\,\ln\left(\sqrt{b}\,\sec(fx+e)+\sqrt{a+b\sec(fx+e)^2}\right)}{f}$	93

```
[In] int(sin(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f/a/sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2)+1/f*b/a*sec(f*x+e)*(a+b*sec(f*x+
e)^2)^(1/2)+1/f*b^(1/2)*ln(b^(1/2)*sec(f*x+e)+(a+b*sec(f*x+e)^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.76

$$\int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx = \left[-\frac{2\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)-\sqrt{b}\log\left(\frac{a\cos(fx+e)^2+2\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2b}{\cos(fx+e)^2}\right)}{2f}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{b}\right)+\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{f}\right]$$

```
[In] integrate(sin(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) - sqrt(b)
*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/f, -(sqrt(-b)*arctan(sqrt(-b)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) + sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/f]
```

Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx = \int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx$$

```
[In] integrate(sin(f*x+e)*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sin(e + f*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.33

$$\int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx = -\frac{2\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)+\sqrt{b}\log\left(\frac{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)-\sqrt{b}}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)+\sqrt{b}}\right)}{2f}$$

```
[In] integrate(sin(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] -1/2*(2*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b)*log((sqrt(a + b/c
os(f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a + b/cos(f*x + e)^2)*cos(f*x
+ e) + sqrt(b))))/f
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

$$\int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx = -\frac{\left(\frac{b\arctan\left(\frac{\sqrt{a\cos(fx+e)^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+\sqrt{a\cos(fx+e)^2+b}\right)\mathrm{sgn}(\cos(fx+e))}{f}$$

```
[In] integrate(sin(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] -(b*arctan(sqrt(a*cos(f*x + e)^2 + b)/sqrt(-b))/sqrt(-b) + sqrt(a*cos(f*x +
 e)^2 + b))*sgn(cos(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 18.36 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.32

$$\int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx = -\frac{\cos(e+f\,x)\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{f} - \frac{\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,1\mathrm{i}}{\sqrt{a}\,\cos(e+f\,x)}\right)\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,1\mathrm{i}}{\sqrt{a}\,f\,\sqrt{\frac{b}{a\cos(e+f\,x)^2}+1}}$$

```
[In] int(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2),x)

[Out] - (cos(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2))/f - (b^(1/2)*asin((b^(1/2)*1i
)/(a^(1/2)*cos(e + f*x)))*(a + b/cos(e + f*x)^2)^(1/2)*1i)/(a^(1/2)*f*(b/(a
*cos(e + f*x)^2) + 1)^(1/2))
```

3.70 $\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}$$

```
[Out] arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-arctanh(sec(
f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*(a+b)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4219, 399, 223, 212, 385, 213}

$$\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}$$

```
[In] Int[Csc[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - (S
qrt[a + b]*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/
f
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{-1-(-a-b)x^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}$$

$$= \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.45

$$\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\sqrt{2}\left(\sqrt{b}\text{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{b}}\right) - \sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{a+b}}\right)\right)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f\sqrt{a+2b+a\cos(2e+2fx)}}$$

```
[In] Integrate[Csc[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[2]*(Sqrt[b]*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[b]] - Sqrt[a
+ b]*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b]])*Cos[e + f*x]*Sqrt
[a + b*Sec[e + f*x]^2])/(f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(70) = 140$.

Time = 0.56 (sec) , antiderivative size = 542, normalized size of antiderivative = 6.61

method	result
default	$\frac{\sqrt{a+b\sec(fx+e)^2}\left(2\sqrt{b}\ln\left(-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sec(fx+e)-4\sec(fx+e)b\right)\sqrt{a+b}-\ln\left(2\sqrt{\frac{b+a\cos(fx+}{(1+\cos(fx+}}\right.\right.}{}$

```
[In] int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f/(a+b)^(1/2)*(a+b*sec(f*x+e)^2)^(1/2)*(2*b^(1/2)*ln(-4*b^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*sec(f*x+e)-4*sec(f*x+e)*b)*(a+b)^(1/2)-ln(2/(a+b)^(1/2)*(((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)
))*a-b*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^
(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-co
s(f*x+e)*a+b)/(1+cos(f*x+e)))-ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
```

```
+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a-ln(-4*(((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*b)*cos(f*x+e)/(1+
cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 496, normalized size of antiderivative = 6.05

$$\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{\sqrt{a+b}\log\left(\frac{2\left(a\cos(fx+e)^2-2\sqrt{a+b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+a+2b\right)}{\cos(fx+e)^2-1}\right)+\sqrt{b}\log\left(\frac{a\cos(fx+e)^2+2\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(f}{\cos(fx+e)^2}\right.}{2f}\right.$$

$$-\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{b}\right)-\sqrt{a+b}\log\left(\frac{2\left(a\cos(fx+e)^2-2\sqrt{a+b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+a+2b\right.}{\cos(fx+e)^2-1}\right.}{2f}$$

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a + b)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) +
sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/f, 1/2*(2*sqrt(-a - b)*arct
an(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a
 + b)) + sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/f, -1/2*(2*sqrt(-b
)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/
b) - sqrt(a + b)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)))/f,
 (sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*cos(f*x + e)/(a + b)) - sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*cos(f*x + e)/b))/f]
```

Sympy [F]

$$\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{a+b\sec^2\left(e+fx\right)}\csc\left(e+fx\right)dx$$

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*csc(e + f*x), x)
```

Maxima [F]

$$\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec\left(fx+e\right)^2+a}\csc\left(fx+e\right)\,dx$$

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(70)=140$.

Time $=0.70$ (sec), antiderivative size $=405$, normalized size of antiderivative $=4.94$

$$\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=$$
$$-\frac{\left(\frac{4\,b\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-\sqrt{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-2\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a+b}-\sqrt{a+b}}{2\,\sqrt{-b}}\right)}{\sqrt{-b}}+\sqrt{a+\cdots}\right.}{\cdots}$$

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] -1/2*(4*b*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*
f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*
b*tan(1/2*f*x + 1/2*e)^2 + a + b) - sqrt(a + b))/sqrt(-b))/sqrt(-b) + sqrt(
a + b)*log(abs(-sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a*tan(1/2*f*x + 1
/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1
/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))) - sqrt(a + b)*log(abs(-sqrt(a +
b)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x +
 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b
) - sqrt(a + b))) - sqrt(a + b)*log(abs((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2
 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f
*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a + b) - sqrt(a + b)*
(a - b))))*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\sin\left(e+f\,x\right)}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x),x)
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x), x)
```

3.71 $\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 124

$$\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{(a+2b)\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2\sqrt{a+b}f} - \frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}$$

```
[Out] arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-1/2*(a+2*b)*
arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/f/(a+b)^(1/2)-1/2*
cot(f*x+e)*csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4219, 478, 537, 223, 212, 385, 213}

$$\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{(a+2b)\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f\sqrt{a+b}} - \frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}$$

```
[In] Int[Csc[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - ((
a + 2*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(2
*Sqrt[a + b]*f) - (Cot[e + f*x]*Csc[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2
*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx^2}}{(-1+x^2)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{a+2bx^2}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{2f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f} + \frac{(a+2b)\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{2f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} + \frac{(a+2b)\text{Subst}\left(\int \frac{1}{-1-(-a-b)x^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}$$

$$= \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{(a+2b)\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2\sqrt{a+b}f} - \frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.31

$$\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(2\sqrt{b}(a+b)\text{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{b}}\right)-\sqrt{a+b}(a+2b)\text{arctanh}\left(\frac{\sqrt{a+b-a}}{\sqrt{}}\right.\right.}{\sqrt{2}(a+b)f\sqrt{a+2b+a\cos(2(e+fx))}}$$

```
[In] Integrate[Csc[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(2*Sqrt[b]*(a + b)*ArcTanh[Sqrt[a
+ b - a*Sin[e + f*x]^2]/Sqrt[b]] - Sqrt[a + b]*(a + 2*b)*ArcTanh[Sqrt[a + b
 - a*Sin[e + f*x]^2]/Sqrt[a + b]] - (a + b)*Csc[e + f*x]^2*Sqrt[a + b - a*S
in[e + f*x]^2]))/(Sqrt[2]*(a + b)*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2128 vs. $2(106) = 212$.

Time = 0.44 (sec) , antiderivative size = 2129, normalized size of antiderivative = 17.17

method	result	size
default	Expression too large to display	2129

```
[In] int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f/(a+b)^(5/2)*(4*(a+b)^(3/2)*cos(f*x+e)*ln(-4*b^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*sec(f*x+e)-4*sec(f*x+e)*b)*b^(3/2)+4*(a+b)^(3/2)*cos(f*x+e)*ln(-4*b^
(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)-4*sec(f*x+e)*b)*b^(1/2)*a-4*(a+b)^(
3/2)*ln(-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*b^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)-4*sec(f*x+e)*b)*b^(3/2
)-4*(a+b)^(3/2)*ln(-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4
*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)-4*sec(f*x+e
)*b)*b^(1/2)*a+2*(a+b)^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a+
2*(a+b)^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b-cos(f*x+e)*ln(2
/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f
*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+
b)/(1+cos(f*x+e)))*a^3-4*cos(f*x+e)*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^2*b-5*cos(f*x+
e)*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2
)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*
```

```
x+e)*a+b)/(1+cos(f*x+e))*a*b^2-2*cos(f*x+e)*ln(2/(a+b)^(1/2)*(((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*b^3-cos
(f*x+e)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(
f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a
+b)/(-1+cos(f*x+e)))*a^3-4*cos(f*x+e)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^2*b-5*cos(f*x+e)*ln(-4
*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f
*x+e)))*a*b^2-2*cos(f*x+e)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)
^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*b^3+ln(2/(a+b)^(1/2)*(((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^3+4*ln
(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos
(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*
a+b)/(1+cos(f*x+e)))*a^2*b+5*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a*b^2+2*ln(2/(a+b)^(1/2
)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f
*x+e)))*b^3+ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*
cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+
e)*a+b)/(-1+cos(f*x+e)))*a^3+4*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(
a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^2*b+5*ln(-4*(((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a*b^2+2*ln
(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+co
s(f*x+e)))*b^3)*(a+b*sec(f*x+e)^2)^(1/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*csc(f*x+e)*cot(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 867, normalized size of antiderivative = 6.99

$$\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) +
((a + 2*b)*cos(f*x + e)^2 - a - 2*b)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 -
2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a
+ 2*b)/(cos(f*x + e)^2 - 1)) + 2*((a + b)*cos(f*x + e)^2 - a - b)*sqrt(b)*l
og((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/((a + b)*f*cos(f*x + e)^2 - (a + b)*
f), 1/2*(((a + 2*b)*cos(f*x + e)^2 - a - 2*b)*sqrt(-a - b)*arctan(sqrt(-a -
 b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) + (a
+ b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + ((a + b)*co
s(f*x + e)^2 - a - b)*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos
(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/((a +
 b)*f*cos(f*x + e)^2 - (a + b)*f), -1/4*(4*((a + b)*cos(f*x + e)^2 - a - b)
*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f
*x + e)/b) - 2*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
+ e) - ((a + 2*b)*cos(f*x + e)^2 - a - 2*b)*sqrt(a + b)*log(2*(a*cos(f*x +
e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e) + a + 2*b)/(cos(f*x + e)^2 - 1)))/((a + b)*f*cos(f*x + e)^2 - (a + b)*f)
, 1/2*(((a + 2*b)*cos(f*x + e)^2 - a - 2*b)*sqrt(-a - b)*arctan(sqrt(-a - b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) - 2*((a
 + b)*cos(f*x + e)^2 - a - b)*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) + (a + b)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*cos(f*x + e))/((a + b)*f*cos(f*x + e)^2 - (a + b)*f)]
```

Sympy [F]

$$\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{a+b\sec^2{(e+fx)}}\csc^3{(e+fx)}\,dx$$

```
[In] integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*csc(e + f*x)**3, x)
```

Maxima [F]

$$\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec{(fx+e)}^2+a}\csc{(fx+e)}^3\,dx$$

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. $2(106) = 212$.

Time = 0.82 (sec) , antiderivative size = 578, normalized size of antiderivative = 4.66

$$\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\left(\frac{4\,(a+2\,b)\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-\sqrt{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-2\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a+b}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}} - \frac{16\,b\arctan}{}\right.}{}$$

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(4*(a + 2*b)*arctan(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))/sqrt(-a - b))/sqrt(-a - b) - 16*b*arc
tan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^
4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x
 + 1/2*e)^2 + a + b) - sqrt(a + b))/sqrt(-b))/sqrt(-b) + 2*(a + 2*b)*log(ab
s(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*
tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2
*e)^2 + a + b))*(a + b) + sqrt(a + b)*(a - b)))/sqrt(a + b) - sqrt(a*tan(1/
2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - 2*((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^
2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*
f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a - b) - (a + b)^(3/
2))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1
/2*e)^2 + a + b))^2 - a - b))*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\sin\left(e+f\,x\right)^3}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^3,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^3, x)
```

3.72 $\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 183

$$\begin{aligned}\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx &= \frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{(3a^2+12ab+8b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{3/2}f}\\ &\quad-\frac{(3a+4b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f}\\ &\quad-\frac{\cot(e+fx)\csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f}\end{aligned}$$

```
[Out] -1/8*(3*a^2+12*a*b+8*b^2)*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)
^(1/2))/(a+b)^(3/2)/f+arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*
b^(1/2)/f-1/8*(3*a+4*b)*cot(f*x+e)*csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/(a+b
)/f-1/4*cot(f*x+e)*csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used

$= \{4219, 478, 592, 537, 223, 212, 385, 213\}$

$$\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= -\frac{(3a^2+12ab+8b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f(a+b)^{3/2}} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}$$
$$- \frac{\cot(e+fx)\csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f}$$
$$- \frac{(3a+4b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f(a+b)}$$

```
[In] Int[Csc[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - ((
3*a^2 + 12*a*b + 8*b^2)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e
 + f*x]^2]])/(8*(a + b)^(3/2)*f) - ((3*a + 4*b)*Cot[e + f*x]*Csc[e + f*x]*S
qrt[a + b*Sec[e + f*x]^2])/(8*(a + b)*f) - (Cot[e + f*x]*Csc[e + f*x]^3*Sqr
t[a + b*Sec[e + f*x]^2])/(4*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
```

```
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4\sqrt{a+bx^2}}{(-1+x^2)^3}\,dx, x, \sec(e+fx)\right)}{f}\\
&= -\frac{\cot(e+fx)\csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f}+\frac{\text{Subst}\left(\int \frac{x^2(3a+4bx^2)}{(-1+x^2)^2\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{4f}\\
&= -\frac{(3a+4b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f}\\
&\quad -\frac{\cot(e+fx)\csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f}\\
&\quad +\frac{\text{Subst}\left(\int \frac{a(3a+4b)+8b(a+b)x^2}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{8(a+b)f}\end{aligned}$$

$$= -\frac{(3a+4b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f} - \frac{\cot(e+fx)\csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f} + \frac{b\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^2}}dx,x,\sec(e+fx)\right)}{f} + \frac{(3a^2+12ab+8b^2)\,\text{Subst}\left(\int\frac{1}{(-1+x^2)\sqrt{a+bx^2}}dx,x,\sec(e+fx)\right)}{8(a+b)f}$$

$$= -\frac{(3a+4b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f} - \frac{\cot(e+fx)\csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f} + \frac{b\text{Subst}\left(\int\frac{1}{1-bx^2}dx,x,\frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} + \frac{(3a^2+12ab+8b^2)\,\text{Subst}\left(\int\frac{1}{-1-(-a-b)x^2}dx,x,\frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)f}$$

$$= \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{(3a^2+12ab+8b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{3/2}f} - \frac{(3a+4b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f} - \frac{\cot(e+fx)\csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f}$$

Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.14

$$\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\cos(e+fx)\left(8\sqrt{b}(a+b)^2\text{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{b}}\right) - \sqrt{a+b}(3a^2+12ab+8b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e+f}}{\sqrt{a+b}}\right.\right.}{4\sqrt{2}(a+b)^2f\sqrt{a+2b+a\cos(}}$$

```
[In] Integrate[Csc[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*(8*Sqrt[b]*(a + b)^2*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/S
qrt[b]] - Sqrt[a + b]*(3*a^2 + 12*a*b + 8*b^2)*ArcTanh[Sqrt[a + b - a*Sin[e
 + f*x]^2]/Sqrt[a + b]] + ((a + b)*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(-7*a
 - 8*b + (3*a + 4*b)*Cos[2*(e + f*x)])*Csc[e + f*x]^4)/(2*Sqrt[2]))*Sqrt[a
+ b*Sec[e + f*x]^2])/(4*Sqrt[2]*(a + b)^2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x
)]])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 4317 vs. $2(161) = 322$.

Time = 0.51 (sec) , antiderivative size = 4318, normalized size of antiderivative = 23.60

method	result	size
default	Expression too large to display	4318

```
[In] int(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/f/(a+b)^(9/2)*((a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc
(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)
^2+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^2)^(1/2)*((1-cos(f*x+e))^2*csc(f*
x+e)^2-1)*(-128*b^(3/2)*ln(4*(b*(1-cos(f*x+e))^2*csc(f*x+e)^2+b^(1/2)*(a*(1
-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+
e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)+b)/((1-cos(
f*x+e))^2*csc(f*x+e)^2-1))*(a+b)^(5/2)*a*(1-cos(f*x+e))^4*csc(f*x+e)^4-18*(
a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(
f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(
5/2)*a*b*(1-cos(f*x+e))^6*csc(f*x+e)^6-64*b^(1/2)*ln(4*(b*(1-cos(f*x+e))^2*
csc(f*x+e)^2+b^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*cs
c(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e
)^2+a+b)^(1/2)+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))*(a+b)^(5/2)*a^2*(1-cos
(f*x+e))^4*csc(f*x+e)^4+14*a*b*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*
x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2
*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(5/2)*(1-cos(f*x+e))^4*csc(f*x+e)^4+212*ln((
a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+(a*(1-cos(f
*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*
csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)-a+b)/
(a+b)^(1/2))*a^3*b^2*(1-cos(f*x+e))^4*csc(f*x+e)^4+252*ln((a*(1-cos(f*x+e))
^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+(a*(1-cos(f*x+e))^4*csc(f*x
+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b
*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)-a+b)/(a+b)^(1/2))*a^2
*b^3*(1-cos(f*x+e))^4*csc(f*x+e)^4+144*ln((a*(1-cos(f*x+e))^2*csc(f*x+e)^2+
b*(1-cos(f*x+e))^2*csc(f*x+e)^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f
*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^
2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)-a+b)/(a+b)^(1/2))*a*b^4*(1-cos(f*x+e)
)^4*csc(f*x+e)^4-64*b^(5/2)*ln(4*(b*(1-cos(f*x+e))^2*csc(f*x+e)^2+b^(1/2)*(
a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(
f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)+b)/((1-
cos(f*x+e))^2*csc(f*x+e)^2-1))*(a+b)^(5/2)*(1-cos(f*x+e))^4*csc(f*x+e)^4+84
*ln(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))^2+b*(1-cos(f*x+e))^2+(a*(1-cos(f*
x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*c
sc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)*sin(f*
x+e)^2+a*sin(f*x+e)^2+b*sin(f*x+e)^2))*a^4*(1-cos(f*x+e))^4*b*csc(f*x+e)^4+
```

212*ln(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))^2+b*(1-cos(f*x+e))^2+(a*(1-cos
(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^
2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)*sin
(f*x+e)^2+a*sin(f*x+e)^2+b*sin(f*x+e)^2))*a^3*b^2*(1-cos(f*x+e))^4*csc(f*x+
e)^4+252*ln(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))^2+b*(1-cos(f*x+e))^2+(a*(
1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x
+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2
)*sin(f*x+e)^2+a*sin(f*x+e)^2+b*sin(f*x+e)^2))*a^2*b^3*(1-cos(f*x+e))^4*csc
(f*x+e)^4+144*ln(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))^2+b*(1-cos(f*x+e))^2
+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-co
s(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)
^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2+b*sin(f*x+e)^2))*b^4*(1-cos(f*x+e))^4*a*
csc(f*x+e)^4-10*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x
+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a
+b)^(1/2)*(1-cos(f*x+e))^6*a^2*(a+b)^(5/2)*csc(f*x+e)^6-8*(a*(1-cos(f*x+e))
^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*
x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(1-cos(f*x+e))^6*b^2*(a
+b)^(5/2)*csc(f*x+e)^6+30*a^2*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x
+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*
csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(5/2)*(1-cos(f*x+e))^4*csc(f*x+e)^4-8*b^2*(a*
(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*
x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(5/
2)*(1-cos(f*x+e))^4*csc(f*x+e)^4+11*a*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1
-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*
x+e))^2*csc(f*x+e)^2+a+b)^(3/2)*(a+b)^(5/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+9
b(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-
cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(3/2)*(a+
b)^(5/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+84*ln((a*(1-cos(f*x+e))^2*csc(f*x+e)
^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-co
s(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e
))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)-a+b)/(a+b)^(1/2))*a^4*b*(1-cos(f*x
+e))^4*csc(f*x+e)^4+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc
(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)
^2+a+b)^(3/2)*(a+b)^(5/2)*a+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e
))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*cs
c(f*x+e)^2+a+b)^(3/2)*(a+b)^(5/2)*b+12*ln((a*(1-cos(f*x+e))^2*csc(f*x+e)^2+
b*(1-cos(f*x+e))^2*csc(f*x+e)^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f
*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^
2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)-a+b)/(a+b)^(1/2))*a^5*(1-cos(f*x+e))^
4*csc(f*x+e)^4+32*ln((a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*cs
c(f*x+e)^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4
-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(
1/2)*(a+b)^(1/2)-a+b)/(a+b)^(1/2))*b^5*(1-cos(f*x+e))^4*csc(f*x+e)^4+12*ln(
2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))^2+b*(1-cos(f*x+e))^2+(a*(1-cos(f*x+e)
)^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f

```
*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)
^2+a*sin(f*x+e)^2+b*sin(f*x+e)^2))*a^5*(1-cos(f*x+e))^4*csc(f*x+e)^4+32*ln(
2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))^2+b*(1-cos(f*x+e))^2+(a*(1-cos(f*x+e)
)^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f
*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)
^2+a*sin(f*x+e)^2+b*sin(f*x+e)^2))*b^5*(1-cos(f*x+e))^4*csc(f*x+e)^4)/(a*(1
-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+
e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)/(1-cos(f*x+
e))^4*sin(f*x+e)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(161) = 322$.

Time = 0.66 (sec) , antiderivative size = 1476, normalized size of antiderivative = 8.07

$$\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((3*a^2 + 12*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 12*a*b + 8*b^2
)*cos(f*x + e)^2 + 3*a^2 + 12*a*b + 8*b^2)*sqrt(a + b)*log(2*(a*cos(f*x + e
)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e
) + a + 2*b)/(cos(f*x + e)^2 - 1)) + 8*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4
- 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(b)*log((a*
cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(
f*x + e) + 2*b)/cos(f*x + e)^2) + 2*((3*a^2 + 7*a*b + 4*b^2)*cos(f*x + e)^3
 - (5*a^2 + 11*a*b + 6*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f
*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f), 1/8*(((3*a^2 + 12*a*b + 8*b^2)*co
s(f*x + e)^4 - 2*(3*a^2 + 12*a*b + 8*b^2)*cos(f*x + e)^2 + 3*a^2 + 12*a*b +
 8*b^2)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*cos(f*x + e)/(a + b)) + 4*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2
*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(b)*log((a*cos
(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
 + e) + 2*b)/cos(f*x + e)^2) + ((3*a^2 + 7*a*b + 4*b^2)*cos(f*x + e)^3 - (5
*a^2 + 11*a*b + 6*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(
f*x + e)^2 + (a^2 + 2*a*b + b^2)*f), -1/16*(16*((a^2 + 2*a*b + b^2)*cos(f*x
 + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b
)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/
b) - ((3*a^2 + 12*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 12*a*b + 8*b^2)*
cos(f*x + e)^2 + 3*a^2 + 12*a*b + 8*b^2)*sqrt(a + b)*log(2*(a*cos(f*x + e)^
2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)
+ a + 2*b)/(cos(f*x + e)^2 - 1)) - 2*((3*a^2 + 7*a*b + 4*b^2)*cos(f*x + e)^
```

```
3 - (5*a^2 + 11*a*b + 6*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*
f*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f), 1/8*(((3*a^2 + 12*a*b + 8*b^2)*c
os(f*x + e)^4 - 2*(3*a^2 + 12*a*b + 8*b^2)*cos(f*x + e)^2 + 3*a^2 + 12*a*b
+ 8*b^2)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*cos(f*x + e)/(a + b)) - 8*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 -
2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b)*arctan(s
qrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) + ((3*a
^2 + 7*a*b + 4*b^2)*cos(f*x + e)^3 - (5*a^2 + 11*a*b + 6*b^2)*cos(f*x + e))
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f*
x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f)]
```

Sympy [F]

$$\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\csc^5(e+fx)\,dx$$

```
[In] integrate(csc(f*x+e)**5*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*csc(e + f*x)**5, x)
```

Maxima [F]

$$\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\,\csc(fx+e)^5\,dx$$

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^5, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 867 vs. $2(161) = 322$.

Time = 1.21 (sec) , antiderivative size = 867, normalized size of antiderivative = 4.74

$$\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] -1/64*(sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1
/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)*(tan(1/2*f*x + 1/2*
e)^2 + (9*a + 11*b)/(a + b)) + 128*b*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x +
```

```
1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*
tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - sqrt(a + b))
/sqrt(-b))/sqrt(-b) - 8*(3*a^2 + 12*a*b + 8*b^2)*arctan(-(sqrt(a + b)*tan(1
/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^
4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))/sqrt(
-a - b))/((a + b)*sqrt(-a - b)) - 4*(3*a^2 + 12*a*b + 8*b^2)*log(abs(-(sqrt
(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*
f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 +
a + b))*(a + b) + sqrt(a + b)*(a - b)))/(a + b)^(3/2) + 4*(2*(sqrt(a + b)*t
an(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2
*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^3
*(2*a^2 - 3*b^2) - (sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*t
an(1/2*f*x + 1/2*e)^2 + a + b))^2*(3*a^2 + 10*a*b + 7*b^2)*sqrt(a + b) - 2*
(3*a^3 + 3*a^2*b - 2*a*b^2 - 2*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)) + 5*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*sqrt(a + b))/(((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - a - b)^2*(a + b)))*sgn(cos(f*x +
e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\sin\left(e+f\,x\right)^5}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^5,x)
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^5, x)
```

3.73 $\int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 240

$$
\begin{aligned}
\int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx
&= \frac{(5a^3-15a^2b-5ab^2-b^3)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{5/2}f} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} \\
&\quad - \frac{(a-b)(5a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16a^2f} \\
&\quad - \frac{(5a-b)\cos(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24af} \\
&\quad - \frac{\cos(e+fx)\sin^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f}
\end{aligned}
$$

```
[Out] 1/16*(5*a^3-15*a^2*b-5*a*b^2-b^3)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+
e)^2)^(1/2))/a^(5/2)/f+arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2
))*b^(1/2)/f-1/16*(a-b)*(5*a+b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^
(1/2)/a^2/f-1/24*(5*a-b)*cos(f*x+e)*sin(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)
/a/f-1/6*cos(f*x+e)*sin(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4217, 478, 592, 537, 223, 212, 385, 209}

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx$$
$$=-\frac{(a-b)(5a+b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16a^2f}$$
$$+\frac{(5a^3-15a^2b-5ab^2-b^3)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{5/2}f}$$
$$+\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\sin^5(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6f}$$
$$-\frac{(5a-b)\sin^3(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{24af}$$

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^6,x]
```

```
[Out] ((5*a^3 - 15*a^2*b - 5*a*b^2 - b^3)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a +
b + b*Tan[e + f*x]^2]])/(16*a^(5/2)*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e +
f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f - ((a - b)*(5*a + b)*Cos[e + f*x]*
Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(16*a^2*f) - ((5*a - b)*Cos[e
+ f*x]*Sin[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(24*a*f) - (Cos[e + f
*x]*Sin[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(6*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^6\sqrt{a+b+bx^2}}{(1+x^2)^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$=-\frac{\cos(e+fx)\sin^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f}+\frac{\text{Subst}\left(\int\frac{x^4(5(a+b)+6bx^2)}{(1+x^2)^3\sqrt{a+b+bx^2}}\,dx,x,\tan(e+fx)\right)}{6f}$$

$$=-\frac{(5a-b)\cos(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24af}-\frac{\cos(e+fx)\sin^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f}+\frac{\text{Subst}\left(\int\frac{x^2(3(5a-b)(a+b)+24abx^2)}{(1+x^2)^2\sqrt{a+b+bx^2}}\,dx,x,\tan(e+fx)\right)}{24af}$$

$$=-\frac{(a-b)(5a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16a^2f}-\frac{(5a-b)\cos(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24af}-\frac{\cos(e+fx)\sin^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f}+\frac{\text{Subst}\left(\int\frac{3(5a+b)(a^2-b^2)+48a^2bx^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx,x,\tan(e+fx)\right)}{48a^2f}$$

$$=-\frac{(a-b)(5a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16a^2f}-\frac{(5a-b)\cos(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24af}-\frac{\cos(e+fx)\sin^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f}+\frac{b\text{Subst}\left(\int\frac{1}{\sqrt{a+b+bx^2}}\,dx,x,\tan(e+fx)\right)}{f}+\frac{(-48a^2b+3(5a+b)(a^2-b^2))\,\text{Subst}\left(\int\frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx,x,\tan(e+fx)\right)}{48a^2f}$$

$$= -\frac{(a-b)(5a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16a^2f} - \frac{(5a-b)\cos(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24af} - \frac{\cos(e+fx)\sin^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f} + \frac{b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(-48a^2b+3(5a+b)(a^2-b^2))\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{48a^2f}$$

$$= -\frac{(16a^2b-(5a+b)(a^2-b^2))\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{5/2}f} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{(a-b)(5a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16a^2f} - \frac{(5a-b)\cos(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24af} - \frac{\cos(e+fx)\sin^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f}$$

Mathematica [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx = \int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx$$

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^6,x]
[Out] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^6, x]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1179 vs. 2(214) = 428.

Time = 12.64 (sec) , antiderivative size = 1180, normalized size of antiderivative = 4.92

method	result	size
default	Expression too large to display	1180

```
[In] int(sin(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/f/a^2/(-a)^(1/2)*(8*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*a^2*cos(f*x+e)^5*sin(f*x+e)+8*(-a)^(1/2)*sin(f*x+e)*cos(f*x+e)^4*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2-26*(-a)^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*a^2*cos(f*x+e)^3*sin(f*x+e)+2*(-a)^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b*cos(f*x+e)^3*sin(f*x+e)-26*(-a)^(1
/2)*sin(f*x+e)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2
+2*(-a)^(1/2)*sin(f*x+e)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*a*b+33*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*co
s(f*x+e)*sin(f*x+e)-14*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*a*b*cos(f*x+e)*sin(f*x+e)-3*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*b^2*cos(f*x+e)*sin(f*x+e)+33*(-a)^(1/2)*sin(f*x+e)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2-14*(-a)^(1/2)*sin(f*x+e)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b-3*(-a)^(1/2)*sin(f*x+e)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*b^2-24*(-a)^(1/2)*b^(1/2)*ln(4*(-((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*a^2-24*(-a)^(1/2)
*b^(1/2)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*
x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/
(sin(f*x+e)+1))*a^2-15*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
-4*sin(f*x+e)*a)*a^3+45*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)-4*sin(f*x+e)*a)*a^2*b+15*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)-4*sin(f*x+e)*a)*a*b^2+3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)-4*sin(f*x+e)*a)*b^3)*(a+b*sec(f*x+e)^2)^(1/2)*cos(f*x+e)/(1+cos(f*x+
e))/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 2.89 (sec) , antiderivative size = 1715, normalized size of antiderivative = 7.15

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx = \text{Too large to display}$$

```
[In] integrate(sin(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(96*a^3*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b
^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x +
 e)^4) + 3*(5*a^3 - 15*a^2*b - 5*a*b^2 - b^3)*sqrt(-a)*log(128*a^4*cos(f*x
+ e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^
2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4
- 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 -
24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x +
e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(8*a^3*cos(f*x + e)^5 - 2*
(13*a^3 - a^2*b)*cos(f*x + e)^3 + (33*a^3 - 14*a^2*b - 3*a*b^2)*cos(f*x + e
))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*f), 1/384
*(192*a^3*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*
sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 +
 b^2)*sin(f*x + e))) + 3*(5*a^3 - 15*a^2*b - 5*a*b^2 - b^3)*sqrt(-a)*log(12
8*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos
(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b
^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(8*a^3*cos(f
*x + e)^5 - 2*(13*a^3 - a^2*b)*cos(f*x + e)^3 + (33*a^3 - 14*a^2*b - 3*a*b^
2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/
(a^3*f), 1/192*(48*a^3*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*
(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*
sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/c
os(f*x + e)^4) - 3*(5*a^3 - 15*a^2*b - 5*a*b^2 - b^3)*sqrt(a)*arctan(1/4*(8
*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*co
s(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos
(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))
) - 4*(8*a^3*cos(f*x + e)^5 - 2*(13*a^3 - a^2*b)*cos(f*x + e)^3 + (33*a^3 -
 14*a^2*b - 3*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(a^3*f), 1/192*(96*a^3*sqrt(-b)*arctan(-1/2*((a - b)*cos(
f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) - 3*(5*a^3 - 15*a^2*b -
```

```
 5*a*b^2 - b^3)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*co
s(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3
*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(8*a^3*cos(f*x + e)^5 - 2*(13*a^
3 - a^2*b)*cos(f*x + e)^3 + (33*a^3 - 14*a^2*b - 3*a*b^2)*cos(f*x + e))*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a + b\sec^2(e + fx)}\sin^6(e + fx)\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**6*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{a + b\sec^2(e + fx)}\sin^6(e + fx)\,dx = \int \sqrt{b\sec\left(fx + e\right)^2 + a}\sin\left(fx + e\right)^6\,dx$$

```
[In] integrate(sin(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^6, x)
```

Giac [F]

$$\int \sqrt{a + b\sec^2(e + fx)}\sin^6(e + fx)\,dx = \int \sqrt{b\sec\left(fx + e\right)^2 + a}\sin\left(fx + e\right)^6\,dx$$

```
[In] integrate(sin(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx = \int \sin\left(e+f\,x\right)^6\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

```
[In] int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.74 $\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 181

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx = \frac{(3a^2-6ab-b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{3/2}f} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8af} - \frac{\cos(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f}$$

```
[Out] 1/8*(3*a^2-6*a*b-b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))
/a^(3/2)/f+arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f
-1/8*(3*a-b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f-1/4*cos(f
*x+e)*sin(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used

= {4217, 478, 592, 537, 223, 212, 385, 209}

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx$$
$$= \frac{(3a^2-6ab-b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{3/2}f} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$
$$- \frac{\sin^3(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4f}$$
$$- \frac{(3a-b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8af}$$

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^4,x]
```

```
[Out] ((3*a^2 - 6*a*b - b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
 f*x]^2]])/(8*a^(3/2)*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a +
 b + b*Tan[e + f*x]^2]])/f - ((3*a - b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a +
b + b*Tan[e + f*x]^2])/(8*a*f) - (Cos[e + f*x]*Sin[e + f*x]^3*Sqrt[a + b +
b*Tan[e + f*x]^2])/(4*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
```

```
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4\sqrt{a+b+bx^2}}{(1+x^2)^3}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{\cos(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f}\\
&\quad+\frac{\text{Subst}\left(\int \frac{x^2(3(a+b)+4bx^2)}{(1+x^2)^2\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{4f}
\end{aligned}$$

$$= -\frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8af} - \frac{\cos(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f} + \frac{\text{Subst}\left(\int \frac{(3a-b)(a+b)+8abx^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{8af}$$

$$= -\frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8af} - \frac{\cos(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} + \frac{(3a^2-6ab-b^2)\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{8af}$$

$$= -\frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8af} - \frac{\cos(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f} + \frac{b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a^2-6ab-b^2)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8af}$$

$$= \frac{(3a^2-6ab-b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{3/2}f} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8af} - \frac{\cos(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f}$$

Mathematica [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx=\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx$$

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^4,x]
[Out] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^4, x]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 790 vs. $2(159) = 318$.

Time = 9.84 (sec) , antiderivative size = 791, normalized size of antiderivative = 4.37

method	result
default	$\left(2\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\sqrt{-a}\,a\cos(fx+e)^3\sin(fx+e)+2\cos(fx+e)^2\sin(fx+e)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\sqrt{-a}\,a-5\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\sqrt{-a}\,a\cos(\ldots\right.$

```
[In] int(sin(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/8/f/a/(-a)^(1/2)*(2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2
)*a*cos(f*x+e)^3*sin(f*x+e)+2*cos(f*x+e)^2*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a-5*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(-a)^(1/2)*a*cos(f*x+e)*sin(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(-a)^(1/2)*b*cos(f*x+e)*sin(f*x+e)+4*b^(1/2)*ln(4*(((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*(-a)^(1/2)*a+4
*b^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x
+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(
sin(f*x+e)-1))*(-a)^(1/2)*a-5*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*(-a)^(1/2)*a+sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*(-a)^(1/2)*b+3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*si
n(f*x+e)*a)*a^2-6*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*si
n(f*x+e)*a)*a*b-ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(
f*x+e)*a)*b^2)*(a+b*sec(f*x+e)^2)^(1/2)*cos(f*x+e)/(1+cos(f*x+e))/((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 1.09 (sec) , antiderivative size = 1565, normalized size of antiderivative = 8.65

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx = \text{Too large to display}$$

```
[In] integrate(sin(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/64*(16*a^2*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^
2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x +
e)^4) + (3*a^2 - 6*a*b - b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^
4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^
2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)
*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*
a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e)) + 8*(2*a^2*cos(f*x + e)^3 - (5*a^2 - a*b)*cos(
f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f)
, 1/64*(32*a^2*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
 e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e
)^2 + b^2)*sin(f*x + e))) + (3*a^2 - 6*a*b - b^2)*sqrt(-a)*log(128*a^4*cos(
f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^
2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(
a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^
7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*
x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*a^2*cos(f*x + e)^3
 - (5*a^2 - a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e))/(a^2*f), 1/32*(8*a^2*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x
 + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos
(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)
 + 8*b^2)/cos(f*x + e)^4) - (3*a^2 - 6*a*b - b^2)*sqrt(a)*arctan(1/4*(8*a^2
*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*
x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x
 + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) +
4*(2*a^2*cos(f*x + e)^3 - (5*a^2 - a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f), 1/32*(16*a^2*sqrt(-b)*arctan(
-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) - (3*
a^2 - 6*a*b - b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)
*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3
```

```
- 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(2*a^2*cos(f*x + e)^3 - (5*a^
2 - a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
+ e))/(a^2*f)]
```

Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx = \int \sqrt{a+b\sec^2{(e+fx)}}\sin^4{(e+fx)}\,dx$$

```
[In] integrate(sin(f*x+e)**4*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sin(e + f*x)**4, x)
```

Maxima [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx = \int \sqrt{b\sec(fx+e)^2+a}\sin(fx+e)^4\,dx$$

```
[In] integrate(sin(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^4, x)
```

Giac [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx = \int \sqrt{b\sec(fx+e)^2+a}\sin(fx+e)^4\,dx$$

```
[In] integrate(sin(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx = \int \sin(e+f\,x)^4\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,dx$$

```
[In] int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.75 $\int \sqrt{a+b\sec^2(e+fx)}\sin^2(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 123

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^2(e+fx)\,dx = \frac{(a-b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{a}f} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

```
[Out] 1/2*(a-b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)+a
rctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f-1/2*cos(f*x
+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4217, 478, 537, 223, 212, 385, 209}

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^2(e+fx)\,dx = \frac{(a-b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{a}f} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f} - \frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}$$

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^2,x]
```

```
[Out] ((a - b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*
Sqrt[a]*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
 f*x]^2]])/f - (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(
2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+b+bx^2}}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{a+b+2bx^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{(a-b)\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2f} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{(a-b)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

$$= \frac{(a-b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{a}f} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.29 (sec) , antiderivative size = 432, normalized size of antiderivative = 3.51

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^2(e+fx)\,dx$$

$$= \frac{e^{-i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos(e+fx)\left(i\left(-1+e^{2i(e+fx)}\right)+\frac{2e^{2i(e+fx)}\left(2afx-2bfx-i(a-b)\log\left(a+\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^2,x]
```

```
[Out] (Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*Cos[e + f*
x]*(I*(-1 + E^((2*I)*(e + f*x))) + (2*E^((2*I)*(e + f*x))*(2*a*f*x - 2*b*f*
x - I*(a - b)*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*
I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + I*(a - b)*Log[a + a*E^((2
*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f
*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 4*Sqrt[a]*Sqrt[b]*Log[((-(Sqrt[b]*
(-1 + E^((2*I)*(e + f*x)))) + I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2
*I)*(e + f*x)))^2])*f)/(2*b*(1 + E^((2*I)*(e + f*x))))]))/(Sqrt[a]*Sqrt[4*b
*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]))*Sqrt[a + b*Sec[e +
f*x]^2])/(4*Sqrt[2]*E^(I*(e + f*x))*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 515 vs. 2(105) = 210.

Time = 7.38 (sec) , antiderivative size = 516, normalized size of antiderivative = 4.20

method	result
default	$-\frac{\left(\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{-a}\cos(fx+e)\sin(fx+e)+\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sin(fx+e)-\sqrt{b}\ln\left(\frac{4\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{b}\cos(fx+e)-4\sin(f\cdots}{\sin(fx+\cdots}\right.\right.}{\cdots}$

```
[In] int(sin(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f/(-a)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*c
os(f*x+e)*sin(f*x+e)+(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*sin(f*x+e)-b^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/
2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e
```

```
)*a-a-b)/(sin(f*x+e)+1))*(-a)^(1/2)-b^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(-a)^(1/2)-ln(4*(-a)^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a+ln(4*(-a)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b)*(a+b*sec(f*x+e)^2)^(1
/2)*cos(f*x+e)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(105) = 210$.

Time = 0.61 (sec) , antiderivative size = 1417, normalized size of antiderivative = 11.52

$$\int \sqrt{a + b\sec^2(e + fx)}\sin^2(e + fx)\,dx = \text{Too large to display}$$

```
[In] integrate(sin(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*
x + e) - sqrt(-a)*(a - b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*co
s(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*
a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3
)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)
^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b
^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*sin(f*x + e)) - 4*a*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(
a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*s
qrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/co
s(f*x + e)^4))/(a*f), -1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*cos(f*x + e)*sin(f*x + e) - 8*a*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e
)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) - sqrt(-a)*(a - b)*log(128*a^4
*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b +
 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 -
 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x
+ e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*c
os(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/(a*f), -1/8*(4*a*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + (a - b
)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 +
 (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*
x + e)^2)*sin(f*x + e))) - 2*a*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e
)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*
```

```
x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) +
8*b^2)/cos(f*x + e)^4))/(a*f), -1/8*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*cos(f*x + e)*sin(f*x + e) + (a - b)*sqrt(a)*arctan(1/4*(8*a^2*cos
(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x +
e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e
)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*a*
sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(
f*x + e))))/(a*f)]
```

Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^2(e+fx)\,dx=\int \sqrt{a+b\sec^2{(e+fx)}}\sin^2{(e+fx)}\,dx$$

```
[In] integrate(sin(f*x+e)**2*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sin(e + f*x)**2, x)
```

Maxima [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^2(e+fx)\,dx=\int \sqrt{b\sec{(fx+e)}^2+a}\sin{(fx+e)}^2\,dx$$

```
[In] integrate(sin(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^2, x)
```

Giac [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^2(e+fx)\,dx=\int \sqrt{b\sec{(fx+e)}^2+a}\sin{(fx+e)}^2\,dx$$

```
[In] integrate(sin(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^2(e+fx)\,dx = \int \sin\left(e+f\,x\right)^2\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

```
[In] int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.76 $\int \sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 79

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx=\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f+arctanh(b^(
1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4213, 399, 223, 212, 385, 209}

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx=\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{a\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}+\frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{a\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}\\
&= \frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.89 (sec) , antiderivative size = 284, normalized size of antiderivative = 3.59

$$\int \sqrt{a + b\sec^2(e + fx)}\, dx = -\frac{i\left(1 + e^{2i(e+fx)}\right)\left(2\sqrt{b}\arctan\left(\frac{\sqrt{b}\left(-1+e^{2i(e+fx)}\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right) + \sqrt{a}\text{arctanh}\left(\frac{a+2b+ae^{2i(e+fx)}}{\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right) - \right.}{2\sqrt{4be^{2i(e+fx)} + a\left(1 + e^{2i(e+fx)}\right)^2} f}$$

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((-1/2*I)*(1 + E^((2*I)*(e + f*x)))*(2*Sqrt[b]*ArcTan[(Sqrt[b]*(-1 + E^((2*
I)*(e + f*x))))/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^
2]] + Sqrt[a]*ArcTanh[(a + 2*b + a*E^((2*I)*(e + f*x)))/(Sqrt[a]*Sqrt[4*b*E
^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])] - Sqrt[a]*ArcTanh[(a
+ a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)))/(Sqrt[a]*Sqrt[4*b*E^((2*
I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])])*Sqrt[a + b*Sec[e + f*x]^2
])/(Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(67) = 134.

Time = 6.39 (sec) , antiderivative size = 353, normalized size of antiderivative = 4.47

method	result
default	$\frac{\sqrt{a+b\sec(fx+e)^2}\left(\sqrt{b}\ln\left(\frac{-4\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{b}\cos(fx+e)-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}+4\sin(fx+e)a-4a-4b}{\sin(fx+e)-1}\right)\sqrt{-a}+\sqrt{b}\ln\left(-\frac{4\left(-\sqrt{\frac{b+a}{(1+c}}\right.}{}\right.\right.}{2f}$

```
[In] int((a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f/(-a)^(1/2)*(a+b*sec(f*x+e)^2)^(1/2)*(b^(1/2)*ln(4*(-((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*(-a)^(1/2)+b^(1/2)
*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin
(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x
+e)+1))*(-a)^(1/2)+2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4
*sin(f*x+e)*a)*a)*cos(f*x+e)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(67) = 134$.

Time = 0.48 (sec) , antiderivative size = 1227, normalized size of antiderivative = 15.53

$$\int \sqrt{a + b\sec^2(e + fx)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*
a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
 e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*c
os(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e)) + 2*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*co
s(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)
)/f, 1/8*(4*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e)
)*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2
 + b^2)*sin(f*x + e))) + sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a
^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a
^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(
f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b
 + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*sin(f*x + e)))/f, -1/4*(sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5
- 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b
+ a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - sqrt(b)*log(((a^
2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))/f, -1/4*(sqrt(a)*arctan
(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b +
b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*
a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*
x + e))) - 2*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e
))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^
2 + b^2)*sin(f*x + e))))/f]
```

Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{a+b\sec^2{(e+fx)}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 3227, normalized size of antiderivative = 40.85

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx=\text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] -1/2*(2*sqrt(a)*b^(3/2)*arctan2(a*sin(2*f*x + 2*e) + (a^2*cos(4*f*x + 4*e)^
2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4
*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*
sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4
*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arcta
n2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) +
2*(a + 2*b)*cos(2*f*x + 2*e) + a)), a*cos(2*f*x + 2*e) + (a^2*cos(4*f*x + 4
*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2
 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b
^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*c
os(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*a
rctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e
) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + a + 2*b) + a^(3/2)*sqrt(b)*arctan2
(2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^
2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b
)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1
/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*
e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f
*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x +
2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b
 + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2
*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos
(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x
 + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 4*a + 4*b) + a*b*log(((a + b
```

```
)*sqrt((16*b^2*cos(2*f*x + 2*e)^4 + 16*b^2*sin(2*f*x + 2*e)^4 + (a^2 + 2*a*
b + b^2)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^4 - 64*b^2*cos(2*f*x + 2*e)^3 + 96*
b^2*cos(2*f*x + 2*e)^2 - 8*((a*b + b^2)*cos(2*f*x + 2*e)^2 - (a*b + b^2)*si
n(2*f*x + 2*e)^2 + a*b + b^2 - 2*(a*b + b^2)*cos(2*f*x + 2*e))*abs(2*e^(2*I
*f*x + 2*I*e) + 2)^2 - 64*b^2*cos(2*f*x + 2*e) + 32*(b^2*cos(2*f*x + 2*e)^2
 - 2*b^2*cos(2*f*x + 2*e) + b^2)*sin(2*f*x + 2*e)^2 + 16*b^2)/(a^2 + 2*a*b
+ b^2))*cos(1/2*arctan2(8*(b*cos(2*f*x + 2*e) - b)*sin(2*f*x + 2*e)/((a + b
)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2), ((a + b)*abs(2*e^(2*I*f*x + 2*I*e) + 2
)^2 - 4*b*cos(2*f*x + 2*e)^2 + 4*b*sin(2*f*x + 2*e)^2 + 8*b*cos(2*f*x + 2*e
) - 4*b)/((a + b)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2)))^2 + (a + b)*sqrt((16*
b^2*cos(2*f*x + 2*e)^4 + 16*b^2*sin(2*f*x + 2*e)^4 + (a^2 + 2*a*b + b^2)*ab
s(2*e^(2*I*f*x + 2*I*e) + 2)^4 - 64*b^2*cos(2*f*x + 2*e)^3 + 96*b^2*cos(2*f
*x + 2*e)^2 - 8*((a*b + b^2)*cos(2*f*x + 2*e)^2 - (a*b + b^2)*sin(2*f*x + 2
*e)^2 + a*b + b^2 - 2*(a*b + b^2)*cos(2*f*x + 2*e))*abs(2*e^(2*I*f*x + 2*I*
e) + 2)^2 - 64*b^2*cos(2*f*x + 2*e) + 32*(b^2*cos(2*f*x + 2*e)^2 - 2*b^2*co
s(2*f*x + 2*e) + b^2)*sin(2*f*x + 2*e)^2 + 16*b^2)/(a^2 + 2*a*b + b^2))*sin
(1/2*arctan2(8*(b*cos(2*f*x + 2*e) - b)*sin(2*f*x + 2*e)/((a + b)*abs(2*e^(
2*I*f*x + 2*I*e) + 2)^2), ((a + b)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2 - 4*b*c
os(2*f*x + 2*e)^2 + 4*b*sin(2*f*x + 2*e)^2 + 8*b*cos(2*f*x + 2*e) - 4*b)/((
a + b)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2)))^2 + 4*b*cos(2*f*x + 2*e)^2 + 4*b
*sin(2*f*x + 2*e)^2 - 4*sqrt(a*b + b^2)*((16*b^2*cos(2*f*x + 2*e)^4 + 16*b^
2*sin(2*f*x + 2*e)^4 + (a^2 + 2*a*b + b^2)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^4
 - 64*b^2*cos(2*f*x + 2*e)^3 + 96*b^2*cos(2*f*x + 2*e)^2 - 8*((a*b + b^2)*c
os(2*f*x + 2*e)^2 - (a*b + b^2)*sin(2*f*x + 2*e)^2 + a*b + b^2 - 2*(a*b + b
^2)*cos(2*f*x + 2*e))*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2 - 64*b^2*cos(2*f*x +
 2*e) + 32*(b^2*cos(2*f*x + 2*e)^2 - 2*b^2*cos(2*f*x + 2*e) + b^2)*sin(2*f*
x + 2*e)^2 + 16*b^2)/(a^2 + 2*a*b + b^2))^(1/4)*cos(1/2*arctan2(8*(b*cos(2*
f*x + 2*e) - b)*sin(2*f*x + 2*e)/((a + b)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2)
, ((a + b)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2 - 4*b*cos(2*f*x + 2*e)^2 + 4*b*
sin(2*f*x + 2*e)^2 + 8*b*cos(2*f*x + 2*e) - 4*b)/((a + b)*abs(2*e^(2*I*f*x
+ 2*I*e) + 2)^2)))*sin(2*f*x + 2*e) - 4*(sqrt(a*b + b^2)*cos(2*f*x + 2*e) -
 sqrt(a*b + b^2))*((16*b^2*cos(2*f*x + 2*e)^4 + 16*b^2*sin(2*f*x + 2*e)^4 +
 (a^2 + 2*a*b + b^2)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^4 - 64*b^2*cos(2*f*x +
2*e)^3 + 96*b^2*cos(2*f*x + 2*e)^2 - 8*((a*b + b^2)*cos(2*f*x + 2*e)^2 - (a
*b + b^2)*sin(2*f*x + 2*e)^2 + a*b + b^2 - 2*(a*b + b^2)*cos(2*f*x + 2*e))*
abs(2*e^(2*I*f*x + 2*I*e) + 2)^2 - 64*b^2*cos(2*f*x + 2*e) + 32*(b^2*cos(2*
f*x + 2*e)^2 - 2*b^2*cos(2*f*x + 2*e) + b^2)*sin(2*f*x + 2*e)^2 + 16*b^2)/(
a^2 + 2*a*b + b^2))^(1/4)*sin(1/2*arctan2(8*(b*cos(2*f*x + 2*e) - b)*sin(2*
f*x + 2*e)/((a + b)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2), ((a + b)*abs(2*e^(2*
I*f*x + 2*I*e) + 2)^2 - 4*b*cos(2*f*x + 2*e)^2 + 4*b*sin(2*f*x + 2*e)^2 + 8
*b*cos(2*f*x + 2*e) - 4*b)/((a + b)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2))) - 8
*b*cos(2*f*x + 2*e) + 4*b)/((a + b)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2)) - (a
^(3/2) + 2*sqrt(a)*b)*sqrt(b)*arctan2(2*a*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f
*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x +
2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b
```

```
 + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2
*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin
(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x
 + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*a*cos(2*f*x + 2*e) + 2*(a^2
*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(
2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2
 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2
*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqr
t(a)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*c
os(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 2*a + 4*b))/(a*sqrt(
b)*f)
```

Giac [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec\left(fx+e\right)^2+a}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int((a + b/cos(e + f*x)^2)^(1/2), x)
```

3.77 $\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 68

$$\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}$$

```
[Out] arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f-cot(f*x+e)
*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4217, 283, 223, 212}

$$\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

```
[In] Int[Csc[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
- (Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} + \frac{b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

$$= \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$
\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx
= -\frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{b\sin^2(e+fx)}{a+b-a\sin^2(e+fx)}\right)\sqrt{a+b\sec^2(e+fx)}}{f}
$$

```
[In] Integrate[Csc[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, (b*Sin[e + f*x]^2)/(a + b -
 a*Sin[e + f*x]^2)]*Sqrt[a + b*Sec[e + f*x]^2])/f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(60) = 120.

Time = 5.34 (sec) , antiderivative size = 326, normalized size of antiderivative = 4.79

method	result
default	$-\frac{\left(-\sqrt{b}\,\ln\left(\frac{4\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\sqrt{b}\,\cos(fx+e)-4\sin(fx+e)a+4\sqrt{b}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4a-4b}{\sin(fx+e)+1}\right)\sin(fx+e)-\sqrt{b}\,\ln\left(-\frac{4\left(\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\right.}{}\right.\right.}{2f(1+c}$

```
[In] int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(-b^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*
cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a
-a-b)/(sin(f*x+e)+1))*sin(f*x+e)-b^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*sin(f*x+e)+2*cos(f*x+e)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2))*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*cot(f*x+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(60) = 120$.

Time = 0.31 (sec) , antiderivative size = 306, normalized size of antiderivative = 4.50

$$\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \left[\frac{\sqrt{b}\log\left(\frac{(a^2-6\,ab+b^2)\cos(fx+e)^4+8\,(ab-b^2)\cos(fx+e)^2+4\,\left((a-b)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)+8\,b^2}{\cos(fx+e)^4}\right)\mathrm{s}}{4\,f\sin(fx+e)}\right.$$

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f
*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*si
n(f*x + e) - 4*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/(f
*sin(f*x + e)), 1/2*(sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos
(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f
*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x + e) - 2*sqrt((a*cos(f*x + e)^2 + b
)/cos(f*x + e)^2)*cos(f*x + e))/(f*sin(f*x + e))]
```

Sympy [F]

$$\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\csc^2(e+fx)\,dx$$

```
[In] integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*csc(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right) - \frac{\sqrt{b\tan(fx+e)^2+a+b}}{\tan(fx+e)}}{f}$$

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) - sqrt(b*tan(f*x + e)^2 +
a + b)/tan(f*x + e))/f
```

Giac [F]

$$\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)^2+a}\csc\left(fx+e\right)^2\,dx$$

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\sin\left(e+f\,x\right)^2}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^2,x)
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^2, x)
```

3.78 $\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 105

$$\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}-\frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{3(a+b)f}$$

```
[Out] arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f-cot(f*x+e)
*(a+b+b*tan(f*x+e)^2)^(1/2)/f-1/3*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(3/2)/(
a+b)/f
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4217, 462, 283, 223, 212}

$$\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{3f(a+b)}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

```
[In] Int[Csc[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
- (Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f - (Cot[e + f*x]^3*(a + b
+ b*Tan[e + f*x]^2)^(3/2))/(3*(a + b)*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(1+x^2)\sqrt{a+b+bx^2}}{x^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\,(a+b+b\tan^2(e+fx))^{3/2}}{3(a+b)f} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{3(a+b)f} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{3(a+b)f} + \frac{b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

$$= \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{3(a+b)f}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.76 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.71

$$\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{\sqrt{2}\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)\left(\frac{4b\,\text{Hypergeometric2F1}\left(2,2,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)\text{se}}{}\right.}{3f\sqrt{a+2b+a\cos}}$$

```
[In] Integrate[Csc[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/3*(Sqrt[2]*Cot[e + f*x]*Csc[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2]*(1 - (
a*Sin[e + f*x]^2)/(a + b))*((4*b*Hypergeometric2F1[2, 2, 3/2, -((b*Tan[e +
f*x]^2)/(a + b))]*Sec[e + f*x]^2*Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)]*(a +
b - a*Sin[e + f*x]^2)^2*Tan[e + f*x]^2)/(a + b)^2 + (a + b + 2*a*Sin[e + f*
x]^2)*(Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)] + ArcSin[Sqrt[-((b*Tan[e + f*x]
^2)/(a + b))]]*Sqrt[-((b*Tan[e + f*x]^2)/(a + b))])))/(f*Sqrt[a + 2*b + a*C
os[2*e + 2*f*x]]*Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)]*Sqrt[a + b - a*Sin[e
+ f*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1036 vs. 2(93) = 186.

Time = 7.32 (sec) , antiderivative size = 1037, normalized size of antiderivative = 9.88

method	result	size
default	Expression too large to display	1037

```
[In] int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/f/(a+b)*(3*sin(f*x+e)*b^(3/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*cos(f*x+e)+3*sin(f*x+e)*b^(3/2)*ln(
-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1
))*cos(f*x+e)-3*sin(f*x+e)*b^(3/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))-3*sin(f*x+e)*b^(3/2)*ln(-4*(((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))+3*sin(f*
x+e)*b^(1/2)*a*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*co
s(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a
-b)/(sin(f*x+e)+1))*cos(f*x+e)+3*sin(f*x+e)*b^(1/2)*a*ln(-4*(((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*cos(f*x+e)-3*si
n(f*x+e)*b^(1/2)*a*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2
)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)
*a-a-b)/(sin(f*x+e)+1))-3*sin(f*x+e)*b^(1/2)*a*ln(-4*(((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))-4*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^2*a-6*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*b+6*a*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+8*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b)*(a+b*sec(f*x+e)^2)^(1/2)/((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cot(f*x+e)*csc(f*x+e)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(93) = 186$.

Time = 0.47 (sec) , antiderivative size = 436, normalized size of antiderivative = 4.15

$$\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{3\left((a+b)\cos\left(fx+e\right)^2 - a - b\right)\sqrt{b}\log\left(\frac{(a^2-6\,ab+b^2)\cos(fx+e)^4+8\,(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2\,b\cos(fx+e\right.}{\cos(fx+e)^4}\right.}{12\left((a+b)f\cos\left(fx\right.\right.} \right.$$

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a + b)*cos(f*x + e)^2 - a - b)*sqrt(b)*log(((a^2 - 6*a*b + b^2)*
cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 +
 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(
f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((2*a + 3*b)*cos(f*x + e
)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
)/(((a + b)*f*cos(f*x + e)^2 - (a + b)*f)*sin(f*x + e)), 1/6*(3*((a + b)*co
s(f*x + e)^2 - a - b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*co
s(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(
f*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x + e) - 2*((2*a + 3*b)*cos(f*x + e)
^3 - (3*a + 4*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))
/(((a + b)*f*cos(f*x + e)^2 - (a + b)*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2\left(e+fx\right)}\csc^4\left(e+fx\right)dx$$

```
[In] integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*csc(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.78

$$\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{3\sqrt{b}\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right) - \frac{3\sqrt{b\tan(fx+e)^2+a+b}}{\tan(fx+e)} - \frac{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}}{(a+b)\tan(fx+e)^3}}{3f}$$

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) - 3*sqrt(b*tan(f*x +
 e)^2 + a + b)/tan(f*x + e) - (b*tan(f*x + e)^2 + a + b)^(3/2)/((a + b)*tan
(f*x + e)^3))/f
```

Giac [F]

$$\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\csc(fx+e)^4\,dx$$

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\sin(e+f\,x)^4}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^4,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^4, x)
```

3.79 $\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 149

$$\begin{aligned}\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx &= \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}\\ &\quad-\frac{2(5a+4b)\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{15(a+b)^2f}\\ &\quad-\frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{5(a+b)f}\end{aligned}$$

```
[Out] arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f-cot(f*x+e)
*(a+b+b*tan(f*x+e)^2)^(1/2)/f-2/15*(5*a+4*b)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)
^2)^(3/2)/(a+b)^2/f-1/5*cot(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(3/2)/(a+b)/f
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used

= {4217, 473, 462, 283, 223, 212}

$$\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$=\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{5f(a+b)}$$
$$-\frac{2(5a+4b)\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{15f(a+b)^2}$$
$$-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

```
[In] Int[Csc[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
- (Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f - (2*(5*a + 4*b)*Cot[e +
f*x]^3*(a + b + b*Tan[e + f*x]^2)^(3/2))/(15*(a + b)^2*f) - (Cot[e + f*x]^5
*(a + b + b*Tan[e + f*x]^2)^(3/2))/(5*(a + b)*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2\sqrt{a+b+bx^2}}{x^6}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{5(a+b)f} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}\,(2(5a+4b)+5(a+b)x^2)}{x^4}\,dx, x, \tan(e+fx)\right)}{5(a+b)f}$$

$$= -\frac{2(5a+4b)\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{15(a+b)^2f} - \frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{5(a+b)f} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{2(5a+4b)\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{15(a+b)^2f} - \frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{5(a+b)f} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{2(5a+4b)\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{15(a+b)^2f} - \frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{5(a+b)f} + \frac{b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

$$= \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{2(5a+4b)\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{15(a+b)^2f} - \frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{5(a+b)f}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.73 (sec) , antiderivative size = 941, normalized size of antiderivative = 6.32

$$\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{\sqrt{2}\csc^5(e+fx)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)\left(-3(a+b)\cos^2(e+fx)-4a\cos^2(e+\right.}{}$$

```
[In] Integrate[Csc[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[2]*Csc[e + f*x]^5*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(1 - (a*Sin
[e + f*x]^2)/(a + b))*(-3*(a + b)*Cos[e + f*x]^2 - 4*a*Cos[e + f*x]^2*Sin[e
 + f*x]^2 - 16*b*Hypergeometric2F1[2, 2, 3/2, -((b*Tan[e + f*x]^2)/(a + b))
]*Sin[e + f*x]^2 + 8*b*HypergeometricPFQ[{2, 2, 2}, {1, 3/2}, -((b*Tan[e +
f*x]^2)/(a + b))]*Sin[e + f*x]^2 - (8*a^2*Cos[e + f*x]^2*Sin[e + f*x]^4)/(a
 + b) - (8*a*b*Hypergeometric2F1[2, 2, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*
Sin[e + f*x]^4)/(a + b) - (16*a*b*HypergeometricPFQ[{2, 2, 2}, {1, 3/2}, -(
(b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^4)/(a + b) + (24*a^2*b*Hypergeome
tric2F1[2, 2, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^6)/(a + b)^2
 + (8*a^2*b*HypergeometricPFQ[{2, 2, 2}, {1, 3/2}, -((b*Tan[e + f*x]^2)/(a
+ b))]*Sin[e + f*x]^6)/(a + b)^2 - (16*b^2*Hypergeometric2F1[2, 2, 3/2, -((
```

```
b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(a + b) + (8*b^2
*HypergeometricPFQ[{2, 2, 2}, {1, 3/2}, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[
e + f*x]^2*Tan[e + f*x]^2)/(a + b) - (8*a*b^2*Hypergeometric2F1[2, 2, 3/2,
-((b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^4*Tan[e + f*x]^2)/(a + b)^2 - (
16*a*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 3/2}, -((b*Tan[e + f*x]^2)/(a + b
))]*Sin[e + f*x]^4*Tan[e + f*x]^2)/(a + b)^2 + (24*a^2*b^2*Hypergeometric2F
1[2, 2, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^6*Tan[e + f*x]^2)/
(a + b)^3 + (8*a^2*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 3/2}, -((b*Tan[e +
f*x]^2)/(a + b))]*Sin[e + f*x]^6*Tan[e + f*x]^2)/(a + b)^3 - (4*a*ArcSin[Sq
rt[-((b*Tan[e + f*x]^2)/(a + b))]]*Cos[e + f*x]^2*Sin[e + f*x]^2*Sqrt[-((b*
Tan[e + f*x]^2)/(a + b))])/Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)] + (3*b*ArcS
in[Sqrt[-((b*Tan[e + f*x]^2)/(a + b))]]*Sin[e + f*x]^2)/Sqrt[-((b*Sec[e + f
*x]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(a + b)^2)] + (8*a^2*b*Arc
Sin[Sqrt[-((b*Tan[e + f*x]^2)/(a + b))]]*Sin[e + f*x]^6)/((a + b)^2*Sqrt[-(
(b*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(a + b)^2)])))
/(15*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1679 vs. $2(133) = 266$.

Time = 7.18 (sec) , antiderivative size = 1680, normalized size of antiderivative = 11.28

method	result	size
default	Expression too large to display	1680

```
[In] int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/30/f/(a+b)^2*(15*sin(f*x+e)^3*b^(5/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*cos(f*x+e)+15*sin(f*x+e)^3*b^
(5/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)
+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin
(f*x+e)-1))*cos(f*x+e)+30*sin(f*x+e)^3*b^(3/2)*a*ln(4*(((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*cos(f*x+e)+30*sin(f*x
+e)^3*b^(3/2)*a*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*
cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a
+a+b)/(sin(f*x+e)-1))*cos(f*x+e)-15*sin(f*x+e)^3*b^(5/2)*ln(4*(((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))-15*sin(f*x+e)
^3*b^(5/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f
*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)
/(sin(f*x+e)-1))+15*sin(f*x+e)^3*b^(1/2)*a^2*ln(4*(((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*cos(f*x+e)+15*sin(f*x+e)^
```

```
3*b^(1/2)*a^2*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*co
s(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a
+b)/(sin(f*x+e)-1))*cos(f*x+e)-30*sin(f*x+e)^3*b^(3/2)*a*ln(4*(((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))-30*sin(f*x+e)
^3*b^(3/2)*a*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos
(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+
b)/(sin(f*x+e)-1))+16*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*a^2+50*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b+30*
cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2-15*sin(f*x+e)^
3*b^(1/2)*a^2*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos
(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-
b)/(sin(f*x+e)+1))-15*sin(f*x+e)^3*b^(1/2)*a^2*ln(-4*(((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))-40*cos(f*x+e)^2*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2-118*cos(f*x+e)^2*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*a*b-70*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*b^2+30*a^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+80*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b+46*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*b^2)*(a+b*sec(f*x+e)^2)^(1/2)/((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*cot(f*x+e)*csc(f*x+e)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 312 vs. 2(133) = 266.

Time = 1.39 (sec) , antiderivative size = 656, normalized size of antiderivative = 4.40

$$\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{15\left(\left(a^2+2\,ab+b^2\right)\cos\left(fx+e\right)^4-2\left(a^2+2\,ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2\right)\sqrt{b}\log\left(\frac{(a^2-6\,ab+b^2}{}\right.}{}\right.$$

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/60*(15*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f
*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e
)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*
x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) +
8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((8*a^2 + 25*a*b + 15*b^2)*cos(f*x
+ e)^5 - (20*a^2 + 59*a*b + 35*b^2)*cos(f*x + e)^3 + (15*a^2 + 40*a*b + 23*
```

```
b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^2 + 2*
a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2
 + 2*a*b + b^2)*f)*sin(f*x + e)), 1/30*(15*((a^2 + 2*a*b + b^2)*cos(f*x + e
)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b)*ar
ctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*
sin(f*x + e) - 2*((8*a^2 + 25*a*b + 15*b^2)*cos(f*x + e)^5 - (20*a^2 + 59*a
*b + 35*b^2)*cos(f*x + e)^3 + (15*a^2 + 40*a*b + 23*b^2)*cos(f*x + e))*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^2 + 2*a*b + b^2)*f*cos(f*x +
e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f)*sin(
f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.96

$$\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{15\sqrt{b}\,\text{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right) - \frac{15\sqrt{b\tan(fx+e)^2+a+b}}{\tan(fx+e)} - \frac{10\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}}{(a+b)\tan(fx+e)^3} + \frac{2\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}b}{(a+b)^2\tan(fx+e)^3} - \frac{3\left(b\tan(fx+e)^2 \cdots\right.}{(a+b)\tan(f\cdots}}{15f}$$

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] 1/15*(15*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) - 15*sqrt(b*tan(f*
x + e)^2 + a + b)/tan(f*x + e) - 10*(b*tan(f*x + e)^2 + a + b)^(3/2)/((a +
b)*tan(f*x + e)^3) + 2*(b*tan(f*x + e)^2 + a + b)^(3/2)*b/((a + b)^2*tan(f*
x + e)^3) - 3*(b*tan(f*x + e)^2 + a + b)^(3/2)/((a + b)*tan(f*x + e)^5))/f
```

Giac [F]

$$\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)^2+a}\csc\left(fx+e\right)^6\,dx$$

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\sin\left(e+f\,x\right)^6}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^6,x)
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^6, x)
```

3.80 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^5(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 196

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^5(e+fx)\,dx=\frac{(3a-4b)\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}+\frac{(3a-4b)b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af}-\frac{(3a-4b)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af}+\frac{2\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5af}$$

```
[Out] -1/3*(3*a-4*b)*cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2)/a/f+2/3*cos(f*x+e)^3*(a+
b*sec(f*x+e)^2)^(5/2)/a/f-1/5*cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(5/2)/a/f+1/2
*(3*a-4*b)*arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f+1
/2*(3*a-4*b)*b*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used

$= \{4219, 473, 464, 283, 201, 223, 212\}$

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^5(e+fx)\,dx = \frac{\sqrt{b}(3a-4b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} + \frac{b(3a-4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af} - \frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5af} + \frac{2\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af} - \frac{(3a-4b)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^5,x]
```

```
[Out] ((3*a - 4*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]
^2]])/(2*f) + ((3*a - 4*b)*b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*a*
f) - ((3*a - 4*b)*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2))/(3*a*f) + (2*C
os[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(5/2))/(3*a*f) - (Cos[e + f*x]^5*(a +
b*Sec[e + f*x]^2)^(5/2))/(5*a*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(-1+x^2)^2(a+bx^2)^{3/2}}{x^6}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5af} + \frac{\text{Subst}\left(\int \frac{(-10a+5ax^2)(a+bx^2)^{3/2}}{x^4}\,dx, x, \sec(e+fx)\right)}{5af}$$

$$= \frac{2\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af} - \frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5af} + \frac{(3a-4b)\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{x^2}\,dx, x, \sec(e+fx)\right)}{3af}$$

$$= -\frac{(3a-4b)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af} + \frac{2\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af} - \frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5af} + \frac{((3a-4b)b)\text{Subst}\left(\int \sqrt{a+bx^2}\,dx, x, \sec(e+fx)\right)}{af}$$

$$= \frac{(3a-4b)b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af} - \frac{(3a-4b)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af} + \frac{2\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af} - \frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5af} + \frac{((3a-4b)b)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{2f}$$

$$= \frac{(3a-4b)b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af} - \frac{(3a-4b)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af} + \frac{2\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af} - \frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5af} + \frac{((3a-4b)b)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}$$

$$= \frac{(3a-4b)\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} + \frac{(3a-4b)b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af} - \frac{(3a-4b)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af} + \frac{2\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af} - \frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5af}$$

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.96

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^5(e+fx)\,dx = \frac{\sqrt{2}\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\left(-\frac{6b\left(a+b-a\sin^2(e+fx)\right)^{5/2}}{a} + 15\sec^2(e+fx)\left(a+b-a\sin^2\right.\right.}{15bf(a}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^5,x]
```

```
[Out] (Sqrt[2]*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2)*((-6*b*(a + b - a*Sin[
e + f*x]^2)^(5/2))/a + 15*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)^(5/2) -
 5*(3*a - 4*b)*(-3*b^(3/2)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[b]]
+ Sqrt[a + b - a*Sin[e + f*x]^2]*(a + 4*b - a*Sin[e + f*x]^2))))/(15*b*f*(a
 + 2*b + a*Cos[2*(e + f*x)])^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 801 vs. $2(172) = 344$.

Time = 7.03 (sec) , antiderivative size = 802, normalized size of antiderivative = 4.09

method	result
default	$-\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}\left(6\cos(fx+e)^8\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,a^2b+6\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,a^2b\cos(fx+e)^7+60\cos(fx+e)^3b^{\frac{5}{2}}\ln\left(-4\sqrt{b}\sqrt{\frac{b+a\,\cos}{(1+\cos}}\right.\right.}{}$

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/30/f/a/b*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(6*cos(f*x+e)^8*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*a^2*b+6*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*a^2*b*cos(f*x+e)^7+60*cos(f*x+e)^3*b^(5/2)*ln(-4*b^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*sec(f*x+e)-4*sec(f*x+e)*b)*a-20*cos(f*x+e)^6*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*a^2*b+12*cos(f*x+e)^6*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*a*b^2-45*cos(f*x+e)^3*b^(3/2)*ln(-4*b^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*sec(f*x+e)-4*sec(f*x+e)*b)*a^2-20*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*a^2*b*cos(f*x+e)^5+12*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*
b^2*cos(f*x+e)^5+30*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*a^2*b-80*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^2+6
*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3+30*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^3*a^2*b-80*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^3*b^2*a+6*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*b^3*cos(f*x+e)^3-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)^2*b^2*a-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f
*x+e)*a*b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.53 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.79

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^5(e+fx)\,dx = \left[-\frac{15\,(3\,a^2-4\,ab)\sqrt{b}\cos(fx+e)\log\left(\frac{a\cos(fx+e)^2-2\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2\,b}{\cos(fx+e)^2}\right)+2\,(6\,a^2\cos(f}{60\,af}\right.$$

$$-\frac{15\,(3\,a^2-4\,ab)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{b}\right)\cos(fx+e)+\left(6\,a^2\cos(fx+e)^6-4\,(5\,a^2-3\,ab}{30\,af\cos(fx+e)}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] [-1/60*(15*(3*a^2 - 4*a*b)*sqrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2 - 2*s
qrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(
f*x + e)^2) + 2*(6*a^2*cos(f*x + e)^6 - 4*(5*a^2 - 3*a*b)*cos(f*x + e)^4 +
2*(15*a^2 - 40*a*b + 3*b^2)*cos(f*x + e)^2 - 15*a*b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2))/(a*f*cos(f*x + e)), -1/30*(15*(3*a^2 - 4*a*b)*sqrt(-
b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)
/b)*cos(f*x + e) + (6*a^2*cos(f*x + e)^6 - 4*(5*a^2 - 3*a*b)*cos(f*x + e)^4
 + 2*(15*a^2 - 40*a*b + 3*b^2)*cos(f*x + e)^2 - 15*a*b)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2))/(a*f*cos(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^5(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**5,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.41

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^5(e+fx)\,dx =$$

$$-\frac{\frac{12\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{5}{2}}\cos(fx+e)^5}{a}-40\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3+60\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,a\cos(fx+e)-120\sqrt{a}}{}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(12*(a + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5/a - 40*(a + b/cos(f*x
 + e)^2)^(3/2)*cos(f*x + e)^3 + 60*sqrt(a + b/cos(f*x + e)^2)*a*cos(f*x + e
) - 120*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e) - 30*sqrt(a + b/cos(f*x +
 e)^2)*a*b*cos(f*x + e)/((a + b/cos(f*x + e)^2)*cos(f*x + e)^2 - b) + 45*a*
sqrt(b)*log((sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a + b
/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b))) - 60*b^(3/2)*log((sqrt(a + b/cos(
f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e
) + sqrt(b))))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1952 vs. 2(172) = 344.

Time = 1.95 (sec) , antiderivative size = 1952, normalized size of antiderivative = 9.96

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^5(e+fx)\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^5,x, algorithm="giac")
```

```
[Out] -1/15*(15*(3*a*b - 4*b^2)*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 -
 sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x
 + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - sqrt(a + b))/sqrt(-b))*
sgn(cos(f*x + e))/sqrt(-b) + 30*((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt
(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/
2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^3*(a*b + 2*b^2)*sgn(cos(f*x +
 e)) - (sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4
+ b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x +
 1/2*e)^2 + a + b))^2*(3*a*b - 2*b^2)*sqrt(a + b)*sgn(cos(f*x + e)) + (3*a^
2*b - 3*a*b^2 - 2*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2
*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sgn(cos(f*x + e)) - (a^2*b - a*b^2 + 2*
```

b^3)*sqrt(a + b)*sgn(cos(f*x + e)))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a + b) + a - 3*b)^2 - 4*(15*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^9*(2*a*b - b^2)*sgn(cos(f*x + e)) + 15*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^8*(22*a*b - 7*b^2)*sqrt(a + b)*sgn(cos(f*x + e)) - 20*(16*a^3 - 6*a^2*b - 51*a*b^2 + 15*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^7*sgn(cos(f*x + e)) + 20*(32*a^3 - 150*a^2*b + 105*a*b^2 - 21*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^6*sqrt(a + b)*sgn(cos(f*x + e)) + 2*(416*a^4 - 30*a^3*b - 2685*a^2*b^2 + 840*a*b^3 - 105*b^4)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^5*sgn(cos(f*x + e)) - 10*(256*a^4 - 942*a^3*b + 567*a^2*b^2 - 21*b^4)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^4*sqrt(a + b)*sgn(cos(f*x + e)) + 20*(16*a^5 - 178*a^4*b + 531*a^3*b^2 - 63*a^2*b^3 - 63*a*b^4 + 21*b^5)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^3*sgn(cos(f*x + e)) + 20*(160*a^5 - 590*a^4*b + 81*a^3*b^2 + 171*a^2*b^3 - 69*a*b^4 + 15*b^5)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2*sqrt(a + b)*sgn(cos(f*x + e)) - 5*(576*a^6 - 2230*a^5*b + 739*a^4*b^2 + 632*a^3*b^3 - 486*a^2*b^4 + 102*a*b^5 - 21*b^6)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sgn(cos(f*x + e)) + (768*a^6 - 3654*a^5*b + 4103*a^4*b^2 - 2200*a^3*b^3 + 690*a^2*b^4 - 90*a*b^5 + 15*b^6)*sqrt(a + b)*sgn(cos(f*x + e)))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 + 2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a + b) - 3*a + b)^5)/f

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^5(e+fx)\,dx=\int \sin\left(e+f\,x\right)^5\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

```
[In] int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.81 $\int (a+b\sec^2(e+fx))^{3/2}\sin^3(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 162

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^3(e+fx)\,dx = \frac{(3a-2b)\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} + \frac{(3a-2b)b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af} - \frac{(3a-2b)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af} + \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af}$$

```
[Out] -1/3*(3*a-2*b)*cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2)/a/f+1/3*cos(f*x+e)^3*(a+
b*sec(f*x+e)^2)^(5/2)/a/f+1/2*(3*a-2*b)*arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec
(f*x+e)^2)^(1/2))*b^(1/2)/f+1/2*(3*a-2*b)*b*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(
1/2)/a/f
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4219, 464, 283, 201, 223, 212}

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^3(e+fx)\,dx = \frac{\sqrt{b}(3a-2b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} + \frac{b(3a-2b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af} + \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af} - \frac{(3a-2b)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^3,x]
```

```
[Out] ((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]
^2]])/(2*f) + ((3*a - 2*b)*b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*a*
f) - ((3*a - 2*b)*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2))/(3*a*f) + (Cos
[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(5/2))/(3*a*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
```

```
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a+bx^2)^{3/2}}{x^4} \, dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af} + \frac{(3a-2b)\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{x^2} \, dx, x, \sec(e+fx)\right)}{3af}$$

$$= -\frac{(3a-2b)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af} + \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af} + \frac{((3a-2b)b)\text{Subst}\left(\int \sqrt{a+bx^2} \, dx, x, \sec(e+fx)\right)}{af}$$

$$= \frac{(3a-2b)b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af} - \frac{(3a-2b)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af} + \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af} + \frac{((3a-2b)b)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}} \, dx, x, \sec(e+fx)\right)}{2f}$$

$$= \frac{(3a-2b)b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af} - \frac{(3a-2b)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af} + \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af} + \frac{((3a-2b)b)\text{Subst}\left(\int \frac{1}{1-bx^2} \, dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}$$

$$= \frac{(3a-2b)\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} + \frac{(3a-2b)b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af} - \frac{(3a-2b)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af} + \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.01

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^3(e+fx)\,dx = \frac{\sqrt{2}\cos^3(e+fx)\,(a+b\sec^2(e+fx))^{3/2}\left(3\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))^{5/2}-(3a-2b)\left(-\right.\right.}{3bf(a+2b+a\cos(2(}$$

[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^3,x]

[Out] (Sqrt[2]*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2)*(3*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)^(5/2) - (3*a - 2*b)*(-3*b^(3/2)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[b]] + Sqrt[a + b - a*Sin[e + f*x]^2]*(a + 4*b - a*Sin[e + f*x]^2))))/(3*b*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. 2(142) = 284.

Time = 6.93 (sec) , antiderivative size = 552, normalized size of antiderivative = 3.41

method	result
default	$-\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}\left(6\ln\left(-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sec(fx+e)-4\sec(fx+e)b\right)\cos(fx+e)^3b^{\frac{5}{2}}-2\cos(fx+e\right.}{}$

[In] int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^3,x,method=_RETURNVERBOSE)

[Out] -1/6/f/b*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(6*ln(-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)-4*sec(f*x+e)*b)*cos(f*x+e)^3*b^(5/2)-2*cos(f*x+e)^6*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b-9*ln(-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)-4*sec(f*x+e)*b)*cos(f*x+e)^3*b^(3/2)*a-2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b*cos(f*x+e)^5+6*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b-8*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2+6*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b-8*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*cos(f*x+e)^3-3*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2-3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*cos(f*x+e))

Fricas [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.72

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^3(e+fx)\,dx = \left[-\frac{3\,(3\,a-2\,b)\sqrt{b}\cos\left(fx+e\right)\log\left(\frac{a\cos(fx+e)^2-2\,\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2\,b}{\cos(fx+e)^2}\right)-2\,\left(2\,a\cos\left(fx+\right.\right.}{12\,f\cos\left(fx+e\right)}\right.$$

$$-\frac{3\,(3\,a-2\,b)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{b}\right)\cos\left(fx+e\right)-\left(2\,a\cos\left(fx+e\right)^4-2\,(3\,a-4\,b)\cos\left(\right.\right.}{6\,f\cos\left(fx+e\right)}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(3*a - 2*b)*sqrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2 - 2*sqrt(b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x +
 e)^2) - 2*(2*a*cos(f*x + e)^4 - 2*(3*a - 4*b)*cos(f*x + e)^2 + 3*b)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)), -1/6*(3*(3*a - 2*b
)*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(
f*x + e)/b)*cos(f*x + e) - (2*a*cos(f*x + e)^4 - 2*(3*a - 4*b)*cos(f*x + e)
^2 + 3*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^3(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.54

$$\int \left(a + b\sec^2(e + fx)\right)^{3/2}\sin^3(e + fx)\,dx = \frac{4\left(a + \frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3 - 12\sqrt{a + \frac{b}{\cos(fx+e)^2}}a\cos(fx+e) + 12\sqrt{a + \frac{b}{\cos(fx+e)^2}}b\cos(fx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(a + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3 - 12*sqrt(a + b/cos(f*x
 + e)^2)*a*cos(f*x + e) + 12*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e) + 6*
sqrt(a + b/cos(f*x + e)^2)*a*b*cos(f*x + e)/((a + b/cos(f*x + e)^2)*cos(f*x
 + e)^2 - b) - 9*a*sqrt(b)*log((sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) - s
qrt(b))/(sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b))) + 6*b^(3/2)*lo
g((sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a + b/cos(f*x +
 e)^2)*cos(f*x + e) + sqrt(b))))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1388 vs. 2(142) = 284.

Time = 1.24 (sec) , antiderivative size = 1388, normalized size of antiderivative = 8.57

$$\int \left(a + b\sec^2(e + fx)\right)^{3/2}\sin^3(e + fx)\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^3,x, algorithm="giac")
```

```
[Out] -1/3*(3*(3*a*b - 2*b^2)*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - sqrt(a + b))/sqrt(-b))*sg
n(cos(f*x + e))/sqrt(-b) + 6*((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*
tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e
)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^3*(a*b + 2*b^2)*sgn(cos(f*x + e)
) - (sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b
*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/
2*e)^2 + a + b))^2*(3*a*b - 2*b^2)*sqrt(a + b)*sgn(cos(f*x + e)) + (3*a^2*b
 - 3*a*b^2 - 2*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*
x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*
tan(1/2*f*x + 1/2*e)^2 + a + b))*sgn(cos(f*x + e)) - (a^2*b - a*b^2 + 2*b^3
)*sqrt(a + b)*sgn(cos(f*x + e)))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqr
```

```
t(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1
/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*(sqrt(a + b)*tan(1/2*f
*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 -
2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a +
b) + a - 3*b)^2 - 8*(3*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2
*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^5*a*b*sgn(cos(f*x + e)) - 3*(sqrt(a + b
)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x +
1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)
)^4*(2*a^2 - 3*a*b)*sqrt(a + b)*sgn(cos(f*x + e)) + 2*(4*a^3 - 9*a^2*b + 3*
a*b^2)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4
+ b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x +
 1/2*e)^2 + a + b))^3*sgn(cos(f*x + e)) + 6*(2*a^3 - 3*a^2*b - a*b^2)*(sqrt
(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*
f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 +
a + b))^2*sqrt(a + b)*sgn(cos(f*x + e)) - 3*(8*a^4 - 13*a^3*b - 2*a^2*b^2 +
 3*a*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)
^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*
x + 1/2*e)^2 + a + b))*sgn(cos(f*x + e)) + (10*a^4 - 23*a^3*b + 12*a^2*b^2
- 3*a*b^3)*sqrt(a + b)*sgn(cos(f*x + e)))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e
)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/
2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 + 2*(sqrt(a + b)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/
2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*
sqrt(a + b) - 3*a + b)^3)/f
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^3(e+fx)\,dx=\int \sin\left(e+f\,x\right)^3\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

```
[In] int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.82 $\int (a+b\sec^2(e+fx))^{3/2}\sin(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 100

$$\int (a+b\sec^2(e+fx))^{3/2}\sin(e+fx)\,dx=\frac{3a\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}+\frac{3b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}-\frac{\cos(e+fx)\,(a+b\sec^2(e+fx))^{3/2}}{f}$$

```
[Out] -cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2)/f+3/2*a*arctanh(sec(f*x+e)*b^(1/2)/(a+
b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f+3/2*b*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/
f
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4219, 283, 201, 223, 212}

$$\int (a+b\sec^2(e+fx))^{3/2}\sin(e+fx)\,dx=\frac{3a\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}+\frac{3b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}-\frac{\cos(e+fx)\,(a+b\sec^2(e+fx))^{3/2}}{f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x],x]
```

```
[Out] (3*a*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(2
*f) + (3*b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*f) - (Cos[e + f*x]*(
a + b*Sec[e + f*x]^2)^(3/2))/f
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{x^2}\,dx, x, \sec(e+fx)\right)}{f}\\ &= -\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{f}+\frac{(3b)\text{Subst}\left(\int \sqrt{a+bx^2}\,dx, x, \sec(e+fx)\right)}{f}\end{aligned}$$

$$=\frac{3b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}-\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{f}+\frac{(3ab)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^2}}\,dx,x,\sec(e+fx)\right)}{2f}$$

$$=\frac{3b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}-\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{f}+\frac{(3ab)\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}$$

$$=\frac{3a\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}+\frac{3b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}-\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.48 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.73

$$\int\left(a+b\sec^2(e+fx)\right)^{3/2}\sin(e+fx)\,dx=$$
$$-\frac{a\cos(e+fx)(a+2b+a\cos(2(e+fx)))^2\,\text{Hypergeometric2F1}\left(2,\frac{5}{2},\frac{7}{2},1+\frac{a\cos^2(e+fx)}{b}\right)\sqrt{a+b\sec^2(e+fx}}{20b^2f}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x],x]
[Out] -1/20*(a*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^2*Hypergeometric2F1[2,
 5/2, 7/2, 1 + (a*Cos[e + f*x]^2)/b]*Sqrt[a + b*Sec[e + f*x]^2])/(b^2*f)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.21

method	result
derivativedivides	$-\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{5}{2}}}{fa\sec(fx+e)}+\frac{b\sec(fx+e)\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}{fa}+\frac{3b\sec(fx+e)\sqrt{a+b\sec(fx+e)^2}}{2f}+\frac{3\sqrt{b}\,a\ln\left(\sqrt{b}\sec(f\right.}{}$
default	$-\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{5}{2}}}{fa\sec(fx+e)}+\frac{b\sec(fx+e)\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}{fa}+\frac{3b\sec(fx+e)\sqrt{a+b\sec(fx+e)^2}}{2f}+\frac{3\sqrt{b}\,a\ln\left(\sqrt{b}\sec(f\right.}{}$

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f/a/sec(f*x+e)*(a+b*sec(f*x+e)^2)^(5/2)+1/f*b/a*sec(f*x+e)*(a+b*sec(f*x+
e)^2)^(3/2)+3/2*b*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f+3/2/f*b^(1/2)*a*ln(
b^(1/2)*sec(f*x+e)+(a+b*sec(f*x+e)^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.31

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin(e+fx)\,dx = \left[\frac{3\,a\sqrt{b}\cos(fx+e)\log\left(\frac{a\cos(fx+e)^2+2\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2\,b}{\cos(fx+e)^2}\right)-2\left(2\,a\cos(fx+e)^2-b\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\,f\cos(fx+e)}, -\frac{3\,a\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{b}\right)\cos(fx+e)+\left(2\,a\cos(fx+e)^2-b\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\,f\cos(fx+e)}\right]$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e),x, algorithm="fricas")
```

```
[Out] [1/4*(3*a*sqrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) - 2*(
2*a*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos
(f*x + e)), -1/2*(3*a*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)*cos(f*x + e)/b)*cos(f*x + e) + (2*a*cos(f*x + e)^2 - b)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e))]
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin(e+fx)\,dx = \int \left(a+b\sec^2(e+fx)\right)^{\frac{3}{2}}\sin(e+fx)\,dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*sin(e + f*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.42

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin(e+fx)\,dx = -\frac{4\sqrt{a+\frac{b}{\cos(fx+e)^2}}a\cos(fx+e)-\frac{2\sqrt{a+\frac{b}{\cos(fx+e)^2}}ab\cos(fx+e)}{\left(a+\frac{b}{\cos(fx+e)^2}\right)\cos(fx+e)^2-b}+3a\sqrt{b}\log\left(\frac{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)-\sqrt{b}}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)+\sqrt{b}}\right)}{4f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e),x, algorithm="maxima")
```

```
[Out] -1/4*(4*sqrt(a + b/cos(f*x + e)^2)*a*cos(f*x + e) - 2*sqrt(a + b/cos(f*x +
e)^2)*a*b*cos(f*x + e)/((a + b/cos(f*x + e)^2)*cos(f*x + e)^2 - b) + 3*a*sq
rt(b)*log((sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a + b/c
os(f*x + e)^2)*cos(f*x + e) + sqrt(b))))/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.87

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin(e+fx)\,dx = -\frac{\left(\frac{3b\arctan\left(\frac{\sqrt{a\cos(fx+e)^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+2\sqrt{a\cos(fx+e)^2+b}-\frac{\sqrt{a\cos(fx+e)^2+b}b}{a\cos(fx+e)^2}\right)a\mathrm{sgn}(\cos(fx+e))}{2f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e),x, algorithm="giac")
```

```
[Out] -1/2*(3*b*arctan(sqrt(a*cos(f*x + e)^2 + b)/sqrt(-b))/sqrt(-b) + 2*sqrt(a*c
os(f*x + e)^2 + b) - sqrt(a*cos(f*x + e)^2 + b)*b/(a*cos(f*x + e)^2))*a*sgn
(cos(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 20.41 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.61

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin(e+fx)\,dx = -\frac{\cos\left(e+f\,x\right)\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b}{a\cos(e+f\,x)^2}\right)}{f\,\left(\frac{b}{a\cos(e+f\,x)^2}+1\right)^{3/2}}$$

```
[In] int(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] -(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -b
/(a*cos(e + f*x)^2)))/(f*(b/(a*cos(e + f*x)^2) + 1)^(3/2))
```

3.83 $\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 122

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{\sqrt{b}(3a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}-\frac{(a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}+\frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}$$

```
[Out] -(a+b)^(3/2)*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/f+1/2
*(3*a+2*b)*arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f+1
/2*b*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4219, 427, 537, 223, 212, 385, 213}

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=-\frac{(a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}(3a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}+\frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}$$

```
[In] Int[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]
^2]])/(2*f) - ((a + b)^(3/2)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*
Sec[e + f*x]^2]])/f + (b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n}, x]
```

```
&& IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{a(2a+b)+b(3a+2b)x^2}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{2f}$$

$$= \frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{(a+b)^2\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f} + \frac{(b(3a+2b))\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{2f}$$

$$= \frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{(a+b)^2\text{Subst}\left(\int \frac{1}{-1-(-a-b)x^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} + \frac{(b(3a+2b))\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}$$

$$= \frac{\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} - \frac{(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} + \frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.40

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\left(2\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{b}}\right)\cos^2(e+fx) - 4(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{a+b}}\right.\right.}{2\sqrt{2}f\sqrt{a+2b+a\cos}}$$

```
[In] Integrate[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((2*Sqrt[b]*(3*a + 2*b)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[b]]*Cos
[e + f*x]^2 - 4*(a + b)^(3/2)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a
 + b]]*Cos[e + f*x]^2 + Sqrt[2]*b*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])*Sec[e
 + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*Sqrt[2]*f*Sqrt[a + 2*b + a*Cos[2*(e
+ f*x)]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1436 vs. $2(104) = 208$.

Time = 6.76 (sec) , antiderivative size = 1437, normalized size of antiderivative = 11.78

method	result	size
default	Expression too large to display	1437

```
[In] int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f/b/(a+b)^(5/2)*(a+b*sec(f*x+e)^2)^(3/2)/(1+cos(f*x+e))/(b+a*cos(f*x+e)
^2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(2*cos(f*x+e)^3*b^(5/2)*(a+
b)^(5/2)*ln(-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*b^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)-4*sec(f*x+e)*b)+3*
cos(f*x+e)^3*b^(3/2)*(a+b)^(5/2)*ln(-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f
*x+e)-4*sec(f*x+e)*b)*a+cos(f*x+e)^2*(a+b)^(5/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*b^2-cos(f*x+e)^3*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^4*b-4*cos(f*x+e
)^3*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/
2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f
*x+e)*a+b)/(1+cos(f*x+e)))*a^3*b^2-6*cos(f*x+e)^3*ln(2/(a+b)^(1/2)*(((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^
2*b^3-4*cos(f*x+e)^3*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a*b^4-cos(f*x+e)^3*ln(2/(a+b)^(
1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+co
s(f*x+e)))*b^5-cos(f*x+e)^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b
)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^4*b-4*cos(f*x+e)^3*ln(-4*(((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*
a^3*b^2-6*cos(f*x+e)^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(
a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/
2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^2*b^3-4*cos(f*x+e)^3*ln(-4*(((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^
```

```
2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a*b
^4-cos(f*x+e)^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1
/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(
f*x+e)*a+b)/(-1+cos(f*x+e)))*b^5+cos(f*x+e)*(a+b)^(5/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 715, normalized size of antiderivative = 5.86

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left[\frac{2\,(a+b)^{\frac{3}{2}}\cos(fx+e)\log\left(\frac{2\left(a\cos(fx+e)^2-2\sqrt{a+b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+a+2\,b\right)}{\cos(fx+e)^2-1}\right)+(3\,a+2\,b)\sqrt{\ldots}}{4\,f\cos(fx+\ldots}\right.$$

$$-\frac{(3\,a+2\,b)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{b}\right)\cos(fx+e)-(a+b)^{\frac{3}{2}}\cos(fx+e)\log\left(\frac{2\left(a\cos(fx+e)^2-\ldots\right.}{\ldots}\right.}{2\,f\cos(fx+e)}$$

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a + b)^(3/2)*cos(f*x + e)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*
x + e)^2 - 1)) + (3*a + 2*b)*sqrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2 + 2
*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/co
s(f*x + e)^2) + 2*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x
 + e)), 1/4*(4*(a + b)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b))*cos(f*x + e) + (3*a + 2*b)*s
qrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*b*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)), -1/2*((3*a + 2*b)*sqrt
(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e)/b)*cos(f*x + e) - (a + b)^(3/2)*cos(f*x + e)*log(2*(a*cos(f*x + e)^2 - 2
*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a +
 2*b)/(cos(f*x + e)^2 - 1)) - b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
)/(f*cos(f*x + e)), 1/2*(2*(a + b)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b))*cos(f*x + e) - (
```

```
3*a + 2*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*cos(f*x + e)/b)*cos(f*x + e) + b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2))/(f*cos(f*x + e))]
```

Sympy [F]

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\csc\left(e+fx\right)dx$$

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*csc(e + f*x), x)
```

Maxima [F]

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)\,dx$$

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e), x)
```

Giac [F(-2)]

Exception generated.

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\sin\left(e+f\,x\right)}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x),x)
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x), x)
```

3.84 $\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 161

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{\sqrt{b}(3a+4b)\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}-\frac{\sqrt{a+b}(a+4b)\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}+\frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{\cot(e+fx)\csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{2f}$$

```
[Out] -1/2*cot(f*x+e)*csc(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2)/f+1/2*(3*a+4*b)*arctanh
(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-1/2*(a+4*b)*arctanh
(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*(a+b)^(1/2)/f+b*sec(f*x+e
)*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4219, 478, 542, 537, 223, 212, 385, 213}

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{\sqrt{b}(3a+4b)\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}-\frac{\sqrt{a+b}(a+4b)\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}+\frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{\cot(e+fx)\csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{2f}$$

```
[In] Int[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(3*a + 4*b)*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]
^2]])/(2*f) - (Sqrt[a + b]*(a + 4*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqr
t[a + b*Sec[e + f*x]^2]])/(2*f) + (b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2
])/f - (Cot[e + f*x]*Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2))/(2*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)^{3/2}}{(-1+x^2)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{2f} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}(a+4bx^2)}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{2f}$$

$$= \frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{\cot(e+fx)\csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{2f} + \frac{\text{Subst}\left(\int \frac{2a(a+2b)+2b(3a+4b)x^2}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{4f}$$

$$= \frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{\cot(e+fx)\csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{2f} + \frac{((a+b)(a+4b))\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{2f} + \frac{(b(3a+4b))\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{2f}$$

$$= \frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{\cot(e+fx)\csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{2f} + \frac{((a+b)(a+4b))\text{Subst}\left(\int \frac{1}{-1-(-a-b)x^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} + \frac{(b(3a+4b))\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}$$

$$= \frac{\sqrt{b}(3a+4b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} - \frac{\sqrt{a+b}(a+4b)\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} + \frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{\cot(e+fx)\csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{2f}$$

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.25

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = -\frac{\csc^2(e+fx)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\sqrt{2}\sqrt{a+2b+a\cos(2(e+fx))}(a+(a+2b)\cos(2(e+fx)\right.}{4\sqrt{2}f\sqrt{}}$$

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/4*(Csc[e + f*x]^2*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(Sqrt[2]*Sqrt[
a + 2*b + a*Cos[2*(e + f*x)]]*(a + (a + 2*b)*Cos[2*(e + f*x)]) - Sqrt[b]*(3
*a + 4*b)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[b]]*Sin[2*(e + f*x)]^
2 + Sqrt[a + b]*(a + 4*b)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b
]]*Sin[2*(e + f*x)]^2))/(Sqrt[2]*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3469 vs. 2(139) = 278.

Time = 6.92 (sec) , antiderivative size = 3470, normalized size of antiderivative = 21.55

method	result	size
default	Expression too large to display	3470

```
[In] int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f/b/(a+b)^(7/2)*(14*ln(-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)-4*
```

sec(f*x+e)*b)*cos(f*x+e)^3*(a+b)^(5/2)*b^(5/2)*a+6*ln(-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)-4*sec(f*x+e)*b)*cos(f*x+e)^3*(a+b)^(5/2)*b^(3/2)*a^2-14*ln(-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)-4*sec(f*x+e)*b)*cos(f*x+e)^2*(a+b)^(5/2)*b^(5/2)*a-6*ln(-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)-4*sec(f*x+e)*b)*cos(f*x+e)^2*(a+b)^(5/2)*b^(3/2)*a^2+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^2*(a+b)^(5/2)*a^2*b+6*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^2*(a+b)^(5/2)*a*b^2-2*(a+b)^(5/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3-4*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*cos(f*x+e)^3*b^6-4*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*cos(f*x+e)^3*b^6+4*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*cos(f*x+e)^2*b^6+4*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*cos(f*x+e)^2*b^6+8*ln(-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)-4*sec(f*x+e)*b)*cos(f*x+e)^3*(a+b)^(5/2)*b^(7/2)-8*ln(-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)-4*sec(f*x+e)*b)*cos(f*x+e)^2*(a+b)^(5/2)*b^(7/2)-ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*cos(f*x+e)^3*a^5*b-8*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*cos(f*x+e)^3*a^4*b^2-22*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*cos(f*x+e)^3*a^3*b^3-28*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*cos(f*x+e)^3*a^2*b^4-17*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*cos(f*x+e)^3*a*b^5-ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*cos(f*x+e)^3*a^5*b-8*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*cos(f*x+e)^3*a^4*b^2-22*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos

```
(f*x+e)*a+b)/(-1+cos(f*x+e)))*cos(f*x+e)^3*a^3*b^3-28*ln(-4*(((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*cos(f*x+
e)^3*a^2*b^4-17*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1
/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(
f*x+e)*a+b)/(-1+cos(f*x+e)))*cos(f*x+e)^3*a*b^5+ln(2/(a+b)^(1/2)*(((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*cos(
f*x+e)^2*a^5*b+8*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b
)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*cos(f*x+e)^2*a^4*b^2+22*ln(2/(a+b)^
(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+c
os(f*x+e)))*cos(f*x+e)^2*a^3*b^3+28*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*cos(f*x+e)^2*a^2
*b^4+17*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)
^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-c
os(f*x+e)*a+b)/(1+cos(f*x+e)))*cos(f*x+e)^2*a*b^5+ln(-4*(((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*cos(f*x+e)^2
*a^5*b+8*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos
(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*
a+b)/(-1+cos(f*x+e)))*cos(f*x+e)^2*a^4*b^2+22*ln(-4*(((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*cos(f*x+e)^2*a^3
*b^3+28*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(
f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a
+b)/(-1+cos(f*x+e)))*cos(f*x+e)^2*a^2*b^4+17*ln(-4*(((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*cos(f*x+e)^2*a*b^
5+4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^2*(a+b)^(5/2)*b^
3-2*(a+b)^(5/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^2)*(a+b*sec
(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/(b+a*cos(f*x+e
)^2)*cot(f*x+e)*csc(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 984, normalized size of antiderivative = 6.11

$$\int \csc^3(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx = \text{Too large to display}$$

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a + 4*b)*cos(f*x + e)^3 - (a + 4*b)*cos(f*x + e))*sqrt(a + b)*log(2
*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) + ((3*a + 4*b)*cos(f*x +
e)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e
)^2) + 2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2))/(f*cos(f*x + e)^3 - f*cos(f*x + e)), 1/4*(2*((a + 4*b)*cos(f*x +
e)^3 - (a + 4*b)*cos(f*x + e))*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos
(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) + ((3*a + 4*b)*cos(f
*x + e)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqr
t(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*
x + e)^2) + 2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2))/(f*cos(f*x + e)^3 - f*cos(f*x + e)), -1/4*(2*((3*a + 4*b)*co
s(f*x + e)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) - ((a + 4*b)*cos(f*x + e
)^3 - (a + 4*b)*cos(f*x + e))*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(
a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/
(cos(f*x + e)^2 - 1)) - 2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 - f*cos(f*x + e)), 1/2*(((a +
4*b)*cos(f*x + e)^3 - (a + 4*b)*cos(f*x + e))*sqrt(-a - b)*arctan(sqrt(-a -
 b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) - ((3
*a + 4*b)*cos(f*x + e)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt(-b)*arctan(sqrt(-
b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) + ((a + 2*b)
*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*
x + e)^3 - f*cos(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^3\,dx$$

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%{[%%%{16384,[5,4]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{-1,[0,1]
%%%}]%
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \frac{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}}{{\sin\left(e+f\,x\right)}^3}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^3,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^3, x)
```

3.85 $\int \csc^5(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx$

Optimal result

Integrand size = 25, antiderivative size = 218

$$\int \csc^5(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \frac{3\sqrt{b}(a+2b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} - \frac{3(a^2+8ab+8b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8\sqrt{a+b}f} + \frac{3(a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{3(a+2b)\csc^2(e+fx)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{\cot(e+fx)\csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{4f}$$

```
[Out] -1/4*cot(f*x+e)*csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2)/f+3/2*(a+2*b)*arctanh
(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-3/8*(a^2+8*a*b+8*b^
2)*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/f/(a+b)^(1/2)+3
/8*(a+4*b)*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f-3/8*(a+2*b)*csc(f*x+e)^2*s
ec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4219, 478, 591, 596, 537, 223, 212, 385, 213}

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = -\frac{3(a^2+8ab+8b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f\sqrt{a+b}} + \frac{3\sqrt{b}(a+2b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} + \frac{3(a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{3(a+2b)\csc^2(e+fx)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{\cot(e+fx)\csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{4f}$$

```
[In] Int[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*Sqrt[b]*(a + 2*b)*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]
^2]])/(2*f) - (3*(a^2 + 8*a*b + 8*b^2)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/S
qrt[a + b*Sec[e + f*x]^2]])/(8*Sqrt[a + b]*f) + (3*(a + 4*b)*Sec[e + f*x]*S
qrt[a + b*Sec[e + f*x]^2])/(8*f) - (3*(a + 2*b)*Csc[e + f*x]^2*Sec[e + f*x]
*Sqrt[a + b*Sec[e + f*x]^2])/(8*f) - (Cot[e + f*x]*Csc[e + f*x]^3*(a + b*Se
c[e + f*x]^2)^(3/2))/(4*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 591

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Dist[1/(
a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*
(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m +
 n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n,
 0] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e -
 a*f])
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
```

```
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4(a+bx^2)^{3/2}}{(-1+x^2)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{4f} + \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx^2}(3a+6bx^2)}{(-1+x^2)^2}\,dx, x, \sec(e+fx)\right)}{4f}$$

$$= -\frac{3(a+2b)\csc^2(e+fx)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{\cot(e+fx)\csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{4f} + \frac{\text{Subst}\left(\int \frac{x^2(3a(a+6b)+6b(a+4b)x^2)}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{8f}$$

$$= \frac{3(a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{3(a+2b)\csc^2(e+fx)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{\cot(e+fx)\csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{4f} - \frac{\text{Subst}\left(\int \frac{-6ab(a+4b)-24b^2(a+2b)x^2}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{16bf}$$

$$= \frac{3(a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{3(a+2b)\csc^2(e+fx)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{\cot(e+fx)\csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{4f} + \frac{(3b(a+2b))\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{2f} + \frac{(3(a^2+8ab+8b^2))\,\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{8f}$$

$$= \frac{3(a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{3(a+2b)\csc^2(e+fx)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{\cot(e+fx)\csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{4f} + \frac{(3b(a+2b))\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} + \frac{(3(a^2+8ab+8b^2))\,\text{Subst}\left(\int \frac{1}{-1-(-a-b)x^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f}$$

$$= \frac{3\sqrt{b}(a+2b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} - \frac{3(a^2+8ab+8b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8\sqrt{a+b}f} + \frac{3(a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{3(a+2b)\csc^2(e+fx)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{\cot(e+fx)\csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{4f}$$

Mathematica [A] (verified)

Time = 2.60 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.20

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = -\frac{(b+a\cos^2(e+fx))\left(-12b^{3/2}(a^2+3ab+2b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{b}}\right)\cos^2(e+fx)+3b\sqrt{a+b}(a^2+8a\right.}{}$$

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*((b + a*Cos[e + f*x]^2)*(-12*b^(3/2)*(a^2 + 3*a*b + 2*b^2)*ArcTanh[Sqr
t[a + b - a*Sin[e + f*x]^2]/Sqrt[b]]*Cos[e + f*x]^2 + 3*b*Sqrt[a + b]*(a^2
+ 8*a*b + 8*b^2)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b]]*Cos[e
+ f*x]^2 + (b*(a + b)*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(11*a + 4*b + 8*(a
 + 3*b)*Cos[2*(e + f*x)] - 3*(a + 4*b)*Cos[4*(e + f*x)])*Csc[e + f*x]^4)/(8
*Sqrt[2]))*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(Sqrt[2]*b*(a + b)*f*(a
 + 2*b + a*Cos[2*(e + f*x)])^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5130 vs. 2(190) = 380.

Time = 6.61 (sec) , antiderivative size = 5131, normalized size of antiderivative = 23.54

method	result	size
default	Expression too large to display	5131

```
[In] int(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 1511, normalized size of antiderivative = 6.93

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] [1/16*(3*((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^5 - 2*(a^2 + 8*a*b + 8*b^2)*co
s(f*x + e)^3 + (a^2 + 8*a*b + 8*b^2)*cos(f*x + e))*sqrt(a + b)*log(2*(a*cos
(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos
(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) + 12*((a^2 + 3*a*b + 2*b^2)*cos(
f*x + e)^5 - 2*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 + 3*a*b + 2*b^2)
*cos(f*x + e))*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*(3*(a^2 +
 5*a*b + 4*b^2)*cos(f*x + e)^4 - (5*a^2 + 23*a*b + 18*b^2)*cos(f*x + e)^2 +
 4*a*b + 4*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a + b)*f*cos
(f*x + e)^5 - 2*(a + b)*f*cos(f*x + e)^3 + (a + b)*f*cos(f*x + e)), 1/8*(3*
((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^5 - 2*(a^2 + 8*a*b + 8*b^2)*cos(f*x + e
)^3 + (a^2 + 8*a*b + 8*b^2)*cos(f*x + e))*sqrt(-a - b)*arctan(sqrt(-a - b)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) + 6*((a^2
 + 3*a*b + 2*b^2)*cos(f*x + e)^5 - 2*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 +
 (a^2 + 3*a*b + 2*b^2)*cos(f*x + e))*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt
(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x
 + e)^2) + (3*(a^2 + 5*a*b + 4*b^2)*cos(f*x + e)^4 - (5*a^2 + 23*a*b + 18*b
^2)*cos(f*x + e)^2 + 4*a*b + 4*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2))/((a + b)*f*cos(f*x + e)^5 - 2*(a + b)*f*cos(f*x + e)^3 + (a + b)*f*co
s(f*x + e)), -1/16*(24*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^5 - 2*(a^2 + 3*a
*b + 2*b^2)*cos(f*x + e)^3 + (a^2 + 3*a*b + 2*b^2)*cos(f*x + e))*sqrt(-b)*a
rctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b)
- 3*((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^5 - 2*(a^2 + 8*a*b + 8*b^2)*cos(f*x
```

```
 + e)^3 + (a^2 + 8*a*b + 8*b^2)*cos(f*x + e))*sqrt(a + b)*log(2*(a*cos(f*x
+ e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
+ e) + a + 2*b)/(cos(f*x + e)^2 - 1)) - 2*(3*(a^2 + 5*a*b + 4*b^2)*cos(f*x
+ e)^4 - (5*a^2 + 23*a*b + 18*b^2)*cos(f*x + e)^2 + 4*a*b + 4*b^2)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a + b)*f*cos(f*x + e)^5 - 2*(a + b)*
f*cos(f*x + e)^3 + (a + b)*f*cos(f*x + e)), 1/8*(3*((a^2 + 8*a*b + 8*b^2)*c
os(f*x + e)^5 - 2*(a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^3 + (a^2 + 8*a*b + 8*b
^2)*cos(f*x + e))*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) - 12*((a^2 + 3*a*b + 2*b^2)*cos(f
*x + e)^5 - 2*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 + 3*a*b + 2*b^2)*
cos(f*x + e))*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*cos(f*x + e)/b) + (3*(a^2 + 5*a*b + 4*b^2)*cos(f*x + e)^4 - (5*a^2
+ 23*a*b + 18*b^2)*cos(f*x + e)^2 + 4*a*b + 4*b^2)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2))/((a + b)*f*cos(f*x + e)^5 - 2*(a + b)*f*cos(f*x + e)^3
 + (a + b)*f*cos(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate(csc(f*x+e)**5*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^5\,dx$$

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^5, x)
```

Giac [F(-2)]

Exception generated.

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{262144,[6,6]%%%},[12]%%%}+%%%{%%{[%%%{1572864,[6,6]%%%},0
]:[1,0
```

Mupad [F(-1)]

Timed out.

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\sin\left(e+f\,x\right)^5}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^5,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^5, x)
```

3.86 $\int (a+b\sec^2(e+fx))^{3/2}\sin^6(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 298

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^6(e+fx)\,dx = \frac{(5a^3-45a^2b+15ab^2+b^3)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{3/2}f} + \frac{(3a-5b)\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} - \frac{(5a^2-26ab+b^2)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16af} + \frac{(5a^2-40ab+3b^2)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48af} + \frac{(5a-3b)\sin^4(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24f} - \frac{\cos(e+fx)\sin^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{6f}$$

```
[Out] 1/16*(5*a^3-45*a^2*b+15*a*b^2+b^3)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x
+e)^2)^(1/2))/a^(3/2)/f+1/2*(3*a-5*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan
(f*x+e)^2)^(1/2))*b^(1/2)/f-1/16*(5*a^2-26*a*b+b^2)*(a+b+b*tan(f*x+e)^2)^(1
/2)*tan(f*x+e)/a/f+1/48*(5*a^2-40*a*b+3*b^2)*sin(f*x+e)^2*(a+b+b*tan(f*x+e)
^2)^(1/2)*tan(f*x+e)/a/f+1/24*(5*a-3*b)*sin(f*x+e)^4*(a+b+b*tan(f*x+e)^2)^(
1/2)*tan(f*x+e)/f-1/6*cos(f*x+e)*sin(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(3/2)/f
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4217, 478, 591, 592, 596, 537, 223, 212, 385, 209}

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^6(e+fx)\,dx = -\frac{(5a^2-26ab+b^2)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16af} + \frac{(5a^2-40ab+3b^2)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{48af} + \frac{(5a^3-45a^2b+15ab^2+b^3)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{3/2}f} + \frac{\sqrt{b}(3a-5b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f} + \frac{(5a-3b)\sin^4(e+fx)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{24f} - \frac{\sin^5(e+fx)\cos(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{6f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^6,x]
```

```
[Out] ((5*a^3 - 45*a^2*b + 15*a*b^2 + b^3)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a +
 b + b*Tan[e + f*x]^2]])/(16*a^(3/2)*f) + ((3*a - 5*b)*Sqrt[b]*ArcTanh[(Sqr
t[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*f) - ((5*a^2 - 26*a*
b + b^2)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(16*a*f) + ((5*a^2 -
40*a*b + 3*b^2)*Sin[e + f*x]^2*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])
/(48*a*f) + ((5*a - 3*b)*Sin[e + f*x]^4*Tan[e + f*x]*Sqrt[a + b + b*Tan[e +
 f*x]^2])/(24*f) - (Cos[e + f*x]*Sin[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^
(3/2))/(6*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 591

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Dist[1/(
a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*
(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m +
 n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n,
 0] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e -
 a*f])
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
```

```
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^6(a+b+bx^2)^{3/2}}{(1+x^2)^4}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{\cos(e+fx)\sin^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{6f} \\
&\quad + \frac{\text{Subst}\left(\int \frac{x^4\sqrt{a+b+bx^2}\,(5(a+b)+8bx^2)}{(1+x^2)^3}\,dx, x, \tan(e+fx)\right)}{6f} \\
&= \frac{(5a-3b)\sin^4(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24f} \\
&\quad - \frac{\cos(e+fx)\sin^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{6f} \\
&\quad - \frac{\text{Subst}\left(\int \frac{x^4(5(a-7b)(a+b)+2(5a-19b)bx^2)}{(1+x^2)^2\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{24f}
\end{aligned}$$

$$= \frac{(5a^2-40ab+3b^2)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48af} + \frac{(5a-3b)\sin^4(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24f} - \frac{\cos(e+fx)\sin^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{6f} - \frac{\text{Subst}\left(\int \frac{x^2\left(3(a+b)(5a^2-40ab+3b^2)+6b(5a^2-26ab+b^2)x^2\right)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{48af}$$

$$= -\frac{(5a^2-26ab+b^2)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16af} + \frac{(5a^2-40ab+3b^2)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48af} + \frac{(5a-3b)\sin^4(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24f} - \frac{\cos(e+fx)\sin^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{6f} + \frac{\text{Subst}\left(\int \frac{6b(a+b)(5a^2-26ab+b^2)+48a(3a-5b)b^2x^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{96abf}$$

$$= -\frac{(5a^2-26ab+b^2)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16af} + \frac{(5a^2-40ab+3b^2)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48af} + \frac{(5a-3b)\sin^4(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24f} - \frac{\cos(e+fx)\sin^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{6f} + \frac{((3a-5b)b)\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2f} + \frac{(5a^3-45a^2b+15ab^2+b^3)\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{16af}$$

$$= -\frac{(5a^2 - 26ab + b^2)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16af} + \frac{(5a^2 - 40ab + 3b^2)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48af} + \frac{(5a-3b)\sin^4(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24f} - \frac{\cos(e+fx)\sin^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{6f} + \frac{((3a-5b)b)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{(5a^3 - 45a^2b + 15ab^2 + b^3)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16af}$$

$$= \frac{(5a^3 - 45a^2b + 15ab^2 + b^3)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{3/2}f} + \frac{(3a-5b)\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} - \frac{(5a^2 - 26ab + b^2)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16af} + \frac{(5a^2 - 40ab + 3b^2)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48af} + \frac{(5a-3b)\sin^4(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24f} - \frac{\cos(e+fx)\sin^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{6f}$$

Mathematica [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^6(e+fx)\,dx = \int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^6(e+fx)\,dx$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^6,x]
[Out] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^6, x]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1601 vs. 2(266) = 532.

Time = 16.25 (sec) , antiderivative size = 1602, normalized size of antiderivative = 5.38

method	result	size
default	Expression too large to display	1602

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/f/b/(-a)^(1/2)/a*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(8*cos(f*x+e)^8*sin(f*x+
e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a^2*b+8*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a^2*b*cos(f*x+e)^7*sin(f*x+e)
-26*cos(f*x+e)^6*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a
)^(1/2)*a^2*b+14*cos(f*x+e)^6*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*(-a)^(1/2)*a*b^2+60*cos(f*x+e)^3*(-a)^(1/2)*b^(5/2)*ln(4*(((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a+60*cos
(f*x+e)^3*(-a)^(1/2)*b^(5/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a-26*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(-a)^(1/2)*a^2*b*cos(f*x+e)^5*sin(f*x+e)+14*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a*b^2*cos(f*x+e)^5*sin(f*x+e)-36*cos(f*x+e
)^3*(-a)^(1/2)*b^(3/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^
(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*
x+e)*a-a-b)/(sin(f*x+e)+1))*a^2-36*cos(f*x+e)^3*(-a)^(1/2)*b^(3/2)*ln(-4*((
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a^
2+33*cos(f*x+e)^4*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-
a)^(1/2)*a^2*b-68*cos(f*x+e)^4*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(-a)^(1/2)*a*b^2+3*cos(f*x+e)^4*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*b^3+33*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*(-a)^(1/2)*a^2*b*cos(f*x+e)^3*sin(f*x+e)-68*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a*b^2*cos(f*x+e)^3*sin(f*x+e)+3*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*b^3*cos(f*x+e)^3*sin(f*x+e)-1
5*cos(f*x+e)^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f
*x+e)*a)*a^3*b+135*cos(f*x+e)^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)-4*sin(f*x+e)*a)*a^2*b^2-45*cos(f*x+e)^3*ln(4*(-a)^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^3-3*cos(f*x+e)^3*ln(4*(-a)^
(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^4-24*cos(f*x+e
```

```
)^2*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a*b^2
-24*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a*b^2*cos(f*x+e)
*sin(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 10.53 (sec) , antiderivative size = 1855, normalized size of antiderivative = 6.22

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^6(e+fx)\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] [-1/384*(3*(5*a^3 - 45*a^2*b + 15*a*b^2 + b^3)*sqrt(-a)*cos(f*x + e)*log(12
8*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos
(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b
^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 48*(3*a^3 - 5*
a^2*b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*
b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqr
t(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(
f*x + e)^4) + 8*(8*a^3*cos(f*x + e)^6 - 2*(13*a^3 - 7*a^2*b)*cos(f*x + e)^4
 - 24*a^2*b + (33*a^3 - 68*a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f*cos(f*x + e)), 1/384*(96*
(3*a^3 - 5*a^2*b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*
x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x
+ e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) - 3*(5*a^3 - 45*a^2*b + 15*a*b^2
+ b^3)*sqrt(-a)*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)
*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*
b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x +
 e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*
a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*sin(f*x + e)) - 8*(8*a^3*cos(f*x + e)^6 - 2*(13*a^3 - 7*a^2*b)*cos(f*x
 + e)^4 - 24*a^2*b + (33*a^3 - 68*a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f*cos(f*x + e)), -1/
192*(3*(5*a^3 - 45*a^2*b + 15*a*b^2 + b^3)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*
x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))
*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4
 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x +
 e) + 24*(3*a^3 - 5*a^2*b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*co
s(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2
```

```
*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e) + 8*b^2)/cos(f*x + e)^4) + 4*(8*a^3*cos(f*x + e)^6 - 2*(13*a^3 - 7*a
^2*b)*cos(f*x + e)^4 - 24*a^2*b + (33*a^3 - 68*a^2*b + 3*a*b^2)*cos(f*x + e
)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f*cos(f
*x + e)), -1/192*(3*(5*a^3 - 45*a^2*b + 15*a*b^2 + b^3)*sqrt(a)*arctan(1/4*
(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*
cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*c
os(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e
)))*cos(f*x + e) - 48*(3*a^3 - 5*a^2*b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f
*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) + 4*(8*a^3*
cos(f*x + e)^6 - 2*(13*a^3 - 7*a^2*b)*cos(f*x + e)^4 - 24*a^2*b + (33*a^3 -
 68*a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e))/(a^2*f*cos(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^6(e+fx)\,dx=\text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**6,x)
[Out] Timed out
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^6(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^6\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^6,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^6, x)
```

Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^6(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^6\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^6,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^6(e+fx)\,dx=\int \sin(e+f\,x)^6\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}\,dx$$

```
[In] int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.87 $\int (a+b\sec^2(e+fx))^{3/2}\sin^4(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 217

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^4(e+fx)\,dx = \frac{3(a^2-6ab+b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{a}f} + \frac{3(a-b)\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} - \frac{3(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{3(a-b)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} - \frac{\cos(e+fx)\sin^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f}$$

```
[Out] 3/8*(a^2-6*a*b+b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f
/a^(1/2)+3/2*(a-b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b
^(1/2)/f-3/8*(a-3*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f+3/8*(a-b)*sin(
f*x+e)^2*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f-1/4*cos(f*x+e)*sin(f*x+e)^
3*(a+b+b*tan(f*x+e)^2)^(3/2)/f
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4217, 478, 591, 596, 537, 223, 212, 385, 209}

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^4(e+fx)\,dx = \frac{3(a^2-6ab+b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8\sqrt{a}f} + \frac{3\sqrt{b}(a-b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f} - \frac{3(a-3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8f} + \frac{3(a-b)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8f} - \frac{\sin^3(e+fx)\cos(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^4,x]
```

```
[Out] (3*(a^2 - 6*a*b + b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
 f*x]^2]])/(8*Sqrt[a]*f) + (3*(a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x]
)/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*f) - (3*(a - 3*b)*Tan[e + f*x]*Sqrt[a
 + b + b*Tan[e + f*x]^2])/(8*f) + (3*(a - b)*Sin[e + f*x]^2*Tan[e + f*x]*Sq
rt[a + b + b*Tan[e + f*x]^2])/(8*f) - (Cos[e + f*x]*Sin[e + f*x]^3*(a + b +
 b*Tan[e + f*x]^2)^(3/2))/(4*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 591

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Dist[1/(
a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*
(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m +
 n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n,
 0] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e -
 a*f])
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4(a+b+bx^2)^{3/2}}{(1+x^2)^3}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{\cos(e+fx)\sin^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f}\\
&\quad+\frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+b+bx^2}\,(3(a+b)+6bx^2)}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{4f}\\
&= \frac{3(a-b)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f}\\
&\quad-\frac{\cos(e+fx)\sin^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f}\\
&\quad-\frac{\text{Subst}\left(\int \frac{x^2(3(a-5b)(a+b)+6(a-3b)bx^2)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{8f}\\
&= -\frac{3(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f}\\
&\quad+\frac{3(a-b)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f}\\
&\quad-\frac{\cos(e+fx)\sin^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f}\\
&\quad+\frac{\text{Subst}\left(\int \frac{6(a-3b)b(a+b)+24(a-b)b^2x^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{16bf}
\end{aligned}
$$

$$= -\frac{3(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{3(a-b)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} - \frac{\cos(e+fx)\sin^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f} + \frac{(3(a-b)b)\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2f} + \frac{(3(a^2-6ab+b^2))\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{8f}$$

$$= -\frac{3(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{3(a-b)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} - \frac{\cos(e+fx)\sin^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f} + \frac{(3(a-b)b)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{(3(a^2-6ab+b^2))\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8f}$$

$$= \frac{3(a^2-6ab+b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{a}f} + \frac{3(a-b)\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} - \frac{3(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{3(a-b)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} - \frac{\cos(e+fx)\sin^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f}$$

Mathematica [A] (verified)

Time = 3.57 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.04

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^4(e+fx)\,dx = \frac{3\left((a^2-6ab+b^2)\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)+4\sqrt{a}(a-b)\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\right)\cos(e-}{2\sqrt{2}\sqrt{a}f(a+2b+a\cos(2(e+fx)))^{3/2}} + \frac{(-7a+26b+(-6a+10b)\cos(2(e+fx))+a\cos(4(e+fx)))\sqrt{a+b\sec^2(e+fx)}\tan(e+fx)}{32f}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^4,x]
```

```
[Out] (3*((a^2 - 6*a*b + b^2)*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e
+ f*x]^2]] + 4*Sqrt[a]*(a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[
a + b - a*Sin[e + f*x]^2]])*Cos[e + f*x]*(b + a*Cos[e + f*x]^2)*Sqrt[a + b*
Sec[e + f*x]^2])/(2*Sqrt[2]*Sqrt[a]*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2))
 + ((-7*a + 26*b + (-6*a + 10*b)*Cos[2*(e + f*x)] + a*Cos[4*(e + f*x)])*Sqr
t[a + b*Sec[e + f*x]^2]*Tan[e + f*x])/(32*f)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1185 vs. 2(189) = 378.

Time = 16.91 (sec) , antiderivative size = 1186, normalized size of antiderivative = 5.47

method	result	size
default	Expression too large to display	1186

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f/(-a)^(1/2)/b*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(2*cos(f*x+e)^6*sin(f*x+e)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a*b-6*ln(4*(((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*cos(f*x+e)
^3*(-a)^(1/2)*b^(5/2)-6*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(
f*x+e)*a+a+b)/(sin(f*x+e)-1))*cos(f*x+e)^3*(-a)^(1/2)*b^(5/2)+2*cos(f*x+e)^
5*sin(f*x+e)*a*b*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)+6*l
n(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+
1))*cos(f*x+e)^3*(-a)^(1/2)*b^(3/2)*a+6*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*cos(f*x+e)^3*(-a)^(1/2)*b^(3/
```

```
2)*a-5*cos(f*x+e)^4*sin(f*x+e)*a*b*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*(-a)^(1/2)+5*sin(f*x+e)*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*(-a)^(1/2)*b^2-5*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*a*b*cos(f*x+e)^3*sin(f*x+e)+5*sin(f*x+e)*cos(f*x+e)^3*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*b^2+3*ln(4*(-a)^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*cos(f*x+e)^3*a^2*b-18*ln(4*(-a)^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*cos(f*x+e)^3*a*b^2
+3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4
*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*cos
(f*x+e)^3*b^3+4*sin(f*x+e)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(-a)^(1/2)*b^2+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*b^2*cos(f*x+e)*sin(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 3.35 (sec) , antiderivative size = 1667, normalized size of antiderivative = 7.68

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^4(e+fx)\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] [-1/64*(3*(a^2 - 6*a*b + b^2)*sqrt(-a)*cos(f*x + e)*log(128*a^4*cos(f*x + e
)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*
cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7
*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*
(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^
3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 24*(a^2 - a*b)*sqrt(b)*cos(f*x
+ e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2
 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 8*(2*a^2*
cos(f*x + e)^4 - 5*(a^2 - a*b)*cos(f*x + e)^2 + 4*a*b)*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*f*cos(f*x + e)), 1/64*(48*(a^2 - a
*b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-
b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*
sin(f*x + e)))*cos(f*x + e) - 3*(a^2 - 6*a*b + b^2)*sqrt(-a)*cos(f*x + e)*l
og(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 -
14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b
^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^
3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b +
5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqr
```

```
t(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*a^2*
cos(f*x + e)^4 - 5*(a^2 - a*b)*cos(f*x + e)^2 + 4*a*b)*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*f*cos(f*x + e)), -1/32*(3*(a^2 - 6
*a*b + b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*
x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2
*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) + 12*(a^2 - a*b)*sqrt(b)*co
s(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x
+ e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*(
2*a^2*cos(f*x + e)^4 - 5*(a^2 - a*b)*cos(f*x + e)^2 + 4*a*b)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*f*cos(f*x + e)), -1/32*(3*(a
^2 - 6*a*b + b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*
cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 -
 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) - 24*(a^2 - a*b)*sqrt
(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x
+ e)))*cos(f*x + e) - 4*(2*a^2*cos(f*x + e)^4 - 5*(a^2 - a*b)*cos(f*x + e)^
2 + 4*a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*f*c
os(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^4(e+fx)\,dx=\text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**4,x)
[Out] Timed out
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^4(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^4\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^4,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^4, x)
```

Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^4(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^4\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^4,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^4(e+fx)\,dx=\int \sin\left(e+f\,x\right)^4\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

```
[In] int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.88 $\int (a+b\sec^2(e+fx))^{3/2}\sin^2(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 161

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^2(e+fx)\,dx = \frac{\sqrt{a}(a-3b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{(3a-b)\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{\cos(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{2f}$$

```
[Out] 1/2*(a-3*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f
+1/2*(3*a-b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)
/f+b*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f-1/2*cos(f*x+e)*sin(f*x+e)*(a+b
+b*tan(f*x+e)^2)^(3/2)/f
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4217, 478, 542, 537, 223, 212, 385, 209}

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^2(e+fx)\,dx = \frac{\sqrt{a}(a-3b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f} + \frac{\sqrt{b}(3a-b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f} + \frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f} - \frac{\sin(e+fx)\cos(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{2f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^2,x]
```

```
[Out] (Sqrt[a]*(a - 3*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x
]^2]])/(2*f) + ((3*a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b
 + b*Tan[e + f*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^
2])/f - (Cos[e + f*x]*Sin[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2(a+b+bx^2)^{3/2}}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{\cos(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{2f} \\
&\quad + \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}(a+b+4bx^2)}{1+x^2}\,dx, x, \tan(e+fx)\right)}{2f} \\
&= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} \\
&\quad - \frac{\cos(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{2f} \\
&\quad + \frac{\text{Subst}\left(\int \frac{2(a^2-b^2)+2(3a-b)bx^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{4f} \\
&= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} \\
&\quad - \frac{\cos(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{2f} \\
&\quad + \frac{(a(a-3b))\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2f} \\
&\quad + \frac{((3a-b)b)\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2f}
\end{aligned}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{\cos(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{2f} + \frac{(a(a-3b))\text{Subst}\left(\int\frac{1}{1+ax^2}\,dx,x,\frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{((3a-b)b)\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}$$

$$= \frac{\sqrt{a}(a-3b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{(3a-b)\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{\cos(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{2f}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.78 (sec) , antiderivative size = 493, normalized size of antiderivative = 3.06

$$\int\left(a+b\sec^2(e+fx)\right)^{3/2}\sin^2(e+fx)\,dx = \frac{e^{-i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(\frac{i\left(-1+e^{2i(e+fx)}\right)\left(-4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2\right)}{\left(1+e^{2i(e+fx)}\right)^2}+\right.}{}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^2,x]
```

```
[Out] (Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*Cos[e + f*
x]^3*((I*(-1 + E^((2*I)*(e + f*x)))*(-4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((
2*I)*(e + f*x)))^2))/(1 + E^((2*I)*(e + f*x)))^2 + (2*E^((2*I)*(e + f*x))*(
2*Sqrt[a]*(a - 3*b)*f*x - I*Sqrt[a]*(a - 3*b)*Log[a + 2*b + a*E^((2*I)*(e +
 f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))
^2]] + I*Sqrt[a]*(a - 3*b)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e
+ f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x))
)^2]] + 2*Sqrt[b]*(-3*a + b)*Log[((Sqrt[b]*(-1 + E^((2*I)*(e + f*x))) - I*S
qrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*f)/(b*(-3*a +
 b)*(1 + E^((2*I)*(e + f*x))))]))/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^(
```

```
(2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(2*Sqrt[2]*E^(I*(e + f*
x))*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(139) = 278$.

Time = 11.54 (sec) , antiderivative size = 900, normalized size of antiderivative = 5.59

method	result	size
default	Expression too large to display	900

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f/b/(-a)^(1/2)*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(cos(f*x+e)^3*b^(5/2)*ln(4*
(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)
*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))
*(-a)^(1/2)+cos(f*x+e)^3*b^(5/2)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*(-a)^(1/2)-3*cos(f*x+e)^3*b^(3/2)*l
n(4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*
x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)
-1))*(-a)^(1/2)*a-3*cos(f*x+e)^3*b^(3/2)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*(-a)^(1/2)*a+2*cos(f*x+e)^4
*sin(f*x+e)*a*b*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)+2*(-
a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b*cos(f*x+e)^3*sin(f
*x+e)-2*sin(f*x+e)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*(-a)^(1/2)*b^2-2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*si
n(f*x+e)*a)*cos(f*x+e)^3*a^2*b+6*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)-4*sin(f*x+e)*a)*cos(f*x+e)^3*a*b^2-2*(-a)^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*cos(f*x+e)*sin(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(139) = 278$.

Time = 1.18 (sec) , antiderivative size = 1535, normalized size of antiderivative = 9.53

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^2(e+fx)\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] [-1/16*(sqrt(-a)*(a - 3*b)*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a
^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)
^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a
^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b
)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7
*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)*sin(f*x + e)) + 2*(3*a - b)*sqrt(b)*cos(f*x + e)*log(((a^2
- 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*c
os(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 8*(a*cos(f*x + e)^2 - b)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e))
, 1/16*(4*(3*a - b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(
f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*
x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) - sqrt(-a)*(a - 3*b)*cos(f*x +
e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^
4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28
*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(1
6*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*
b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))
*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(a*
cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))/(f*cos(f*x + e)), -1/8*((a - 3*b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)
^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(
a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2
*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) +
(3*a - b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*
(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*
sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/c
os(f*x + e)^4) + 4*(a*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e)), -1/8*((a - 3*b)*sqrt(a)*arctan(1
/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^
2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^
3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x
+ e)))*cos(f*x + e) - 2*(3*a - b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e
)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) + 4*(a*cos(f*x +
e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(f*cos(
f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^2(e+fx)\,dx=\text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^2(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^2\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^2,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^2, x)
```

Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^2(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^2\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^2,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^2(e+fx)\,dx=\int \sin\left(e+f\,x\right)^2\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

```
[In] int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.89 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 118

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}(3a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}+\frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

```
[Out] a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/2*(3*a+b)
*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f+1/2*b*(a+
b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4213, 427, 537, 223, 212, 385, 209}

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\sqrt{b}(3a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}+\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*(3*a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f
*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{(a+b)(2a+b)+b(3a+b)x^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{a^2\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} + \frac{(b(3a+b))\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{a^2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(b(3a+b))\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}$$

$$= \frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}(3a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.46 (sec) , antiderivative size = 527, normalized size of antiderivative = 4.47

$$\int (a+b\sec^2(e+fx))^{3/2}\,dx = \frac{\sqrt{2}e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{ib\left(-1+e^{2i(e+fx)}\right)}{\left(1+e^{2i(e+fx)}\right)^2}+\frac{2a^{3/2}fx-ia^{3/2}\log\left(a\right.}{}\right.}{}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*(((-I)*b*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((
2*I)*(e + f*x)))^2 + (2*a^(3/2)*f*x - I*a^(3/2)*Log[a + 2*b + a*E^((2*I)*(e
 + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)
))^2]] + I*a^(3/2)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x))
+ Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] -
3*a*Sqrt[b]*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (2*I)*Sqrt[4*b*E
^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(b*(3*a + b)*(1 + E^
((2*I)*(e + f*x))))] - b^(3/2)*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f
 + (2*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(
b*(3*a + b)*(1 + E^((2*I)*(e + f*x))))])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(
1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(f*(a + 2*b + a*
Cos[2*e + 2*f*x])^(3/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 708 vs. $2(100) = 200$.

Time = 9.88 (sec) , antiderivative size = 709, normalized size of antiderivative = 6.01

method	result
default	$\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}\left(\cos(fx+e)^3 b^{\frac{5}{2}}\ln\left(\frac{-4\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{b}\cos(fx+e)-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}+4\sin(fx+e)a-4a-4b}{\sin(fx+e)-1}\right)\sqrt{-a}+\cos(f\right.$

```
[In] int((a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f/(-a)^(1/2)/b*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(cos(f*x+e)^3*b^(5/2)*ln(4*(
-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*
a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*
(-a)^(1/2)+cos(f*x+e)^3*b^(5/2)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*(-a)^(1/2)+3*cos(f*x+e)^3*b^(3/2)*ln
(4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x
+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-
1))*(-a)^(1/2)*a+3*cos(f*x+e)^3*b^(3/2)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*(-a)^(1/2)*a+2*sin(f*x+e)*co
s(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*b^2+4*ln(
4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^
(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*cos(f*x+e
)^3*a^2*b+2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*cos(
f*x+e)*sin(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(100) = 200$.

Time = 0.61 (sec) , antiderivative size = 1457, normalized size of antiderivative = 12.35

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b
)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a
*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
+ e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7
*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e)) + (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^
2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^
3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e)), 1/8*(2*(3*a + b)*sqrt(-b)*arcta
n(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos
(f*x + e) + sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 -
 a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 +
 a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b
^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*co
s(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2
*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e)) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e))/(f*cos(f*x + e)), -1/8*(2*a^(3/2)*arctan(1/4*(8*a^2*cos(f*x
+ e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*s
qrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 -
 a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e
) - (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
+ 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^
2)/cos(f*x + e)^4) - 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e))/(f*cos(f*x + e)), -1/4*(a^(3/2)*arctan(1/4*(8*a^2*cos(f*x + e)^5 -
8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b +
a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) - (3*a
+ b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(
-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)
*sin(f*x + e)))*cos(f*x + e) - 2*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(f*cos(f*x + e))]
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\sec^2{(e+fx)}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec{(fx+e)}^2+a\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec{(fx+e)}^2+a\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(a+\frac{b}{\cos{(e+f\,x)}^2}\right)^{3/2}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int((a + b/cos(e + f*x)^2)^(3/2), x)
```

3.90 $\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 105

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{3\sqrt{b}(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}+\frac{3b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}-\frac{\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{f}$$

```
[Out] 3/2*(a+b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f+
3/2*b*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f-cot(f*x+e)*(a+b+b*tan(f*x+e)^
2)^(3/2)/f
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4217, 283, 201, 223, 212}

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{3\sqrt{b}(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}+\frac{3b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{f}$$

```
[In] Int[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*
x]^2]])/(2*f) + (3*b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f) - (
Cot[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/f
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^{3/2}}{x^2}\,dx, x, \tan(e+fx)\right)}{f}\\ &= -\frac{\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{f}+\frac{(3b)\text{Subst}\left(\int \sqrt{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{f}\end{aligned}$$

$$= \frac{3b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} - \frac{\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{f} + \frac{(3b(a+b))\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2f}$$

$$= \frac{3b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} - \frac{\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{f} + \frac{(3b(a+b))\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}$$

$$= \frac{3\sqrt{b}(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{3b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} - \frac{\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.61

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = -\frac{(a+b)\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2}, 2, \frac{1}{2}, \frac{b\sin^2(e+fx)}{a+b-a\sin^2(e+fx)}\right)\sqrt{a+b\sec^2(e+fx)}}{f}$$

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] -(((a + b)*Cot[e + f*x]*Hypergeometric2F1[-1/2, 2, 1/2, (b*Sin[e + f*x]^2)/
(a + b - a*Sin[e + f*x]^2)]*Sqrt[a + b*Sec[e + f*x]^2])/f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 740 vs. 2(91) = 182.

Time = 10.17 (sec) , antiderivative size = 741, normalized size of antiderivative = 7.06

method	result
default	$\left(3b^{\frac{5}{2}}\ln\left(\frac{-4\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{b}\cos(fx+e)-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}+4\sin(fx+e)a-4a-4b}{\sin(fx+e)-1}\right)\cos(fx+e)^2\sin(fx+e)+3b^{\frac{5}{2}}\ln\left(-\frac{4\left(-\sqrt{\frac{b+a}{(1+}}\right.}{}\right.\right.$

```
[In] int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f/b*(3*b^(5/2)*ln(4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/
2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)-a-b)/(sin(f*x+e)-1))*cos(f*x+e)^2*sin(f*x+e)+3*b^(5/2)*ln(-4*(-((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*cos(f*x+e
)^2*sin(f*x+e)+3*b^(3/2)*ln(4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)-a-b)/(sin(f*x+e)-1))*a*cos(f*x+e)^2*sin(f*x+e)+3*b^(3/2)*ln(-4*(-
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a
-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*a
*cos(f*x+e)^2*sin(f*x+e)-4*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*a*b-6*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*cos(f*x+e)^3
-4*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b-6*cos(f*x+e
)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2+2*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*b^2*cos(f*x+e)+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*b^2)*(a+b*sec(f*x+e)^2)^(3/2)/(b+a*cos(f*x+e)^2)/((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)/(1+cos(f*x+e))*cot(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.52

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left[\frac{3\,(a+b)\sqrt{b}\cos\left(fx+e\right)\log\left(\frac{(a^2-6\,ab+b^2)\cos(fx+e)^4+8\,(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2\,b\cos(f\cdots\right.}{\cos(fx+e)^4}\right.}{8\,f\cos\left(fx\cdots\right.}\right.$$

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^
4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x
+ e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*
b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((2*a + 3*b)*cos(f*x + e)^2 - b)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)*sin(f*x + e)), 1/4
*(3*(a + b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e)
)*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2
 + b^2)*sin(f*x + e)))*cos(f*x + e)*sin(f*x + e) - 2*((2*a + 3*b)*cos(f*x +
 e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)*sin
(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.93

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{3\,a\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)+3\,b^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)+3\,\sqrt{b\tan\left(fx+e\right)^2+a+b}\,b\tan\left(fx+e\right)}{2\,f}$$

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] 1/2*(3*a*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 3*b^(3/2)*arcsin
h(b*tan(f*x + e)/sqrt((a + b)*b)) + 3*sqrt(b*tan(f*x + e)^2 + a + b)*b*tan(
f*x + e) - 2*(b*tan(f*x + e)^2 + a + b)^(3/2)/tan(f*x + e))/f
```

Giac [F]

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^2\,dx$$

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\sin\left(e+f\,x\right)^2}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^2,x)
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^2, x)
```

3.91 $\int \csc^4(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx$

Optimal result

Integrand size = 25, antiderivative size = 172

$$\int \csc^4(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \frac{\sqrt{b}(3a+5b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b(3a+5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2(a+b)f} - \frac{(3a+5b)\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{3(a+b)f} - \frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{3(a+b)f}$$

```
[Out] 1/2*(3*a+5*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2
)/f+1/2*b*(3*a+5*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/(a+b)/f-1/3*(3*a+
5*b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/(a+b)/f-1/3*cot(f*x+e)^3*(a+b+b*
tan(f*x+e)^2)^(5/2)/(a+b)/f
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used

$= \{4217, 464, 283, 201, 223, 212\}$

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\sqrt{b}(3a+5b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}$$
$$+\frac{b(3a+5b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f(a+b)}$$
$$-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{3f(a+b)}$$
$$-\frac{(3a+5b)\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{3f(a+b)}$$

```
[In] Int[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(3*a + 5*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
f*x]^2]])/(2*f) + (b*(3*a + 5*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2
])/(2*(a + b)*f) - ((3*a + 5*b)*Cot[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/
2))/(3*(a + b)*f) - (Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(5/2))/(3*(a
 + b)*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)(a+b+bx^2)^{3/2}}{x^4}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{3(a+b)f}\\
&\quad+\frac{(3a+5b)\text{Subst}\left(\int \frac{(a+b+bx^2)^{3/2}}{x^2}\,dx, x, \tan(e+fx)\right)}{3(a+b)f}\\
&= -\frac{(3a+5b)\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{3(a+b)f}\\
&\quad-\frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{3(a+b)f}\\
&\quad+\frac{(b(3a+5b))\text{Subst}\left(\int \sqrt{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{(a+b)f}\\
&= \frac{b(3a+5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2(a+b)f}\\
&\quad-\frac{(3a+5b)\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{3(a+b)f}\\
&\quad-\frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{3(a+b)f}\\
&\quad+\frac{(b(3a+5b))\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2f}
\end{aligned}
$$

$$= \frac{b(3a+5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2(a+b)f} - \frac{(3a+5b)\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{3(a+b)f} - \frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{3(a+b)f} + \frac{(b(3a+5b))\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}$$

$$= \frac{\sqrt{b}(3a+5b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b(3a+5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2(a+b)f} - \frac{(3a+5b)\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{3(a+b)f} - \frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{3(a+b)f}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.69 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.15

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\sqrt{2}e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{i\left(4a\left(1+e^{2i(e+fx)}\right)^2\left(1-4e^{2i(e+fx)}+e^{4i(e+\cdots}\right.\right.}{(-1\cdots}\right.}{\cdots}$$

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*(((-I)*(4*a*(1 + E^((2*I)*(e + f*x)))^2*(1 - 4
*E^((2*I)*(e + f*x)) + E^((4*I)*(e + f*x))) + b*(15 - 20*E^((2*I)*(e + f*x)
) - 22*E^((4*I)*(e + f*x)) - 20*E^((6*I)*(e + f*x)) + 15*E^((8*I)*(e + f*x)
))))/((-1 + E^((2*I)*(e + f*x)))^3*(1 + E^((2*I)*(e + f*x)))^2) - (3*Sqrt[b
]*(3*a + 5*b)*Log[(-4*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b
*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e +
 f*x)))])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a
 + b*Sec[e + f*x]^2)^(3/2))/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1140 vs. $2(152) = 304$.

Time = 10.65 (sec) , antiderivative size = 1141, normalized size of antiderivative = 6.63

method	result	size
default	Expression too large to display	1141

```
[In] int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12/f/b*(15*cos(f*x+e)^3*sin(f*x+e)*b^(5/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))+15*cos(f*x+e)^3*sin(f*x+
e)*b^(5/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f
*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)
/(sin(f*x+e)-1))-15*cos(f*x+e)^2*sin(f*x+e)*b^(5/2)*ln(4*(((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))-15*cos(f*x+e)^2*si
n(f*x+e)*b^(5/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)
*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*
a+a+b)/(sin(f*x+e)-1))+9*cos(f*x+e)^3*sin(f*x+e)*b^(3/2)*a*ln(4*(((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))+9*cos(f*x+e
)^3*sin(f*x+e)*b^(3/2)*a*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin
(f*x+e)*a+a+b)/(sin(f*x+e)-1))-9*cos(f*x+e)^2*sin(f*x+e)*b^(3/2)*a*ln(4*(((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))-9*c
os(f*x+e)^2*sin(f*x+e)*b^(3/2)*a*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))-8*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*a*b-30*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*b^2+12*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*a*b+40*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2-6*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2)*(a+b*sec(f*x+e)^2)^(3/2)/(b+a*
cos(f*x+e)^2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cot(f*x+e)*csc(f*
x+e)^2
```

Fricas [A] (verification not implemented)

none

Time = 1.50 (sec) , antiderivative size = 472, normalized size of antiderivative = 2.74

$$\int \csc^4(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \left[\frac{3\left((3a+5b)\cos(fx+e)^3-(3a+5b)\cos(fx+e)\right)\sqrt{b}\log\left(\frac{(a^2-6ab+b^2)\cos(fx+e)^4+8(ab-b^2)\,c}{}\right.}{}\right.$$

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*((3*a + 5*b)*cos(f*x + e)^3 - (3*a + 5*b)*cos(f*x + e))*sqrt(b)*lo
g(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((
a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*(
(4*a + 15*b)*cos(f*x + e)^4 - 2*(3*a + 10*b)*cos(f*x + e)^2 + 3*b)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((f*cos(f*x + e)^3 - f*cos(f*x + e))*s
in(f*x + e)), 1/12*(3*((3*a + 5*b)*cos(f*x + e)^3 - (3*a + 5*b)*cos(f*x + e
))*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*s
in(f*x + e)))*sin(f*x + e) - 2*((4*a + 15*b)*cos(f*x + e)^4 - 2*(3*a + 10*b
)*cos(f*x + e)^2 + 3*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((f*co
s(f*x + e)^3 - f*cos(f*x + e))*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \csc^4(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.41

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{9\,a\sqrt{b}\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)+\frac{6\,ab^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a+b}+9\,b^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)+\frac{6\,b^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a+b}$$

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*(9*a*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 6*a*b^(3/2)*arcs
inh(b*tan(f*x + e)/sqrt((a + b)*b))/(a + b) + 9*b^(3/2)*arcsinh(b*tan(f*x +
 e)/sqrt((a + b)*b)) + 6*b^(5/2)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/(a
 + b) + 9*sqrt(b*tan(f*x + e)^2 + a + b)*b*tan(f*x + e) + 6*sqrt(b*tan(f*x
+ e)^2 + a + b)*b^2*tan(f*x + e)/(a + b) - 6*(b*tan(f*x + e)^2 + a + b)^(3/
2)/tan(f*x + e) - 4*(b*tan(f*x + e)^2 + a + b)^(3/2)*b/((a + b)*tan(f*x + e
)) - 2*(b*tan(f*x + e)^2 + a + b)^(5/2)/((a + b)*tan(f*x + e)^3))/f
```

Giac [F]

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^4\,dx$$

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\sin\left(e+f\,x\right)^4}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^4,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^4, x)
```

3.92 $\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 209

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\sqrt{b}(3a+7b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b(3a+7b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2(a+b)f} - \frac{(3a+7b)\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{3(a+b)f} - \frac{2\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{3(a+b)f} - \frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{5(a+b)f}$$

```
[Out] 1/2*(3*a+7*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2
)/f+1/2*b*(3*a+7*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/(a+b)/f-1/3*(3*a+
7*b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/(a+b)/f-2/3*cot(f*x+e)^3*(a+b+b*
tan(f*x+e)^2)^(5/2)/(a+b)/f-1/5*cot(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(5/2)/(a+
b)/f
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4217, 473, 464, 283, 201, 223, 212}

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\sqrt{b}(3a+7b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f} + \frac{b(3a+7b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f(a+b)} - \frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{5f(a+b)} - \frac{2\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{3f(a+b)} - \frac{(3a+7b)\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{3f(a+b)}$$

```
[In] Int[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(3*a + 7*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + 
f*x]^2]])/(2*f) + (b*(3*a + 7*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2
])/(2*(a + b)*f) - ((3*a + 7*b)*Cot[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/
2))/(3*(a + b)*f) - (2*Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(5/2))/(3*
(a + b)*f) - (Cot[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^(5/2))/(5*(a + b)*f
)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2(a+b+bx^2)^{3/2}}{x^6}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{5(a+b)f} \\
&\quad + \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^{3/2}(10(a+b)+5(a+b)x^2)}{x^4}\,dx, x, \tan(e+fx)\right)}{5(a+b)f}
\end{aligned}
$$

$$= -\frac{2\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{3(a+b)f} - \frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{5(a+b)f} + \frac{(3a+7b)\text{Subst}\left(\int \frac{(a+b+bx^2)^{3/2}}{x^2}\,dx, x, \tan(e+fx)\right)}{3(a+b)f}$$

$$= -\frac{(3a+7b)\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{3(a+b)f} - \frac{2\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{3(a+b)f} - \frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{5(a+b)f} + \frac{(b(3a+7b))\text{Subst}\left(\int \sqrt{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{(a+b)f}$$

$$= \frac{b(3a+7b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2(a+b)f} - \frac{(3a+7b)\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{3(a+b)f} - \frac{2\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{3(a+b)f} - \frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{5(a+b)f} + \frac{(b(3a+7b))\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2f}$$

$$= \frac{b(3a+7b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2(a+b)f} - \frac{(3a+7b)\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{3(a+b)f} - \frac{2\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{3(a+b)f} - \frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{5(a+b)f} + \frac{(b(3a+7b))\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}$$

$$
\begin{aligned}
&= \frac{\sqrt{b}(3a+7b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} \\
&\quad + \frac{b(3a+7b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2(a+b)f} \\
&\quad - \frac{(3a+7b)\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{3(a+b)f} \\
&\quad - \frac{2\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{3(a+b)f} \\
&\quad - \frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{5(a+b)f}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.26 (sec) , antiderivative size = 512, normalized size of antiderivative = 2.45

$$
\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\sqrt{2}e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{i\left(16a^2\left(1+e^{2i(e+fx)}\right)^2\left(1-6e^{2i(e+fx)}+16e\right.\right.}{}\right.}{}
$$

```
[In] Integrate[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*(((-I)*(16*a^2*(1 + E^((2*I)*(e + f*x)))^2*(1
- 6*E^((2*I)*(e + f*x)) + 16*E^((4*I)*(e + f*x)) - 6*E^((6*I)*(e + f*x)) +
E^((8*I)*(e + f*x))) + b^2*(105 - 350*E^((2*I)*(e + f*x)) + 231*E^((4*I)*(e
 + f*x)) + 412*E^((6*I)*(e + f*x)) + 231*E^((8*I)*(e + f*x)) - 350*E^((10*I
)*(e + f*x)) + 105*E^((12*I)*(e + f*x))) + a*b*(115 - 402*E^((2*I)*(e + f*x
)) + 317*E^((4*I)*(e + f*x)) + 708*E^((6*I)*(e + f*x)) + 317*E^((8*I)*(e +
f*x)) - 402*E^((10*I)*(e + f*x)) + 115*E^((12*I)*(e + f*x)))))/((a + b)*(-1
 + E^((2*I)*(e + f*x)))^5*(1 + E^((2*I)*(e + f*x)))^2) - (15*Sqrt[b]*(3*a +
 7*b)*Log[(-4*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b*E^((2*I
)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e + f*x)))]
)/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec
[e + f*x]^2)^(3/2))/(15*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1850 vs. 2(185) = 370.

Time = 11.45 (sec) , antiderivative size = 1851, normalized size of antiderivative = 8.86

method	result	size
default	Expression too large to display	1851

```
[In] int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/60/f/(a+b)/b*(105*cos(f*x+e)^3*sin(f*x+e)^3*b^(7/2)*ln(4*(((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))+105*cos(f*x+e)^
3*sin(f*x+e)^3*b^(7/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b
^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f
*x+e)*a+a+b)/(sin(f*x+e)-1))-105*cos(f*x+e)^2*sin(f*x+e)^3*b^(7/2)*ln(4*(((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))-105
*cos(f*x+e)^2*sin(f*x+e)^3*b^(7/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))+150*cos(f*x+e)^3*sin(f*x+e)^3*b^(5
/2)*a*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+
b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(
f*x+e)+1))+150*cos(f*x+e)^3*sin(f*x+e)^3*b^(5/2)*a*ln(-4*(((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))-150*cos(f*x+e)^2*s
in(f*x+e)^3*b^(5/2)*a*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(
1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x
+e)*a-a-b)/(sin(f*x+e)+1))-150*cos(f*x+e)^2*sin(f*x+e)^3*b^(5/2)*a*ln(-4*((
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))+45
*cos(f*x+e)^3*sin(f*x+e)^3*b^(3/2)*a^2*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))+45*cos(f*x+e)^3*sin(f*x+e)^3*b^
(3/2)*a^2*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*
x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/
(sin(f*x+e)-1))-45*cos(f*x+e)^2*sin(f*x+e)^3*b^(3/2)*a^2*ln(4*(((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))-45*cos(f*x+e)
^2*sin(f*x+e)^3*b^(3/2)*a^2*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-
sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))+32*cos(f*x+e)^6*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*a^2*b+230*cos(f*x+e)^6*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*a*b^2+210*cos(f*x+e)^6*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*b^3-80*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b-54
```

```
6*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^2-490*cos(f*
x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3+60*cos(f*x+e)^2*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b+370*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*cos(f*x+e)^2*b^2*a+322*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*b^3-30*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
a*b^2-30*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3)*(a+b*sec(f*x+e)^2
)^(3/2)/(b+a*cos(f*x+e)^2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cot(
f*x+e)*csc(f*x+e)^4
```

Fricas [A] (verification not implemented)

none

Time = 6.34 (sec) , antiderivative size = 682, normalized size of antiderivative = 3.26

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left[\frac{15\left(\left(3a^2+10ab+7b^2\right)\cos\left(fx+e\right)^5-2\left(3a^2+10ab+7b^2\right)\cos\left(fx+e\right)^3+\left(3a^2+10ab\right.\right.}{}\right.$$

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/120*(15*((3*a^2 + 10*a*b + 7*b^2)*cos(f*x + e)^5 - 2*(3*a^2 + 10*a*b + 7
*b^2)*cos(f*x + e)^3 + (3*a^2 + 10*a*b + 7*b^2)*cos(f*x + e))*sqrt(b)*log((
(a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a -
 b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((16
*a^2 + 115*a*b + 105*b^2)*cos(f*x + e)^6 - (40*a^2 + 273*a*b + 245*b^2)*cos
(f*x + e)^4 + (30*a^2 + 185*a*b + 161*b^2)*cos(f*x + e)^2 - 15*a*b - 15*b^2
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a + b)*f*cos(f*x + e)^5 -
 2*(a + b)*f*cos(f*x + e)^3 + (a + b)*f*cos(f*x + e))*sin(f*x + e)), 1/60*(
15*((3*a^2 + 10*a*b + 7*b^2)*cos(f*x + e)^5 - 2*(3*a^2 + 10*a*b + 7*b^2)*co
s(f*x + e)^3 + (3*a^2 + 10*a*b + 7*b^2)*cos(f*x + e))*sqrt(-b)*arctan(-1/2*
((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x +
e) - 2*((16*a^2 + 115*a*b + 105*b^2)*cos(f*x + e)^6 - (40*a^2 + 273*a*b + 2
45*b^2)*cos(f*x + e)^4 + (30*a^2 + 185*a*b + 161*b^2)*cos(f*x + e)^2 - 15*a
*b - 15*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a + b)*f*cos(f
*x + e)^5 - 2*(a + b)*f*cos(f*x + e)^3 + (a + b)*f*cos(f*x + e))*sin(f*x +
e))]
```

Sympy [F(-1)]

Timed out.

$$\int \csc^6(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.31

$$\int \csc^6(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx = \frac{45 \, a\sqrt{b} \operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right) + \frac{60 \, ab^{\frac{3}{2}} \operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a+b} + 45 \, b^{\frac{3}{2}} \operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right) + \frac{60 \, b^{\frac{5}{2}} \operatorname{arsinh}\left(\frac{b\tan}{\sqrt{(}}\right.}{a+b}}{}$$

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] 1/30*(45*a*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 60*a*b^(3/2)*a
rcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/(a + b) + 45*b^(3/2)*arcsinh(b*tan(f
*x + e)/sqrt((a + b)*b)) + 60*b^(5/2)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b
))/(a + b) + 45*sqrt(b*tan(f*x + e)^2 + a + b)*b*tan(f*x + e) + 60*sqrt(b*t
an(f*x + e)^2 + a + b)*b^2*tan(f*x + e)/(a + b) - 30*(b*tan(f*x + e)^2 + a
+ b)^(3/2)/tan(f*x + e) - 40*(b*tan(f*x + e)^2 + a + b)^(3/2)*b/((a + b)*ta
n(f*x + e)) - 20*(b*tan(f*x + e)^2 + a + b)^(5/2)/((a + b)*tan(f*x + e)^3)
- 6*(b*tan(f*x + e)^2 + a + b)^(5/2)/((a + b)*tan(f*x + e)^5))/f
```

Giac [F]

$$\int \csc^6(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \csc\left(fx+e\right)^6 \, dx$$

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\sin\left(e+f\,x\right)^6}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^6,x)
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^6, x)
```

3.93 $\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 123

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(15a^2+20ab+8b^2)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^3f} + \frac{2(5a+2b)\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^2f} - \frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}}{5af}$$

```
[Out] -1/15*(15*a^2+20*a*b+8*b^2)*cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/a^3/f+2/15*
(5*a+2*b)*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2)/a^2/f-1/5*cos(f*x+e)^5*(a+b
*sec(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4219, 473, 464, 270}

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{2(5a+2b)\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^2f} - \frac{(15a^2+20ab+8b^2)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^3f} - \frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}}{5af}$$

```
[In] Int[Sin[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/15*((15*a^2 + 20*a*b + 8*b^2)*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(
a^3*f) + (2*(5*a + 2*b)*Cos[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2])/(15*a^2*
f) - (Cos[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2])/(5*a*f)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(-1+x^2)^2}{x^6\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}}{5af} + \frac{\text{Subst}\left(\int \frac{-2(5a+2b)+5ax^2}{x^4\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{5af}$$

$$= \frac{2(5a+2b)\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^2f} - \frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}}{5af} + \frac{(15a^2+20ab+8b^2)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{15a^2f}$$

$$= -\frac{(15a^2+20ab+8b^2)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^3f} + \frac{2(5a+2b)\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^2f} - \frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}}{5af}$$

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.76

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left(89a^2+144ab+64b^2-4a(7a+4b)\cos(2(e+fx))+3a^2\cos(4(e+fx))\right)\text{se}}{240a^3f\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Integrate[Sin[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/240*((a + 2*b + a*Cos[2*(e + f*x)])*(89*a^2 + 144*a*b + 64*b^2 - 4*a*(7*
a + 4*b)*Cos[2*(e + f*x)] + 3*a^2*Cos[4*(e + f*x)])*Sec[e + f*x])/(a^3*f*Sq
rt[a + b*Sec[e + f*x]^2])
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{\left(b+a\cos(fx+e)^2\right)\left(3\cos(fx+e)^4a^2-10\cos(fx+e)^2a^2-4\cos(fx+e)^2ab+15a^2+20ab+8b^2\right)\sec(fx+e)}{15fa^3\sqrt{a+b\sec(fx+e)^2}}$	94

```
[In] int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/f/a^3*(b+a*cos(f*x+e)^2)*(3*cos(f*x+e)^4*a^2-10*cos(f*x+e)^2*a^2-4*co
s(f*x+e)^2*a*b+15*a^2+20*a*b+8*b^2)/(a+b*sec(f*x+e)^2)^(1/2)*sec(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.71

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\left(3\,a^2\cos\left(fx+e\right)^5-2\left(5\,a^2+2\,ab\right)\cos\left(fx+e\right)^3+\left(15\,a^2+20\,ab+8\,b^2\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{15\,a^3f}$$

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*a^2*cos(f*x + e)^5 - 2*(5*a^2 + 2*a*b)*cos(f*x + e)^3 + (15*a^2 +
20*a*b + 8*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(
a^3*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.32

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\frac{15\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a}-\frac{10\left(\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3-3\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)\right)}{a^2}+\frac{3\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{5}{2}}\cos(fx+e)^5\cdots}{\cdots}}{15\,f}$$

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a - 10*((a + b/cos(f*x +
e)^2)^(3/2)*cos(f*x + e)^3 - 3*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))/a
^2 + (3*(a + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 10*(a + b/cos(f*x + e
)^2)^(3/2)*b*cos(f*x + e)^3 + 15*sqrt(a + b/cos(f*x + e)^2)*b^2*cos(f*x + e
))/a^3)/f
```

Giac [F]

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin\left(fx+e\right)^5}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin\left(e+f\,x\right)^5}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

```
[In] int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.94 $\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 74

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(3a+2b)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a^2f} + \frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3af}$$

```
[Out] -1/3*(3*a+2*b)*cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/a^2/f+1/3*cos(f*x+e)^3*(
a+b*sec(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {4219, 464, 270}

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3af} - \frac{(3a+2b)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a^2f}$$

```
[In] Int[Sin[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/3*((3*a + 2*b)*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(a^2*f) + (Cos[e
 + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2])/(3*a*f)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f} \\
&= \frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3af} + \frac{(3a+2b)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{3af} \\
&= -\frac{(3a+2b)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a^2f} + \frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3af}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{(-5a-4b+a\cos(2(e+fx)))(a+2b+a\cos(2(e+fx)))\sec(e+fx)}{12a^2f\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Integrate[Sin[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] ((-5*a - 4*b + a*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f
*x])/(12*a^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{\left(b+a\cos(fx+e)^2\right)\left(a\cos(fx+e)^2-3a-2b\right)\sec(fx+e)}{3f\,a^2\sqrt{a+b\sec(fx+e)^2}}$	58

```
[In] int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3/f/a^2*(b+a*cos(f*x+e)^2)*(a*cos(f*x+e)^2-3*a-2*b)/(a+b*sec(f*x+e)^2)^(1
/2)*sec(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\left(a\cos\left(fx+e\right)^3-\left(3\,a+2\,b\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{3\,a^2f}$$

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*(a*cos(f*x + e)^3 - (3*a + 2*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)/(a^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.12

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\frac{3\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a}-\frac{\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3-3\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)}{a^2}}{3f}$$

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] -1/3*(3*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a - ((a + b/cos(f*x + e)^2)
^(3/2)*cos(f*x + e)^3 - 3*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))/a^2)/f
```

Giac [F]

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin(fx+e)^3}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin(e+f\,x)^3}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

```
[In] int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.95 $\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 30

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}} \, dx = -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{af}$$

```
[Out] -cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4219, 270}

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}} \, dx = -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{af}$$

```
[In] Int[Sin[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(a*f))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
```

```
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{af}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.60

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(a+2b+a\cos(2e+2fx))\sec(e+fx)}{2af\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Integrate[Sin[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] -1/2*((a + 2*b + a*Cos[2*e + 2*f*x])*Sec[e + f*x])/(a*f*Sqrt[a + b*Sec[e +
f*x]^2])
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$-\frac{\sqrt{a+b\sec(fx+e)^2}}{fa\sec(fx+e)}$	31
default	$-\frac{\sqrt{a+b\sec(fx+e)^2}}{fa\sec(fx+e)}$	31

```
[In] int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/f/a/sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{af}$$

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] -sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a*f)
```

Sympy [F]

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sin(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{af}$$

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] -sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/(a*f)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\sqrt{a\cos(fx+e)^2+b}}{af\operatorname{sgn}(\cos(fx+e))}$$

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] -sqrt(a*cos(f*x + e)^2 + b)/(a*f*sgn(cos(f*x + e)))
```

Mupad [B] (verification not implemented)

Time = 17.83 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\cos\left(e+f\,x\right)\,\sqrt{\frac{a+2\,b+a\,\cos\left(2\,e+2\,f\,x\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}{a\,f}$$

```
[In] int(sin(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] -(cos(e + f*x)*((a + 2*b + a*cos(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2
))/(a*f)
```

3.96 $\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 43

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{\sqrt{a+b}f}$$

```
[Out] -arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/f/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4219, 385, 213}

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f\sqrt{a+b}}$$

```
[In] Int[Csc[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]]/(Sqrt[a +
b]*f))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-1-(-a-b)x^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}$$
$$= -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{\sqrt{a+b}f}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.00

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$
$$= -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{a+b}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)}{\sqrt{2}\sqrt{a+b}f\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Integrate[Csc[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] -((ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b]]*Sqrt[a + 2*b + a*Cos
[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a + b]*f*Sqrt[a + b*Sec[e + f*x]
^2]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(37) = 74$.

Time = 1.22 (sec) , antiderivative size = 247, normalized size of antiderivative = 5.74

method	result
default	$-\frac{\left(\ln\left(\frac{2\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{a+b}\cos(fx+e)+2\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{a+b}-2\cos(fx+e)a+2b}{\sqrt{a+b}\,(1+\cos(fx+e))}\right)+\ln\left(-\frac{4\left(\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{a+b}\cos(fx+e)+\right.}{-1+\text{co}}\right.\right.}{2f\sqrt{a+b}\sqrt{a+b\sec(fx+e)^2}}$

```
[In] int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f/(a+b)^(1/2)*(ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(
a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))+ln(-4*(((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e))))*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)*(sec(f*x+e)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 140, normalized size of antiderivative = 3.26

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \left[\frac{\log\left(\frac{2\left(a\cos(fx+e)^2-2\sqrt{a+b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+a+2b\right)}{\cos(fx+e)^2-1}\right)}{2\sqrt{a+b}f}, \frac{\sqrt{-a-b}\arctan\left(\frac{\sqrt{-a-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{a+b}\right)}{(a+b)f}\right.$$

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1))/(sqrt(a + b)*f)
, sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*cos(f*x + e)/(a + b))/((a + b)*f)]
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\csc(fx+e)}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(37) = 74.

Time = 0.63 (sec) , antiderivative size = 305, normalized size of antiderivative = 7.09

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx =$$
$$-\frac{\frac{\log\left(\left|-\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}+\sqrt{a+b}\right|\right)}{\sqrt{a+b}} - \frac{\log\left(\left|-\right.\right.}{}}{}$$

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(log(abs(-sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a*tan(1/2*f*x + 1/
2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/
2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b)))/sqrt(a + b) - log(abs(-sqrt(a + b
)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x +
1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)
 - sqrt(a + b)))/sqrt(a + b) - log(abs(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2
 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f
*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a + b) + sqrt(a + b)*
(a - b)))/sqrt(a + b))/(f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\sin(e+f\,x)\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

```
[In] int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2)),x)
[Out] int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

3.97 $\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 87

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{a\,\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^{3/2}f} - \frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f}$$

```
[Out] -1/2*a*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b)^(3/2)
/f-1/2*cot(f*x+e)*csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4219, 482, 12, 385, 213}

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{a\,\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f(a+b)^{3/2}} - \frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f(a+b)}$$

```
[In] Int[Csc[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/2*(a*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/((a
 + b)^(3/2)*f) - (Cot[e + f*x]*Csc[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*
(a + b)*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{(-1+x^2)^2\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}\\
&= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f} + \frac{\text{Subst}\left(\int \frac{a}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{2(a+b)f}\\
&= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f} + \frac{a\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{2(a+b)f}
\end{aligned}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f} + \frac{a\text{Subst}\left(\int \frac{1}{-1-(-a-b)x^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)f}$$

$$= -\frac{a\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^{3/2}f} - \frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f}$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.61

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx =$$

$$-\frac{a\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\left(\frac{(a+b)\csc^2(e+fx)}{a}+\frac{\text{arctanh}\left(\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)}{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}\right)}{2\sqrt{2}(a+b)^2f\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Integrate[Csc[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/2*(a*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x]*Sqrt[a + b - a*Sin[
e + f*x]^2]*(((a + b)*Csc[e + f*x]^2)/a + ArcTanh[Sqrt[1 - (a*Sin[e + f*x]^
2)/(a + b)]]/Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]))/(Sqrt[2]*(a + b)^2*f*Sq
rt[a + b*Sec[e + f*x]^2])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1214 vs. 2(75) = 150.

Time = 1.28 (sec) , antiderivative size = 1215, normalized size of antiderivative = 13.97

method	result	size
default	Expression too large to display	1215

```
[In] int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f/(a+b)^(5/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f
*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2
+a+b)^(1/2)/((a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)
^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)
/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^2)^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-
1)/(1-cos(f*x+e))^2*((a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*cs
c(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e
)^2+a+b)^(1/2)*(a+b)^(3/2)*(1-cos(f*x+e))^2+2*ln((a*(1-cos(f*x+e))^2*csc(f*
x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(
```

```
1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f
*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)-a+b)/(a+b)^(1/2))*a^2*(1-cos(f
*x+e))^2+2*ln((a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e
)^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1
-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a
+b)^(1/2)-a+b)/(a+b)^(1/2))*(1-cos(f*x+e))^2*a*b+2*ln(2/(1-cos(f*x+e))^2*(-
a*(1-cos(f*x+e))^2+b*(1-cos(f*x+e))^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1
-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*
x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2+b*s
in(f*x+e)^2))*a^2*(1-cos(f*x+e))^2+2*ln(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e
))^2+b*(1-cos(f*x+e))^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4
*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*
x+e)^2+a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2+b*sin(f*x+e)^2))*
(1-cos(f*x+e))^2*a*b+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*cs
c(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e
)^2+a+b)^(1/2)*(a+b)^(3/2)*sin(f*x+e)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.51

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \left[\frac{2\,(a+b)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos\left(fx+e\right)+\left(a\cos\left(fx+e\right)^2-a\right)\sqrt{a+b}\log\left(\frac{2\left(a\cos(fx+e)^2-2\sqrt{a+b}\sqrt{\frac{a\cos(fx+e)^2+}{\cos(fx+e)^2}}\right.}{\cos(fx+e)^2-1}\right.}{4\left((a^2+2\,ab+b^2)f\cos\left(fx+e\right)^2-(a^2+2\,ab+b^2)f\right)}\right.$$

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) +
(a*cos(f*x + e)^2 - a)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*
x + e)^2 - 1)))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 - (a^2 + 2*a*b + b^2)
*f), 1/2*((a*cos(f*x + e)^2 - a)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) + (a + b)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/((a^2 + 2*a*b + b^2)*f*co
s(f*x + e)^2 - (a^2 + 2*a*b + b^2)*f)]
```

Sympy [F]

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

```
[In] integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(csc(e + f*x)**3/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\csc(fx+e)^3}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(csc(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\csc(fx+e)^3}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\sin(e+f\,x)^3\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

```
[In] int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2)),x)
[Out] int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

3.98 $\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 138

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{3a^2\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{5/2}f} - \frac{(5a+2b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \frac{\cot^3(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)f}$$

```
[Out] -3/8*a^2*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b)^(5/
2)/f-1/8*(5*a+2*b)*cot(f*x+e)*csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/(a+b)^2/f
-1/4*cot(f*x+e)^3*csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4219, 481, 541, 12, 385, 213}

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{3a^2\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f(a+b)^{5/2}} - \frac{\cot^3(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f(a+b)} - \frac{(5a+2b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f(a+b)^2}$$

```
[In] Int[Csc[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (-3*a^2*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(8*
(a + b)^(5/2)*f) - ((5*a + 2*b)*Cot[e + f*x]*Csc[e + f*x]*Sqrt[a + b*Sec[e
+ f*x]^2])/(8*(a + b)^2*f) - (Cot[e + f*x]^3*Csc[e + f*x]*Sqrt[a + b*Sec[e
+ f*x]^2])/(4*(a + b)*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
```

```
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^3\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)f} - \frac{\text{Subst}\left(\int \frac{-a-2(2a+b)x^2}{(-1+x^2)^2\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{4(a+b)f}$$

$$= -\frac{(5a+2b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \frac{\cot^3(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)f} - \frac{\text{Subst}\left(\int -\frac{3a^2}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{8(a+b)^2f}$$

$$= -\frac{(5a+2b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \frac{\cot^3(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)f} + \frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{8(a+b)^2f}$$

$$= -\frac{(5a+2b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \frac{\cot^3(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)f} + \frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{-1-(-a-b)x^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^2f}$$

$$= -\frac{3a^2\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{5/2}f} - \frac{(5a+2b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \frac{\cot^3(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.57

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{a^2(a+2b+a\cos(2(e+fx)))\,\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},1-\frac{a\sin^2(e+fx)}{a+b}\right)\sec(e+fx)}{2(a+b)^3f\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Integrate[Csc[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/2*(a^2*(a + 2*b + a*Cos[2*(e + f*x)])*Hypergeometric2F1[1/2, 3, 3/2, 1 -
 (a*Sin[e + f*x]^2)/(a + b)]*Sec[e + f*x])/((a + b)^3*f*Sqrt[a + b*Sec[e +
f*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1759 vs. 2(122) = 244.

Time = 1.14 (sec) , antiderivative size = 1760, normalized size of antiderivative = 12.75

method	result	size
default	Expression too large to display	1760

```
[In] int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/f/(a+b)^(9/2)*(6*(a+b)^(5/2)*cos(f*x+e)^4*a^2-3*sin(f*x+e)^4*ln(-4*(((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e
)))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^4*cos(f*x+e)-6*sin(f*x+e)
^4*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e
)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(
-1+cos(f*x+e)))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b*cos(f*x+e
)-3*sin(f*x+e)^4*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(
1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos
(f*x+e)*a+b)/(-1+cos(f*x+e)))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a
^2*b^2*cos(f*x+e)-3*sin(f*x+e)^4*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*a^4*cos(f*x+e)-6*sin(f*x+e)^4*ln(2/(a+b)^(1/2)*(((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)
))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b*cos(f*x+e)-3*sin(f*x+e
```

```
)^4*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/
2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f
*x+e)*a+b)/(1+cos(f*x+e)))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*
b^2*cos(f*x+e)-3*sin(f*x+e)^4*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*a^4-6*sin(f*x+e)^4*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(
a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*a^3*b-3*sin(f*x+e)^4*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*a^2*b^2-3*sin(f*x+e)^4*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^4-6*sin(f*x+e)^4*ln(2/(a+b)^(1/2)*(((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b-3*sin(f*x+e)^4*ln(2/(a+b)^(
1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+co
s(f*x+e)))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b^2-10*(a+b)^(5/
2)*cos(f*x+e)^2*a^2+2*cos(f*x+e)^2*(a+b)^(5/2)*a*b-10*(a+b)^(5/2)*a*b-4*(a+
b)^(5/2)*b^2)/(a+b*sec(f*x+e)^2)^(1/2)/(-1+cos(f*x+e))^2/(1+cos(f*x+e))^2*s
ec(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 491, normalized size of antiderivative = 3.56

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[\frac{3\left(a^2\cos\left(fx+e\right)^4-2\,a^2\cos\left(fx+e\right)^2+a^2\right)\sqrt{a+b}\log\left(\frac{2\left(a\cos(fx+e)^2-2\sqrt{a+b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+a+2\,b\right.}{\cos(fx+e)^2-1}\right.}{16\left(\left(a^3+3\,a^2b+3\,ab^2+b^3\right)f\cos\left(fx+e\right)^4-2\left(a^3+3\,a^2b+3\,ab^2+\right.\right.} \right.$$

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt(a + b)*log(
2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
```

```
)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) + 2*(3*(a^2 + a*b)*cos(f
*x + e)^3 - (5*a^2 + 7*a*b + 2*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^4 - 2*(
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*f), 1/8*(3*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt(-a
 - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f
*x + e)/(a + b)) + (3*(a^2 + a*b)*cos(f*x + e)^3 - (5*a^2 + 7*a*b + 2*b^2)*
cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*f*cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos
(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f)]
```

Sympy [F]

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\csc^5{(e+fx)}}{\sqrt{a+b\sec^2{(e+fx)}}}\,dx$$

```
[In] integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(csc(e + f*x)**5/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\csc{(fx+e)}^5}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(csc(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\csc{(fx+e)}^5}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^5\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

```
[In] int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2)),x)
[Out] int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

3.99 $\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 193

$$\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{5(a+b)^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{7/2}f} - \frac{(33a^2+40ab+15b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} + \frac{(9a+5b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24a^2f} + \frac{\cos^3(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6af}$$

```
[Out] 5/16*(a+b)^3*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(7/2)/
f-1/48*(33*a^2+40*a*b+15*b^2)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1
/2)/a^3/f+1/24*(9*a+5*b)*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)
/a^2/f+1/6*cos(f*x+e)^3*sin(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4217, 481, 592, 541, 12, 385, 209}

$$\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx = \frac{5(a+b)^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{7/2}f} + \frac{(9a+5b)\sin(e+fx)\cos^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{24a^2f} - \frac{(33a^2+40ab+15b^2)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{48a^3f} + \frac{\sin^3(e+fx)\cos^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6af}$$

```
[In] Int[Sin[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (5*(a + b)^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])
/(16*a^(7/2)*f) - ((33*a^2 + 40*a*b + 15*b^2)*Cos[e + f*x]*Sin[e + f*x]*Sqr
t[a + b + b*Tan[e + f*x]^2])/(48*a^3*f) + ((9*a + 5*b)*Cos[e + f*x]^3*Sin[e
 + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(24*a^2*f) + (Cos[e + f*x]^3*Sin[e
+ f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(6*a*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
```

```
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^4\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6af} - \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)-2(3a+b)x^2)}{(1+x^2)^3\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{6af}$$

$$= \frac{(9a+5b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24a^2f} + \frac{\cos^3(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6af} - \frac{\text{Subst}\left(\int \frac{(a+b)(9a+5b)-2(12a^2+13ab+5b^2)x^2}{(1+x^2)^2\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{24a^2f}$$

$$= -\frac{(33a^2+40ab+15b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} + \frac{(9a+5b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24a^2f} + \frac{\cos^3(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6af} + \frac{\text{Subst}\left(\int \frac{15(a+b)^3}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{48a^3f}$$

$$= -\frac{(33a^2+40ab+15b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} + \frac{(9a+5b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24a^2f} + \frac{\cos^3(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6af} + \frac{(5(a+b)^3)\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{16a^3f}$$

$$= -\frac{(33a^2+40ab+15b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} + \frac{(9a+5b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24a^2f} + \frac{\cos^3(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6af} + \frac{(5(a+b)^3)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^3f}$$

$$= \frac{5(a+b)^3 \arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{7/2}f} - \frac{(33a^2+40ab+15b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} + \frac{(9a+5b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24a^2f} + \frac{\cos^3(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6af}$$

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.84

$$\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\sqrt{a+2b+a\cos(2(e+fx))}\sec(e+fx)\left(15(a+b)^3\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)-\sqrt{a}\sin(e+fx)\sqrt{a+b-}\right.}{48\sqrt{2}a^{7/2}f\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Integrate[Sin[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sec[e + f*x]*(15*(a + b)^3*ArcTan[(Sqrt
[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]] - Sqrt[a]*Sin[e + f*x]*Sq
rt[a + b - a*Sin[e + f*x]^2]*(15*(a + b)^2 + 10*a*(a + b)*Sin[e + f*x]^2 +
8*a^2*Sin[e + f*x]^4)))/(48*Sqrt[2]*a^(7/2)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1135 vs. 2(173) = 346.

Time = 6.76 (sec) , antiderivative size = 1136, normalized size of antiderivative = 5.89

method	result	size
default	Expression too large to display	1136

```
[In] int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/f/a^3/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)*(-8*cos(f*x+e)^5*sin(f*x+e)*
(-a)^(1/2)*a^3+26*cos(f*x+e)^3*sin(f*x+e)*(-a)^(1/2)*a^3+2*cos(f*x+e)^3*sin
(f*x+e)*(-a)^(1/2)*a^2*b-33*cos(f*x+e)*sin(f*x+e)*(-a)^(1/2)*a^3-14*cos(f*x
+e)*sin(f*x+e)*(-a)^(1/2)*a^2*b-5*cos(f*x+e)*sin(f*x+e)*(-a)^(1/2)*a*b^2+15
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3+45*((b+a*cos(f*x+e)^2)/(1+cos(f*
```

```
x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f
*x+e)*a)*a^2*b+45*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^2+15*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)-4*sin(f*x+e)*a)*b^3+15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+
4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^
3*sec(f*x+e)+45*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2*b*sec(f*x+e)+45*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^2*sec(f*x+e)+15*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)-4*sin(f*x+e)*a)*b^3*sec(f*x+e)-33*(-a)^(1/2)*a^2*b*tan(f*x+e)-40*(-a)^(
1/2)*a*b^2*tan(f*x+e)-15*(-a)^(1/2)*b^3*tan(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 1.28 (sec) , antiderivative size = 639, normalized size of antiderivative = 3.31

$$\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \Bigg[-\frac{15\,(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{-a}\log\Big(128\,a^4\cos(fx+e)^8-256\,(a^4-a^3b)\cos(fx+e)^6+32\,(5\,a^4-1}{}$$

$$-\frac{15\,(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\,(a^2-ab)\cos(fx+e)^3+(a^2-6\,ab+b^2)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e}{\cos(fx+e}}}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3\,a^2b)\cos(fx+e)^2\right)\sin(fx+e)}\right.}{}$$

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(-a)*log(128*a^4*cos(f*x +
e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)
```

```
*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 -
7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24
*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)
^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^3*cos(f*x + e)^5 - 2*(1
3*a^3 + 5*a^2*b)*cos(f*x + e)^3 + (33*a^3 + 40*a^2*b + 15*a*b^2)*cos(f*x +
e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^4*f), -1/1
92*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x +
e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqr
t(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a
^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(8*a^3*co
s(f*x + e)^5 - 2*(13*a^3 + 5*a^2*b)*cos(f*x + e)^3 + (33*a^3 + 40*a^2*b + 1
5*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
+ e))/(a^4*f)]
```

Sympy [F]

$$\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sin^6{(e+fx)}}{\sqrt{a+b\sec^2{(e+fx)}}}\,dx$$

```
[In] integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sin(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sin{(fx+e)}^6}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sin\left(fx+e\right)^6}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sin\left(e+f\,x\right)^6}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

```
[In] int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.100 $\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{3(a+b)^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{5/2}f} - \frac{(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4af}$$

```
[Out] 3/8*(a+b)^2*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f
-1/8*(5*a+3*b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a^2/f+1/4*c
os(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4217, 481, 541, 12, 385, 209}

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{3(a+b)^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{5/2}f} - \frac{(5a+3b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8a^2f} + \frac{\sin(e+fx)\cos^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4af}$$

```
[In] Int[Sin[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (3*(a + b)^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])
/(8*a^(5/2)*f) - ((5*a + 3*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[
e + f*x]^2])/(8*a^2*f) + (Cos[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b + b*Tan[e
+ f*x]^2])/(4*a*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
```

```
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^3\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4af}\\
&\quad - \frac{\text{Subst}\left(\int \frac{a+b-2(2a+b)x^2}{(1+x^2)^2\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{4af}\\
&= -\frac{(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f}\\
&\quad + \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4af}\\
&\quad + \frac{\text{Subst}\left(\int \frac{3(a+b)^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{8a^2f}\\
&= -\frac{(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f}\\
&\quad + \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4af}\\
&\quad + \frac{(3(a+b)^2)\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{8a^2f}\\
&= -\frac{(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f}\\
&\quad + \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4af}\\
&\quad + \frac{(3(a+b)^2)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^2f}\\
&= \frac{3(a+b)^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{5/2}f}\\
&\quad - \frac{(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f}\\
&\quad + \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4af}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.07

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\sqrt{a+2b+a\cos(2(e+fx))}\sec(e+fx)\left(3(a+b)^2\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)-\sqrt{a}\sin(e+fx)\sqrt{a+b-\right.}{8\sqrt{2}a^{5/2}f\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Integrate[Sin[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sec[e + f*x]*(3*(a + b)^2*ArcTan[(Sqrt[
a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]] - Sqrt[a]*Sin[e + f*x]*Sqr
t[a + b - a*Sin[e + f*x]^2]*(3*(a + b) + 2*a*Sin[e + f*x]^2)))/(8*Sqrt[2]*a
^(5/2)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. 2(119) = 238.

Time = 5.56 (sec) , antiderivative size = 809, normalized size of antiderivative = 5.99

method	result
default	$\frac{2\sin(fx+e)\cos(fx+e)^3\sqrt{-a}\,a^2-5\sin(fx+e)\cos(fx+e)\sqrt{-a}\,a^2-\cos(fx+e)\sqrt{-a}\,\sin(fx+e)ab+3\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\ln\left(4\sqrt{-a}\,\sqrt{\frac{b+}{(1+}}\right.}{}$

```
[In] int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f/a^2/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)*(2*sin(f*x+e)*cos(f*x+e)^3*(-
a)^(1/2)*a^2-5*sin(f*x+e)*cos(f*x+e)*(-a)^(1/2)*a^2-cos(f*x+e)*(-a)^(1/2)*s
in(f*x+e)*a*b+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2+6*ln(4*(-a)^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*a*b+3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2-5*(-
a)^(1/2)*a*b*tan(f*x+e)-3*(-a)^(1/2)*b^2*tan(f*x+e)+3*sec(f*x+e)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)-4*sin(f*x+e)*a)*a^2+6*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b
```

```
*sec(f*x+e)+3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*c
os(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*
x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*sec(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.56 (sec) , antiderivative size = 565, normalized size of antiderivative = 4.19

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[-\frac{3\,(a^2+2\,ab+b^2)\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8 - 256\,(a^4-a^3b)\cos\left(fx+e\right)^6 + 32\,(5\,a^4 - 14\,a^3b + 5\,a\right.}{} \right.$$

$$-\frac{3\,(a^2+2\,ab+b^2)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5 - 8\,(a^2-ab)\cos(fx+e)^3 + (a^2-6\,ab+b^2)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4 - a^2b + ab^2 - (a^3-3\,a^2b)\cos(fx+e)^2\right)\sin(fx+e)}\right) - 4\,(}{32\,a^3f}$$

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(3*(a^2 + 2*a*b + b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^
4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^
2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)
*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*
a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e)) - 8*(2*a^2*cos(f*x + e)^3 - (5*a^2 + 3*a*b)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*
f), -1/32*(3*(a^2 + 2*a*b + b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 -
 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b +
 a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2*a^2*cos(f*x
+ e)^3 - (5*a^2 + 3*a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e))/(a^3*f)]
```

Sympy [F]

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin^4{(e+fx)}}{\sqrt{a+b\sec^2{(e+fx)}}}\,dx$$

```
[In] integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sin(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin{(fx+e)}^4}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin{(fx+e)}^4}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin{(e+f\,x)}^4}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

```
[In] int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.101 $\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 85

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{3/2}f} - \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af}$$

```
[Out] 1/2*(a+b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-1
/2*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4217, 482, 12, 385, 209}

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{3/2}f} - \frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2af}$$

```
[In] Int[Sin[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*
a^(3/2)*f) - (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*
a*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}\\ &= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af}\\ &\quad+\frac{\text{Subst}\left(\int \frac{a+b}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2af}\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af} \\
&\quad + \frac{(a+b)\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2af} \\
&= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af} \\
&\quad + \frac{(a+b)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2af} \\
&= \frac{(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{3/2}f} - \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.47

$$
\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx
= \frac{\sqrt{a+2b+a\cos(2(e+fx))}\sec(e+fx)\left((a+b)\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)-\sqrt{a}\sin(e+fx)\sqrt{a+b-a\,\mathrm{s}}\right.}{2\sqrt{2}a^{3/2}f\sqrt{a+b\sec^2(e+fx)}}
$$

```
[In] Integrate[Sin[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sec[e + f*x]*((a + b)*ArcTan[(Sqrt[a]*S
in[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]] - Sqrt[a]*Sin[e + f*x]*Sqrt[a
+ b - a*Sin[e + f*x]^2]))/(2*Sqrt[2]*a^(3/2)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(73) = 146$.

Time = 4.49 (sec) , antiderivative size = 509, normalized size of antiderivative = 5.99

method	result
default	$-\frac{\sin(fx+e)\cos(fx+e)\sqrt{-a}\,a-\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)a}{}$

```
[In] int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f/a/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)*(sin(f*x+e)*cos(f*x+e)*(-a)^(1
/2)*a-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos
```

```
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a-((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f
*x+e)*a)*b+(-a)^(1/2)*b*tan(f*x+e)-sec(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f
*x+e)*a)*a-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b*sec(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(73) = 146$.

Time = 0.41 (sec) , antiderivative size = 497, normalized size of antiderivative = 5.85

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[-\frac{8\,a\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e)+\sqrt{-a}(a+b)\log\left(128\,a^4\cos(fx+e)^8-256\,(a^4-a^3b)\cos\right.}{}\right.$$

$$-\frac{4\,a\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e)+(a+b)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\,(a^2-ab)\cos(fx+e)^3+(a^2-6\,a\right.}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3\,a\right.}\right.}{8\,a^2f}$$

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*
x + e) + sqrt(-a)*(a + b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*co
s(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*
a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3
)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)
^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b
^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*sin(f*x + e)))/(a^2*f), -1/8*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*cos(f*x + e)*sin(f*x + e) + (a + b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x
+ e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*s
qrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 -
 a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))))/(a^2*f)]
```

Sympy [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin^2{(e+fx)}}{\sqrt{a+b\sec^2{(e+fx)}}}\,dx$$

```
[In] integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sin(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin{(fx+e)}^2}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin{(fx+e)}^2}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin{(e+f\,x)}^2}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

```
[In] int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.102 $\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4213, 385, 209}

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}$$

```
[In] Int[1/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}\\ &= \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(39) = 78.

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.23

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)}{\sqrt{2}\sqrt{a}f\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Integrate[1/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2*b
 + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[e +
f*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(33) = 66$.

Time = 2.73 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.54

method	result	size
default	$\frac{\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)(\sec(fx+e)+1)}{f\sqrt{-a}\,\sqrt{a+b\sec(fx+e)^2}}$	138

```
[In] int(1/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/f/(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)/(a+b*sec(f*x+e)^2)^(1/
2)*(sec(f*x+e)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(33) = 66$.

Time = 0.39 (sec) , antiderivative size = 408, normalized size of antiderivative = 10.46

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$
$$= \left[-\frac{\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6+32\left(5\,a^4-14\,a^3b+5\,a^2b^2\right)\cos\left(fx+e\right.\right.}{}\right.$$
$$\left.-\frac{\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\left(a^2-ab\right)\cos(fx+e)^3+\left(a^2-6\,ab+b^2\right)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-\left(a^3-3\,a^2b\right)\cos(fx+e)^2\right)\sin(fx+e)}\right)}{4\sqrt{a}f}\right]$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] [-1/8*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*
a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
 e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*c
```

```
os(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e))/(a*f), -1/4*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x +
 e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)
*cos(f*x + e)^2)*sin(f*x + e)))/(sqrt(a)*f)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{1}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(1/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. $2(33)=66$.

Time = 0.43 (sec) , antiderivative size = 992, normalized size of antiderivative = 25.44

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\text{Too large to display}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*(arctan2(2*a*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f
*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*
sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e
)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4
*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x
+ 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos
(2*f*x + 2*e) + a)), 2*a*cos(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2
*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 +
 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f
*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x +
4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*si
n(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a +
2*b)*cos(2*f*x + 2*e) + a)) + 2*a + 4*b) - arctan2(2*(a^2*cos(4*f*x + 4*e)^
2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4
*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*
sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4
*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arcta
n2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) +
```

```
2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f
*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*
sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e
)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4
*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x
+ 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos
(2*f*x + 2*e) + a)) + 4*a + 4*b))/(sqrt(a)*f)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{1}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

```
[In] int(1/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(1/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.103 $\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 33

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=-\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{(a+b)f}$$

```
[Out] -cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {4217, 270}

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f(a+b)}$$

```
[In] Int[Csc[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/((a + b)*f))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
```

```
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{(a+b)f}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.67

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\csc(e+fx)\sec(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Integrate[Csc[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] -1/2*((a + 2*b + a*Cos[2*(e + f*x)])*Csc[e + f*x]*Sec[e + f*x])/((a + b)*f*
Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

method	result	size
default	$-\frac{\cot(fx+e)a+b\sec(fx+e)\csc(fx+e)}{f(a+b)\sqrt{a+b\sec(fx+e)^2}}$	48

```
[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/f/(a+b)/(a+b*sec(f*x+e)^2)^(1/2)*(cot(f*x+e)*a+b*sec(f*x+e)*csc(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.42

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{(a+b)f\sin(fx+e)}$$

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] -sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/((a + b)*f*sin(f*
x + e))
```

Sympy [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(csc(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\sqrt{b\tan(fx+e)^2+a+b}}{(a+b)f\tan(fx+e)}$$

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] -sqrt(b*tan(f*x + e)^2 + a + b)/((a + b)*f*tan(f*x + e))
```

Giac [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\csc\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 18.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.24

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=-\frac{\left(2\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(4\,e+4\,f\,x\right)\right)\,\sqrt{\frac{a+2\,b+a\,\cos\left(2\,e+2\,f\,x\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}{2\,f\,\sin\left(2\,e+2\,f\,x\right)^2\,\left(a+b\right)}$$

```
[In] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2)),x)
[Out] -((2*sin(2*e + 2*f*x) + sin(4*e + 4*f*x))*((a + 2*b + a*cos(2*e + 2*f*x))/(
cos(2*e + 2*f*x) + 1))^(1/2))/(2*f*sin(2*e + 2*f*x)^2*(a + b))
```

3.104 $\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 78

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(3a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^2f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f}$$

```
[Out] -1/3*(3*a+b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^2/f-1/3*cot(f*x+e)
^3*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {4217, 464, 270}

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f(a+b)} - \frac{(3a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f(a+b)^2}$$

```
[In] Int[Csc[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/3*((3*a + b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/((a + b)^2*f)
- (Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*(a + b)*f)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} + \frac{(3a+b)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{3(a+b)f}$$

$$= -\frac{(3a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^2f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{(-2a-b+a\cos(2(e+fx)))(a+2b+a\cos(2(e+fx)))\csc^3(e+fx)\sec(e+fx)}{6(a+b)^2f\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Integrate[Csc[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((-2*a - b + a*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*(e + f*x)])*Csc[e + f*x
]^3*Sec[e + f*x])/(6*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [A] (verified)

Time = 3.64 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99

method	result	size
default	$\frac{\left(b+a\cos(fx+e)^2\right)\left(2a\cos(fx+e)^2-3a-b\right)\sec(fx+e)\csc(fx+e)^3}{3f(a^2+2ab+b^2)\sqrt{a+b\sec(fx+e)^2}}$	77

[In] int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/3/f/(a^2+2*a*b+b^2)*(b+a*cos(f*x+e)^2)*(2*a*cos(f*x+e)^2-3*a-b)/(a+b*sec(f*x+e)^2)^(1/2)*sec(f*x+e)*csc(f*x+e)^3

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.23

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\left(2\,a\cos\left(fx+e\right)^3-\left(3\,a+b\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{3\,\left(\left(a^2+2\,ab+b^2\right)f\cos\left(fx+e\right)^2-\left(a^2+2\,ab+b^2\right)f\right)\sin\left(fx+e\right)}$$

[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] -1/3*(2*a*cos(f*x + e)^3 - (3*a + b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 - (a^2 + 2*a*b + b^2)*f)*sin(f*x + e))

Sympy [F]

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\csc^4\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

[In] integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)

[Out] Integral(csc(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.23

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\frac{3\sqrt{b\tan(fx+e)^2+a+b}}{(a+b)\tan(fx+e)} - \frac{2\sqrt{b\tan(fx+e)^2+a+b}b}{(a+b)^2\tan(fx+e)} + \frac{\sqrt{b\tan(fx+e)^2+a+b}}{(a+b)\tan(fx+e)^3}}{3f}$$

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(b*tan(f*x + e)^2 + a + b)/((a + b)*tan(f*x + e)) - 2*sqrt(b*ta
n(f*x + e)^2 + a + b)*b/((a + b)^2*tan(f*x + e)) + sqrt(b*tan(f*x + e)^2 +
a + b)/((a + b)*tan(f*x + e)^3))/f
```

Giac [F]

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\csc(fx+e)^4}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 22.95 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.58

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{2\left(e^{e\,2i+f\,x\,2i}+1\right)\sqrt{a+\frac{b}{\left(\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}\right)^2}}\left(a\,1i-a\,e^{e\,2i+f\,x\,2i}\,4i+a\,e^{e\,4i+f\,x\,4i}\,1i-b\,e^{e\,2i+f\,x\,2i}\,2i\right)}{3\,f\,(a+b)^2\left(e^{e\,2i+f\,x\,2i}-1\right)^3}$$

```
[In] int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

```
[Out] -(2*(exp(e*2i + f*x*2i) + 1)*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*
x*1i)/2)^2)^(1/2)*(a*1i - a*exp(e*2i + f*x*2i)*4i + a*exp(e*4i + f*x*4i)*1i
 - b*exp(e*2i + f*x*2i)*2i))/(3*f*(a + b)^2*(exp(e*2i + f*x*2i) - 1)^3)
```

3.105 $\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 132

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(15a^2+10ab+3b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^3f} - \frac{2(5a+3b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} - \frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5(a+b)f}$$

```
[Out] -1/15*(15*a^2+10*a*b+3*b^2)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^3/f
-2/15*(5*a+3*b)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^2/f-1/5*cot(f
*x+e)^5*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4217, 473, 464, 270}

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(15a^2+10ab+3b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)^3} - \frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5f(a+b)} - \frac{2(5a+3b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)^2}$$

```
[In] Int[Csc[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/15*((15*a^2 + 10*a*b + 3*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2
])/((a + b)^3*f) - (2*(5*a + 3*b)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x
]^2])/(15*(a + b)^2*f) - (Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(5
*(a + b)*f)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^6\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5(a+b)f} + \frac{\text{Subst}\left(\int \frac{2(5a+3b)+5(a+b)x^2}{x^4\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{5(a+b)f}$$

$$= -\frac{2(5a+3b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} - \frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5(a+b)f} + \frac{(15a^2+10ab+3b^2)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{15(a+b)^2f}$$

$$= -\frac{(15a^2+10ab+3b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^3f} - \frac{2(5a+3b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} - \frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5(a+b)f}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.76

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left(8a^2+8ab+3b^2-2a(3a+b)\cos(2(e+fx))+a^2\cos(4(e+fx))\right)\csc^5(e+}{30(a+b)^3f\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Integrate[Csc[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/30*((a + 2*b + a*Cos[2*(e + f*x)])*(8*a^2 + 8*a*b + 3*b^2 - 2*a*(3*a + b
)*Cos[2*(e + f*x)] + a^2*Cos[4*(e + f*x)])*Csc[e + f*x]^5*Sec[e + f*x])/((a
 + b)^3*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [A] (verified)

Time = 5.02 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{\left(b+a\cos(fx+e)^2\right)\left(8\cos(fx+e)^4a^2-20\cos(fx+e)^2a^2-4\cos(fx+e)^2ab+15a^2+10ab+3b^2\right)\sec(fx+e)\csc(fx+e)^5}{15f(a^3+3a^2b+3a\,b^2+b^3)\sqrt{a+b\sec(fx+e)^2}}$	120

```
[In] int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/f/(a^3+3*a^2*b+3*a*b^2+b^3)*(b+a*cos(f*x+e)^2)*(8*cos(f*x+e)^4*a^2-20
*cos(f*x+e)^2*a^2-4*cos(f*x+e)^2*a*b+15*a^2+10*a*b+3*b^2)/(a+b*sec(f*x+e)^2
)^(1/2)*sec(f*x+e)*csc(f*x+e)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.59 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.30

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\left(8\,a^2\cos\left(fx+e\right)^5-4\left(5\,a^2+ab\right)\cos\left(fx+e\right)^3+\left(15\,a^2+10\,ab+3\,b^2\right)\cos\left(fx+e\right)\right)\sqrt{}}{15\left(\left(a^3+3\,a^2b+3\,ab^2+b^3\right)f\cos\left(fx+e\right)^4-2\left(a^3+3\,a^2b+3\,ab^2+b^3\right)f\cos\left(fx+e\right)^2+\left(a^3+3\,a^2b+\right.\right.}$$

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*a^2*cos(f*x + e)^5 - 4*(5*a^2 + a*b)*cos(f*x + e)^3 + (15*a^2 + 10
*a*b + 3*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*f*cos(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f)*sin(f*x + e)
)
```

Sympy [F]

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\csc^6\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

```
[In] integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.45

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\frac{15\sqrt{b\tan(fx+e)^2+a+b}}{(a+b)\tan(fx+e)}-\frac{20\sqrt{b\tan(fx+e)^2+a+b}b}{(a+b)^2\tan(fx+e)}+\frac{8\sqrt{b\tan(fx+e)^2+a+b}b^2}{(a+b)^3\tan(fx+e)}+\frac{10\sqrt{b\tan(fx+e)^2+a+b}}{(a+b)\tan(fx+e)^3}-\frac{4\sqrt{b\tan(fx+e)^2+a+}}{(a+b)^2\tan(fx+e)^3}}{15\,f}$$

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*sqrt(b*tan(f*x + e)^2 + a + b)/((a + b)*tan(f*x + e)) - 20*sqrt(b
*tan(f*x + e)^2 + a + b)*b/((a + b)^2*tan(f*x + e)) + 8*sqrt(b*tan(f*x + e)
```

```
^2 + a + b)*b^2/((a + b)^3*tan(f*x + e)) + 10*sqrt(b*tan(f*x + e)^2 + a + b
)/((a + b)*tan(f*x + e)^3) - 4*sqrt(b*tan(f*x + e)^2 + a + b)*b/((a + b)^2*
tan(f*x + e)^3) + 3*sqrt(b*tan(f*x + e)^2 + a + b)/((a + b)*tan(f*x + e)^5)
)/f
```

Giac [F]

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\csc\left(fx+e\right)^6}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 30.64 (sec) , antiderivative size = 723, normalized size of antiderivative = 5.48

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\text{Too large to display}$$

```
[In] int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2)),x)
[Out] (((32*a + 16*b)/(5*f*(6*a + 6*b)*(a*1i + b*1i)) + (32*a + 80*b)/(5*f*(6*a +
 6*b)*(a*1i + b*1i)))*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2
)^2)^(1/2)*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1))/((exp(e*2i + f*
x*2i) - 1)^3*(exp(e*2i + f*x*2i) + 1)) - ((a + b/(exp(- e*1i - f*x*1i)/2 +
exp(e*1i + f*x*1i)/2)^2)^(1/2)*((a*(2*a + b)*32i)/(15*f*(a*1i + b*1i)^2*(8*
a + 8*b)) + (a*(2*a + 3*b)*32i)/(15*f*(a*1i + b*1i)^2*(8*a + 8*b)))*(2*exp(
e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1))/((exp(e*2i + f*x*2i) - 1)^2*(exp(
e*2i + f*x*2i) + 1)) + ((a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)
/2)^2)^(1/2)*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1)*((96*a + 32*b)
/(5*f*(a*1i + b*1i)*(16*a + 16*b)) + (160*a + 160*b)/(5*f*(a*1i + b*1i)*(16
*a + 16*b)) + (256*a + 320*b)/(5*f*(a*1i + b*1i)*(16*a + 16*b))))/((exp(e*2
i + f*x*2i) - 1)^4*(exp(e*2i + f*x*2i) + 1)) - ((a + b/(exp(- e*1i - f*x*1i
)/2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x
*4i) + 1)*32i)/(f*(exp(e*2i + f*x*2i) - 1)^5*(exp(e*2i + f*x*2i) + 1)*(10*a
 + 10*b)) - (8*a^2*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2)^2
)^(1/2)*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1))/(15*f*(exp(e*2i +
f*x*2i) - 1)*(exp(e*2i + f*x*2i) + 1)*(a*1i + b*1i)^3)
```

3.106 $\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 171

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(15a^2+40ab+24b^2)\cos(e+fx)}{15a^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{2(5a+3b)\cos^3(e+fx)}{15a^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cos^5(e+fx)}{5af\sqrt{a+b\sec^2(e+fx)}} - \frac{2b(15a^2+40ab+24b^2)\sec(e+fx)}{15a^4f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -1/15*(15*a^2+40*a*b+24*b^2)*cos(f*x+e)/a^3/f/(a+b*sec(f*x+e)^2)^(1/2)+2/15
*(5*a+3*b)*cos(f*x+e)^3/a^2/f/(a+b*sec(f*x+e)^2)^(1/2)-1/5*cos(f*x+e)^5/a/f
/(a+b*sec(f*x+e)^2)^(1/2)-2/15*b*(15*a^2+40*a*b+24*b^2)*sec(f*x+e)/a^4/f/(a
+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4219, 473, 464, 277, 197}

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{2(5a+3b)\cos^3(e+fx)}{15a^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\left(\frac{8b(5a+3b)}{a^2}+15\right)\cos(e+fx)}{15af\sqrt{a+b\sec^2(e+fx)}} - \frac{2b(15a^2+40ab+24b^2)\sec(e+fx)}{15a^4f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cos^5(e+fx)}{5af\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/15*((15 + (8*b*(5*a + 3*b))/a^2)*Cos[e + f*x])/(a*f*Sqrt[a + b*Sec[e + f
*x]^2]) + (2*(5*a + 3*b)*Cos[e + f*x]^3)/(15*a^2*f*Sqrt[a + b*Sec[e + f*x]^
2]) - Cos[e + f*x]^5/(5*a*f*Sqrt[a + b*Sec[e + f*x]^2]) - (2*b*(15*a^2 + 40
*a*b + 24*b^2)*Sec[e + f*x])/(15*a^4*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(-1+x^2)^2}{x^6(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)}{5af\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{-2(5a+3b)+5ax^2}{x^4(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{5af}$$

$$= \frac{2(5a+3b)\cos^3(e+fx)}{15a^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cos^5(e+fx)}{5af\sqrt{a+b\sec^2(e+fx)}} - \frac{(-15a^2-8b(5a+3b))\,\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{15a^2f}$$

$$= -\frac{(15a^2+8b(5a+3b))\cos(e+fx)}{15a^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{2(5a+3b)\cos^3(e+fx)}{15a^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cos^5(e+fx)}{5af\sqrt{a+b\sec^2(e+fx)}} + \frac{(2b(-15a^2-8b(5a+3b)))\,\text{Subst}\left(\int \frac{1}{(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{15a^3f}$$

$$= -\frac{(15a^2+8b(5a+3b))\cos(e+fx)}{15a^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{2(5a+3b)\cos^3(e+fx)}{15a^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cos^5(e+fx)}{5af\sqrt{a+b\sec^2(e+fx)}} - \frac{2b(15a^2+8b(5a+3b))\sec(e+fx)}{15a^4f\sqrt{a+b\sec^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 6.91 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.76

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\,(150a^3+1528a^2b+2944ab^2+1536b^3+a(125a^2+544ab+384b^2)\cos(2(e+ \ldots}{960a^4f\,(a+b\sec^2(e+fx))^{3/2}}$$

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/960*((a + 2*b + a*Cos[2*(e + f*x)])*(150*a^3 + 1528*a^2*b + 2944*a*b^2 +
 1536*b^3 + a*(125*a^2 + 544*a*b + 384*b^2)*Cos[2*(e + f*x)] - 2*a^2*(11*a
+ 12*b)*Cos[4*(e + f*x)] + 3*a^3*Cos[6*(e + f*x)])*Sec[e + f*x]^3)/(a^4*f*(
a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [A] (verified)

Time = 5.04 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99

method	result
default	$-\frac{(a+b)^{6} a^{2} \left(b+a \cos(fx+e)^{2}\right) \left(3a^{3} \cos(fx+e)^{6}-10 \cos(fx+e)^{4} a^{3}-6 \cos(fx+e)^{4} a^{2} b+15 \cos(fx+e)^{2} a^{3}+40 \cos(fx+e)^{2} a^{2} b+24 \cos(f}{15 f \left(\sqrt{-ab}-a\right)^{6} \left(\sqrt{-ab}+a\right)^{6} \left(a+b \sec(fx+e)^{2}\right)^{\frac{3}{2}}}$

```
[In] int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/f*(a+b)^6*a^2/((-a*b)^(1/2)-a)^6/((-a*b)^(1/2)+a)^6*(b+a*cos(f*x+e)^2
)*(3*a^3*cos(f*x+e)^6-10*cos(f*x+e)^4*a^3-6*cos(f*x+e)^4*a^2*b+15*cos(f*x+e
)^2*a^3+40*cos(f*x+e)^2*a^2*b+24*cos(f*x+e)^2*a*b^2+30*a^2*b+80*a*b^2+48*b^
3)/(a+b*sec(f*x+e)^2)^(3/2)*sec(f*x+e)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.80

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx =$$

$$-\frac{\left(3\,a^{3} \cos\left(fx+e\right)^{7} - 2\left(5\,a^{3} + 3\,a^{2} b\right) \cos\left(fx+e\right)^{5} + \left(15\,a^{3} + 40\,a^{2} b + 24\,a b^{2}\right) \cos\left(fx+e\right)^{3} + 2\left(15\,a^{2} b + 40\right.}{15\left(a^{5} f \cos\left(fx+e\right)^{2} + a^{4} b f\right)}$$

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*a^3*cos(f*x + e)^7 - 2*(5*a^3 + 3*a^2*b)*cos(f*x + e)^5 + (15*a^3
+ 40*a^2*b + 24*a*b^2)*cos(f*x + e)^3 + 2*(15*a^2*b + 40*a*b^2 + 24*b^3)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(a^5*f*cos(f*x + e)
^2 + a^4*b*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.46

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx =$$

$$-\frac{\frac{15\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^2} - \frac{10\left(\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3 - 6\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)\right)}{a^3} + \frac{15\,b}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,a^2\cos(fx+e)} + \cdots}{\cdots}$$

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^2 - 10*((a + b/cos(f*x
+ e)^2)^(3/2)*cos(f*x + e)^3 - 6*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))
/a^3 + 15*b/(sqrt(a + b/cos(f*x + e)^2)*a^2*cos(f*x + e)) + 30*b^2/(sqrt(a
+ b/cos(f*x + e)^2)*a^3*cos(f*x + e)) + 15*b^3/(sqrt(a + b/cos(f*x + e)^2)*
a^4*cos(f*x + e)) + 3*((a + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 5*(a +
 b/cos(f*x + e)^2)^(3/2)*b*cos(f*x + e)^3 + 15*sqrt(a + b/cos(f*x + e)^2)*b
^2*cos(f*x + e))/a^4)/f
```

Giac [F]

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(fx+e\right)^5}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(e+f\,x\right)^5}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.107 $\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 114

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}} \, dx = -\frac{(3a+4b)\cos(e+fx)}{3a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\cos^3(e+fx)}{3af\sqrt{a+b\sec^2(e+fx)}} - \frac{2b(3a+4b)\sec(e+fx)}{3a^3f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -1/3*(3*a+4*b)*cos(f*x+e)/a^2/f/(a+b*sec(f*x+e)^2)^(1/2)+1/3*cos(f*x+e)^3/a
/f/(a+b*sec(f*x+e)^2)^(1/2)-2/3*b*(3*a+4*b)*sec(f*x+e)/a^3/f/(a+b*sec(f*x+e
)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4219, 464, 277, 197}

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}} \, dx = -\frac{2b(3a+4b)\sec(e+fx)}{3a^3f\sqrt{a+b\sec^2(e+fx)}} - \frac{(3a+4b)\cos(e+fx)}{3a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\cos^3(e+fx)}{3af\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/3*((3*a + 4*b)*Cos[e + f*x])/(a^2*f*Sqrt[a + b*Sec[e + f*x]^2]) + Cos[e
+ f*x]^3/(3*a*f*Sqrt[a + b*Sec[e + f*x]^2]) - (2*b*(3*a + 4*b)*Sec[e + f*x]
)/(3*a^3*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f} \\
&= \frac{\cos^3(e+fx)}{3af\sqrt{a+b\sec^2(e+fx)}} + \frac{(3a+4b)\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{3af} \\
&= -\frac{(3a+4b)\cos(e+fx)}{3a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\cos^3(e+fx)}{3af\sqrt{a+b\sec^2(e+fx)}} \\
&\quad - \frac{(2b(3a+4b))\text{Subst}\left(\int \frac{1}{(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{3a^2f} \\
&= -\frac{(3a+4b)\cos(e+fx)}{3a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\cos^3(e+fx)}{3af\sqrt{a+b\sec^2(e+fx)}} - \frac{2b(3a+4b)\sec(e+fx)}{3a^3f\sqrt{a+b\sec^2(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.61 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.82

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left(9a^2+64ab+64b^2+8a(a+2b)\cos(2(e+fx))-a^2\cos(4(e+fx))\right)\sec^3(e+}{48a^3f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/48*((a + 2*b + a*Cos[2*(e + f*x)])*(9*a^2 + 64*a*b + 64*b^2 + 8*a*(a + 2
*b)*Cos[2*(e + f*x)] - a^2*Cos[4*(e + f*x)])*Sec[e + f*x]^3)/(a^3*f*(a + b*
Sec[e + f*x]^2)^(3/2))
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{a(a+b)^4\left(b+a\cos(fx+e)^2\right)\left(\cos(fx+e)^4a^2-3\cos(fx+e)^2a^2-4\cos(fx+e)^2ab-6ab-8b^2\right)\sec(fx+e)^3}{3f\left(\sqrt{-ab}-a\right)^4\left(\sqrt{-ab}+a\right)^4\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}$	115

```
[In] int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f*a/((-a*b)^(1/2)-a)^4/((-a*b)^(1/2)+a)^4*(a+b)^4*(b+a*cos(f*x+e)^2)*(c
os(f*x+e)^4*a^2-3*cos(f*x+e)^2*a^2-4*cos(f*x+e)^2*a*b-6*a*b-8*b^2)/(a+b*sec
(f*x+e)^2)^(3/2)*sec(f*x+e)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.86

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\left(a^2\cos(fx+e)^5-(3\,a^2+4\,ab)\cos(fx+e)^3-2\,(3\,ab+4\,b^2)\cos(fx+e)\right)\sqrt{}}{3\left(a^4f\cos(fx+e)^2+a^3bf\right)}$$

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*cos(f*x + e)^5 - (3*a^2 + 4*a*b)*cos(f*x + e)^3 - 2*(3*a*b + 4*b^2
)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(a^4*f*cos(f*x
+ e)^2 + a^3*b*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.24

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx =$$
$$-\frac{\frac{3\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^2} - \frac{\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3 - 6\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)}{a^3} + \frac{3\,b}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,a^2\cos(fx+e)} + \frac{3\,b^2}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,a^3\cos(fx+e)}}{3\,f}$$

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] -1/3*(3*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^2 - ((a + b/cos(f*x + e)^
2)^(3/2)*cos(f*x + e)^3 - 6*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))/a^3
+ 3*b/(sqrt(a + b/cos(f*x + e)^2)*a^2*cos(f*x + e)) + 3*b^2/(sqrt(a + b/cos
(f*x + e)^2)*a^3*cos(f*x + e)))/f
```

Giac [F]

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(e+f\,x)^3}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.108 $\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 62

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)}{af\sqrt{a+b\sec^2(e+fx)}} - \frac{2b\sec(e+fx)}{a^2f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -cos(f*x+e)/a/f/(a+b*sec(f*x+e)^2)^(1/2)-2*b*sec(f*x+e)/a^2/f/(a+b*sec(f*x+
e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4219, 277, 197}

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{2b\sec(e+fx)}{a^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cos(e+fx)}{af\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Cos[e + f*x]/(a*f*Sqrt[a + b*Sec[e + f*x]^2])) - (2*b*Sec[e + f*x])/(a^2*
f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)}{af\sqrt{a+b\sec^2(e+fx)}} - \frac{(2b)\text{Subst}\left(\int \frac{1}{(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{af}$$

$$= -\frac{\cos(e+fx)}{af\sqrt{a+b\sec^2(e+fx)}} - \frac{2b\sec(e+fx)}{a^2f\sqrt{a+b\sec^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))(a+4b+a\cos(2(e+fx)))\sec^3(e+fx)}{4a^2f\,(a+b\sec^2(e+fx))^{3/2}}$$

```
[In] Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] -1/4*((a + 2*b + a*Cos[2*(e + f*x)])*(a + 4*b + a*Cos[2*(e + f*x)])*Sec[e +
 f*x]^3)/(a^2*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{-\frac{1}{a\sec(fx+e)\sqrt{a+b\sec(fx+e)^2}}-\frac{2b\sec(fx+e)}{a^2\sqrt{a+b\sec(fx+e)^2}}}{f}$	59
default	$\frac{-\frac{1}{a\sec(fx+e)\sqrt{a+b\sec(fx+e)^2}}-\frac{2b\sec(fx+e)}{a^2\sqrt{a+b\sec(fx+e)^2}}}{f}$	59

```
[In] int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/f*(-1/a/sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2)-2*b/a^2*sec(f*x+e)/(a+b*sec(f
*x+e)^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=-\frac{\left(a\cos\left(fx+e\right)^3+2\,b\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{a^3f\cos\left(fx+e\right)^2+a^2bf}$$

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] -(a*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/(a^3*f*cos(f*x + e)^2 + a^2*b*f)
```

Sympy [F]

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{\sin\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(sin(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = -\frac{\frac{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^2}+\frac{b}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,a^2\cos(fx+e)}}{f}$$

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] -(sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^2 + b/(sqrt(a + b/cos(f*x + e)^
2)*a^2*cos(f*x + e)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.77

$$\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = -\frac{\sqrt{a\cos(fx+e)^2+b}+\frac{b}{\sqrt{a\cos(fx+e)^2+b}}}{a^2 f\,\mathrm{sgn}\left(\cos(fx+e)\right)}$$

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] -(sqrt(a*cos(f*x + e)^2 + b) + b/sqrt(a*cos(f*x + e)^2 + b))/(a^2*f*sgn(cos
(f*x + e)))
```

Mupad [B] (verification not implemented)

Time = 24.73 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.50

$$\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = -\frac{e^{-e\,1i-f\,x\,1i}\left(e^{e\,2i+f\,x\,2i}+1\right)\sqrt{a+\frac{b}{\left(\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}\right)^2}}\left(a+2\,a\,e^{e\,2i+f\,x\,2i}+a\,e^{e\,4i+f\,x\,4i}+8\,b\,e^{e\,2i+f\,x\,2i}\right)}{2\,a^2\,f\,\left(a+2\,a\,e^{e\,2i+f\,x\,2i}+a\,e^{e\,4i+f\,x\,4i}+4\,b\,e^{e\,2i+f\,x\,2i}\right)}$$

```
[In] int(sin(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] -(exp(- e*1i - f*x*1i)*(exp(e*2i + f*x*2i) + 1)*(a + b/(exp(- e*1i - f*x*1i
)/2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*(a + 2*a*exp(e*2i + f*x*2i) + a*exp(e*
4i + f*x*4i) + 8*b*exp(e*2i + f*x*2i)))/(2*a^2*f*(a + 2*a*exp(e*2i + f*x*2i
) + a*exp(e*4i + f*x*4i) + 4*b*exp(e*2i + f*x*2i)))
```

3.109 $\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 80

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)^{3/2}f}-\frac{b\sec(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b)^(3/2)/f-b*s
ec(f*x+e)/a/(a+b)/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4219, 390, 385, 213}

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f(a+b)^{3/2}}-\frac{b\sec(e+fx)}{af(a+b)\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]]/((a + b)^(
3/2)*f)) - (b*Sec[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}\\
&= -\frac{b\sec(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{(a+b)f}\\
&= -\frac{b\sec(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-1-(-a-b)x^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)f}\\
&= -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)^{3/2}f} - \frac{b\sec(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.41

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(b\sqrt{a+b}+a\text{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{a+b}}\right)\sqrt{a+b-a\sin^2(e+fx}\right.}{2a(a+b)^{3/2}f\,(a+b\sec^2(e+fx))^{3/2}}$$

```
[In] Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(b*Sqrt[a + b] + a*ArcT
anh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b]]*Sqrt[a + b - a*Sin[e + f*x]
^2]))/(a*(a + b)^(3/2)*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1060 vs. 2(72) = 144.

Time = 1.18 (sec) , antiderivative size = 1061, normalized size of antiderivative = 13.26

method	result	size
default	Expression too large to display	1061

```
[In] int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f/(a+b)^(5/2)/a*(b+a*cos(f*x+e)^2)*(cos(f*x+e)*ln(2/(a+b)^(1/2)*(((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+cos(f*x+e)*ln(2/(a+b)^(1/2)
*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*
x+e)))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b+cos(f*x+e)*ln(-4*(((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e
)))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+cos(f*x+e)*ln(-4*(((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b+2*(a+b)^(3/2)*b+ln(2/(a+b)
^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+
cos(f*x+e)))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+ln(2/(a+b)^(1/
2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(
f*x+e)))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b+ln(-4*(((b+a*cos(f
```

```
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+ln(-4*(((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*a*b)/(a+b*sec(f*x+e)^2)^(3/2)*sec(f*x+e)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(72) = 144$.

Time = 0.33 (sec) , antiderivative size = 344, normalized size of antiderivative = 4.30

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[-\frac{2\,(ab+b^2)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e) - \left(a^2\cos(fx+e)^2+ab\right)\sqrt{a+b}\log}{2\left((a^4+2\,a^3b+a^2b^2)f\cos(fx+e)^2+(a\right.}\right.$$

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a*b + b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e) - (a^2*cos(f*x + e)^2 + a*b)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 - 2*sqr
t(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b
)/(cos(f*x + e)^2 - 1)))/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3
*b + 2*a^2*b^2 + a*b^3)*f), ((a^2*cos(f*x + e)^2 + a*b)*sqrt(-a - b)*arctan
(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a +
 b)) - (a*b + b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)
)/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b + 2*a^2*b^2 + a*b^3)
*f)]
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\csc\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(csc(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(72) = 144$.

Time = 0.86 (sec) , antiderivative size = 503, normalized size of antiderivative = 6.29

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx =$$

$$-\frac{\frac{2\left(\frac{\left(ab^2+b^3\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}{a^3b\,\mathrm{sgn}(\cos(fx+e))+2a^2b^2\,\mathrm{sgn}(\cos(fx+e))+ab^3\,\mathrm{sgn}(\cos(fx+e))}+\frac{ab^2+b^3}{a^3b\,\mathrm{sgn}(\cos(fx+e))+2a^2b^2\,\mathrm{sgn}(\cos(fx+e))+ab^3\,\mathrm{sgn}(\cos(fx+e))}\right)}{\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}} - \frac{\log\left(\left|-\sqrt{a+b}\tan(\right.\right.}{}}{}$$

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] -1/2*(2*((a*b^2 + b^3)*tan(1/2*f*x + 1/2*e)^2/(a^3*b*sgn(cos(f*x + e)) + 2*
a^2*b^2*sgn(cos(f*x + e)) + a*b^3*sgn(cos(f*x + e))) + (a*b^2 + b^3)/(a^3*b
*sgn(cos(f*x + e)) + 2*a^2*b^2*sgn(cos(f*x + e)) + a*b^3*sgn(cos(f*x + e)))
)/sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*
x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - log(abs(-sqrt(a + b)*t
an(1/2*f*x + 1/2*e)^2 + sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2
*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) -
sqrt(a + b)))/((a + b)^(3/2)*sgn(cos(f*x + e))) - log(abs((sqrt(a + b)*tan(
1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)
^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt
(a + b) - a + b))/((a + b)^(3/2)*sgn(cos(f*x + e))) + log(abs((sqrt(a + b)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/
2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*
sqrt(a + b) - a - b))/((a + b)^(3/2)*sgn(cos(f*x + e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin(e+f\,x)\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2)),x)
[Out] int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

3.110 $\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 126

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a-2b)\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^{5/2}f} - \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{3b\sec(e+fx)}{2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -1/2*(a-2*b)*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b)
^(5/2)/f-1/2*cot(f*x+e)*csc(f*x+e)/(a+b)/f/(a+b*sec(f*x+e)^2)^(1/2)-3/2*b*s
ec(f*x+e)/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4219, 482, 541, 12, 385, 213}

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a-2b)\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f(a+b)^{5/2}} - \frac{3b\sec(e+fx)}{2f(a+b)^2\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot(e+fx)\csc(e+fx)}{2f(a+b)\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*((a - 2*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^
2]])/((a + b)^(5/2)*f) - (Cot[e + f*x]*Csc[e + f*x])/(2*(a + b)*f*Sqrt[a +
```

```
b*Sec[e + f*x]^2]) - (3*b*Sec[e + f*x])/(2*(a + b)^2*f*Sqrt[a + b*Sec[e + f
*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2}{(-1+x^2)^2(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-2bx^2}{(-1+x^2)(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{2(a+b)f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{3b\sec(e+fx)}{2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a(a-2b)}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{2a(a+b)^2f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{3b\sec(e+fx)}{2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{(a-2b)\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{2(a+b)^2f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{3b\sec(e+fx)}{2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{(a-2b)\text{Subst}\left(\int \frac{1}{-1-(-a-b)x^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^2f}$$

$$= -\frac{(a-2b)\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^{5/2}f} - \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{3b\sec(e+fx)}{2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.77

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left((a+b)\csc^2(e+fx)-(a-2b)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1-\frac{a\sin^2(e+fx)}{a+b}\right)\right.}{4(a+b)^2f\,(a+b\sec^2(e+fx))^{3/2}}$$

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] -1/4*((a + 2*b + a*Cos[2*(e + f*x)])*((a + b)*Csc[e + f*x]^2 - (a - 2*b)*Hy
pergeometric2F1[-1/2, 1, 1/2, 1 - (a*Sin[e + f*x]^2)/(a + b)])*Sec[e + f*x]
^3)/((a + b)^2*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1713 vs. 2(110) = 220.

Time = 1.09 (sec) , antiderivative size = 1714, normalized size of antiderivative = 13.60

method	result	size
default	Expression too large to display	1714

```
[In] int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f/(a+b)^(9/2)*(b+a*cos(f*x+e)^2)*(2*cos(f*x+e)^2*(a+b)^(5/2)*a-4*cos(f
*x+e)^2*(a+b)^(5/2)*b+cos(f*x+e)*sin(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^
(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^3-3*cos(f*x+e)*sin(f*x+e)^2*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a*b^2-2*cos(f*x
+e)*sin(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(2/(a+b)^(1/
2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(
f*x+e)))*b^3+cos(f*x+e)*sin(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*
x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b
)/(-1+cos(f*x+e)))*a^3-3*cos(f*x+e)*sin(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^
(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+co
s(f*x+e)*a+b)/(-1+cos(f*x+e)))*a*b^2-2*cos(f*x+e)*sin(f*x+e)^2*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*b^3+((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+
b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^3*sin(f*x+e)^2-3*b^2*ln(2/(a+b)^
(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+c
os(f*x+e)))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*sin(f*x+e)^2-2*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*b^3*sin(f*
x+e)^2+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^3*sin(f*x
+e)^2-3*b^2*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*
cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+
```

```
e)*a+b)/(-1+cos(f*x+e))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*sin(
f*x+e)^2-2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*b^3*sin
(f*x+e)^2+6*(a+b)^(5/2)*b)/(a+b*sec(f*x+e)^2)^(3/2)*sec(f*x+e)^3*csc(f*x+e)
^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(110) = 220$.

Time = 0.38 (sec) , antiderivative size = 547, normalized size of antiderivative = 4.34

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[-\frac{\left((a^2-2\,ab)\cos\left(fx+e\right)^4-(a^2-3\,ab+2\,b^2)\cos\left(fx+e\right)^2-ab+2\,b^2\right)\cdots}{4\left((a^4+3\,a^3b+3\,a^2b^2+a\cdots\right.}\right.$$

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((a^2 - 2*a*b)*cos(f*x + e)^4 - (a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^2
 - a*b + 2*b^2)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 + 2*sqrt(a + b)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^
2 - 1)) - 2*((a^2 - a*b - 2*b^2)*cos(f*x + e)^3 + 3*(a*b + b^2)*cos(f*x + e
))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 + 3*a^3*b + 3*a^2*b^2
 + a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*f*cos(f*x + e)
^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f), 1/2*(((a^2 - 2*a*b)*cos(f*x +
e)^4 - (a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^2 - a*b + 2*b^2)*sqrt(-a - b)*arc
tan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(
a + b)) + ((a^2 - a*b - 2*b^2)*cos(f*x + e)^3 + 3*(a*b + b^2)*cos(f*x + e))
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 + 3*a^3*b + 3*a^2*b^2 +
 a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*f*cos(f*x + e)^2
 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f)]
```

Sympy [F]

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc^3\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^3\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2)),x)
[Out] int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

3.111 $\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 177

$$\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = -\frac{3a(a-4b)\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{7/2}f} - \frac{5a\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{(13a-2b)b\sec(e+fx)}{8(a+b)^3f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -3/8*a*(a-4*b)*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/(a+
b)^(7/2)/f-5/8*a*cot(f*x+e)*csc(f*x+e)/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(1/2)-1
/4*cot(f*x+e)^3*csc(f*x+e)/(a+b)/f/(a+b*sec(f*x+e)^2)^(1/2)-1/8*(13*a-2*b)*
b*sec(f*x+e)/(a+b)^3/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4219, 481, 541, 12, 385, 213}

$$\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = -\frac{3a(a-4b)\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f(a+b)^{7/2}} - \frac{b(13a-2b)\sec(e+fx)}{8f(a+b)^3\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4f(a+b)\sqrt{a+b\sec^2(e+fx)}} - \frac{5a\cot(e+fx)\csc(e+fx)}{8f(a+b)^2\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*a*(a - 4*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^
2]])/(8*(a + b)^(7/2)*f) - (5*a*Cot[e + f*x]*Csc[e + f*x])/(8*(a + b)^2*f*S
qrt[a + b*Sec[e + f*x]^2]) - (Cot[e + f*x]^3*Csc[e + f*x])/(4*(a + b)*f*Sqr
t[a + b*Sec[e + f*x]^2]) - ((13*a - 2*b)*b*Sec[e + f*x])/(8*(a + b)^3*f*Sqr
t[a + b*Sec[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^3(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{-a-4ax^2}{(-1+x^2)^2(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{4(a+b)f}$$

$$= -\frac{5a\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{-a(3a-2b)+10abx^2}{(-1+x^2)(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{8(a+b)^2f}$$

$$= -\frac{5a\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{(13a-2b)b\sec(e+fx)}{8(a+b)^3f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int -\frac{3a^2(a-4b)}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{8a(a+b)^3f}$$

$$= -\frac{5a\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{(13a-2b)b\sec(e+fx)}{8(a+b)^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{(3a(a-4b))\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{8(a+b)^3f}$$

$$= -\frac{5a\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{(13a-2b)b\sec(e+fx)}{8(a+b)^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{(3a(a-4b))\text{Subst}\left(\int \frac{1}{-1-(-a-b)x^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^3f}$$

$$= -\frac{3a(a-4b)\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{7/2}f} - \frac{5a\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{(13a-2b)b\sec(e+fx)}{8(a+b)^3f\sqrt{a+b\sec^2(e+fx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.56

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left((a+b)^2\csc^4(e+fx)-a(a-4b)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},1-\frac{a\sin^2(e+fx)}{a+b}\right.\right.}{8(a+b)^3f\,(a+b\sec^2(e+fx))^{3/2}}$$

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/8*((a + 2*b + a*Cos[2*(e + f*x)])*((a + b)^2*Csc[e + f*x]^4 - a*(a - 4*b
)*Hypergeometric2F1[-1/2, 2, 1/2, 1 - (a*Sin[e + f*x]^2)/(a + b)])*Sec[e +
f*x]^3)/((a + b)^3*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3611 vs. 2(157) = 314.

Time = 1.23 (sec) , antiderivative size = 3612, normalized size of antiderivative = 20.41

method	result	size
default	Expression too large to display	3612

```
[In] int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/f/(a+b)^(13/2)*(2*a*b*(a+b)^(7/2)+(a+b)^(7/2)*a^2+(a+b)^(7/2)*b^2+7*a^
2*(1-cos(f*x+e))^2*(a+b)^(7/2)*csc(f*x+e)^2-3*b^2*(1-cos(f*x+e))^2*(a+b)^(7
/2)*csc(f*x+e)^2+(a+b)^(7/2)*a^2*(1-cos(f*x+e))^10*csc(f*x+e)^10+(a+b)^(7/2
)*b^2*(1-cos(f*x+e))^10*csc(f*x+e)^10-12*ln(2/(1-cos(f*x+e))^2*(-a*(1-cos(f
*x+e))^2+b*(1-cos(f*x+e))^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e
))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*cs
c(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2+b*sin(f*x+e)^
2))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1
-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*a^
4*b*(1-cos(f*x+e))^4*csc(f*x+e)^4-108*ln(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+
e))^2+b*(1-cos(f*x+e))^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^
```

```
4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f
*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2+b*sin(f*x+e)^2))
*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-co
s(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*a^3*b
^2*(1-cos(f*x+e))^4*csc(f*x+e)^4-132*ln(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e
))^2+b*(1-cos(f*x+e))^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4
*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*
x+e)^2+a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2+b*sin(f*x+e)^2))*
(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos
(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*a^2*b^
3*(1-cos(f*x+e))^4*csc(f*x+e)^4-48*ln(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))
^2+b*(1-cos(f*x+e))^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*c
sc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+
e)^2+a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2+b*sin(f*x+e)^2))*(a
*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f
*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*a*b^4*(1
-cos(f*x+e))^4*csc(f*x+e)^4-12*ln((a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos
(f*x+e))^2*csc(f*x+e)^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4
*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*
x+e)^2+a+b)^(1/2)*(a+b)^(1/2)-a+b)/(a+b)^(1/2))*(a*(1-cos(f*x+e))^4*csc(f*x
+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b
*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*a^4*b*(1-cos(f*x+e))^4*csc(f*x+e)
^4-108*ln((a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+
(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos
(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^
(1/2)-a+b)/(a+b)^(1/2))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4
*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*
x+e)^2+a+b)^(1/2)*a^3*b^2*(1-cos(f*x+e))^4*csc(f*x+e)^4-132*ln((a*(1-cos(f*
x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+(a*(1-cos(f*x+e))^4*cs
c(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^
2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)-a+b)/(a+b)^(1/2)
)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-c
os(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*a^2*
b^3*(1-cos(f*x+e))^4*csc(f*x+e)^4-48*ln((a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*
(1-cos(f*x+e))^2*csc(f*x+e)^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x
+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*
csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)-a+b)/(a+b)^(1/2))*(a*(1-cos(f*x+e))^4*c
sc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)
^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*a*b^4*(1-cos(f*x+e))^4*csc(
f*x+e)^4+7*(a+b)^(7/2)*a^2*(1-cos(f*x+e))^8*csc(f*x+e)^8-3*(a+b)^(7/2)*b^2*
(1-cos(f*x+e))^8*csc(f*x+e)^8-8*a^2*(1-cos(f*x+e))^6*(a+b)^(7/2)*csc(f*x+e)
^6-14*b^2*(1-cos(f*x+e))^6*(a+b)^(7/2)*csc(f*x+e)^6-8*a^2*(1-cos(f*x+e))^4*
(a+b)^(7/2)*csc(f*x+e)^4-14*b^2*(1-cos(f*x+e))^4*(a+b)^(7/2)*csc(f*x+e)^4+1
2*ln(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))^2+b*(1-cos(f*x+e))^2+(a*(1-cos(f
*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*
```

```
csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)*sin(f
*x+e)^2+a*sin(f*x+e)^2+b*sin(f*x+e)^2))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*
(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(
f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*a^5*(1-cos(f*x+e))^4*csc(f*x+e)^4+12*ln((
a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+(a*(1-cos(f
*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*
csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)-a+b)/
(a+b)^(1/2))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)
^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)
^(1/2)*a^5*(1-cos(f*x+e))^4*csc(f*x+e)^4+4*a*b*(1-cos(f*x+e))^2*(a+b)^(7/2)
*csc(f*x+e)^2+2*(1-cos(f*x+e))^10*a*b*(a+b)^(7/2)*csc(f*x+e)^10+4*(1-cos(f*
x+e))^8*a*b*(a+b)^(7/2)*csc(f*x+e)^8+98*(1-cos(f*x+e))^6*a*b*(a+b)^(7/2)*cs
c(f*x+e)^6+98*a*b*(1-cos(f*x+e))^4*(a+b)^(7/2)*csc(f*x+e)^4)*(a*(1-cos(f*x+
e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc
(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(1-cos(f*x+e))^4*sin(f*x+e
)^4/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^3/((a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b
*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos
(f*x+e))^2*csc(f*x+e)^2+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^2)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(157) = 314$.

Time = 0.42 (sec) , antiderivative size = 873, normalized size of antiderivative = 4.93

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[-\frac{3\left((a^3-4a^2b)\cos(fx+e)^6-(2a^3-9a^2b+4ab^2)\cos(fx+e)^4+a^2b-\right.}{16\left((a^5+4a^4b+6a^3b\right.}\right.$$

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*((a^3 - 4*a^2*b)*cos(f*x + e)^6 - (2*a^3 - 9*a^2*b + 4*a*b^2)*cos
(f*x + e)^4 + a^2*b - 4*a*b^2 + (a^3 - 6*a^2*b + 8*a*b^2)*cos(f*x + e)^2)*s
qrt(a + b)*log(2*(a*cos(f*x + e)^2 + 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) - 2*(3*(a
^3 - 3*a^2*b - 4*a*b^2)*cos(f*x + e)^5 - (5*a^3 - 16*a^2*b - 17*a*b^2 + 4*b
^3)*cos(f*x + e)^3 - (13*a^2*b + 11*a*b^2 - 2*b^3)*cos(f*x + e))*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3
+ a*b^4)*f*cos(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*
b^4 - b^5)*f*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*
b^4 - 2*b^5)*f*cos(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 +
b^5)*f), 1/8*(3*((a^3 - 4*a^2*b)*cos(f*x + e)^6 - (2*a^3 - 9*a^2*b + 4*a*b^
```

```
2)*cos(f*x + e)^4 + a^2*b - 4*a*b^2 + (a^3 - 6*a^2*b + 8*a*b^2)*cos(f*x + e
)^2)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)*cos(f*x + e)/(a + b)) + (3*(a^3 - 3*a^2*b - 4*a*b^2)*cos(f*x + e)^5
- (5*a^3 - 16*a^2*b - 17*a*b^2 + 4*b^3)*cos(f*x + e)^3 - (13*a^2*b + 11*a*b
^2 - 2*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^
5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*f*cos(f*x + e)^6 - (2*a^5 + 7*
a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*f*cos(f*x + e)^4 + (a^5 + 2*
a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*f*cos(f*x + e)^2 + (a^4*b
+ 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f)]
```

Sympy [F]

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(csc(e + f*x)**5/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc(fx+e)^5}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{1}{\sin(e+f\,x)^5\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2)),x)
[Out] int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

3.112 $\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}} dx$

Optimal result

Integrand size = 25, antiderivative size = 242

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}} dx = \frac{5(a+b)^2(a+7b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{9/2}f} - \frac{(a+b)(33a+35b)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(9a+7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{b(81a^2+190ab+105b^2)\tan(e+fx)}{48a^4f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] 5/16*(a+b)^2*(a+7*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/
a^(9/2)/f-1/48*(a+b)*(33*a+35*b)*cos(f*x+e)*sin(f*x+e)/a^3/f/(a+b+b*tan(f*x
+e)^2)^(1/2)+1/24*(9*a+7*b)*cos(f*x+e)^3*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)
^2)^(1/2)+1/6*cos(f*x+e)^3*sin(f*x+e)^3/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/48
*b*(81*a^2+190*a*b+105*b^2)*tan(f*x+e)/a^4/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used

$= \{4217, 481, 592, 541, 12, 385, 209\}$

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{5(a+b)^2(a+7b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{9/2}f} - \frac{(a+b)(33a+35b)\sin(e+fx)\cos(e+fx)}{48a^3f\sqrt{a+b\tan^2(e+fx)+b}} + \frac{(9a+7b)\sin(e+fx)\cos^3(e+fx)}{24a^2f\sqrt{a+b\tan^2(e+fx)+b}} - \frac{b(81a^2+190ab+105b^2)\tan(e+fx)}{48a^4f\sqrt{a+b\tan^2(e+fx)+b}} + \frac{\sin^3(e+fx)\cos^3(e+fx)}{6af\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[In] Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (5*(a + b)^2*(a + 7*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
 f*x]^2]])/(16*a^(9/2)*f) - ((a + b)*(33*a + 35*b)*Cos[e + f*x]*Sin[e + f*x
])/(48*a^3*f*Sqrt[a + b + b*Tan[e + f*x]^2]) + ((9*a + 7*b)*Cos[e + f*x]^3*
Sin[e + f*x])/(24*a^2*f*Sqrt[a + b + b*Tan[e + f*x]^2]) + (Cos[e + f*x]^3*S
in[e + f*x]^3)/(6*a*f*Sqrt[a + b + b*Tan[e + f*x]^2]) - (b*(81*a^2 + 190*a*
b + 105*b^2)*Tan[e + f*x])/(48*a^4*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^4(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)+2(b-3(a+b))x^2)}{(1+x^2)^3(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{6af}$$

$$= \frac{(9a+7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{(a+b)(9a+7b)-4(a+b)(6a+7b)x^2}{(1+x^2)^2(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{24a^2f}$$

$$= -\frac{(a+b)(33a+35b)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(9a+7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{(a+b)(15a^2+54ab+35b^2)-2b(a+b)(33a+35b)x^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{48a^3f}$$

$$= -\frac{(a+b)(33a+35b)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(9a+7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{b(81a^2+190ab+105b^2)\tan(e+fx)}{48a^4f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{15(a+b)^3(a+7b)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{48a^4(a+b)f}$$

$$= -\frac{(a+b)(33a+35b)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(9a+7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{b(81a^2+190ab+105b^2)\tan(e+fx)}{48a^4f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5(a+b)^2(a+7b))\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{16a^4f}$$

$$= -\frac{(a+b)(33a+35b)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(9a+7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{b(81a^2+190ab+105b^2)\tan(e+fx)}{48a^4f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5(a+b)^2(a+7b))\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^4f}$$

$$= \frac{5(a+b)^2(a+7b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{9/2}f} - \frac{(a+b)(33a+35b)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(9a+7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{b(81a^2+190ab+105b^2)\tan(e+fx)}{48a^4f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 6.30 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.06

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(120(a+b)^2(a+7b)\arcsin\left(\frac{\sqrt{a}\sin}{\sqrt{a}}\right.\right.}{}$$

```
[In] Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(120*(a + b)^2*(a + 7*b)*Arc
Sin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - 2*
Sqrt[2]*Sqrt[a]*Sqrt[a + b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*(3
7*a^3 + 439*a^2*b + 830*a*b^2 + 420*b^3 + a*(29*a^2 + 108*a*b + 70*b^2)*Cos
[2*(e + f*x)] - 7*a^2*(a + b)*Cos[4*(e + f*x)] + a^3*Cos[6*(e + f*x)])*Sin[
e + f*x]))/(1536*a^(9/2)*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a
 + b - a*Sin[e + f*x]^2)/(a + b)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1161 vs. 2(218) = 436.

Time = 9.33 (sec) , antiderivative size = 1162, normalized size of antiderivative = 4.80

method	result	size
default	Expression too large to display	1162

```
[In] int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/f/a^4/(-a)^(1/2)*(b+a*cos(f*x+e)^2)*(8*(-a)^(1/2)*a^3*cos(f*x+e)^6*si
n(f*x+e)-26*(-a)^(1/2)*a^3*cos(f*x+e)^4*sin(f*x+e)-14*(-a)^(1/2)*a^2*b*cos(
f*x+e)^4*sin(f*x+e)+33*(-a)^(1/2)*a^3*cos(f*x+e)^2*sin(f*x+e)+68*(-a)^(1/2)
*a^2*b*cos(f*x+e)^2*sin(f*x+e)+35*(-a)^(1/2)*a*b^2*cos(f*x+e)^2*sin(f*x+e)-
15*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4
*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*cos(f*x+e)-135*ln(4*(-a)^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*a^2*b*cos(f*x+e)-225*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*a*b^2*cos(f*x+e)-105*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3*cos(f
*x+e)+81*(-a)^(1/2)*a^2*b*sin(f*x+e)+190*(-a)^(1/2)*a*b^2*sin(f*x+e)+105*(-
```

```
a)^(1/2)*b^3*sin(f*x+e)-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4
*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3-135*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2*b-225*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f
*x+e)*a)*a*b^2-105*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^3)/(a+b*sec(f*x+
e)^2)^(3/2)*sec(f*x+e)^3
```

Fricas [A] (verification not implemented)

none

Time = 5.91 (sec) , antiderivative size = 813, normalized size of antiderivative = 3.36

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[-\frac{15\left(a^3b+9\,a^2b^2+15\,ab^3+7\,b^4+\left(a^4+9\,a^3b+15\,a^2b^2+7\,ab^3\right)\cos\left(fx+\right.\right.}{}\right.$$

$$-\frac{15\left(a^3b+9\,a^2b^2+15\,ab^3+7\,b^4+\left(a^4+9\,a^3b+15\,a^2b^2+7\,ab^3\right)\cos\left(fx+e\right)^2\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\,(\right.}{4\left(2\,a^3\cos\right.}\right.}{}$$

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(15*(a^3*b + 9*a^2*b^2 + 15*a*b^3 + 7*b^4 + (a^4 + 9*a^3*b + 15*a^2
*b^2 + 7*a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(
a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e
)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*
a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*
b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 -
7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^4*cos(f*x + e)^7 - 2*(13*a^4 + 7*a^
3*b)*cos(f*x + e)^5 + (33*a^4 + 68*a^3*b + 35*a^2*b^2)*cos(f*x + e)^3 + (81
*a^3*b + 190*a^2*b^2 + 105*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e))/(a^6*f*cos(f*x + e)^2 + a^5*b*f), -1/192*(15
```

```
*(a^3*b + 9*a^2*b^2 + 15*a*b^3 + 7*b^4 + (a^4 + 9*a^3*b + 15*a^2*b^2 + 7*a*
b^3)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*
b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^
3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(8*a^4*cos(f*x + e)^7 - 2*(
13*a^4 + 7*a^3*b)*cos(f*x + e)^5 + (33*a^4 + 68*a^3*b + 35*a^2*b^2)*cos(f*x
 + e)^3 + (81*a^3*b + 190*a^2*b^2 + 105*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^6*f*cos(f*x + e)^2 + a^5*b*f
)]
```

Sympy [F]

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin^6\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(sin(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(fx+e\right)^6}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(fx+e\right)^6}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(e+f\,x\right)^6}{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}}\,dx$$

```
[In] int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.113 $\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 175

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{3(a+b)(a+5b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{7/2}f} - \frac{5(a+b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{b(13a+15b)\tan(e+fx)}{8a^3f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] 3/8*(a+b)*(a+5*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(
7/2)/f-5/8*(a+b)*cos(f*x+e)*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/4
*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/8*b*(13*a+15*b)*t
an(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4217, 481, 541, 12, 385, 209}

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{3(a+b)(a+5b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{7/2}f} - \frac{b(13a+15b)\tan(e+fx)}{8a^3f\sqrt{a+b\tan^2(e+fx)+b}} - \frac{5(a+b)\sin(e+fx)\cos(e+fx)}{8a^2f\sqrt{a+b\tan^2(e+fx)+b}} + \frac{\sin(e+fx)\cos^3(e+fx)}{4af\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[In] Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a + b)*(a + 5*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f
*x]^2]])/(8*a^(7/2)*f) - (5*(a + b)*Cos[e + f*x]*Sin[e + f*x])/(8*a^2*f*Sqr
t[a + b + b*Tan[e + f*x]^2]) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f*Sqrt[a
+ b + b*Tan[e + f*x]^2]) - (b*(13*a + 15*b)*Tan[e + f*x])/(8*a^3*f*Sqrt[a +
 b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^3(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{a+b-4(a+b)x^2}{(1+x^2)^2(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= -\frac{5(a+b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{(a+b)(3a+5b)-10b(a+b)x^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{8a^2f}$$

$$= -\frac{5(a+b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{b(13a+15b)\tan(e+fx)}{8a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{3(a+b)^2(a+5b)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{8a^3(a+b)f}$$

$$= -\frac{5(a+b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{b(13a+15b)\tan(e+fx)}{8a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3(a+b)(a+5b))\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{8a^3f}$$

$$= -\frac{5(a+b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{b(13a+15b)\tan(e+fx)}{8a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3(a+b)(a+5b))\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^3f}$$

$$= \frac{3(a+b)(a+5b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{7/2}f} - \frac{5(a+b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{b(13a+15b)\tan(e+fx)}{8a^3f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 3.33 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.31

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(24(a^2+6ab+5b^2)\arcsin\left(\frac{\sqrt{a}\sin(e+}{\sqrt{a+b}}\right.\right.}{}$$

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(24*(a^2 + 6*a*b + 5*b^2)*Ar
cSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - 2
*Sqrt[2]*Sqrt[a]*Sqrt[a + b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*(
7*a^2 + 62*a*b + 60*b^2 + 2*a*(3*a + 5*b)*Cos[2*(e + f*x)] - a^2*Cos[4*(e +
 f*x)])*Sin[e + f*x]))/(256*a^(7/2)*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)^(3
/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 832 vs. $2(155) = 310$.

Time = 7.29 (sec) , antiderivative size = 833, normalized size of antiderivative = 4.76

method	result
default	$\frac{\left(b+a\cos(fx+e)^2\right)\left(2\cos(fx+e)^4\sin(fx+e)\sqrt{-a}\,a^2-5\cos(fx+e)^2\sin(fx+e)\sqrt{-a}\,a^2-5\sqrt{-a}\,ab\cos(fx+e)^2\sin(fx+e)+3\cos(fx+e)\right.}{}$

```
[In] int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f/a^3/(-a)^(1/2)*(b+a*cos(f*x+e)^2)*(2*cos(f*x+e)^4*sin(f*x+e)*(-a)^(1/
2)*a^2-5*cos(f*x+e)^2*sin(f*x+e)*(-a)^(1/2)*a^2-5*(-a)^(1/2)*a*b*cos(f*x+e)
^2*sin(f*x+e)+3*cos(f*x+e)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+18*ln(
4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^
(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b*cos(f*x+e)+15*ln(4*(-a)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x
```

```
+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*b^2*cos(f*x+e)-13*(-a)^(1/2)*a*b*sin(f*x+e)-15*(-a)^(1/2)*b
^2*sin(f*x+e)+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2+18*ln(4*(-a)^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*a*b+15*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2)/(
a+b*sec(f*x+e)^2)^(3/2)*sec(f*x+e)^3
```

Fricas [A] (verification not implemented)

none

Time = 1.83 (sec) , antiderivative size = 703, normalized size of antiderivative = 4.02

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[-\frac{3\left(a^2b+6\,ab^2+5\,b^3+\left(a^3+6\,a^2b+5\,ab^2\right)\cos\left(fx+e\right)^2\right)\sqrt{-a}\log\left(128\,a\right.}{}\right.$$

$$-\frac{3\left(a^2b+6\,ab^2+5\,b^3+\left(a^3+6\,a^2b+5\,ab^2\right)\cos\left(fx+e\right)^2\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\left(a^2-ab\right)\cos(fx+e)^3+(a^2-\right.}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-\right.}\right.}{}$$

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(3*(a^2*b + 6*a*b^2 + 5*b^3 + (a^3 + 6*a^2*b + 5*a*b^2)*cos(f*x + e)
^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6
+ 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^
2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e
)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3
 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos
(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
)) - 8*(2*a^3*cos(f*x + e)^5 - 5*(a^3 + a^2*b)*cos(f*x + e)^3 - (13*a^2*b +
 15*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e))/(a^5*f*cos(f*x + e)^2 + a^4*b*f), -1/32*(3*(a^2*b + 6*a*b^2 + 5*b^3
 + (a^3 + 6*a^2*b + 5*a*b^2)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(
f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e
```

```
))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)
^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2*
a^3*cos(f*x + e)^5 - 5*(a^3 + a^2*b)*cos(f*x + e)^3 - (13*a^2*b + 15*a*b^2)
*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a
^5*f*cos(f*x + e)^2 + a^4*b*f)]
```

Sympy [F]

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin^4{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(sin(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(fx+e)^4}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(fx+e)^4}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(e+f\,x)^4}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.114 $\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 121

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a+3b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{5/2}f} - \frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{3b\tan(e+fx)}{2a^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] 1/2*(a+3*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f
-1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)-3/2*b*tan(f*x+e)/
a^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4217, 482, 541, 12, 385, 209}

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a+3b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{5/2}f} - \frac{3b\tan(e+fx)}{2a^2f\sqrt{a+b\tan^2(e+fx)+b}} - \frac{\sin(e+fx)\cos(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[In] Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 3*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(
2*a^(5/2)*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*Sqrt[a + b + b*Tan[e + f*
x]^2]) - (3*b*Tan[e + f*x])/(2*a^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a+b-2bx^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{3b\tan(e+fx)}{2a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{(a+b)(a+3b)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2a^2(a+b)f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{3b\tan(e+fx)}{2a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a+3b)\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2a^2f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{3b\tan(e+fx)}{2a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a+3b)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^2f}$$

$$= \frac{(a+3b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{5/2}f} - \frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{3b\tan(e+fx)}{2a^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.57

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(4(a+3b)\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a+\right.}{32a^{5/2}\sqrt{a+b}f}$$

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(4*(a + 3*b)*ArcSin[(Sqrt[a]
*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - 2*Sqrt[2]*Sqrt
[a]*Sqrt[a + b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*(a + 6*b + a*C
os[2*(e + f*x)])*Sin[e + f*x]))/(32*a^(5/2)*Sqrt[a + b]*f*(a + b*Sec[e + f*
x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(105) = 210$.

Time = 5.31 (sec) , antiderivative size = 532, normalized size of antiderivative = 4.40

method	result
default	$-\frac{\left(b+a\cos(fx+e)^2\right)\left(\cos(fx+e)^2\sin(fx+e)\sqrt{-a}\,a-\cos(fx+e)\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\right.\right.}{}$

```
[In] int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/2/f/a^2/(-a)^(1/2)*(b+a*cos(f*x+e)^2)*(cos(f*x+e)^2*sin(f*x+e)*(-a)^(1/2
)*a-cos(f*x+e)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f
*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a-3*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)-4*sin(f*x+e)*a)*b*cos(f*x+e)+3*(-a)^(1/2)*b*sin(f*x+e)-((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)-4*sin(f*x+e)*a)*a-3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*
(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b)/(a+b*sec
(f*x+e)^2)^(3/2)*sec(f*x+e)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(105) = 210$.

Time = 0.67 (sec) , antiderivative size = 607, normalized size of antiderivative = 5.02

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \left[-\frac{\left(\left(a^2+3\,ab\right)\cos\left(fx+e\right)^2+ab+3\,b^2\right)\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8-2\right.}{}\right.$$

$$-\frac{\left(\left(a^2+3\,ab\right)\cos\left(fx+e\right)^2+ab+3\,b^2\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\left(a^2-ab\right)\cos(fx+e)^3+\left(a^2-6\,ab+b^2\right)\cos(fx+e)\right)\sqrt{a}\sqrt{}}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-\left(a^3-3\,a^2b\right)\cos(fx+e)^2\right)\sin(fx}\right.}{8\left(a^4f\cos\left(fx+e\right)^2+a^3b\right.}$$

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((a^2 + 3*a*b)*cos(f*x + e)^2 + a*b + 3*b^2)*sqrt(-a)*log(128*a^4*c
os(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5
*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 3
2*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x +
e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos
(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(a^2*cos(f*x + e)^3
 + 3*a*b*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
+ e))/(a^4*f*cos(f*x + e)^2 + a^3*b*f), -1/8*(((a^2 + 3*a*b)*cos(f*x + e)^2
 + a*b + 3*b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*co
s(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3
*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(a^2*cos(f*x + e)^3 + 3*a*b*cos(
f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^4*f*
cos(f*x + e)^2 + a^3*b*f)]
```

Sympy [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin^2{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(sin(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin{(fx+e)}^2}{(b\sec{(fx+e)}^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(e+f\,x\right)^2}{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}}\,dx$$

```
[In] int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.115 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 77

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f}-\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*tan(f*x+e
)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4213, 390, 385, 209}

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{b\tan(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f) -
 (b*Tan[e + f*x])/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{af}$$

$$= -\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{af}$$

$$= \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 168 vs. 2(77) = 154.

Time = 1.05 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.18

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(\sqrt{a+b}\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a+2b\right.}{4a^{3/2}(a+b)f\,(a+b\sec^2(e+fx))}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(Sqrt[a + b]*ArcSin[(Sqrt[a]
*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - Sqrt[2]*Sqrt[a
]*b*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(4*a^(3/2)*
(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a +
 b)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. 2(69) = 138.

Time = 3.69 (sec) , antiderivative size = 515, normalized size of antiderivative = 6.69

method	result
default	$-\frac{\left(b+a\cos(fx+e)^2\right)\left(-\cos(fx+e)\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)\sqrt{\frac{b+a\cos(fx+e}{(1+\cos(fx+e)}}\right.}{}$

```
[In] int(1/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f/(a+b)/a/(-a)^(1/2)*(b+a*cos(f*x+e)^2)*(-cos(f*x+e)*ln(4*(-a)^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*a-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^
(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b*cos(f*x+e)+(-a
)^(1/2)*b*sin(f*x+e)-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^
(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a-((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)-4*sin(f*x+e)*a)*b)/(a+b*sec(f*x+e)^2)^(3/2)*sec(f*x+e)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(69) = 138.

Time = 0.43 (sec) , antiderivative size = 601, normalized size of antiderivative = 7.81

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \left[-\frac{8\,ab\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(\left(a^2+ab\right)\cos\left(fx+e\right)^2+\right.}{}\right.$$

$$-\frac{4\,ab\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(\left(a^2+ab\right)\cos\left(fx+e\right)^2+ab+b^2\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e\right.}{4\left(\right.}\right.}{4\left(\left(a^4+a^3b\right)f\cos\left(fx+e\right)^2+\left(a^3b+a^2b^2\right)f\right)}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(8*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f
*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*sqrt(-a)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/((a^4 + a^3*b)*f*cos(f*x
 + e)^2 + (a^3*b + a^2*b^2)*f), -1/4*(4*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b
+ b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e
)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*c
os(f*x + e)^2)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a
^2*b^2)*f)]
```

Sympy [F]

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \int \frac{1}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2055 vs. $2(69) = 138$.

Time = 0.54 (sec) , antiderivative size = 2055, normalized size of antiderivative = 26.69

$$\int \frac{1}{(a + b\sec^2(e + fx))^{3/2}} \, dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*b*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*
e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))*sin(2*f*x + 2*e
) - 2*(a^2 + a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*
x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^3 - 2*(a*
b*cos(2*f*x + 2*e) + (a^2 + a*b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a
+ 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e)
+ a))^2 - a^2 - 2*a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin
(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) - (a
^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*co
s(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a
^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos
(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*(
((a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e),
 a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a + b)*sin(1/
2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x +
4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*a*sin(2*f*x + 2*e) +
 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^
2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b
)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1
/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*
e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*a*cos(2*f*x
+ 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*
b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x
+ 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2
 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x +
2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*
f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 2*a +
 4*b) - ((a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x
 + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a + b
)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(
4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*(a^2*cos(4*f
*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x +
2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b
 + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2
```

```
*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin
(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x
 + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f*x + 4*e)^2 + a
^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2
 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2
*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x
+ 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*
sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a
+ 2*b)*cos(2*f*x + 2*e) + a)) + 4*a + 4*b))*sqrt(a))/((a^2*cos(4*f*x + 4*e)
^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 +
4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)
*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(
4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*((a^3 + a^2*b)*cos(1
/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x +
 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a^3 + a^2*b)*sin(1/2*arctan
2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2
*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*f)
```

Giac [F]

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=\int \frac{1}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(1/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(1/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.116 $\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 68

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\cot(e+fx)}{(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{2b\tan(e+fx)}{(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] -cot(f*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-2*b*tan(f*x+e)/(a+b)^2/f/(a+
b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {4217, 277, 197}

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{2b\tan(e+fx)}{f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}} - \frac{\cot(e+fx)}{f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[In] Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Cot[e + f*x]/((a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])) - (2*b*Tan[e + f
*x])/((a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)}{(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(2b)\text{Subst}\left(\int \frac{1}{(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{(a+b)f}$$

$$= -\frac{\cot(e+fx)}{(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{2b\tan(e+fx)}{(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))(a+3b+(a-b)\cos(2(e+fx)))\csc(e+fx)\sec^3(e+fx)}{4(a+b)^2f\,(a+b\sec^2(e+fx))^{3/2}}$$

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] -1/4*((a + 2*b + a*Cos[2*(e + f*x)])*(a + 3*b + (a - b)*Cos[2*(e + f*x)])*C
sc[e + f*x]*Sec[e + f*x]^3)/((a + b)^2*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [A] (verified)

Time = 3.99 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.24

method	result	size
default	$-\frac{\left(b+a\cos(fx+e)^{2}\right)\left(a\cos(fx+e)^{2}-\cos(fx+e)^{2}b+2b\right)\sec(fx+e)^{3}\csc(fx+e)}{f(a^{2}+2ab+b^{2})\left(a+b\sec(fx+e)^{2}\right)^{\frac{3}{2}}}$	84

[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/f/(a^2+2*a*b+b^2)*(b+a*cos(f*x+e)^2)*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+2*b)
/(a+b*sec(f*x+e)^2)^(3/2)*sec(f*x+e)^3*csc(f*x+e)

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.50

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\left((a-b)\cos(fx+e)^{3}+2\,b\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^{2}+b}{\cos(fx+e)^{2}}}}{\left((a^{3}+2\,a^{2}b+ab^{2})f\cos(fx+e)^{2}+(a^{2}b+2\,ab^{2}+b^{3})f\right)\sin(fx+e)}$$

[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] -((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)/(((a^3 + 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 + (a^2*b + 2*a*b^2
 + b^3)*f)*sin(f*x + e))

Sympy [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc^{2}\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)

[Out] Integral(csc(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.94

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\frac{2\,b\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^2}+\frac{1}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)\tan(fx+e)}}{f}$$

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(2*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) + 1/(sqrt(b*t
an(f*x + e)^2 + a + b)*(a + b)*tan(f*x + e)))/f
```

Giac [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc(fx+e)^2}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 26.10 (sec) , antiderivative size = 2151, normalized size of antiderivative = 31.63

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

```
[Out] -((a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*(2*exp(e*
2i + f*x*2i) + exp(e*4i + f*x*4i) + 1)*(exp(e*4i + f*x*4i)*(((a + 4*b)*(((a
 + 4*b)*((((a^2*(a + 3*b)*(a*(a - b) - (a + 3*b)^2))/(a*b + a^2) + (a*(a +
3*b)^2*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2))*1i)/(8*f*(a*b^2 + a^2*b)*(
a + 3*b)) - (a^3*(a + 3*b)*3i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*b)) + (a^2*(a
+ 3*b)*(a + 4*b)*1i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*b))))/a - (((a + 3*b)^3
- ((a + 3*b)*(a*(a - b) - (a + 3*b)^2)*(a*(a + 3*b) - a*(a + 4*b)))/(a*b +
a^2))*1i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) - (((a^2*(a + 3*b)*(a*(a - b) - (
a + 3*b)^2))/(a*b + a^2) + (a*(a + 3*b)^2*(a*(a + 3*b) - a*(a + 4*b)))/(a*b
 + a^2))*3i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) + (a^3*(a + 3*b)*3i)/(8*f*(a*b
 + a^2)*(a*b^2 + a^2*b)) + (a^2*(a + 3*b)*(a + 4*b)*1i)/(8*f*(a*b + a^2)*(a
```

```
*b^2 + a^2*b))))/a + ((a + 4*b)*((((a^2*(a + 3*b)*(a*(a - b) - (a + 3*b)^2)
)/(a*b + a^2) + (a*(a + 3*b)^2*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2))*1i
)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) - (a^3*(a + 3*b)*3i)/(8*f*(a*b + a^2)*(a*
b^2 + a^2*b)) + (a^2*(a + 3*b)*(a + 4*b)*1i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*
b))))/a + (((a + 3*b)^3 - ((a + 3*b)*(a*(a - b) - (a + 3*b)^2)*(a*(a + 3*b)
 - a*(a + 4*b)))/(a*b + a^2))*3i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) + (((a^2*
(a + 3*b)*(a*(a - b) - (a + 3*b)^2))/(a*b + a^2) + (a*(a + 3*b)^2*(a*(a + 3
*b) - a*(a + 4*b)))/(a*b + a^2))*3i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) - (a^3
*(a + 3*b)*1i)/(4*f*(a*b + a^2)*(a*b^2 + a^2*b))) + exp(e*2i + f*x*2i)*((((
a + 3*b)^3 - ((a + 3*b)*(a*(a - b) - (a + 3*b)^2)*(a*(a + 3*b) - a*(a + 4*b
)))/(a*b + a^2))*3i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) - ((a + 4*b)*(((a + 4*
b)*((((a^2*(a + 3*b)*(a*(a - b) - (a + 3*b)^2))/(a*b + a^2) + (a*(a + 3*b)^
2*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2))*1i)/(8*f*(a*b^2 + a^2*b)*(a + 3
*b)) - (a^3*(a + 3*b)*3i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*b)) + (a^2*(a + 3*b
)*(a + 4*b)*1i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*b))))/a - (((a + 3*b)^3 - ((a
 + 3*b)*(a*(a - b) - (a + 3*b)^2)*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2))
*1i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) - (((a^2*(a + 3*b)*(a*(a - b) - (a + 3
*b)^2))/(a*b + a^2) + (a*(a + 3*b)^2*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^
2))*3i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) + (a^3*(a + 3*b)*3i)/(8*f*(a*b + a^
2)*(a*b^2 + a^2*b)) + (a^2*(a + 3*b)*(a + 4*b)*1i)/(8*f*(a*b + a^2)*(a*b^2
+ a^2*b))))/a + (((a^2*(a + 3*b)*(a*(a - b) - (a + 3*b)^2))/(a*b + a^2) + (
a*(a + 3*b)^2*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2))*1i)/(4*f*(a*b^2 + a
^2*b)*(a + 3*b)) - (a^3*(a + 3*b)*3i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*b)) + (
a^2*(a + 3*b)*(a + 4*b)*1i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*b))) - ((a + 4*b)
*((((a^2*(a + 3*b)*(a*(a - b) - (a + 3*b)^2))/(a*b + a^2) + (a*(a + 3*b)^2*
(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2))*1i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b
)) - (a^3*(a + 3*b)*3i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*b)) + (a^2*(a + 3*b)*
(a + 4*b)*1i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*b))))/a + (((a + 3*b)^3 - ((a +
 3*b)*(a*(a - b) - (a + 3*b)^2)*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2))*1
i)/(4*f*(a*b^2 + a^2*b)*(a + 3*b)) + (((a^2*(a + 3*b)*(a*(a - b) - (a + 3*b
)^2))/(a*b + a^2) + (a*(a + 3*b)^2*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2)
)*3i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) - (a^3*(a + 3*b)*3i)/(8*f*(a*b + a^2)
*(a*b^2 + a^2*b)) - (a^2*(a + 3*b)*(a + 4*b)*1i)/(8*f*(a*b + a^2)*(a*b^2 +
a^2*b))))/((exp(e*2i + f*x*2i) + 1)*(a - a*exp(e*6i + f*x*6i) + exp(e*2i +
f*x*2i)*(a + 4*b) - exp(e*4i + f*x*4i)*(a + 4*b)))
```

3.117 $\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 123

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = -\frac{(3a-b)\cot(e+fx)}{3(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\cot^3(e+fx)}{3(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{2(3a-b)b\tan(e+fx)}{3(a+b)^3f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] -1/3*(3*a-b)*cot(f*x+e)/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*cot(f*x+e)
^3/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-2/3*(3*a-b)*b*tan(f*x+e)/(a+b)^3/f/(a
+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4217, 464, 277, 197}

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = -\frac{2b(3a-b)\tan(e+fx)}{3f(a+b)^3\sqrt{a+b\tan^2(e+fx)+b}} - \frac{\cot^3(e+fx)}{3f(a+b)\sqrt{a+b\tan^2(e+fx)+b}} - \frac{(3a-b)\cot(e+fx)}{3f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[In] Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/3*((3*a - b)*Cot[e + f*x])/((a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
- Cot[e + f*x]^3/(3*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2]) - (2*(3*a - b
)*b*Tan[e + f*x])/(3*(a + b)^3*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)}{3(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a-b)\text{Subst}\left(\int \frac{1}{x^2(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3(a+b)f}$$

$$= -\frac{(3a-b)\cot(e+fx)}{3(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\cot^3(e+fx)}{3(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(2(3a-b)b)\text{Subst}\left(\int \frac{1}{(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3(a+b)^2f}$$

$$= -\frac{(3a-b)\cot(e+fx)}{3(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\cot^3(e+fx)}{3(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{2(3a-b)b\tan(e+fx)}{3(a+b)^3f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.83

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left(-2a(a-3b)+(a^2-2ab-3b^2)\csc^2(e+fx)+(a+b)^2\csc^4(e+fx)\right)\sec^2(e}{6(a+b)^3f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] -1/6*((a + 2*b + a*Cos[2*(e + f*x)])*(-2*a*(a - 3*b) + (a^2 - 2*a*b - 3*b^2
)*Csc[e + f*x]^2 + (a + b)^2*Csc[e + f*x]^4)*Sec[e + f*x]^2*Tan[e + f*x])/(
(a + b)^3*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [A] (verified)

Time = 4.53 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.15

method	result
default	$\frac{\left(b+a\cos(fx+e)^2\right)\left(2\cos(fx+e)^4a^2-6\cos(fx+e)^4ab-3\cos(fx+e)^2a^2+10\cos(fx+e)^2ab-3\cos(fx+e)^2b^2-6ab+2b^2\right)\sec(fx+e)^3}{3f(a^3+3a^2b+3a\,b^2+b^3)\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}$

```
[In] int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/3/f/(a^3+3*a^2*b+3*a*b^2+b^3)*(b+a*cos(f*x+e)^2)*(2*cos(f*x+e)^4*a^2-6*co
s(f*x+e)^4*a*b-3*cos(f*x+e)^2*a^2+10*cos(f*x+e)^2*a*b-3*cos(f*x+e)^2*b^2-6*
a*b+2*b^2)/(a+b*sec(f*x+e)^2)^(3/2)*sec(f*x+e)^3*csc(f*x+e)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.67 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.54

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\left(2\left(a^2-3ab\right)\cos\left(fx+e\right)^5-\left(3a^2-10ab+3b^2\right)\cos\left(fx+e\right)^3-2\left(3ab-b^2\right)\cos\left(fx+e\right)\right)\sqrt{}}{3\left(\left(a^4+3a^3b+3a^2b^2+ab^3\right)f\cos\left(fx+e\right)^4-\left(a^4+2a^3b-2ab^3-b^4\right)f\cos\left(fx+e\right)^2-\left(a^3b+3a^2b^2+3\right.\right.}$$

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] -1/3*(2*(a^2 - 3*a*b)*cos(f*x + e)^5 - (3*a^2 - 10*a*b + 3*b^2)*cos(f*x + e
)^3 - 2*(3*a*b - b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
```

```
)^2)/(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2*a^3*
b - 2*a*b^3 - b^4)*f*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f
)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(csc(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.27

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\frac{6\,b\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^2} - \frac{8\,b^2\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^3} + \frac{3}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)\tan(fx+e)} - \frac{4\,b}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^2\tan(fx}}{3\,f}$$

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] -1/3*(6*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) - 8*b^2*t
an(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3) + 3/(sqrt(b*tan(f*x
+ e)^2 + a + b)*(a + b)*tan(f*x + e)) - 4*b/(sqrt(b*tan(f*x + e)^2 + a + b)
*(a + b)^2*tan(f*x + e)) + 1/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(f*
x + e)^3))/f
```

Giac [F]

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc(fx+e)^4}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 40.46 (sec) , antiderivative size = 124682, normalized size of antiderivative = 1013.67

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\text{Too large to display}$$

```
[In] int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

```
[Out] (a^2*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*128i)
/(3*(32*a^4*f + 32*b^4*f + 192*a^2*b^2*f + 128*a*b^3*f + 128*a^3*b*f - 32*a
^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*a^4*f*(cos(
4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 32*a^4*f*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 32*b^4*f*(cos(2*f*x) + sin(2*f*x)*
1i)*(cos(2*e) + sin(2*e)*1i) - 32*b^4*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4
*e) + sin(4*e)*1i) + 32*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - 128*a*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
) - 128*a^3*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 128
*a*b^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) - 128*a^3*b*
f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 128*a*b^3*f*(cos(
6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) + 128*a^3*b*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 192*a^2*b^2*f*(cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 192*a^2*b^2*f*(cos(4*f*x) + sin(4*f*
x)*1i)*(cos(4*e) + sin(4*e)*1i) + 192*a^2*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i))) - (5*a*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(
cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(
2*e)*1i))/4 + 1/2))^(1/2))/(3*(a^3*f*1i + b^3*f*1i + a*b^2*f*3i + a^2*b*f*3
i - a^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*1i - b^3*f*
(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*1i - a*b^2*f*(cos(4*f
*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*3i - a^2*b*f*(cos(4*f*x) + si
n(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*3i)) - (5*b*(a + b/(((cos(2*f*x) - si
n(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2))/(3*(a^3*f*1i + b^3*f*1i + a*b^2*f*
3i + a^2*b*f*3i - a^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1
i)*1i - b^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*1i - a*
b^2*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*3i - a^2*b*f*(c
os(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*3i)) - (a^8*(a + b/(((c
os(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1i)/(4*(a^3*b^7*f + 6
*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9
*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*
a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(co
s(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x
) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + si
```

n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - ((a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*32i)/(24*a^2*f + 24*b^2*f + 48*a*b*f - 48*a^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 48*a^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 24*a^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 48*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 48*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 24*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 96*a*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 96*a*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 48*a*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) + (b^2*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*128i)/(3*(32*a^4*f + 32*b^4*f + 192*a^2*b^2*f + 128*a*b^3*f + 128*a^3*b*f - 32*a^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*a^4*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 32*a^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 32*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*b^4*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 32*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 128*a*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 128*a^3*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 128*a*b^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) - 128*a^3*b*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 128*a*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) + 128*a^3*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 192*a^2*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 192*a^2*b^2*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 192*a^2*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i))) + (a^7*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*f*

x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1i)/(4*(3*a^2*b^7*f + 16*a
^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b
*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a
^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(
cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f
*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i
)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e
) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
- 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*
a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b
^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*
(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6
*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8
*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*
1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(c
os(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e
) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + si
n(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*
1i))) - ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((co
s(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(
2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*64i)/(24*a^2*f + 24*b^
2*f + 48*a*b*f - 48*a^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 48*a^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 24*
a^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 48*b^2*f*(cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 48*b^2*f*(cos(6*f*x) +
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 24*b^2*f*(cos(8*f*x) + sin(8*f*x)
1i)(cos(8*e) + sin(8*e)*1i) - 96*a*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(
2*e) + sin(2*e)*1i) + 96*a*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin
(6*e)*1i) - 48*a*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))
 - ((cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f
*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x
)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*32i)/(24*a^2*f + 24*b^2*f +
 48*a*b*f - 48*a^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
+ 48*a^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 24*a^2*f
*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 48*b^2*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 48*b^2*f*(cos(6*f*x) + sin(6
*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 24*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*
(cos(8*e) + sin(8*e)*1i) - 96*a*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
+ sin(2*e)*1i) + 96*a*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i) - 48*a*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) - (a

```
^7*b*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*5i)/(
3*(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6
*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1
i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*
b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2
*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x)
 + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)
*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6
*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*
e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i
) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 5
4*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*
b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(co
s(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x
) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + si
n(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*
x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)
*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8
*e) + sin(8*e)*1i))) - (a^2*b^7*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(
2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i))/4 + 1/2))^(1/2)*117i)/(4*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f +
 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^
10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(
cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x)
 + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x
)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)
*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos
(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + si
n(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
- 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a
^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^
7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(
cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6
*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x)
 + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + si
n(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*
```

x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*
(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8
*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*
e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
- 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a
^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^3*b^6
*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(
2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*515i)/(4*
(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f +
33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)
1i)(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(co
s(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + s
in(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
+ 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402
*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*
b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f
*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x
) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + si
n(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*
x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(co
s(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + s
in(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*
e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
- 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a
^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6
f(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(co
s(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*
x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + s
in(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*
x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^4*b^5*(a + b/(((cos(2*f*x) - sin(2*f
*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*
e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1357i)/(6*(3*a^3*b^8*f + 19*a^4*b^7*f +
51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a
^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
- 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b
f(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(co
s(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2
*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)

1i)(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(
2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
 sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6
*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i
) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 4
02*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^
6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4
f(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(c
os(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f
*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + s
in(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f
*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i
)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos
(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
+ sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(
8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i
))) - (a^5*b^4*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1
i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(
1/2)*1207i)/(6*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f +
65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i
)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos
(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
+ sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
+ 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102
*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b
^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(
cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*
f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*
x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(co
s(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
 + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(
8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1
i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 7
5*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*
b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*
(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*
f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^6*b^3*(a + b/(((cos(2

*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1135i)/(12*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^7*b^2*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*265i)/(12*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6 *f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a*b^6*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*45i)/(4*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^6*b*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*19i)/(6*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*

b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2
*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*
x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
+ sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8
*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i
) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40
*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b
^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(c
os(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^2*b^6*(a + b/(((
cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*6i)/(a^3*b^7*f + 6*a
^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b
*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^
9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(
8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x)
+ sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(
2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)
1i)(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(
2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
- 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*
a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b
^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*
(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6
*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f
*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)
*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(
8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8
*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
) + (a^3*b^5*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i)
)/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/
2)*20i)/(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3
*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*
e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) +
4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*

b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(
2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x
) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*
(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6
*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6
*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1
i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 1
6*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7
f(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos
(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x
) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + si
n(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*
x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*
(cos(8*e) + sin(8*e)*1i)) + (a^4*b^4*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*
(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i))/4 + 1/2))^(1/2)*91i)/(4*(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f
+ 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(
6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*
(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*
e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
 + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54
*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b
^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(c
os(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f
*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) +
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6
*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)
1i)(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(
cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*
e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(
8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i
) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20
*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b
^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(c
os(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^5*b^3*(a + b/(((
cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*25i)/(3*(a^3*b^7*f +
 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a

^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^6*b^2*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*11i)/(6*(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f

```
*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^2*b^5*(a + b/(((cos(2*f*x) - sin(2*
f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2
*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*81i)/(2*(3*a^2*b^7*f + 16*a^3*b^6*f + 3
5*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^
8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(
2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) +
 sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*
x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos
(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
+ sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
+ 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a
^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7
*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(c
os(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*
f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(
6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)
*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(c
os(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e
) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + si
n(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)
*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^3
*b^4*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*231i)
/(4*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*
f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*
e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(
2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
 - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*
f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2
*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*
1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + s
in(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*
e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i
) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20
*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^
7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(c
```

os(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f
*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) +
sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*
f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i
)*(cos(8*e) + sin(8*e)*1i))) + (a^4*b^3*(a + b/(((cos(2*f*x) - sin(2*f*x)*1
i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
sin(2*e)*1i))/4 + 1/2))^(1/2)*125i)/(3*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4
*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x
) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(
6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
+ 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*
a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^
2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(c
os(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*
f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x
)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*
(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*
e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + s
in(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e
)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
- 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^
7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^5*b^2*
(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2
*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*193i)/(12*
(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f +
8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
 sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2
*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(c
os(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f
*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) +
 sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(
2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x
)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2
*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6
*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1

i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^8*b*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*2i)/(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) + (a*b*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*256i)/(3*(32*a^4*f + 32*b^4*f + 192*a^2*b^2*f + 128*a*b^3*f + 128*a^3*b*f - 32*a^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*a^4*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 32*a^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 32*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*b^4*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 32*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 128*a*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 128*a^3* b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 128*a*b^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) - 128*a^3*b*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 128*a*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) + 128*a^3*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 192*a^2*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 192*a^2*b^2*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 192*a^2*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i))) + (a^2*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*256i)/(3*(32*a^4*f + 32*b^4*f + 192*a^2*b^2*f + 128*a*b^3*f + 128*a^3*b*f - 32*a^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*a^4*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 32*a^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 32*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*b^4*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 32*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 128*a*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 128*a^3*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 128*a*b^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) - 128*a^3*b*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 128*a*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) + 128*a^3*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 192*a^2*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 192*a^2*b^2*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 192*a^2*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i))) + (a^2*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*128i)/(3*(32*a^4*f + 32*b^4*f + 192*a^2*b^2*f + 128*a*b^3*f + 128*a^3*b*f - 32*a^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*a^4*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 32*a^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 32*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*b^4*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 32*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 128*a*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 128*a^3*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 128*a*b^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) - 128*a^3*b*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 128*a*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) + 128*a^3*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 192*a^2*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 192*a^2*b^2*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 192*a^2*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i))) + (b^2*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*256i)/(3*(32*a^4*f + 32*b^4*f + 192*a^2*b^2*f + 128*a*b^3*f +

```
128*a^3*b*f - 32*a^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
) - 32*a^4*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 32*a^4
*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 32*b^4*f*(cos(2*
f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*b^4*f*(cos(4*f*x) + sin
(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 32*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - 128*a*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(
2*e) + sin(2*e)*1i) - 128*a^3*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
sin(2*e)*1i) - 128*a*b^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e
)*1i) - 128*a^3*b*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) +
 128*a*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) + 128*a^
3*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 192*a^2*b^2*f
*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 192*a^2*b^2*f*(cos
(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 192*a^2*b^2*f*(cos(6*f*
x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i))) + (b^2*(cos(4*f*x) + sin(4*f
*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos
(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i))/4 + 1/2))^(1/2)*128i)/(3*(32*a^4*f + 32*b^4*f + 192*a^2*b^2*f + 128*
a*b^3*f + 128*a^3*b*f - 32*a^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i) - 32*a^4*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i
) + 32*a^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 32*b^4
*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*b^4*f*(cos(4*
f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 32*b^4*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 128*a*b^3*f*(cos(2*f*x) + sin(2*f*x)
*1i)*(cos(2*e) + sin(2*e)*1i) - 128*a^3*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i) - 128*a*b^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e)
 + sin(4*e)*1i) - 128*a^3*b*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(
4*e)*1i) + 128*a*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i
) + 128*a^3*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 192
*a^2*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 192*a^2*
b^2*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 192*a^2*b^2*f
*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i))) + (a^7*(cos(2*f*x)
 + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x
)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i))/4 + 1/2))^(1/2)*1i)/(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b
^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(co
s(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x)
+ sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*
f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*
(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
 sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1
i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^
6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*
f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos
```

(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) + (a^7*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*3i)/(2*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^7*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1i)/(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(

2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) + (a^7*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1i)/(4*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (b^7*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*117i)/(2*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2

*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12
*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*
(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x)
 + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i
)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*
1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*
a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^
3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(c
os(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*
x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + s
in(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f
*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i
)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos
(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
 sin(8*e)*1i))) + (b^7*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i
)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*261i)/(2
*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f +
 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
+ sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(
cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*
f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x)
+ sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)
*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(
2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
 sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*
1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^
7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f
*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(
8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x)
 + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin
(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x

```
)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(
cos(8*e) + sin(8*e)*1i))) + (b^7*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + s
in(6*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))
/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2
)*171i)/(2*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a
^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6
*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
 a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7
*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*
f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x)
+ sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(
2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)
*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(
cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6
*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
 sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1
i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3
*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b
^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(
cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*
f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) +
 sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*
f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (b^7*(cos(8*f*x) + sin(8*f*x)*1i)*(co
s(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin
(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 +
1/2))^(1/2)*27i)/(2*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4
*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*
f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6
*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 7
0*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3
*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*
f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(co
s(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + s
in(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1
i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(c
os(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*
e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + s
```

in(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (10*a*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2))/(3*(a^3*f*1i + b^3*f*1i + a*b^2*f*3i + a^2*b*f*3i - a^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*1i - b^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*1i - a*b^2*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*3i - a^2*b*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*3i)) - (5*a*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2))/(3*(a^3*f*1i + b^3*f*1i + a*b^2*f*3i + a^2*b*f*3i - a^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*1i - b^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*1i - a*b^2*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*3i - a^2*b*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*3i)) - (10*b*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2))/(3*(a^3*f*1i + b^3*f*1i + a*b^2*f*3i + a^2*b*f*3i - a^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*1i - b^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*1i - a*b^2*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*3i - a^2*b*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*3i)) - (5*b*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2))/(3*(a^3*f*1i + b^3*f*1i + a*b^2*f*3i + a^2*b*f*3i - a^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*1i - b^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*1i - a*b^2*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*3i - a^2*b*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*3i)) - (a^8*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1i)/(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2

*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*
1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
 + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin
(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*
1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a
^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2
f(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8
*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) +
 sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8
*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*
1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(c
os(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i)) - (a^8*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1
i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((co
s(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*3i)/(2*
(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a
^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
 - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^
8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f
*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) +
 sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(
2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)
1i)(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(
cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + si
n(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
- 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*
a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^
2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(
8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x)
+ sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(
8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)
1i)(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(
cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e
) + sin(8*e)*1i))) - (a^8*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
1i)(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1i)/(
a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^
8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(
2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)

```
- a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8
*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(co
s(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2
*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*
1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
 + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin
(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*
1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a
^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2
*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8
*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) +
 sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8
*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*
1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(c
os(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i)) - (a^8*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1
i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((co
s(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1i)/(4*
(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a
^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
 - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^
8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f
*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) +
 sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(
2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)
*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(
cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + si
n(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
- 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*
a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^
2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(
8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x)
+ sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(
8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)
*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(
cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e
) + sin(8*e)*1i))) + (a*b*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
```

```
*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*512i)
/(3*(32*a^4*f + 32*b^4*f + 192*a^2*b^2*f + 128*a*b^3*f + 128*a^3*b*f - 32*a
^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*a^4*f*(cos(
4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 32*a^4*f*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 32*b^4*f*(cos(2*f*x) + sin(2*f*x)*
1i)*(cos(2*e) + sin(2*e)*1i) - 32*b^4*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4
*e) + sin(4*e)*1i) + 32*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - 128*a*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
) - 128*a^3*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 128
*a*b^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) - 128*a^3*b*
f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 128*a*b^3*f*(cos(
6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) + 128*a^3*b*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 192*a^2*b^2*f*(cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 192*a^2*b^2*f*(cos(4*f*x) + sin(4*f*
x)*1i)*(cos(4*e) + sin(4*e)*1i) + 192*a^2*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i))) + (a*b*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e)
+ sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1
i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(
1/2)*256i)/(3*(32*a^4*f + 32*b^4*f + 192*a^2*b^2*f + 128*a*b^3*f + 128*a^3*
b*f - 32*a^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*a
^4*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 32*a^4*f*(cos(
6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 32*b^4*f*(cos(2*f*x) + s
in(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*b^4*f*(cos(4*f*x) + sin(4*f*x)*
1i)*(cos(4*e) + sin(4*e)*1i) + 32*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6
*e) + sin(6*e)*1i) - 128*a*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i) - 128*a^3*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) - 128*a*b^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) -
128*a^3*b*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 128*a*b
^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) + 128*a^3*b*f*(c
os(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 192*a^2*b^2*f*(cos(2*
f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 192*a^2*b^2*f*(cos(4*f*x)
+ sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 192*a^2*b^2*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i))) + (a*b^7*(cos(2*f*x) + sin(2*f*x)*1i
)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e)
- sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))
/4 + 1/2))^(1/2)*24i)/(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*
f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(c
os(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + s
in(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a
^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^
4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(c
```

os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f
*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(co
s(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + si
n(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*
a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^
5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(c
os(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f
*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + s
in(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) - (a^7*b*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e)
 - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
)/4 + 1/2))^(1/2)*49i)/(6*(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*
b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(
2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(
2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1
i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 1
10*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^
6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*
f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x)
 + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x
)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)
*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
+ sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
- 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^
5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*
f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos
(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x)
 + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a*b^7*(cos(4*f*x) + sin(4*f
*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos
(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i))/4 + 1/2))^(1/2)*24i)/(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^
6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*
1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*

e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + si
n(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*
a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^
3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*
x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1
i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(c
os(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + si
n(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i
) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*
a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^
4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(c
os(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*
x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) - (a^7*b*(cos(4*f*x) + sin(4*
f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(co
s(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*
e)*1i))/4 + 1/2))^(1/2)*27i)/(2*(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 2
0*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x)
+ sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(co
s(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
+ sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(
2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1
i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^
7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*
f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(
6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*
x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
+ sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e
)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^
6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*
f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(
8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a*b^7*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i

)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i))/4 + 1/2))^(1/2)*24i)/(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f +
 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x
) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6
*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(
cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e
) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + si
n(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
+ 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*
a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^
2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(co
s(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*
x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*
f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*
1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(c
os(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8
*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
 - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*
a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^
3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(co
s(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) - (a^7*b*(cos(6*f*x) +
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1
i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
sin(2*e)*1i))/4 + 1/2))^(1/2)*55i)/(6*(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5
*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2
*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1
i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos
(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
+ sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(
2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*
1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^
8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f
*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(
6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x)
 + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + si
n(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + s
in(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)

*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a*b^7*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*24i)/(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) - (a^7*b*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*13i)/(6*(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x

) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*
e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + si
n(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)
*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8
*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^2*b^7*(
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) -
 sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)
*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1339i)/(2*(3*a^3*b^8*f + 19*a^4*
b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b
^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
- a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b
^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2
*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x)
 + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f
*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
 + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin
(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
- 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262
*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*
b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*
(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*
f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) +
 sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8
*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*
1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(c
os(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e
) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin
(8*e)*1i))) - (a^3*b^6*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*4310i)/(
3*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f
+ 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(
cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
 sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2

*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 4
02*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^
6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4
f(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f
*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) +
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*
f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*
1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(
cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6
*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1
i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19
*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b
^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(
cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*
f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) +
sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*
f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^4*b^5*(cos(2*f*x) + sin(2*f*x)*1i)
*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) -
sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/
4 + 1/2))^(1/2)*4999i)/(3*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a
^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*
f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6
*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + si
n(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)
*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos
(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
+ sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + si
n(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
+ 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*
a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^
8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(
cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6
*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + si
n(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(
8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8

*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i
) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33
*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^
2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^5*b^4*(cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - si
n(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*2249i)/(2*(3*a^3*b^8*f + 19*a^4*b^7
*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*
f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a
^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*
f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2
*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)
1i)(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2
*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*
e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i
) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 4
10*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^
7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3
f(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(co
s(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x
) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + si
n(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*
x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)
*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(
8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*
e)*1i))) - (a^6*b^3*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(
a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*
f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*2641i)/(6*(
3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 3
3*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*
1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + si
n(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*
a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b
^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(

2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin (6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i) *(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos (6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + si n(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^ 4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6* f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos (8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + si n(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^7*b^2*(cos(2*f*x) + sin(2*f*x)*1i)*(c os(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - si n(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*277i)/(3*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b ^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(c os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8* f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1 i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co s(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s in(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2* e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2 2*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2* b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f* (cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + s in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6* f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)* 1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(c os(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin (8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)* 1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8 *b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f* (cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^2*b^7*(cos(4*f *x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*

f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2
*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1030i)/(3*a^3*b^8*f + 19*a^4*b^7*f + 51
*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^1
0*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) -
2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f
*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(
2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x)
 + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f
*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*
e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e
)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
 - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402
*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*
b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f
*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos
(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin
(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x
)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*
(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8
*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*
e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))
 - (a^3*b^6*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/((
(cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + s
in(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*13109i)/(6*(3*a^3*b
^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b
^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
 sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*
1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^
4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6
f(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*
f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x)
+ sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(
2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)
1i)(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(
cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6
*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +

sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*
1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3
*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f
*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(
8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x)
+ sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin
(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x
)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(
cos(8*e) + sin(8*e)*1i))) - (a^4*b^5*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e)
+ sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*
1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^
(1/2)*7504i)/(3*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f +
65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f
*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + si
n(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1
i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
+ sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + si
n(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
+ 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 10
2*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*
b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*
(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6
*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + si
n(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1
i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(c
os(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin
(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*
1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7
*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f
*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8
*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^5*b^4*(cos(4*f*x) +
sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1
i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
sin(2*e)*1i))/4 + 1/2))^(1/2)*10009i)/(6*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a
^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*
b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*
a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(

cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*
f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(
2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x
)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)
*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos
(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
+ sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*
1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a
^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^
5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(
cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6
*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8
*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*
1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(c
os(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e
) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + si
n(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)
*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)))
- (a^6*b^3*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((
cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1946i)/(3*(3*a^3*b^8
*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3
*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(
2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + s
in(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i
) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82
*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*
b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f
*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*
f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1
i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(co
s(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + s
in(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*
e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i
) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2
2*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b
^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(

cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*
f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) +
sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8
*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*
1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(co
s(8*e) + sin(8*e)*1i))) - (a^7*b^2*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) +
sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i
))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1
/2)*273i)/(2*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65
*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x)
+ sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6
*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*
(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2
*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1
i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a
^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2
f(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(co
s(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*
x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) +
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6
*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)
1i)(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*
(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(
6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*
e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
- 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*
a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^
4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(c
os(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*
x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^2*b^7*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*
(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i))/4 + 1/2))^(1/2)*337i)/(2*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^
6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f +
2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*
b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8
*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x)
+ sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(
2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x
)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)
*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos

(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^3*b^6*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1724i)/(3*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e

```
) + sin(8*e)*1i))) - (a^4*b^5*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4
+ ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*2
725i)/(3*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7
*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + s
in(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x
)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos
(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
+ sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(
2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*
1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*
a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b
^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*
f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) +
 sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(
6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x
)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)
*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
 + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin
(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1
i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 5
1*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*
b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*
(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8
*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) +
 sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^5*b^4*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos
(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i))/4 + 1/2))^(1/2)*1535i)/(2*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f
 + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*
a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f
*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*
x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + s
in(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f
*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*
e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82
*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*
```

b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f
*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos
(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*
x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) +
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*
f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i
)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos
(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
+ sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(
8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1
i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9
*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^6*b
^3*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/(((cos(2*f*
x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)
1i)(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*2131i)/(6*(3*a^3*b^8*f + 19*
a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a
^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*
1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a
^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8
f(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*
f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x)
+ sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)
*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(
6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*
1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*
a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^
2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(co
s(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*
x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + s
in(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f
*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i
)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos
(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
sin(8*e)*1i))) - (a^7*b^2*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e
)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + (
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*253i
)/(3*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4
*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2

```
*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e
) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + si
n(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
+ 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410
*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*
b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f
*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(
2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x)
 + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*
x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(co
s(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + s
in(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e
)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^
5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*
f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos
(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x
) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin
(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^2*b^7*(cos(8*f*x) + sin(8*f*x)*
1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e
) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
))/4 + 1/2))^(1/2)*885i)/(4*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75
*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*
b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos
(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) +
sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*
f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*
e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1
i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 1
2*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*
b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f
*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos
(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*
x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) +
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6
*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*
```

1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^3*b^6*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1209i)/(4*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^4*b^5*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*917i)/(6*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402

```
*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*
b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f
*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x
) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + si
n(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*
x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(co
s(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + s
in(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*
e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
 - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a
^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6
*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(co
s(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*
x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + s
in(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*
x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^5*b^4*(cos(8*f*x) + sin(8*f*x)*1i)*(
cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - s
in(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4
+ 1/2))^(1/2)*68i)/(3*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b
^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*
f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e
) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*
e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2
2*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*
b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*
(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(
6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*
f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*
1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(c
os(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin
(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*
1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8
```

*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*
(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^6*b^3*(cos(8*f
*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*
f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2
*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*625i)/(12*(3*a^3*b^8*f + 19*a^4*b^7*f +
 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f +
a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
 - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*
b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f
*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) +
 sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(
2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x
)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)
*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos
(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
+ sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1
i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a
^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^
4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(
cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*
f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) +
sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*
f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1
i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(co
s(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin
(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1
i))) - (a^7*b^2*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)*(a +
b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x)
 + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*217i)/(12*(3*a^
3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^
8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*
e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
 + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242
*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*
b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f
*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + s
in(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f

*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(co
s(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + s
in(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*
e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i
) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*
a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^
7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(c
os(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f
*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) +
sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*
f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i
)*(cos(8*e) + sin(8*e)*1i))) + (a*b^6*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e
) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)
*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))
^(1/2)*261i)/(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25
*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i
)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*
e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin
(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
 - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b
^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(
2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x
) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)
*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
 + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + si
n(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3
*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f
*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(
8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x)
 + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(
8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) + (a^6*b*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) -
sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4
 + 1/2))^(1/2)*85i)/(6*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*
b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(

```
6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + si
n(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
+ 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*
a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b
^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2
*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x)
+ sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(
6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x
)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)
*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
+ sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i
) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35
*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b
^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(
cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f
*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a*b^6*(cos(4*f*x) + sin(
4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(
cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(
2*e)*1i))/4 + 1/2))^(1/2)*1107i)/(2*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^
5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) +
 sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(
cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*
e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 1
80*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6
*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f
*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(
6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*
f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1
i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(co
s(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
+ sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(
8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1
i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 2
5*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b
^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^6*b*(cos(
4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin
```

(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*45i)/(2*(3*a^2*b^7*f + 16*a^3*b^6*f
+ 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a
*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6
*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(
cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*
e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1
i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2
0*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*
b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f
*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos
(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*
x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)
*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(
8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
 sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8
*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i
) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (
a*b^6*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/(((cos(2
*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f
*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*369i)/(3*a^2*b^7*f + 16*a
^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b
*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a
^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(
cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f
*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i
)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e
) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
- 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*
a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b
^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*
(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6
*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8
*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*

1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(c
os(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e
) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + si
n(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*
1i)) + (a^6*b*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/
(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) +
 sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*91i)/(6*(3*a^2*b^
7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2
*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12
*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*
(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x)
 + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i
)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*
1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*
a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^
3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(c
os(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*
x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + s
in(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f
*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i
)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos
(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
 sin(8*e)*1i))) + (a*b^6*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*
1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*261i)/
(4*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f
 + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e
) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
- 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f
*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(
2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x
) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + s
in(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*
f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
 + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + si

n(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e
)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
 - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*
a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7
f(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(co
s(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*
x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + s
in(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f
*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)
*(cos(8*e) + sin(8*e)*1i))) + (a^6*b*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*
1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^
(1/2)*11i)/(3*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 2
5*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2
*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + si
n(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i
) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*
b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f
*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + s
in(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f
*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(co
s(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + s
in(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e
)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
- 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^
3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*
f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos
(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x
) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin
(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^2*b^6*(cos(2*f*x) + sin(2*f*x)*
1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e
) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
))/4 + 1/2))^(1/2)*98i)/(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^
4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*
f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*
(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
 sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*
e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
 + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110
*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*

```
b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2
*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) +
 sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6
*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*
1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(
cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6
*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*
e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*
b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*
(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8
*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) +
 sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) + (a^3*b^5*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(
2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i))/4 + 1/2))^(1/2)*299i)/(2*(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20
*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) +
 sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*
x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos
(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
 sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 1
00*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7
*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f
*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6
*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(
6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x
)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)
*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(
6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
 sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)
*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6
*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f
*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8
*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^4*b^4*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1
i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
sin(2*e)*1i))/4 + 1/2))^(1/2)*286i)/(3*(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^
5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(
2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) +
```

```
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*
1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
+ 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a
^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*
f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos
(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*
f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1
i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(co
s(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e
)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
- 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a
^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*
f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^5*b^3*(cos(2
*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(
2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos
(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*8i)/(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5
*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x)
 + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*
x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2
*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1
i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 1
6*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b
^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(
cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*
f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x
)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*
(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(
8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1
i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 1
5*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b
^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) - (a^6*b^2*(cos
```

(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - si
n(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*19i)/(a^3*b^7*f + 6*a^4*b^6*f + 15*
a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f
*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f
*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8
*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + si
n(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
 + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^
2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*
f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos
(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*
x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) +
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*
f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1
i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8
*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + s
in(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e
)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
 - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^
8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) + (a^2*b^6*(
cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) -
 sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)
*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*86i)/(a^3*b^7*f + 6*a^4*b^6*f +
15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*
b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(
6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + si
n(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x
)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2
*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
 sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(
2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1
i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4
*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b
^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(
cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6
*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x)
 + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x
)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(co

s(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) + (a^3*b^5*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*203i)/(2*(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^4*b^4*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*121i)/(6*(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -

72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a
^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^
4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(c
os(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f
*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin
(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)
1i)(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(
cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*
e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + s
in(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)
*1i))) - (a^5*b^3*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a
+ b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*152i)/(3*(a^3
*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b
^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a
^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2
*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x)
+ sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)
*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(
2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*
e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
 - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 10
0*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*
b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*
(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*
x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + si
n(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*
x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)
*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(
8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
sin(8*e)*1i))) - (a^6*b^2*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)
1i)(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*130i)
/(3*(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f +
 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
 sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^
2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*
f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*

x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*
f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
+ sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*
1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^
8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(
cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f
*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) +
sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*
f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1
i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos
(8*e) + sin(8*e)*1i))) - (a^2*b^6*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
sin(6*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i)
)/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/
2)*98i)/(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3
*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*
e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) +
4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*
b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(
2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x
) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*
(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6
*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6
*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1
i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 1
6*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7
f(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos
(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x
) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + si
n(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*
x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*
(cos(8*e) + sin(8*e)*1i)) - (a^3*b^5*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
 + sin(6*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*
1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^
(1/2)*331i)/(2*(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*
a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*

(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^4*b^4*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*464i)/(3*(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) +

sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^5*b^3*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*92i)/(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) - (a^6*b^2*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*37i)/(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +

```
sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*
1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 1
5*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*
b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*
(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*
f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) - (a^2*b^6*(cos(8*f*x) + si
n(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)
*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + si
n(2*e)*1i))/4 + 1/2))^(1/2)*92i)/(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f +
20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x)
 + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*
f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(c
os(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*
1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a
^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2
*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos
(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + si
n(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1
i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(co
s(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
 + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*
e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
- 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a
^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3
*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos
(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) - (a^3*b^5*(cos(8*f*x) +
 sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*
1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
 sin(2*e)*1i))/4 + 1/2))^(1/2)*275i)/(2*(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b
^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) +
 sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)
*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e
) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + si
n(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
 + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*
a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8
*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(co
```

s(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^4*b^4*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*409i)/(4*(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^5*b^3*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*125i)/(3*(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i

)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e
) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + si
n(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4
*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*
f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(co
s(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*
x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)
1i)(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(c
os(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e
) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + si
n(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)
*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)))
- (a^6*b^2*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((
cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*65i)/(6*(a^3*b^7*f +
6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a
^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) -
2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(
cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f
*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*
f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*
1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(
cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*
e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a
^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^
4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(c
os(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f
*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin
(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)
1i)(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(
cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*
e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + s
in(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)
*1i))) + (a^2*b^5*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a
+ b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*957i)/(2*(3*a
^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^

7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^3*b^4*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1397i)/(3*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) +

sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^4*b^3*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*519i)/(2*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^5*b^2*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*83i)/(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i

)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
 sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8
*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i
) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25
*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^
2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) + (a^2*b^5*(cos(
4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin
(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1923i)/(2*(3*a^2*b^7*f + 16*a^3*b^6*
f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12
*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f
*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)
*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(
2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
+ sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^
2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6
f(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(c
os(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*
f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) +
 sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6
*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1
i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(co
s(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin
(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*
1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) +
 (a^3*b^4*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((c
os(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*5303i)/(6*(3*a^2*b^7*
f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f
 + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1
i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a
*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(c
os(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) +
 sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2
*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)
1i)(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(
2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +

sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i
) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2
30*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^
5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*
f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos
(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x)
 + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin
(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x
)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*
(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8
*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + s
in(8*e)*1i))) + (a^4*b^3*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*
1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*2785i)
/(6*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*
f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*
e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(
2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
 - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*
f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2
*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*
1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + s
in(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*
e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i
) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20
*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^
7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(c
os(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f
*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) +
sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*
f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i
)*(cos(8*e) + sin(8*e)*1i))) + (a^5*b^2*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4
*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*
e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2
))^(1/2)*839i)/(6*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f
 + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
+ sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e

```
)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*
a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b
^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(
2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x)
 + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x
)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)
*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
 + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin
(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*
1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 1
6*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*
b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*
(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8
*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) +
 sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^2*b^5*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos
(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i))/4 + 1/2))^(1/2)*1299i)/(2*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f
 + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*
f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)
*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin
(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*
1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*
a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^
3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f
*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) +
 sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(
6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x
)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*
(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6
*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + s
in(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e
)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
- 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a
^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*
f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^3*b^4*(cos(6
*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/(((cos(2*f*x) - sin(
2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos
```

(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1805i)/(3*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^4*b^3*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*633i)/(2*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*

1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(c
os(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e
) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin
(8*e)*1i))) + (a^5*b^2*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i
)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*95i)/(3*
a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a
^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + si
n(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^
8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(
8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x)
 + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f
*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin
(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
 - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*
a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^
2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(co
s(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*
x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + s
in(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f
*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i
)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(
8*e) + sin(8*e)*1i)) + (a^2*b^5*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + si
n(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/
4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)
*126i)/(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b
^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos
(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*
1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8
*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2
*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x)
 + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)
*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(
6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
+ sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin

(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2
*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f
*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(
8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x)
+ sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin
(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)
1i)(cos(8*e) + sin(8*e)*1i)) + (a^3*b^4*(cos(8*f*x) + sin(8*f*x)*1i)*(cos
(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(
2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1
/2))^(1/2)*503i)/(4*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4
*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*
f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6
*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 7
0*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3
*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*
f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(co
s(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + s
in(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1
i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(c
os(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*
e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + s
in(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e
)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^
4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*
f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos
(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x)
+ sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^4*b^3*(cos(8*f*x) + sin(8
*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(c
os(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i))/4 + 1/2))^(1/2)*421i)/(6*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*
f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2
*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + s
in(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x
)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(co
s(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + si
n(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
+ 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180

*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b
^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*
f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*
x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)
*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(
6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*
e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
 - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*
a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2
f(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^5*b^2*(cos(
8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin
(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*265i)/(12*(3*a^2*b^7*f + 16*a^3*b^6*
f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12
*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f
*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)
*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(
2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
+ sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^
2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6
f(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(c
os(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*
f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) +
 sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6
*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1
i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(co
s(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin
(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*
1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) -
 (a*b^8*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos
(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2
*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*261i)/(2*(3*a^3*b^8*f +
 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f +
 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6

*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^8*b*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*8i)/(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19

*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b
^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(
cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*
f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) +
 sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*
f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) - (a*b^8*(cos(4*f*x) + sin(4*f*x)*1i)*(c
os(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - si
n(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 +
 1/2))^(1/2)*405i)/(2*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b
^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*
f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e
) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*
e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2
2*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*
b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*
(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(
6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*
f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*
1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(c
os(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin
(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*
1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8
*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*
(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^8*b*(cos(4*f*x
) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*
x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e
) + sin(2*e)*1i))/4 + 1/2))^(1/2)*12i)/(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5
*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*
f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^
10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(co
s(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + s
in(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*
f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*
1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(

cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2
*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
 sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i
) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 24
2*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5
*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*
f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(co
s(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f
*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) +
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f
*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i
)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos
(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
+ sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(
8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1
i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) - (
a*b^8*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/(((cos(2
*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f
*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*27i)/(2*(3*a^3*b^8*f + 19
*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*
a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*
a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^
8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2
*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x)
 + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f
*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i
)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
+ sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin
(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
- 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102
*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b
^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(c
os(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f
*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) +
sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*
f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1
i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(co
s(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)

```
+ sin(8*e)*1i))) - (a^8*b*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*8i)/(
3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 3
3*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*
1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + si
n(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*
a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b
^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(
2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x)
 + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x
)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)
*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
 + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + si
n(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e
)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
- 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^
4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*
f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos
(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x
) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + si
n(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x
)*1i)*(cos(8*e) + sin(8*e)*1i)) + (a*b^8*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(
8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2
*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/
2))^(1/2)*117i)/(2*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*
f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(
2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) +
 sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x
)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2
*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
 sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(
2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*
1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a
^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9
*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(co
s(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f
```

```
*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) +
 sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(
6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x
)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)
*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(
6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*
e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
 - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*
a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^
3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(co
s(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^8*b*(cos(8*f*x) +
 sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*
1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
 sin(2*e)*1i))/4 + 1/2))^(1/2)*2i)/(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6
*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f +
2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b
*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*
f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) +
 sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2
*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)
*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(
2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
+ sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*
1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^
4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6
*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(c
os(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*
f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(
6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*
1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(c
os(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e
) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + si
n(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)
*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))
```

3.118 $\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 183

$$\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = -\frac{(15a^2-10ab-b^2)\cot(e+fx)}{15(a+b)^3f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{2(5a+2b)\cot^3(e+fx)}{15(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\cot^5(e+fx)}{5(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{2b(15a^2-10ab-b^2)\tan(e+fx)}{15(a+b)^4f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] -1/15*(15*a^2-10*a*b-b^2)*cot(f*x+e)/(a+b)^3/f/(a+b+b*tan(f*x+e)^2)^(1/2)-2
/15*(5*a+2*b)*cot(f*x+e)^3/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/5*cot(f*x
+e)^5/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-2/15*b*(15*a^2-10*a*b-b^2)*tan(f*x
+e)/(a+b)^4/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4217, 473, 464, 277, 197}

$$\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = -\frac{2b(15a^2-10ab-b^2)\tan(e+fx)}{15f(a+b)^4\sqrt{a+b\tan^2(e+fx)+b}} - \frac{(15a^2-10ab-b^2)\cot(e+fx)}{15f(a+b)^3\sqrt{a+b\tan^2(e+fx)+b}} - \frac{\cot^5(e+fx)}{5f(a+b)\sqrt{a+b\tan^2(e+fx)+b}} - \frac{2(5a+2b)\cot^3(e+fx)}{15f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[In] Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/15*((15*a^2 - 10*a*b - b^2)*Cot[e + f*x])/((a + b)^3*f*Sqrt[a + b + b*Ta
n[e + f*x]^2]) - (2*(5*a + 2*b)*Cot[e + f*x]^3)/(15*(a + b)^2*f*Sqrt[a + b
+ b*Tan[e + f*x]^2]) - Cot[e + f*x]^5/(5*(a + b)*f*Sqrt[a + b + b*Tan[e + f
*x]^2]) - (2*b*(15*a^2 - 10*a*b - b^2)*Tan[e + f*x])/(15*(a + b)^4*f*Sqrt[a
 + b + b*Tan[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^6(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{2(5a+2b)+5(a+b)x^2}{x^4(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{5(a+b)f}$$

$$= -\frac{2(5a+2b)\cot^3(e+fx)}{15(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\cot^5(e+fx)}{5(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(15a^2-10ab-b^2)\,\text{Subst}\left(\int \frac{1}{x^2(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{15(a+b)^2f}$$

$$= -\frac{(15a^2-10ab-b^2)\cot(e+fx)}{15(a+b)^3f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{2(5a+2b)\cot^3(e+fx)}{15(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\cot^5(e+fx)}{5(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(2b(15a^2-10ab-b^2))\,\text{Subst}\left(\int \frac{1}{(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{15(a+b)^3f}$$

$$= -\frac{(15a^2-10ab-b^2)\cot(e+fx)}{15(a+b)^3f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{2(5a+2b)\cot^3(e+fx)}{15(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\cot^5(e+fx)}{5(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{2b(15a^2-10ab-b^2)\tan(e+fx)}{15(a+b)^4f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.69

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx =$$

$$-\frac{(a+2b+a\cos(2(e+fx)))\left(-8a^2(a-5b)+4a(a^2-4ab-5b^2)\csc^2(e+fx)+(a-5b)(a+b)^2\csc^4(e+f\right.}{30(a+b)^4f\,(a+b\sec^2(e+fx))^{3/2}}$$

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/30*((a + 2*b + a*Cos[2*(e + f*x)])*(-8*a^2*(a - 5*b) + 4*a*(a^2 - 4*a*b
- 5*b^2)*Csc[e + f*x]^2 + (a - 5*b)*(a + b)^2*Csc[e + f*x]^4 + 3*(a + b)^3*
Csc[e + f*x]^6)*Sec[e + f*x]^2*Tan[e + f*x])/((a + b)^4*f*(a + b*Sec[e + f*
x]^2)^(3/2))
```

Maple [A] (verified)

Time = 7.08 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.19

method	result
default	$-\frac{\left(b+a\cos(fx+e)^2\right)\left(8a^3\cos(fx+e)^6-40\cos(fx+e)^6a^2b-20\cos(fx+e)^4a^3+104\cos(fx+e)^4a^2b-20\cos(fx+e)^4a\,b^2+15\cos(fx+e\right.}{15f\left(a^4+4a^3b+6a^2b^2+4a\,b^3+b^4\right)\left(a+\right.}$

[In] int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/15/f/(a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*(b+a*cos(f*x+e)^2)*(8*a^3*cos(f
*x+e)^6-40*cos(f*x+e)^6*a^2*b-20*cos(f*x+e)^4*a^3+104*cos(f*x+e)^4*a^2*b-20
*cos(f*x+e)^4*a*b^2+15*cos(f*x+e)^2*a^3-85*cos(f*x+e)^2*a^2*b+49*cos(f*x+e)
^2*a*b^2+5*cos(f*x+e)^2*b^3+30*a^2*b-20*a*b^2-2*b^3)/(a+b*sec(f*x+e)^2)^(3/
2)*sec(f*x+e)^3*csc(f*x+e)^5

Fricas [A] (verification not implemented)

none

Time = 2.34 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.72

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx =$$

$$-\frac{\left(8\,(a^3-5\,a^2b)\cos\left(fx+e\right)^7-4\,(5\,a^3-26\,a^2b+5\,ab^2)\cos\left(fx+e\right)^5+(15\,a\right.}{15\,\left((a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4)f\cos\left(fx+e\right)^6-(2\,a^5+7\,a^4b+8\,a^3b^2+2\,a^2b^3-2\,ab^4-b^5)f\cos\right.}$$

[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] -1/15*(8*(a^3 - 5*a^2*b)*cos(f*x + e)^7 - 4*(5*a^3 - 26*a^2*b + 5*a*b^2)*co
s(f*x + e)^5 + (15*a^3 - 85*a^2*b + 49*a*b^2 + 5*b^3)*cos(f*x + e)^3 + 2*(1
5*a^2*b - 10*a*b^2 - b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)/(((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*f*cos(f*x + e)^6
 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*f*cos(f*x + e)
^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*f*cos(f*x +
e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f)*sin(f*x + e))

Sympy [F]

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.54

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\frac{30\,b\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^2} - \frac{80\,b^2\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^3} + \frac{48\,b^3\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^4} + \frac{15}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)\tan(fx+e)} - \cdots}{\cdots}$$

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/15*(30*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) - 80*b^
2*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3) + 48*b^3*tan(f*x
+ e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^4) + 15/(sqrt(b*tan(f*x + e)^2
 + a + b)*(a + b)*tan(f*x + e)) - 40*b/(sqrt(b*tan(f*x + e)^2 + a + b)*(a +
 b)^2*tan(f*x + e)) + 24*b^2/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3*tan(
f*x + e)) + 10/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(f*x + e)^3) - 6*
b/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2*tan(f*x + e)^3) + 3/(sqrt(b*tan
(f*x + e)^2 + a + b)*(a + b)*tan(f*x + e)^5))/f
```

Giac [F]

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc(fx+e)^6}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \text{Hanged}$$

```
[In] int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2)),x)
[Out] \text{Hanged}
```

3.119 $\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 219

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{(5a^2+20ab+16b^2)\cos(e+fx)}{5a^3f\,(a+b\sec^2(e+fx))^{3/2}} + \frac{2(5a+4b)\cos^3(e+fx)}{15a^2f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{\cos^5(e+fx)}{5af\,(a+b\sec^2(e+fx))^{3/2}} - \frac{4b(5a^2+20ab+16b^2)\sec(e+fx)}{15a^4f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{8b(5a^2+20ab+16b^2)\sec(e+fx)}{15a^5f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -1/5*(5*a^2+20*a*b+16*b^2)*cos(f*x+e)/a^3/f/(a+b*sec(f*x+e)^2)^(3/2)+2/15*(
5*a+4*b)*cos(f*x+e)^3/a^2/f/(a+b*sec(f*x+e)^2)^(3/2)-1/5*cos(f*x+e)^5/a/f/(
a+b*sec(f*x+e)^2)^(3/2)-4/15*b*(5*a^2+20*a*b+16*b^2)*sec(f*x+e)/a^4/f/(a+b*
sec(f*x+e)^2)^(3/2)-8/15*b*(5*a^2+20*a*b+16*b^2)*sec(f*x+e)/a^5/f/(a+b*sec(
f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4219, 473, 464, 277, 198, 197}

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{2(5a+4b)\cos^3(e+fx)}{15a^2f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{\left(\frac{4b(5a+4b)}{a^2}+5\right)\cos(e+fx)}{5af\,(a+b\sec^2(e+fx))^{3/2}} - \frac{8b(5a^2+20ab+16b^2)\sec(e+fx)}{15a^5f\sqrt{a+b\sec^2(e+fx)}} - \frac{4b(5a^2+20ab+16b^2)\sec(e+fx)}{15a^4f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{\cos^5(e+fx)}{5af\,(a+b\sec^2(e+fx))^{3/2}}$$

```
[In] Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/5*((5 + (4*b*(5*a + 4*b))/a^2)*Cos[e + f*x])/(a*f*(a + b*Sec[e + f*x]^2)
^(3/2)) + (2*(5*a + 4*b)*Cos[e + f*x]^3)/(15*a^2*f*(a + b*Sec[e + f*x]^2)^(
3/2)) - Cos[e + f*x]^5/(5*a*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (4*b*(5*a^2 +
 20*a*b + 16*b^2)*Sec[e + f*x])/(15*a^4*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (
8*b*(5*a^2 + 20*a*b + 16*b^2)*Sec[e + f*x])/(15*a^5*f*Sqrt[a + b*Sec[e + f*
x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)^2}{x^6(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}\\
&= -\frac{\cos^5(e+fx)}{5af\,(a+b\sec^2(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{-2(5a+4b)+5ax^2}{x^4(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{5af}\\
&= \frac{2(5a+4b)\cos^3(e+fx)}{15a^2f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{\cos^5(e+fx)}{5af\,(a+b\sec^2(e+fx))^{3/2}}\\
&\quad + \frac{(5a^2+20ab+16b^2)\,\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{5a^2f}\\
&= -\frac{(5a^2+20ab+16b^2)\cos(e+fx)}{5a^3f\,(a+b\sec^2(e+fx))^{3/2}}\\
&\quad + \frac{2(5a+4b)\cos^3(e+fx)}{15a^2f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{\cos^5(e+fx)}{5af\,(a+b\sec^2(e+fx))^{3/2}}\\
&\quad - \frac{(4b(5a^2+20ab+16b^2))\,\text{Subst}\left(\int \frac{1}{(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{5a^3f}\\
&= -\frac{(5a^2+20ab+16b^2)\cos(e+fx)}{5a^3f\,(a+b\sec^2(e+fx))^{3/2}} + \frac{2(5a+4b)\cos^3(e+fx)}{15a^2f\,(a+b\sec^2(e+fx))^{3/2}}\\
&\quad - \frac{\cos^5(e+fx)}{5af\,(a+b\sec^2(e+fx))^{3/2}} - \frac{4b(5a^2+20ab+16b^2)\sec(e+fx)}{15a^4f\,(a+b\sec^2(e+fx))^{3/2}}\\
&\quad - \frac{(8b(5a^2+20ab+16b^2))\,\text{Subst}\left(\int \frac{1}{(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{15a^4f}\\
&= -\frac{(5a^2+20ab+16b^2)\cos(e+fx)}{5a^3f\,(a+b\sec^2(e+fx))^{3/2}} + \frac{2(5a+4b)\cos^3(e+fx)}{15a^2f\,(a+b\sec^2(e+fx))^{3/2}}\\
&\quad - \frac{\cos^5(e+fx)}{5af\,(a+b\sec^2(e+fx))^{3/2}} - \frac{4b(5a^2+20ab+16b^2)\sec(e+fx)}{15a^4f\,(a+b\sec^2(e+fx))^{3/2}}\\
&\quad - \frac{8b(5a^2+20ab+16b^2)\sec(e+fx)}{15a^5f\sqrt{a+b\sec^2(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 3.11 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.83

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left(425a^4+6400a^3b+22784a^2b^2+32768ab^3+16384b^4+48a(11a^3+150a^2b+\cdots\right.}{\cdots}$$

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/3840*((a + 2*b + a*Cos[2*(e + f*x)])*(425*a^4 + 6400*a^3*b + 22784*a^2*b
^2 + 32768*a*b^3 + 16384*b^4 + 48*a*(11*a^3 + 150*a^2*b + 384*a*b^2 + 256*b
^3)*Cos[2*(e + f*x)] + 12*a^2*(7*a^2 + 64*a*b + 64*b^2)*Cos[4*(e + f*x)] -
16*a^4*Cos[6*(e + f*x)] - 32*a^3*b*Cos[6*(e + f*x)] + 3*a^4*Cos[8*(e + f*x)
])*Sec[e + f*x]^5)/(a^5*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [A] (verified)

Time = 5.67 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.99

method	result
default	$\frac{a^2\left(b+a\cos(fx+e)^2\right)(a+b)^7\left(3\cos(fx+e)^8a^4-10a^4\cos(fx+e)^6-8a^3b\cos(fx+e)^6+15a^4\cos(fx+e)^4+60\cos(fx+e)^4a^3b+48a^2b^2\cdots\right.}{15f\left(\sqrt{-ab}+a\right)^7\left(\sqrt{-ab}-a\right)^7\left(a+b\cdots\right.}$

```
[In] int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15/f*a^2/((-a*b)^(1/2)+a)^7/((-a*b)^(1/2)-a)^7*(b+a*cos(f*x+e)^2)*(a+b)^7
*(3*cos(f*x+e)^8*a^4-10*a^4*cos(f*x+e)^6-8*a^3*b*cos(f*x+e)^6+15*a^4*cos(f*
x+e)^4+60*cos(f*x+e)^4*a^3*b+48*a^2*b^2*cos(f*x+e)^4+60*cos(f*x+e)^2*a^3*b+
240*a^2*b^2*cos(f*x+e)^2+192*a*b^3*cos(f*x+e)^2+40*a^2*b^2+160*a*b^3+128*b^
4)/(a+b*sec(f*x+e)^2)^(5/2)*sec(f*x+e)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.86

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\left(3\,a^4\cos\left(fx+e\right)^9-2\left(5\,a^4+4\,a^3b\right)\cos\left(fx+e\right)^7+3\left(5\,a^4+20\,a^3b+16\,a^2b^2\right)\cos\left(fx+e\right)^5+12\left(5\,a^3b+\cdots\right.}{15\left(a^7f\cos\left(fx+e\right)^4+2\,a^6bf\cos\left(\cdots\right.\right.}$$

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*a^4*cos(f*x + e)^9 - 2*(5*a^4 + 4*a^3*b)*cos(f*x + e)^7 + 3*(5*a^4
 + 20*a^3*b + 16*a^2*b^2)*cos(f*x + e)^5 + 12*(5*a^3*b + 20*a^2*b^2 + 16*a*
b^3)*cos(f*x + e)^3 + 8*(5*a^2*b^2 + 20*a*b^3 + 16*b^4)*cos(f*x + e))*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*f*co
s(f*x + e)^2 + a^5*b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.53

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\frac{15\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^3} - \frac{10\left(\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3 - 9\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)\right)}{a^4} + \frac{3\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{5}{2}}\cos(fx+e)^5 - 20\,(}{}}{}$$

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^3 - 10*((a + b/cos(f*x
+ e)^2)^(3/2)*cos(f*x + e)^3 - 9*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))
/a^4 + (3*(a + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 20*(a + b/cos(f*x +
 e)^2)^(3/2)*b*cos(f*x + e)^3 + 90*sqrt(a + b/cos(f*x + e)^2)*b^2*cos(f*x +
 e))/a^5 + 5*(6*(a + b/cos(f*x + e)^2)*b*cos(f*x + e)^2 - b^2)/((a + b/cos(
f*x + e)^2)^(3/2)*a^3*cos(f*x + e)^3) + 10*(9*(a + b/cos(f*x + e)^2)*b^2*co
s(f*x + e)^2 - b^3)/((a + b/cos(f*x + e)^2)^(3/2)*a^4*cos(f*x + e)^3) + 5*(
12*(a + b/cos(f*x + e)^2)*b^3*cos(f*x + e)^2 - b^4)/((a + b/cos(f*x + e)^2)
^(3/2)*a^5*cos(f*x + e)^3))/f
```

Giac [F]

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\sin\left(fx+e\right)^5}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\sin\left(e+f\,x\right)^5}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

```
[In] int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.120 $\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 146

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = -\frac{(a+2b)\cos(e+fx)}{a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{4b(a+2b)\sec(e+fx)}{3a^3f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{8b(a+2b)\sec(e+fx)}{3a^4f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -(a+2*b)*cos(f*x+e)/a^2/f/(a+b*sec(f*x+e)^2)^(3/2)+1/3*cos(f*x+e)^3/a/f/(a+
b*sec(f*x+e)^2)^(3/2)-4/3*b*(a+2*b)*sec(f*x+e)/a^3/f/(a+b*sec(f*x+e)^2)^(3/
2)-8/3*b*(a+2*b)*sec(f*x+e)/a^4/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4219, 464, 277, 198, 197}

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = -\frac{8b(a+2b)\sec(e+fx)}{3a^4f\sqrt{a+b\sec^2(e+fx)}} - \frac{4b(a+2b)\sec(e+fx)}{3a^3f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(a+2b)\cos(e+fx)}{a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

```
[In] Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(((a + 2*b)*Cos[e + f*x])/(a^2*f*(a + b*Sec[e + f*x]^2)^(3/2))) + Cos[e +
f*x]^3/(3*a*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (4*b*(a + 2*b)*Sec[e + f*x])/
```

```
(3*a^3*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (8*b*(a + 2*b)*Sec[e + f*x])/(3*a^
4*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{(a+2b)\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{af}$$

$$= -\frac{(a+2b)\cos(e+fx)}{a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(4b(a+2b))\text{Subst}\left(\int \frac{1}{(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{a^2f}$$

$$= -\frac{(a+2b)\cos(e+fx)}{a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{4b(a+2b)\sec(e+fx)}{3a^3f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(8b(a+2b))\text{Subst}\left(\int \frac{1}{(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{3a^3f}$$

$$= -\frac{(a+2b)\cos(e+fx)}{a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{4b(a+2b)\sec(e+fx)}{3a^3f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{8b(a+2b)\sec(e+fx)}{3a^4f\sqrt{a+b\sec^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 2.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.88

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left(26a^3+264a^2b+640ab^2+512b^3+3a(11a^2+96ab+128b^2)\cos(2(e+fx))+\right.}{192a^4f\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/192*((a + 2*b + a*Cos[2*(e + f*x)])*(26*a^3 + 264*a^2*b + 640*a*b^2 + 51
2*b^3 + 3*a*(11*a^2 + 96*a*b + 128*b^2)*Cos[2*(e + f*x)] + 6*a^2*(a + 4*b)*
Cos[4*(e + f*x)] - a^3*Cos[6*(e + f*x)])*Sec[e + f*x]^5)/(a^4*f*(a + b*Sec[
e + f*x]^2)^(5/2))
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01

method	result
default	$-\frac{a\left(b+a\cos(fx+e)^2\right)(a+b)^5\left(a^3\cos(fx+e)^6-3\cos(fx+e)^4a^3-6\cos(fx+e)^4a^2b-12\cos(fx+e)^2a^2b-24\cos(fx+e)^2ab^2-8ab^2-16b\right.}{3f\left(\sqrt{-ab}+a\right)^5\left(\sqrt{-ab}-a\right)^5\left(a+b\sec(fx+e)^2\right)^{\frac{5}{2}}}$

```
[In] int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*a/((-a*b)^(1/2)+a)^5/((-a*b)^(1/2)-a)^5*(b+a*cos(f*x+e)^2)*(a+b)^5*(
a^3*cos(f*x+e)^6-3*cos(f*x+e)^4*a^3-6*cos(f*x+e)^4*a^2*b-12*cos(f*x+e)^2*a^
2*b-24*cos(f*x+e)^2*a*b^2-8*a*b^2-16*b^3)/(a+b*sec(f*x+e)^2)^(5/2)*sec(f*x+
e)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.95

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\left(a^3\cos\left(fx+e\right)^7-3\left(a^3+2a^2b\right)\cos\left(fx+e\right)^5-12\left(a^2b+2ab^2\right)\cos\left(fx+\right.\right.}{3\left(a^6f\cos\left(fx+e\right)^4+2a^5bf\cos\left(fx+e\right.\right.}$$

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(a^3*cos(f*x + e)^7 - 3*(a^3 + 2*a^2*b)*cos(f*x + e)^5 - 12*(a^2*b + 2*
a*b^2)*cos(f*x + e)^3 - 8*(a*b^2 + 2*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)/(a^6*f*cos(f*x + e)^4 + 2*a^5*b*f*cos(f*x + e)^2 +
 a^4*b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.34

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\frac{3\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^3}-\frac{\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3-9\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)}{a^4}+\frac{6\left(a+\frac{b}{\cos(fx+e)^2}\right)b\cos(fx+e)^2-b^2}{\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}a^3\cos(fx+e)^3}+\frac{9\left(\right.}{\left(\right.}}{3f}$$

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^3 - ((a + b/cos(f*x + e)^
2)^(3/2)*cos(f*x + e)^3 - 9*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))/a^4
+ (6*(a + b/cos(f*x + e)^2)*b*cos(f*x + e)^2 - b^2)/((a + b/cos(f*x + e)^2)
^(3/2)*a^3*cos(f*x + e)^3) + (9*(a + b/cos(f*x + e)^2)*b^2*cos(f*x + e)^2 -
 b^3)/((a + b/cos(f*x + e)^2)^(3/2)*a^4*cos(f*x + e)^3))/f
```

Giac [F]

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin\left(e+f\,x\right)^3}{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{5/2}}\,dx$$

```
[In] int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.121 $\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{5/2}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 97

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{5/2}} \, dx = -\frac{\cos(e+fx)}{af(a+b\sec^2(e+fx))^{3/2}} - \frac{4b\sec(e+fx)}{3a^2f(a+b\sec^2(e+fx))^{3/2}} - \frac{8b\sec(e+fx)}{3a^3f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -cos(f*x+e)/a/f/(a+b*sec(f*x+e)^2)^(3/2)-4/3*b*sec(f*x+e)/a^2/f/(a+b*sec(f*
x+e)^2)^(3/2)-8/3*b*sec(f*x+e)/a^3/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4219, 277, 198, 197}

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{5/2}} \, dx = -\frac{8b\sec(e+fx)}{3a^3f\sqrt{a+b\sec^2(e+fx)}} - \frac{4b\sec(e+fx)}{3a^2f(a+b\sec^2(e+fx))^{3/2}} - \frac{\cos(e+fx)}{af(a+b\sec^2(e+fx))^{3/2}}$$

```
[In] Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(Cos[e + f*x]/(a*f*(a + b*Sec[e + f*x]^2)^(3/2))) - (4*b*Sec[e + f*x])/(3*
a^2*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (8*b*Sec[e + f*x])/(3*a^3*f*Sqrt[a +
b*Sec[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}\\
&= -\frac{\cos(e+fx)}{af\,(a+b\sec^2(e+fx))^{3/2}} - \frac{(4b)\text{Subst}\left(\int \frac{1}{(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{af}\\
&= -\frac{\cos(e+fx)}{af\,(a+b\sec^2(e+fx))^{3/2}} - \frac{4b\sec(e+fx)}{3a^2f\,(a+b\sec^2(e+fx))^{3/2}}\\
&\quad - \frac{(8b)\text{Subst}\left(\int \frac{1}{(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{3a^2f}\\
&= -\frac{\cos(e+fx)}{af\,(a+b\sec^2(e+fx))^{3/2}} - \frac{4b\sec(e+fx)}{3a^2f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{8b\sec(e+fx)}{3a^3f\sqrt{a+b\sec^2(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.91

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left((3a+8b)^2+12a(a+4b)\cos(2(e+fx))+3a^2\cos(4(e+fx))\right)\sec^5(e+fx)}{48a^3f\,(a+b\sec^2(e+fx))^{5/2}}$$

```
[In] Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/48*((a + 2*b + a*Cos[2*(e + f*x)])*((3*a + 8*b)^2 + 12*a*(a + 4*b)*Cos[2
*(e + f*x)] + 3*a^2*Cos[4*(e + f*x)])*Sec[e + f*x]^5)/(a^3*f*(a + b*Sec[e +
 f*x]^2)^(5/2))
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{-\frac{1}{a\sec(fx+e)\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}-\frac{4b\left(\frac{\sec(fx+e)}{3a\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}+\frac{2\sec(fx+e)}{3a^2\sqrt{a+b\sec(fx+e)^2}}\right)}{a}}{f}$	90
default	$\frac{-\frac{1}{a\sec(fx+e)\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}-\frac{4b\left(\frac{\sec(fx+e)}{3a\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}+\frac{2\sec(fx+e)}{3a^2\sqrt{a+b\sec(fx+e)^2}}\right)}{a}}{f}$	90

```
[In] int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a/sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2)-4*b/a*(1/3*sec(f*x+e)/a/(a+b*
sec(f*x+e)^2)^(3/2)+2/3/a^2*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\left(3\,a^2\cos\left(fx+e\right)^5+12\,ab\cos\left(fx+e\right)^3+8\,b^2\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{3\left(a^5f\cos\left(fx+e\right)^4+2\,a^4bf\cos\left(fx+e\right)^2+a^3b^2f\right)}$$

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^2*cos(f*x + e)^5 + 12*a*b*cos(f*x + e)^3 + 8*b^2*cos(f*x + e))*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f
*cos(f*x + e)^2 + a^3*b^2*f)
```

Sympy [F]

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sin(e + f*x)/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.89

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\frac{3\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^3} + \frac{6\left(a+\frac{b}{\cos(fx+e)^2}\right)b\cos(fx+e)^2-b^2}{\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}a^3\cos(fx+e)^3}}{3f}$$

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^3 + (6*(a + b/cos(f*x + e
)^2)*b*cos(f*x + e)^2 - b^2)/((a + b/cos(f*x + e)^2)^(3/2)*a^3*cos(f*x + e)
^3))/f
```

Giac [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.72

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{3\sqrt{a\cos(fx+e)^2+b} + \frac{6\left(a\cos(fx+e)^2+b\right)b-b^2}{\left(a\cos(fx+e)^2+b\right)^{\frac{3}{2}}}}{3a^3f\mathrm{sgn}\left(\cos(fx+e)\right)}$$

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*sqrt(a*cos(f*x + e)^2 + b) + (6*(a*cos(f*x + e)^2 + b)*b - b^2)/(a*
cos(f*x + e)^2 + b)^(3/2))/(a^3*f*sgn(cos(f*x + e)))
```

Mupad [B] (verification not implemented)

Time = 30.61 (sec) , antiderivative size = 26927, normalized size of antiderivative = 277.60

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] int(sin(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] ((a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*(exp(e*3i
+ f*x*3i)*(((2*a + 4*b)*(((2*a + 4*b)*((((32*a*b^2 + 30*a^2*b + 3*a^3)/(48*
a^3*b*f*(a + b)) - ((2*a + 4*b)*(8*a*b^2 + 8*a^2*b + a^3))/(48*a^4*b*f*(a +
 b)))*(2*a + 4*b))/a + (8*a*b^2 + 8*a^2*b + a^3)/(48*a^3*b*f*(a + b)) - (16
*a*b + a^2 + 20*b^2)/(24*a^2*b*f*(a + b))))/a - (a + 6*b)/(24*a*b*f*(a + b)
) - (32*a*b^2 + 30*a^2*b + 3*a^3)/(48*a^3*b*f*(a + b)) + ((2*a + 4*b)*(8*a*
b^2 + 8*a^2*b + a^3))/(48*a^4*b*f*(a + b))))/a - (((32*a*b^2 + 30*a^2*b + 3
*a^3)/(48*a^3*b*f*(a + b)) - ((2*a + 4*b)*(8*a*b^2 + 8*a^2*b + a^3))/(48*a^
4*b*f*(a + b)))*(2*a + 4*b))/a - (8*a*b^2 + 8*a^2*b + a^3)/(48*a^3*b*f*(a +
 b)) + (16*a*b + a^2 + 20*b^2)/(24*a^2*b*f*(a + b)) + (32*a*b^2 + 40*a^2*b
+ 3*a^3)/(48*a^3*b*f*(a + b))) + exp(e*1i + f*x*1i)*(((2*a + 4*b)*((((32*a*
b^2 + 30*a^2*b + 3*a^3)/(48*a^3*b*f*(a + b)) - ((2*a + 4*b)*(8*a*b^2 + 8*a^
2*b + a^3))/(48*a^4*b*f*(a + b)))*(2*a + 4*b))/a + (8*a*b^2 + 8*a^2*b + a^3
)/(48*a^3*b*f*(a + b)) - (16*a*b + a^2 + 20*b^2)/(24*a^2*b*f*(a + b))))/a +
 (48*a*b^2 + 18*a^2*b + a^3 + 32*b^3)/(48*a^3*b*f*(a + b)) - (a + 6*b)/(24*
a*b*f*(a + b)) - (32*a*b^2 + 30*a^2*b + 3*a^3)/(48*a^3*b*f*(a + b)) + ((2*a
 + 4*b)*(8*a*b^2 + 8*a^2*b + a^3))/(48*a^4*b*f*(a + b))))*(2*exp(e*2i + f*x
*2i) + exp(e*4i + f*x*4i) + 1))/((exp(e*2i + f*x*2i) + 1)*(a + exp(e*2i + f
*x*2i)*(2*a + 4*b) + a*exp(e*4i + f*x*4i))) - ((a + b/(exp(- e*1i - f*x*1i)
/2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*(exp(e*1i + f*x*1i)*(((a*2i + b*4i)*(((
a*2i + b*4i)*(((a + 2*b)*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)*1i)/(48*f*(2*a
*b + a^2)*(a*b^2 + a^2*b)) - ((a*1i + ((8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^
2 + a^2*b)))*1i)/(12*f*(2*a*b + a^2)) + (a*(a + 2*b)*(8*a*b + a^2 + 8*b^2))
/(12*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a - ((a*1i + ((8*a*b + a^2 + 8*b
^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(3*f*(2*a*b + a^2)) + (a*(a + 2*b)*(8*a*
b + a^2 + 8*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a - ((a*1i + ((
8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*2i)/(3*f*(2*a*b + a^2)) + (
(a*2i + b*4i)*(((a*2i + b*4i)*(((a + 2*b)*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^
2)*1i)/(48*f*(2*a*b + a^2)*(a*b^2 + a^2*b)) - ((a*1i + ((8*a*b + a^2 + 8*b^
2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(12*f*(2*a*b + a^2)) + (a*(a + 2*b)*(8*a*
b + a^2 + 8*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a + ((a*2i + b*
4i)*(((a*1i + ((8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(3*f*(2
*a*b + a^2)) - ((a*2i + b*4i)*(((a*2i + b*4i)*(((a + 2*b)*(a*2i + b*4i)*(8*
a*b + a^2 + 8*b^2)*1i)/(48*f*(2*a*b + a^2)*(a*b^2 + a^2*b)) - ((a*1i + ((8*
a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(12*f*(2*a*b + a^2)) + (a
*(a + 2*b)*(8*a*b + a^2 + 8*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)
```

```
/a - ((a*1i + ((8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(3*f*(2
*a*b + a^2)) + (a*(a + 2*b)*(8*a*b + a^2 + 8*b^2))/(12*f*(2*a*b + a^2)*(a*b
^2 + a^2*b)))*1i)/a + ((a + 2*b)*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)*1i)/(4
8*f*(2*a*b + a^2)*(a*b^2 + a^2*b)) + (a*(a + 2*b)*(8*a*b + a^2 + 8*b^2))/(1
2*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a - ((a*1i + ((8*a*b + a^2 + 8*b^2)
^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(3*f*(2*a*b + a^2)) + (3*a*(a + 2*b)*(8*a*b
 + a^2 + 8*b^2))/(16*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a - ((a + 2*b)*(
a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)*1i)/(48*f*(2*a*b + a^2)*(a*b^2 + a^2*b))
) + exp(e*3i + f*x*3i)*(((a*2i + b*4i)*(((a + 2*b)*(a*2i + b*4i)*(8*a*b + a
^2 + 8*b^2)*1i)/(48*f*(2*a*b + a^2)*(a*b^2 + a^2*b)) - ((a*1i + ((8*a*b + a
^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(12*f*(2*a*b + a^2)) + (a*(a + 2
*b)*(8*a*b + a^2 + 8*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a + ((
a*2i + b*4i)*(((a*1i + ((8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i
)/(3*f*(2*a*b + a^2)) - ((a*2i + b*4i)*(((a*2i + b*4i)*(((a + 2*b)*(a*2i +
b*4i)*(8*a*b + a^2 + 8*b^2)*1i)/(48*f*(2*a*b + a^2)*(a*b^2 + a^2*b)) - ((a*
1i + ((8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(12*f*(2*a*b + a
^2)) + (a*(a + 2*b)*(8*a*b + a^2 + 8*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 + a^2
*b)))*1i)/a - ((a*1i + ((8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i
)/(3*f*(2*a*b + a^2)) + (a*(a + 2*b)*(8*a*b + a^2 + 8*b^2))/(12*f*(2*a*b +
a^2)*(a*b^2 + a^2*b)))*1i)/a + ((a + 2*b)*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^
2)*1i)/(48*f*(2*a*b + a^2)*(a*b^2 + a^2*b)) + (a*(a + 2*b)*(8*a*b + a^2 + 8
*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a - ((a*2i + b*4i)*(((a*2i
 + b*4i)*(((a*2i + b*4i)*(((a + 2*b)*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)*1i
)/(48*f*(2*a*b + a^2)*(a*b^2 + a^2*b)) - ((a*1i + ((8*a*b + a^2 + 8*b^2)^2*
1i)/(4*(a*b^2 + a^2*b)))*1i)/(12*f*(2*a*b + a^2)) + (a*(a + 2*b)*(8*a*b + a
^2 + 8*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a - ((a*1i + ((8*a*b
 + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(3*f*(2*a*b + a^2)) + (a*(a
+ 2*b)*(8*a*b + a^2 + 8*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a -
 ((a*1i + ((8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*3i)/(4*f*(2*a*b
 + a^2)) + ((a*2i + b*4i)*(((a*2i + b*4i)*(((a + 2*b)*(a*2i + b*4i)*(8*a*b
+ a^2 + 8*b^2)*1i)/(48*f*(2*a*b + a^2)*(a*b^2 + a^2*b)) - ((a*1i + ((8*a*b
+ a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(12*f*(2*a*b + a^2)) + (a*(a
+ 2*b)*(8*a*b + a^2 + 8*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a +
 ((a*2i + b*4i)*(((a*1i + ((8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))
*1i)/(3*f*(2*a*b + a^2)) - ((a*2i + b*4i)*(((a*2i + b*4i)*(((a + 2*b)*(a*2i
 + b*4i)*(8*a*b + a^2 + 8*b^2)*1i)/(48*f*(2*a*b + a^2)*(a*b^2 + a^2*b)) - (
(a*1i + ((8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(12*f*(2*a*b
+ a^2)) + (a*(a + 2*b)*(8*a*b + a^2 + 8*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 +
a^2*b)))*1i)/a - ((a*1i + ((8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))
*1i)/(3*f*(2*a*b + a^2)) + (a*(a + 2*b)*(8*a*b + a^2 + 8*b^2))/(12*f*(2*a*b
 + a^2)*(a*b^2 + a^2*b)))*1i)/a + ((a + 2*b)*(a*2i + b*4i)*(8*a*b + a^2 + 8
*b^2)*1i)/(48*f*(2*a*b + a^2)*(a*b^2 + a^2*b)) + (a*(a + 2*b)*(8*a*b + a^2
+ 8*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a - ((a*1i + ((8*a*b +
a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(3*f*(2*a*b + a^2)) + (3*a*(a +
 2*b)*(8*a*b + a^2 + 8*b^2))/(16*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a -
```

((a + 2*b)*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)*1i)/(48*f*(2*a*b + a^2)*(a*b
^2 + a^2*b)))*1i)/a + (a*(a + 2*b)*(8*a*b + a^2 + 8*b^2))/(6*f*(2*a*b + a^2
)*(a*b^2 + a^2*b))))*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1))/((exp
(e*2i + f*x*2i) + 1)*(a*1i + exp(e*2i + f*x*2i)*(a*2i + b*4i) + a*exp(e*4i
+ f*x*4i)*1i)^2) - exp(- e*1i - f*x*1i)*(a + b/(exp(- e*1i - f*x*1i)/2 + ex
p(e*1i + f*x*1i)/2)^2)^(1/2)*(1/(2*a^3*f) + exp(e*2i + f*x*2i)/(2*a^3*f)) -
((a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*(exp(e*3i
+ f*x*3i)*(((a*2i + b*4i)*(((a*2i + b*4i)*(((a*2i + b*4i)*((((8*a*b + a^2
+ 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a
^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(a*2i +
b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*
1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 +
10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i +
b*1i)*(a*1i + b*2i)))*1i)/a - ((a*2i + b*4i)*(((a*2i + b*4i)*(((a*2i + b*4
i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i
+ b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((
a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*
(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 +
8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b
^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + (((8*a*b + a^2 + 8*b^2)^2
/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^2*b
+ a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(8
*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*
1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*
b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*
1i + b*1i)*(a*1i + b*2i)))*1i)/a - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^
2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2
*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b))
+ (a*(a + 2*b)*1i)/(a*1i + b*2i))*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^
2))/(48*a^3*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + ((a + 2*b)^2*(8*a*b + a^2 +
8*b^2)*(13*a*b + 3*a^2 + 10*b^2)*1i)/(24*a*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*
(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*
b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i
)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)
*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*
1i)/a - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a
*1i + b*2i))*(13*a*b + 3*a^2 + 10*b^2))/(6*a^2*f*(2*a*b + a^2)*(a*1i + b*1i
)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i
+ b*2i))*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a
*1i + b*1i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b + a
^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^
2 + 8*b^2)*(44*a*b + 5*a^2 + 44*b^2)*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^
2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*
a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b
^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + ((a*2i + b*4i)*(((a*2i +

b*4i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*
1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) -
((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^
2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2
+ 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(
a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + (((8*a*b + a^2 + 8*b^2
)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^
2*b + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2
*(8*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i +
b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a
^3*b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*
(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 +
a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(44*a*b + 5*a^2 + 44*b^2))/(48*a
*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a
^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(
2*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b))
+ (a*(a + 2*b)*1i)/(a*1i + b*2i))*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b
^2))/(48*a^3*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + ((a + 2*b)^2*(8*a*b + a^2 +
8*b^2)*(13*a*b + 3*a^2 + 10*b^2)*1i)/(24*a*f*(2*a*b + a^2)*(a*b^2 + a^2*b)
*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8
*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2
i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3
)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i))
+ ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(46*a*b^2 + 10*a^2*b + a^3 + 36*b^3)*1
i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i
)/a - ((a*2i + b*4i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a
+ 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(
a*1i + b*1i)) - ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*
b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)
^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*
(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + ((a*2i
+ b*4i)*(((a*2i + b*4i)*(((a*2i + b*4i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^
2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*
b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2
+ 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i +
b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*
b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i
)))*1i)/a - ((a*2i + b*4i)*(((a*2i + b*4i)*(((a*2i + b*4i)*((((8*a*b + a^2
+ 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a
^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(a*2i +
b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*
1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 +
10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i +
b*1i)*(a*1i + b*2i)))*1i)/a + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)
) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3))/(1

2*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)
^2*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i))
+ ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2
*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i +
b*2i)))*1i)/a - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b
)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i +
b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/
(a*1i + b*2i))*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2))/(48*a^3*b*f*(2*
a*b + a^2)*(a*1i + b*1i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(13*a*b + 3*
a^2 + 10*b^2)*1i)/(24*a*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i
+ b*2i)) + ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*
(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) - ((a + 2*b)^2*(
8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a
b + a^2)(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a - (((8*a*b +
a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(13*a*
b + 3*a^2 + 10*b^2))/(6*a^2*f*(2*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2
+ 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(14*a*b^2
+ 10*a^2*b + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a
+ 2*b)^2*(8*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)
*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(44*a*b
+ 5*a^2 + 44*b^2)*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*
(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*b +
5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i +
b*1i)*(a*1i + b*2i)))*1i)/a - ((a*2i + b*4i)*(((a*2i + b*4i)*(((a*2i + b*4
i)*(((a*2i + b*4i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a +
2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*
1i + b*1i)) - ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*
f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2
*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2
*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a - ((a*2i +
b*4i)*(((a*2i + b*4i)*(((a*2i + b*4i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2
+ a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*
f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 +
8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*
2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^
3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i))
)*1i)/a + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/
(a*1i + b*2i))*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^
2)*(a*1i + b*1i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*
b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b
+ a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*
f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a - (((8*
a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*
(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b +
a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(64*a*

b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2))/(48*a^3*b*f*(2*a*b + a^2)*(a*1i + b*
1i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(13*a*b + 3*a^2 + 10*b^2)*1i)/(24
*a*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b
)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2
+ a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*
(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^
2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*
b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(13*a*b + 3*a^2 + 10*b^2))/
(6*a^2*f*(2*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2
 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^2*b + a^3 + 4
*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(8*a*b + a^2
 + 8*b^2)^2*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i
+ b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(44*a*b + 5*a^2 + 44*b^2)*1i)
/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a
+ 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2)*
1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i))
)*1i)/a + ((a*2i + b*4i)*(((a*2i + b*4i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b
^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a
*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2
 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i +
 b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4
*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2
i)))*1i)/a + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1
i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b +
 a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2
*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*
a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2
*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + ((
(8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i
))*(44*a*b + 5*a^2 + 44*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - (((8
*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))
*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b +
 a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(64*a
*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2))/(48*a^3*b*f*(2*a*b + a^2)*(a*1i + b
*1i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(13*a*b + 3*a^2 + 10*b^2)*1i)/(2
4*a*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*
b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2
 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)
*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a
^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(46
*a*b^2 + 10*a^2*b + a^3 + 36*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*
b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a - ((a*2i + b*4i)*((((8*a*b + a^2 + 8
b^2)^2/(4(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2
+ 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(a*2i + b*4
i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i

+ b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*
a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*
1i)*(a*1i + b*2i)))*1i)/a + ((a*2i + b*4i)*(((a*2i + b*4i)*(((a*2i + b*4i)*
((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b
2i))(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a +
 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*
b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b
^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2
+ a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + (((8*a*b + a^2 + 8*b^2)^2/(4
*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^2*b +
a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(8*a*
b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)
*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*b +
 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i
+ b*1i)*(a*1i + b*2i)))*1i)/a - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b
)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*
b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (
a*(a + 2*b)*1i)/(a*1i + b*2i))*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2))
/(48*a^3*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b
^2)*(13*a*b + 3*a^2 + 10*b^2)*1i)/(24*a*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*
1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2
)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i))
- ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i
)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)
/a + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i
 + b*2i))*(13*a*b + 3*a^2 + 10*b^2))/(6*a^2*f*(2*a*b + a^2)*(a*1i + b*1i))
- (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i +
b*2i))*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i
 + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i
)/(a*1i + b*2i))*(46*a*b^2 + 10*a^2*b + a^3 + 36*b^3))/(12*a^2*b*f*(2*a*b +
 a^2)*(a*1i + b*1i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2
*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) - ((a + 2*b)^2*(8*
a*b + a^2 + 8*b^2)*(44*a*b + 5*a^2 + 44*b^2)*1i)/(192*b*f*(2*a*b + a^2)*(a*
b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^4*(8*a*b + a^2 + 8*b
^2)*(16*a*b + a^2 + 16*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*
(a*1i + b*1i)*(a*1i + b*2i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*a*b^3
 + 68*a^3*b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 +
a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + ((a*2i + b*4i)*(((a*2i + b*4i)
*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i +
b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a
+ 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a
*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*
b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2
 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + (((8*a*b + a^2 + 8*b^2)^2/(
4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^2*b +

a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(8*a
*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i
)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*b
+ 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i
+ b*1i)*(a*1i + b*2i)))*1i)/a + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*
b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(44*a*b + 5*a^2 + 44*b^2))/(48*a*b*f*
(2*a*b + a^2)*(a*1i + b*1i)) - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)
) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b
+ a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a
*(a + 2*b)*1i)/(a*1i + b*2i))*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2))/
(48*a^3*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*((8*a*b + a^2 + 8*b
^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(16*a*b + a^2 +
16*b^2))/(48*a^3*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + ((a + 2*b)^2*(8*a*b +
a^2 + 8*b^2)*(a*b - a^2 + 2*b^2)*1i)/(8*a*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(
a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(13*a*b +
3*a^2 + 10*b^2)*1i)/(24*a*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*
1i + b*2i)) + ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*
f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) - ((a + 2*b)^2
*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2
*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*
a*b + a^2 + 8*b^2)*(46*a*b^2 + 10*a^2*b + a^3 + 36*b^3)*1i)/(48*a*b*f*(2*a*
b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + ((a*2i + b*4
i)*(((a*2i + b*4i)*(((a*2i + b*4i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a
^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(
2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b
^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)
) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*
1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1
i)/a + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*
1i + b*2i))*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*
(a*1i + b*1i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b +
a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b +
a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(
2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a - (((8*a*b
+ a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*
a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2
+ 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(64*a*b^3
+ 68*a^3*b + 5*a^4 + 132*a^2*b^2))/(48*a^3*b*f*(2*a*b + a^2)*(a*1i + b*1i)
) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(13*a*b + 3*a^2 + 10*b^2)*1i)/(24*a*
f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2
*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a
^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14
*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b
)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2
+ a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(a*b - a^2 + 2*b^2))/(2*a^2*f*

(2*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)
) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(13*a*b + 3*a^2 + 10*b^2))/(6*a^2*f*(2*
a*b + a^2)*(a*1i + b*1i)) - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) +
 (a*(a + 2*b)*1i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3))/(12*a
^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 +
 a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(46*a*b^2 + 10*a^2*b + a^3 + 36*
b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + ((a + 2*b)^2*(8*a*b + a^2
+ 8*b^2)^2*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i +
 b*2i)) - ((a + b)*(a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*1i)/(16*f*(2*a*b + a^2
)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) - ((a + 2*b)^2*(8*a*b + a^2
+ 8*b^2)*(44*a*b + 5*a^2 + 44*b^2)*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*
b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^4*(8*a*b + a^2 + 8*b^2)*(16*a*
b + a^2 + 16*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*
1i)*(a*1i + b*2i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*
b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*
1i + b*1i)*(a*1i + b*2i))) - exp(e*1i + f*x*1i)*(((a*2i + b*4i)*(((a*2i + b
4i)((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1
i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) -
((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2
)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2
+ 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a
*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a - ((a*2i + b*4i)*(((a*2i
+ b*4i)*(((a*2i + b*4i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a
*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2
)*(a*1i + b*1i)) - ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192
*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2
b)^2(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b
f(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + (((8
*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))
*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1
i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b + a^2)*(a*b^
2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2
)*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2
)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a - (((8*a*b + a^2 + 8*
b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 +
 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2
/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(64*a*b^3 + 68*a^3*b
 + 5*a^4 + 132*a^2*b^2))/(48*a^3*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + ((a + 2
b)^2(8*a*b + a^2 + 8*b^2)*(13*a*b + 3*a^2 + 10*b^2)*1i)/(24*a*f*(2*a*b +
a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(a*2i + b*
4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i
 + b*1i)*(a*1i + b*2i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10
*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b
1i)(a*1i + b*2i)))*1i)/a - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b))
+ (a*(a + 2*b)*1i)/(a*1i + b*2i))*(13*a*b + 3*a^2 + 10*b^2))/(6*a^2*f*(2*a*

b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (
a*(a + 2*b)*1i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3))/(12*a^2
*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)^2*1i
)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a
 + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(44*a*b + 5*a^2 + 44*b^2)*1i)/(192*b*f*(2*a
b + a^2)(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*
b + a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b
f(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a - ((a*
2i + b*4i)*(((a*2i + b*4i)*(((a*2i + b*4i)*(((a*2i + b*4i)*((((8*a*b + a^2
+ 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a
^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(a*2i +
b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*
1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 +
10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i +
 b*1i)*(a*1i + b*2i)))*1i)/a - ((a*2i + b*4i)*(((a*2i + b*4i)*(((a*2i + b*4
i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i
+ b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((
a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*
(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 +
8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b
^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + (((8*a*b + a^2 + 8*b^2)^2
/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^2*b
 + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(8
*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*
1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*
b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*
1i + b*1i)*(a*1i + b*2i)))*1i)/a - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^
2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2
*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b))
+ (a*(a + 2*b)*1i)/(a*1i + b*2i))*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^
2))/(48*a^3*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + ((a + 2*b)^2*(8*a*b + a^2 +
8*b^2)*(13*a*b + 3*a^2 + 10*b^2)*1i)/(24*a*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*
(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*
b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i
)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)
*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*
1i)/a - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a
*1i + b*2i))*(13*a*b + 3*a^2 + 10*b^2))/(6*a^2*f*(2*a*b + a^2)*(a*1i + b*1i
)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i
 + b*2i))*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a
*1i + b*1i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b + a
^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^
2 + 8*b^2)*(44*a*b + 5*a^2 + 44*b^2)*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^
2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*
a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b

^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + ((a*2i + b*4i)*(((a*2i + b*4i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(44*a*b + 5*a^2 + 44*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2))/(48*a^3*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(13*a*b + 3*a^2 + 10*b^2)*1i)/(24*a*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(46*a*b^2 + 10*a^2*b + a^3 + 36*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a - ((a*2i + b*4i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + ((a*2i + b*4i)*(((a*2i + b*4i)*(((a*2i + b*4i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i)

)*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2))/(48*a^3*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(13*a*b + 3*a^2 + 10*b^2)*1i)/(24*a*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(13*a*b + 3*a^2 + 10*b^2))/(6*a^2*f*(2*a*b + a^2)*(a*1i + b*1i)) - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(46*a*b^2 + 10*a^2*b + a^3 + 36*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(44*a*b + 5*a^2 + 44*b^2)*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^4*(8*a*b + a^2 + 8*b^2)*(16*a*b + a^2 + 16*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + ((a*2i + b*4i)*(((a*2i + b*4i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + ((a + b)*((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i)))/(4*a*f*(2*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(44*a*b + 5*a^2 + 44*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2))/(48*a^3*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(16*a*b + a^2 + 16*b^2))/(48*a^3*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(a*b - a^2 + 2*b^2)*1i)/(8*a*f*(2*a*b + a^2)*

```
(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 +
8*b^2)*(13*a*b + 3*a^2 + 10*b^2)*1i)/(24*a*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*
(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*
b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i
)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)
*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) +
 ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(46*a*b^2 + 10*a^2*b + a^3 + 36*b^3)*1i
)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i))))*(2
*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1))/((exp(e*2i + f*x*2i) + 1)*(a
*1i + exp(e*2i + f*x*2i)*(a*2i + b*4i) + a*exp(e*4i + f*x*4i)*1i))
```

3.122 $\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)^{5/2}f} - \frac{b\sec(e+fx)}{3a(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{b(5a+2b)\sec(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b)^(5/2)/f-1/3
*b*sec(f*x+e)/a/(a+b)/f/(a+b*sec(f*x+e)^2)^(3/2)-1/3*b*(5*a+2*b)*sec(f*x+e)
/a^2/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4219, 425, 541, 12, 385, 213}

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{b(5a+2b)\sec(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\sec^2(e+fx)}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f(a+b)^{5/2}} - \frac{b\sec(e+fx)}{3af(a+b)\,(a+b\sec^2(e+fx))^{3/2}}$$

```
[In] Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]]/((a + b)^(
5/2)*f)) - (b*Sec[e + f*x])/(3*a*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)) -
(b*(5*a + 2*b)*Sec[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{b\sec(e+fx)}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a+2b-2bx^2}{(-1+x^2)(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{3a(a+b)f}$$

$$= -\frac{b\sec(e+fx)}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b(5a+2b)\sec(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{3a^2}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{3a^2(a+b)^2f}$$

$$= -\frac{b\sec(e+fx)}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b(5a+2b)\sec(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{(a+b)^2f}$$

$$= -\frac{b\sec(e+fx)}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b(5a+2b)\sec(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-1-(-a-b)x^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)^2f}$$

$$= -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)^{5/2}f} - \frac{b\sec(e+fx)}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b(5a+2b)\sec(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.86 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.85

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^5(e+fx)\left(a^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\right.\right.}{6a^2(a+b)f\left(a+b\sec^2(e+\right.}$$

```
[In] Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^5*(a^2*Hypergeometric2F1[-3/2,
 1, -1/2, 1 - (a*Sin[e + f*x]^2)/(a + b)] + (a + b)*(-2*(2*a + b) + 3*a*Sin
[e + f*x]^2)))/(6*a^2*(a + b)*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3289 vs. 2(113) = 226.

Time = 5.23 (sec) , antiderivative size = 3290, normalized size of antiderivative = 25.91

method	result	size
default	Expression too large to display	3290

```
[In] int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/f/a^2/(2*(-a*b)^(1/2)+a-b)^2/(2*(-a*b)^(1/2)-a+b)^2/(a+b)^(1/2)*(b+a*c
os(f*x+e)^2)*(10*(a+b)^(5/2)*a*b^2+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*cos(f*x+e)^3*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^5+3*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*cos(f*x+e)^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^5+12*cos(f*x+e)^2*(a+b)^(5/2)*
a^2*b+6*cos(f*x+e)^2*(a+b)^(5/2)*a*b^2+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*cos(f*x+e)^2*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^5+3*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*cos(f*x+e)^2*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^5+3*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^4*b+6*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^3*b^2+3*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^2*b^3+3*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^
(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^4*b+6*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1
/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(
f*x+e)*a+b)/(-1+cos(f*x+e)))*a^3*b^2+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(
f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a
+b)/(-1+cos(f*x+e)))*a^2*b^3+4*(a+b)^(5/2)*b^3+6*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*cos(f*x+e)^3*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
```

```
^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^4*b+3*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^3*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^3*b^2+
6*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^3*ln(-4*(((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^4
*b+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^3*ln(-4*(((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*
a^3*b^2+6*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^2*ln(2/(a+
b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e
)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(
1+cos(f*x+e)))*a^4*b+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+
e)^2*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1
/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(
f*x+e)*a+b)/(1+cos(f*x+e)))*a^3*b^2+6*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*cos(f*x+e)^2*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b
)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+
cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^4*b+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*cos(f*x+e)^2*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(
a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/
2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^3*b^2+3*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*cos(f*x+e)*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^4*b+6*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^3*b^2+3*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*ln(2/(a+b)^(1/2)*(((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^
2*b^3+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*ln(-4*(((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))
*a^4*b+6*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*ln(-4*(((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e))
)*a^3*b^2+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*ln(-4*((
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+
e)))*a^2*b^3)/(a+b*sec(f*x+e)^2)^(5/2)*sec(f*x+e)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(113) = 226$.

Time = 0.36 (sec) , antiderivative size = 592, normalized size of antiderivative = 4.66

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \left[\frac{3\left(a^4\cos\left(fx+e\right)^4+2\,a^3b\cos\left(fx+e\right)^2+a^2b^2\right)\sqrt{a+b}\log\left(\frac{2\left(a\cos(fx+e)^2-\cdots\right.}{\cdots}\right.}{6\left(\left(a^7+3\,a^6b+3\,a^5b^2+a^4b^3\right)f\cos\left(\cdots\right.\right.}\right.$$

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^4*cos(f*x + e)^4 + 2*a^3*b*cos(f*x + e)^2 + a^2*b^2)*sqrt(a + b)
*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) - 2*(3*(2*a^3*b + 3
*a^2*b^2 + a*b^3)*cos(f*x + e)^3 + (5*a^2*b^2 + 7*a*b^3 + 2*b^4)*cos(f*x +
e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 3*a^6*b + 3*a^5*b^
2 + a^4*b^3)*f*cos(f*x + e)^4 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)
*f*cos(f*x + e)^2 + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f), 1/3*(3*
(a^4*cos(f*x + e)^4 + 2*a^3*b*cos(f*x + e)^2 + a^2*b^2)*sqrt(-a - b)*arctan
(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a +
 b)) - (3*(2*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^3 + (5*a^2*b^2 + 7*a*b
^3 + 2*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^
7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^4 + 2*(a^6*b + 3*a^5*b^2
+ 3*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4
+ a^2*b^5)*f)]
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\csc\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(csc(e + f*x)/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\csc(fx+e)}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(csc(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1097 vs. $2(113) = 226$.

Time = 1.29 (sec) , antiderivative size = 1097, normalized size of antiderivative = 8.64

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] -1/6*(4*(((((3*a^6*b^3 + 16*a^5*b^4 + 35*a^4*b^5 + 40*a^3*b^6 + 25*a^2*b^7 +
 8*a*b^8 + b^9)*tan(1/2*f*x + 1/2*e)^2/(a^8*b^2*sgn(cos(f*x + e)) + 6*a^7*b
^3*sgn(cos(f*x + e)) + 15*a^6*b^4*sgn(cos(f*x + e)) + 20*a^5*b^5*sgn(cos(f*
x + e)) + 15*a^4*b^6*sgn(cos(f*x + e)) + 6*a^3*b^7*sgn(cos(f*x + e)) + a^2*
b^8*sgn(cos(f*x + e))) - 3*(a^6*b^3 + 2*a^5*b^4 - 3*a^4*b^5 - 12*a^3*b^6 -
13*a^2*b^7 - 6*a*b^8 - b^9)/(a^8*b^2*sgn(cos(f*x + e)) + 6*a^7*b^3*sgn(cos(
f*x + e)) + 15*a^6*b^4*sgn(cos(f*x + e)) + 20*a^5*b^5*sgn(cos(f*x + e)) + 1
5*a^4*b^6*sgn(cos(f*x + e)) + 6*a^3*b^7*sgn(cos(f*x + e)) + a^2*b^8*sgn(cos
(f*x + e))))*tan(1/2*f*x + 1/2*e)^2 - 3*(a^6*b^3 + 2*a^5*b^4 - 3*a^4*b^5 -
12*a^3*b^6 - 13*a^2*b^7 - 6*a*b^8 - b^9)/(a^8*b^2*sgn(cos(f*x + e)) + 6*a^7
*b^3*sgn(cos(f*x + e)) + 15*a^6*b^4*sgn(cos(f*x + e)) + 20*a^5*b^5*sgn(cos(
f*x + e)) + 15*a^4*b^6*sgn(cos(f*x + e)) + 6*a^3*b^7*sgn(cos(f*x + e)) + a^
2*b^8*sgn(cos(f*x + e))))*tan(1/2*f*x + 1/2*e)^2 + (3*a^6*b^3 + 16*a^5*b^4
+ 35*a^4*b^5 + 40*a^3*b^6 + 25*a^2*b^7 + 8*a*b^8 + b^9)/(a^8*b^2*sgn(cos(f*
x + e)) + 6*a^7*b^3*sgn(cos(f*x + e)) + 15*a^6*b^4*sgn(cos(f*x + e)) + 20*a
^5*b^5*sgn(cos(f*x + e)) + 15*a^4*b^6*sgn(cos(f*x + e)) + 6*a^3*b^7*sgn(cos
(f*x + e)) + a^2*b^8*sgn(cos(f*x + e))))/(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(
1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^
2 + a + b)^(3/2) - 3*log(abs(-sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a*t
an(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)
^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - sqrt(a + b)))/((a^2 + 2*a*b + b^
2)*sqrt(a + b)*sgn(cos(f*x + e))) - 3*log(abs((sqrt(a + b)*tan(1/2*f*x + 1/
2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan
(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a + b) - a
```

```
+ b))/((a^2 + 2*a*b + b^2)*sqrt(a + b)*sgn(cos(f*x + e))) + 3*log(abs((sqrt
(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*
f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 +
a + b))*sqrt(a + b) - a - b))/((a^2 + 2*a*b + b^2)*sqrt(a + b)*sgn(cos(f*x
+ e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

```
[In] int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(5/2)),x)
[Out] int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

3.123 $\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 171

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{(a-4b)\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^{7/2}f} - \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{5b\sec(e+fx)}{6(a+b)^2f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{(13a-2b)b\sec(e+fx)}{6a(a+b)^3f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -1/2*(a-4*b)*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b)
^(7/2)/f-1/2*cot(f*x+e)*csc(f*x+e)/(a+b)/f/(a+b*sec(f*x+e)^2)^(3/2)-5/6*b*s
ec(f*x+e)/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(3/2)-1/6*(13*a-2*b)*b*sec(f*x+e)/a/
(a+b)^3/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4219, 482, 541, 12, 385, 213}

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{(a-4b)\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f(a+b)^{7/2}} - \frac{b(13a-2b)\sec(e+fx)}{6af(a+b)^3\sqrt{a+b\sec^2(e+fx)}} - \frac{5b\sec(e+fx)}{6f(a+b)^2\,(a+b\sec^2(e+fx))^{3/2}} - \frac{\cot(e+fx)\csc(e+fx)}{2f(a+b)\,(a+b\sec^2(e+fx))^{3/2}}$$

```
[In] Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/2*((a - 4*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^
2]])/((a + b)^(7/2)*f) - (Cot[e + f*x]*Csc[e + f*x])/(2*(a + b)*f*(a + b*Se
c[e + f*x]^2)^(3/2)) - (5*b*Sec[e + f*x])/(6*(a + b)^2*f*(a + b*Sec[e + f*x
]^2)^(3/2)) - ((13*a - 2*b)*b*Sec[e + f*x])/(6*a*(a + b)^3*f*Sqrt[a + b*Sec
[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
```

```
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{(-1+x^2)^2(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f} \\
&= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a-4bx^2}{(-1+x^2)(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{2(a+b)f} \\
&= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{5b\sec(e+fx)}{6(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} \\
&\quad + \frac{\text{Subst}\left(\int \frac{a(3a-2b)-10abx^2}{(-1+x^2)(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{6a(a+b)^2f} \\
&= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{5b\sec(e+fx)}{6(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} \\
&\quad - \frac{(13a-2b)b\sec(e+fx)}{6a(a+b)^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{3a^2(a-4b)}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{6a^2(a+b)^3f} \\
&= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{5b\sec(e+fx)}{6(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} \\
&\quad - \frac{(13a-2b)b\sec(e+fx)}{6a(a+b)^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{(a-4b)\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{2(a+b)^3f} \\
&= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{5b\sec(e+fx)}{6(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} \\
&\quad - \frac{(13a-2b)b\sec(e+fx)}{6a(a+b)^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{(a-4b)\text{Subst}\left(\int \frac{1}{-1-(-a-b)x^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^3f} \\
&= -\frac{(a-4b)\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^{7/2}f} - \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} \\
&\quad - \frac{5b\sec(e+fx)}{6(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(13a-2b)b\sec(e+fx)}{6a(a+b)^3f\sqrt{a+b\sec^2(e+fx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.03 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.88

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left((a+b)\left(3a^2+6ab-2b^2+(3a^2+2b^2)\cos(2(e+fx))\right)\csc^2(e+fx)-3a(a-\right.}{24a(a+b)^3f\,(a+b\sec^2}$$

[In] Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]

[Out] -1/24*((a + 2*b + a*Cos[2*(e + f*x)])*((a + b)*(3*a^2 + 6*a*b - 2*b^2 + (3*a^2 + 2*b^2)*Cos[2*(e + f*x)])*Csc[e + f*x]^2 - 3*a*(a - 4*b)*(a + 2*b + a*Cos[2*(e + f*x)])*Hypergeometric2F1[-1/2, 1, 1/2, 1 - (a*Sin[e + f*x]^2)/(a + b)])*Sec[e + f*x]^5)/(a*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^(5/2))

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 8688 vs. 2(151) = 302.

Time = 5.54 (sec) , antiderivative size = 8689, normalized size of antiderivative = 50.81

method	result	size
default	Expression too large to display	8689

[In] int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] result too large to display

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(151) = 302.

Time = 0.43 (sec) , antiderivative size = 941, normalized size of antiderivative = 5.50

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \left[-\frac{3\left((a^4-4a^3b)\cos(fx+e)^6-(a^4-6a^3b+8a^2b^2)\cos(fx+e)^4-a^2b^2\right.}{12\left((a^7+4a^6b+6a^5b^2\right.}\right.$$

[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")

```
[Out] [-1/12*(3*((a^4 - 4*a^3*b)*cos(f*x + e)^6 - (a^4 - 6*a^3*b + 8*a^2*b^2)*cos
(f*x + e)^4 - a^2*b^2 + 4*a*b^3 - (2*a^3*b - 9*a^2*b^2 + 4*a*b^3)*cos(f*x +
 e)^2)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 + 2*sqrt(a + b)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) -
 2*(3*(a^4 - 3*a^3*b - 4*a^2*b^2)*cos(f*x + e)^5 + 2*(9*a^3*b + 4*a^2*b^2 -
 4*a*b^3 + b^4)*cos(f*x + e)^3 + (13*a^2*b^2 + 11*a*b^3 - 2*b^4)*cos(f*x +
e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 4*a^6*b + 6*a^5*b^
2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*
a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^4 - (2*a^6*b + 7*a^5*b^2 +
8*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - a*b^6)*f*cos(f*x + e)^2 - (a^5*b^2 + 4*
a^4*b^3 + 6*a^3*b^4 + 4*a^2*b^5 + a*b^6)*f), 1/6*(3*((a^4 - 4*a^3*b)*cos(f*
x + e)^6 - (a^4 - 6*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 - a^2*b^2 + 4*a*b^3 -
 (2*a^3*b - 9*a^2*b^2 + 4*a*b^3)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(sqrt(-
a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) +
(3*(a^4 - 3*a^3*b - 4*a^2*b^2)*cos(f*x + e)^5 + 2*(9*a^3*b + 4*a^2*b^2 - 4*
a*b^3 + b^4)*cos(f*x + e)^3 + (13*a^2*b^2 + 11*a*b^3 - 2*b^4)*cos(f*x + e))
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 4*a^6*b + 6*a^5*b^2 +
 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4
*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^4 - (2*a^6*b + 7*a^5*b^2 + 8*a
^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - a*b^6)*f*cos(f*x + e)^2 - (a^5*b^2 + 4*a^4
*b^3 + 6*a^3*b^4 + 4*a^2*b^5 + a*b^6)*f)]
```

Sympy [F]

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\csc^3{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(csc(e + f*x)**3/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\csc(fx+e)^3}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\sin(e+f\,x)^3\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

```
[In] int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(5/2)),x)
[Out] int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

3.124 $\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 234

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{(3a^2-24ab+8b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{9/2}f} - \frac{(5a-2b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{(23a-12b)b\sec(e+fx)}{24(a+b)^3f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{5(11a-10b)b\sec(e+fx)}{24(a+b)^4f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -1/8*(3*a^2-24*a*b+8*b^2)*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)
^(1/2))/(a+b)^(9/2)/f-1/8*(5*a-2*b)*cot(f*x+e)*csc(f*x+e)/(a+b)^2/f/(a+b*se
c(f*x+e)^2)^(3/2)-1/4*cot(f*x+e)^3*csc(f*x+e)/(a+b)/f/(a+b*sec(f*x+e)^2)^(3
/2)-1/24*(23*a-12*b)*b*sec(f*x+e)/(a+b)^3/f/(a+b*sec(f*x+e)^2)^(3/2)-5/24*(
11*a-10*b)*b*sec(f*x+e)/(a+b)^4/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4219, 481, 541, 12, 385, 213}

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{(3a^2-24ab+8b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f(a+b)^{9/2}} - \frac{5b(11a-10b)\sec(e+fx)}{24f(a+b)^4\sqrt{a+b\sec^2(e+fx)}} - \frac{b(23a-12b)\sec(e+fx)}{24f(a+b)^3\,(a+b\sec^2(e+fx))^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4f(a+b)\,(a+b\sec^2(e+fx))^{3/2}} - \frac{(5a-2b)\cot(e+fx)\csc(e+fx)}{8f(a+b)^2\,(a+b\sec^2(e+fx))^{3/2}}$$

```
[In] Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/8*((3*a^2 - 24*a*b + 8*b^2)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a +
b*Sec[e + f*x]^2]])/((a + b)^(9/2)*f) - ((5*a - 2*b)*Cot[e + f*x]*Csc[e + f
*x])/(8*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (Cot[e + f*x]^3*Csc[e +
 f*x])/(4*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)) - ((23*a - 12*b)*b*Sec[e
+ f*x])/(24*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (5*(11*a - 10*b)*b*
Sec[e + f*x])/(24*(a + b)^4*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^3(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-a-2(2a-b)x^2}{(-1+x^2)^2(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{4(a+b)f}$$

$$= -\frac{(5a-2b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-a(3a-4b)+4(5a-2b)bx^2}{(-1+x^2)(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{8(a+b)^2f}$$

$$= -\frac{(5a-2b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(23a-12b)b\sec(e+fx)}{24(a+b)^3f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-a^2(9a-26b)+2a(23a-12b)bx^2}{(-1+x^2)(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{24a(a+b)^3f}$$

$$= -\frac{(5a-2b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(23a-12b)b\sec(e+fx)}{24(a+b)^3f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{5(11a-10b)b\sec(e+fx)}{24(a+b)^4f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int -\frac{3a^2(3a^2-24ab+8b^2)}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{24a^2(a+b)^4f}$$

$$= -\frac{(5a-2b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(23a-12b)b\sec(e+fx)}{24(a+b)^3f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{5(11a-10b)b\sec(e+fx)}{24(a+b)^4f\sqrt{a+b\sec^2(e+fx)}} + \frac{(3a^2-24ab+8b^2)\,\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{8(a+b)^4f}$$

$$= -\frac{(5a-2b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(23a-12b)b\sec(e+fx)}{24(a+b)^3f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{5(11a-10b)b\sec(e+fx)}{24(a+b)^4f\sqrt{a+b\sec^2(e+fx)}} + \frac{(3a^2-24ab+8b^2)\,\text{Subst}\left(\int \frac{1}{-1-(-a-b)x^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^4f}$$

$$= -\frac{(3a^2-24ab+8b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{9/2}f} - \frac{(5a-2b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(23a-12b)b\sec(e+fx)}{24(a+b)^3f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{5(11a-10b)b\sec(e+fx)}{24(a+b)^4f\sqrt{a+b\sec^2(e+fx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.55

$$\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left(3(a+b)(3a-4b+(a+8b)\cos(2(e+fx)))\csc^4(e+fx)-2(3a^2-24ab+8\right.}{96(a+b)^3f\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/96*((a + 2*b + a*Cos[2*(e + f*x)])*(3*(a + b)*(3*a - 4*b + (a + 8*b)*Cos
[2*(e + f*x)])*Csc[e + f*x]^4 - 2*(3*a^2 - 24*a*b + 8*b^2)*Hypergeometric2F
1[-3/2, 1, -1/2, 1 - (a*Sin[e + f*x]^2)/(a + b)])*Sec[e + f*x]^5)/((a + b)^
3*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 8114 vs. 2(210) = 420.

Time = 6.06 (sec) , antiderivative size = 8115, normalized size of antiderivative = 34.68

method	result	size
default	Expression too large to display	8115

```
[In] int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 644 vs. $2(210) = 420$.

Time = 0.56 (sec) , antiderivative size = 1307, normalized size of antiderivative = 5.59

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((3*a^4 - 24*a^3*b + 8*a^2*b^2)*cos(f*x + e)^8 - 2*(3*a^4 - 27*a^3
*b + 32*a^2*b^2 - 8*a*b^3)*cos(f*x + e)^6 + (3*a^4 - 36*a^3*b + 107*a^2*b^2
 - 56*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 3*a^2*b^2 - 24*a*b^3 + 8*b^4 + 2*(3*a
^3*b - 27*a^2*b^2 + 32*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqrt(a + b)*log(2*(a*
cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) + 2*(3*(3*a^4 - 21*a^3*b - 16
*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^7 - (15*a^4 - 117*a^3*b + 4*a^2*b^2 + 104*
a*b^3 - 32*b^4)*cos(f*x + e)^5 - (78*a^3*b - 71*a^2*b^2 - 61*a*b^3 + 88*b^4
)*cos(f*x + e)^3 - 5*(11*a^2*b^2 + a*b^3 - 10*b^4)*cos(f*x + e))*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^
3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 -
5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2
- 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*cos(f*x + e)^4 + 2*(
a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*cos(f*x + e)^2
 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f), 1/24
*(3*((3*a^4 - 24*a^3*b + 8*a^2*b^2)*cos(f*x + e)^8 - 2*(3*a^4 - 27*a^3*b +
32*a^2*b^2 - 8*a*b^3)*cos(f*x + e)^6 + (3*a^4 - 36*a^3*b + 107*a^2*b^2 - 56
*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 3*a^2*b^2 - 24*a*b^3 + 8*b^4 + 2*(3*a^3*b
- 27*a^2*b^2 + 32*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(sqrt(-
a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) +
(3*(3*a^4 - 21*a^3*b - 16*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^7 - (15*a^4 - 117
*a^3*b + 4*a^2*b^2 + 104*a*b^3 - 32*b^4)*cos(f*x + e)^5 - (78*a^3*b - 71*a^
2*b^2 - 61*a*b^3 + 88*b^4)*cos(f*x + e)^3 - 5*(11*a^2*b^2 + a*b^3 - 10*b^4)
*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 5*a^6*b
 + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7
 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 +
(a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^
7)*f*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^
6 - b^7)*f*cos(f*x + e)^2 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5
+ 5*a*b^6 + b^7)*f)]
```

Sympy [F]

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(csc(e + f*x)**5/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\csc(fx+e)^5}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Hanged}$$

```
[In] int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(5/2)),x)
[Out] \text{Hanged}
```

3.125 $\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 288

$$\begin{aligned}
\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = {} & \frac{5(a+b)\,(a^2+14ab+21b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{11/2}f} \\
& - \frac{(a+b)(11a+21b)\cos(e+fx)\sin(e+fx)}{16a^3f\,(a+b+b\tan^2(e+fx))^{3/2}} \\
& + \frac{3(a+b)\cos^3(e+fx)\sin(e+fx)}{8a^2f\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\,(a+b+b\tan^2(e+fx))^{3/2}} \\
& - \frac{7b(a+b)(7a+15b)\tan(e+fx)}{48a^4f\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{b(113a^2+420ab+315b^2)\tan(e+fx)}{48a^5f\sqrt{a+b+b\tan^2(e+fx)}}
\end{aligned}$$

```
[Out] 5/16*(a+b)*(a^2+14*a*b+21*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^
2)^(1/2))/a^(11/2)/f-1/48*b*(113*a^2+420*a*b+315*b^2)*tan(f*x+e)/a^5/f/(a+b
+b*tan(f*x+e)^2)^(1/2)-1/16*(a+b)*(11*a+21*b)*cos(f*x+e)*sin(f*x+e)/a^3/f/(
a+b+b*tan(f*x+e)^2)^(3/2)+3/8*(a+b)*cos(f*x+e)^3*sin(f*x+e)/a^2/f/(a+b+b*ta
n(f*x+e)^2)^(3/2)+1/6*cos(f*x+e)^3*sin(f*x+e)^3/a/f/(a+b+b*tan(f*x+e)^2)^(3
/2)-7/48*b*(a+b)*(7*a+15*b)*tan(f*x+e)/a^4/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4217, 481, 592, 541, 12, 385, 209}

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{7b(a+b)(7a+15b)\tan(e+fx)}{48a^4f\left(a+b\tan^2(e+fx)+b\right)^{3/2}} - \frac{(a+b)(11a+21b)\sin(e+fx)\cos(e+fx)}{16a^3f\left(a+b\tan^2(e+fx)+b\right)^{3/2}} + \frac{3(a+b)\sin(e+fx)\cos^3(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)+b\right)^{3/2}} + \frac{5(a+b)\left(a^2+14ab+21b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{11/2}f} - \frac{b\left(113a^2+420ab+315b^2\right)\tan(e+fx)}{48a^5f\sqrt{a+b\tan^2(e+fx)+b}} + \frac{\sin^3(e+fx)\cos^3(e+fx)}{6af\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[In] Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (5*(a + b)*(a^2 + 14*a*b + 21*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b
 + b*Tan[e + f*x]^2]])/(16*a^(11/2)*f) - ((a + b)*(11*a + 21*b)*Cos[e + f*x
]*Sin[e + f*x])/(16*a^3*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (3*(a + b)*Co
s[e + f*x]^3*Sin[e + f*x])/(8*a^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (Co
s[e + f*x]^3*Sin[e + f*x]^3)/(6*a*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (7*
b*(a + b)*(7*a + 15*b)*Tan[e + f*x])/(48*a^4*f*(a + b + b*Tan[e + f*x]^2)^(
3/2)) - (b*(113*a^2 + 420*a*b + 315*b^2)*Tan[e + f*x])/(48*a^5*f*Sqrt[a + b
 + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^4(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)-6(a+b)x^2)}{(1+x^2)^3(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{6af}$$

$$=\frac{3(a+b)\cos^3(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}+\frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}}-\frac{\text{Subst}\left(\int\frac{9(a+b)^2-6(a+b)(4a+9b)x^2}{(1+x^2)^2(a+b+bx^2)^{5/2}}\,dx,x,\tan(e+fx)\right)}{24a^2f}$$

$$=-\frac{(a+b)(11a+21b)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}+\frac{3(a+b)\cos^3(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}+\frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}}+\frac{\text{Subst}\left(\int\frac{3(a+b)^2(5a+21b)-12b(a+b)(11a+21b)x^2}{(1+x^2)(a+b+bx^2)^{5/2}}\,dx,x,\tan(e+fx)\right)}{48a^3f}$$

$$=-\frac{(a+b)(11a+21b)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}+\frac{3(a+b)\cos^3(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}+\frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}}-\frac{7b(a+b)(7a+15b)\tan(e+fx)}{48a^4f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}+\frac{\text{Subst}\left(\int\frac{3(a+b)^2(15a^2+112ab+105b^2)-42b(a+b)^2(7a+15b)x^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx,x,\tan(e+fx)\right)}{144a^4(a+b)f}$$

$$=-\frac{(a+b)(11a+21b)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}+\frac{3(a+b)\cos^3(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}+\frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}}-\frac{7b(a+b)(7a+15b)\tan(e+fx)}{48a^4f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}-\frac{b(113a^2+420ab+315b^2)\tan(e+fx)}{48a^5f\sqrt{a+b+b\tan^2(e+fx)}}+\frac{\text{Subst}\left(\int\frac{45(a+b)^3(a^2+14ab+21b^2)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx,x,\tan(e+fx)\right)}{144a^5(a+b)^2f}$$

$$=-\frac{(a+b)(11a+21b)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}+\frac{3(a+b)\cos^3(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}+\frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}}-\frac{7b(a+b)(7a+15b)\tan(e+fx)}{48a^4f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}-\frac{b(113a^2+420ab+315b^2)\tan(e+fx)}{48a^5f\sqrt{a+b+b\tan^2(e+fx)}}+\frac{(5(a+b)\left(a^2+14ab+21b^2\right))\text{Subst}\left(\int\frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx,x,\tan(e+fx)\right)}{16a^5f}$$

$$= -\frac{(a+b)(11a+21b)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{3(a+b)\cos^3(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{7b(a+b)(7a+15b)\tan(e+fx)}{48a^4f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(113a^2+420ab+315b^2)\tan(e+fx)}{48a^5f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5(a+b)\left(a^2+14ab+21b^2\right))\operatorname{Subst}\left(\int\frac{1}{1+ax^2}\,dx,x,\frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^5f}$$

$$= \frac{5(a+b)\left(a^2+14ab+21b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{11/2}f} - \frac{(a+b)(11a+21b)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{3(a+b)\cos^3(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{7b(a+b)(7a+15b)\tan(e+fx)}{48a^4f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(113a^2+420ab+315b^2)\tan(e+fx)}{48a^5f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1705 vs. $2(288) = 576$.

Time = 16.24 (sec) , antiderivative size = 1705, normalized size of antiderivative = 5.92

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx =$$

$$-\frac{\left(\frac{a+2b+a\cos(2(e+fx))}{a+b}\right)^{3/2}(a+2b+a\cos(2e+2fx))^{5/2}\sec^5(e+fx)\left(-60\sqrt{a+b}(3a^3+17a^2b+28ab^2+14\right.}{}$$

$$+\frac{\left(\frac{a+2b+a\cos(2(e+fx))}{a+b}\right)^{3/2}(a+2b+a\cos(2e+2fx))^{5/2}\sec^5(e+fx)\left(420\sqrt{a+b}(a^4+9a^3b+26a^2b^2+30ab^3\right.}{}$$

$$-\frac{5(a+2b+a\cos(2e+2fx))^{5/2}\csc(e+fx)\sec^5(e+fx)\left(\frac{\sin^2(e+fx)}{a+b}+\frac{(a+2b+a\cos(2(e+fx)))\sin^2(e+fx)}{(a+b)^2}-\frac{12\sin^4(}{a+}\right.}{12288\sqrt{2}f\left(a+b\sec^2(e+fx\right.}$$

$$+\frac{5(a+2b+a\cos(2e+2fx))^{5/2}\csc(e+fx)\sec^5(e+fx)\left(\frac{\sin^2(e+fx)}{a+b}+\frac{(a+2b+a\cos(2(e+fx)))\sin^2(e+fx)}{(a+b)^2}-\frac{24\sin^4(e}{a+}\right.}{}$$

$$+\frac{5(2a+3b+a\cos(2(e+fx)))(a+2b+a\cos(2e+2fx))^{5/2}\sec^4(e+fx)\tan(e+fx)}{3072(a+b)^2f(a+2b+a\cos(2(e+fx)))^{3/2}\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

$$-\frac{5(b+(3a+2b)\cos(2(e+fx)))(a+2b+a\cos(2e+2fx))^{5/2}\sec^4(e+fx)\tan(e+fx)}{3072(a+b)^2f(a+2b+a\cos(2(e+fx)))^{3/2}\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

```
[In] Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/3072*(((a + 2*b + a*Cos[2*(e + f*x)])/(a + b))^(3/2)*(a + 2*b + a*Cos[2*
e + 2*f*x])^(5/2)*Sec[e + f*x]^5*(-60*Sqrt[a + b]*(3*a^3 + 17*a^2*b + 28*a*
b^2 + 14*b^3)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2
*(e + f*x)])^2 + Sqrt[a]*Sin[e + f*x]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a +
b)]*(3*(239*a^5 + 1839*a^4*b + 5200*a^3*b^2 + 6960*a^2*b^3 + 4480*a*b^4 + 1
120*b^5) - 2*a*(459*a^4 + 3180*a^3*b + 7200*a^2*b^2 + 6720*a*b^3 + 2240*b^4
)*Sin[e + f*x]^2 + 672*a^2*b*(a + b)^2*Sin[e + f*x]^4 + 192*a^3*(a + b)^2*S
in[e + f*x]^6)))/(Sqrt[2]*a^(9/2)*f*(a + 2*b + a*Cos[2*(e + f*x)])^(7/2)*(a
 + b*Sec[e + f*x]^2)^(5/2)) + (((a + 2*b + a*Cos[2*(e + f*x)])/(a + b))^(3/
2)*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5*(420*Sqrt[a + b]*(a^
4 + 9*a^3*b + 26*a^2*b^2 + 30*a*b^3 + 12*b^4)*ArcSin[(Sqrt[a]*Sin[e + f*x])
/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 - Sqrt[a]*Sin[e + f*x]*Sqrt[
(a + b - a*Sin[e + f*x]^2)/(a + b)]*(3*(561*a^6 + 6161*a^5*b + 25200*a^4*b^
2 + 50960*a^3*b^3 + 54880*a^2*b^4 + 30240*a*b^5 + 6720*b^6) - 2*a*(1151*a^5
```

```
 + 11230*a^4*b + 39200*a^3*b^2 + 62720*a^2*b^3 + 47040*a*b^4 + 13440*b^5)*S
in[e + f*x]^2 + 672*a^2*(a + b)^2*(a^2 + 3*a*b + 6*b^2)*Sin[e + f*x]^4 - 57
6*a^3*(a - 2*b)*(a + b)^2*Sin[e + f*x]^6 + 512*a^4*(a + b)^2*Sin[e + f*x]^8
)))/(3072*Sqrt[2]*a^(11/2)*f*(a + 2*b + a*Cos[2*(e + f*x)])^(7/2)*(a + b*Se
c[e + f*x]^2)^(5/2)) - (5*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Csc[e + f*x]
*Sec[e + f*x]^5*(Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*S
in[e + f*x]^2)/(a + b)^2 - (12*Sin[e + f*x]^4)/(a + b) + (16*(a + b - a*Sin
[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*x]^2
)/(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b - a*
Sin[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sq
rt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^3))/(
12288*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a + b - a*Sin[e + f*x]^2)^(3/
2)) + (5*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Csc[e + f*x]*Sec[e + f*x]^5*(
Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*Sin[e + f*x]^2)/(a
 + b)^2 - (24*Sin[e + f*x]^4)/(a + b) + (96*Sin[e + f*x]^6)/a + (80*(a + b
- a*Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e +
 f*x]^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a +
 b - a*Sin[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f
*x])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/
a^3 - (160*(a + b - a*Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6
*a*(a + b)^2*Sin[e + f*x]^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (3*Sqrt[a]*(a
 + b)^(3/2)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(
a + b - a*Sin[e + f*x]^2)/(a + b)] + (a^2*Sin[e + f*x]^4)/(-1 + (a*Sin[e +
f*x]^2)/(a + b))^2))/a^4))/(12288*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a
 + b - a*Sin[e + f*x]^2)^(3/2)) + (5*(2*a + 3*b + a*Cos[2*(e + f*x)])*(a +
2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^4*Tan[e + f*x])/(3072*(a + b)^
2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(a + b*Sec[e + f*x]^2)^(5/2)) - (5
*(b + (3*a + 2*b)*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Se
c[e + f*x]^4*Tan[e + f*x])/(3072*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f*x)])
^(3/2)*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2210 vs. $2(260) = 520$.

Time = 10.46 (sec) , antiderivative size = 2211, normalized size of antiderivative = 7.68

method	result	size
default	Expression too large to display	2211

```
[In] int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/f/a^5/(-a)^(1/2)*(b+a*cos(f*x+e)^2)*(-315*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin
(f*x+e)*a)*a*b^3*cos(f*x+e)^2+162*(-a)^(1/2)*a^3*b*cos(f*x+e)^2*sin(f*x+e)+
```

574*(-a)^(1/2)*a^2*b^2*cos(f*x+e)^2*sin(f*x+e)+420*(-a)^(1/2)*a*b^3*cos(f*x+e)^2*sin(f*x+e)-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3*b*cos(f*x+e)-225*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2*b^2*cos(f*x+e)-525*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^3*cos(f*x+e)-18*(-a)^(1/2)*a^3*b*cos(f*x+e)^6*sin(f*x+e)+96*(-a)^(1/2)*a^3*b*cos(f*x+e)^4*sin(f*x+e)+63*(-a)^(1/2)*a^2*b^2*cos(f*x+e)^4*sin(f*x+e)-225*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3*b*cos(f*x+e)^3-525*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2*b^2*cos(f*x+e)^3-315*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^3*cos(f*x+e)^3-225*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3*b*cos(f*x+e)^2+315*(-a)^(1/2)*b^4*sin(f*x+e)-315*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^4+113*(-a)^(1/2)*a^2*b^2*sin(f*x+e)+420*(-a)^(1/2)*a*b^3*sin(f*x+e)-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^4*cos(f*x+e)^3-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^4*cos(f*x+e)^2-315*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^4*cos(f*x+e)-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3*b-225*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2*b^2-525*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^3+8*(-a)^(1/2)*a^4*cos(f*x+e)^8*sin(f*x+e)-26*(-a)^(1/2)*a^4*cos(f*x+e)^6*sin(f*x+e)+33*(-a)^(1/2)*a^4*cos(f*x+e)^4*sin(f*x+e)-525*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2

```
)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x
+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a
)*a^2*b^2*cos(f*x+e)^2)/(a+b*sec(f*x+e)^2)^(5/2)*sec(f*x+e)^5
```

Fricas [A] (verification not implemented)

none

Time = 24.11 (sec) , antiderivative size = 1003, normalized size of antiderivative = 3.48

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(15*(a^3*b^2 + 15*a^2*b^3 + 35*a*b^4 + 21*b^5 + (a^5 + 15*a^4*b + 3
5*a^3*b^2 + 21*a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 15*a^3*b^2 + 35*a^2*b^3
 + 21*a*b^4)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4
 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4
 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2
*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*
cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a
^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^5*cos(f*x + e)^9 - 2*(13*a^5 + 9*a^4*b
)*cos(f*x + e)^7 + 3*(11*a^5 + 32*a^4*b + 21*a^3*b^2)*cos(f*x + e)^5 + 2*(8
1*a^4*b + 287*a^3*b^2 + 210*a^2*b^3)*cos(f*x + e)^3 + (113*a^3*b^2 + 420*a^
2*b^3 + 315*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*sin(f*x + e))/(a^8*f*cos(f*x + e)^4 + 2*a^7*b*f*cos(f*x + e)^2 + a^6*b^2*
f), -1/192*(15*(a^3*b^2 + 15*a^2*b^3 + 35*a*b^4 + 21*b^5 + (a^5 + 15*a^4*b
+ 35*a^3*b^2 + 21*a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 15*a^3*b^2 + 35*a^2*
b^3 + 21*a*b^4)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 -
8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b +
a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(8*a^5*cos(f*x +
 e)^9 - 2*(13*a^5 + 9*a^4*b)*cos(f*x + e)^7 + 3*(11*a^5 + 32*a^4*b + 21*a^3
*b^2)*cos(f*x + e)^5 + 2*(81*a^4*b + 287*a^3*b^2 + 210*a^2*b^3)*cos(f*x + e
)^3 + (113*a^3*b^2 + 420*a^2*b^3 + 315*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^8*f*cos(f*x + e)^4 + 2*a^7*b*
f*cos(f*x + e)^2 + a^6*b^2*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\sin(fx+e)^6}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\sin(fx+e)^6}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\sin(e+f\,x)^6}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

```
[In] int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.126 $\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 227

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{(3a^2+30ab+35b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{9/2}f} - \frac{(5a+7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(23a+35b)\tan(e+fx)}{24a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b(11a+21b)\tan(e+fx)}{24a^4f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] 1/8*(3*a^2+30*a*b+35*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1
/2))/a^(9/2)/f-5/24*b*(11*a+21*b)*tan(f*x+e)/a^4/f/(a+b+b*tan(f*x+e)^2)^(1/
2)-1/8*(5*a+7*b)*cos(f*x+e)*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^(3/2)+1/4
*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(3/2)-1/24*b*(23*a+35*b)*
tan(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used

$= \{4217, 481, 541, 12, 385, 209\}$

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{5b(11a+21b)\tan(e+fx)}{24a^4f\sqrt{a+b\tan^2(e+fx)+b}} - \frac{b(23a+35b)\tan(e+fx)}{24a^3f\left(a+b\tan^2(e+fx)+b\right)^{3/2}} - \frac{(5a+7b)\sin(e+fx)\cos(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)+b\right)^{3/2}} + \frac{(3a^2+30ab+35b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{9/2}f} + \frac{\sin(e+fx)\cos^3(e+fx)}{4af\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[In] Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((3*a^2 + 30*a*b + 35*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan
[e + f*x]^2]])/(8*a^(9/2)*f) - ((5*a + 7*b)*Cos[e + f*x]*Sin[e + f*x])/(8*a
^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a
*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*(23*a + 35*b)*Tan[e + f*x])/(24*a
^3*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (5*b*(11*a + 21*b)*Tan[e + f*x])/(
24*a^4*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^3(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{a+b-2(2a+3b)x^2}{(1+x^2)^2(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= -\frac{(5a+7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{(a+b)(3a+7b)-4b(5a+7b)x^2}{(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{8a^2f}$$

$$= -\frac{(5a+7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(23a+35b)\tan(e+fx)}{24a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{(a+b)^2(9a+35b)-2b(a+b)(23a+35b)x^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{24a^3(a+b)f}$$

$$= -\frac{(5a+7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(23a+35b)\tan(e+fx)}{24a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b(11a+21b)\tan(e+fx)}{24a^4f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{3(a+b)^2(3a^2+30ab+35b^2)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{24a^4(a+b)^2f}$$

$$= -\frac{(5a+7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(23a+35b)\tan(e+fx)}{24a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b(11a+21b)\tan(e+fx)}{24a^4f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a^2+30ab+35b^2)\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{8a^4f}$$

$$= -\frac{(5a+7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(23a+35b)\tan(e+fx)}{24a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b(11a+21b)\tan(e+fx)}{24a^4f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a^2+30ab+35b^2)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^4f}$$

$$= \frac{(3a^2+30ab+35b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{9/2}f} - \frac{(5a+7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(23a+35b)\tan(e+fx)}{24a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b(11a+21b)\tan(e+fx)}{24a^4f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1315 vs. $2(227) = 454$.

Time = 12.11 (sec) , antiderivative size = 1315, normalized size of antiderivative = 5.79

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$

$$-\frac{\left(\frac{a+2b+a\cos(2(e+fx))}{a+b}\right)^{3/2}(a+2b+a\cos(2e+2fx))^{5/2}\sec^5(e+fx)\left(-60\sqrt{a+b}(3a^3+17a^2b+28ab^2+14b^3\right.}{}$$

$$-\frac{(a+2b+a\cos(2e+2fx))^{5/2}\csc(e+fx)\sec^5(e+fx)\left(\frac{\sin^2(e+fx)}{a+b}+\frac{(a+2b+a\cos(2(e+fx)))\sin^2(e+fx)}{(a+b)^2}-\frac{12\sin^4(e+\right.}{a+b}}{768\sqrt{2}f\,(a+b\sec^2(e+fx))^{5/}}$$

$$+\frac{(a+2b+a\cos(2e+2fx))^{5/2}\csc(e+fx)\sec^5(e+fx)\left(\frac{\sin^2(e+fx)}{a+b}+\frac{(a+2b+a\cos(2(e+fx)))\sin^2(e+fx)}{(a+b)^2}-\frac{24\sin^4(e+f}{a+b}\right.}{}$$

$$+\frac{(2a+3b+a\cos(2(e+fx)))(a+2b+a\cos(2e+2fx))^{5/2}\sec^4(e+fx)\tan(e+fx)}{256(a+b)^2f(a+2b+a\cos(2(e+fx)))^{3/2}\,(a+b\sec^2(e+fx))^{5/2}}$$

$$-\frac{(b+(3a+2b)\cos(2(e+fx)))(a+2b+a\cos(2e+2fx))^{5/2}\sec^4(e+fx)\tan(e+fx)}{384(a+b)^2f(a+2b+a\cos(2(e+fx)))^{3/2}\,(a+b\sec^2(e+fx))^{5/2}}$$

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/768*(((a + 2*b + a*Cos[2*(e + f*x)])/(a + b))^(3/2)*(a + 2*b + a*Cos[2*e
 + 2*f*x])^(5/2)*Sec[e + f*x]^5*(-60*Sqrt[a + b]*(3*a^3 + 17*a^2*b + 28*a*b
^2 + 14*b^3)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*
(e + f*x)])^2 + Sqrt[a]*Sin[e + f*x]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b
)]*(3*(239*a^5 + 1839*a^4*b + 5200*a^3*b^2 + 6960*a^2*b^3 + 4480*a*b^4 + 11
20*b^5) - 2*a*(459*a^4 + 3180*a^3*b + 7200*a^2*b^2 + 6720*a*b^3 + 2240*b^4)
*Sin[e + f*x]^2 + 672*a^2*b*(a + b)^2*Sin[e + f*x]^4 + 192*a^3*(a + b)^2*Si
n[e + f*x]^6)))/(Sqrt[2]*a^(9/2)*f*(a + 2*b + a*Cos[2*(e + f*x)])^(7/2)*(a
+ b*Sec[e + f*x]^2)^(5/2)) - ((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Csc[e +
f*x]*Sec[e + f*x]^5*(Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)
])*Sin[e + f*x]^2)/(a + b)^2 - (12*Sin[e + f*x]^4)/(a + b) + (16*(a + b - a
*Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*
x]^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b
- a*Sin[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x]
)/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^3
))/(768*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a + b - a*Sin[e + f*x]^2)^(
3/2)) + ((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Csc[e + f*x]*Sec[e + f*x]^5*(
Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*Sin[e + f*x]^2)/(a
```

```
 + b)^2 - (24*Sin[e + f*x]^4)/(a + b) + (96*Sin[e + f*x]^6)/a + (80*(a + b
- a*Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e +
 f*x]^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a +
 b - a*Sin[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f
*x])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/
a^3 - (160*(a + b - a*Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6
*a*(a + b)^2*Sin[e + f*x]^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (3*Sqrt[a]*(a
 + b)^(3/2)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(
a + b - a*Sin[e + f*x]^2)/(a + b)] + (a^2*Sin[e + f*x]^4)/(-1 + (a*Sin[e +
f*x]^2)/(a + b))^2))/a^4))/(3072*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a
+ b - a*Sin[e + f*x]^2)^(3/2)) + ((2*a + 3*b + a*Cos[2*(e + f*x)])*(a + 2*b
 + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^4*Tan[e + f*x])/(256*(a + b)^2*f*
(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(a + b*Sec[e + f*x]^2)^(5/2)) - ((b +
(3*a + 2*b)*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e +
f*x]^4*Tan[e + f*x])/(384*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*
(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1610 vs. $2(203) = 406$.

Time = 8.52 (sec) , antiderivative size = 1611, normalized size of antiderivative = 7.10

method	result	size
default	Expression too large to display	1611

```
[In] int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/f/a^4/(-a)^(1/2)*(b+a*cos(f*x+e)^2)*(6*(-a)^(1/2)*a^3*cos(f*x+e)^6*sin
(f*x+e)-15*(-a)^(1/2)*a^3*cos(f*x+e)^4*sin(f*x+e)-21*(-a)^(1/2)*a^2*b*cos(f
*x+e)^4*sin(f*x+e)+9*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4
*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3
+90*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2*b*cos(f*x+e)^3+105*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^2*cos(f*x+e)^3+9*cos(f*x+e)^2*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)-4*sin(f*x+e)*a)*a^3+90*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(
-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2*b
*cos(f*x+e)^2+105*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+
```

```
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^2*cos(f*x+e)^2-
78*(-a)^(1/2)*a^2*b*cos(f*x+e)^2*sin(f*x+e)-140*(-a)^(1/2)*a*b^2*cos(f*x+e)
^2*sin(f*x+e)+9*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(
f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b*cos(f*x+e)+90*l
n(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a
)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^2*cos(f*x+e)+105*ln(4*(-a)^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*b^3*cos(f*x+e)+9*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+
e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)
*a^2*b+90*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^2+105*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)-4*sin(f*x+e)*a)*b^3-55*(-a)^(1/2)*a*b^2*sin(f*x+e)-105*(-a)^(1/2)*b^3*
sin(f*x+e))/(a+b*sec(f*x+e)^2)^(5/2)*sec(f*x+e)^5
```

Fricas [A] (verification not implemented)

none

Time = 7.53 (sec) , antiderivative size = 873, normalized size of antiderivative = 3.85

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \left[-\frac{3\left((3\,a^4+30\,a^3b+35\,a^2b^2)\cos{(fx+e)}^4+3\,a^2b^2+30\,ab^3+35\,b^4+2\,(3\,a\right.}{}\right.$$

$$-\frac{3\left((3\,a^4+30\,a^3b+35\,a^2b^2)\cos{(fx+e)}^4+3\,a^2b^2+30\,ab^3+35\,b^4+2\,(3\,a^3b+30\,a^2b^2+35\,ab^3)\cos{(fx+e}\right.}{}$$

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*((3*a^4 + 30*a^3*b + 35*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 30
*a*b^3 + 35*b^4 + 2*(3*a^3*b + 30*a^2*b^2 + 35*a*b^3)*cos(f*x + e)^2)*sqrt(
-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a
```

```
^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 2
8*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(
16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2
*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e)
)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(6
*a^4*cos(f*x + e)^7 - 3*(5*a^4 + 7*a^3*b)*cos(f*x + e)^5 - 2*(39*a^3*b + 70
*a^2*b^2)*cos(f*x + e)^3 - 5*(11*a^2*b^2 + 21*a*b^3)*cos(f*x + e))*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^7*f*cos(f*x + e)^4 + 2
*a^6*b*f*cos(f*x + e)^2 + a^5*b^2*f), -1/96*(3*((3*a^4 + 30*a^3*b + 35*a^2*
b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 30*a*b^3 + 35*b^4 + 2*(3*a^3*b + 30*a^2*b
^2 + 35*a*b^3)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8
*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a
*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(6*a^4*cos(f*x +
e)^7 - 3*(5*a^4 + 7*a^3*b)*cos(f*x + e)^5 - 2*(39*a^3*b + 70*a^2*b^2)*cos(f
*x + e)^3 - 5*(11*a^2*b^2 + 21*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*sin(f*x + e))/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*f*cos(f*
x + e)^2 + a^5*b^2*f)]
```

Sympy [F]

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin^4{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

[In] `integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)`

[Out] `Integral(sin(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)`

Maxima [F]

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin{(fx+e)}^4}{(b\sec{(fx+e)}^2+a)^{\frac{5}{2}}}\,dx$$

[In] `integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")`

[Out] `integrate(sin(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)`

Giac [F]

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin(fx+e)^4}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin(e+f\,x)^4}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

```
[In] int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.127 $\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 167

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{(a+5b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{7/2}f} - \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan(e+fx)}{6a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(13a+15b)\tan(e+fx)}{6a^3(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] 1/2*(a+5*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(7/2)/f
-1/6*b*(13*a+15*b)*tan(f*x+e)/a^3/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/2*co
s(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(3/2)-5/6*b*tan(f*x+e)/a^2/f/(
a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4217, 482, 541, 12, 385, 209}

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{(a+5b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{7/2}f} - \frac{b(13a+15b)\tan(e+fx)}{6a^3f(a+b)\sqrt{a+b\tan^2(e+fx)+b}} - \frac{5b\tan(e+fx)}{6a^2f\left(a+b\tan^2(e+fx)+b\right)^{3/2}} - \frac{\sin(e+fx)\cos(e+fx)}{2af\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[In] Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 5*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(
2*a^(7/2)*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*(a + b + b*Tan[e + f*x]^2
)^(3/2)) - (5*b*Tan[e + f*x])/(6*a^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) -
(b*(13*a + 15*b)*Tan[e + f*x])/(6*a^3*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]
^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
```

```
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a+b-4bx^2}{(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan(e+fx)}{6a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{(a+b)(3a+5b)-10b(a+b)x^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{6a^2(a+b)f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan(e+fx)}{6a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(13a+15b)\tan(e+fx)}{6a^3(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{3(a+b)^2(a+5b)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{6a^3(a+b)^2f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan(e+fx)}{6a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(13a+15b)\tan(e+fx)}{6a^3(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a+5b)\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2a^3f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan(e+fx)}{6a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(13a+15b)\tan(e+fx)}{6a^3(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a+5b)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^3f}$$

$$= \frac{(a+5b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{7/2}f} - \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan(e+fx)}{6a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(13a+15b)\tan(e+fx)}{6a^3(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 983 vs. $2(167) = 334$.

Time = 8.40 (sec) , antiderivative size = 983, normalized size of antiderivative = 5.89

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx =$$

$$-\frac{(a+2b+a\cos(2e+2fx))^{5/2}\csc(e+fx)\sec^5(e+fx)\left(\frac{\sin^2(e+fx)}{a+b}+\frac{(a+2b+a\cos(2(e+fx)))\sin^2(e+fx)}{(a+b)^2}-\frac{12\sin^4(e+\ldots}{a+b}\right.}{256\sqrt{2}f\left(a+b\sec^2(e+fx)\right)^{5/\ldots}}$$

$$-\frac{(a+2b+a\cos(2e+2fx))^{5/2}\csc(e+fx)\sec^5(e+fx)\left(\frac{\sin^2(e+fx)}{a+b}+\frac{(a+2b+a\cos(2(e+fx)))\sin^2(e+fx)}{(a+b)^2}-\frac{24\sin^4(e+\ldots}{a+b}\right.}{\ldots}$$

$$+\frac{5(2a+3b+a\cos(2(e+fx)))(a+2b+a\cos(2e+2fx))^{5/2}\sec^4(e+fx)\tan(e+fx)}{384(a+b)^2f(a+2b+a\cos(2(e+fx)))^{3/2}\left(a+b\sec^2(e+fx)\right)^{5/2}} - \frac{(b+(3a+2b)\cos(2(e+fx)))(a+2b+a\cos(2e+2fx))^{5/2}\sec^4(e+fx)\tan(e+fx)}{384(a+b)^2f(a+2b+a\cos(2(e+fx)))^{3/2}\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/256*((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Csc[e + f*x]*Sec[e + f*x]^5*(S
in[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*Sin[e + f*x]^2)/(a
+ b)^2 - (12*Sin[e + f*x]^4)/(a + b) + (16*(a + b - a*Sin[e + f*x]^2)*(1 -
(a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*x]^2)/(a + 2*b + a*Cos
[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b - a*Sin[e + f*x]^2)^2
+ (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e +
 f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^3))/(Sqrt[2]*f*(a + b*S
ec[e + f*x]^2)^(5/2)*(a + b - a*Sin[e + f*x]^2)^(3/2)) - ((a + 2*b + a*Cos[
2*e + 2*f*x])^(5/2)*Csc[e + f*x]*Sec[e + f*x]^5*(Sin[e + f*x]^2/(a + b) + (
(a + 2*b + a*Cos[2*(e + f*x)])*Sin[e + f*x]^2)/(a + b)^2 - (24*Sin[e + f*x]
^4)/(a + b) + (96*Sin[e + f*x]^6)/a + (80*(a + b - a*Sin[e + f*x]^2)*(1 - (
a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*x]^2)/(a + 2*b + a*Cos[
```

```
2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b - a*Sin[e + f*x]^2)^2 +
 (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e +
f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^3 - (160*(a + b - a*Sin[
e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)^2*Sin[e + f*x]^
2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (3*Sqrt[a]*(a + b)^(3/2)*ArcSin[(Sqrt[a
]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/
(a + b)] + (a^2*Sin[e + f*x]^4)/(-1 + (a*Sin[e + f*x]^2)/(a + b))^2))/a^4))
/(768*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a + b - a*Sin[e + f*x]^2)^(3/
2)) + (5*(2*a + 3*b + a*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5
/2)*Sec[e + f*x]^4*Tan[e + f*x])/(384*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f
*x)])^(3/2)*(a + b*Sec[e + f*x]^2)^(5/2)) - ((b + (3*a + 2*b)*Cos[2*(e + f*
x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^4*Tan[e + f*x])/(384
*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(a + b*Sec[e + f*x]^2)^(5
/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1591 vs. $2(147) = 294$.

Time = 7.01 (sec) , antiderivative size = 1592, normalized size of antiderivative = 9.53

method	result	size
default	Expression too large to display	1592

```
[In] int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/f/(a+b)/a^3/(-a)^(1/2)*(b+a*cos(f*x+e)^2)*(3*(-a)^(1/2)*a^3*cos(f*x+e)
^4*sin(f*x+e)+3*(-a)^(1/2)*a^2*b*cos(f*x+e)^4*sin(f*x+e)-3*cos(f*x+e)^3*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3-18*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(
f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e
)*a)*a^2*b*cos(f*x+e)^3-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4
*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^2*cos(
f*x+e)^3+18*(-a)^(1/2)*a^2*b*cos(f*x+e)^2*sin(f*x+e)+20*(-a)^(1/2)*a*b^2*co
s(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e
)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*
a^3-18*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2*b*cos(f*x+e)^2-15*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^2*cos(f*x+e)^2-3*ln(4*(-a)^(1/2)*((b+a*co
```

```
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*a^2*b*cos(f*x+e)-18*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b
^2*cos(f*x+e)-15*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin
(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3*cos(f*x+e)+13*(-
a)^(1/2)*a*b^2*sin(f*x+e)+15*(-a)^(1/2)*b^3*sin(f*x+e)-3*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)-4*sin(f*x+e)*a)*a^2*b-18*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln
(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^2-15
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^3)/(a+b*sec(f*x+e)^2)^(5/2)*sec(f*
x+e)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(147) = 294$.

Time = 2.09 (sec) , antiderivative size = 879, normalized size of antiderivative = 5.26

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \left[-\frac{3\left((a^4+6a^3b+5a^2b^2)\cos(fx+e)^4+a^2b^2+6ab^3+5b^4+2(a^3b+6a^2b\right.}{}\right.$$

$$-\frac{3\left((a^4+6a^3b+5a^2b^2)\cos(fx+e)^4+a^2b^2+6ab^3+5b^4+2(a^3b+6a^2b^2+5ab^3)\cos(fx+e)^2\right)\sqrt{a}\,\mathrm{arcta}}{}$$

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*((a^4 + 6*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 6*a*b^3 +
 5*b^4 + 2*(a^3*b + 6*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a
^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b
 + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4
 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*
```

```
x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)
*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(3*(a^4 + a^3*b
)*cos(f*x + e)^5 + 2*(9*a^3*b + 10*a^2*b^2)*cos(f*x + e)^3 + (13*a^2*b^2 +
15*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
 + e))/((a^7 + a^6*b)*f*cos(f*x + e)^4 + 2*(a^6*b + a^5*b^2)*f*cos(f*x + e)
^2 + (a^5*b^2 + a^4*b^3)*f), -1/24*(3*((a^4 + 6*a^3*b + 5*a^2*b^2)*cos(f*x
+ e)^4 + a^2*b^2 + 6*a*b^3 + 5*b^4 + 2*(a^3*b + 6*a^2*b^2 + 5*a*b^3)*cos(f*
x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x
+ e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b
)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(3*(a^4 + a^3*b)*cos(f*x + e)^5 + 2*(9
*a^3*b + 10*a^2*b^2)*cos(f*x + e)^3 + (13*a^2*b^2 + 15*a*b^3)*cos(f*x + e))
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 + a^6*b)*f
*cos(f*x + e)^4 + 2*(a^6*b + a^5*b^2)*f*cos(f*x + e)^2 + (a^5*b^2 + a^4*b^3
)*f)]
```

Sympy [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(sin(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin(fx+e)^2}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin\left(e+f\,x\right)^2}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

```
[In] int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.128 $\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 125

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\tan(e+fx)}{3a(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*(5*a+
3*b)*tan(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*b*tan(f*x+e)/a
/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4213, 425, 541, 12, 385, 209}

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f} - \frac{b(5a+3b)\tan(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}} - \frac{b\tan(e+fx)}{3af(a+b)\,(a+b\tan^2(e+fx)+b)^{3/2}}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f) -
 (b*Tan[e + f*x])/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*(5*
a + 3*b)*Tan[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a+b-2bx^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3a(a+b)f}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{3(a+b)^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{3a^2(a+b)^2f}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{a^2f}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^2f}$$

$$= \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 15.75 (sec) , antiderivative size = 1927, normalized size of antiderivative = 15.42

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{}{4\sqrt{2}f\left(a+b\sec^2(e+fx)\right)^{5/2}\left(a+b-a\sin^2(e+fx)\right)^{5/2}\left(3(a+b)\,\text{AppellF1}\right.}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^4*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5
/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2,
5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2,
-1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*
((15*a*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^
2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Sin[e + f*
x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*S
in[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x
]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1/
2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5
)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2,
5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
-2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Appel
lF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e +
 f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt[2]*(a + b - a*Sin[
e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a
*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos[e
 + f*x]^4*Sin[e + f*x]*((5*a*f*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - (4*f*A
ppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[
e + f*x]*Sin[e + f*x])/3))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3
*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
+ b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)
/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e
+ f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/
2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^4*Sin[e + f*x]*
(2*f*(5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
 + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f
*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a + b)*((5*a*f*AppellF1[3/2
, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Si
n[e + f*x])/(3*(a + b)) - (4*f*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3) + Sin[e + f*x]^2*
(5*a*((21*a*f*AppellF1[5/2, -2, 9/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b)) - (12*f*AppellF1[5/2, -1,
 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e +
 f*x])/5) - 4*(a + b)*((3*a*f*AppellF1[5/2, -1, 7/2, 7/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (6*(a + b)^
3*f*Cot[e + f*x]*Csc[e + f*x]^4*(-1 + (a*Sin[e + f*x]^2)/(a + b))^2*((Sqrt[
a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/(Sqrt[a + b]*Sq
```

```
rt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + (a^2*Sin[e + f*x]^4)/(3*(a + b)^2*(-1
 + (a*Sin[e + f*x]^2)/(a + b))^2) + (a*Sin[e + f*x]^2)/((a + b)*(-1 + (a*Si
n[e + f*x]^2)/(a + b)))))/(a^3*(1 - (a*Sin[e + f*x]^2)/(a + b))^(3/2))))))/
(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2,
5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
-2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Appel
lF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e +
 f*x]^2)^2)))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1262 vs. $2(111) = 222$.

Time = 5.54 (sec) , antiderivative size = 1263, normalized size of antiderivative = 10.10

method	result	size
default	Expression too large to display	1263

```
[In] int(1/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f/(a+b)^2/a^2/(-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x
+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*
csc(f*x+e)^2+a+b)*(-12*(-a)^(1/2)*a^2*b*(1-cos(f*x+e))^5*csc(f*x+e)^5-18*(-
a)^(1/2)*a*b^2*(1-cos(f*x+e))^5*csc(f*x+e)^5-6*(-a)^(1/2)*b^3*(1-cos(f*x+e)
)^5*csc(f*x+e)^5+24*(1-cos(f*x+e))^3*a^2*(-a)^(1/2)*b*csc(f*x+e)^3-4*(-a)^(
1/2)*a*b^2*(1-cos(f*x+e))^3*csc(f*x+e)^3-12*(-a)^(1/2)*b^3*(1-cos(f*x+e))^3
*csc(f*x+e)^3+3*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x
+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a
+b)^(3/2)*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e)
)^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc
(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x
+e)^2+1))*a^2+6*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x
+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a
+b)^(3/2)*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e)
)^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc
(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x
+e)^2+1))*a*b+3*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x
+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a
+b)^(3/2)*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e)
)^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc
(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x
+e)^2+1))*b^2-12*(-a)^(1/2)*a^2*b*(csc(f*x+e)-cot(f*x+e))-18*(-a)^(1/2)*a*b
^2*(csc(f*x+e)-cot(f*x+e))-6*(-a)^(1/2)*b^3*(csc(f*x+e)-cot(f*x+e)))/((a*(1
-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+
e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/((1-cos(f*x+e))^2
*csc(f*x+e)^2-1)^2)^(5/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. 2(111) = 222.

Time = 0.71 (sec) , antiderivative size = 881, normalized size of antiderivative = 7.05

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \left[-\frac{3\left(\left(a^4+2a^3b+a^2b^2\right)\cos(fx+e)^4+a^2b^2+2ab^3+b^4+2\left(a^3b+2a^2b^2+\right.\right.}{}\right.$$

$$-\frac{3\left(\left(a^4+2a^3b+a^2b^2\right)\cos(fx+e)^4+a^2b^2+2ab^3+b^4+2\left(a^3b+2a^2b^2+ab^3\right)\cos(fx+e)^2\right)\sqrt{a}\arctan\left(\right.}{12\left(\left(a^7+2a^6b+a^5b^2\right)f\right.}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 2*a*b^3 + b
^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*(3*a^3*b + 2*a^2*b
^2)*cos(f*x + e)^3 + (5*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*
x + e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*
a^4*b^3 + a^3*b^4)*f), -1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 +
 a^2*b^2 + 2*a*b^3 + b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sq
rt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^
2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x +
e)^2)*sin(f*x + e))) + 4*(2*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (5*a^2*b
^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b + 2*a^5*b
^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*f)]
```

Sympy [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{(a+b\sec^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral((a + b*sec(e + f*x)**2)**(-5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\left(b\sec{(fx+e)}^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

```
[In] int(1/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(1/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.129 $\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 106

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = -\frac{\cot(e+fx)}{(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{4b\tan(e+fx)}{3(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{8b\tan(e+fx)}{3(a+b)^3f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] -8/3*b*tan(f*x+e)/(a+b)^3/f/(a+b+b*tan(f*x+e)^2)^(1/2)-cot(f*x+e)/(a+b)/f/(
a+b+b*tan(f*x+e)^2)^(3/2)-4/3*b*tan(f*x+e)/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(
3/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4217, 277, 198, 197}

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = -\frac{8b\tan(e+fx)}{3f(a+b)^3\sqrt{a+b\tan^2(e+fx)+b}} - \frac{4b\tan(e+fx)}{3f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)^{3/2}} - \frac{\cot(e+fx)}{f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[In] Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(Cot[e + f*x]/((a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2))) - (4*b*Tan[e +
 f*x])/(3*(a + b)^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (8*b*Tan[e + f*x]
)/(3*(a + b)^3*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)}{(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{(4b)\text{Subst}\left(\int \frac{1}{(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{(a+b)f}$$

$$= -\frac{\cot(e+fx)}{(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{4b\tan(e+fx)}{3(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{(8b)\text{Subst}\left(\int \frac{1}{(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3(a+b)^2f}$$

$$= -\frac{\cot(e+fx)}{(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{4b\tan(e+fx)}{3(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{8b\tan(e+fx)}{3(a+b)^3f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.02

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left(3a^2-6ab-b^2-6(a^2-b^2)\csc^2(e+fx)+3(a+b)^2\csc^4(e+fx)\right)\sec^2(e+fx)}{6(a+b)^3f\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/6*((a + 2*b + a*Cos[2*(e + f*x)])*(3*a^2 - 6*a*b - b^2 - 6*(a^2 - b^2)*C
sc[e + f*x]^2 + 3*(a + b)^2*Csc[e + f*x]^4)*Sec[e + f*x]^2*Tan[e + f*x]^3)/
((a + b)^3*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [A] (verified)

Time = 5.67 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.28

method	result
default	$-\frac{\left(b+a\cos(fx+e)^2\right)\left(3\cos(fx+e)^4a^2-6\cos(fx+e)^4ab-\cos(fx+e)^4b^2+12\cos(fx+e)^2ab-4\cos(fx+e)^2b^2+8b^2\right)\sec(fx+e)^5\csc(fx+e)}{3f(a^3+3a^2b+3ab^2+b^3)\left(a+b\sec(fx+e)^2\right)^{\frac{5}{2}}}$

```
[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f/(a^3+3*a^2*b+3*a*b^2+b^3)*(b+a*cos(f*x+e)^2)*(3*cos(f*x+e)^4*a^2-6*c
os(f*x+e)^4*a*b-cos(f*x+e)^4*b^2+12*cos(f*x+e)^2*a*b-4*cos(f*x+e)^2*b^2+8*b
^2)/(a+b*sec(f*x+e)^2)^(5/2)*sec(f*x+e)^5*csc(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.71 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.81

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\left((3a^2-6ab-b^2)\cos(fx+e)^5+4(3ab-b^2)\cos(fx+e)^3+8b^2\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{3\left((a^5+3a^4b+3a^3b^2+a^2b^3)f\cos(fx+e)^4+2(a^4b+3a^3b^2+3a^2b^3+ab^4)f\cos(fx+e)^2+(a^3b^2+3a\cdots\right)}$$

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*((3*a^2 - 6*a*b - b^2)*cos(f*x + e)^5 + 4*(3*a*b - b^2)*cos(f*x + e)^3
 + 8*b^2*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a^5 +
```

```
3*a^4*b + 3*a^3*b^2 + a^2*b^3)*f*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3
*a^2*b^3 + a*b^4)*f*cos(f*x + e)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*
f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\csc^2{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(csc(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\frac{8\,b\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^3} + \frac{4\,b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)^2} + \frac{3}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)\tan(fx+e)}}{3\,f}$$

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] -1/3*(8*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3) + 4*b*tan
(f*x + e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2) + 3/((b*tan(f*x + e)
^2 + a + b)^(3/2)*(a + b)*tan(f*x + e)))/f
```

Giac [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\csc{(fx+e)}^2}{\left(b\sec{(fx+e)}^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 29.97 (sec) , antiderivative size = 336, normalized size of antiderivative = 3.17

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\left(e^{e\,2i+f\,x\,2i}+1\right)\sqrt{a+\frac{b}{\left(\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}\right)^2}}\left(-a\,b\,6i+a^2\,3i-b^2\,1i+a^2\,e^{e\,2i+f\,x\,2i}\,12i+a^2\,e^{e\,4i+f\,x\,4i}\,18i+a^2\cdots\right)}{3\,f\,(a\cdots}$$

```
[In] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

```
[Out] -((exp(e*2i + f*x*2i) + 1)*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*
1i)/2)^2)^(1/2)*(a^2*3i - a*b*6i - b^2*1i + a^2*exp(e*2i + f*x*2i)*12i + a^
2*exp(e*4i + f*x*4i)*18i + a^2*exp(e*6i + f*x*6i)*12i + a^2*exp(e*8i + f*x*
8i)*3i - b^2*exp(e*2i + f*x*2i)*20i + b^2*exp(e*4i + f*x*4i)*90i - b^2*exp(
e*6i + f*x*6i)*20i - b^2*exp(e*8i + f*x*8i)*1i + a*b*exp(e*2i + f*x*2i)*24i
 + a*b*exp(e*4i + f*x*4i)*60i + a*b*exp(e*6i + f*x*6i)*24i - a*b*exp(e*8i +
 f*x*8i)*6i))/(3*f*(a + b)^3*(exp(e*2i + f*x*2i) - 1)*(a + 2*a*exp(e*2i + f
*x*2i) + a*exp(e*4i + f*x*4i) + 4*b*exp(e*2i + f*x*2i))^2)
```

3.130 $\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 158

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=-\frac{(a-b)\cot(e+fx)}{(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}-\frac{\cot^3(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}-\frac{4(a-b)b\tan(e+fx)}{3(a+b)^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}-\frac{8(a-b)b\tan(e+fx)}{3(a+b)^4f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] -8/3*(a-b)*b*tan(f*x+e)/(a+b)^4/f/(a+b+b*tan(f*x+e)^2)^(1/2)-(a-b)*cot(f*x+
e)/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(3/2)-1/3*cot(f*x+e)^3/(a+b)/f/(a+b+b*tan
(f*x+e)^2)^(3/2)-4/3*(a-b)*b*tan(f*x+e)/(a+b)^3/f/(a+b+b*tan(f*x+e)^2)^(3/2
)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4217, 464, 277, 198, 197}

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=-\frac{8b(a-b)\tan(e+fx)}{3f(a+b)^4\sqrt{a+b\tan^2(e+fx)+b}}-\frac{4b(a-b)\tan(e+fx)}{3f(a+b)^3\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\cot^3(e+fx)}{3f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{(a-b)\cot(e+fx)}{f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[In] Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(((a - b)*Cot[e + f*x])/((a + b)^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2))) -
Cot[e + f*x]^3/(3*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (4*(a - b)*
b*Tan[e + f*x])/(3*(a + b)^3*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (8*(a -
b)*b*Tan[e + f*x])/(3*(a + b)^4*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-b)\text{Subst}\left(\int \frac{1}{x^2(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{(a+b)f}$$

$$= -\frac{(a-b)\cot(e+fx)}{(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{(4(a-b)b)\text{Subst}\left(\int \frac{1}{(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{(a+b)^2f}$$

$$= -\frac{(a-b)\cot(e+fx)}{(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{4(a-b)b\tan(e+fx)}{3(a+b)^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{(8(a-b)b)\text{Subst}\left(\int \frac{1}{(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3(a+b)^3f}$$

$$= -\frac{(a-b)\cot(e+fx)}{(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{4(a-b)b\tan(e+fx)}{3(a+b)^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{8(a-b)b\tan(e+fx)}{3(a+b)^4f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 4.51 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.87

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))^3\left(\frac{4b^2(a+b)}{(a+2b+a\cos(2(e+fx)))^2} + \frac{4b(-3a+b)}{a+2b+a\cos(2(e+fx))} - 2(a\right.}{24(a+b)^4f\left(a+b\sec\right.}$$

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^3*((4*b^2*(a + b))/(a + 2*b + a*Cos[2*(e +
f*x)])^2 + (4*b*(-3*a + b))/(a + 2*b + a*Cos[2*(e + f*x)]) - 2*(a - 3*b)*Cs
c[e + f*x]^2 - (a + b)*Csc[e + f*x]^4)*Sec[e + f*x]^4*Tan[e + f*x])/(24*(a
+ b)^4*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [A] (verified)

Time = 7.22 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.42

method	result
default	$\frac{\left(b+a\cos(fx+e)^2\right)\left(2a^3\cos(fx+e)^6-12\cos(fx+e)^6a^2b+2\cos(fx+e)^6ab^2-3\cos(fx+e)^4a^3+21\cos(fx+e)^4a^2b-21\cos(fx+e)^4ab^2+\right.}{3f(a^4+4a^3b+6a^2b^2+4ab^3+b^4)\left(a+b\right.}$

```
[In] int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f/(a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*(b+a*cos(f*x+e)^2)*(2*a^3*cos(f*x
+e)^6-12*cos(f*x+e)^6*a^2*b+2*cos(f*x+e)^6*a*b^2-3*cos(f*x+e)^4*a^3+21*cos(
f*x+e)^4*a^2*b-21*cos(f*x+e)^4*a*b^2+3*cos(f*x+e)^4*b^3-12*cos(f*x+e)^2*a^2
*b+24*cos(f*x+e)^2*a*b^2-12*cos(f*x+e)^2*b^3-8*a*b^2+8*b^3)/(a+b*sec(f*x+e)
^2)^(5/2)*sec(f*x+e)^5*csc(f*x+e)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(144) = 288.

Time = 2.35 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.03

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$

$$-\frac{\left(2\left(a^3-6a^2b+ab^2\right)\cos\left(fx+e\right)^7-3\left(a^3-7a^2b+7ab^2-b^3\right)\cos\left(\right.\right.}{3\left(\left(a^6+4a^5b+6a^4b^2+4a^3b^3+a^2b^4\right)f\cos\left(fx+e\right)^6-\left(a^6+2a^5b-2a^4b^2-8a^3b^3-7a^2b^4-2ab^5\right)f\cos\right.}$$

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a^3 - 6*a^2*b + a*b^2)*cos(f*x + e)^7 - 3*(a^3 - 7*a^2*b + 7*a*b^2
 - b^3)*cos(f*x + e)^5 - 12*(a^2*b - 2*a*b^2 + b^3)*cos(f*x + e)^3 - 8*(a*b
^2 - b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a^6
+ 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^6 - (a^6 + 2*a^
5*b - 2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*f*cos(f*x + e)^4 - (2*a^
5*b + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*f*cos(f*x + e)^2 -
 (a^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6)*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(csc(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.37

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\frac{8\,b\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^3} + \frac{4\,b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)^2} - \frac{16\,b^2\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^4} - \frac{8\,b^2\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)^3} + \frac{\cdots}{(b\tan\cdots}}{3\,f}$$

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(8*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3) + 4*b*tan
(f*x + e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2) - 16*b^2*tan(f*x + e
)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^4) - 8*b^2*tan(f*x + e)/((b*tan(f
*x + e)^2 + a + b)^(3/2)*(a + b)^3) + 3/((b*tan(f*x + e)^2 + a + b)^(3/2)*(
a + b)*tan(f*x + e)) - 6*b/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2*tan(
f*x + e)) + 1/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)*tan(f*x + e)^3))/f
```

Giac [F]

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\csc(fx+e)^4}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Hanged}$$

```
[In] int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(5/2)),x)
[Out] \text{Hanged}
```

3.131 $\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 226

$$\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = -\frac{(5a^2-10ab+b^2)\cot(e+fx)}{5(a+b)^3 f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{2(5a+b)\cot^3(e+fx)}{15(a+b)^2 f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{4b(5a^2-10ab+b^2)\tan(e+fx)}{15(a+b)^4 f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{8b(5a^2-10ab+b^2)\tan(e+fx)}{15(a+b)^5 f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] -8/15*b*(5*a^2-10*a*b+b^2)*tan(f*x+e)/(a+b)^5/f/(a+b+b*tan(f*x+e)^2)^(1/2)-
1/5*(5*a^2-10*a*b+b^2)*cot(f*x+e)/(a+b)^3/f/(a+b+b*tan(f*x+e)^2)^(3/2)-2/15
*(5*a+b)*cot(f*x+e)^3/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(3/2)-1/5*cot(f*x+e)^5
/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)-4/15*b*(5*a^2-10*a*b+b^2)*tan(f*x+e)/(a
+b)^4/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4217, 473, 464, 277, 198, 197}

$$\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = -\frac{8b(5a^2-10ab+b^2)\tan(e+fx)}{15f(a+b)^5\sqrt{a+b\tan^2(e+fx)+b}} - \frac{4b(5a^2-10ab+b^2)\tan(e+fx)}{15f(a+b)^4\left(a+b\tan^2(e+fx)+b\right)^{3/2}} - \frac{(5a^2-10ab+b^2)\cot(e+fx)}{5f(a+b)^3\left(a+b\tan^2(e+fx)+b\right)^{3/2}} - \frac{\cot^5(e+fx)}{5f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}} - \frac{2(5a+b)\cot^3(e+fx)}{15f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[In] Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/5*((5*a^2 - 10*a*b + b^2)*Cot[e + f*x])/((a + b)^3*f*(a + b + b*Tan[e +
f*x]^2)^(3/2)) - (2*(5*a + b)*Cot[e + f*x]^3)/(15*(a + b)^2*f*(a + b + b*Ta
n[e + f*x]^2)^(3/2)) - Cot[e + f*x]^5/(5*(a + b)*f*(a + b + b*Tan[e + f*x]^
2)^(3/2)) - (4*b*(5*a^2 - 10*a*b + b^2)*Tan[e + f*x])/(15*(a + b)^4*f*(a +
b + b*Tan[e + f*x]^2)^(3/2)) - (8*b*(5*a^2 - 10*a*b + b^2)*Tan[e + f*x])/(1
5*(a + b)^5*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^6(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{2(5a+b)+5(a+b)x^2}{x^4(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{5(a+b)f}$$

$$= -\frac{2(5a+b)\cot^3(e+fx)}{15(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(5a^2-10ab+b^2)\,\text{Subst}\left(\int \frac{1}{x^2(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{5(a+b)^2f}$$

$$= -\frac{(5a^2-10ab+b^2)\cot(e+fx)}{5(a+b)^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{2(5a+b)\cot^3(e+fx)}{15(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{(4b(5a^2-10ab+b^2))\,\text{Subst}\left(\int \frac{1}{(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{5(a+b)^3f}$$

$$= -\frac{(5a^2-10ab+b^2)\cot(e+fx)}{5(a+b)^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{2(5a+b)\cot^3(e+fx)}{15(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{4b(5a^2-10ab+b^2)\tan(e+fx)}{15(a+b)^4f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{(8b(5a^2-10ab+b^2))\,\text{Subst}\left(\int \frac{1}{(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{15(a+b)^4f}$$

$$= -\frac{(5a^2-10ab+b^2)\cot(e+fx)}{5(a+b)^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{2(5a+b)\cot^3(e+fx)}{15(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{4b(5a^2-10ab+b^2)\tan(e+fx)}{15(a+b)^4f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{8b(5a^2-10ab+b^2)\tan(e+fx)}{15(a+b)^5f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 4.93 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.77

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))^3\left(\frac{20ab^2(a+b)}{(a+2b+a\cos(2(e+fx)))^2}+\frac{10ab(-6a+5b)}{a+2b+a\cos(2(e+fx))}+(-8a^2 \cdots\right.}{\cdots}$$

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^3*((20*a*b^2*(a + b))/(a + 2*b + a*Cos[2*(e
 + f*x)])^2 + (10*a*b*(-6*a + 5*b))/(a + 2*b + a*Cos[2*(e + f*x)]) + (-8*a^
2 + 50*a*b - 15*b^2)*Csc[e + f*x]^2 + 2*(a + b)*(-2*a + 5*b)*Csc[e + f*x]^4
 - 3*(a + b)^2*Csc[e + f*x]^6)*Sec[e + f*x]^4*Tan[e + f*x])/(120*(a + b)^5*
f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [A] (verified)

Time = 6.23 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.47

method	result
default	$-\frac{\left(b+a\cos(fx+e)^2\right)\left(8\cos(fx+e)^8a^4-80\cos(fx+e)^8a^3b+40\cos(fx+e)^8a^2b^2-20a^4\cos(fx+e)^6+212a^3b\cos(fx+e)^6-220\cos(fx+e\cdots\right.}{\cdots}$

```
[In] int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/f/(a^5+5*a^4*b+10*a^3*b^2+10*a^2*b^3+5*a*b^4+b^5)*(b+a*cos(f*x+e)^2)*
(8*cos(f*x+e)^8*a^4-80*cos(f*x+e)^8*a^3*b+40*cos(f*x+e)^8*a^2*b^2-20*a^4*co
s(f*x+e)^6+212*a^3*b*cos(f*x+e)^6-220*cos(f*x+e)^6*a^2*b^2+60*cos(f*x+e)^6*
a*b^3+15*a^4*cos(f*x+e)^4-180*cos(f*x+e)^4*a^3*b+378*a^2*b^2*cos(f*x+e)^4-1
80*cos(f*x+e)^4*a*b^3+15*cos(f*x+e)^4*b^4+60*cos(f*x+e)^2*a^3*b-220*a^2*b^2
*cos(f*x+e)^2+212*a*b^3*cos(f*x+e)^2-20*cos(f*x+e)^2*b^4+40*a^2*b^2-80*a*b^
3+8*b^4)/(a+b*sec(f*x+e)^2)^(5/2)*sec(f*x+e)^5*csc(f*x+e)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 460 vs. $2(206) = 412$.

Time = 9.34 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.04

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$

$$-\frac{\left(8\left(a^4-10a^3b+5a^2b^2\right)\cos(fx+e)^9-4\left(5a^4-53a^3b+55a^2b^2-\cdots\right.\right.}{15\left(\left(a^7+5a^6b+10a^5b^2+10a^4b^3+5a^3b^4+a^2b^5\right)f\cos(fx+e)^8-2\left(a^7+4a^6b+5a^5b^2-5a^3b^4-4a^2b^5\cdots\right.\right.}$$

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a^4 - 10*a^3*b + 5*a^2*b^2)*cos(f*x + e)^9 - 4*(5*a^4 - 53*a^3*b
+ 55*a^2*b^2 - 15*a*b^3)*cos(f*x + e)^7 + 3*(5*a^4 - 60*a^3*b + 126*a^2*b^2
 - 60*a*b^3 + 5*b^4)*cos(f*x + e)^5 + 4*(15*a^3*b - 55*a^2*b^2 + 53*a*b^3 -
 5*b^4)*cos(f*x + e)^3 + 8*(5*a^2*b^2 - 10*a*b^3 + b^4)*cos(f*x + e))*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a
^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b + 5*a^5*b
^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5
*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*cos(f*x + e)^4
+ 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*cos(f*x +
 e)^2 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f)*
sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.65

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\frac{40\,b\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^3}+\frac{20\,b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)^2}-\frac{160\,b^2\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^4}-\frac{80\,b^2\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)^3}+\frac{1\cdots}{\sqrt{b\tan\cdots}}}{\cdots}$$

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/15*(40*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3) + 20*b*
tan(f*x + e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2) - 160*b^2*tan(f*x
 + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^4) - 80*b^2*tan(f*x + e)/((b*
tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^3) + 128*b^3*tan(f*x + e)/(sqrt(b*tan
(f*x + e)^2 + a + b)*(a + b)^5) + 64*b^3*tan(f*x + e)/((b*tan(f*x + e)^2 +
a + b)^(3/2)*(a + b)^4) + 15/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)*tan(
f*x + e)) - 60*b/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2*tan(f*x + e))
```

```
+ 48*b^2/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^3*tan(f*x + e)) + 10/((b
*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)*tan(f*x + e)^3) - 8*b/((b*tan(f*x +
e)^2 + a + b)^(3/2)*(a + b)^2*tan(f*x + e)^3) + 3/((b*tan(f*x + e)^2 + a +
b)^(3/2)*(a + b)*tan(f*x + e)^5))/f
```

Giac [F]

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\csc(fx+e)^6}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\text{Hanged}$$

```
[In] int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(5/2)),x)
[Out] \text{Hanged}
```

3.132 $\int (a+b\sec^2(e+fx))^p (d\sin(e+fx))^m \, dx$

Optimal result

Integrand size = 25, antiderivative size = 123

$$\int (a+b\sec^2(e+fx))^p (d\sin(e+fx))^m \, dx = \frac{\text{AppellF1}\left(\frac{1+m}{2},\frac{1}{2}+p,-p,\frac{3+m}{2},\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^{\frac{1}{2}+p}(a+b\sec^2(e+fx))^p (d\sin(e+}{f(1+m)}$$

```
[Out] AppellF1(1/2+1/2*m,1/2+p,-p,3/2+1/2*m,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(c
os(f*x+e)^2)^(1/2+p)*(a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m*tan(f*x+e)/f/(1+
m)/(((a+b-a*sin(f*x+e)^2)/(a+b))^p)
```

Rubi [F]

$$\int (a+b\sec^2(e+fx))^p (d\sin(e+fx))^m \, dx = \int (a+b\sec^2(e+fx))^p (d\sin(e+fx))^m \, dx$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*(d*Sin[e + f*x])^m,x]
```

```
[Out] Defer[Int][(a + b*Sec[e + f*x]^2)^p*(d*Sin[e + f*x])^m, x]
```

Rubi steps

$$\text{integral} = \int (a+b\sec^2(e+fx))^p (d\sin(e+fx))^m \, dx$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 286 vs. $2(123) = 246$.

Time = 4.20 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.33

$$\int \left(a+b\sec^2(e+fx)\right)^p(d\sin(e+fx))^m\,dx$$
$$=\frac{\text{AppellF1}\left(\frac{1+m}{2},\frac{2+m}{2},-p,\frac{3+m}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\cos(e+f}{f(1+m)\left(\text{AppellF1}\left(\frac{1+m}{2},\frac{2+m}{2},-p,\frac{3+m}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)-\frac{\left(-2bp\,\text{AppellF1}\left(\frac{3+m}{2},\frac{2+m}{2},1-p,\frac{5+m}{2},-\right.\right.}{}\right.}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*(d*Sin[e + f*x])^m,x]
```

```
[Out] (AppellF1[(1 + m)/2, (2 + m)/2, -p, (3 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e
+ f*x]^2)/(a + b))]*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]*(d*S
in[e + f*x])^m)/(f*(1 + m)*(AppellF1[(1 + m)/2, (2 + m)/2, -p, (3 + m)/2, -
Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - ((-2*b*p*AppellF1[(3 + m)/
2, (2 + m)/2, 1 - p, (5 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
b))] + (a + b)*(2 + m)*AppellF1[(3 + m)/2, (4 + m)/2, -p, (5 + m)/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)/((a + b)*(3 + m)
)))
```

Maple [F]

$$\int \left(a+b\sec\left(fx+e\right)^{2}\right)^{p}\left(d\sin\left(fx+e\right)\right)^{m}dx$$

```
[In] int((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x)
```

Fricas [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p(d\sin(e+fx))^m\,dx=\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\left(d\sin\left(fx+e\right)\right)^{m}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p (d\sin(e+fx))^m\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e)**2)**p*(d*sin(f*x+e))**m,x)
[Out] Timed out
```

Maxima [F]

$$\int (a+b\sec^2(e+fx))^p (d\sin(e+fx))^m\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p (d\sin\left(fx+e\right))^m\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

Giac [F]

$$\int (a+b\sec^2(e+fx))^p (d\sin(e+fx))^m\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p (d\sin\left(fx+e\right))^m\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p (d\sin(e+fx))^m\,dx = \int (d\,\sin\left(e+f\,x\right))^m \left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

```
[In] int((d*sin(e + f*x))^m*(a + b/cos(e + f*x)^2)^p,x)
[Out] int((d*sin(e + f*x))^m*(a + b/cos(e + f*x)^2)^p, x)
```

3.133 $\int (a+b\sec^2(e+fx))^p \sin^5(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 182

$$\int (a+b\sec^2(e+fx))^p \sin^5(e+fx)\,dx$$
$$=\frac{(10a+b(3-2p))\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{15a^2f}$$
$$-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{5af}$$
$$-\frac{(15a^2+10ab(1-2p)+b^2(3-8p+4p^2))\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+fx)}{a}\right)(a-}{15a^2f}$$

```
[Out] 1/15*(10*a+b*(3-2*p))*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(p+1)/a^2/f-1/5*cos(f
*x+e)^5*(a+b*sec(f*x+e)^2)^(p+1)/a/f-1/15*(15*a^2+10*a*b*(1-2*p)+b^2*(4*p^2
-8*p+3))*cos(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*sec(f*x+e)^2/a)*(a+b*sec(
f*x+e)^2)^p/a^2/f/((1+b*sec(f*x+e)^2/a)^p)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used

$= \{4219, 473, 464, 372, 371\}$

$$\int \left(a+b\sec^2(e+fx)\right)^p \sin^5(e+fx)\,dx =$$

$$-\frac{\left(15a^2+10ab(1-2p)+b^2(4p^2-8p+3)\right)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}\text{Hyperg}}{15a^2f}$$
$$+\frac{(10a+b(3-2p))\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{p+1}}{15a^2f}$$
$$-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{p+1}}{5af}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^5,x]
```

```
[Out] ((10*a + b*(3 - 2*p))*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(1 + p))/(15*a^
2*f) - (Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(1 + p))/(5*a*f) - ((15*a^2 +
 10*a*b*(1 - 2*p) + b^2*(3 - 8*p + 4*p^2))*Cos[e + f*x]*Hypergeometric2F1[-
1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)]*(a + b*Sec[e + f*x]^2)^p)/(15*a^2*f*
(1 + (b*Sec[e + f*x]^2)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
```

```
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)^2(a+bx^2)^p}{x^6}\,dx, x, \sec(e+fx)\right)}{f} \\
&= -\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{5af} \\
&\quad + \frac{\text{Subst}\left(\int \frac{(-10a-b(3-2p)+5ax^2)(a+bx^2)^p}{x^4}\,dx, x, \sec(e+fx)\right)}{5af} \\
&= \frac{(10a+b(3-2p))\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{15a^2f} \\
&\quad - \frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{5af} \\
&\quad + \frac{(15a^2+10ab(1-2p)+b^2(3-8p+4p^2))\,\text{Subst}\left(\int \frac{(a+bx^2)^p}{x^2}\,dx, x, \sec(e+fx)\right)}{15a^2f} \\
&= \frac{(10a+b(3-2p))\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{15a^2f} \\
&\quad - \frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{5af} \\
&\quad + \frac{\left((15a^2+10ab(1-2p)+b^2(3-8p+4p^2))\left(a+b\sec^2(e+fx)\right)^p\left(1+\frac{b\sec^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int\right.}{15a^2f}
\end{aligned}
$$

$$= \frac{(10a+b(3-2p))\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{15a^2f} - \frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{5af} - \frac{(15a^2+10ab(1-2p)+b^2(3-8p+4p^2))\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e}{a}\right.}{15a^2f}$$

Mathematica [A] (verified)

Time = 3.66 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.39

$$\int \left(a+b\sec^2(e+fx)\right)^p \sin^5(e+fx)\,dx$$

$$= \frac{2\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\sin^4(e+fx)\left(4(15a^2+10ab(1-2p)+b^2(3-8p+4p^2))\,\text{Hypergeome}\right.}{15a^2f\left(4\cos(2(e+fx))\left(\frac{a+b+b\tan^2(e+fx)}{a}\right)^p-2^{-p}\left(3\right.\right.}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^5,x]
```

```
[Out] (2*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^4*(4*(15*a^2 + 10*a*b
*(1 - 2*p) + b^2*(3 - 8*p + 4*p^2))*Hypergeometric2F1[-1/2, -p, 1/2, -((b*S
ec[e + f*x]^2)/a)] + (a + 2*b + a*Cos[2*(e + f*x)])*(-17*a - 6*b + 4*b*p +
3*a*Cos[2*(e + f*x)])*((a + b + b*Tan[e + f*x]^2)/a)^p))/(15*a^2*f*(4*Cos[2
*(e + f*x)]*((a + b + b*Tan[e + f*x]^2)/a)^p - (3*(((a + 2*b + a*Cos[2*(e +
 f*x)])*Sec[e + f*x]^2)/a)^p + 2^p*Cos[4*(e + f*x)]*((a + b + b*Tan[e + f*x
]^2)/a)^p)/2^p))
```

Maple [F]

$$\int \left(a+b\sec\left(fx+e\right)^2\right)^p \sin\left(fx+e\right)^5\,dx$$

```
[In] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x)
```

Fricas [F]

$$\int (a+b\sec^2(e+fx))^p \sin^5(e+fx)\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)^5\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x, algorithm="fricas")
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(b*sec(f*x + e)^2 + a)^p*s
in(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p \sin^5(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e)**5,x)
[Out] Timed out
```

Maxima [F]

$$\int (a+b\sec^2(e+fx))^p \sin^5(e+fx)\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)^5\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^5, x)
```

Giac [F]

$$\int (a+b\sec^2(e+fx))^p \sin^5(e+fx)\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)^5\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + b\sec^2(e + fx))^p \sin^5(e + fx)\,dx = \int \sin(e + f\,x)^5 \left(a + \frac{b}{\cos(e + f\,x)^2}\right)^p\,dx$$

```
[In] int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^p, x)
```

3.134 $\int (a+b\sec^2(e+fx))^p \sin^3(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 117

$$\int (a+b\sec^2(e+fx))^p \sin^3(e+fx)\,dx = \frac{\cos^3(e+fx)\,(a+b\sec^2(e+fx))^{1+p}}{3af} - \frac{(3a+b-2bp)\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^p\left(1+\frac{b\sec^2}{}\right.}{3af}$$

```
[Out] 1/3*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(p+1)/a/f-1/3*(-2*b*p+3*a+b)*cos(f*x+e)
*hypergeom([-1/2, -p],[1/2],-b*sec(f*x+e)^2/a)*(a+b*sec(f*x+e)^2)^p/a/f/((1
+b*sec(f*x+e)^2/a)^p)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4219, 464, 372, 371}

$$\int (a+b\sec^2(e+fx))^p \sin^3(e+fx)\,dx = \frac{\cos^3(e+fx)\,(a+b\sec^2(e+fx))^{p+1}}{3af} - \frac{(3a-2bp+b)\cos(e+fx)\,(a+b\sec^2(e+fx))^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b}{}\right.}{3af}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^3,x]
```

```
[Out] (Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(1 + p))/(3*a*f) - ((3*a + b - 2*b*p
)*Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)]*(a
 + b*Sec[e + f*x]^2)^p)/(3*a*f*(1 + (b*Sec[e + f*x]^2)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a+bx^2)^p}{x^4}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{3af} + \frac{(3a+b-2bp)\text{Subst}\left(\int \frac{(a+bx^2)^p}{x^2}\,dx, x, \sec(e+fx)\right)}{3af}$$

$$= \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{3af} + \frac{\left((3a+b-2bp)\left(a+b\sec^2(e+fx)\right)^p\left(1+\frac{b\sec^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2}\,dx, x, \sec(e+fx\right.}{3af}$$

$$= \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{3af} - \frac{(3a+b-2bp)\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+fx)}{a}\right)\left(a+b\sec^2(e+fx)\right)^p\left(\right.}{3af}$$

Mathematica [A] (verified)

Time = 2.40 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.52

$$\int \left(a+b\sec^2(e+fx)\right)^p \sin^3(e+fx)\,dx = -\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\sin^2(e+fx)\left(-2(3a+b-2bp)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec}{}\right.\right.}{3af\left(-2\left(1+\frac{b\sec^2(e+fx)}{a}\right)^p+\left(\frac{a+b+b\tan^2(e+fx)}{a}\right)^p+\cos(2(e+fx))\right.}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^3,x]
```

```
[Out] -1/3*(Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^2*(-2*(3*a + b - 2
*b*p)*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)] + (a + 2*b
+ a*Cos[2*(e + f*x)])*((a + b + b*Tan[e + f*x]^2)/a)^p))/(a*f*(-2*(1 + (b*S
ec[e + f*x]^2)/a)^p + ((a + b + b*Tan[e + f*x]^2)/a)^p + Cos[2*(e + f*x)]*(
(a + b + b*Tan[e + f*x]^2)/a)^p))
```

Maple [F]

$$\int \left(a+b\sec\left(fx+e\right)^2\right)^p \sin\left(fx+e\right)^3 dx$$

```
[In] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x)
```

Fricas [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p \sin^3(e+fx)\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)^3\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*(b*sec(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p \sin^3(e+fx)\, dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int (a+b\sec^2(e+fx))^p \sin^3(e+fx)\, dx = \int (b\sec(fx+e)^2+a)^p \sin(fx+e)^3\, dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^3, x)
```

Giac [F]

$$\int (a+b\sec^2(e+fx))^p \sin^3(e+fx)\, dx = \int (b\sec(fx+e)^2+a)^p \sin(fx+e)^3\, dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p \sin^3(e+fx)\, dx = \int \sin(e+f\,x)^3 \left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p dx$$

```
[In] int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^p, x)
```

3.135 $\int (a+b\sec^2(e+fx))^p \sin(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 68

$$\int (a+b\sec^2(e+fx))^p \sin(e+fx)\,dx = -\frac{\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^p\left(1+\frac{b\sec^2(e+fx)}{a}\right)^{-p}}{f}$$

```
[Out] -cos(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*sec(f*x+e)^2/a)*(a+b*sec(f*x+e)^2
)^p/f/((1+b*sec(f*x+e)^2/a)^p)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4219, 372, 371}

$$\int (a+b\sec^2(e+fx))^p \sin(e+fx)\,dx = -\frac{\cos(e+fx)(a+b\sec^2(e+fx))^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+fx)}{a}\right)}{f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x],x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)]*(
a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/a)^p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\left((a+b\sec^2(e+fx))^p\left(1+\frac{b\sec^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^p\left(1+\frac{b\sec^2(e+fx)}{a}\right.}{f}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int (a+b\sec^2(e+fx))^p\sin(e+fx)\,dx = -\frac{\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^p\left(1+\frac{b\sec^2(e+fx)}{a}\right)^{-p}}{f}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x],x]
[Out] -((Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)]*(
a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/a)^p))
```

Maple [F]

$$\int \left(a+b\sec\left(fx+e\right)^{2}\right)^{p}\sin\left(fx+e\right)dx$$

```
[In] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x)
[Out] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x)
```

Fricas [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p\sin(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\sin\left(fx+e\right)\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p\sin(e+fx)\,dx=\int \left(a+b\sec^{2}\left(e+fx\right)\right)^{p}\sin\left(e+fx\right)dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e),x)
[Out] Integral((a + b*sec(e + f*x)**2)**p*sin(e + f*x), x)
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p\sin(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\sin\left(fx+e\right)\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

Giac [F]

$$\int (a + b \sec^2(e + fx))^p \sin(e + fx) \, dx = \int \left(b \sec\left(fx + e\right)^2 + a\right)^p \sin\left(fx + e\right) \, dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

Mupad [B] (verification not implemented)

Time = 19.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.16

$$\int (a + b \sec^2(e + fx))^p \sin(e + fx) \, dx = \frac{\cos\left(e + f\,x\right)\,\left(a + \frac{b}{\cos(e+f\,x)^2}\right)^p {}_2\mathrm{F}_1\left(\frac{1}{2} - p, -p;\ \frac{3}{2} - p;\ -\frac{a\cos(e+f\,x)^2}{b}\right)}{f\,\left(2\,p - 1\right)\,\left(\frac{a\cos(e+f\,x)^2}{b} + 1\right)^p}$$

```
[In] int(sin(e + f*x)*(a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] (cos(e + f*x)*(a + b/cos(e + f*x)^2)^p*hypergeom([1/2 - p, -p], 3/2 - p, -(
a*cos(e + f*x)^2)/b))/(f*(2*p - 1)*((a*cos(e + f*x)^2)/b + 1)^p)
```

3.136 $\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = -\frac{\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)\sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(1+\frac{b\sec^2(e+fx)}{a}\right)^{-p}}{f}$$

```
[Out] -AppellF1(1/2,1,-p,3/2,sec(f*x+e)^2,-b*sec(f*x+e)^2/a)*sec(f*x+e)*(a+b*sec(
f*x+e)^2)^p/f/((1+b*sec(f*x+e)^2/a)^p)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4219, 441, 440}

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = -\frac{\sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)}{f}$$

```
[In] Int[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[1/2, 1, -p, 3/2, Sec[e + f*x]^2, -((b*Sec[e + f*x]^2)/a)]*Sec[e
 + f*x]*(a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/a)^p))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
```

```
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\left((a+b\sec^2(e+fx))^p \left(1+\frac{b\sec^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)\sec(e+fx)\,(a+b\sec^2(e+fx))^p\left(1+\frac{b\sec^2(e+fx)}{a}\right.}{f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1532 vs. 2(77) = 154.

Time = 16.05 (sec) , antiderivative size = 1532, normalized size of antiderivative = 19.90

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$= \frac{}{2f\left(-ap(a+2b+a\cos(2(e+fx)))^{-1+p}\sec^2(e+fx)^p\sin(2(e+fx))\left(\frac{2\,\text{AppellF1}\left(-\frac{1}{2}-p,-\frac{1}{2},-p,\frac{1}{2}-p,-\cot^2(e+f\right.}{(1}\right.\right.}$$

```
[In] Integrate[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^p*Csc[e + f*x]*(Sec[e + f*x]^2)^p*(a + b*Se
c[e + f*x]^2)^p*((2*AppellF1[-1/2 - p, -1/2, -p, 1/2 - p, -Cot[e + f*x]^2,
-(((a + b)*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e + f*x]^2])/((1 + 2*p)*(1 + ((a +
b)*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]) - (AppellF1[1, 1/2, -p, 2, -T
an[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2)/((a + b + b*T
an[e + f*x]^2)/(a + b))^p))/(2*f*(-(a*p*(a + 2*b + a*Cos[2*(e + f*x)])^(-1
+ p)*(Sec[e + f*x]^2)^p*Sin[2*(e + f*x)]*((2*AppellF1[-1/2 - p, -1/2, -p, 1
/2 - p, -Cot[e + f*x]^2, -(((a + b)*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e + f*x]^2
])/((1 + 2*p)*(1 + ((a + b)*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]) - (A
ppellF1[1, 1/2, -p, 2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[
e + f*x]^2)/((a + b + b*Tan[e + f*x]^2)/(a + b))^p)) + p*(a + 2*b + a*Cos[2
*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Tan[e + f*x]*((2*AppellF1[-1/2 - p, -1/2,
 -p, 1/2 - p, -Cot[e + f*x]^2, -(((a + b)*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e +
f*x]^2])/((1 + 2*p)*(1 + ((a + b)*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]
) - (AppellF1[1, 1/2, -p, 2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))
]*Tan[e + f*x]^2)/((a + b + b*Tan[e + f*x]^2)/(a + b))^p) + ((a + 2*b + a*C
os[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*((2*AppellF1[-1/2 - p, -1/2, -p, 1/2
- p, -Cot[e + f*x]^2, -(((a + b)*Cot[e + f*x]^2)/b)]*Cot[e + f*x]*Sqrt[Sec[
e + f*x]^2])/((1 + 2*p)*(1 + ((a + b)*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x
]^2]) + (4*(a + b)*p*AppellF1[-1/2 - p, -1/2, -p, 1/2 - p, -Cot[e + f*x]^2,
 -(((a + b)*Cot[e + f*x]^2)/b)]*Cot[e + f*x]*(1 + ((a + b)*Cot[e + f*x]^2)/
b)^(-1 - p)*Sqrt[Csc[e + f*x]^2]*Sqrt[Sec[e + f*x]^2])/(b*(1 + 2*p)) + (2*(
(-2*(a + b)*(-1/2 - p)*p*AppellF1[1/2 - p, -1/2, 1 - p, 3/2 - p, -Cot[e + f
*x]^2, -(((a + b)*Cot[e + f*x]^2)/b)]*Cot[e + f*x]*Csc[e + f*x]^2)/(b*(1/2
- p)) - ((-1/2 - p)*AppellF1[1/2 - p, 1/2, -p, 3/2 - p, -Cot[e + f*x]^2, -(
((a + b)*Cot[e + f*x]^2)/b)]*Cot[e + f*x]*Csc[e + f*x]^2)/(1/2 - p))*Sqrt[S
ec[e + f*x]^2])/((1 + 2*p)*(1 + ((a + b)*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e +
f*x]^2]) + (2*AppellF1[-1/2 - p, -1/2, -p, 1/2 - p, -Cot[e + f*x]^2, -(((a
+ b)*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e + f*x]^2]*Tan[e + f*x])/((1 + 2*p)*(1 +
 ((a + b)*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]) + (2*b*p*AppellF1[1, 1
/2, -p, 2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*T
an[e + f*x]^3*((a + b + b*Tan[e + f*x]^2)/(a + b))^(-1 - p))/(a + b) - (2*A
ppellF1[1, 1/2, -p, 2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[
e + f*x]^2*Tan[e + f*x])/((a + b + b*Tan[e + f*x]^2)/(a + b))^p - (Tan[e +
f*x]^2*((b*p*AppellF1[2, 1/2, 1 - p, 3, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(a + b) - (AppellF1[2, 3/2, -p, 3
, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*
x])/2))/((a + b + b*Tan[e + f*x]^2)/(a + b))^p))/2))
```

Maple [F]

$$\int \csc(fx+e)\left(a+b\sec(fx+e)^2\right)^p dx$$

```
[In] int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec(fx+e)^2+a\right)^p \csc(fx+e)\,dx$$

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2)**p,x)
[Out] Timed out
```

Maxima [F]

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec(fx+e)^2+a\right)^p \csc(fx+e)\,dx$$

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

Giac [F]

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)\,dx$$

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\sin\left(e+f\,x\right)}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x),x)
[Out] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x), x)
```

3.137 $\int \csc^3(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 81

$$\int \csc^3(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \frac{\text{AppellF1}\left(\frac{3}{2},2,-p,\frac{5}{2},\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)\sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(1+\frac{b\sec^2(e+fx)}{a}\right)^{-p}}{3f}$$

```
[Out] 1/3*AppellF1(3/2,2,-p,5/2,sec(f*x+e)^2,-b*sec(f*x+e)^2/a)*sec(f*x+e)^3*(a+b
*sec(f*x+e)^2)^p/f/((1+b*sec(f*x+e)^2/a)^p)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4219, 525, 524}

$$\int \csc^3(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \frac{\sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},2,-p,\frac{5}{2},\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)}{3f}$$

```
[In] Int[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[3/2, 2, -p, 5/2, Sec[e + f*x]^2, -((b*Sec[e + f*x]^2)/a)]*Sec[e +
 f*x]^3*(a + b*Sec[e + f*x]^2)^p)/(3*f*(1 + (b*Sec[e + f*x]^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
```

```
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)^p}{(-1+x^2)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\left((a+b\sec^2(e+fx))^p\left(1+\frac{b\sec^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^2\left(1+\frac{bx^2}{a}\right)^p}{(-1+x^2)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{AppellF1}\left(\frac{3}{2},2,-p,\frac{5}{2},\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)\sec^3(e+fx)\,(a+b\sec^2(e+fx))^p\left(1+\frac{b\sec^2(e+fx}{a}\right.}{3f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 266 vs. $2(81) = 162$.

Time = 3.22 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.28

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$= \frac{\text{AppellF1}\left(\frac{1}{2}-p,-\frac{1}{2},-p,\frac{3}{2}-p,-\cot^2(e+fx)\right.}{f(-1+2p)\left(-\frac{\left(2(a+b)p\,\text{AppellF1}\left(\frac{3}{2}-p,-\frac{1}{2},1-p,\frac{5}{2}-p,-\cot^2(e+fx),-\frac{(a+b)\cot^2(e+fx)}{b}\right)+b\,\text{AppellF1}\left(\frac{3}{2}-p,\frac{1}{2},-p,\frac{5}{2}-p,-\cot^2(e+fx),-\frac{(a}{}\right.\right.}{b(-3+2p)}\right.}$$

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2 - p, -1/2, -p, 3/2 - p, -Cot[e + f*x]^2, -(((a + b)*Cot[e + f
*x]^2)/b)]*Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p)/(f*(-1 + 2*p)*(-(((2*(a
 + b)*p*AppellF1[3/2 - p, -1/2, 1 - p, 5/2 - p, -Cot[e + f*x]^2, -(((a + b)
*Cot[e + f*x]^2)/b)] + b*AppellF1[3/2 - p, 1/2, -p, 5/2 - p, -Cot[e + f*x]^
2, -(((a + b)*Cot[e + f*x]^2)/b)])*Cot[e + f*x]*Csc[e + f*x])/(b*(-3 + 2*p)
)) + AppellF1[1/2 - p, -1/2, -p, 3/2 - p, -Cot[e + f*x]^2, -(((a + b)*Cot[e
 + f*x]^2)/b)]*Sec[e + f*x]))
```

Maple [F]

$$\int \csc\left(fx+e\right)^{3}\left(a+b\sec\left(fx+e\right)^{2}\right)^{p}dx$$

```
[In] int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^3\,dx$$

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^3\,dx$$

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

Giac [F]

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^3\,dx$$

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\sin\left(e+f\,x\right)^3}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^3,x)
[Out] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^3, x)
```

3.138 $\int (a+b\sec^2(e+fx))^p \sin^4(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int (a+b\sec^2(e+fx))^p \sin^4(e+fx)\,dx = \frac{\operatorname{AppellF1}\left(\frac{5}{2},3,-p,\frac{7}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^5(e+fx)\,(a+b+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+}{a+b}\right.}{5f}$$

```
[Out] 1/5*AppellF1(5/2,3,-p,7/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)^5
*(a+b+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4217, 525, 524}

$$\int (a+b\sec^2(e+fx))^p \sin^4(e+fx)\,dx = \frac{\tan^5(e+fx)\,(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{5}{2},3,-p,\frac{7}{2},-\tan^2(e+fx),-\frac{b\tan^2(}{a+}\right.}{5f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^4,x]
```

```
[Out] (AppellF1[5/2, 3, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^p)/(5*f*(1 + (b*Tan[e + f*x]^2)/(
a + b))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4(a+b+bx^2)^p}{(1+x^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((a+b+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^4\left(1+\frac{bx^2}{a+b}\right)^p}{(1+x^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{AppellF1}\left(\frac{5}{2},3,-p,\frac{7}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^5(e+fx)\,(a+b+b\tan^2(e+fx))^p\left(1+\frac{b}{}\right.}{5f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 5878 vs. 2(88) = 176.

Time = 25.84 (sec) , antiderivative size = 5878, normalized size of antiderivative = 66.80

$$\int (a+b\sec^2(e+fx))^p\sin^4(e+fx)\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^4,x]
[Out] Result too large to show
```

Maple [F]

$$\int \left(a + b\sec\left(fx+e\right)^2\right)^p \sin\left(fx+e\right)^4 dx$$

```
[In] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x)
[Out] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x)
```

Fricas [F]

$$\int \left(a + b\sec^2(e+fx)\right)^p \sin^4(e+fx)\,dx = \int \left(b\sec\left(fx+e\right)^2 + a\right)^p \sin\left(fx+e\right)^4\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x, algorithm="fricas")
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(b*sec(f*x + e)^2 + a)^p,
x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a + b\sec^2(e+fx)\right)^p \sin^4(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e)**4,x)
[Out] Timed out
```

Maxima [F]

$$\int \left(a + b\sec^2(e+fx)\right)^p \sin^4(e+fx)\,dx = \int \left(b\sec\left(fx+e\right)^2 + a\right)^p \sin\left(fx+e\right)^4\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^4, x)
```

Giac [F]

$$\int (a+b\sec^2(e+fx))^p \sin^4(e+fx)\,dx = \int \left(b\sec(fx+e)^2+a\right)^p \sin(fx+e)^4\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p \sin^4(e+fx)\,dx = \int \sin(e+f\,x)^4 \left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p\,dx$$

```
[In] int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^p, x)
```

3.139 $\int (a+b\sec^2(e+fx))^p \sin^2(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int (a+b\sec^2(e+fx))^p \sin^2(e+fx)\,dx = \frac{\operatorname{AppellF1}\left(\frac{3}{2},2,-p,\frac{5}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^3(e+fx)\,(a+b+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{3f}$$

```
[Out] 1/3*AppellF1(3/2,2,-p,5/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)^3
*(a+b+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4217, 525, 524}

$$\int (a+b\sec^2(e+fx))^p \sin^2(e+fx)\,dx = \frac{\tan^3(e+fx)\,(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{2},2,-p,\frac{5}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{3f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^2,x]
```

```
[Out] (AppellF1[3/2, 2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2)/(
a + b))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2(a+b+bx^2)^p}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((a+b+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^2\left(1+\frac{bx^2}{a+b}\right)^p}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{AppellF1}\left(\frac{3}{2}, 2, -p, \frac{5}{2}, -\tan^2(e+fx), -\frac{b\tan^2(e+fx)}{a+b}\right)\tan^3(e+fx)\,(a+b+b\tan^2(e+fx))^p\left(1+\frac{b}{\ }\right.}{3f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3781 vs. 2(88) = 176.

Time = 20.19 (sec) , antiderivative size = 3781, normalized size of antiderivative = 42.97

$$\int (a+b\sec^2(e+fx))^p\sin^2(e+fx)\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^2,x]
```

```
[Out] (3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-2 + p)*(a +
b*Sec[e + f*x]^2)^p*Sin[e + f*x]^2*Tan[e + f*x]*(AppellF1[1/2, 2, -p, 3/2,
-Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]/(-3*(a + b)*AppellF1[1/2, 2
, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(-(b*p*Appel
lF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]) +
2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))])*Tan[e + f*x]^2) + (AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)
/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1,
 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2
, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*
AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*
Tan[e + f*x]^2)))/(f*(3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f
*x]^2)^(-1 + p)*(AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))]/(-3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b
*Tan[e + f*x]^2)/(a + b))] + 2*(-(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e +
 f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]) + 2*(a + b)*AppellF1[3/2, 3, -p, 5
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (App
ellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[
e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a +
b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*
x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*T
an[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) - 6*a*(a + b)*p
*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^(-2 + p)*Sin[2*(e
 + f*x)]*Tan[e + f*x]*(AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))]/(-3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2
, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(-(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -T
an[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]) + 2*(a + b)*AppellF1[3/2, 3,
 -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)
+ (AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2
]*Sec[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)
/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[
e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2,
-((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) + 6*(a +
b)*(-2 + p)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-2 + p)*Tan[
e + f*x]^2*(AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/(a + b))]/(-3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))] + 2*(-(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]
^2, -((b*Tan[e + f*x]^2)/(a + b))]) + 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -
Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (AppellF1
[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f
*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)),
-Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)
/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e
+ f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) + 3*(a + b)*(a + 2*b
 + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-2 + p)*Tan[e + f*x]*(((2*b*p*Ap
```

pellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (4*AppellF1[3/2, 3, -p, 5/2, -Ta
n[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3
)/(-3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))] + 2*(-(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*
Tan[e + f*x]^2)/(a + b))]) + 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f
*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (2*AppellF1[1/2, -
p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*T
an[e + f*x])/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a +
 b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f
*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*
Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (Sec[e + f*x]
^2*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan
[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p
, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Ta
n[e + f*x])/3))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(
a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e
+ f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -(
(b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (AppellF1[
1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(4*(-(b*p*
AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))
]) + 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] - 3*(a + b)*((2*b*p*AppellF1[3/2
, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*
x]^2*Tan[e + f*x])/(3*(a + b)) - (4*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]
^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[
e + f*x]^2*(-(b*p*((-6*b*(1 - p)*AppellF1[5/2, 2, 2 - p, 7/2, -Tan[e + f*x]
^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b))
 - (12*AppellF1[5/2, 3, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5)) + 2*(a + b)*((6*b*p*AppellF1[5/2,
 3, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x
]^2*Tan[e + f*x])/(5*(a + b)) - (18*AppellF1[5/2, 4, -p, 7/2, -Tan[e + f*x]
^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(-3*(
a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
 b))] + 2*(-(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e +
 f*x]^2)/(a + b))]) + 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2 - (AppellF1[1/2, -p, 1, 3
/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*(4*(b*p*
AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2
] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
 + f*x]^2])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1
 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^
2*Tan[e + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x
]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e +
 f*x]^2*(b*p*((-6*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)

```
), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5 - (6*b*(1 - p)*AppellF1[
5/2, 2 - p, 1, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e +
 f*x]^2*Tan[e + f*x])/(5*(a + b))) - (a + b)*((6*b*p*AppellF1[5/2, 1 - p, 2
, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e
 + f*x])/(5*(a + b)) - (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/(
a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*App
ellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*
(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f
*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -
Tan[e + f*x]^2])*Tan[e + f*x]^2)^2)))
```

Maple [F]

$$\int \left(a + b\sec\left(fx + e\right)^{2}\right)^{p} \sin\left(fx + e\right)^{2} dx$$

```
[In] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x)
[Out] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x)
```

Fricas [F]

$$\int \left(a + b\sec^2(e + fx)\right)^p \sin^2(e + fx)\, dx = \int \left(b\sec\left(fx + e\right)^2 + a\right)^p \sin\left(fx + e\right)^2\, dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x, algorithm="fricas")
[Out] integral(-(cos(f*x + e)^2 - 1)*(b*sec(f*x + e)^2 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a + b\sec^2(e + fx)\right)^p \sin^2(e + fx)\, dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p \sin^2(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)^2\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^2, x)
```

Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p \sin^2(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)^2\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^p \sin^2(e+fx)\,dx=\int \sin\left(e+f\,x\right)^2\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

```
[In] int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^p, x)
```

3.140 $\int (a+b\sec^2(e+fx))^p\,dx$

Optimal result

Integrand size = 14, antiderivative size = 83

$$\int (a+b\sec^2(e+fx))^p\,dx = \frac{\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{f}$$

```
[Out] AppellF1(1/2,1,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b+
b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4213, 441, 440}

$$\int (a+b\sec^2(e+fx))^p\,dx = \frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((a+b+b\tan^2(e+fx))^p \left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{AppellF1}\left(\frac{1}{2}, 1, -p, \frac{3}{2}, -\tan^2(e+fx), -\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\,(a+b+b\tan^2(e+fx))^p \left(1+\frac{b\,\text{ta}}{}\right.}{f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2137 vs. 2(83) = 166.

Time = 14.98 (sec) , antiderivative size = 2137, normalized size of antiderivative = 25.75

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*
(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, -p, 1, 3
```

/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-1 + p))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (6*(a + b)*p*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (6*a*(a + b)*p*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^p*Sin[e + f*x]*Sin[2*(e + f*x)])/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (3*(a + b)*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]*(4*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5 - (6*b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -(

```
(b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/
(5*(a + b))) - (a + b)*((6*b*p*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*
x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) -
 (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]
^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2,
 -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 -
 p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*Appel
lF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e
 + f*x]^2)^2))
```

Maple [F]

$$\int \left(a + b\sec\left(fx+e\right)^{2}\right)^{p} dx$$

[In] int((a+b*sec(f*x+e)^2)^p,x)

[Out] int((a+b*sec(f*x+e)^2)^p,x)

Fricas [F]

$$\int \left(a + b\sec^2(e+fx)\right)^p \, dx = \int \left(b\sec\left(fx+e\right)^2 + a\right)^p dx$$

[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")

[Out] integral((b*sec(f*x + e)^2 + a)^p, x)

Sympy [F]

$$\int \left(a + b\sec^2(e+fx)\right)^p \, dx = \int \left(a + b\sec^2\left(e+fx\right)\right)^p \, dx$$

[In] integrate((a+b*sec(f*x+e)**2)**p,x)

[Out] Integral((a + b*sec(e + f*x)**2)**p, x)

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p, x)
```

Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^p,x)
[Out] int((a + b/cos(e + f*x)^2)^p, x)
```

3.141 $\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 73

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = -\frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{f}$$

```
[Out] -cot(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*tan(f*x+e)^2/(a+b))*(a+b+b*tan(f*
x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4217, 372, 371}

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = -\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

```
[In] Int[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -((Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a +
b))]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^p}{x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((a+b+b\tan^2(e+fx))^p \left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right)^p}{x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)(a+b+b\tan^2(e+fx))^p \left(1+\frac{b\tan^2}{a}\right.}{f}$$

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = -\frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)(a+b\sec^2(e+fx))^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{f}$$

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -((Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a +
b))]*(a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p))
```

Maple [F]

$$\int \csc\left(fx+e\right)^{2} \left(a+b\sec\left(fx+e\right)^{2}\right)^{p} dx$$

```
[In] int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \csc^2(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \csc\left(fx+e\right)^2 \, dx$$

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2)**p,x)
[Out] Timed out
```

Maxima [F]

$$\int \csc^2(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \csc\left(fx+e\right)^2 \, dx$$

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

Giac [F]

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^2\,dx$$

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\sin\left(e+f\,x\right)^2}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^2,x)
[Out] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^2, x)
```

3.142 $\int \csc^4(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 128

$$\int \csc^4(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = -\frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{3(a+b)f} - \frac{(3a+2b(1+p))\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)\left(a+b+b\tan^2(e+fx)\right)^p \left(1\right.}{3(a+b)f}$$

```
[Out] -1/3*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(p+1)/(a+b)/f-1/3*(3*a+2*b*(p+1))*co
t(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*tan(f*x+e)^2/(a+b))*(a+b+b*tan(f*x+e
)^2)^p/(a+b)/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4217, 464, 372, 371}

$$\int \csc^4(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = -\frac{(3a+2b(p+1))\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p \left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2},-p\right.}{3f(a+b)} - \frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{3f(a+b)}$$

```
[In] Int[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -1/3*(Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(1 + p))/((a + b)*f) - ((3*
a + 2*b*(1 + p))*Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e +
```

```
f*x]^2)/(a + b))]*(a + b + b*Tan[e + f*x]^2)^p)/(3*(a + b)*f*(1 + (b*Tan[e
 + f*x]^2)/(a + b))^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)(a+b+bx^2)^p}{x^4}\,dx, x, \tan(e+fx)\right)}{f}\\ &= -\frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{3(a+b)f}\\ &\quad+\frac{(3a+2b(1+p))\text{Subst}\left(\int \frac{(a+b+bx^2)^p}{x^2}\,dx, x, \tan(e+fx)\right)}{3(a+b)f}\end{aligned}$$

$$= -\frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{3(a+b)f} + \frac{\left((3a+2b(1+p))\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\operatorname{Subst}\left(\int\frac{\left(1+\frac{bx^2}{a+b}\right)^p}{x^2}\,dx,x,\tan(e}{3(a+b)f}$$

$$= -\frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{3(a+b)f} - \frac{(3a+2b(1+p))\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)\left(a+b+b\tan^2(e+f\right.}{3(a+b)f}$$

Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.03

$$\int\csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = -\frac{\cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\left((3a+2b(1+p))\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\right.\right.}{3(a+b)f}$$

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -1/3*(Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^p*((3*a + 2*b*(1 + p))*Hypergeome
tric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))] + Cot[e + f*x]^2*(a +
 b + b*Tan[e + f*x]^2)*(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/((a + b)*f*(1 +
 (b*Tan[e + f*x]^2)/(a + b))^p)
```

Maple [F]

$$\int\csc(fx+e)^4\left(a+b\sec(fx+e)^2\right)^p\,dx$$

```
[In] int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^4\,dx$$

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2)**p,x)
[Out] Timed out
```

Maxima [F]

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^4\,dx$$

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

Giac [F]

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^4\,dx$$

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\sin\left(e+f\,x\right)^4}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^4,x)
[Out] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^4, x)
```

3.143 $\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 192

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$= -\frac{(10a+b(7+2p))\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{15(a+b)^2f}$$
$$-\frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{5(a+b)f}$$
$$-\frac{(15a^2+20ab(1+p)+4b^2(2+3p+p^2))\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)(a}{15(a+b)^2f}$$

```
[Out] -1/15*(10*a+b*(7+2*p))*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(p+1)/(a+b)^2/f-1/
5*cot(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(p+1)/(a+b)/f-1/15*(15*a^2+20*a*b*(p+1)
+4*b^2*(p^2+3*p+2))*cot(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*tan(f*x+e)^2/(
a+b))*(a+b+b*tan(f*x+e)^2)^p/(a+b)^2/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used

$= \{4217, 473, 464, 372, 371\}$

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx =$$
$$-\frac{\left(15a^2+20ab(p+1)+4b^2(p^2+3p+2)\right)\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{Hype}}{15f(a+b)^2}$$
$$-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{5f(a+b)}$$
$$-\frac{(10a+b(2p+7))\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{15f(a+b)^2}$$

```
[In] Int[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -1/15*((10*a + b*(7 + 2*p))*Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(1 +
p))/((a + b)^2*f) - (Cot[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(5*
(a + b)*f) - ((15*a^2 + 20*a*b*(1 + p) + 4*b^2*(2 + 3*p + p^2))*Cot[e + f*x
]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + b +
b*Tan[e + f*x]^2)^p)/(15*(a + b)^2*f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
```

```
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2(a+b+bx^2)^p}{x^6}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{5(a+b)f} + \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^p(10a+b(7+2p)+5(a+b)x^2)}{x^4}\,dx, x, \tan(e+fx)\right)}{5(a+b)f}$$

$$= -\frac{(10a+b(7+2p))\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{15(a+b)^2f} - \frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{5(a+b)f} + \frac{(15a^2+20ab(1+p)+4b^2(2+3p+p^2))\,\text{Subst}\left(\int \frac{(a+b+bx^2)^p}{x^2}\,dx, x, \tan(e+fx)\right)}{15(a+b)^2f}$$

$$= -\frac{(10a+b(7+2p))\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{15(a+b)^2f} - \frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{5(a+b)f} + \frac{\left((15a^2+20ab(1+p)+4b^2(2+3p+p^2))\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subs}}{15(a+b)^2f}$$

$$= -\frac{(10a+b(7+2p))\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{15(a+b)^2f} - \frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{5(a+b)f} - \frac{(15a^2+20ab(1+p)+4b^2(2+3p+p^2))\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)}{15(a+b)^2f}$$

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.78

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = -\frac{\cot(e+fx)\left(3\cot^4(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-p,-\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)+10\cot^2(e+fx)\,\text{Hyperge}\right.}{}$$

```
[In] Integrate[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -1/15*(Cot[e + f*x]*(3*Cot[e + f*x]^4*Hypergeometric2F1[-5/2, -p, -3/2, -((
b*Tan[e + f*x]^2)/(a + b))] + 10*Cot[e + f*x]^2*Hypergeometric2F1[-3/2, -p,
 -1/2, -((b*Tan[e + f*x]^2)/(a + b))] + 15*Hypergeometric2F1[-1/2, -p, 1/2,
 -((b*Tan[e + f*x]^2)/(a + b))])*(a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Tan[e
 + f*x]^2)/(a + b))^p)
```

Maple [F]

$$\int \csc\left(fx+e\right)^6\left(a+b\sec\left(fx+e\right)^2\right)^p\,dx$$

```
[In] int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^6\,dx$$

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2)**p,x)
[Out] Timed out
```

Maxima [F]

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^6\,dx$$

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

Giac [F]

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^6\,dx$$

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\sin\left(e+f\,x\right)^6}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^6,x)
[Out] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^6, x)
```

3.144 $\int (a - a\sec^2(c+dx))^4 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int (a - a\sec^2(c+dx))^4 \, dx = a^4x - \frac{a^4\tan(c+dx)}{d} + \frac{a^4\tan^3(c+dx)}{3d} - \frac{a^4\tan^5(c+dx)}{5d} + \frac{a^4\tan^7(c+dx)}{7d}$$

```
[Out] a^4*x-a^4*tan(d*x+c)/d+1/3*a^4*tan(d*x+c)^3/d-1/5*a^4*tan(d*x+c)^5/d+1/7*a^
4*tan(d*x+c)^7/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4205, 3554, 8}

$$\int (a - a\sec^2(c+dx))^4 \, dx = \frac{a^4\tan^7(c+dx)}{7d} - \frac{a^4\tan^5(c+dx)}{5d} + \frac{a^4\tan^3(c+dx)}{3d} - \frac{a^4\tan(c+dx)}{d} + a^4x$$

```
[In] Int[(a - a*Sec[c + d*x]^2)^4,x]
```

```
[Out] a^4*x - (a^4*Tan[c + d*x])/d + (a^4*Tan[c + d*x]^3)/(3*d) - (a^4*Tan[c + d*
x]^5)/(5*d) + (a^4*Tan[c + d*x]^7)/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4205

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= a^4\int \tan^8(c+dx)\,dx\\
&= \frac{a^4\tan^7(c+dx)}{7d}-a^4\int \tan^6(c+dx)\,dx\\
&= -\frac{a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^7(c+dx)}{7d}+a^4\int \tan^4(c+dx)\,dx\\
&= \frac{a^4\tan^3(c+dx)}{3d}-\frac{a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^7(c+dx)}{7d}-a^4\int \tan^2(c+dx)\,dx\\
&= -\frac{a^4\tan(c+dx)}{d}+\frac{a^4\tan^3(c+dx)}{3d}-\frac{a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^7(c+dx)}{7d}+a^4\int 1\,dx\\
&= a^4x-\frac{a^4\tan(c+dx)}{d}+\frac{a^4\tan^3(c+dx)}{3d}-\frac{a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^7(c+dx)}{7d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

$$\int \left(a-a\sec^2(c+dx)\right)^4\,dx = a^4\left(\frac{\arctan(\tan(c+dx))}{d}-\frac{\tan(c+dx)}{d}+\frac{\tan^3(c+dx)}{3d}-\frac{\tan^5(c+dx)}{5d}+\frac{\tan^7(c+dx)}{7d}\right)$$

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^4,x]
[Out] a^4*(ArcTan[Tan[c + d*x]]/d - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d) - Tan[c
 + d*x]^5/(5*d) + Tan[c + d*x]^7/(7*d))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.31

method	result
risch	$a^4x-\frac{8ia^4(105\,e^{12i(dx+c)}+315\,e^{10i(dx+c)}+770\,e^{8i(dx+c)}+770\,e^{6i(dx+c)}+609\,e^{4i(dx+c)}+203\,e^{2i(dx+c)}+44)}{105d(e^{2i(dx+c)}+1)^7}$
derivativedivides	$\frac{a^4(dx+c)-4a^4\tan(dx+c)-6a^4\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+4a^4\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)-a^4\left(-\right.}{d}$
default	$\frac{a^4(dx+c)-4a^4\tan(dx+c)-6a^4\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+4a^4\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)-a^4\left(-\right.}{d}$
parts	$a^4x-\frac{a^4\left(-\frac{16}{35}-\frac{\sec(dx+c)^6}{7}-\frac{6\sec(dx+c)^4}{35}-\frac{8\sec(dx+c)^2}{35}\right)\tan(dx+c)}{d}-\frac{4a^4\tan(dx+c)}{d}-\frac{6a^4\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan}{d}$
parallelrisch	$\frac{a^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}xd-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}xd+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}+21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}xd-\frac{44\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{3}-35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}x\right.}{}$
norman	$\frac{a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}-a^4x+\frac{2a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{44a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+\frac{706a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{15d}-\frac{3048a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{35d}+\frac{706a^4\tan\left(\frac{dx}{2}+\right.}{15d}}{}$

```
[In] int((a-a*sec(d*x+c)^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*x-8/105*I*a^4*(105*exp(12*I*(d*x+c))+315*exp(10*I*(d*x+c))+770*exp(8*I*
(d*x+c))+770*exp(6*I*(d*x+c))+609*exp(4*I*(d*x+c))+203*exp(2*I*(d*x+c))+44)
/d/(exp(2*I*(d*x+c))+1)^7
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int \left(a-a\sec^2(c+dx)\right)^4\,dx$$
$$=\frac{105\,a^4dx\cos\left(dx+c\right)^7-\left(176\,a^4\cos\left(dx+c\right)^6-122\,a^4\cos\left(dx+c\right)^4+66\,a^4\cos\left(dx+c\right)^2-15\,a^4\right)\sin\left(dx+\right.}{105\,d\cos\left(dx+c\right)^7}$$

```
[In] integrate((a-a*sec(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(105*a^4*d*x*cos(d*x + c)^7 - (176*a^4*cos(d*x + c)^6 - 122*a^4*cos(d
*x + c)^4 + 66*a^4*cos(d*x + c)^2 - 15*a^4)*sin(d*x + c))/(d*cos(d*x + c)^7
)
```

Sympy [F]

$$\int \left(a-a\sec^2(c+dx)\right)^4\,dx=a^4\left(\int 1\,dx+\int \left(-4\sec^2(c+dx)\right)\,dx+\int 6\sec^4(c+dx)\,dx+\int \left(-4\sec^6(c+dx)\right)\,dx+\int \sec^8(c+dx)\,dx\right)$$

```
[In] integrate((a-a*sec(d*x+c)**2)**4,x)
```

```
[Out] a**4*(Integral(1, x) + Integral(-4*sec(c + d*x)**2, x) + Integral(6*sec(c +
 d*x)**4, x) + Integral(-4*sec(c + d*x)**6, x) + Integral(sec(c + d*x)**8,
x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.74

$$\begin{aligned}\int \left(a-a\sec^2(c+dx)\right)^4\,dx &= a^4x+\frac{\left(5\tan(dx+c)^7+21\tan(dx+c)^5+35\tan(dx+c)^3+35\tan(dx+c)\right)a^4}{35\,d}\\&\quad-\frac{4\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)a^4}{15\,d}\\&\quad+\frac{2\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^4}{d}-\frac{4\,a^4\tan(dx+c)}{d}\end{aligned}$$

```
[In] integrate((a-a*sec(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 1/35*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35
*tan(d*x + c))*a^4/d - 4/15*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(
d*x + c))*a^4/d + 2*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4/d - 4*a^4*tan(d*x
 + c)/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

$$\int \left(a-a\sec^2(c+dx)\right)^4\,dx=\frac{15\,a^4\tan(dx+c)^7-21\,a^4\tan(dx+c)^5+35\,a^4\tan(dx+c)^3+105\,(dx+c)a^4-105\,a^4\tan(dx+c)}{105\,d}$$

```
[In] integrate((a-a*sec(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] 1/105*(15*a^4*tan(d*x + c)^7 - 21*a^4*tan(d*x + c)^5 + 35*a^4*tan(d*x + c)^
3 + 105*(d*x + c)*a^4 - 105*a^4*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 19.40 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

$$\int \left(a-a\sec^2(c+dx)\right)^4 \, dx = \frac{\frac{a^4\tan(c+d\,x)^7}{7}-\frac{a^4\tan(c+d\,x)^5}{5}+\frac{a^4\tan(c+d\,x)^3}{3}-a^4\tan(c+d\,x)+d\,x\,a^4}{d}$$

```
[In] int((a - a/cos(c + d*x)^2)^4,x)
```

```
[Out] ((a^4*tan(c + d*x)^3)/3 - a^4*tan(c + d*x) - (a^4*tan(c + d*x)^5)/5 + (a^4*
tan(c + d*x)^7)/7 + a^4*d*x)/d
```

3.145 $\int (a-a\sec^2(c+dx))^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int (a-a\sec^2(c+dx))^3\,dx=a^3x-\frac{a^3\tan(c+dx)}{d}+\frac{a^3\tan^3(c+dx)}{3d}-\frac{a^3\tan^5(c+dx)}{5d}$$

```
[Out] a^3*x-a^3*tan(d*x+c)/d+1/3*a^3*tan(d*x+c)^3/d-1/5*a^3*tan(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4205, 3554, 8}

$$\int (a-a\sec^2(c+dx))^3\,dx=-\frac{a^3\tan^5(c+dx)}{5d}+\frac{a^3\tan^3(c+dx)}{3d}-\frac{a^3\tan(c+dx)}{d}+a^3x$$

```
[In] Int[(a - a*Sec[c + d*x]^2)^3,x]
```

```
[Out] a^3*x - (a^3*Tan[c + d*x])/d + (a^3*Tan[c + d*x]^3)/(3*d) - (a^3*Tan[c + d*
x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4205

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(a^3\int \tan^6(c+dx)\,dx\right)\\ &= -\frac{a^3\tan^5(c+dx)}{5d}+a^3\int \tan^4(c+dx)\,dx\\ &= \frac{a^3\tan^3(c+dx)}{3d}-\frac{a^3\tan^5(c+dx)}{5d}-a^3\int \tan^2(c+dx)\,dx\\ &= -\frac{a^3\tan(c+dx)}{d}+\frac{a^3\tan^3(c+dx)}{3d}-\frac{a^3\tan^5(c+dx)}{5d}+a^3\int 1\,dx\\ &= a^3x-\frac{a^3\tan(c+dx)}{d}+\frac{a^3\tan^3(c+dx)}{3d}-\frac{a^3\tan^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04

$$\int \left(a-a\sec^2(c+dx)\right)^3\,dx=-a^3\left(-\frac{\arctan(\tan(c+dx))}{d}+\frac{\tan(c+dx)}{d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan^5(c+dx)}{5d}\right)$$

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^3,x]
[Out] -(a^3*(-(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d - Tan[c + d*x]^3/(3*d) +
Tan[c + d*x]^5/(5*d)))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.34

method	result
risch	$a^{3}x-\frac{2ia^{3}\left(45\,{\mathrm e}^{8i(dx+c)}+90\,{\mathrm e}^{6i(dx+c)}+140\,{\mathrm e}^{4i(dx+c)}+70\,{\mathrm e}^{2i(dx+c)}+23\right)}{15d\left({\mathrm e}^{2i(dx+c)}+1\right)^{5}}$
derivativedivides	$\frac{a^{3}(dx+c)-3a^{3}\tan(dx+c)-3a^{3}\left(-\frac{2}{3}-\frac{\sec(dx+c)^{2}}{3}\right)\tan(dx+c)+a^{3}\left(-\frac{8}{15}-\frac{\sec(dx+c)^{4}}{5}-\frac{4\sec(dx+c)^{2}}{15}\right)\tan(dx+c)}{d}$
default	$\frac{a^{3}(dx+c)-3a^{3}\tan(dx+c)-3a^{3}\left(-\frac{2}{3}-\frac{\sec(dx+c)^{2}}{3}\right)\tan(dx+c)+a^{3}\left(-\frac{8}{15}-\frac{\sec(dx+c)^{4}}{5}-\frac{4\sec(dx+c)^{2}}{15}\right)\tan(dx+c)}{d}$
parts	$a^{3}x-\frac{3a^{3}\tan(dx+c)}{d}-\frac{3a^{3}\left(-\frac{2}{3}-\frac{\sec(dx+c)^{2}}{3}\right)\tan(dx+c)}{d}+\frac{a^{3}\left(-\frac{8}{15}-\frac{\sec(dx+c)^{4}}{5}-\frac{4\sec(dx+c)^{2}}{15}\right)\tan(dx+c)}{d}$
parallelrisch	$\frac{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}xd-5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}xd+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}+10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}xd-\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{3}-10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}xd+\frac{35}{}\right.}{d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}$
norman	$\frac{a^{3}x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}-a^{3}x+\frac{2a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{32a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3d}+\frac{356a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{15d}-\frac{32a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{3d}+\frac{2a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)^{5}}$

```
[In] int((a-a*sec(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*x-2/15*I*a^3*(45*exp(8*I*(d*x+c))+90*exp(6*I*(d*x+c))+140*exp(4*I*(d*x+
c))+70*exp(2*I*(d*x+c))+23)/d/(exp(2*I*(d*x+c))+1)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.23

$$\int \left(a-a\sec^2(c+dx)\right)^3\,dx = \frac{15\,a^3dx\cos\left(dx+c\right)^5-\left(23\,a^3\cos\left(dx+c\right)^4-11\,a^3\cos\left(dx+c\right)^2+3\,a^3\right)\sin\left(dx+c\right)}{15\,d\cos\left(dx+c\right)^5}$$

```
[In] integrate((a-a*sec(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*a^3*d*x*cos(d*x + c)^5 - (23*a^3*cos(d*x + c)^4 - 11*a^3*cos(d*x +
 c)^2 + 3*a^3)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F]

$$\int \left(a - a\sec^2(c+dx)\right)^3 \, dx = -a^3 \left(\int (-1)\, dx + \int 3\sec^2(c+dx)\, dx + \int \left(-3\sec^4(c+dx)\right)\, dx + \int \sec^6(c+dx)\, dx\right)$$

```
[In] integrate((a-a*sec(d*x+c)**2)**3,x)
```

```
[Out] -a**3*(Integral(-1, x) + Integral(3*sec(c + d*x)**2, x) + Integral(-3*sec(c
 + d*x)**4, x) + Integral(sec(c + d*x)**6, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.45

$$\int \left(a - a\sec^2(c+dx)\right)^3 \, dx = a^3 x - \frac{\left(3\,\tan\left(dx+c\right)^5 + 10\,\tan\left(dx+c\right)^3 + 15\,\tan\left(dx+c\right)\right)a^3}{15\,d} + \frac{\left(\tan\left(dx+c\right)^3 + 3\,\tan\left(dx+c\right)\right)a^3}{d} - \frac{3\,a^3\tan\left(dx+c\right)}{d}$$

```
[In] integrate((a-a*sec(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*x - 1/15*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3/d
 + (tan(d*x + c)^3 + 3*tan(d*x + c))*a^3/d - 3*a^3*tan(d*x + c)/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int \left(a - a\sec^2(c+dx)\right)^3 \, dx = -\frac{3\,a^3\tan\left(dx+c\right)^5 - 5\,a^3\tan\left(dx+c\right)^3 - 15\,(dx+c)a^3 + 15\,a^3\tan\left(dx+c\right)}{15\,d}$$

```
[In] integrate((a-a*sec(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/15*(3*a^3*tan(d*x + c)^5 - 5*a^3*tan(d*x + c)^3 - 15*(d*x + c)*a^3 + 15*
a^3*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 19.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int \left(a-a\sec^2(c+dx)\right)^3\,dx=-\frac{\frac{a^3\tan(c+d\,x)^5}{5}-\frac{a^3\tan(c+d\,x)^3}{3}+a^3\tan(c+d\,x)-d\,x\,a^3}{d}$$

```
[In] int((a - a/cos(c + d*x)^2)^3,x)
[Out] -(a^3*tan(c + d*x) - (a^3*tan(c + d*x)^3)/3 + (a^3*tan(c + d*x)^5)/5 - a^3*
d*x)/d
```

3.146 $\int (a - a\sec^2(c + dx))^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int (a - a\sec^2(c + dx))^2 \, dx = a^2x - \frac{a^2\tan(c + dx)}{d} + \frac{a^2\tan^3(c + dx)}{3d}$$

```
[Out] a^2*x-a^2*tan(d*x+c)/d+1/3*a^2*tan(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4205, 3554, 8}

$$\int (a - a\sec^2(c + dx))^2 \, dx = \frac{a^2\tan^3(c + dx)}{3d} - \frac{a^2\tan(c + dx)}{d} + a^2x$$

```
[In] Int[(a - a*Sec[c + d*x]^2)^2,x]
```

```
[Out] a^2*x - (a^2*Tan[c + d*x])/d + (a^2*Tan[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4205

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= a^2 \int \tan^4(c+dx)\,dx \\ &= \frac{a^2\tan^3(c+dx)}{3d} - a^2 \int \tan^2(c+dx)\,dx \\ &= -\frac{a^2\tan(c+dx)}{d} + \frac{a^2\tan^3(c+dx)}{3d} + a^2 \int 1\,dx \\ &= a^2x - \frac{a^2\tan(c+dx)}{d} + \frac{a^2\tan^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \left(a - a\sec^2(c+dx)\right)^2\,dx = a^2\left(\frac{\arctan(\tan(c+dx))}{d} - \frac{\tan(c+dx)}{d} + \frac{\tan^3(c+dx)}{3d}\right)$$

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^2,x]
[Out] a^2*(ArcTan[Tan[c + d*x]]/d - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d))
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

method	result
parts	$a^{2} x -\frac{a^{2} \left(-\frac{2}{3}-\frac{\sec \left(d x +c \right)^{2}}{3}\right) \tan \left(d x +c \right)}{d}-\frac{2 a^{2} \tan \left(d x +c \right)}{d}$
derivativedivides	$\frac{a^{2} \left(d x +c \right)-2 a^{2} \tan \left(d x +c \right)-a^{2} \left(-\frac{2}{3}-\frac{\sec \left(d x +c \right)^{2}}{3}\right) \tan \left(d x +c \right)}{d}$
default	$\frac{a^{2} \left(d x +c \right)-2 a^{2} \tan \left(d x +c \right)-a^{2} \left(-\frac{2}{3}-\frac{\sec \left(d x +c \right)^{2}}{3}\right) \tan \left(d x +c \right)}{d}$
risch	$a^{2} x -\frac{4 i a^{2} \left(3 \,{\mathrm e}^{4 i \left(d x +c \right)}+3 \,{\mathrm e}^{2 i \left(d x +c \right)}+2\right)}{3 d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{3}}$
parallelrisch	$\frac{a^{2} \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{6} x d -3 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{4} x d +2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{5}+3 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2} x d -\frac{20 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{3}-d x +2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{d \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^{3} \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{3}}$
norman	$\frac{a^{2} x \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{6}-a^{2} x +\frac{2 a^{2} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{d}-\frac{20 a^{2} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{3 d}+\frac{2 a^{2} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{5}}{d}+3 a^{2} x \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}-3 a^{2} x \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{4}}{\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}-1\right)^{3}}$

```
[In] int((a-a*sec(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x-a^2/d*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)-2*a^2*tan(d*x+c)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int \left(a-a\sec^2(c+dx)\right)^2\,dx=\frac{3\,a^{2} d x \cos \left(d x + c\right)^{3} - {\left(4 \, a^{2} \cos \left(d x + c\right)^{2} - a^{2}\right)} \sin \left(d x + c\right)}{3 \, d \cos \left(d x + c\right)^{3}}$$

```
[In] integrate((a-a*sec(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^2*d*x*cos(d*x + c)^3 - (4*a^2*cos(d*x + c)^2 - a^2)*sin(d*x + c))/
(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \left(a - a\sec^2(c+dx)\right)^2\,dx = a^2\left(\int 1\,dx + \int \left(-2\sec^2\left(c+dx\right)\right)\,dx + \int \sec^4\left(c+dx\right)dx\right)$$

```
[In] integrate((a-a*sec(d*x+c)**2)**2,x)
[Out] a**2*(Integral(1, x) + Integral(-2*sec(c + d*x)**2, x) + Integral(sec(c + d
*x)**4, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int \left(a - a\sec^2(c+dx)\right)^2\,dx = a^2x + \frac{\left(\tan\left(dx+c\right)^3 + 3\,\tan\left(dx+c\right)\right)a^2}{3\,d} - \frac{2\,a^2\tan\left(dx+c\right)}{d}$$

```
[In] integrate((a-a*sec(d*x+c)^2)^2,x, algorithm="maxima")
[Out] a^2*x + 1/3*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2/d - 2*a^2*tan(d*x + c)/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \left(a - a\sec^2(c+dx)\right)^2\,dx = \frac{a^2\tan\left(dx+c\right)^3 + 3\left(dx+c\right)a^2 - 3\,a^2\tan\left(dx+c\right)}{3\,d}$$

```
[In] integrate((a-a*sec(d*x+c)^2)^2,x, algorithm="giac")
[Out] 1/3*(a^2*tan(d*x + c)^3 + 3*(d*x + c)*a^2 - 3*a^2*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 18.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \left(a - a\sec^2(c+dx)\right)^2\,dx = a^2\,x - \frac{a^2\,\left(3\tan(c+d\,x) - \tan(c+d\,x)^3\right)}{3\,d}$$

```
[In] int((a - a/cos(c + d*x)^2)^2,x)
[Out] a^2*x - (a^2*(3*tan(c + d*x) - tan(c + d*x)^3))/(3*d)
```

3.147 $\int (a - a\sec^2(c + dx))\ dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int (a - a\sec^2(c + dx))\ dx = ax - \frac{a\tan(c + dx)}{d}$$

```
[Out] a*x-a*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3852, 8}

$$\int (a - a\sec^2(c + dx))\ dx = ax - \frac{a\tan(c + dx)}{d}$$

```
[In] Int[a - a*Sec[c + d*x]^2,x]
```

```
[Out] a*x - (a*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= ax - a\int \sec^2(c+dx)\,dx\\ &= ax + \frac{a\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d}\\ &= ax - \frac{a\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \left(a - a\sec^2(c+dx)\right)\,dx = -a\left(-\frac{\arctan(\tan(c+dx))}{d} + \frac{\tan(c+dx)}{d}\right)$$

```
[In] Integrate[a - a*Sec[c + d*x]^2,x]
[Out] -(a*(-(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
default	$ax - \frac{a\tan(dx+c)}{d}$	17
parts	$ax - \frac{a\tan(dx+c)}{d}$	17
derivativedivides	$\frac{(dx+c)a - a\tan(dx+c)}{d}$	22
risch	$ax - \frac{2ia}{d\left(e^{2i(dx+c)}+1\right)}$	25
parallelrisch	$\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)} + ax$	35
norman	$\frac{ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 - ax + \frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}$	51

```
[In] int(a-a*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
[Out] a*x-a*tan(d*x+c)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \left(a-a\sec^2(c+dx)\right)\,dx=\frac{adx\cos\left(dx+c\right)-a\sin\left(dx+c\right)}{d\cos\left(dx+c\right)}$$

```
[In] integrate(a-a*sec(d*x+c)^2,x, algorithm="fricas")
[Out] (a*d*x*cos(d*x + c) - a*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \left(a-a\sec^2(c+dx)\right)\,dx=-a\left(\int (-1)\,dx+\int \sec^2\left(c+dx\right)dx\right)$$

```
[In] integrate(a-a*sec(d*x+c)**2,x)
[Out] -a*(Integral(-1, x) + Integral(sec(c + d*x)**2, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \left(a-a\sec^2(c+dx)\right)\,dx=ax-\frac{a\tan\left(dx+c\right)}{d}$$

```
[In] integrate(a-a*sec(d*x+c)^2,x, algorithm="maxima")
[Out] a*x - a*tan(d*x + c)/d
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \left(a-a\sec^2(c+dx)\right)\,dx=ax-\frac{a\tan\left(dx+c\right)}{d}$$

```
[In] integrate(a-a*sec(d*x+c)^2,x, algorithm="giac")
[Out] a*x - a*tan(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 18.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \left(a - a\sec^2(c + dx)\right) \, dx = a\,x - \frac{a\tan(c + d\,x)}{d}$$

```
[In] int(a - a/cos(c + d*x)^2,x)
[Out] a*x - (a*tan(c + d*x))/d
```

3.148 $\int \frac{1}{a-a\sec^2(c+dx)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{1}{a-a\sec^2(c+dx)}\,dx = \frac{x}{a}+\frac{\cot(c+dx)}{ad}$$

```
[Out] x/a+cot(d*x+c)/a/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4205, 3554, 8}

$$\int \frac{1}{a-a\sec^2(c+dx)}\,dx = \frac{\cot(c+dx)}{ad}+\frac{x}{a}$$

```
[In] Int[(a - a*Sec[c + d*x]^2)^(-1),x]
```

```
[Out] x/a + Cot[c + d*x]/(a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4205

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\int \cot^2(c+dx)\,dx}{a}\\ &= \frac{\cot(c+dx)}{ad}+\frac{\int 1\,dx}{a}\\ &= \frac{x}{a}+\frac{\cot(c+dx)}{ad}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int \frac{1}{a-a\sec^2(c+dx)}\,dx = \frac{\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right)}{ad}$$

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(-1),x]
[Out] (Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2])/(a*d)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

method	result	size
derivativedivides	$\frac{\frac{1}{\tan(dx+c)}+\arctan(\tan(dx+c))}{da}$	24
default	$\frac{\frac{1}{\tan(dx+c)}+\arctan(\tan(dx+c))}{da}$	24
risch	$\frac{x}{a}+\frac{2i}{da\left(e^{2i(dx+c)}-1\right)}$	29
parallelrisch	$\frac{2dx-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\cot\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}$	34
norman	$\frac{\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+\frac{1}{2ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}$	55

```
[In] int(1/(a-a*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(1/tan(d*x+c)+arctan(tan(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int \frac{1}{a - a\sec^2(c + dx)}\,dx = \frac{dx\sin(dx + c) + \cos(dx + c)}{ad\sin(dx + c)}$$

```
[In] integrate(1/(a-a*sec(d*x+c)^2),x, algorithm="fricas")
[Out] (d*x*sin(d*x + c) + cos(d*x + c))/(a*d*sin(d*x + c))
```

Sympy [F]

$$\int \frac{1}{a - a\sec^2(c + dx)}\,dx = -\frac{\int \frac{1}{\sec^2(c+dx)-1}\,dx}{a}$$

```
[In] integrate(1/(a-a*sec(d*x+c)**2),x)
[Out] -Integral(1/(sec(c + d*x)**2 - 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{1}{a - a\sec^2(c + dx)}\,dx = \frac{\frac{dx+c}{a} + \frac{1}{a\tan(dx+c)}}{d}$$

```
[In] integrate(1/(a-a*sec(d*x+c)^2),x, algorithm="maxima")
[Out] ((d*x + c)/a + 1/(a*tan(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(19) = 38.

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.37

$$\int \frac{1}{a - a\sec^2(c + dx)}\,dx = \frac{\frac{2(dx+c)}{a} - \frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a} + \frac{1}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{2d}$$

```
[In] integrate(1/(a-a*sec(d*x+c)^2),x, algorithm="giac")
[Out] 1/2*(2*(d*x + c)/a - tan(1/2*d*x + 1/2*c)/a + 1/(a*tan(1/2*d*x + 1/2*c)))/d
```

Mupad [B] (verification not implemented)

Time = 18.65 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{a-a\sec^2(c+dx)}\,dx=\frac{x}{a}+\frac{\cot(c+d\,x)}{a\,d}$$

```
[In] int(1/(a - a/cos(c + d*x)^2),x)
[Out] x/a + cot(c + d*x)/(a*d)
```

3.149 $\int \frac{1}{(a-a\sec^2(c+dx))^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{1}{(a-a\sec^2(c+dx))^2}\,dx = \frac{x}{a^2}+\frac{\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}$$

```
[Out] x/a^2+cot(d*x+c)/a^2/d-1/3*cot(d*x+c)^3/a^2/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4205, 3554, 8}

$$\int \frac{1}{(a-a\sec^2(c+dx))^2}\,dx = -\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)}{a^2d}+\frac{x}{a^2}$$

```
[In] Int[(a - a*Sec[c + d*x]^2)^(-2),x]
```

```
[Out] x/a^2 + Cot[c + d*x]/(a^2*d) - Cot[c + d*x]^3/(3*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4205

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \cot^4(c+dx)\,dx}{a^2}\\
&= -\frac{\cot^3(c+dx)}{3a^2d} - \frac{\int \cot^2(c+dx)\,dx}{a^2}\\
&= \frac{\cot(c+dx)}{a^2d} - \frac{\cot^3(c+dx)}{3a^2d} + \frac{\int 1\,dx}{a^2}\\
&= \frac{x}{a^2} + \frac{\cot(c+dx)}{a^2d} - \frac{\cot^3(c+dx)}{3a^2d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a-a\sec^2(c+dx))^2}\,dx = -\frac{\cot^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(c+dx)\right)}{3a^2d}$$

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(-2),x]
[Out] -1/3*(Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(a^
2*d)
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\arctan(\tan(dx+c))-\frac{1}{3\tan(dx+c)^3}+\frac{1}{\tan(dx+c)}}{d\,a^2}$	34
default	$\frac{\arctan(\tan(dx+c))-\frac{1}{3\tan(dx+c)^3}+\frac{1}{\tan(dx+c)}}{d\,a^2}$	34
risch	$\frac{x}{a^2}+\frac{4i(3\,\mathrm{e}^{4i(dx+c)}-3\,\mathrm{e}^{2i(dx+c)}+2)}{3d\,a^2\left(\mathrm{e}^{2i(dx+c)}-1\right)^3}$	53
parallelrisch	$\frac{-\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+24dx+15\cot\left(\frac{dx}{2}+\frac{c}{2}\right)-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d\,a^2}$	60
norman	$\frac{\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{a}-\frac{1}{24ad}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8ad}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{24ad}}{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}$	98

```
[In] int(1/(a-a*sec(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/d/a^2*(arctan(tan(d*x+c))-1/3/tan(d*x+c)^3+1/tan(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(35) = 70.

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.03

$$\int \frac{1}{(a-a\sec^2(c+dx))^2}\,dx = \frac{4\cos(dx+c)^3+3\left(dx\cos(dx+c)^2-dx\right)\sin(dx+c)-3\cos(dx+c)}{3\left(a^2d\cos(dx+c)^2-a^2d\right)\sin(dx+c)}$$

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^2,x, algorithm="fricas")
[Out] 1/3*(4*cos(d*x + c)^3 + 3*(d*x*cos(d*x + c)^2 - d*x)*sin(d*x + c) - 3*cos(d
*x + c))/((a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{1}{(a-a\sec^2(c+dx))^2}\,dx = \frac{\int \frac{1}{\sec^4(c+dx)-2\sec^2(c+dx)+1}\,dx}{a^2}$$

```
[In] integrate(1/(a-a*sec(d*x+c)**2)**2,x)
[Out] Integral(1/(sec(c + d*x)**4 - 2*sec(c + d*x)**2 + 1), x)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{1}{(a-a\sec^2(c+dx))^2}\,dx = \frac{\frac{3\,(dx+c)}{a^2}+\frac{3\,\tan(dx+c)^2-1}{a^2\tan(dx+c)^3}}{3\,d}$$

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(d*x + c)/a^2 + (3*tan(d*x + c)^2 - 1)/(a^2*tan(d*x + c)^3))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(35) = 70.

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.16

$$\int \frac{1}{(a-a\sec^2(c+dx))^2}\,dx = \frac{\frac{24\,(dx+c)}{a^2}+\frac{15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}+\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{24\,d}$$

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(24*(d*x + c)/a^2 + (15*tan(1/2*d*x + 1/2*c)^2 - 1)/(a^2*tan(1/2*d*x +
 1/2*c)^3) + (a^4*tan(1/2*d*x + 1/2*c)^3 - 15*a^4*tan(1/2*d*x + 1/2*c))/a^6
)/d
```

Mupad [B] (verification not implemented)

Time = 18.37 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a-a\sec^2(c+dx))^2}\,dx = \frac{x}{a^2}+\frac{\tan(c+d\,x)^2-\frac{1}{3}}{a^2\,d\tan\left(c+d\,x\right)^3}$$

```
[In] int(1/(a - a/cos(c + d*x)^2)^2,x)
```

```
[Out] x/a^2 + (tan(c + d*x)^2 - 1/3)/(a^2*d*tan(c + d*x)^3)
```

3.150 $\int \frac{1}{(a-a\sec^2(c+dx))^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{1}{(a-a\sec^2(c+dx))^3}\,dx=\frac{x}{a^3}+\frac{\cot(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot^5(c+dx)}{5a^3d}$$

```
[Out] x/a^3+cot(d*x+c)/a^3/d-1/3*cot(d*x+c)^3/a^3/d+1/5*cot(d*x+c)^5/a^3/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4205, 3554, 8}

$$\int \frac{1}{(a-a\sec^2(c+dx))^3}\,dx=\frac{\cot^5(c+dx)}{5a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot(c+dx)}{a^3d}+\frac{x}{a^3}$$

```
[In] Int[(a - a*Sec[c + d*x]^2)^(-3),x]
```

```
[Out] x/a^3 + Cot[c + d*x]/(a^3*d) - Cot[c + d*x]^3/(3*a^3*d) + Cot[c + d*x]^5/(5
*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4205

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\int \cot^6(c+dx)\,dx}{a^3}\\ &= \frac{\cot^5(c+dx)}{5a^3d}+\frac{\int \cot^4(c+dx)\,dx}{a^3}\\ &= -\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot^5(c+dx)}{5a^3d}-\frac{\int \cot^2(c+dx)\,dx}{a^3}\\ &= \frac{\cot(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot^5(c+dx)}{5a^3d}+\frac{\int 1\,dx}{a^3}\\ &= \frac{x}{a^3}+\frac{\cot(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot^5(c+dx)}{5a^3d}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^3}\,dx=\frac{\cot^5(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(c+dx)\right)}{5a^3d}$$

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(-3),x]
[Out] (Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2])/(5*a^3*d
)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

method	result	si
derivativedivides	$\frac{\arctan(\tan(dx+c))-\frac{1}{3\tan(dx+c)^3}+\frac{1}{5\tan(dx+c)^5}+\frac{1}{\tan(dx+c)}}{d\,a^3}$	44
default	$\frac{\arctan(\tan(dx+c))-\frac{1}{3\tan(dx+c)^3}+\frac{1}{5\tan(dx+c)^5}+\frac{1}{\tan(dx+c)}}{d\,a^3}$	44
risch	$\frac{x}{a^3}+\frac{2i\left(45\,\mathrm{e}^{8i(dx+c)}-90\,\mathrm{e}^{6i(dx+c)}+140\,\mathrm{e}^{4i(dx+c)}-70\,\mathrm{e}^{2i(dx+c)}+23\right)}{15d\,a^3\left(\mathrm{e}^{2i(dx+c)}-1\right)^5}$	75
parallelrisch	$\frac{-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+3\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^5+35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-35\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^3+480dx-330\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+330\cot\left(\frac{dx}{2}+\frac{c}{2}\right)}{480d\,a^3}$	88
norman	$\frac{\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{a}+\frac{1}{160ad}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{96ad}+\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{16ad}-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{16ad}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{96ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{160ad}}{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}$	13

```
[In] int(1/(a-a*sec(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
[Out] 1/d/a^3*(arctan(tan(d*x+c))-1/3/tan(d*x+c)^3+1/5/tan(d*x+c)^5+1/tan(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(51) = 102.

Time = 0.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.98

$$\int \frac{1}{(a-a\sec^2(c+dx))^3}\,dx$$
$$=\frac{23\cos(dx+c)^5-35\cos(dx+c)^3+15\left(dx\cos(dx+c)^4-2\,dx\cos(dx+c)^2+dx\right)\sin(dx+c)+15\cos(}{15\left(a^3d\cos(dx+c)^4-2\,a^3d\cos(dx+c)^2+a^3d\right)\sin(dx+c)}$$

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^3,x, algorithm="fricas")
[Out] 1/15*(23*cos(d*x + c)^5 - 35*cos(d*x + c)^3 + 15*(d*x*cos(d*x + c)^4 - 2*d*
x*cos(d*x + c)^2 + d*x)*sin(d*x + c) + 15*cos(d*x + c))/((a^3*d*cos(d*x + c
)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{1}{(a-a\sec^2(c+dx))^3}\,dx = -\frac{\int \frac{1}{\sec^6(c+dx)-3\sec^4(c+dx)+3\sec^2(c+dx)-1}\,dx}{a^3}$$

```
[In] integrate(1/(a-a*sec(d*x+c)**2)**3,x)
[Out] -Integral(1/(sec(c + d*x)**6 - 3*sec(c + d*x)**4 + 3*sec(c + d*x)**2 - 1),
x)/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a-a\sec^2(c+dx))^3}\,dx = \frac{\frac{15\,(dx+c)}{a^3} + \frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{a^3\tan(dx+c)^5}}{15\,d}$$

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^3,x, algorithm="maxima")
[Out] 1/15*(15*(d*x + c)/a^3 + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/(a^3*ta
n(d*x + c)^5))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(51) = 102.

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.02

$$\int \frac{1}{(a-a\sec^2(c+dx))^3}\,dx = \frac{\frac{480\,(dx+c)}{a^3} + \frac{330\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5} - \frac{3\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-35\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+330\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{480\,d}$$

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^3,x, algorithm="giac")
[Out] 1/480*(480*(d*x + c)/a^3 + (330*tan(1/2*d*x + 1/2*c)^4 - 35*tan(1/2*d*x + 1
/2*c)^2 + 3)/(a^3*tan(1/2*d*x + 1/2*c)^5) - (3*a^12*tan(1/2*d*x + 1/2*c)^5
- 35*a^12*tan(1/2*d*x + 1/2*c)^3 + 330*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

Mupad [B] (verification not implemented)

Time = 18.41 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^3}\,dx=\frac{x}{a^3}+\frac{\tan(c+d\,x)^4-\frac{\tan(c+d\,x)^2}{3}+\frac{1}{5}}{a^3\,d\tan\left(c+d\,x\right)^5}$$

```
[In] int(1/(a - a/cos(c + d*x)^2)^3,x)
[Out] x/a^3 + (tan(c + d*x)^4 - tan(c + d*x)^2/3 + 1/5)/(a^3*d*tan(c + d*x)^5)
```

3.151 $\int \frac{1}{(a-a\sec^2(c+dx))^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 73

$$\int \frac{1}{(a-a\sec^2(c+dx))^4}\,dx=\frac{x}{a^4}+\frac{\cot(c+dx)}{a^4d}-\frac{\cot^3(c+dx)}{3a^4d}+\frac{\cot^5(c+dx)}{5a^4d}-\frac{\cot^7(c+dx)}{7a^4d}$$

```
[Out] x/a^4+cot(d*x+c)/a^4/d-1/3*cot(d*x+c)^3/a^4/d+1/5*cot(d*x+c)^5/a^4/d-1/7*co
t(d*x+c)^7/a^4/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4205, 3554, 8}

$$\int \frac{1}{(a-a\sec^2(c+dx))^4}\,dx=-\frac{\cot^7(c+dx)}{7a^4d}+\frac{\cot^5(c+dx)}{5a^4d}-\frac{\cot^3(c+dx)}{3a^4d}+\frac{\cot(c+dx)}{a^4d}+\frac{x}{a^4}$$

```
[In] Int[(a - a*Sec[c + d*x]^2)^(-4),x]
```

```
[Out] x/a^4 + Cot[c + d*x]/(a^4*d) - Cot[c + d*x]^3/(3*a^4*d) + Cot[c + d*x]^5/(5
*a^4*d) - Cot[c + d*x]^7/(7*a^4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4205

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \cot^8(c+dx)\,dx}{a^4}\\ &= -\frac{\cot^7(c+dx)}{7a^4d} - \frac{\int \cot^6(c+dx)\,dx}{a^4}\\ &= \frac{\cot^5(c+dx)}{5a^4d} - \frac{\cot^7(c+dx)}{7a^4d} + \frac{\int \cot^4(c+dx)\,dx}{a^4}\\ &= -\frac{\cot^3(c+dx)}{3a^4d} + \frac{\cot^5(c+dx)}{5a^4d} - \frac{\cot^7(c+dx)}{7a^4d} - \frac{\int \cot^2(c+dx)\,dx}{a^4}\\ &= \frac{\cot(c+dx)}{a^4d} - \frac{\cot^3(c+dx)}{3a^4d} + \frac{\cot^5(c+dx)}{5a^4d} - \frac{\cot^7(c+dx)}{7a^4d} + \frac{\int 1\,dx}{a^4}\\ &= \frac{x}{a^4} + \frac{\cot(c+dx)}{a^4d} - \frac{\cot^3(c+dx)}{3a^4d} + \frac{\cot^5(c+dx)}{5a^4d} - \frac{\cot^7(c+dx)}{7a^4d}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.49

$$\int \frac{1}{(a-a\sec^2(c+dx))^4}\,dx = -\frac{\cot^7(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},-\tan^2(c+dx)\right)}{7a^4d}$$

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(-4),x]
```

```
[Out] -1/7*(Cot[c + d*x]^7*Hypergeometric2F1[-7/2, 1, -5/2, -Tan[c + d*x]^2])/(a^
4*d)
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{-\frac{1}{7\tan(dx+c)^7}-\frac{1}{3\tan(dx+c)^3}+\frac{1}{5\tan(dx+c)^5}+\frac{1}{\tan(dx+c)}+\arctan(\tan(dx+c))}{d\,a^4}$
default	$\frac{-\frac{1}{7\tan(dx+c)^7}-\frac{1}{3\tan(dx+c)^3}+\frac{1}{5\tan(dx+c)^5}+\frac{1}{\tan(dx+c)}+\arctan(\tan(dx+c))}{d\,a^4}$
risch	$\frac{x}{a^4}+\frac{8i\left(105\,\mathrm{e}^{12i(dx+c)}-315\,\mathrm{e}^{10i(dx+c)}+770\,\mathrm{e}^{8i(dx+c)}-770\,\mathrm{e}^{6i(dx+c)}+609\,\mathrm{e}^{4i(dx+c)}-203\,\mathrm{e}^{2i(dx+c)}+44\right)}{105d\,a^4\left(\mathrm{e}^{2i(dx+c)}-1\right)^7}$
parallelrisch	$\frac{-15\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^7+15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7+189\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^5-189\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5-1295\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^3+1295\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+1}{13440d\,a^4}$
norman	$\frac{\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{a}-\frac{1}{896ad}+\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{640ad}-\frac{37\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{384ad}+\frac{93\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{128ad}-\frac{93\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{128ad}+\frac{37\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{384ad}-\frac{9\tan\left(\frac{d}{2}\right.}{640}}{a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}$

```
[In] int(1/(a-a*sec(d*x+c)^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(-1/7/tan(d*x+c)^7-1/3/tan(d*x+c)^3+1/5/tan(d*x+c)^5+1/tan(d*x+c)+a
rctan(tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(67) = 134.

Time = 0.24 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.01

$$\int \frac{1}{(a-a\sec^2(c+dx))^4}\,dx$$
$$=\frac{176\cos(dx+c)^7-406\cos(dx+c)^5+350\cos(dx+c)^3+105\left(dx\cos(dx+c)^6-3\,dx\cos(dx+c)^4+3\right.}{105\left(a^4d\cos(dx+c)^6-3\,a^4d\cos(dx+c)^4+3\,a^4d\cos(dx+c)^2-\right.}$$

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(176*cos(d*x + c)^7 - 406*cos(d*x + c)^5 + 350*cos(d*x + c)^3 + 105*(
d*x*cos(d*x + c)^6 - 3*d*x*cos(d*x + c)^4 + 3*d*x*cos(d*x + c)^2 - d*x)*sin
(d*x + c) - 105*cos(d*x + c))/((a^4*d*cos(d*x + c)^6 - 3*a^4*d*cos(d*x + c)
^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^4}\,dx = \frac{\int \frac{1}{\sec^8(c+dx)-4\sec^6(c+dx)+6\sec^4(c+dx)-4\sec^2(c+dx)+1}\,dx}{a^4}$$

```
[In] integrate(1/(a-a*sec(d*x+c)**2)**4,x)
```

```
[Out] Integral(1/(sec(c + d*x)**8 - 4*sec(c + d*x)**6 + 6*sec(c + d*x)**4 - 4*sec
(c + d*x)**2 + 1), x)/a**4
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^4}\,dx = \frac{\frac{105\,(dx+c)}{a^4} + \frac{105\,\tan(dx+c)^6-35\,\tan(dx+c)^4+21\,\tan(dx+c)^2-15}{a^4\tan(dx+c)^7}}{105\,d}$$

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] 1/105*(105*(d*x + c)/a^4 + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*tan
(d*x + c)^2 - 15)/(a^4*tan(d*x + c)^7))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(67) = 134$.

Time = 0.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.90

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^4}\,dx$$
$$= \frac{\frac{13440\,(dx+c)}{a^4} + \frac{9765\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-1295\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+189\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15}{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7} + \frac{15\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-189\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+1}{a^{28}}}{13440\,d}$$

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] 1/13440*(13440*(d*x + c)/a^4 + (9765*tan(1/2*d*x + 1/2*c)^6 - 1295*tan(1/2*
d*x + 1/2*c)^4 + 189*tan(1/2*d*x + 1/2*c)^2 - 15)/(a^4*tan(1/2*d*x + 1/2*c)
^7) + (15*a^24*tan(1/2*d*x + 1/2*c)^7 - 189*a^24*tan(1/2*d*x + 1/2*c)^5 + 1
295*a^24*tan(1/2*d*x + 1/2*c)^3 - 9765*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

Mupad [B] (verification not implemented)

Time = 19.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int \frac{1}{(a-a\sec^2(c+dx))^4}\,dx=\frac{x}{a^4}+\frac{\tan(c+d\,x)^6-\frac{\tan(c+d\,x)^4}{3}+\frac{\tan(c+d\,x)^2}{5}-\frac{1}{7}}{a^4\,d\tan\left(c+d\,x\right)^7}$$

```
[In] int(1/(a - a/cos(c + d*x)^2)^4,x)
```

```
[Out] x/a^4 + (tan(c + d*x)^2/5 - tan(c + d*x)^4/3 + tan(c + d*x)^6 - 1/7)/(a^4*d
*tan(c + d*x)^7)
```

3.152 $\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(6a+5b)\text{arctanh}(\sin(e+fx))}{16f}+\frac{(6a+5b)\sec(e+fx)\tan(e+fx)}{16f}+\frac{(6a+5b)\sec^3(e+fx)\tan(e+fx)}{24f}+\frac{b\sec^5(e+fx)\tan(e+fx)}{6f}$$

```
[Out] 1/16*(6*a+5*b)*arctanh(sin(f*x+e))/f+1/16*(6*a+5*b)*sec(f*x+e)*tan(f*x+e)/f
+1/24*(6*a+5*b)*sec(f*x+e)^3*tan(f*x+e)/f+1/6*b*sec(f*x+e)^5*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4131, 3853, 3855}

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(6a+5b)\text{arctanh}(\sin(e+fx))}{16f}+\frac{(6a+5b)\tan(e+fx)\sec^3(e+fx)}{24f}+\frac{(6a+5b)\tan(e+fx)\sec(e+fx)}{16f}+\frac{b\tan(e+fx)\sec^5(e+fx)}{6f}$$

```
[In] Int[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((6*a + 5*b)*ArcTanh[Sin[e + f*x]])/(16*f) + ((6*a + 5*b)*Sec[e + f*x]*Tan[
e + f*x])/(16*f) + ((6*a + 5*b)*Sec[e + f*x]^3*Tan[e + f*x])/(24*f) + (b*Se
c[e + f*x]^5*Tan[e + f*x])/(6*f)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\sec^5(e+fx)\tan(e+fx)}{6f}+\frac{1}{6}(6a+5b)\int\sec^5(e+fx)\,dx\\
&= \frac{(6a+5b)\sec^3(e+fx)\tan(e+fx)}{24f}\\
&\quad+\frac{b\sec^5(e+fx)\tan(e+fx)}{6f}+\frac{1}{8}(6a+5b)\int\sec^3(e+fx)\,dx\\
&= \frac{(6a+5b)\sec(e+fx)\tan(e+fx)}{16f}+\frac{(6a+5b)\sec^3(e+fx)\tan(e+fx)}{24f}\\
&\quad+\frac{b\sec^5(e+fx)\tan(e+fx)}{6f}+\frac{1}{16}(6a+5b)\int\sec(e+fx)\,dx\\
&= \frac{(6a+5b)\text{arctanh}(\sin(e+fx))}{16f}+\frac{(6a+5b)\sec(e+fx)\tan(e+fx)}{16f}\\
&\quad+\frac{(6a+5b)\sec^3(e+fx)\tan(e+fx)}{24f}+\frac{b\sec^5(e+fx)\tan(e+fx)}{6f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.40

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{3a\operatorname{arctanh}(\sin(e+fx))}{8f}+\frac{5b\operatorname{arctanh}(\sin(e+fx))}{16f}+\frac{3a\sec(e+fx)\tan(e+fx)}{8f}+\frac{5b\sec(e+fx)\tan(e+fx)}{16f}+\frac{a\sec^3(e+fx)\tan(e+fx)}{4f}+\frac{5b\sec^3(e+fx)\tan(e+fx)}{24f}+\frac{b\sec^5(e+fx)\tan(e+fx)}{6f}$$

```
[In] Integrate[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (3*a*ArcTanh[Sin[e + f*x]])/(8*f) + (5*b*ArcTanh[Sin[e + f*x]])/(16*f) + (3
*a*Sec[e + f*x]*Tan[e + f*x])/(8*f) + (5*b*Sec[e + f*x]*Tan[e + f*x])/(16*f
) + (a*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) + (5*b*Sec[e + f*x]^3*Tan[e + f*x
])/(24*f) + (b*Sec[e + f*x]^5*Tan[e + f*x])/(6*f)
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{a\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)+b\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx+e}{16}\right.\right.}{f}$
default	$\frac{a\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)+b\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx+e}{16}\right.\right.}{f}$
parts	$\frac{a\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)}{f}+\frac{b\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(f}{16}\right.\right.}{}$
parallelrisch	$\frac{-270\left(a+\frac{5b}{6}\right)\left(\frac{2}{3}+\frac{\cos(6fx+6e)}{15}+\frac{2\cos(4fx+4e)}{5}+\cos(2fx+2e)\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+270\left(a+\frac{5b}{6}\right)\left(\frac{2}{3}+\frac{\cos(6fx+6e)}{15}+\frac{2\cos(}{}\right.}{48f(10+\cos(6fx+6e)+6\,\mathrm{co}}$
norman	$\frac{\frac{(2a+15b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{4f}+\frac{(2a+15b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{4f}+\frac{(10a+11b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8f}+\frac{(10a+11b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{8f}-\frac{(42a-5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{24f}-}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^6}$
risch	$-\frac{i\,e^{i(fx+e)}\left(18a\,e^{10i(fx+e)}+15b\,e^{10i(fx+e)}+102a\,e^{8i(fx+e)}+85b\,e^{8i(fx+e)}+84a\,e^{6i(fx+e)}+198b\,e^{6i(fx+e)}-84a\,e^{4i(fx+e)}\right.}{24f\left(e^{2i(fx+e)}+1\right)^6}$

[In] int(sec(f*x+e)^5*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)

[Out] 1/f*(a*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))+b*(-(-1/6*sec(f*x+e)^5-5/24*sec(f*x+e)^3-5/16*sec(f*x+e))*tan(f*x+e)+5/16*ln(sec(f*x+e)+tan(f*x+e))))

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.16

$$\int \sec^5(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx = \frac{3 (6 a+5 b) \cos (f x+e)^6 \log (\sin (f x+e)+1)-3 (6 a+5 b) \cos (f x+e)^6 \log (-\sin (f x+e)+1)+2 (3}{96 f \cos (f x+e)^6}$$

[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="fricas")

[Out] 1/96*(3*(6*a + 5*b)*cos(f*x + e)^6*log(sin(f*x + e) + 1) - 3*(6*a + 5*b)*cos(f*x + e)^6*log(-sin(f*x + e) + 1) + 2*(3*(6*a + 5*b)*cos(f*x + e)^4 + 2*(6*a + 5*b)*cos(f*x + e)^2 + 8*b)*sin(f*x + e))/(f*cos(f*x + e)^6)

Sympy [F]

$$\int \sec^5(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx = \int \left(a+b\sec^2(e+fx)\right) \sec^5(e+fx) \, dx$$

[In] integrate(sec(f*x+e)**5*(a+b*sec(f*x+e)**2),x)

[Out] Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**5, x)

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.29

$$\int \sec^5(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx = \frac{3 (6 a+5 b) \log (\sin (f x+e)+1)-3 (6 a+5 b) \log (\sin (f x+e)-1)-\frac{2 \left(3 (6 a+5 b) \sin (f x+e)^5-8 (6 a+5 b) \sin (f x}{\sin (f x+e)^6-3 \sin (f x+e)^4+}}{96 f}$$

[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="maxima")

```
[Out] 1/96*(3*(6*a + 5*b)*log(sin(f*x + e) + 1) - 3*(6*a + 5*b)*log(sin(f*x + e)
- 1) - 2*(3*(6*a + 5*b)*sin(f*x + e)^5 - 8*(6*a + 5*b)*sin(f*x + e)^3 + 3*(
10*a + 11*b)*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x +
 e)^2 - 1))/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.23

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{3\,(6\,a+5\,b)\log\left(|\sin\left(fx+e\right)+1|\right)-3\,(6\,a+5\,b)\log\left(|\sin\left(fx+e\right)-1|\right)-\frac{2\left(18\,a\sin(fx+e)^5+15\,b\sin(fx+e)^5-48\,a\right.}{}}{96\,f}$$

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/96*(3*(6*a + 5*b)*log(abs(sin(f*x + e) + 1)) - 3*(6*a + 5*b)*log(abs(sin(
f*x + e) - 1)) - 2*(18*a*sin(f*x + e)^5 + 15*b*sin(f*x + e)^5 - 48*a*sin(f*
x + e)^3 - 40*b*sin(f*x + e)^3 + 30*a*sin(f*x + e) + 33*b*sin(f*x + e))/(si
n(f*x + e)^2 - 1)^3)/f
```

Mupad [B] (verification not implemented)

Time = 18.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{\mathrm{atanh}(\sin\left(e+f\,x\right))\,\left(\frac{3\,a}{8}+\frac{5\,b}{16}\right)}{f}-\frac{\left(\frac{3\,a}{8}+\frac{5\,b}{16}\right)\,\sin\left(e+f\,x\right)^5+\left(-a-\frac{5\,b}{6}\right)\,\sin\left(e+f\,x\right)^3+\left(\frac{5\,a}{8}+\frac{11\,b}{16}\right)\,\sin\left(e+f\,x\right)}{f\,\left(\sin\left(e+f\,x\right)^6-3\,\sin\left(e+f\,x\right)^4+3\,\sin\left(e+f\,x\right)^2-1\right)}$$

```
[In] int((a + b/cos(e + f*x)^2)/cos(e + f*x)^5,x)
```

```
[Out] (atanh(sin(e + f*x))*((3*a)/8 + (5*b)/16))/f - (sin(e + f*x)^5*((3*a)/8 + (
5*b)/16) + sin(e + f*x)*((5*a)/8 + (11*b)/16) - sin(e + f*x)^3*(a + (5*b)/6
))/(f*(3*sin(e + f*x)^2 - 3*sin(e + f*x)^4 + sin(e + f*x)^6 - 1))
```

3.153 $\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(4a+3b)\text{arctanh}(\sin(e+fx))}{8f}+\frac{(4a+3b)\sec(e+fx)\tan(e+fx)}{8f}+\frac{b\sec^3(e+fx)\tan(e+fx)}{4f}$$

```
[Out] 1/8*(4*a+3*b)*arctanh(sin(f*x+e))/f+1/8*(4*a+3*b)*sec(f*x+e)*tan(f*x+e)/f+1
/4*b*sec(f*x+e)^3*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4131, 3853, 3855}

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(4a+3b)\text{arctanh}(\sin(e+fx))}{8f}+\frac{(4a+3b)\tan(e+fx)\sec(e+fx)}{8f}+\frac{b\tan(e+fx)\sec^3(e+fx)}{4f}$$

```
[In] Int[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((4*a + 3*b)*ArcTanh[Sin[e + f*x]])/(8*f) + ((4*a + 3*b)*Sec[e + f*x]*Tan[e
 + f*x])/(8*f) + (b*Sec[e + f*x]^3*Tan[e + f*x])/(4*f)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\sec^3(e+fx)\tan(e+fx)}{4f} + \frac{1}{4}(4a+3b)\int \sec^3(e+fx)\,dx \\
&= \frac{(4a+3b)\sec(e+fx)\tan(e+fx)}{8f} \\
&\quad + \frac{b\sec^3(e+fx)\tan(e+fx)}{4f} + \frac{1}{8}(4a+3b)\int \sec(e+fx)\,dx \\
&= \frac{(4a+3b)\text{arctanh}(\sin(e+fx))}{8f} \\
&\quad + \frac{(4a+3b)\sec(e+fx)\tan(e+fx)}{8f} + \frac{b\sec^3(e+fx)\tan(e+fx)}{4f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.33

$$\begin{aligned}
\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= \frac{a\text{arctanh}(\sin(e+fx))}{2f} + \frac{3b\text{arctanh}(\sin(e+fx))}{8f} \\
&\quad + \frac{a\sec(e+fx)\tan(e+fx)}{2f} \\
&\quad + \frac{3b\sec(e+fx)\tan(e+fx)}{8f} \\
&\quad + \frac{b\sec^3(e+fx)\tan(e+fx)}{4f}
\end{aligned}$$

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (a*ArcTanh[Sin[e + f*x]])/(2*f) + (3*b*ArcTanh[Sin[e + f*x]])/(8*f) + (a*Se
c[e + f*x]*Tan[e + f*x])/(2*f) + (3*b*Sec[e + f*x]*Tan[e + f*x])/(8*f) + (b
*Sec[e + f*x]^3*Tan[e + f*x])/(4*f)
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{a\left(\frac{\tan(fx+e)\sec(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+b\left(-\left(-\frac{\sec(fx+e)^{3}}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx}{8}\right.}{f}$
default	$\frac{a\left(\frac{\tan(fx+e)\sec(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+b\left(-\left(-\frac{\sec(fx+e)^{3}}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx}{8}\right.}{f}$
parts	$\frac{a\left(\frac{\tan(fx+e)\sec(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}+\frac{b\left(-\left(-\frac{\sec(fx+e)^{3}}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan}{8}\right.}{f}$
parallelrisch	$\frac{-8\left(\frac{3}{4}+\frac{\cos(4fx+4e)}{4}+\cos(2fx+2e)\right)\left(a+\frac{3b}{4}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+8\left(\frac{3}{4}+\frac{\cos(4fx+4e)}{4}+\cos(2fx+2e)\right)\left(a+\frac{3b}{4}\right)\ln\left(\tan\left(\right.\right.}{4f(\cos(4fx+4e)+4\cos(2fx+2e)+3)}$
norman	$\frac{-\frac{(4a-3b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{4f}-\frac{(4a-3b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{4f}+\frac{(4a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}+\frac{(4a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{4f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{4}}-\frac{(4a+3b)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}{8f}$
risch	$-\frac{i\,e^{i(fx+e)}\left(4a\,e^{6i(fx+e)}+3b\,e^{6i(fx+e)}+4a\,e^{4i(fx+e)}+11b\,e^{4i(fx+e)}-4a\,e^{2i(fx+e)}-11b\,e^{2i(fx+e)}-4a-3b\right)}{4f\left(e^{2i(fx+e)}+1\right)^{4}}+\frac{\ln(e^{i(fx}}{2}$

```
[In] int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*(1/2*tan(f*x+e)*sec(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+b*(-(-1/4*
sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e))))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.36

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{(4a+3b)\cos(fx+e)^4\log(\sin(fx+e)+1)-(4a+3b)\cos(fx+e)^4\log(-\sin(fx+e)+1)+2\left((4a\right.}{16f\cos(fx+e)^4}$$

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/16*((4*a + 3*b)*cos(f*x + e)^4*log(sin(f*x + e) + 1) - (4*a + 3*b)*cos(f*
x + e)^4*log(-sin(f*x + e) + 1) + 2*((4*a + 3*b)*cos(f*x + e)^2 + 2*b)*sin(
f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F]

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\int\left(a+b\sec^2\left(e+fx\right)\right)\sec^3\left(e+fx\right)dx$$

```
[In] integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2),x)
[Out] Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.39

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$=\frac{(4a+3b)\log\left(\sin\left(fx+e\right)+1\right)-(4a+3b)\log\left(\sin\left(fx+e\right)-1\right)-\frac{2\left((4a+3b)\sin(fx+e)^3-(4a+5b)\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}}{16f}$$

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] 1/16*((4*a + 3*b)*log(sin(f*x + e) + 1) - (4*a + 3*b)*log(sin(f*x + e) - 1)
 - 2*((4*a + 3*b)*sin(f*x + e)^3 - (4*a + 5*b)*sin(f*x + e))/(sin(f*x + e)^
4 - 2*sin(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.40

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$=\frac{(4a+3b)\log\left(\left|\sin\left(fx+e\right)+1\right|\right)-(4a+3b)\log\left(\left|\sin\left(fx+e\right)-1\right|\right)-\frac{2\left(4a\sin(fx+e)^3+3b\sin(fx+e)^3-4a\sin(fx+\cdots\right.}{\left(\sin(fx+e)^2-1\right)^2}}{16f}$$

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] 1/16*((4*a + 3*b)*log(abs(sin(f*x + e) + 1)) - (4*a + 3*b)*log(abs(sin(f*x
+ e) - 1)) - 2*(4*a*sin(f*x + e)^3 + 3*b*sin(f*x + e)^3 - 4*a*sin(f*x + e)
- 5*b*sin(f*x + e))/(sin(f*x + e)^2 - 1)^2)/f
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{\operatorname{atanh}(\sin(e+f\,x))\,\left(\frac{a}{2}+\frac{3\,b}{8}\right)}{f}-\frac{\sin(e+f\,x)^3\,\left(\frac{a}{2}+\frac{3\,b}{8}\right)-\sin(e+f\,x)\,\left(\frac{a}{2}+\frac{5\,b}{8}\right)}{f\,\left(\sin(e+f\,x)^4-2\sin(e+f\,x)^2+1\right)}$$

```
[In] int((a + b/cos(e + f*x)^2)/cos(e + f*x)^3,x)
[Out] (atanh(sin(e + f*x))*(a/2 + (3*b)/8))/f - (sin(e + f*x)^3*(a/2 + (3*b)/8) -
 sin(e + f*x)*(a/2 + (5*b)/8))/(f*(sin(e + f*x)^4 - 2*sin(e + f*x)^2 + 1))
```

3.154 $\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 40

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(2a+b)\text{arctanh}(\sin(e+fx))}{2f}+\frac{b\sec(e+fx)\tan(e+fx)}{2f}$$

```
[Out] 1/2*(2*a+b)*arctanh(sin(f*x+e))/f+1/2*b*sec(f*x+e)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4131, 3855}

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(2a+b)\text{arctanh}(\sin(e+fx))}{2f}+\frac{b\tan(e+fx)\sec(e+fx)}{2f}$$

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((2*a + b)*ArcTanh[Sin[e + f*x]])/(2*f) + (b*Sec[e + f*x]*Tan[e + f*x])/(2*
f)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\sec(e+fx)\tan(e+fx)}{2f}+\frac{1}{2}(2a+b)\int\sec(e+fx)\,dx\\ &= \frac{(2a+b)\text{arctanh}(\sin(e+fx))}{2f}+\frac{b\sec(e+fx)\tan(e+fx)}{2f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int\sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{a\text{arctanh}(\sin(e+fx))}{f}+\frac{b\text{arctanh}(\sin(e+fx))}{2f}+\frac{b\sec(e+fx)\tan(e+fx)}{2f}$$

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2),x]
[Out] (a*ArcTanh[Sin[e + f*x]])/f + (b*ArcTanh[Sin[e + f*x]])/(2*f) + (b*Sec[e +
f*x]*Tan[e + f*x])/(2*f)
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

method	result	size
derivativedivides	$\frac{a\ln(\sec(fx+e)+\tan(fx+e))+b\left(\frac{\tan(fx+e)\sec(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$	55
default	$\frac{a\ln(\sec(fx+e)+\tan(fx+e))+b\left(\frac{\tan(fx+e)\sec(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$	55
parts	$\frac{b\left(\frac{\tan(fx+e)\sec(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}+\frac{a\ln(\sec(fx+e)+\tan(fx+e))}{f}$	57
parallelrisch	$\frac{-\left(a+\frac{b}{2}\right)(1+\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\left(a+\frac{b}{2}\right)(1+\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\sin(fx+e)b}{f(1+\cos(2fx+2e))}$	86
norman	$\frac{\frac{b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{2}}-\frac{(2a+b)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2f}+\frac{(2a+b)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2f}$	93
risch	$-\frac{ib\left(e^{3i(fx+e)}-e^{i(fx+e)}\right)}{f\left(e^{2i(fx+e)}+1\right)^{2}}+\frac{\ln\left(e^{i(fx+e)}+i\right)a}{f}+\frac{\ln\left(e^{i(fx+e)}+i\right)b}{2f}-\frac{\ln\left(e^{i(fx+e)}-i\right)a}{f}-\frac{\ln\left(e^{i(fx+e)}-i\right)b}{2f}$	118

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*ln(sec(f*x+e)+tan(f*x+e))+b*(1/2*tan(f*x+e)*sec(f*x+e)+1/2*ln(sec(f*
x+e)+tan(f*x+e))))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.80

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{(2a+b)\cos(fx+e)^2\log(\sin(fx+e)+1)-(2a+b)\cos(fx+e)^2\log(-\sin(fx+e)+1)+2b\sin(fx+}{4f\cos(fx+e)^2}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/4*((2*a + b)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (2*a + b)*cos(f*x + e
)^2*log(-sin(f*x + e) + 1) + 2*b*sin(f*x + e))/(f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\int \left(a+b\sec^2{(e+fx)}\right)\sec{(e+fx)}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2),x)
[Out] Integral((a + b*sec(e + f*x)**2)*sec(e + f*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(2\,a+b)\log\left(\sin\left(fx+e\right)+1\right)-(2\,a+b)\log\left(\sin\left(fx+e\right)-1\right)-\frac{2\,b\sin(fx+e)}{\sin(fx+e)^2-1}}{4\,f}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] 1/4*((2*a + b)*log(sin(f*x + e) + 1) - (2*a + b)*log(sin(f*x + e) - 1) - 2*
b*sin(f*x + e)/(sin(f*x + e)^2 - 1))/f
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.50

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(2\,a+b)\log\left(\left|\sin\left(fx+e\right)+1\right|\right)-(2\,a+b)\log\left(\left|\sin\left(fx+e\right)-1\right|\right)-\frac{2\,b\sin(fx+e)}{\sin(fx+e)^2-1}}{4\,f}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] 1/4*((2*a + b)*log(abs(sin(f*x + e) + 1)) - (2*a + b)*log(abs(sin(f*x + e)
- 1)) - 2*b*sin(f*x + e)/(sin(f*x + e)^2 - 1))/f
```

Mupad [B] (verification not implemented)

Time = 18.46 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \sec(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx = \frac{\operatorname{atanh}(\sin(e+f\,x)) \, \left(a+\frac{b}{2}\right)}{f} - \frac{b\sin(e+f\,x)}{2\,f\,\left(\sin(e+f\,x)^2-1\right)}$$

```
[In] int((a + b/cos(e + f*x)^2)/cos(e + f*x),x)
```

```
[Out] (atanh(sin(e + f*x))*(a + b/2))/f - (b*sin(e + f*x))/(2*f*(sin(e + f*x)^2 -
 1))
```

3.155 $\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 24

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{b\operatorname{arctanh}(\sin(e+fx))}{f}+\frac{a\sin(e+fx)}{f}$$

```
[Out] b*arctanh(sin(f*x+e))/f+a*sin(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4130, 3855}

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{a\sin(e+fx)}{f}+\frac{b\operatorname{arctanh}(\sin(e+fx))}{f}$$

```
[In] Int[Cos[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (b*ArcTanh[Sin[e + f*x]])/f + (a*Sin[e + f*x])/f
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
```

```
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{a\sin(e+fx)}{f} + b\int \sec(e+fx)\,dx$$
$$= \frac{b\,\text{arctanh}(\sin(e+fx))}{f} + \frac{a\sin(e+fx)}{f}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\,\text{arctanh}(\sin(e+fx))}{f} + \frac{a\cos(fx)\sin(e)}{f} + \frac{a\cos(e)\sin(fx)}{f}$$

```
[In] Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2),x]
[Out] (b*ArcTanh[Sin[e + f*x]])/f + (a*Cos[f*x]*Sin[e])/f + (a*Cos[e]*Sin[f*x])/f
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\frac{\sin(fx+e)a+b\ln(\sec(fx+e)+\tan(fx+e))}{f}$	30
default	$\frac{\sin(fx+e)a+b\ln(\sec(fx+e)+\tan(fx+e))}{f}$	30
parallelrisch	$\frac{\sin(fx+e)a-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)b+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)b}{f}$	43
risch	$-\frac{ia\,e^{i(fx+e)}}{2f}+\frac{ia\,e^{-i(fx+e)}}{2f}+\frac{\ln(e^{i(fx+e)}+i)b}{f}-\frac{\ln(e^{i(fx+e)}-i)b}{f}$	71
norman	$\frac{-\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}+\frac{b\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{f}-\frac{b\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{f}$	101

```
[In] int(cos(f*x+e)*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(sin(f*x+e)*a+b*ln(sec(f*x+e)+tan(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\log\left(\sin\left(fx+e\right)+1\right)-b\log\left(-\sin\left(fx+e\right)+1\right)+2\,a\sin\left(fx+e\right)}{2\,f}$$

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
[Out] 1/2*(b*log(sin(f*x + e) + 1) - b*log(-sin(f*x + e) + 1) + 2*a*sin(f*x + e))
/f
```

Sympy [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)\cos\left(e+fx\right)dx$$

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2),x)
[Out] Integral((a + b*sec(e + f*x)**2)*cos(e + f*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b(\log\left(\sin\left(fx+e\right)+1\right)-\log\left(\sin\left(fx+e\right)-1\right))+2\,a\sin\left(fx+e\right)}{2\,f}$$

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] 1/2*(b*(log(sin(f*x + e) + 1) - log(sin(f*x + e) - 1)) + 2*a*sin(f*x + e))/
f
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\log\left(|\sin\left(fx+e\right)+1|\right)-b\log\left(|\sin\left(fx+e\right)-1|\right)+2\,a\sin\left(fx+e\right)}{2\,f}$$

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*(b*log(abs(sin(f*x + e) + 1)) - b*log(abs(sin(f*x + e) - 1)) + 2*a*sin(
f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 18.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{a\,\sin\left(e+f\,x\right)+b\,\mathrm{atanh}(\sin\left(e+f\,x\right))}{f}$$

```
[In] int(cos(e + f*x)*(a + b/cos(e + f*x)^2),x)
```

```
[Out] (a*sin(e + f*x) + b*atanh(sin(e + f*x)))/f
```

3.156 $\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 30

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(a+b)\sin(e+fx)}{f}-\frac{a\sin^3(e+fx)}{3f}$$

```
[Out] (a+b)*sin(f*x+e)/f-1/3*a*sin(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4129, 3092}

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(a+b)\sin(e+fx)}{f}-\frac{a\sin^3(e+fx)}{3f}$$

```
[In] Int[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + b)*Sin[e + f*x])/f - (a*Sin[e + f*x]^3)/(3*f)
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 4129

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
```

```
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \cos(e+fx)\left(b+a\cos^2(e+fx)\right)\,dx\\ &= -\frac{\text{Subst}\left(\int \left(a+b-ax^2\right)\,dx, x, -\sin(e+fx)\right)}{f}\\ &= \frac{(a+b)\sin(e+fx)}{f}-\frac{a\sin^3(e+fx)}{3f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.67

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\cos(fx)\sin(e)}{f}+\frac{b\cos(e)\sin(fx)}{f}+\frac{a\sin(e+fx)}{f}-\frac{a\sin^3(e+fx)}{3f}$$

```
[In] Integrate[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
[Out] (b*Cos[f*x]*Sin[e])/f + (b*Cos[e]*Sin[f*x])/f + (a*Sin[e + f*x])/f - (a*Sin
[e + f*x]^3)/(3*f)
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

method	result	size
parallelrisch	$\frac{\sin(3fx+3e)a+9\left(a+\frac{4b}{3}\right)\sin(fx+e)}{12f}$	31
derivativedivides	$\frac{\frac{a\left(\cos(fx+e)^2+2\right)\sin(fx+e)}{3}+\sin(fx+e)b}{f}$	33
default	$\frac{\frac{a\left(\cos(fx+e)^2+2\right)\sin(fx+e)}{3}+\sin(fx+e)b}{f}$	33
risch	$\frac{3a\sin(fx+e)}{4f}+\frac{\sin(fx+e)b}{f}+\frac{a\sin(3fx+3e)}{12f}$	40
norman	$\frac{\frac{2(a-3b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}-\frac{2(a-3b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{3f}-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{f}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}$	111

```
[In] int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(sin(3*f*x+3*e)*a+9*(a+4/3*b)*sin(f*x+e))/f
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{\left(a\cos\left(fx+e\right)^2+2\,a+3\,b\right)\sin\left(fx+e\right)}{3\,f}$$

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
[Out] 1/3*(a*cos(f*x + e)^2 + 2*a + 3*b)*sin(f*x + e)/f
```

Sympy [F]

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\int\left(a+b\sec^2\left(e+fx\right)\right)\cos^3\left(e+fx\right)dx$$

```
[In] integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2),x)
[Out] Integral((a + b*sec(e + f*x)**2)*cos(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{a\sin\left(fx+e\right)^3-3\left(a+b\right)\sin\left(fx+e\right)}{3\,f}$$

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] -1/3*(a*sin(f*x + e)^3 - 3*(a + b)*sin(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int \cos^3(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx = -\frac{a \sin\left(fx+e\right)^3 - 3\,a \sin\left(fx+e\right) - 3\,b \sin\left(fx+e\right)}{3\,f}$$

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] -1/3*(a*sin(f*x + e)^3 - 3*a*sin(f*x + e) - 3*b*sin(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \cos^3(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx = -\frac{\frac{a \sin(e+f\,x)^3}{3} - \sin\left(e+f\,x\right)\,\left(a+b\right)}{f}$$

```
[In] int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2),x)
[Out] -((a*sin(e + f*x)^3)/3 - sin(e + f*x)*(a + b))/f
```

3.157 $\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 50

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(a+b)\sin(e+fx)}{f}-\frac{(2a+b)\sin^3(e+fx)}{3f}+\frac{a\sin^5(e+fx)}{5f}$$

```
[Out] (a+b)*sin(f*x+e)/f-1/3*(2*a+b)*sin(f*x+e)^3/f+1/5*a*sin(f*x+e)^5/f
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4129, 3092, 380}

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{(2a+b)\sin^3(e+fx)}{3f}+\frac{(a+b)\sin(e+fx)}{f}+\frac{a\sin^5(e+fx)}{5f}$$

```
[In] Int[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + b)*Sin[e + f*x])/f - ((2*a + b)*Sin[e + f*x]^3)/(3*f) + (a*Sin[e + f*
x]^5)/(5*f)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 4129

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \cos^3(e+fx)\left(b+a\cos^2(e+fx)\right)\,dx \\
&= -\frac{\text{Subst}\left(\int (1-x^2)\left(a+b-ax^2\right)\,dx, x, -\sin(e+fx)\right)}{f} \\
&= -\frac{\text{Subst}\left(\int \left(a\left(1+\frac{b}{a}\right)-(2a+b)x^2+ax^4\right)\,dx, x, -\sin(e+fx)\right)}{f} \\
&= \frac{(a+b)\sin(e+fx)}{f}-\frac{(2a+b)\sin^3(e+fx)}{3f}+\frac{a\sin^5(e+fx)}{5f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.42

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{a\sin(e+fx)}{f}+\frac{b\sin(e+fx)}{f}-\frac{2a\sin^3(e+fx)}{3f}-\frac{b\sin^3(e+fx)}{3f}+\frac{a\sin^5(e+fx)}{5f}$$

```
[In] Integrate[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
[Out] (a*Sin[e + f*x])/f + (b*Sin[e + f*x])/f - (2*a*Sin[e + f*x]^3)/(3*f) - (b*S
in[e + f*x]^3)/(3*f) + (a*Sin[e + f*x]^5)/(5*f)
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result
parallelrisch	$\frac{(25a+20b)\sin(3fx+3e)+3\sin(5fx+5e)a+150\sin(fx+e)\left(a+\frac{6b}{5}\right)}{240f}$
derivativedivides	$\frac{\frac{a\left(\frac{8}{3}+\cos(fx+e)^4+\frac{4\cos(fx+e)^2}{3}\right)\sin(fx+e)}{5}+\frac{b\left(\cos(fx+e)^2+2\right)\sin(fx+e)}{3}}{f}$
default	$\frac{\frac{a\left(\frac{8}{3}+\cos(fx+e)^4+\frac{4\cos(fx+e)^2}{3}\right)\sin(fx+e)}{5}+\frac{b\left(\cos(fx+e)^2+2\right)\sin(fx+e)}{3}}{f}$
risch	$\frac{5a\sin(fx+e)}{8f}+\frac{3\sin(fx+e)b}{4f}+\frac{a\sin(5fx+5e)}{80f}+\frac{5a\sin(3fx+3e)}{48f}+\frac{\sin(3fx+3e)b}{12f}$
norman	$\frac{-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{f}-\frac{2(a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}+\frac{2(a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{3f}-\frac{4(19a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{15f}+4(19}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}$

```
[In] int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/240*((25*a+20*b)*sin(3*f*x+3*e)+3*sin(5*f*x+5*e)*a+150*sin(f*x+e)*(a+6/5*
b))/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\left(3\,a\cos\left(fx+e\right)^4+\left(4\,a+5\,b\right)\cos\left(fx+e\right)^2+8\,a+10\,b\right)\sin\left(fx+e\right)}{15\,f}$$

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*a*cos(f*x + e)^4 + (4*a + 5*b)*cos(f*x + e)^2 + 8*a + 10*b)*sin(f*x
 + e)/f
```

Sympy [F]

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)\cos^5\left(e+fx\right)dx$$

```
[In] integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*cos(e + f*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{3\,a\sin\left(fx+e\right)^5-5\left(2\,a+b\right)\sin\left(fx+e\right)^3+15\left(a+b\right)\sin\left(fx+e\right)}{15\,f}$$

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*a*sin(f*x + e)^5 - 5*(2*a + b)*sin(f*x + e)^3 + 15*(a + b)*sin(f*x
+ e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.14

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{3\,a\sin\left(fx+e\right)^5-10\,a\sin\left(fx+e\right)^3-5\,b\sin\left(fx+e\right)^3+15\,a\sin\left(fx+e\right)+15\,b\sin\left(fx+e\right)}{15\,f}$$

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(3*a*sin(f*x + e)^5 - 10*a*sin(f*x + e)^3 - 5*b*sin(f*x + e)^3 + 15*a*
sin(f*x + e) + 15*b*sin(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 18.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{\frac{a\sin(e+f\,x)^5}{5}+\left(-\frac{2\,a}{3}-\frac{b}{3}\right)\,\sin\left(e+f\,x\right)^3+\left(a+b\right)\,\sin\left(e+f\,x\right)}{f}$$

```
[In] int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2),x)
```

```
[Out] ((a*sin(e + f*x)^5)/5 - sin(e + f*x)^3*((2*a)/3 + b/3) + sin(e + f*x)*(a + b))/f
```

3.158 $\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(7a+6b)\tan(e+fx)}{7f}+\frac{b\sec^6(e+fx)\tan(e+fx)}{7f}+\frac{2(7a+6b)\tan^3(e+fx)}{21f}+\frac{(7a+6b)\tan^5(e+fx)}{35f}$$

```
[Out] 1/7*(7*a+6*b)*tan(f*x+e)/f+1/7*b*sec(f*x+e)^6*tan(f*x+e)/f+2/21*(7*a+6*b)*t
an(f*x+e)^3/f+1/35*(7*a+6*b)*tan(f*x+e)^5/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4131, 3852}

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(7a+6b)\tan^5(e+fx)}{35f}+\frac{2(7a+6b)\tan^3(e+fx)}{21f}+\frac{(7a+6b)\tan(e+fx)}{7f}+\frac{b\tan(e+fx)\sec^6(e+fx)}{7f}$$

```
[In] Int[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((7*a + 6*b)*Tan[e + f*x])/(7*f) + (b*Sec[e + f*x]^6*Tan[e + f*x])/(7*f) +
(2*(7*a + 6*b)*Tan[e + f*x]^3)/(21*f) + ((7*a + 6*b)*Tan[e + f*x]^5)/(35*f)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{b\sec^6(e+fx)\tan(e+fx)}{7f} + \frac{1}{7}(7a+6b)\int \sec^6(e+fx)\,dx$$

$$= \frac{b\sec^6(e+fx)\tan(e+fx)}{7f} - \frac{(7a+6b)\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -\tan(e+fx)\right)}{7f}$$

$$= \frac{(7a+6b)\tan(e+fx)}{7f} + \frac{b\sec^6(e+fx)\tan(e+fx)}{7f} + \frac{2(7a+6b)\tan^3(e+fx)}{21f} + \frac{(7a+6b)\tan^5(e+fx)}{35f}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{a\left(\tan(e+fx)+\frac{2}{3}\tan^3(e+fx)+\frac{1}{5}\tan^5(e+fx)\right)}{f} + \frac{b\left(\tan(e+fx)+\tan^3(e+fx)+\frac{3}{5}\tan^5(e+fx)+\frac{1}{7}\tan^7(e+fx)\right)}{f}$$

```
[In] Integrate[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
[Out] (a*(Tan[e + f*x] + (2*Tan[e + f*x]^3)/3 + Tan[e + f*x]^5/5))/f + (b*(Tan[e
+ f*x] + Tan[e + f*x]^3 + (3*Tan[e + f*x]^5)/5 + Tan[e + f*x]^7/7))/f
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-a\left(-\frac{8}{15}-\frac{\sec(fx+e)^{4}}{5}-\frac{4\sec(fx+e)^{2}}{15}\right)\tan(fx+e)-b\left(-\frac{16}{35}-\frac{\sec(fx+e)^{6}}{7}-\frac{6\sec(fx+e)^{4}}{35}-\frac{8\sec(fx+e)^{2}}{35}\right)\tan(fx+e)}{f}$
default	$\frac{-a\left(-\frac{8}{15}-\frac{\sec(fx+e)^{4}}{5}-\frac{4\sec(fx+e)^{2}}{15}\right)\tan(fx+e)-b\left(-\frac{16}{35}-\frac{\sec(fx+e)^{6}}{7}-\frac{6\sec(fx+e)^{4}}{35}-\frac{8\sec(fx+e)^{2}}{35}\right)\tan(fx+e)}{f}$
parts	$-\frac{a\left(-\frac{8}{15}-\frac{\sec(fx+e)^{4}}{5}-\frac{4\sec(fx+e)^{2}}{15}\right)\tan(fx+e)}{f}-\frac{b\left(-\frac{16}{35}-\frac{\sec(fx+e)^{6}}{7}-\frac{6\sec(fx+e)^{4}}{35}-\frac{8\sec(fx+e)^{2}}{35}\right)\tan(fx+e)}{f}$
risch	$\frac{16i\left(70a\,\mathrm{e}^{8i(fx+e)}+175a\,\mathrm{e}^{6i(fx+e)}+210b\,\mathrm{e}^{6i(fx+e)}+147a\,\mathrm{e}^{4i(fx+e)}+126b\,\mathrm{e}^{4i(fx+e)}+49a\,\mathrm{e}^{2i(fx+e)}+42b\,\mathrm{e}^{2i(fx+e)}+7a+6b\right.}{105f\left(\mathrm{e}^{2i(fx+e)}+1\right)^{7}}$
parallelrisch	$\frac{(1176a+1008b)\sin(3fx+3e)+(392a+336b)\sin(5fx+5e)+(56a+48b)\sin(7fx+7e)+840\sin(fx+e)(a+2b)}{105f(\cos(7fx+7e)+7\cos(5fx+5e)+21\cos(3fx+3e)+35\cos(fx+e))}$
norman	$\frac{-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{13}}{f}+\frac{4(5a+3b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3f}+\frac{4(5a+3b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{3f}+\frac{8(91a+53b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{35f}-\cdots}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{7}}$

```
[In] int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)-b*(-16/35-1/7
*sec(f*x+e)^6-6/35*sec(f*x+e)^4-8/35*sec(f*x+e)^2)*tan(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{\left(8\,(7\,a+6\,b)\cos\left(fx+e\right)^{6}+4\,(7\,a+6\,b)\cos\left(fx+e\right)^{4}+3\,(7\,a+6\,b)\cos\left(fx+e\right)^{2}+15\,b\right)\sin\left(fx+e\right)}{105\,f\cos\left(fx+e\right)^{7}}$$

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(7*a + 6*b)*cos(f*x + e)^6 + 4*(7*a + 6*b)*cos(f*x + e)^4 + 3*(7*a
 + 6*b)*cos(f*x + e)^2 + 15*b)*sin(f*x + e)/(f*cos(f*x + e)^7)
```

Sympy [F]

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\int\left(a+b\sec^2\left(e+fx\right)\right)\sec^6\left(e+fx\right)dx$$

```
[In] integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{15\,b\tan\left(fx+e\right)^7+21\left(a+3\,b\right)\tan\left(fx+e\right)^5+35\left(2\,a+3\,b\right)\tan\left(fx+e\right)^3+105\left(a+b\right)\tan\left(fx+e\right)}{105\,f}$$

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/105*(15*b*tan(f*x + e)^7 + 21*(a + 3*b)*tan(f*x + e)^5 + 35*(2*a + 3*b)*t
an(f*x + e)^3 + 105*(a + b)*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.91

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{15\,b\tan\left(fx+e\right)^7+21\,a\tan\left(fx+e\right)^5+63\,b\tan\left(fx+e\right)^5+70\,a\tan\left(fx+e\right)^3+105\,b\tan\left(fx+e\right)^3+1}{105\,f}$$

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/105*(15*b*tan(f*x + e)^7 + 21*a*tan(f*x + e)^5 + 63*b*tan(f*x + e)^5 + 70
*a*tan(f*x + e)^3 + 105*b*tan(f*x + e)^3 + 105*a*tan(f*x + e) + 105*b*tan(f
*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 18.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{\frac{b\tan(e+f\,x)^7}{7}+\left(\frac{a}{5}+\frac{3\,b}{5}\right)\,\tan(e+f\,x)^5+\left(\frac{2\,a}{3}+b\right)\,\tan(e+f\,x)^3+(a+b)\,\tan(e+f\,x)}{f}$$

```
[In] int((a + b/cos(e + f*x)^2)/cos(e + f*x)^6,x)
[Out] (tan(e + f*x)^5*(a/5 + (3*b)/5) + (b*tan(e + f*x)^7)/7 + tan(e + f*x)^3*((2
*a)/3 + b) + tan(e + f*x)*(a + b))/f
```

3.159 $\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(5a+4b)\tan(e+fx)}{5f}+\frac{b\sec^4(e+fx)\tan(e+fx)}{5f}+\frac{(5a+4b)\tan^3(e+fx)}{15f}$$

```
[Out] 1/5*(5*a+4*b)*tan(f*x+e)/f+1/5*b*sec(f*x+e)^4*tan(f*x+e)/f+1/15*(5*a+4*b)*t
an(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4131, 3852}

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(5a+4b)\tan^3(e+fx)}{15f}+\frac{(5a+4b)\tan(e+fx)}{5f}+\frac{b\tan(e+fx)\sec^4(e+fx)}{5f}$$

```
[In] Int[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((5*a + 4*b)*Tan[e + f*x])/(5*f) + (b*Sec[e + f*x]^4*Tan[e + f*x])/(5*f) +
((5*a + 4*b)*Tan[e + f*x]^3)/(15*f)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\sec^4(e+fx)\tan(e+fx)}{5f}+\frac{1}{5}(5a+4b)\int\sec^4(e+fx)\,dx\\
&= \frac{b\sec^4(e+fx)\tan(e+fx)}{5f}-\frac{(5a+4b)\text{Subst}\left(\int\left(1+x^2\right)\,dx,x,-\tan(e+fx)\right)}{5f}\\
&= \frac{(5a+4b)\tan(e+fx)}{5f}+\frac{b\sec^4(e+fx)\tan(e+fx)}{5f}+\frac{(5a+4b)\tan^3(e+fx)}{15f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int\sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{a\left(\tan(e+fx)+\frac{1}{3}\tan^3(e+fx)\right)}{f}+\frac{b\left(\tan(e+fx)+\frac{2}{3}\tan^3(e+fx)+\frac{1}{5}\tan^5(e+fx)\right)}{f}$$

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
[Out] (a*(Tan[e + f*x] + Tan[e + f*x]^3/3))/f + (b*(Tan[e + f*x] + (2*Tan[e + f*x
]^3)/3 + Tan[e + f*x]^5/5))/f
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

method	result	si
derivativedivides	$\frac{-a\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-b\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f}$	5
default	$\frac{-a\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-b\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f}$	5
parts	$-\frac{a\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}-\frac{b\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f}$	6
parallelrisch	$\frac{(50a+40b)\sin(3fx+3e)+(10a+8b)\sin(5fx+5e)+40\sin(fx+e)(a+2b)}{15f(\cos(5fx+5e)+5\cos(3fx+3e)+10\cos(fx+e))}$	8
risch	$\frac{4i\left(15a\,e^{6i(fx+e)}+35a\,e^{4i(fx+e)}+40b\,e^{4i(fx+e)}+25a\,e^{2i(fx+e)}+20b\,e^{2i(fx+e)}+5a+4b\right)}{15f\left(e^{2i(fx+e)}+1\right)^5}$	8
norman	$\frac{-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{f}+\frac{8(2a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}+\frac{8(2a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{3f}-\frac{4(25a+29b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{15f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^5}$	1

```
[In] int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-b*(-8/15-1/5*sec(f*x+e)^4-4/15*s
ec(f*x+e)^2)*tan(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\left(2\,(5\,a+4\,b)\cos\left(fx+e\right)^4+(5\,a+4\,b)\cos\left(fx+e\right)^2+3\,b\right)\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)^5}$$

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(5*a + 4*b)*cos(f*x + e)^4 + (5*a + 4*b)*cos(f*x + e)^2 + 3*b)*sin(
f*x + e)/(f*cos(f*x + e)^5)
```

Sympy [F]

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\int \left(a+b\sec^2{(e+fx)}\right)\sec^4{(e+fx)}\,dx$$

```
[In] integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{3\,b\tan\left(fx+e\right)^5+5\,(a+2\,b)\tan\left(fx+e\right)^3+15\,(a+b)\tan\left(fx+e\right)}{15\,f}$$

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*b*tan(f*x + e)^5 + 5*(a + 2*b)*tan(f*x + e)^3 + 15*(a + b)*tan(f*x
+ e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{3\,b\tan\left(fx+e\right)^5+5\,a\tan\left(fx+e\right)^3+10\,b\tan\left(fx+e\right)^3+15\,a\tan\left(fx+e\right)+15\,b\tan\left(fx+e\right)}{15\,f}$$

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(3*b*tan(f*x + e)^5 + 5*a*tan(f*x + e)^3 + 10*b*tan(f*x + e)^3 + 15*a*
tan(f*x + e) + 15*b*tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 18.42 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{\frac{b\tan(e+f\,x)^5}{5}+\left(\frac{a}{3}+\frac{2\,b}{3}\right)\,\tan(e+f\,x)^3+(a+b)\,\tan(e+f\,x)}{f}$$

```
[In] int((a + b/cos(e + f*x)^2)/cos(e + f*x)^4,x)
[Out] (tan(e + f*x)^3*(a/3 + (2*b)/3) + (b*tan(e + f*x)^5)/5 + tan(e + f*x)*(a +
b))/f
```

3.160 $\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(3a+2b)\tan(e+fx)}{3f}+\frac{b\sec^2(e+fx)\tan(e+fx)}{3f}$$

```
[Out] 1/3*(3*a+2*b)*tan(f*x+e)/f+1/3*b*sec(f*x+e)^2*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4131, 3852, 8}

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(3a+2b)\tan(e+fx)}{3f}+\frac{b\tan(e+fx)\sec^2(e+fx)}{3f}$$

```
[In] Int[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((3*a + 2*b)*Tan[e + f*x])/(3*f) + (b*Sec[e + f*x]^2*Tan[e + f*x])/(3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b\sec^2(e+fx)\tan(e+fx)}{3f} + \frac{1}{3}(3a+2b)\int \sec^2(e+fx)\,dx \\
&= \frac{b\sec^2(e+fx)\tan(e+fx)}{3f} - \frac{(3a+2b)\text{Subst}(\int 1\,dx, x, -\tan(e+fx))}{3f} \\
&= \frac{(3a+2b)\tan(e+fx)}{3f} + \frac{b\sec^2(e+fx)\tan(e+fx)}{3f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$
\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{a\tan(e+fx)}{f} + \frac{b\left(\tan(e+fx)+\frac{1}{3}\tan^3(e+fx)\right)}{f}
$$

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
[Out] (a*Tan[e + f*x])/f + (b*(Tan[e + f*x] + Tan[e + f*x]^3/3))/f
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{a\tan(fx+e)-b\left(-\frac{2}{3}-\frac{\sec(fx+e)^{2}}{3}\right)\tan(fx+e)}{f}$	35
default	$\frac{a\tan(fx+e)-b\left(-\frac{2}{3}-\frac{\sec(fx+e)^{2}}{3}\right)\tan(fx+e)}{f}$	35
parts	$\frac{a\tan(fx+e)}{f}-\frac{b\left(-\frac{2}{3}-\frac{\sec(fx+e)^{2}}{3}\right)\tan(fx+e)}{f}$	37
parallelrisch	$\frac{(3a+2b)\sin(3fx+3e)+3\sin(fx+e)(a+2b)}{3f(\cos(3fx+3e)+3\cos(fx+e))}$	57
risch	$\frac{2i\left(3a\,e^{4i(fx+e)}+6a\,e^{2i(fx+e)}+6b\,e^{2i(fx+e)}+3a+2b\right)}{3f\left(e^{2i(fx+e)}+1\right)^{3}}$	63
norman	$\frac{-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{f}+\frac{4(3a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{3}}$	75

```
[In] int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(a*tan(f*x+e)-b*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{\left((3\,a+2\,b)\cos\left(fx+e\right)^2+b\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
[Out] 1/3*((3*a + 2*b)*cos(f*x + e)^2 + b)*sin(f*x + e)/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\int\left(a+b\sec^2\left(e+fx\right)\right)\sec^2\left(e+fx\right)dx$$

```
[In] integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2),x)
[Out] Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \sec^2(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx = \frac{\left(\tan\left(fx+e\right)^3 + 3\, \tan\left(fx+e\right)\right)b + 3\, a \tan\left(fx+e\right)}{3\, f}$$

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] 1/3*((tan(f*x + e)^3 + 3*tan(f*x + e))*b + 3*a*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \sec^2(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx = \frac{b \tan\left(fx+e\right)^3 + 3\, a \tan\left(fx+e\right) + 3\, b \tan\left(fx+e\right)}{3\, f}$$

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] 1/3*(b*tan(f*x + e)^3 + 3*a*tan(f*x + e) + 3*b*tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 17.96 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int \sec^2(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx = \frac{b \tan(e+f\, x)^3}{3\, f} + \frac{\tan(e+f\, x)\, (a+b)}{f}$$

```
[In] int((a + b/cos(e + f*x)^2)/cos(e + f*x)^2,x)
[Out] (b*tan(e + f*x)^3)/(3*f) + (tan(e + f*x)*(a + b))/f
```

3.161 $\int (a + b\sec^2(e + fx))\ dx$

Optimal result

Integrand size = 12, antiderivative size = 15

$$\int (a + b\sec^2(e + fx))\ dx = ax + \frac{b\tan(e + fx)}{f}$$

```
[Out] a*x+b*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3852, 8}

$$\int (a + b\sec^2(e + fx))\ dx = ax + \frac{b\tan(e + fx)}{f}$$

```
[In] Int[a + b*Sec[e + f*x]^2,x]
```

```
[Out] a*x + (b*Tan[e + f*x])/f
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= ax + b\int \sec^2(e+fx)\,dx \\
&= ax - \frac{b\text{Subst}(\int 1\,dx, x, -\tan(e+fx))}{f} \\
&= ax + \frac{b\tan(e+fx)}{f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \left(a+b\sec^2(e+fx)\right)\,dx = ax + \frac{b\tan(e+fx)}{f}$$

```
[In] Integrate[a + b*Sec[e + f*x]^2,x]
[Out] a*x + (b*Tan[e + f*x])/f
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$ax+\frac{b\tan(fx+e)}{f}$	16
parts	$ax+\frac{b\tan(fx+e)}{f}$	16
derivativedivides	$\frac{(fx+e)a+b\tan(fx+e)}{f}$	21
risch	$ax+\frac{2ib}{f\left(\mathrm{e}^{2i(fx+e)}+1\right)}$	25
parallelrisch	$-\frac{2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}+ax$	35
norman	$\frac{ax\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-ax-\frac{2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}$	51

```
[In] int(a+b*sec(f*x+e)^2,x,method=_RETURNVERBOSE)
[Out] a*x+b*tan(f*x+e)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(15) = 30$.

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \left(a+b\sec^2(e+fx)\right)\,dx=\frac{afx\cos\left(fx+e\right)+b\sin\left(fx+e\right)}{f\cos\left(fx+e\right)}$$

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="fricas")
[Out] (a*f*x*cos(f*x + e) + b*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)\,dx$$

```
[In] integrate(a+b*sec(f*x+e)**2,x)
[Out] Integral(a + b*sec(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \left(a+b\sec^2(e+fx)\right)\,dx=ax+\frac{b\tan\left(fx+e\right)}{f}$$

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="maxima")
[Out] a*x + b*tan(f*x + e)/f
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \left(a+b\sec^2(e+fx)\right)\,dx=ax+\frac{b\tan\left(fx+e\right)}{f}$$

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="giac")
[Out] a*x + b*tan(f*x + e)/f
```

Mupad [B] (verification not implemented)

Time = 18.42 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \left(a + b\sec^2(e + fx)\right) \, dx = \frac{b\tan(e + f\,x) + a\,f\,x}{f}$$

```
[In] int(a + b/cos(e + f*x)^2,x)
[Out] (b*tan(e + f*x) + a*f*x)/f
```

3.162 $\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 31

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{1}{2}(a+2b)x+\frac{a\cos(e+fx)\sin(e+fx)}{2f}$$

```
[Out] 1/2*(a+2*b)*x+1/2*a*cos(f*x+e)*sin(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4130, 8}

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{1}{2}x(a+2b)+\frac{a\sin(e+fx)\cos(e+fx)}{2f}$$

```
[In] Int[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b)*x)/2 + (a*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{a\cos(e+fx)\sin(e+fx)}{2f} + \frac{1}{2}(a+2b)\int 1\,dx$$
$$= \frac{1}{2}(a+2b)x + \frac{a\cos(e+fx)\sin(e+fx)}{2f}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = bx + \frac{a(e+fx)}{2f} + \frac{a\sin(2(e+fx))}{4f}$$

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
[Out] b*x + (a*(e + f*x))/(2*f) + (a*Sin[2*(e + f*x)])/(4*f)
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result
risch	$\frac{ax}{2}+xb+\frac{a\sin(2fx+2e)}{4f}$
parallelrisch	$\frac{a\sin(2fx+2e)+2(a+2b)xf}{4f}$
derivativedivides	$\frac{a\left(\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b(fx+e)}{f}$
default	$\frac{a\left(\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b(fx+e)}{f}$
norman	$\frac{\left(-\frac{a}{2}-b\right)x+\left(-\frac{a}{2}-b\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\left(\frac{a}{2}+b\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+\left(\frac{a}{2}+b\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6-\frac{a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}-}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}$

```
[In] int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/2*a*x+x*b+1/4*a/f*sin(2*f*x+2*e)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(a+2\,b)fx+a\cos\left(fx+e\right)\sin\left(fx+e\right)}{2\,f}$$

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
[Out] 1/2*((a + 2*b)*f*x + a*cos(f*x + e)*sin(f*x + e))/f
```

Sympy [A] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.65

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=a\left(\begin{cases}\frac{x\sin^2{(e+fx)}}{2}+\frac{x\cos^2{(e+fx)}}{2}+\frac{\sin{(e+fx)}\cos{(e+fx)}}{2f} & \text{for } f\neq 0\\ x\cos^2{(e)} & \text{otherwise}\end{cases}\right)+bx$$

```
[In] integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2),x)
[Out] a*Piecewise((x*sin(e + f*x)**2/2 + x*cos(e + f*x)**2/2 + sin(e + f*x)*cos(e
 + f*x)/(2*f), Ne(f, 0)), (x*cos(e)**2, True)) + b*x
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(fx+e)(a+2\,b)+\frac{a\tan(fx+e)}{\tan(fx+e)^2+1}}{2\,f}$$

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] 1/2*((f*x + e)*(a + 2*b) + a*tan(f*x + e)/(tan(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(fx+e)(a+2b)+\frac{a\tan(fx+e)}{\tan(fx+e)^2+1}}{2f}$$

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] 1/2*((f*x + e)*(a + 2*b) + a*tan(f*x + e)/(tan(f*x + e)^2 + 1))/f
```

Mupad [B] (verification not implemented)

Time = 19.58 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{\frac{a\,\sin(2\,e+2\,f\,x)}{4}+f\,x\,\left(\frac{a}{2}+b\right)}{f}$$

```
[In] int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2),x)
[Out] ((a*sin(2*e + 2*f*x))/4 + f*x*(a/2 + b))/f
```

3.163 $\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{1}{8}(3a+4b)x+\frac{(3a+4b)\cos(e+fx)\sin(e+fx)}{8f}+\frac{a\cos^3(e+fx)\sin(e+fx)}{4f}$$

```
[Out] 1/8*(3*a+4*b)*x+1/8*(3*a+4*b)*cos(f*x+e)*sin(f*x+e)/f+1/4*a*cos(f*x+e)^3*si
n(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4130, 2715, 8}

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(3a+4b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}x(3a+4b)+\frac{a\sin(e+fx)\cos^3(e+fx)}{4f}$$

```
[In] Int[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((3*a + 4*b)*x)/8 + ((3*a + 4*b)*Cos[e + f*x]*Sin[e + f*x])/(8*f) + (a*Cos[
e + f*x]^3*Sin[e + f*x])/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{a\cos^3(e+fx)\sin(e+fx)}{4f}+\frac{1}{4}(3a+4b)\int\cos^2(e+fx)\,dx\\ &= \frac{(3a+4b)\cos(e+fx)\sin(e+fx)}{8f}+\frac{a\cos^3(e+fx)\sin(e+fx)}{4f}+\frac{1}{8}(3a+4b)\int 1\,dx\\ &= \frac{1}{8}(3a+4b)x+\frac{(3a+4b)\cos(e+fx)\sin(e+fx)}{8f}+\frac{a\cos^3(e+fx)\sin(e+fx)}{4f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int\cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{4(3a+4b)(e+fx)+8(a+b)\sin(2(e+fx))+a\sin(4(e+fx))}{32f}$$

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
[Out] (4*(3*a + 4*b)*(e + f*x) + 8*(a + b)*Sin[2*(e + f*x)] + a*Sin[4*(e + f*x)])
/(32*f)
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

method	result
parallelrisch	$\frac{(8a+8b)\sin(2fx+2e)+\sin(4fx+4e)a+12\left(a+\frac{4b}{3}\right)xf}{32f}$
risch	$\frac{3ax}{8}+\frac{xb}{2}+\frac{\sin(4fx+4e)a}{32f}+\frac{a\sin(2fx+2e)}{4f}+\frac{\sin(2fx+2e)b}{4f}$
derivativedivides	$\frac{a\left(\frac{\left(\cos(fx+e)^3+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+b\left(\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$
default	$\frac{a\left(\frac{\left(\cos(fx+e)^3+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+b\left(\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$
norman	$\frac{\left(-\frac{3a}{8}-\frac{b}{2}\right)x+\left(-\frac{9a}{8}-\frac{3b}{2}\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\left(-\frac{3a}{4}-b\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+\left(\frac{3a}{4}+b\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+\left(\frac{3a}{8}+\frac{b}{2}\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}$

```
[In] int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32*((8*a+8*b)*sin(2*f*x+2*e)+sin(4*f*x+4*e)*a+12*(a+4/3*b)*x*f)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(3\,a+4\,b)fx+\left(2\,a\cos\left(fx+e\right)^3+(3\,a+4\,b)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{8\,f}$$

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/8*((3*a + 4*b)*f*x + (2*a*cos(f*x + e)^3 + (3*a + 4*b)*cos(f*x + e))*sin(
f*x + e))/f
```

Sympy [F]

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2{(e+fx)}\right)\cos^4{(e+fx)}\,dx$$

[In] integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2),x)

[Out] Integral((a + b*sec(e + f*x)**2)*cos(e + f*x)**4, x)

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(fx+e)(3\,a+4\,b)+\frac{(3\,a+4\,b)\tan(fx+e)^3+(5\,a+4\,b)\tan(fx+e)}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}}{8\,f}$$

[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="maxima")

[Out] 1/8*((f*x + e)*(3*a + 4*b) + ((3*a + 4*b)*tan(f*x + e)^3 + (5*a + 4*b)*tan(f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(fx+e)(3\,a+4\,b)+\frac{3\,a\tan(fx+e)^3+4\,b\tan(fx+e)^3+5\,a\tan(fx+e)+4\,b\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^2}}{8\,f}$$

[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="giac")

[Out] 1/8*((f*x + e)*(3*a + 4*b) + (3*a*tan(f*x + e)^3 + 4*b*tan(f*x + e)^3 + 5*a*tan(f*x + e) + 4*b*tan(f*x + e))/(tan(f*x + e)^2 + 1)^2)/f

Mupad [B] (verification not implemented)

Time = 18.81 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = x\left(\frac{3\,a}{8}+\frac{b}{2}\right)+\frac{\left(\frac{3\,a}{8}+\frac{b}{2}\right)\,\tan(e+f\,x)^3+\left(\frac{5\,a}{8}+\frac{b}{2}\right)\,\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^4+2\tan\left(e+f\,x\right)^2+1\right)}$$

```
[In] int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2),x)
[Out] x*((3*a)/8 + b/2) + (tan(e + f*x)^3*((3*a)/8 + b/2) + tan(e + f*x)*((5*a)/8
 + b/2))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1))
```

3.164 $\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{1}{16}(5a+6b)x+\frac{(5a+6b)\cos(e+fx)\sin(e+fx)}{16f}+\frac{(5a+6b)\cos^3(e+fx)\sin(e+fx)}{24f}+\frac{a\cos^5(e+fx)\sin(e+fx)}{6f}$$

```
[Out] 1/16*(5*a+6*b)*x+1/16*(5*a+6*b)*cos(f*x+e)*sin(f*x+e)/f+1/24*(5*a+6*b)*cos(
f*x+e)^3*sin(f*x+e)/f+1/6*a*cos(f*x+e)^5*sin(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4130, 2715, 8}

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(5a+6b)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{(5a+6b)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}x(5a+6b)+\frac{a\sin(e+fx)\cos^5(e+fx)}{6f}$$

```
[In] Int[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((5*a + 6*b)*x)/16 + ((5*a + 6*b)*Cos[e + f*x]*Sin[e + f*x])/(16*f) + ((5*a
 + 6*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) + (a*Cos[e + f*x]^5*Sin[e + f*x
])/(6*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{a\cos^5(e+fx)\sin(e+fx)}{6f}+\frac{1}{6}(5a+6b)\int\cos^4(e+fx)\,dx\\
&= \frac{(5a+6b)\cos^3(e+fx)\sin(e+fx)}{24f}\\
&\quad+\frac{a\cos^5(e+fx)\sin(e+fx)}{6f}+\frac{1}{8}(5a+6b)\int\cos^2(e+fx)\,dx\\
&= \frac{(5a+6b)\cos(e+fx)\sin(e+fx)}{16f}+\frac{(5a+6b)\cos^3(e+fx)\sin(e+fx)}{24f}\\
&\quad+\frac{a\cos^5(e+fx)\sin(e+fx)}{6f}+\frac{1}{16}(5a+6b)\int 1\,dx\\
&= \frac{1}{16}(5a+6b)x+\frac{(5a+6b)\cos(e+fx)\sin(e+fx)}{16f}\\
&\quad+\frac{(5a+6b)\cos^3(e+fx)\sin(e+fx)}{24f}+\frac{a\cos^5(e+fx)\sin(e+fx)}{6f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{60ae+72be+60afx+72bfx+(45a+48b)\sin(2(e+fx))+(9a+6b)\sin(4(e+fx))+a\sin(6(e+fx)}{192f}$$

```
[In] Integrate[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
[Out] (60*a*e + 72*b*e + 60*a*f*x + 72*b*f*x + (45*a + 48*b)*Sin[2*(e + f*x)] + (
9*a + 6*b)*Sin[4*(e + f*x)] + a*Sin[6*(e + f*x)])/(192*f)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{(45a+48b)\sin(2fx+2e)+(9a+6b)\sin(4fx+4e)+a\sin(6fx+6e)+60xf\left(a+\frac{6b}{5}\right)}{192f}$
risch	$\frac{5ax}{16}+\frac{3xb}{8}+\frac{a\sin(6fx+6e)}{192f}+\frac{3\sin(4fx+4e)a}{64f}+\frac{\sin(4fx+4e)b}{32f}+\frac{15a\sin(2fx+2e)}{64f}+\frac{\sin(2fx+2e)b}{4f}$
derivativedivides	$\frac{a\left(\frac{\left(\cos(fx+e)^5+\frac{5\cos(fx+e)^3}{4}+\frac{15\cos(fx+e)}{8}\right)\sin(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+b\left(\frac{\left(\cos(fx+e)^3+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right.}{f}$
default	$\frac{a\left(\frac{\left(\cos(fx+e)^5+\frac{5\cos(fx+e)^3}{4}+\frac{15\cos(fx+e)}{8}\right)\sin(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+b\left(\frac{\left(\cos(fx+e)^3+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right.}{f}$
norman	$\left(-\frac{5a}{16}-\frac{3b}{8}\right)x+\left(-\frac{45a}{16}-\frac{27b}{8}\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+\left(-\frac{25a}{16}-\frac{15b}{8}\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\left(-\frac{25a}{16}-\frac{15b}{8}\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+\left(\frac{5a}{16}+\frac{3b}{8}\right)$

```
[In] int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/192*((45*a+48*b)*sin(2*f*x+2*e)+(9*a+6*b)*sin(4*f*x+4*e)+a*sin(6*f*x+6*e)
+60*x*f*(a+6/5*b))/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{3\,(5\,a+6\,b)fx+\left(8\,a\cos\left(fx+e\right)^5+2\,(5\,a+6\,b)\cos\left(fx+e\right)^3+3\,(5\,a+6\,b)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{48\,f}$$

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
[Out] 1/48*(3*(5*a + 6*b)*f*x + (8*a*cos(f*x + e)^5 + 2*(5*a + 6*b)*cos(f*x + e)^
3 + 3*(5*a + 6*b)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [F(-1)]

Timed out.

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{3\,(fx+e)(5\,a+6\,b)+\frac{3\,(5\,a+6\,b)\tan(fx+e)^5+8\,(5\,a+6\,b)\tan(fx+e)^3+3\,(11\,a+10\,b)\tan(fx+e)}{\tan(fx+e)^6+3\,\tan(fx+e)^4+3\,\tan(fx+e)^2+1}}{48\,f}$$

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] 1/48*(3*(f*x + e)*(5*a + 6*b) + (3*(5*a + 6*b)*tan(f*x + e)^5 + 8*(5*a + 6*
b)*tan(f*x + e)^3 + 3*(11*a + 10*b)*tan(f*x + e))/(tan(f*x + e)^6 + 3*tan(f
*x + e)^4 + 3*tan(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.08

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{3\,(fx+e)(5\,a+6\,b)+\frac{15\,a\tan(fx+e)^5+18\,b\tan(fx+e)^5+40\,a\tan(fx+e)^3+48\,b\tan(fx+e)^3+33\,a\tan(fx+e)+30\,b\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^3}}{48\,f}$$

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/48*(3*(f*x + e)*(5*a + 6*b) + (15*a*tan(f*x + e)^5 + 18*b*tan(f*x + e)^5
+ 40*a*tan(f*x + e)^3 + 48*b*tan(f*x + e)^3 + 33*a*tan(f*x + e) + 30*b*tan(
f*x + e))/(tan(f*x + e)^2 + 1)^3)/f
```

Mupad [B] (verification not implemented)

Time = 19.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = x\left(\frac{5\,a}{16}+\frac{3\,b}{8}\right) + \frac{\left(\frac{5\,a}{16}+\frac{3\,b}{8}\right)\,\tan(e+f\,x)^5+\left(\frac{5\,a}{6}+b\right)\,\tan(e+f\,x)^3+\left(\frac{11\,a}{16}+\frac{5\,b}{8}\right)\,\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^6+3\tan\left(e+f\,x\right)^4+3\tan\left(e+f\,x\right)^2+1\right)}$$

```
[In] int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2),x)
```

```
[Out] x*((5*a)/16 + (3*b)/8) + (tan(e + f*x)^5*((5*a)/16 + (3*b)/8) + tan(e + f*x
)*((11*a)/16 + (5*b)/8) + tan(e + f*x)^3*((5*a)/6 + b))/(f*(3*tan(e + f*x)^
2 + 3*tan(e + f*x)^4 + tan(e + f*x)^6 + 1))
```

3.165 $\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 165

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{(48a^2+80ab+35b^2)\,\text{arctanh}(\sin(e+fx))}{128f} + \frac{(48a^2+80ab+35b^2)\sec(e+fx)\tan(e+fx)}{128f} + \frac{(48a^2+80ab+35b^2)\sec^3(e+fx)\tan(e+fx)}{192f} + \frac{b(10a+7b)\sec^5(e+fx)\tan(e+fx)}{48f} + \frac{b\sec^7(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{8f}$$

```
[Out] 1/128*(48*a^2+80*a*b+35*b^2)*arctanh(sin(f*x+e))/f+1/128*(48*a^2+80*a*b+35*
b^2)*sec(f*x+e)*tan(f*x+e)/f+1/192*(48*a^2+80*a*b+35*b^2)*sec(f*x+e)^3*tan(
f*x+e)/f+1/48*b*(10*a+7*b)*sec(f*x+e)^5*tan(f*x+e)/f+1/8*b*sec(f*x+e)^7*(a+
b-a*sin(f*x+e)^2)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used

$= \{4232, 424, 393, 205, 212\}$

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$
$$= \frac{(48a^2+80ab+35b^2)\operatorname{arctanh}(\sin(e+fx))}{128f} + \frac{(48a^2+80ab+35b^2)\tan(e+fx)\sec^3(e+fx)}{192f} + \frac{(48a^2+80ab+35b^2)\tan(e+fx)\sec(e+fx)}{128f} + \frac{b(10a+7b)\tan(e+fx)\sec^5(e+fx)}{48f} + \frac{b\tan(e+fx)\sec^7(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{8f}$$

```
[In] Int[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((48*a^2 + 80*a*b + 35*b^2)*ArcTanh[Sin[e + f*x]])/(128*f) + ((48*a^2 + 80*
a*b + 35*b^2)*Sec[e + f*x]*Tan[e + f*x])/(128*f) + ((48*a^2 + 80*a*b + 35*b
^2)*Sec[e + f*x]^3*Tan[e + f*x])/(192*f) + (b*(10*a + 7*b)*Sec[e + f*x]^5*T
an[e + f*x])/(48*f) + (b*Sec[e + f*x]^7*(a + b - a*Sin[e + f*x]^2)*Tan[e +
f*x])/(8*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
```

```
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b-ax^2)^2}{(1-x^2)^5}\,dx, x, \sin(e+fx)\right)}{f} \\
&= \frac{b\sec^7(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{8f} \\
&\quad - \frac{\text{Subst}\left(\int \frac{-((a+b)(8a+7b))+a(8a+5b)x^2}{(1-x^2)^4}\,dx, x, \sin(e+fx)\right)}{8f} \\
&= \frac{b(10a+7b)\sec^5(e+fx)\tan(e+fx)}{48f} \\
&\quad + \frac{b\sec^7(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{8f} \\
&\quad + \frac{(48a^2+80ab+35b^2)\,\text{Subst}\left(\int \frac{1}{(1-x^2)^3}\,dx, x, \sin(e+fx)\right)}{48f} \\
&= \frac{(48a^2+80ab+35b^2)\sec^3(e+fx)\tan(e+fx)}{192f} \\
&\quad + \frac{b(10a+7b)\sec^5(e+fx)\tan(e+fx)}{48f} \\
&\quad + \frac{b\sec^7(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{8f} \\
&\quad + \frac{(48a^2+80ab+35b^2)\,\text{Subst}\left(\int \frac{1}{(1-x^2)^2}\,dx, x, \sin(e+fx)\right)}{64f}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{(48a^2+80ab+35b^2)\sec(e+fx)\tan(e+fx)}{128f} \\
&\quad + \frac{(48a^2+80ab+35b^2)\sec^3(e+fx)\tan(e+fx)}{192f} \\
&\quad + \frac{b(10a+7b)\sec^5(e+fx)\tan(e+fx)}{48f} \\
&\quad + \frac{b\sec^7(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{8f} \\
&\quad + \frac{(48a^2+80ab+35b^2)\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(e+fx)\right)}{128f} \\
&= \frac{(48a^2+80ab+35b^2)\,\mathrm{arctanh}(\sin(e+fx))}{128f} \\
&\quad + \frac{(48a^2+80ab+35b^2)\sec(e+fx)\tan(e+fx)}{128f} \\
&\quad + \frac{(48a^2+80ab+35b^2)\sec^3(e+fx)\tan(e+fx)}{192f} \\
&\quad + \frac{b(10a+7b)\sec^5(e+fx)\tan(e+fx)}{48f} \\
&\quad + \frac{b\sec^7(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{8f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.56

$$
\begin{aligned}
&\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx \\
&= \frac{3a^2\mathrm{arctanh}(\sin(e+fx))}{8f} + \frac{5ab\mathrm{arctanh}(\sin(e+fx))}{8f} \\
&\quad + \frac{35b^2\mathrm{arctanh}(\sin(e+fx))}{128f} + \frac{3a^2\sec(e+fx)\tan(e+fx)}{8f} \\
&\quad + \frac{5ab\sec(e+fx)\tan(e+fx)}{8f} + \frac{35b^2\sec(e+fx)\tan(e+fx)}{128f} \\
&\quad + \frac{a^2\sec^3(e+fx)\tan(e+fx)}{4f} + \frac{5ab\sec^3(e+fx)\tan(e+fx)}{12f} \\
&\quad + \frac{35b^2\sec^3(e+fx)\tan(e+fx)}{192f} + \frac{ab\sec^5(e+fx)\tan(e+fx)}{3f} \\
&\quad + \frac{7b^2\sec^5(e+fx)\tan(e+fx)}{48f} + \frac{b^2\sec^7(e+fx)\tan(e+fx)}{8f}
\end{aligned}
$$

```
[In] Integrate[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

[Out] (3*a^2*ArcTanh[Sin[e + f*x]])/(8*f) + (5*a*b*ArcTanh[Sin[e + f*x]])/(8*f) + (35*b^2*ArcTanh[Sin[e + f*x]])/(128*f) + (3*a^2*Sec[e + f*x]*Tan[e + f*x])/(8*f) + (5*a*b*Sec[e + f*x]*Tan[e + f*x])/(8*f) + (35*b^2*Sec[e + f*x]*Tan[e + f*x])/(128*f) + (a^2*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) + (5*a*b*Sec[e + f*x]^3*Tan[e + f*x])/(12*f) + (35*b^2*Sec[e + f*x]^3*Tan[e + f*x])/(192*f) + (a*b*Sec[e + f*x]^5*Tan[e + f*x])/(3*f) + (7*b^2*Sec[e + f*x]^5*Tan[e + f*x])/(48*f) + (b^2*Sec[e + f*x]^7*Tan[e + f*x])/(8*f)

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{a^2\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)+2ab\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(f}{16}\right.\right.}{}$
default	$\frac{a^2\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)+2ab\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(f}{16}\right.\right.}{}$
parts	$\frac{a^2\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)}{f}+\frac{b^2\left(-\left(-\frac{\sec(fx+e)^7}{8}-\frac{7\sec(fx+e)^5}{48}-\frac{35\sec}{}\right.\right.}{}$
parallelrisch	$\frac{-8064\left(\frac{\cos(8fx+8e)}{56}+\frac{\cos(6fx+6e)}{7}+\frac{\cos(4fx+4e)}{2}+\cos(2fx+2e)+\frac{5}{8}\right)\left(a^2+\frac{5}{3}ab+\frac{35}{48}b^2\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+8064\left(\frac{\cos(8fx}{56}\right.}{}$
norman	$\frac{\left(80a^2+176ab+93b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{64f}+\frac{\left(80a^2+176ab+93b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{15}}{64f}-\frac{\left(432a^2+1360ab-1085b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{192f}-\frac{\left(432a^2+1360a\right.}{}$
risch	$-\frac{i\,e^{i(fx+e)}\left(144a^2e^{14i(fx+e)}+240ab\,e^{14i(fx+e)}+105b^2e^{14i(fx+e)}+1104a^2e^{12i(fx+e)}+1840ab\,e^{12i(fx+e)}+805b^2e^{12i(fx+}\right.}{}$

[In] int(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/f*(a^2*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))+2*a*b*(-(-1/6*sec(f*x+e)^5-5/24*sec(f*x+e)^3-5/16*sec(f*x+e))*tan(f*x+e)+5/16*ln(sec(f*x+e)+tan(f*x+e)))+b^2*(-(-1/8*sec(f*x+e)^7-7/48*sec(f*x+e)^5-35/192*sec(f*x+e)^3-35/128*sec(f*x+e))*tan(f*x+e)+35/128*ln(sec(f*x+e)+tan(f*x+e))))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.02

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{3\left(48\,a^2+80\,ab+35\,b^2\right)\cos\left(fx+e\right)^8\log\left(\sin\left(fx+e\right)+1\right)-3\left(48\,a^2+80\,ab+35\,b^2\right)\cos\left(fx+e\right)^8\log\left(-\right.}{}$$

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
[Out] 1/768*(3*(48*a^2 + 80*a*b + 35*b^2)*cos(f*x + e)^8*log(sin(f*x + e) + 1) -
3*(48*a^2 + 80*a*b + 35*b^2)*cos(f*x + e)^8*log(-sin(f*x + e) + 1) + 2*(3*(
48*a^2 + 80*a*b + 35*b^2)*cos(f*x + e)^6 + 2*(48*a^2 + 80*a*b + 35*b^2)*cos
(f*x + e)^4 + 8*(16*a*b + 7*b^2)*cos(f*x + e)^2 + 48*b^2)*sin(f*x + e))/(f*
cos(f*x + e)^8)
```

Sympy [F]

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^2\sec^5\left(e+fx\right)dx$$

```
[In] integrate(sec(f*x+e)**5*(a+b*sec(f*x+e)**2)**2,x)
[Out] Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.21

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{3\left(48\,a^2+80\,ab+35\,b^2\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(48\,a^2+80\,ab+35\,b^2\right)\log\left(\sin\left(fx+e\right)-1\right)-\frac{2\left(3\left(48\,a^2\right.\right.}{}}{768}$$

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
[Out] 1/768*(3*(48*a^2 + 80*a*b + 35*b^2)*log(sin(f*x + e) + 1) - 3*(48*a^2 + 80*
a*b + 35*b^2)*log(sin(f*x + e) - 1) - 2*(3*(48*a^2 + 80*a*b + 35*b^2)*sin(f
*x + e)^7 - 11*(48*a^2 + 80*a*b + 35*b^2)*sin(f*x + e)^5 + (624*a^2 + 1168*
a*b + 511*b^2)*sin(f*x + e)^3 - 3*(80*a^2 + 176*a*b + 93*b^2)*sin(f*x + e))
/(sin(f*x + e)^8 - 4*sin(f*x + e)^6 + 6*sin(f*x + e)^4 - 4*sin(f*x + e)^2 +
 1))/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.34

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{3\left(48\,a^2+80\,ab+35\,b^2\right)\log\left(\left|\sin\left(fx+e\right)+1\right|\right)-3\left(48\,a^2+80\,ab+35\,b^2\right)\log\left(\left|\sin\left(fx+e\right)-1\right|\right)-\frac{2\left(144\,a^2 \cdots\right.}{\cdots}}{\cdots}$$

[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")

[Out] 1/768*(3*(48*a^2 + 80*a*b + 35*b^2)*log(abs(sin(f*x + e) + 1)) - 3*(48*a^2 + 80*a*b + 35*b^2)*log(abs(sin(f*x + e) - 1)) - 2*(144*a^2*sin(f*x + e)^7 + 240*a*b*sin(f*x + e)^7 + 105*b^2*sin(f*x + e)^7 - 528*a^2*sin(f*x + e)^5 - 880*a*b*sin(f*x + e)^5 - 385*b^2*sin(f*x + e)^5 + 624*a^2*sin(f*x + e)^3 + 1168*a*b*sin(f*x + e)^3 + 511*b^2*sin(f*x + e)^3 - 240*a^2*sin(f*x + e) - 528*a*b*sin(f*x + e) - 279*b^2*sin(f*x + e))/(sin(f*x + e)^2 - 1)^4)/f

Mupad [B] (verification not implemented)

Time = 18.83 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.03

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{\left(-\frac{3\,a^2}{8}-\frac{5\,a\,b}{8}-\frac{35\,b^2}{128}\right)\sin\left(e+f\,x\right)^7+\left(\frac{11\,a^2}{8}+\frac{55\,a\,b}{24}+\frac{385\,b^2}{384}\right)\sin\left(e+f\,x\right)^5+\left(-\frac{13\,a^2}{8}-\frac{73\,a\,b}{24}-\frac{511\,b^2}{384}\right)\sin\left(\cdots\right.}{f\,\left(\sin\left(e+f\,x\right)^8-4\sin\left(e+f\,x\right)^6+6\sin\left(e+f\,x\right)^4-4\sin\left(e+f\,x\cdots\right.\right.}$$
$$+\frac{\mathrm{atanh}(\sin\left(e+f\,x\right))\,\left(\frac{3\,a^2}{8}+\frac{5\,a\,b}{8}+\frac{35\,b^2}{128}\right)}{f}$$

[In] int((a + b/cos(e + f*x)^2)^2/cos(e + f*x)^5,x)

[Out] (sin(e + f*x)*((11*a*b)/8 + (5*a^2)/8 + (93*b^2)/128) - sin(e + f*x)^7*((5*a*b)/8 + (3*a^2)/8 + (35*b^2)/128) + sin(e + f*x)^5*((55*a*b)/24 + (11*a^2)/8 + (385*b^2)/384) - sin(e + f*x)^3*((73*a*b)/24 + (13*a^2)/8 + (511*b^2)/384))/(f*(6*sin(e + f*x)^4 - 4*sin(e + f*x)^2 - 4*sin(e + f*x)^6 + sin(e + f*x)^8 + 1)) + (atanh(sin(e + f*x))*((5*a*b)/8 + (3*a^2)/8 + (35*b^2)/128))/f

3.166 $\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 129

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{(8a^2+12ab+5b^2)\operatorname{arctanh}(\sin(e+fx))}{16f} + \frac{(8a^2+12ab+5b^2)\sec(e+fx)\tan(e+fx)}{16f} + \frac{b(8a+5b)\sec^3(e+fx)\tan(e+fx)}{24f} + \frac{b\sec^5(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{6f}$$

```
[Out] 1/16*(8*a^2+12*a*b+5*b^2)*arctanh(sin(f*x+e))/f+1/16*(8*a^2+12*a*b+5*b^2)*s
ec(f*x+e)*tan(f*x+e)/f+1/24*b*(8*a+5*b)*sec(f*x+e)^3*tan(f*x+e)/f+1/6*b*sec
(f*x+e)^5*(a+b-a*sin(f*x+e)^2)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4232, 424, 393, 205, 212}

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{(8a^2+12ab+5b^2)\operatorname{arctanh}(\sin(e+fx))}{16f} + \frac{(8a^2+12ab+5b^2)\tan(e+fx)\sec(e+fx)}{16f} + \frac{b(8a+5b)\tan(e+fx)\sec^3(e+fx)}{24f} + \frac{b\tan(e+fx)\sec^5(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{6f}$$

```
[In] Int[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 12*a*b + 5*b^2)*ArcTanh[Sin[e + f*x]])/(16*f) + ((8*a^2 + 12*a*b
+ 5*b^2)*Sec[e + f*x]*Tan[e + f*x])/(16*f) + (b*(8*a + 5*b)*Sec[e + f*x]^3*
Tan[e + f*x])/(24*f) + (b*Sec[e + f*x]^5*(a + b - a*Sin[e + f*x]^2)*Tan[e +
 f*x])/(6*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\left(a+b-ax^2\right)^2}{\left(1-x^2\right)^4}\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{b\sec^5(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{6f}\\
&\quad -\frac{\text{Subst}\left(\int \frac{-((a+b)(6a+5b))+3a(2a+b)x^2}{\left(1-x^2\right)^3}\,dx, x, \sin(e+fx)\right)}{6f}\\
&= \frac{b(8a+5b)\sec^3(e+fx)\tan(e+fx)}{24f}\\
&\quad +\frac{b\sec^5(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{6f}\\
&\quad +\frac{\left(8a^2+12ab+5b^2\right)\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2}\,dx, x, \sin(e+fx)\right)}{8f}\\
&= \frac{\left(8a^2+12ab+5b^2\right)\sec(e+fx)\tan(e+fx)}{16f}+\frac{b(8a+5b)\sec^3(e+fx)\tan(e+fx)}{24f}\\
&\quad +\frac{b\sec^5(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{6f}\\
&\quad +\frac{\left(8a^2+12ab+5b^2\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(e+fx)\right)}{16f}\\
&= \frac{\left(8a^2+12ab+5b^2\right)\text{arctanh}(\sin(e+fx))}{16f}\\
&\quad +\frac{\left(8a^2+12ab+5b^2\right)\sec(e+fx)\tan(e+fx)}{16f}\\
&\quad +\frac{b(8a+5b)\sec^3(e+fx)\tan(e+fx)}{24f}\\
&\quad +\frac{b\sec^5(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{6f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.45

$$
\begin{aligned}
&\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx\\
&=\frac{a^2\text{arctanh}(\sin(e+fx))}{2f}+\frac{3ab\text{arctanh}(\sin(e+fx))}{4f}+\frac{5b^2\text{arctanh}(\sin(e+fx))}{16f}\\
&\quad+\frac{a^2\sec(e+fx)\tan(e+fx)}{2f}+\frac{3ab\sec(e+fx)\tan(e+fx)}{4f}\\
&\quad+\frac{5b^2\sec(e+fx)\tan(e+fx)}{16f}+\frac{ab\sec^3(e+fx)\tan(e+fx)}{2f}\\
&\quad+\frac{5b^2\sec^3(e+fx)\tan(e+fx)}{24f}+\frac{b^2\sec^5(e+fx)\tan(e+fx)}{6f}
\end{aligned}
$$

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[e + f*x]])/(2*f) + (3*a*b*ArcTanh[Sin[e + f*x]])/(4*f) + (
5*b^2*ArcTanh[Sin[e + f*x]])/(16*f) + (a^2*Sec[e + f*x]*Tan[e + f*x])/(2*f)
 + (3*a*b*Sec[e + f*x]*Tan[e + f*x])/(4*f) + (5*b^2*Sec[e + f*x]*Tan[e + f*
x])/(16*f) + (a*b*Sec[e + f*x]^3*Tan[e + f*x])/(2*f) + (5*b^2*Sec[e + f*x]^
3*Tan[e + f*x])/(24*f) + (b^2*Sec[e + f*x]^5*Tan[e + f*x])/(6*f)
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{a^2\left(\frac{\tan(fx+e)\sec(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+2ab\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(}{8}\right.}{f}$
default	$\frac{a^2\left(\frac{\tan(fx+e)\sec(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+2ab\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(}{8}\right.}{f}$
parts	$\frac{a^2\left(\frac{\tan(fx+e)\sec(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}+\frac{b^2\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx+e)}{16}\right)\tan(fx+e)+\frac{5\ln}{}\right.}{f}$
parallelrisch	$\frac{-360\left(a^2+\frac{3}{2}ab+\frac{5}{8}b^2\right)\left(\frac{2}{3}+\frac{\cos(6fx+6e)}{15}+\frac{2\cos(4fx+4e)}{5}+\cos(2fx+2e)\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+360\left(a^2+\frac{3}{2}ab+\frac{5}{8}b^2\right)\left(\frac{2}{3}+\frac{\cos(}{}\right.}{48f(10}$
norman	$\frac{\frac{\left(8a^2+4ab+15b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{4f}+\frac{\left(8a^2+4ab+15b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{4f}+\frac{\left(8a^2+20ab+11b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8f}+\frac{\left(8a^2+20ab+11b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}{8f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^6}$
risch	$-\frac{ie^{i(fx+e)}\left(24a^2e^{10i(fx+e)}+36ab\,e^{10i(fx+e)}+15b^2e^{10i(fx+e)}+72a^2e^{8i(fx+e)}+204ab\,e^{8i(fx+e)}+85b^2e^{8i(fx+e)}+48a^2e^{6i}\right.}{}$

```
[In] int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(1/2*tan(f*x+e)*sec(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+2*a*b*(-
(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e))
)+b^2*(-(-1/6*sec(f*x+e)^5-5/24*sec(f*x+e)^3-5/16*sec(f*x+e))*tan(f*x+e)+5/
16*ln(sec(f*x+e)+tan(f*x+e))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.11

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{3\,(8\,a^2+12\,ab+5\,b^2)\cos\left(fx+e\right)^6\log\left(\sin\left(fx+e\right)+1\right)-3\,(8\,a^2+12\,ab+5\,b^2)\cos\left(fx+e\right)^6\log\left(-\sin\right.}{96\,f\cos\left(\right.}$$

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/96*(3*(8*a^2 + 12*a*b + 5*b^2)*cos(f*x + e)^6*log(sin(f*x + e) + 1) - 3*(
8*a^2 + 12*a*b + 5*b^2)*cos(f*x + e)^6*log(-sin(f*x + e) + 1) + 2*(3*(8*a^2
 + 12*a*b + 5*b^2)*cos(f*x + e)^4 + 2*(12*a*b + 5*b^2)*cos(f*x + e)^2 + 8*b
^2)*sin(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [F]

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^2\sec^3\left(e+fx\right)dx$$

```
[In] integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.29

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{3\,(8\,a^2+12\,ab+5\,b^2)\log\left(\sin\left(fx+e\right)+1\right)-3\,(8\,a^2+12\,ab+5\,b^2)\log\left(\sin\left(fx+e\right)-1\right)-\frac{2\left(3\,(8\,a^2+12\,ab\right.}{}}{96\,f}$$

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/96*(3*(8*a^2 + 12*a*b + 5*b^2)*log(sin(f*x + e) + 1) - 3*(8*a^2 + 12*a*b
+ 5*b^2)*log(sin(f*x + e) - 1) - 2*(3*(8*a^2 + 12*a*b + 5*b^2)*sin(f*x + e)
^5 - 8*(6*a^2 + 12*a*b + 5*b^2)*sin(f*x + e)^3 + 3*(8*a^2 + 20*a*b + 11*b^2
)*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1))
/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.42

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$

$$= \frac{3\left(8a^2+12ab+5b^2\right)\log\left(\left|\sin\left(fx+e\right)+1\right|\right)-3\left(8a^2+12ab+5b^2\right)\log\left(\left|\sin\left(fx+e\right)-1\right|\right)-\frac{2\left(24a^2\sin(fx\cdots\right.}{\cdots}}{\cdots}$$

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/96*(3*(8*a^2 + 12*a*b + 5*b^2)*log(abs(sin(f*x + e) + 1)) - 3*(8*a^2 + 12
*a*b + 5*b^2)*log(abs(sin(f*x + e) - 1)) - 2*(24*a^2*sin(f*x + e)^5 + 36*a*
b*sin(f*x + e)^5 + 15*b^2*sin(f*x + e)^5 - 48*a^2*sin(f*x + e)^3 - 96*a*b*s
in(f*x + e)^3 - 40*b^2*sin(f*x + e)^3 + 24*a^2*sin(f*x + e) + 60*a*b*sin(f*
x + e) + 33*b^2*sin(f*x + e))/(sin(f*x + e)^2 - 1)^3)/f
```

Mupad [B] (verification not implemented)

Time = 18.61 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.04

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\operatorname{atanh}(\sin\left(e+f\,x\right))\,\left(\frac{a^2}{2}+\frac{3\,a\,b}{4}+\frac{5\,b^2}{16}\right)}{f}$$
$$-\frac{\left(\frac{a^2}{2}+\frac{3\,a\,b}{4}+\frac{5\,b^2}{16}\right)\,\sin\left(e+f\,x\right)^5+\left(-a^2-2\,a\,b-\frac{5\,b^2}{6}\right)\,\sin\left(e+f\,x\right)^3+\left(\frac{a^2}{2}+\frac{5\,a\,b}{4}+\frac{11\,b^2}{16}\right)\,\sin\left(e+f\,x\right)}{f\,\left(\sin\left(e+f\,x\right)^6-3\,\sin\left(e+f\,x\right)^4+3\,\sin\left(e+f\,x\right)^2-1\right)}$$

```
[In] int((a + b/cos(e + f*x)^2)^2/cos(e + f*x)^3,x)
```

```
[Out] (atanh(sin(e + f*x))*((3*a*b)/4 + a^2/2 + (5*b^2)/16))/f - (sin(e + f*x)*((
5*a*b)/4 + a^2/2 + (11*b^2)/16) - sin(e + f*x)^3*(2*a*b + a^2 + (5*b^2)/6)
+ sin(e + f*x)^5*((3*a*b)/4 + a^2/2 + (5*b^2)/16))/(f*(3*sin(e + f*x)^2 - 3
*sin(e + f*x)^4 + sin(e + f*x)^6 - 1))
```

3.167 $\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 91

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{(8a^2+8ab+3b^2)\,\mathrm{arctanh}(\sin(e+fx))}{8f} + \frac{3b(2a+b)\sec(e+fx)\tan(e+fx)}{8f} + \frac{b\sec^3(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{4f}$$

```
[Out] 1/8*(8*a^2+8*a*b+3*b^2)*arctanh(sin(f*x+e))/f+3/8*b*(2*a+b)*sec(f*x+e)*tan(
f*x+e)/f+1/4*b*sec(f*x+e)^3*(a+b-a*sin(f*x+e)^2)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4232, 424, 393, 212}

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{(8a^2+8ab+3b^2)\,\mathrm{arctanh}(\sin(e+fx))}{8f} + \frac{3b(2a+b)\tan(e+fx)\sec(e+fx)}{8f} + \frac{b\tan(e+fx)\sec^3(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{4f}$$

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[Sin[e + f*x]])/(8*f) + (3*b*(2*a + b)*Sec[
e + f*x]*Tan[e + f*x])/(8*f) + (b*Sec[e + f*x]^3*(a + b - a*Sin[e + f*x]^2)
*Tan[e + f*x])/(4*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b-ax^2)^2}{(1-x^2)^3}\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{b\sec^3(e+fx)\,(a+b-a\sin^2(e+fx))\tan(e+fx)}{4f}\\
&\quad - \frac{\text{Subst}\left(\int \frac{-((a+b)(4a+3b))+a(4a+b)x^2}{(1-x^2)^2}\,dx, x, \sin(e+fx)\right)}{4f}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{3b(2a+b)\sec(e+fx)\tan(e+fx)}{8f} \\
&\quad + \frac{b\sec^3(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{4f} \\
&\quad + \frac{(8a^2+8ab+3b^2)\operatorname{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\sin(e+fx)\right)}{8f} \\
&= \frac{(8a^2+8ab+3b^2)\operatorname{arctanh}(\sin(e+fx))}{8f} + \frac{3b(2a+b)\sec(e+fx)\tan(e+fx)}{8f} \\
&\quad + \frac{b\sec^3(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{4f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.20

$$
\begin{aligned}
\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx &= \frac{a^2\operatorname{arctanh}(\sin(e+fx))}{f} + \frac{ab\operatorname{arctanh}(\sin(e+fx))}{f} \\
&\quad + \frac{3b^2\operatorname{arctanh}(\sin(e+fx))}{8f} \\
&\quad + \frac{ab\sec(e+fx)\tan(e+fx)}{f} \\
&\quad + \frac{3b^2\sec(e+fx)\tan(e+fx)}{8f} \\
&\quad + \frac{b^2\sec^3(e+fx)\tan(e+fx)}{4f}
\end{aligned}
$$

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[e + f*x]])/f + (a*b*ArcTanh[Sin[e + f*x]])/f + (3*b^2*ArcT
anh[Sin[e + f*x]])/(8*f) + (a*b*Sec[e + f*x]*Tan[e + f*x])/f + (3*b^2*Sec[e
 + f*x]*Tan[e + f*x])/(8*f) + (b^2*Sec[e + f*x]^3*Tan[e + f*x])/(4*f)
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{a^2\ln(\sec(fx+e)+\tan(fx+e))+2ab\left(\frac{\tan(fx+e)\sec(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+b^2\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\mathrm{t}\right.}{f}$
default	$\frac{a^2\ln(\sec(fx+e)+\tan(fx+e))+2ab\left(\frac{\tan(fx+e)\sec(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+b^2\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\mathrm{t}\right.}{f}$
parts	$\frac{a^2\ln(\sec(fx+e)+\tan(fx+e))}{f}+\frac{b^2\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)}{f}+\frac{ab\tan(}{}$
parallelrisch	$\frac{-4\left(a^2+ab+\frac{3}{8}b^2\right)\left(\frac{3}{4}+\frac{\cos(4fx+4e)}{4}+\cos(2fx+2e)\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+4\left(a^2+ab+\frac{3}{8}b^2\right)\left(\frac{3}{4}+\frac{\cos(4fx+4e)}{4}+\cos(2fx+2e)\right.}{f(\cos(4fx+4e)+4\cos(2fx+2e)+3)}$
norman	$\frac{-\frac{b(8a-3b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{4f}-\frac{b(8a-3b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{4f}+\frac{b(8a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}+\frac{b(8a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{4f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4}-\frac{(8a^2+8ab+3b^2)\ln\left(\tan\right.}{8f}$
risch	$-\frac{ib\,e^{i(fx+e)}\left(8a\,e^{6i(fx+e)}+3b\,e^{6i(fx+e)}+8a\,e^{4i(fx+e)}+11b\,e^{4i(fx+e)}-8a\,e^{2i(fx+e)}-11b\,e^{2i(fx+e)}-8a-3b\right)}{4f\left(e^{2i(fx+e)}+1\right)^4}-\frac{\ln(e^{i(fx}}{}$

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*ln(sec(f*x+e)+tan(f*x+e))+2*a*b*(1/2*tan(f*x+e)*sec(f*x+e)+1/2*ln(
sec(f*x+e)+tan(f*x+e)))+b^2*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)
+3/8*ln(sec(f*x+e)+tan(f*x+e))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.27

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{(8a^2+8ab+3b^2)\cos(fx+e)^4\log(\sin(fx+e)+1)-(8a^2+8ab+3b^2)\cos(fx+e)^4\log(-\sin(fx+e)}{16f\cos(fx+e)^4}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/16*((8*a^2 + 8*a*b + 3*b^2)*cos(f*x + e)^4*log(sin(f*x + e) + 1) - (8*a^2
 + 8*a*b + 3*b^2)*cos(f*x + e)^4*log(-sin(f*x + e) + 1) + 2*((8*a*b + 3*b^2
)*cos(f*x + e)^2 + 2*b^2)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\int \left(a+b\sec^2{(e+fx)}\right)^2\sec{(e+fx)}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2)**2,x)
[Out] Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.31

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{(8a^2+8ab+3b^2)\log(\sin(fx+e)+1)-(8a^2+8ab+3b^2)\log(\sin(fx+e)-1)-\frac{2\left((8ab+3b^2)\sin(fx+e)\cdots\right.}{\sin(fx+e)^4-\cdots}}{16f}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
[Out] 1/16*((8*a^2 + 8*a*b + 3*b^2)*log(sin(f*x + e) + 1) - (8*a^2 + 8*a*b + 3*b^
2)*log(sin(f*x + e) - 1) - 2*((8*a*b + 3*b^2)*sin(f*x + e)^3 - (8*a*b + 5*b
^2)*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.32

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{(8a^2+8ab+3b^2)\log(|\sin(fx+e)+1|)-(8a^2+8ab+3b^2)\log(|\sin(fx+e)-1|)-\frac{2\left(8ab\sin(fx+e)^3+\cdots\right.}{\cdots}}{16f}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
[Out] 1/16*((8*a^2 + 8*a*b + 3*b^2)*log(abs(sin(f*x + e) + 1)) - (8*a^2 + 8*a*b +
 3*b^2)*log(abs(sin(f*x + e) - 1)) - 2*(8*a*b*sin(f*x + e)^3 + 3*b^2*sin(f*
x + e)^3 - 8*a*b*sin(f*x + e) - 5*b^2*sin(f*x + e))/(sin(f*x + e)^2 - 1)^2)
/f
```

Mupad [B] (verification not implemented)

Time = 18.69 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{\operatorname{atanh}(\sin(e+f\,x))\,\left(a^2+a\,b+\frac{3\,b^2}{8}\right)}{f}$$
$$+\frac{\sin(e+f\,x)\,\left(\frac{5\,b^2}{8}+a\,b\right)-\sin(e+f\,x)^3\,\left(\frac{3\,b^2}{8}+a\,b\right)}{f\,\left(\sin(e+f\,x)^4-2\sin(e+f\,x)^2+1\right)}$$

```
[In] int((a + b/cos(e + f*x)^2)^2/cos(e + f*x),x)
```

```
[Out] (atanh(sin(e + f*x))*(a*b + a^2 + (3*b^2)/8))/f + (sin(e + f*x)*(a*b + (5*b
^2)/8) - sin(e + f*x)^3*(a*b + (3*b^2)/8))/(f*(sin(e + f*x)^4 - 2*sin(e + f
*x)^2 + 1))
```

3.168 $\int \cos(e+fx) \left(a+b\sec^2(e+fx)\right)^2 dx$

Optimal result

Integrand size = 21, antiderivative size = 56

$$\int \cos(e+fx) \left(a+b\sec^2(e+fx)\right)^2 dx = \frac{b(4a+b)\text{arctanh}(\sin(e+fx))}{2f} + \frac{a^2\sin(e+fx)}{f} + \frac{b^2\sec(e+fx)\tan(e+fx)}{2f}$$

```
[Out] 1/2*b*(4*a+b)*arctanh(sin(f*x+e))/f+a^2*sin(f*x+e)/f+1/2*b^2*sec(f*x+e)*tan
(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4232, 398, 393, 212}

$$\int \cos(e+fx) \left(a+b\sec^2(e+fx)\right)^2 dx = \frac{a^2\sin(e+fx)}{f} + \frac{b(4a+b)\text{arctanh}(\sin(e+fx))}{2f} + \frac{b^2\tan(e+fx)\sec(e+fx)}{2f}$$

```
[In] Int[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (b*(4*a + b)*ArcTanh[Sin[e + f*x]])/(2*f) + (a^2*Sin[e + f*x])/f + (b^2*Sec
[e + f*x]*Tan[e + f*x])/(2*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+b-ax^2)^2}{(1-x^2)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(a^2+\frac{b(2a+b)-2abx^2}{(1-x^2)^2}\right)\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{a^2\sin(e+fx)}{f} + \frac{\text{Subst}\left(\int \frac{b(2a+b)-2abx^2}{(1-x^2)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{a^2\sin(e+fx)}{f} + \frac{b^2\sec(e+fx)\tan(e+fx)}{2f} + \frac{(b(4a+b))\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(e+fx)\right)}{2f}$$

$$= \frac{b(4a+b)\text{arctanh}(\sin(e+fx))}{2f} + \frac{a^2\sin(e+fx)}{f} + \frac{b^2\sec(e+fx)\tan(e+fx)}{2f}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.43

$$\int \cos(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = \frac{2ab\text{arctanh}(\sin(e+fx))}{f} + \frac{b^2\text{arctanh}(\sin(e+fx))}{2f} + \frac{a^2\cos(fx)\sin(e)}{f} + \frac{a^2\cos(e)\sin(fx)}{f} + \frac{b^2\sec(e+fx)\tan(e+fx)}{2f}$$

```
[In] Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (2*a*b*ArcTanh[Sin[e + f*x]])/f + (b^2*ArcTanh[Sin[e + f*x]])/(2*f) + (a^2*
Cos[f*x]*Sin[e])/f + (a^2*Cos[e]*Sin[f*x])/f + (b^2*Sec[e + f*x]*Tan[e + f*
x])/(2*f)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{\sin(fx+e)a^2+2ab\ln(\sec(fx+e)+\tan(fx+e))+b^2\left(\frac{\tan(fx+e)\sec(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$
default	$\frac{\sin(fx+e)a^2+2ab\ln(\sec(fx+e)+\tan(fx+e))+b^2\left(\frac{\tan(fx+e)\sec(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$
parallelrisch	$\frac{-4b(1+\cos(2fx+2e))\left(a+\frac{b}{4}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+4b(1+\cos(2fx+2e))\left(a+\frac{b}{4}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\sin(3fx+3e)a^2+}{2f(1+\cos(2fx+2e))}$
risch	$-\frac{ia^2e^{i(fx+e)}}{2f}+\frac{ia^2e^{-i(fx+e)}}{2f}-\frac{ib^2\left(e^{3i(fx+e)}-e^{i(fx+e)}\right)}{f\left(e^{2i(fx+e)}+1\right)^2}-\frac{2\ln\left(e^{i(fx+e)}-i\right)ab}{f}-\frac{\ln\left(e^{i(fx+e)}-i\right)b^2}{2f}+\frac{2\ln(e^{i(}$
norman	$\frac{\frac{\left(2a^2+b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{f}+\frac{\left(6a^2-b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}-\frac{\left(2a^2+b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{\left(6a^2-b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}-\frac{b(4a+b)\ln\left(\tan\left(\frac{f}{2}\right.\right.}{2f}$

```
[In] int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(sin(f*x+e)*a^2+2*a*b*ln(sec(f*x+e)+tan(f*x+e))+b^2*(1/2*tan(f*x+e)*sec
(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e))))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.68

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{(4\,ab+b^2)\cos\left(fx+e\right)^2\log\left(\sin\left(fx+e\right)+1\right)-(4\,ab+b^2)\cos\left(fx+e\right)^2\log\left(-\sin\left(fx+e\right)+1\right)+2\left(2\,a^2\right.}{4\,f\cos\left(fx+e\right)^2}$$

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
[Out] 1/4*((4*a*b + b^2)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (4*a*b + b^2)*cos
(f*x + e)^2*log(-sin(f*x + e) + 1) + 2*(2*a^2*cos(f*x + e)^2 + b^2)*sin(f*x
 + e))/(f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^2\cos\left(e+fx\right)dx$$

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2)**2,x)
[Out] Integral((a + b*sec(e + f*x)**2)**2*cos(e + f*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.55

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=$$
$$-\frac{b^2\left(\frac{2\,\sin(fx+e)}{\sin(fx+e)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)-4\,ab(\log\left(\sin\left(fx+e\right)+1\right)-\log\left(\sin\right.}{4\,f}$$

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
[Out] -1/4*(b^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + lo
g(sin(f*x + e) - 1)) - 4*a*b*(log(sin(f*x + e) + 1) - log(sin(f*x + e) - 1)
) - 4*a^2*sin(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.41

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{4\,a^2\sin\left(fx+e\right)+\left(4\,ab+b^2\right)\log\left(\left|\sin\left(fx+e\right)+1\right|\right)-\left(4\,ab+b^2\right)\log\left(\left|\sin\left(fx+e\right)-1\right|\right)-\frac{2\,b^2\sin(fx+e)}{\sin(fx+e)^2-1}}{4\,f}$$

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*(4*a^2*sin(f*x + e) + (4*a*b + b^2)*log(abs(sin(f*x + e) + 1)) - (4*a*b
 + b^2)*log(abs(sin(f*x + e) - 1)) - 2*b^2*sin(f*x + e)/(sin(f*x + e)^2 - 1
))/f
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{a^2\,\sin\left(e+f\,x\right)+\frac{b\,\mathrm{atanh}\left(\sin\left(e+f\,x\right)\right)\,\left(4\,a+b\right)}{2}-\frac{b^2\,\sin\left(e+f\,x\right)}{2\,\left({\sin\left(e+f\,x\right)}^2-1\right)}}{f}$$

```
[In] int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (a^2*sin(e + f*x) + (b*atanh(sin(e + f*x))*(4*a + b))/2 - (b^2*sin(e + f*x)
)/(2*(sin(e + f*x)^2 - 1)))/f
```

3.169 $\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 49

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{b^2\operatorname{arctanh}(\sin(e+fx))}{f}+\frac{a(a+2b)\sin(e+fx)}{f}-\frac{a^2\sin^3(e+fx)}{3f}$$

```
[Out] b^2*arctanh(sin(f*x+e))/f+a*(a+2*b)*sin(f*x+e)/f-1/3*a^2*sin(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4232, 398, 212}

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=-\frac{a^2\sin^3(e+fx)}{3f}+\frac{a(a+2b)\sin(e+fx)}{f}+\frac{b^2\operatorname{arctanh}(\sin(e+fx))}{f}$$

```
[In] Int[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (b^2*ArcTanh[Sin[e + f*x]])/f + (a*(a + 2*b)*Sin[e + f*x])/f - (a^2*Sin[e +
 f*x]^3)/(3*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b-ax^2)^2}{1-x^2}\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(a(a+2b)-a^2x^2+\frac{b^2}{1-x^2}\right)\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{a(a+2b)\sin(e+fx)}{f}-\frac{a^2\sin^3(e+fx)}{3f}+\frac{b^2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{b^2\text{arctanh}(\sin(e+fx))}{f}+\frac{a(a+2b)\sin(e+fx)}{f}-\frac{a^2\sin^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.47

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{b^2\text{arctanh}(\sin(e+fx))}{f} + \frac{2ab\cos(fx)\sin(e)}{f} + \frac{2ab\cos(e)\sin(fx)}{f} + \frac{a^2\sin(e+fx)}{f} - \frac{a^2\sin^3(e+fx)}{3f}$$

```
[In] Integrate[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (b^2*ArcTanh[Sin[e + f*x]])/f + (2*a*b*Cos[f*x]*Sin[e])/f + (2*a*b*Cos[e]*S
in[f*x])/f + (a^2*Sin[e + f*x])/f - (a^2*Sin[e + f*x]^3)/(3*f)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{\frac{a^{2}\left(\cos(fx+e)^{2}+2\right)\sin(fx+e)}{3}+2\sin(fx+e)ab+b^{2}\ln(\sec(fx+e)+\tan(fx+e))}{f}$
default	$\frac{\frac{a^{2}\left(\cos(fx+e)^{2}+2\right)\sin(fx+e)}{3}+2\sin(fx+e)ab+b^{2}\ln(\sec(fx+e)+\tan(fx+e))}{f}$
parallelrisch	$\frac{-12\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)b^{2}+12\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)b^{2}+\sin(3fx+3e)a^{2}+9\left(a+\frac{8b}{3}\right)a\sin(fx+e)}{12f}$
risch	$-\frac{3ia^{2}e^{i(fx+e)}}{8f}-\frac{ie^{i(fx+e)}ab}{f}+\frac{3ia^{2}e^{-i(fx+e)}}{8f}+\frac{ie^{-i(fx+e)}ab}{f}+\frac{\ln(e^{i(fx+e)}+i)b^{2}}{f}-\frac{\ln(e^{i(fx+e)}-i)b^{2}}{f}+\frac{a^{2}s}{}$
norman	$\frac{-\frac{4a(a-2b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{f}+\frac{4a(a-2b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{f}-\frac{2a(a+2b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2a(a+2b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{f}+\frac{2a(7a+6b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3f}-}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)^{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{3}}$

```
[In] int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/f*(1/3*a^2*(cos(f*x+e)^2+2)*sin(f*x+e)+2*sin(f*x+e)*a*b+b^2*ln(sec(f*x+e)
+tan(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.35

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{3\,b^2\log\left(\sin\left(fx+e\right)+1\right)-3\,b^2\log\left(-\sin\left(fx+e\right)+1\right)+2\left(a^2\cos\left(fx+e\right)^2+2\,a^2+6\,ab\right)\sin\left(fx+e\right)}{6\,f}$$

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
[Out] 1/6*(3*b^2*log(sin(f*x + e) + 1) - 3*b^2*log(-sin(f*x + e) + 1) + 2*(a^2*co
s(f*x + e)^2 + 2*a^2 + 6*a*b)*sin(f*x + e))/f
```

Sympy [F]

$$\int \cos^3(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = \int \left(a+b\sec^2{(e+fx)}\right)^2 \cos^3{(e+fx)} \, dx$$

```
[In] integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*cos(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.29

$$\int \cos^3(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = -\frac{2\,a^2\sin\left(fx+e\right)^3 - 3\,b^2\log\left(\sin\left(fx+e\right)+1\right) + 3\,b^2\log\left(\sin\left(fx+e\right)-1\right) - 6\,(a^2+2\,ab)\sin\left(fx+e\right)}{6\,f}$$

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^2*sin(f*x + e)^3 - 3*b^2*log(sin(f*x + e) + 1) + 3*b^2*log(sin(f*
x + e) - 1) - 6*(a^2 + 2*a*b)*sin(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\int \cos^3(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = -\frac{2\,a^2\sin\left(fx+e\right)^3 - 3\,b^2\log\left(|\sin\left(fx+e\right)+1|\right) + 3\,b^2\log\left(|\sin\left(fx+e\right)-1|\right) - 6\,a^2\sin\left(fx+e\right) - 12\,ab}{6\,f}$$

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(2*a^2*sin(f*x + e)^3 - 3*b^2*log(abs(sin(f*x + e) + 1)) + 3*b^2*log(a
bs(sin(f*x + e) - 1)) - 6*a^2*sin(f*x + e) - 12*a*b*sin(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 17.97 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \cos^3(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = -\frac{\sin\left(e+f\,x\right)\,\left(a^2-2\,a\,\left(a+b\right)\right)+\frac{a^2\,\sin\left(e+f\,x\right)^3}{3}-b^2\,\mathrm{atanh}(\sin\left(e+f\,x\right))}{f}$$

```
[In] int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] -(sin(e + f*x)*(a^2 - 2*a*(a + b)) + (a^2*sin(e + f*x)^3)/3 - b^2*atanh(sin
(e + f*x)))/f
```

3.170 $\int \cos^5(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \cos^5(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = \frac{(a+b)^2 \sin(e+fx)}{f} - \frac{2a(a+b)\sin^3(e+fx)}{3f} + \frac{a^2 \sin^5(e+fx)}{5f}$$

```
[Out] (a+b)^2*sin(f*x+e)/f-2/3*a*(a+b)*sin(f*x+e)^3/f+1/5*a^2*sin(f*x+e)^5/f
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4232, 200}

$$\int \cos^5(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = \frac{a^2 \sin^5(e+fx)}{5f} - \frac{2a(a+b)\sin^3(e+fx)}{3f} + \frac{(a+b)^2 \sin(e+fx)}{f}$$

```
[In] Int[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Sin[e + f*x])/f - (2*a*(a + b)*Sin[e + f*x]^3)/(3*f) + (a^2*Sin[
e + f*x]^5)/(5*f)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (a+b-ax^2)^2\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(a^2\left(1+\frac{b(2a+b)}{a^2}\right)-2a^2\left(1+\frac{b}{a}\right)x^2+a^2x^4\right)\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{(a+b)^2\sin(e+fx)}{f}-\frac{2a(a+b)\sin^3(e+fx)}{3f}+\frac{a^2\sin^5(e+fx)}{5f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.00

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{b^2\cos(fx)\sin(e)}{f}+\frac{b^2\cos(e)\sin(fx)}{f}+\frac{a^2\sin(e+fx)}{f}+\frac{2ab\sin(e+fx)}{f}-\frac{2a^2\sin^3(e+fx)}{3f}-\frac{2ab\sin^3(e+fx)}{3f}+\frac{a^2\sin^5(e+fx)}{5f}$$

```
[In] Integrate[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (b^2*Cos[f*x]*Sin[e])/f + (b^2*Cos[e]*Sin[f*x])/f + (a^2*Sin[e + f*x])/f +
(2*a*b*Sin[e + f*x])/f - (2*a^2*Sin[e + f*x]^3)/(3*f) - (2*a*b*Sin[e + f*x]
^3)/(3*f) + (a^2*Sin[e + f*x]^5)/(5*f)
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{\frac{a^2\left(\frac{8}{3}+\cos(fx+e)^4+\frac{4\cos(fx+e)^2}{3}\right)\sin(fx+e)}{5}+\frac{2ab\left(\cos(fx+e)^2+2\right)\sin(fx+e)}{3}+\sin(fx+e)b^2}{f}$
default	$\frac{\frac{a^2\left(\frac{8}{3}+\cos(fx+e)^4+\frac{4\cos(fx+e)^2}{3}\right)\sin(fx+e)}{5}+\frac{2ab\left(\cos(fx+e)^2+2\right)\sin(fx+e)}{3}+\sin(fx+e)b^2}{f}$
parallelrisch	$\frac{150\sin(fx+e)a^2+360\sin(fx+e)ab+240\sin(fx+e)b^2+3\sin(5fx+5e)a^2+25\sin(3fx+3e)a^2+40\sin(3fx+3e)ab}{240f}$
risch	$\frac{5a^2\sin(fx+e)}{8f}+\frac{3\sin(fx+e)ab}{2f}+\frac{\sin(fx+e)b^2}{f}+\frac{a^2\sin(5fx+5e)}{80f}+\frac{5a^2\sin(3fx+3e)}{48f}+\frac{\sin(3fx+3e)ab}{6f}$
norman	$\frac{-\frac{2\left(a^2+2ab+b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2\left(a^2+2ab+b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{15}}{f}+\frac{2\left(5a^2+2ab-3b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}-\frac{2\left(5a^2+2ab-3b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}{3f}}{\left(1+\tan\left(\right.\right.}$

[In] int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/f*(1/5*a^2*(8/3+cos(f*x+e)^4+4/3*cos(f*x+e)^2)*sin(f*x+e)+2/3*a*b*(cos(f*x+e)^2+2)*sin(f*x+e)+sin(f*x+e)*b^2)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\left(3\,a^2\cos\left(fx+e\right)^4+2\left(2\,a^2+5\,ab\right)\cos\left(fx+e\right)^2+8\,a^2+20\,ab+15\,b^2\right)\sin\left(fx+e\right)}{15\,f}$$

[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")

[Out] 1/15*(3*a^2*cos(f*x + e)^4 + 2*(2*a^2 + 5*a*b)*cos(f*x + e)^2 + 8*a^2 + 20*a*b + 15*b^2)*sin(f*x + e)/f

Sympy [F(-1)]

Timed out.

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{3\,a^2\sin\left(fx+e\right)^5 - 10\left(a^2+ab\right)\sin\left(fx+e\right)^3 + 15\left(a^2+2\,ab+b^2\right)\sin\left(fx+e\right)}{15\,f}$$

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(3*a^2*sin(f*x + e)^5 - 10*(a^2 + a*b)*sin(f*x + e)^3 + 15*(a^2 + 2*a*
b + b^2)*sin(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.43

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{3\,a^2\sin\left(fx+e\right)^5 - 10\,a^2\sin\left(fx+e\right)^3 - 10\,ab\sin\left(fx+e\right)^3 + 15\,a^2\sin\left(fx+e\right) + 30\,ab\sin\left(fx+e\right) + 15\,b}{15\,f}$$

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*a^2*sin(f*x + e)^5 - 10*a^2*sin(f*x + e)^3 - 10*a*b*sin(f*x + e)^3
+ 15*a^2*sin(f*x + e) + 30*a*b*sin(f*x + e) + 15*b^2*sin(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 18.54 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \cos^5(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = \frac{\sin\left(e+f\,x\right)\,\left(a+b\right)^2+\frac{a^2\,\sin\left(e+f\,x\right)^5}{5}-\frac{2\,a\,\sin\left(e+f\,x\right)^3\,\left(a+b\right)}{3}}{f}$$

```
[In] int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^2,x)
[Out] (sin(e + f*x)*(a + b)^2 + (a^2*sin(e + f*x)^5)/5 - (2*a*sin(e + f*x)^3*(a +
 b))/3)/f
```

3.171 $\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 106

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{(a+b)^2\tan(e+fx)}{f}+\frac{2(a+b)(a+2b)\tan^3(e+fx)}{3f}+\frac{(a^2+6ab+6b^2)\tan^5(e+fx)}{5f}+\frac{2b(a+2b)\tan^7(e+fx)}{7f}+\frac{b^2\tan^9(e+fx)}{9f}$$

```
[Out] (a+b)^2*tan(f*x+e)/f+2/3*(a+b)*(a+2*b)*tan(f*x+e)^3/f+1/5*(a^2+6*a*b+6*b^2)
*tan(f*x+e)^5/f+2/7*b*(a+2*b)*tan(f*x+e)^7/f+1/9*b^2*tan(f*x+e)^9/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4231, 380}

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{(a^2+6ab+6b^2)\tan^5(e+fx)}{5f}+\frac{2b(a+2b)\tan^7(e+fx)}{7f}+\frac{2(a+b)(a+2b)\tan^3(e+fx)}{3f}+\frac{(a+b)^2\tan(e+fx)}{f}+\frac{b^2\tan^9(e+fx)}{9f}$$

```
[In] Int[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Tan[e + f*x])/f + (2*(a + b)*(a + 2*b)*Tan[e + f*x]^3)/(3*f) + (
(a^2 + 6*a*b + 6*b^2)*Tan[e + f*x]^5)/(5*f) + (2*b*(a + 2*b)*Tan[e + f*x]^7
)/(7*f) + (b^2*Tan[e + f*x]^9)/(9*f)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (1+x^2)^2 \left(a+b+bx^2\right)^2 \, dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left((a+b)^2+2(a+b)(a+2b)x^2+\left(a^2+6ab+6b^2\right)x^4+2b(a+2b)x^6+b^2x^8\right) \, dx, x, \tan(e\right.}{f}$$

$$= \frac{(a+b)^2\tan(e+fx)}{f} + \frac{2(a+b)(a+2b)\tan^3(e+fx)}{3f} + \frac{\left(a^2+6ab+6b^2\right)\tan^5(e+fx)}{5f} + \frac{2b(a+2b)\tan^7(e+fx)}{7f} + \frac{b^2\tan^9(e+fx)}{9f}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2 \, dx = \frac{315(a+b)^2\tan(e+fx)+210\left(a^2+3ab+2b^2\right)\tan^3(e+fx)+63\left(a^2+6ab+6b^2\right)\tan^5(e+fx)+90b(a-}{315f}$$

```
[In] Integrate[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (315*(a + b)^2*Tan[e + f*x] + 210*(a^2 + 3*a*b + 2*b^2)*Tan[e + f*x]^3 + 63
*(a^2 + 6*a*b + 6*b^2)*Tan[e + f*x]^5 + 90*b*(a + 2*b)*Tan[e + f*x]^7 + 35*
b^2*Tan[e + f*x]^9)/(315*f)
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{-a^2\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)-2ab\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(fx+e)^4}{35}-\frac{8\sec(fx+e)^2}{35}\right)\tan(fx+e)-b^2\left(\right.}{f}$
default	$\frac{-a^2\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)-2ab\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(fx+e)^4}{35}-\frac{8\sec(fx+e)^2}{35}\right)\tan(fx+e)-b^2\left(\right.}{f}$
parts	$-\frac{a^2\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f}-\frac{b^2\left(-\frac{128}{315}-\frac{\sec(fx+e)^8}{9}-\frac{8\sec(fx+e)^6}{63}-\frac{16\sec(fx+e)^4}{105}-\frac{64\sec(fx+e)}{315}\right.}{f}$
parallelrisch	$\frac{(10752a^2+24192ab+10752b^2)\sin(3fx+3e)+(6048a^2+10368ab+4608b^2)\sin(5fx+5e)+(1512a^2+2592ab+1152b^2)\sin(7f}{315f(\cos(9fx+9e)+9\cos(7fx+7e)+36\cos(5fx+5e)+84\cos(3fx}$
risch	$\frac{16i(210a^2e^{12i(fx+e)}+945a^2e^{10i(fx+e)}+1260ab\,e^{10i(fx+e)}+1701a^2e^{8i(fx+e)}+3276ab\,e^{8i(fx+e)}+2016b^2e^{8i(fx+e)}+1554a}{}$

```
[In] int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a^2*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)-2*a*b*(-16/
35-1/7*sec(f*x+e)^6-6/35*sec(f*x+e)^4-8/35*sec(f*x+e)^2)*tan(f*x+e)-b^2*(-1
28/315-1/9*sec(f*x+e)^8-8/63*sec(f*x+e)^6-16/105*sec(f*x+e)^4-64/315*sec(f*
x+e)^2)*tan(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.13

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{\left(8\left(21\,a^2+36\,ab+16\,b^2\right)\cos\left(fx+e\right)^8+4\left(21\,a^2+36\,ab+16\,b^2\right)\cos\left(fx+e\right)^6+3\left(21\,a^2+36\,ab+16\,b^2\right)\right.}{315\,f\cos\left(fx+e\right)^9}$$

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/315*(8*(21*a^2 + 36*a*b + 16*b^2)*cos(f*x + e)^8 + 4*(21*a^2 + 36*a*b + 1
6*b^2)*cos(f*x + e)^6 + 3*(21*a^2 + 36*a*b + 16*b^2)*cos(f*x + e)^4 + 10*(9
*a*b + 4*b^2)*cos(f*x + e)^2 + 35*b^2)*sin(f*x + e)/(f*cos(f*x + e)^9)
```

Sympy [F]

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^2\sec^6\left(e+fx\right)dx$$

```
[In] integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.97

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{35\,b^2\tan\left(fx+e\right)^9+90\,(ab+2\,b^2)\tan\left(fx+e\right)^7+63\,(a^2+6\,ab+6\,b^2)\tan\left(fx+e\right)^5+210\,(a^2+3\,ab+2}{315\,f}$$

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/315*(35*b^2*tan(f*x + e)^9 + 90*(a*b + 2*b^2)*tan(f*x + e)^7 + 63*(a^2 +
6*a*b + 6*b^2)*tan(f*x + e)^5 + 210*(a^2 + 3*a*b + 2*b^2)*tan(f*x + e)^3 +
315*(a^2 + 2*a*b + b^2)*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.43

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{35\,b^2\tan\left(fx+e\right)^9+90\,ab\tan\left(fx+e\right)^7+180\,b^2\tan\left(fx+e\right)^7+63\,a^2\tan\left(fx+e\right)^5+378\,ab\tan\left(fx+e\right.}{}$$

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/315*(35*b^2*tan(f*x + e)^9 + 90*a*b*tan(f*x + e)^7 + 180*b^2*tan(f*x + e)
^7 + 63*a^2*tan(f*x + e)^5 + 378*a*b*tan(f*x + e)^5 + 378*b^2*tan(f*x + e)^
5 + 210*a^2*tan(f*x + e)^3 + 630*a*b*tan(f*x + e)^3 + 420*b^2*tan(f*x + e)^
3 + 315*a^2*tan(f*x + e) + 630*a*b*tan(f*x + e) + 315*b^2*tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 18.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{\tan(e+f\,x)\,\left(a+b\right)^2+\frac{b^2\tan(e+f\,x)^9}{9}+\tan(e+f\,x)^3\,\left(\frac{2\,a^2}{3}+2\,a\,b+\frac{4\,b^2}{3}\right)+\tan(e+f\,x)^5\,\left(\frac{a^2}{5}+\frac{6\,a\,b}{5}+\frac{6\,b^2}{5}\right)}{f}$$

```
[In] int((a + b/cos(e + f*x)^2)^2/cos(e + f*x)^6,x)
```

```
[Out] (tan(e + f*x)*(a + b)^2 + (b^2*tan(e + f*x)^9)/9 + tan(e + f*x)^3*(2*a*b +
(2*a^2)/3 + (4*b^2)/3) + tan(e + f*x)^5*((6*a*b)/5 + a^2/5 + (6*b^2)/5) + (
2*b*tan(e + f*x)^7*(a + 2*b))/7)/f
```

3.172 $\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 80

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{(a+b)^2\tan(e+fx)}{f}+\frac{(a+b)(a+3b)\tan^3(e+fx)}{3f}+\frac{b(2a+3b)\tan^5(e+fx)}{5f}+\frac{b^2\tan^7(e+fx)}{7f}$$

```
[Out] (a+b)^2*tan(f*x+e)/f+1/3*(a+b)*(a+3*b)*tan(f*x+e)^3/f+1/5*b*(2*a+3*b)*tan(f
*x+e)^5/f+1/7*b^2*tan(f*x+e)^7/f
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4231, 380}

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{b(2a+3b)\tan^5(e+fx)}{5f}+\frac{(a+b)(a+3b)\tan^3(e+fx)}{3f}+\frac{(a+b)^2\tan(e+fx)}{f}+\frac{b^2\tan^7(e+fx)}{7f}$$

```
[In] Int[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Tan[e + f*x])/f + ((a + b)*(a + 3*b)*Tan[e + f*x]^3)/(3*f) + (b*
(2*a + 3*b)*Tan[e + f*x]^5)/(5*f) + (b^2*Tan[e + f*x]^7)/(7*f)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int (1+x^2)\,(a+b+bx^2)^2\,dx, x, \tan(e+fx)\right)}{f} \\ &= \frac{\text{Subst}\left(\int \left((a+b)^2+(a+b)(a+3b)x^2+b(2a+3b)x^4+b^2x^6\right)\,dx, x, \tan(e+fx)\right)}{f} \\ &= \frac{(a+b)^2\tan(e+fx)}{f}+\frac{(a+b)(a+3b)\tan^3(e+fx)}{3f} \\ &\quad+\frac{b(2a+3b)\tan^5(e+fx)}{5f}+\frac{b^2\tan^7(e+fx)}{7f} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{105(a+b)^2\tan(e+fx)+35(a^2+4ab+3b^2)\tan^3(e+fx)+21b(2a+3b)\tan^5(e+fx)+15b^2\tan^7(e+f}{105f}$$

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (105*(a + b)^2*Tan[e + f*x] + 35*(a^2 + 4*a*b + 3*b^2)*Tan[e + f*x]^3 + 21*
b*(2*a + 3*b)*Tan[e + f*x]^5 + 15*b^2*Tan[e + f*x]^7)/(105*f)
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{-a^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-2ab\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)-b^2\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(fx}{35}\right.}{f}$
default	$\frac{-a^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-2ab\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)-b^2\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(fx}{35}\right.}{f}$
parts	$-\frac{a^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}-\frac{b^2\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(fx+e)^4}{35}-\frac{8\sec(fx+e)^2}{35}\right)\tan(fx+e)}{f}-\frac{2ab\left(-\frac{8}{15}-\right.}{}$
parallelrisch	$\frac{(1050a^2+2352ab+1008b^2)\sin(3fx+3e)+(490a^2+784ab+336b^2)\sin(5fx+5e)+(70a^2+112ab+48b^2)\sin(7fx+7e)+630}{105f(\cos(7fx+7e)+7\cos(5fx+5e)+21\cos(3fx+3e)+35\cos(fx+e))}$
risch	$\frac{4i(105a^2e^{10i(fx+e)}+455a^2e^{8i(fx+e)}+560ab\,e^{8i(fx+e)}+770a^2e^{6i(fx+e)}+1400ab\,e^{6i(fx+e)}+840b^2e^{6i(fx+e)}+630a^2e^{4i(f}}{105f(e^{2i(fx+}}$
norman	$\frac{-\frac{2(a^2+2ab+b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2(a^2+2ab+b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{13}}{f}+\frac{4(7a^2+10ab+3b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}+\frac{4(7a^2+10ab+3b^2)\tan\left(\frac{fx}{2}+\right.}{3f}}{\left(\tan\left(\frac{fx}{2}+\right.\right.}$

[In] int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/f*(-a^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-2*a*b*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)-b^2*(-16/35-1/7*sec(f*x+e)^6-6/35*sec(f*x+e)^4-8/35*sec(f*x+e)^2)*tan(f*x+e))

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.18

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx = \frac{(2(35a^2+56ab+24b^2)\cos(fx+e)^6+(35a^2+56ab+24b^2)\cos(fx+e)^4+6(7ab+3b^2)\cos(fx+e}{105f\cos(fx+e)^7}$$

[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")

[Out] 1/105*(2*(35*a^2 + 56*a*b + 24*b^2)*cos(f*x + e)^6 + (35*a^2 + 56*a*b + 24*b^2)*cos(f*x + e)^4 + 6*(7*a*b + 3*b^2)*cos(f*x + e)^2 + 15*b^2)*sin(f*x + e)/(f*cos(f*x + e)^7)

Sympy [F]

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^2\sec^4\left(e+fx\right)dx$$

[In] integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2)**2,x)

[Out] Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**4, x)

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{15\,b^2\tan\left(fx+e\right)^7+21\left(2\,ab+3\,b^2\right)\tan\left(fx+e\right)^5+35\left(a^2+4\,ab+3\,b^2\right)\tan\left(fx+e\right)^3+105\left(a^2+2\,ab+\right.}{105\,f}$$

[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")

[Out] 1/105*(15*b^2*tan(f*x + e)^7 + 21*(2*a*b + 3*b^2)*tan(f*x + e)^5 + 35*(a^2 + 4*a*b + 3*b^2)*tan(f*x + e)^3 + 105*(a^2 + 2*a*b + b^2)*tan(f*x + e))/f

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.42

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{15\,b^2\tan\left(fx+e\right)^7+42\,ab\tan\left(fx+e\right)^5+63\,b^2\tan\left(fx+e\right)^5+35\,a^2\tan\left(fx+e\right)^3+140\,ab\tan\left(fx+e\right)^3}{105\,f}$$

[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")

[Out] 1/105*(15*b^2*tan(f*x + e)^7 + 42*a*b*tan(f*x + e)^5 + 63*b^2*tan(f*x + e)^5 + 35*a^2*tan(f*x + e)^3 + 140*a*b*tan(f*x + e)^3 + 105*b^2*tan(f*x + e)^3 + 105*a^2*tan(f*x + e) + 210*a*b*tan(f*x + e) + 105*b^2*tan(f*x + e))/f

Mupad [B] (verification not implemented)

Time = 18.59 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{\tan(e+f\,x)\,\left(a+b\right)^2+\tan(e+f\,x)^3\,\left(\frac{a^2}{3}+\frac{4\,a\,b}{3}+b^2\right)+\frac{b^2\,\tan(e+f\,x)^7}{7}+\frac{b\,\tan(e+f\,x)^5\,\left(2\,a+3\,b\right)}{5}}{f}$$

```
[In] int((a + b/cos(e + f*x)^2)^2/cos(e + f*x)^4,x)
[Out] (tan(e + f*x)*(a + b)^2 + tan(e + f*x)^3*((4*a*b)/3 + a^2/3 + b^2) + (b^2*t
an(e + f*x)^7)/7 + (b*tan(e + f*x)^5*(2*a + 3*b))/5)/f
```

3.173 $\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{(a+b)^2\tan(e+fx)}{f}+\frac{2b(a+b)\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+fx)}{5f}$$

```
[Out] (a+b)^2*tan(f*x+e)/f+2/3*b*(a+b)*tan(f*x+e)^3/f+1/5*b^2*tan(f*x+e)^5/f
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4231, 200}

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{2b(a+b)\tan^3(e+fx)}{3f}+\frac{(a+b)^2\tan(e+fx)}{f}+\frac{b^2\tan^5(e+fx)}{5f}$$

```
[In] Int[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Tan[e + f*x])/f + (2*b*(a + b)*Tan[e + f*x]^3)/(3*f) + (b^2*Tan[
e + f*x]^5)/(5*f)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (a+b+bx^2)^2\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(a^2\left(1+\frac{b(2a+b)}{a^2}\right)+2ab\left(1+\frac{b}{a}\right)x^2+b^2x^4\right)\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{(a+b)^2\tan(e+fx)}{f}+\frac{2b(a+b)\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+fx)}{5f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$
\begin{aligned}
&\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx\\
&= \frac{15(a+b)^2\tan(e+fx)+10b(a+b)\tan^3(e+fx)+3b^2\tan^5(e+fx)}{15f}
\end{aligned}
$$

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
[Out] (15*(a + b)^2*Tan[e + f*x] + 10*b*(a + b)*Tan[e + f*x]^3 + 3*b^2*Tan[e + f*
x]^5)/(15*f)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{a^2\tan(fx+e)-2ab\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-b^2\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f}$
default	$\frac{a^2\tan(fx+e)-2ab\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-b^2\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f}$
parts	$\frac{a^2\tan(fx+e)}{f}-\frac{b^2\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f}-\frac{2ab\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}$
parallelrisch	$\frac{(45a^2+100ab+40b^2)\sin(3fx+3e)+(15a^2+20ab+8b^2)\sin(5fx+5e)+30\sin(fx+e)\left(a^2+\frac{8}{3}ab+\frac{8}{3}b^2\right)}{15f(\cos(5fx+5e)+5\cos(3fx+3e)+10\cos(fx+e))}$
risch	$\frac{2i(15a^2e^{8i(fx+e)}+60a^2e^{6i(fx+e)}+60ab\,e^{6i(fx+e)}+90a^2e^{4i(fx+e)}+140ab\,e^{4i(fx+e)}+80b^2e^{4i(fx+e)}+60a^2e^{2i(fx+e)}+100a}{15f\left(e^{2i(fx+e)}+1\right)^5}$
norman	$\frac{-\frac{2(a^2+2ab+b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2(a^2+2ab+b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{f}+\frac{8(3a^2+4ab+b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}+\frac{8(3a^2+4ab+b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{3f}-}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^5}$

```
[In] int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*tan(f*x+e)-2*a*b*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-b^2*(-8/15-1/5
*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.30

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\left((15\,a^2+20\,ab+8\,b^2)\cos\left(fx+e\right)^4+2\,(5\,ab+2\,b^2)\cos\left(fx+e\right)^2+3\,b^2\right)\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)^5}$$

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*((15*a^2 + 20*a*b + 8*b^2)*cos(f*x + e)^4 + 2*(5*a*b + 2*b^2)*cos(f*x
+ e)^2 + 3*b^2)*sin(f*x + e)/(f*cos(f*x + e)^5)
```

Sympy [F]

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\int \left(a+b\sec^2{(e+fx)}\right)^2\sec^2{(e+fx)}\,dx$$

```
[In] integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2)**2,x)
[Out] Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.34

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{10\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)ab+\left(3\,\tan\left(fx+e\right)^5+10\,\tan\left(fx+e\right)^3+15\,\tan\left(fx+e\right)\right)b^2+15\,a}{15\,f}$$

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
[Out] 1/15*(10*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*b + (3*tan(f*x + e)^5 + 10*tan
(f*x + e)^3 + 15*tan(f*x + e))*b^2 + 15*a^2*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.43

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{3\,b^2\tan\left(fx+e\right)^5+10\,ab\tan\left(fx+e\right)^3+10\,b^2\tan\left(fx+e\right)^3+15\,a^2\tan\left(fx+e\right)+30\,ab\tan\left(fx+e\right)+}{15\,f}$$

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
[Out] 1/15*(3*b^2*tan(f*x + e)^5 + 10*a*b*tan(f*x + e)^3 + 10*b^2*tan(f*x + e)^3
+ 15*a^2*tan(f*x + e) + 30*a*b*tan(f*x + e) + 15*b^2*tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 18.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\tan(e+f\,x)\,\left(a+b\right)^2+\frac{b^2\tan(e+f\,x)^5}{5}+\frac{2\,b\tan(e+f\,x)^3\,\left(a+b\right)}{3}}{f}$$

```
[In] int((a + b/cos(e + f*x)^2)^2/cos(e + f*x)^2,x)
[Out] (tan(e + f*x)*(a + b)^2 + (b^2*tan(e + f*x)^5)/5 + (2*b*tan(e + f*x)^3*(a +
 b))/3)/f
```

3.174 $\int \left(a+b\sec^2(e+fx)\right)^2 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 40

$$\int \left(a+b\sec^2(e+fx)\right)^2 \, dx = a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] a^2*x+b*(2*a+b)*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4213, 398, 209}

$$\int \left(a+b\sec^2(e+fx)\right)^2 \, dx = a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] a^2*x + (b*(2*a + b)*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(b(2a+b)+b^2x^2+\frac{a^2}{1+x^2}\right)\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}+\frac{a^2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{3a^2fx+3b(2a+b)\tan(e+fx)+b^2\tan^3(e+fx)}{3f}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2,x]
[Out] (3*a^2*f*x + 3*b*(2*a + b)*Tan[e + f*x] + b^2*Tan[e + f*x]^3)/(3*f)
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

method	result
parts	$a^2x-\frac{b^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}+\frac{2ab\tan(fx+e)}{f}$
derivativedivides	$\frac{a^2(fx+e)+2ab\tan(fx+e)-b^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}$
default	$\frac{a^2(fx+e)+2ab\tan(fx+e)-b^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}$
risch	$a^2x+\frac{4ib\left(3a\,e^{4i(fx+e)}+6a\,e^{2i(fx+e)}+3b\,e^{2i(fx+e)}+3a+b\right)}{3f\left(e^{2i(fx+e)}+1\right)^3}$
norman	$\frac{a^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6-a^2x+3a^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-3a^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-\frac{2b(2a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2b(2a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}+\frac{4b(6a \cdots}{\cdots}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}$
parallelrisch	$\frac{3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6a^2f+(-12ab-6b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5-9x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4a^2f+(24ab+4b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3+9x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 \cdots}{3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}$

```
[In] int((a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
[Out] a^2*x-b^2/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+2*a*b/f*tan(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{3\,a^2fx\cos\left(fx+e\right)^3+\left(2\left(3\,ab+b^2\right)\cos\left(fx+e\right)^2+b^2\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
[Out] 1/3*(3*a^2*f*x*cos(f*x + e)^3 + (2*(3*a*b + b^2)*cos(f*x + e)^2 + b^2)*sin(
f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx=\int \left(a+b\sec^2{(e+fx)}\right)^2\,dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx=a^2x+\frac{\left(\tan{(fx+e)}^3+3\,\tan{(fx+e)}\right)b^2}{3\,f}+\frac{2\,ab\tan{(fx+e)}}{f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/3*(tan(f*x + e)^3 + 3*tan(f*x + e))*b^2/f + 2*a*b*tan(f*x + e)/f
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{b^2\tan{(fx+e)}^3+3\,(fx+e)a^2+6\,ab\tan{(fx+e)}+3\,b^2\tan{(fx+e)}}{3\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 3*(f*x + e)*a^2 + 6*a*b*tan(f*x + e) + 3*b^2*tan(
f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 18.49 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{\frac{b^2\tan(e+f\,x)^3}{3}-\tan(e+f\,x)\,\left(b^2-2\,b\,(a+b)\right)+a^2\,f\,x}{f}$$

```
[In] int((a + b/cos(e + f*x)^2)^2,x)
[Out] ((b^2*tan(e + f*x)^3)/3 - tan(e + f*x)*(b^2 - 2*b*(a + b)) + a^2*f*x)/f
```

3.175 $\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 47

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{1}{2}a(a+4b)x+\frac{a^2\cos(e+fx)\sin(e+fx)}{2f}+\frac{b^2\tan(e+fx)}{f}$$

```
[Out] 1/2*a*(a+4*b)*x+1/2*a^2*cos(f*x+e)*sin(f*x+e)/f+b^2*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4231, 398, 393, 209}

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{a^2\sin(e+fx)\cos(e+fx)}{2f}+\frac{1}{2}ax(a+4b)+\frac{b^2\tan(e+fx)}{f}$$

```
[In] Int[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (a*(a + 4*b)*x)/2 + (a^2*Cos[e + f*x]*Sin[e + f*x])/(2*f) + (b^2*Tan[e + f*
x])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^2}{\left(1+x^2\right)^2} \, dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \left(b^2+\frac{a(a+2b)+2abx^2}{\left(1+x^2\right)^2}\right) \, dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{b^2 \tan(e+fx)}{f}+\frac{\text{Subst}\left(\int \frac{a(a+2b)+2abx^2}{\left(1+x^2\right)^2} \, dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{a^2 \cos(e+fx) \sin(e+fx)}{2f}+\frac{b^2 \tan(e+fx)}{f}+\frac{(a(a+4b))\text{Subst}\left(\int \frac{1}{1+x^2} \, dx, x, \tan(e+fx)\right)}{2f} \\
&= \frac{1}{2}a(a+4b)x+\frac{a^2 \cos(e+fx) \sin(e+fx)}{2f}+\frac{b^2 \tan(e+fx)}{f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = 2abx+\frac{a^2(e+fx)}{2f}+\frac{a^2\sin(2(e+fx))}{4f}+\frac{b^2\tan(e+fx)}{f}$$

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] 2*a*b*x + (a^2*(e + f*x))/(2*f) + (a^2*Sin[2*(e + f*x)])/(4*f) + (b^2*Tan[e
 + f*x])/f
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{a^2\left(\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+2ab(fx+e)+b^2\tan(fx+e)}{f}$
default	$\frac{a^2\left(\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+2ab(fx+e)+b^2\tan(fx+e)}{f}$
parallelrisch	$\frac{\sin(3fx+3e)a^2+4afx(a+4b)\cos(fx+e)+\sin(fx+e)(a^2+8b^2)}{8f\cos(fx+e)}$
risch	$\frac{a^2x}{2}+2xab-\frac{i e^{2i(fx+e)}a^2}{8f}+\frac{i e^{-2i(fx+e)}a^2}{8f}+\frac{2ib^2}{f\left(e^{2i(fx+e)}+1\right)}$
norman	$\frac{(-\frac{1}{2}a^2-2ab)x+(-\frac{1}{2}a^2-2ab)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8+(\frac{1}{2}a^2+2ab)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+(\frac{1}{2}a^2+2ab)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}+(-a^2-4ab)x\,\mathrm{ta}}{\left(1+\mathrm{t}\right.}$

```
[In] int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+2*a*b*(f*x+e)+b^2*tan(f*
x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.19

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{(a^2+4\,ab)fx\cos\left(fx+e\right)+\left(a^2\cos\left(fx+e\right)^2+2\,b^2\right)\sin\left(fx+e\right)}{2\,f\cos\left(fx+e\right)}$$

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
[Out] 1/2*((a^2 + 4*a*b)*f*x*cos(f*x + e) + (a^2*cos(f*x + e)^2 + 2*b^2)*sin(f*x
+ e))/(f*cos(f*x + e))
```

Sympy [F]

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^2\cos^2\left(e+fx\right)dx$$

```
[In] integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2)**2,x)
[Out] Integral((a + b*sec(e + f*x)**2)**2*cos(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{2\,b^2\tan\left(fx+e\right)+(a^2+4\,ab)(fx+e)+\frac{a^2\tan(fx+e)}{\tan(fx+e)^2+1}}{2\,f}$$

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
[Out] 1/2*(2*b^2*tan(f*x + e) + (a^2 + 4*a*b)*(f*x + e) + a^2*tan(f*x + e)/(tan(f
*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

$$\int \cos^2(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = \frac{2\,b^2\tan\left(fx+e\right)+(a^2+4\,ab)(fx+e)+\frac{a^2\tan(fx+e)}{\tan(fx+e)^2+1}}{2\,f}$$

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^2*tan(f*x + e) + (a^2 + 4*a*b)*(f*x + e) + a^2*tan(f*x + e)/(tan(f
*x + e)^2 + 1))/f
```

Mupad [B] (verification not implemented)

Time = 18.78 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.40

$$\int \cos^2(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = \frac{b^2\tan(e+f\,x)}{f}+\frac{a^2\,\sin\left(2\,e+2\,f\,x\right)}{4\,f}+\frac{a\,\mathrm{atan}\left(\frac{a\tan(e+f\,x)\,(a+4\,b)}{2\,\left(\frac{a^2}{2}+2\,b\,a\right)}\right)\,(a+4\,b)}{2\,f}$$

```
[In] int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (b^2*tan(e + f*x))/f + (a^2*sin(2*e + 2*f*x))/(4*f) + (a*atan((a*tan(e + f*
x)*(a + 4*b))/(2*(2*a*b + a^2/2)))*(a + 4*b))/(2*f)
```

3.176 $\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 81

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{1}{8}\left(3a^2+8ab+8b^2\right)x+\frac{3a(a+2b)\cos(e+fx)\sin(e+fx)}{8f}+\frac{a\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)}{4f}$$

```
[Out] 1/8*(3*a^2+8*a*b+8*b^2)*x+3/8*a*(a+2*b)*cos(f*x+e)*sin(f*x+e)/f+1/4*a*cos(f
*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4231, 424, 393, 209}

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{1}{8}x\left(3a^2+8ab+8b^2\right)+\frac{3a(a+2b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{a\sin(e+fx)\cos^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)}{4f}$$

```
[In] Int[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((3*a^2 + 8*a*b + 8*b^2)*x)/8 + (3*a*(a + 2*b)*Cos[e + f*x]*Sin[e + f*x])/(
8*f) + (a*Cos[e + f*x]^3*Sin[e + f*x]*(a + b + b*Tan[e + f*x]^2))/(4*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^2}{(1+x^2)^3}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{a\cos^3(e+fx)\sin(e+fx)\,(a+b+b\tan^2(e+fx))}{4f}\\ &\quad+\frac{\text{Subst}\left(\int \frac{(a+b)(3a+4b)+b(a+4b)x^2}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{4f}\end{aligned}$$

$$= \frac{3a(a+2b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)}{4f} + \frac{(3a^2+8ab+8b^2)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{8f}$$

$$= \frac{1}{8}\left(3a^2+8ab+8b^2\right)x + \frac{3a(a+2b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)}{4f}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{4(3a^2+8ab+8b^2)(e+fx)+8a(a+2b)\sin(2(e+fx))+a^2\sin(4(e+fx))}{32f}$$

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
[Out] (4*(3*a^2 + 8*a*b + 8*b^2)*(e + f*x) + 8*a*(a + 2*b)*Sin[2*(e + f*x)] + a^2
*Sin[4*(e + f*x)])/(32*f)
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.67

method	result
parallelrisch	$\frac{8a(a+2b)\sin(2fx+2e)+a^2\sin(4fx+4e)+12xf\left(a^2+\frac{8}{3}ab+\frac{8}{3}b^2\right)}{32f}$
risch	$\frac{3a^2x}{8}+xab+x\,b^2+\frac{a^2\sin(4fx+4e)}{32f}+\frac{\sin(2fx+2e)a^2}{4f}+\frac{\sin(2fx+2e)ab}{2f}$
derivativedivides	$\frac{a^2\left(\frac{\left(\cos(fx+e)^3+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b^2(fx+e)}{f}$
default	$\frac{a^2\left(\frac{\left(\cos(fx+e)^3+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b^2(fx+e)}{f}$
norman	$\frac{\left(-\frac{3}{8}a^2-ab-b^2\right)x+\left(-\frac{9}{8}a^2-3ab-3b^2\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8+\left(-\frac{9}{8}a^2-3ab-3b^2\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}+\left(-\frac{3}{8}a^2-ab-b^2\right)x\tan\left(\frac{fx}{2}+\right.}{}$

```
[In] int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32*(8*a*(a+2*b)*sin(2*f*x+2*e)+a^2*sin(4*f*x+4*e)+12*x*f*(a^2+8/3*a*b+8/3
*b^2))/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.77

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{(3\,a^2+8\,ab+8\,b^2)fx+\left(2\,a^2\cos\left(fx+e\right)^3+(3\,a^2+8\,ab)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{8\,f}$$

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((3*a^2 + 8*a*b + 8*b^2)*f*x + (2*a^2*cos(f*x + e)^3 + (3*a^2 + 8*a*b)*
cos(f*x + e))*sin(f*x + e))/f
```

Sympy [F]

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^2\cos^4\left(e+fx\right)dx$$

```
[In] integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*cos(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{(3\,a^2+8\,ab+8\,b^2)(fx+e)+\frac{(3\,a^2+8\,ab)\tan(fx+e)^3+(5\,a^2+8\,ab)\tan(fx+e)}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}}{8\,f}$$

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^2 + 8*a*b + 8*b^2)*(f*x + e) + ((3*a^2 + 8*a*b)*tan(f*x + e)^3 +
(5*a^2 + 8*a*b)*tan(f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07

$$\int \cos^4(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = \frac{(3\,a^2+8\,ab+8\,b^2)(fx+e)+\frac{3\,a^2\tan(fx+e)^3+8\,ab\tan(fx+e)^3+5\,a^2\tan(fx+e)+8\,ab\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^2}}{8\,f}$$

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*((3*a^2 + 8*a*b + 8*b^2)*(f*x + e) + (3*a^2*tan(f*x + e)^3 + 8*a*b*tan(
f*x + e)^3 + 5*a^2*tan(f*x + e) + 8*a*b*tan(f*x + e))/(tan(f*x + e)^2 + 1)^
2)/f
```

Mupad [B] (verification not implemented)

Time = 18.62 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94

$$\int \cos^4(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = x\left(\frac{3\,a^2}{8}+a\,b+b^2\right)+\frac{\left(\frac{3\,a^2}{8}+b\,a\right)\,\tan(e+f\,x)^3+\left(\frac{5\,a^2}{8}+b\,a\right)\,\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^4+2\tan\left(e+f\,x\right)^2+1\right)}$$

```
[In] int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] x*(a*b + (3*a^2)/8 + b^2) + (tan(e + f*x)*(a*b + (5*a^2)/8) + tan(e + f*x)^
3*(a*b + (3*a^2)/8))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1))
```

3.177 $\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 119

$$\begin{aligned}\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx &= \frac{1}{16}\left(5a^2+12ab+8b^2\right)x+\frac{(5a^2+12ab+8b^2)\cos(e+fx)\sin(e+fx)}{16f}\\&\quad+\frac{a(5a+8b)\cos^3(e+fx)\sin(e+fx)}{24f}\\&\quad+\frac{a\cos^5(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)}{6f}\end{aligned}$$

```
[Out] 1/16*(5*a^2+12*a*b+8*b^2)*x+1/16*(5*a^2+12*a*b+8*b^2)*cos(f*x+e)*sin(f*x+e)
/f+1/24*a*(5*a+8*b)*cos(f*x+e)^3*sin(f*x+e)/f+1/6*a*cos(f*x+e)^5*sin(f*x+e)
*(a+b+b*tan(f*x+e)^2)/f
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4231, 424, 393, 205, 209}

$$\begin{aligned}\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx &= \frac{(5a^2+12ab+8b^2)\sin(e+fx)\cos(e+fx)}{16f}\\&\quad+\frac{1}{16}x\left(5a^2+12ab+8b^2\right)+\frac{a(5a+8b)\sin(e+fx)\cos^3(e+fx)}{24f}\\&\quad+\frac{a\sin(e+fx)\cos^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)}{6f}\end{aligned}$$

```
[In] Int[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((5*a^2 + 12*a*b + 8*b^2)*x)/16 + ((5*a^2 + 12*a*b + 8*b^2)*Cos[e + f*x]*Si
n[e + f*x])/(16*f) + (a*(5*a + 8*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) + (
a*Cos[e + f*x]^5*Sin[e + f*x]*(a + b + b*Tan[e + f*x]^2))/(6*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^2}{(1+x^2)^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{a\cos^5(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)}{6f} + \frac{\text{Subst}\left(\int \frac{(a+b)(5a+6b)+3b(a+2b)x^2}{(1+x^2)^3}\,dx, x, \tan(e+fx)\right)}{6f}$$

$$= \frac{a(5a+8b)\cos^3(e+fx)\sin(e+fx)}{24f} + \frac{a\cos^5(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)}{6f} + \frac{(5a^2+12ab+8b^2)\,\text{Subst}\left(\int \frac{1}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{8f}$$

$$= \frac{(5a^2+12ab+8b^2)\cos(e+fx)\sin(e+fx)}{16f} + \frac{a(5a+8b)\cos^3(e+fx)\sin(e+fx)}{24f} + \frac{a\cos^5(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)}{6f} + \frac{(5a^2+12ab+8b^2)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{16f}$$

$$= \frac{1}{16}\left(5a^2+12ab+8b^2\right)x + \frac{(5a^2+12ab+8b^2)\cos(e+fx)\sin(e+fx)}{16f} + \frac{a(5a+8b)\cos^3(e+fx)\sin(e+fx)}{24f} + \frac{a\cos^5(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)}{6f}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.83

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{60a^2e+144abe+96b^2e+60a^2fx+144abfx+96b^2fx+(45a^2+96ab+48b^2)\sin(2(e+fx))+3a(3a+4}{192f}$$

```
[In] Integrate[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (60*a^2*e + 144*a*b*e + 96*b^2*e + 60*a^2*f*x + 144*a*b*f*x + 96*b^2*f*x +
(45*a^2 + 96*a*b + 48*b^2)*Sin[2*(e + f*x)] + 3*a*(3*a + 4*b)*Sin[4*(e + f*
x)] + a^2*Sin[6*(e + f*x)])/(192*f)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{(45a^2+96ab+48b^2)\sin(2fx+2e)+(9a^2+12ab)\sin(4fx+4e)+a^2\sin(6fx+6e)+60x(a^2+\frac{12}{5}ab+\frac{8}{5}b^2)f}{192f}$
derivativedivides	$\frac{a^2\left(\frac{\left(\cos(fx+e)^5+\frac{5\cos(fx+e)^3}{4}+\frac{15\cos(fx+e)}{8}\right)\sin(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+2ab\left(\frac{\left(\cos(fx+e)^3+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\right.}{f}$
default	$\frac{a^2\left(\frac{\left(\cos(fx+e)^5+\frac{5\cos(fx+e)^3}{4}+\frac{15\cos(fx+e)}{8}\right)\sin(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+2ab\left(\frac{\left(\cos(fx+e)^3+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\right.}{f}$
risch	$\frac{5a^2x}{16}+\frac{3xab}{4}+\frac{x\,b^2}{2}+\frac{a^2\sin(6fx+6e)}{192f}+\frac{3a^2\sin(4fx+4e)}{64f}+\frac{\sin(4fx+4e)ab}{16f}+\frac{15\sin(2fx+2e)a^2}{64f}+\frac{\sin(2fx}{2}$

[In] int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/192*((45*a^2+96*a*b+48*b^2)*sin(2*f*x+2*e)+(9*a^2+12*a*b)*sin(4*f*x+4*e)+
a^2*sin(6*f*x+6*e)+60*x*(a^2+12/5*a*b+8/5*b^2)*f)/f

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{3\,(5\,a^2+12\,ab+8\,b^2)fx+\left(8\,a^2\cos\left(fx+e\right)^5+2\,(5\,a^2+12\,ab)\cos\left(fx+e\right)^3+3\,(5\,a^2+12\,ab+8\,b^2)\cos\right.}{48\,f}$$

[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")

[Out] 1/48*(3*(5*a^2 + 12*a*b + 8*b^2)*f*x + (8*a^2*cos(f*x + e)^5 + 2*(5*a^2 + 1
2*a*b)*cos(f*x + e)^3 + 3*(5*a^2 + 12*a*b + 8*b^2)*cos(f*x + e))*sin(f*x +
e))/f

Sympy [F(-1)]

Timed out.

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\text{Timed out}$$

```
[In] integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.13

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{3\,(5\,a^2+12\,ab+8\,b^2)(fx+e)+\frac{3\,(5\,a^2+12\,ab+8\,b^2)\tan(fx+e)^5+8\,(5\,a^2+12\,ab+6\,b^2)\tan(fx+e)^3+3\,(11\,a^2+20\,ab+8\,b^2)\tan(fx+e)}{\tan(fx+e)^6+3\,\tan(fx+e)^4+3\,\tan(fx+e)^2+1}}{48\,f}$$

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
[Out] 1/48*(3*(5*a^2 + 12*a*b + 8*b^2)*(f*x + e) + (3*(5*a^2 + 12*a*b + 8*b^2)*ta
n(f*x + e)^5 + 8*(5*a^2 + 12*a*b + 6*b^2)*tan(f*x + e)^3 + 3*(11*a^2 + 20*a
*b + 8*b^2)*tan(f*x + e))/(tan(f*x + e)^6 + 3*tan(f*x + e)^4 + 3*tan(f*x +
e)^2 + 1))/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.26

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{3\,(5\,a^2+12\,ab+8\,b^2)(fx+e)+\frac{15\,a^2\tan(fx+e)^5+36\,ab\tan(fx+e)^5+24\,b^2\tan(fx+e)^5+40\,a^2\tan(fx+e)^3+96\,ab\tan(fx+e)^3+48\,b^2 \cdots}{\left(\tan(fx+e)^2+1\right)^3}}{48\,f}$$

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
[Out] 1/48*(3*(5*a^2 + 12*a*b + 8*b^2)*(f*x + e) + (15*a^2*tan(f*x + e)^5 + 36*a*
b*tan(f*x + e)^5 + 24*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 + 96*a*b*t
an(f*x + e)^3 + 48*b^2*tan(f*x + e)^3 + 33*a^2*tan(f*x + e) + 60*a*b*tan(f*
x + e) + 24*b^2*tan(f*x + e))/(tan(f*x + e)^2 + 1)^3)/f
```

Mupad [B] (verification not implemented)

Time = 19.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.03

$$\int \cos^6(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = x \left(\frac{5\,a^2}{16}+\frac{3\,a\,b}{4}+\frac{b^2}{2}\right) + \frac{\left(\frac{5\,a^2}{16}+\frac{3\,a\,b}{4}+\frac{b^2}{2}\right)\,\tan(e+f\,x)^5+\left(\frac{5\,a^2}{6}+2\,a\,b+b^2\right)\,\tan(e+f\,x)^3+\left(\frac{11\,a^2}{16}+\frac{5\,a\,b}{4}+\frac{b^2}{2}\right)\,\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^6+3\,\tan\left(e+f\,x\right)^4+3\,\tan\left(e+f\,x\right)^2+1\right)}$$

```
[In] int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] x*((3*a*b)/4 + (5*a^2)/16 + b^2/2) + (tan(e + f*x)*((5*a*b)/4 + (11*a^2)/16
 + b^2/2) + tan(e + f*x)^3*(2*a*b + (5*a^2)/6 + b^2) + tan(e + f*x)^5*((3*a
*b)/4 + (5*a^2)/16 + b^2/2))/(f*(3*tan(e + f*x)^2 + 3*tan(e + f*x)^4 + tan(
e + f*x)^6 + 1))
```

3.178 $\int (a+b\sec^2(c+dx))^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 73

$$\int (a+b\sec^2(c+dx))^3\,dx=a^3x+\frac{b(3a^2+3ab+b^2)\tan(c+dx)}{d}+\frac{b^2(3a+2b)\tan^3(c+dx)}{3d}+\frac{b^3\tan^5(c+dx)}{5d}$$

```
[Out] a^3*x+b*(3*a^2+3*a*b+b^2)*tan(d*x+c)/d+1/3*b^2*(3*a+2*b)*tan(d*x+c)^3/d+1/5
*b^3*tan(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4213, 398, 209}

$$\int (a+b\sec^2(c+dx))^3\,dx=a^3x+\frac{b(3a^2+3ab+b^2)\tan(c+dx)}{d}+\frac{b^2(3a+2b)\tan^3(c+dx)}{3d}+\frac{b^3\tan^5(c+dx)}{5d}$$

```
[In] Int[(a + b*Sec[c + d*x]^2)^3,x]
```

```
[Out] a^3*x + (b*(3*a^2 + 3*a*b + b^2)*Tan[c + d*x])/d + (b^2*(3*a + 2*b)*Tan[c +
 d*x]^3)/(3*d) + (b^3*Tan[c + d*x]^5)/(5*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^3}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(b(3a^2+3ab+b^2)+b^2(3a+2b)x^2+b^3x^4+\frac{a^3}{1+x^2}\right)\,dx, x, \tan(c+dx)\right)}{d} \\
&= \frac{b(3a^2+3ab+b^2)\tan(c+dx)}{d}+\frac{b^2(3a+2b)\tan^3(c+dx)}{3d} \\
&\quad +\frac{b^3\tan^5(c+dx)}{5d}+\frac{a^3\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d} \\
&= a^3x+\frac{b(3a^2+3ab+b^2)\tan(c+dx)}{d}+\frac{b^2(3a+2b)\tan^3(c+dx)}{3d}+\frac{b^3\tan^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int \left(a+b\sec^2(c+dx)\right)^3\,dx = \frac{15a^3dx+15b(3a^2+3ab+b^2)\tan(c+dx)+5b^2(3a+2b)\tan^3(c+dx)+3b^3\tan^5(c+dx)}{15d}$$

```
[In] Integrate[(a + b*Sec[c + d*x]^2)^3,x]
[Out] (15*a^3*d*x + 15*b*(3*a^2 + 3*a*b + b^2)*Tan[c + d*x] + 5*b^2*(3*a + 2*b)*T
an[c + d*x]^3 + 3*b^3*Tan[c + d*x]^5)/(15*d)
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{a^3(dx+c)+3a^2b\tan(dx+c)-3a\,b^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)-b^3\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}$
default	$\frac{a^3(dx+c)+3a^2b\tan(dx+c)-3a\,b^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)-b^3\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}$
parts	$a^3x-\frac{b^3\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}+\frac{3a^2b\tan(dx+c)}{d}-\frac{3a\,b^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
risch	$a^3x+\frac{2ib(45a^2e^{8i(dx+c)}+180a^2e^{6i(dx+c)}+90ab\,e^{6i(dx+c)}+270a^2e^{4i(dx+c)}+210ab\,e^{4i(dx+c)}+80b^2e^{4i(dx+c)}+180a^2e^{2i(}}{15d\left(e^{2i(dx+c)}+1\right)^5}$
norman	$\frac{a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}-a^3x+5a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-10a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+10a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6-5a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8-\frac{2b\left(3a^2+\right.}{}}{}$
parallelrisch	$\frac{15a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}xd+\left(-90a^2b-90a\,b^2-30b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9-75a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8xd+360b\left(a+\frac{b}{3}\right)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7+150a}{}$

```
[In] int((a+b*sec(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(d*x+c)+3*a^2*b*tan(d*x+c)-3*a*b^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x
+c)-b^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.23

$$\int \left(a+b\sec^2(c+dx)\right)^3\,dx$$
$$=\frac{15\,a^3dx\cos\left(dx+c\right)^5+\left(\left(45\,a^2b+30\,ab^2+8\,b^3\right)\cos\left(dx+c\right)^4+3\,b^3+\left(15\,ab^2+4\,b^3\right)\cos\left(dx+c\right)^2\right)\sin\left(dx\right.}{15\,d\cos\left(dx+c\right)^5}$$

```
[In] integrate((a+b*sec(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*a^3*d*x*cos(d*x + c)^5 + ((45*a^2*b + 30*a*b^2 + 8*b^3)*cos(d*x +
c)^4 + 3*b^3 + (15*a*b^2 + 4*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x
+ c)^5)
```

Sympy [F]

$$\int \left(a+b\sec^2(c+dx)\right)^3\,dx=\int \left(a+b\sec^2\left(c+dx\right)\right)^3\,dx$$

```
[In] integrate((a+b*sec(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x)**2)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14

$$\int \left(a+b\sec^2(c+dx)\right)^3\,dx=a^3x+\frac{\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)ab^2}{d}+\frac{\left(3\,\tan\left(dx+c\right)^5+10\,\tan\left(dx+c\right)^3+15\,\tan\left(dx+c\right)\right)b^3}{15\,d}+\frac{3\,a^2b\tan\left(dx+c\right)}{d}$$

```
[In] integrate((a+b*sec(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*x + (tan(d*x + c)^3 + 3*tan(d*x + c))*a*b^2/d + 1/15*(3*tan(d*x + c)^5
+ 10*tan(d*x + c)^3 + 15*tan(d*x + c))*b^3/d + 3*a^2*b*tan(d*x + c)/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \left(a+b\sec^2(c+dx)\right)^3\,dx=\frac{3\,b^3\tan\left(dx+c\right)^5+15\,ab^2\tan\left(dx+c\right)^3+10\,b^3\tan\left(dx+c\right)^3+15\,(dx+c)a^3+45\,a^2b\tan\left(dx+c\right)+45\,a}{15\,d}$$

```
[In] integrate((a+b*sec(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/15*(3*b^3*tan(d*x + c)^5 + 15*a*b^2*tan(d*x + c)^3 + 10*b^3*tan(d*x + c)^
3 + 15*(d*x + c)*a^3 + 45*a^2*b*tan(d*x + c) + 45*a*b^2*tan(d*x + c) + 15*b
^3*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 18.69 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \left(a+b\sec^2(c+dx)\right)^3\,dx$$
$$=\frac{\tan(c+d\,x)\,\left(3\,b\,(a+b)^2-3\,b^2\,(a+b)+b^3\right)+\frac{b^3\tan(c+d\,x)^5}{5}+\tan(c+d\,x)^3\,\left(b^2\,(a+b)-\frac{b^3}{3}\right)+a^3\,d\,x}{d}$$

```
[In] int((a + b/cos(c + d*x)^2)^3,x)
[Out] (tan(c + d*x)*(3*b*(a + b)^2 - 3*b^2*(a + b) + b^3) + (b^3*tan(c + d*x)^5)/
5 + tan(c + d*x)^3*(b^2*(a + b) - b^3/3) + a^3*d*x)/d
```

3.179 $\int (a+b\sec^2(c+dx))^4 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 111

$$\int (a+b\sec^2(c+dx))^4 \, dx = a^4x + \frac{b(2a+b)\,(2a^2+2ab+b^2)\tan(c+dx)}{d} + \frac{b^2(6a^2+8ab+3b^2)\tan^3(c+dx)}{3d} + \frac{b^3(4a+3b)\tan^5(c+dx)}{5d} + \frac{b^4\tan^7(c+dx)}{7d}$$

```
[Out] a^4*x+b*(2*a+b)*(2*a^2+2*a*b+b^2)*tan(d*x+c)/d+1/3*b^2*(6*a^2+8*a*b+3*b^2)*
tan(d*x+c)^3/d+1/5*b^3*(4*a+3*b)*tan(d*x+c)^5/d+1/7*b^4*tan(d*x+c)^7/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4213, 398, 209}

$$\int (a+b\sec^2(c+dx))^4 \, dx = a^4x + \frac{b^2(6a^2+8ab+3b^2)\tan^3(c+dx)}{3d} + \frac{b(2a+b)\,(2a^2+2ab+b^2)\tan(c+dx)}{d} + \frac{b^3(4a+3b)\tan^5(c+dx)}{5d} + \frac{b^4\tan^7(c+dx)}{7d}$$

```
[In] Int[(a + b*Sec[c + d*x]^2)^4,x]
```

```
[Out] a^4*x + (b*(2*a + b)*(2*a^2 + 2*a*b + b^2)*Tan[c + d*x])/d + (b^2*(6*a^2 +
8*a*b + 3*b^2)*Tan[c + d*x]^3)/(3*d) + (b^3*(4*a + 3*b)*Tan[c + d*x]^5)/(5*
d) + (b^4*Tan[c + d*x]^7)/(7*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^4}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(b(2a+b)\left(2a^2+2ab+b^2\right)+b^2\left(6a^2+8ab+3b^2\right)x^2+b^3(4a+3b)x^4+b^4x^6+\frac{a^4}{1+x^2}\right)\,dx,\right.}{d}$$

$$= \frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan(c+dx)}{d}+\frac{b^2\left(6a^2+8ab+3b^2\right)\tan^3(c+dx)}{3d}+\frac{b^3(4a+3b)\tan^5(c+dx)}{5d}+\frac{b^4\tan^7(c+dx)}{7d}+\frac{a^4\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= a^4x+\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan(c+dx)}{d}+\frac{b^2\left(6a^2+8ab+3b^2\right)\tan^3(c+dx)}{3d}+\frac{b^3(4a+3b)\tan^5(c+dx)}{5d}+\frac{b^4\tan^7(c+dx)}{7d}$$

Mathematica [A] (verified)

Time = 3.60 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$\int \left(a+b\sec^2(c+dx)\right)^4\,dx$$
$$=\frac{105a^4dx+105b(4a^3+6a^2b+4ab^2+b^3)\tan(c+dx)+35b^2(6a^2+8ab+3b^2)\tan^3(c+dx)+21b^3(4a+3}{105d}$$

```
[In] Integrate[(a + b*Sec[c + d*x]^2)^4,x]
```

```
[Out] (105*a^4*d*x + 105*b*(4*a^3 + 6*a^2*b + 4*a*b^2 + b^3)*Tan[c + d*x] + 35*b^
2*(6*a^2 + 8*a*b + 3*b^2)*Tan[c + d*x]^3 + 21*b^3*(4*a + 3*b)*Tan[c + d*x]^
5 + 15*b^4*Tan[c + d*x]^7)/(105*d)
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{a^4(dx+c)+4a^3b\tan(dx+c)-6a^2b^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)-4a\,b^3\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)-}{d}$
default	$\frac{a^4(dx+c)+4a^3b\tan(dx+c)-6a^2b^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)-4a\,b^3\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)-}{d}$
parts	$a^4x-\frac{b^4\left(-\frac{16}{35}-\frac{\sec(dx+c)^6}{7}-\frac{6\sec(dx+c)^4}{35}-\frac{8\sec(dx+c)^2}{35}\right)\tan(dx+c)}{d}+\frac{4a^3b\tan(dx+c)}{d}-\frac{6a^2b^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right.}{d}$
risch	$a^4x+\frac{8ib(105a^3e^{12i(dx+c)}+630a^3e^{10i(dx+c)}+315a^2b\,e^{10i(dx+c)}+1575a^3e^{8i(dx+c)}+1365a^2b\,e^{8i(dx+c)}+560a\,b^2e^{8i(dx}}{}$
norman	$\frac{a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}-a^4x+7a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-21a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+35a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6-35a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+21a^4x}{}$
parallelrisch	$\frac{105a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}xd+(-840a^3b-1260a^2b^2-840a\,b^3-210b^4)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}-735a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}xd+(5040a^3b+5}{}$

```
[In] int((a+b*sec(d*x+c)^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(d*x+c)+4*a^3*b*tan(d*x+c)-6*a^2*b^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d
*x+c)-4*a*b^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)-b^4*(-1
6/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.17

$$\int \left(a+b\sec^2(c+dx)\right)^4\,dx$$
$$=\frac{105\,a^4dx\cos\left(dx+c\right)^7+\left(4\left(105\,a^3b+105\,a^2b^2+56\,ab^3+12\,b^4\right)\cos\left(dx+c\right)^6+2\left(105\,a^2b^2+56\,ab^3+12\,b\right.\right.}{105\,d\cos\left(dx+c\right)^7}$$

```
[In] integrate((a+b*sec(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(105*a^4*d*x*cos(d*x + c)^7 + (4*(105*a^3*b + 105*a^2*b^2 + 56*a*b^3
+ 12*b^4)*cos(d*x + c)^6 + 2*(105*a^2*b^2 + 56*a*b^3 + 12*b^4)*cos(d*x + c)
^4 + 15*b^4 + 6*(14*a*b^3 + 3*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x
 + c)^7)
```

Sympy [F]

$$\int \left(a+b\sec^2(c+dx)\right)^4\,dx=\int \left(a+b\sec^2\left(c+dx\right)\right)^4\,dx$$

```
[In] integrate((a+b*sec(d*x+c)**2)**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x)**2)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.21

$$\begin{aligned}\int \left(a+b\sec^2(c+dx)\right)^4\,dx &= a^4x+\frac{2\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^2b^2}{d}\\&+\frac{4\left(3\,\tan\left(dx+c\right)^5+10\,\tan\left(dx+c\right)^3+15\,\tan\left(dx+c\right)\right)ab^3}{15\,d}\\&+\frac{\left(5\,\tan\left(dx+c\right)^7+21\,\tan\left(dx+c\right)^5+35\,\tan\left(dx+c\right)^3+35\,\tan\left(dx+c\right)\right)b^4}{35\,d}\\&+\frac{4\,a^3b\tan\left(dx+c\right)}{d}\end{aligned}$$

```
[In] integrate((a+b*sec(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 2*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2*b^2/d + 4/15*(3*tan(d*x + c
)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a*b^3/d + 1/35*(5*tan(d*x + c)^7
 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*tan(d*x + c))*b^4/d + 4*a^3*b
*tan(d*x + c)/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.33

$$\int \left(a + b\sec^2(c + dx)\right)^4 \, dx = \frac{15\, b^4 \tan\left(dx + c\right)^7 + 84\, ab^3 \tan\left(dx + c\right)^5 + 63\, b^4 \tan\left(dx + c\right)^5 + 210\, a^2b^2 \tan\left(dx + c\right)^3 + 280\, ab^3 \tan\left(dx + \right.}{}$$

```
[In] integrate((a+b*sec(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] 1/105*(15*b^4*tan(d*x + c)^7 + 84*a*b^3*tan(d*x + c)^5 + 63*b^4*tan(d*x + c
)^5 + 210*a^2*b^2*tan(d*x + c)^3 + 280*a*b^3*tan(d*x + c)^3 + 105*b^4*tan(d
*x + c)^3 + 105*(d*x + c)*a^4 + 420*a^3*b*tan(d*x + c) + 630*a^2*b^2*tan(d*
x + c) + 420*a*b^3*tan(d*x + c) + 105*b^4*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 18.71 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.07

$$\int \left(a + b\sec^2(c + dx)\right)^4 \, dx = \frac{\tan(c + d\,x)\,\left(4\,b\,\left(a + b\right)^3 + 4\,b^3\,\left(a + b\right) - 6\,b^2\,\left(a + b\right)^2 - b^4\right) + \tan(c + d\,x)^3\,\left(2\,b^2\,\left(a + b\right)^2 - \frac{4\,b^3\,\left(a+b\right)}{3} + \frac{b^4}{3}\right.}{d}$$

```
[In] int((a + b/cos(c + d*x)^2)^4,x)
```

```
[Out] (tan(c + d*x)*(4*b*(a + b)^3 + 4*b^3*(a + b) - 6*b^2*(a + b)^2 - b^4) + tan
(c + d*x)^3*(2*b^2*(a + b)^2 - (4*b^3*(a + b))/3 + b^4/3) + (b^4*tan(c + d*
x)^7)/7 + tan(c + d*x)^5*((4*b^3*(a + b))/5 - b^4/5) + a^4*d*x)/d
```

3.180 $\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{(2a-b)\text{arctanh}(\sin(e+fx))}{2b^2f} + \frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{b^2\sqrt{a+b}f} + \frac{\sec(e+fx)\tan(e+fx)}{2bf}$$

```
[Out] -1/2*(2*a-b)*arctanh(sin(f*x+e))/b^2/f+a^(3/2)*arctanh(sin(f*x+e)*a^(1/2)/(
a+b)^(1/2))/b^2/f/(a+b)^(1/2)+1/2*sec(f*x+e)*tan(f*x+e)/b/f
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4232, 425, 536, 212, 214}

$$\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{b^2f\sqrt{a+b}} - \frac{(2a-b)\text{arctanh}(\sin(e+fx))}{2b^2f} + \frac{\tan(e+fx)\sec(e+fx)}{2bf}$$

```
[In] Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/2*((2*a - b)*ArcTanh[Sin[e + f*x]])/(b^2*f) + (a^(3/2)*ArcTanh[(Sqrt[a]*
Sin[e + f*x])/Sqrt[a + b]])/(b^2*Sqrt[a + b]*f) + (Sec[e + f*x]*Tan[e + f*x
])/(2*b*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a+b-ax^2)}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sec(e+fx)\tan(e+fx)}{2bf} + \frac{\text{Subst}\left(\int \frac{-a+b-ax^2}{(1-x^2)(a+b-ax^2)}\,dx, x, \sin(e+fx)\right)}{2bf}$$

$$= \frac{\sec(e+fx)\tan(e+fx)}{2bf} + \frac{a^2\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{b^2f} - \frac{(2a-b)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(e+fx)\right)}{2b^2f}$$

$$=-\frac{(2a-b)\operatorname{arctanh}(\sin(e+fx))}{2b^2f}+\frac{a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{b^2\sqrt{a+b}f}+\frac{\sec(e+fx)\tan(e+fx)}{2bf}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.77 (sec) , antiderivative size = 1195, normalized size of antiderivative = 13.90

$$\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$=\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(4a\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-2b\log\left(\cos\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

```
[In] Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(4*a*Log[Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]] - 2*b*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 4*a*Log[
Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 2*b*Log[Cos[(e + f*x)/2] + Sin[(e +
f*x)/2]] + (a^(3/2)*Cos[e]*Log[a + 2*(a + b)*Cos[2*e] - a*Cos[2*(e + f*x)]
- (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e]
- I*Sin[e])^2]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]
*Sin[2*e + f*x]])/(Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]) - (a^(3/2)*Cos[
e]*Log[-a - 2*(a + b)*Cos[2*e] + a*Cos[2*(e + f*x)] + (2*I)*a*Sin[2*e] + (2
*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f*x]
 + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]])/(Sqrt
[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]) + ((2*I)*a^(3/2)*ArcTan[(2*Sin[e]*(I*a
 + I*b + I*(a + b)*Cos[2*e] + Sqrt[a]*Sqrt[a + b]*Cos[f*x]*Sqrt[(Cos[e] - I
*Sin[e])^2] - Sqrt[a]*Sqrt[a + b]*Cos[2*e + f*x]*Sqrt[(Cos[e] - I*Sin[e])^2
] + a*Sin[2*e] + b*Sin[2*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e]
)^2]*Sin[f*x] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e +
 f*x]))/(I*(a + 3*b)*Cos[e] + I*(a + b)*Cos[3*e] + I*a*Cos[e + 2*f*x] + I*a
*Cos[3*e + 2*f*x] + 3*a*Sin[e] + b*Sin[e] + a*Sin[3*e] + b*Sin[3*e] + a*Sin
[e + 2*f*x] - a*Sin[3*e + 2*f*x])]*Sqrt[(Cos[e] - I*Sin[e])^2]*(Cos[e] + I*
Sin[e]))/Sqrt[a + b] - (I*a^(3/2)*Log[a + 2*(a + b)*Cos[2*e] - a*Cos[2*(e +
 f*x)] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(
Cos[e] - I*Sin[e])^2]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin
[e])^2]*Sin[2*e + f*x]]*Sin[e])/(Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]) +
 (I*a^(3/2)*Log[-a - 2*(a + b)*Cos[2*e] + a*Cos[2*(e + f*x)] + (2*I)*a*Sin[
2*e] + (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]
*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x
]]*Sin[e])/(Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]) + (2*a^(3/2)*ArcTan[((
```

```
a + b)*Sin[e])/((a + b)*Cos[e] - Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e
])^2]*(Cos[2*e] + I*Sin[2*e])*Sin[e + f*x])]*Sqrt[(Cos[e] - I*Sin[e])^2]*((
-I)*Cos[e] + Sin[e]))/Sqrt[a + b] + b/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^2 - b/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2))/(8*b^2*f*(a + b*Sec[e + f*
x]^2))
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{\frac{a^2 \operatorname{arctanh}\left(\frac{a \sin(fx+e)}{\sqrt{a(a+b)}}\right)}{b^2\sqrt{a(a+b)}}-\frac{1}{4b(\sin(fx+e)+1)}+\frac{(-2a+b)\ln(\sin(fx+e)+1)}{4b^2}-\frac{1}{4b(\sin(fx+e)-1)}+\frac{(2a-b)\ln(\sin(fx+e)-1)}{4b^2}}{f}$
default	$\frac{\frac{a^2 \operatorname{arctanh}\left(\frac{a \sin(fx+e)}{\sqrt{a(a+b)}}\right)}{b^2\sqrt{a(a+b)}}-\frac{1}{4b(\sin(fx+e)+1)}+\frac{(-2a+b)\ln(\sin(fx+e)+1)}{4b^2}-\frac{1}{4b(\sin(fx+e)-1)}+\frac{(2a-b)\ln(\sin(fx+e)-1)}{4b^2}}{f}$
risch	$-\frac{i\left(e^{3i(fx+e)}-e^{i(fx+e)}\right)}{fb\left(e^{2i(fx+e)}+1\right)^2}+\frac{\ln\left(e^{i(fx+e)}-i\right)a}{b^2 f}-\frac{\ln\left(e^{i(fx+e)}-i\right)}{2bf}-\frac{\ln\left(e^{i(fx+e)}+i\right)a}{b^2 f}+\frac{\ln\left(e^{i(fx+e)}+i\right)}{2bf}+\frac{\sqrt{a(a+}}{}$

```
[In] int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2/b^2/(a*(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2))-1/4/b/(s
in(f*x+e)+1)+1/4/b^2*(-2*a+b)*ln(sin(f*x+e)+1)-1/4/b/(sin(f*x+e)-1)+1/4*(2*
a-b)/b^2*ln(sin(f*x+e)-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 272, normalized size of antiderivative = 3.16

$$\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \left[\frac{2\,a\sqrt{\frac{a}{a+b}}\cos\left(fx+e\right)^2\log\left(-\frac{a\cos(fx+e)^2-2\,(a+b)\sqrt{\frac{a}{a+b}}\sin(fx+e)-2\,a-b}{a\cos(fx+e)^2+b}\right)-(2\,a-b)\cos\left(fx+e\right)^2\log\left(\sin\left(fx\right.\right.}{4\,b^2 f\cos\left(fx+e\right)^2}\right.$$

$$-\frac{4\,a\sqrt{-\frac{a}{a+b}}\arctan\left(\sqrt{-\frac{a}{a+b}}\sin\left(fx+e\right)\right)\cos\left(fx+e\right)^2+(2\,a-b)\cos\left(fx+e\right)^2\log\left(\sin\left(fx+e\right)+1\right)}{4\,b^2 f\cos\left(fx+e\right)^2}$$

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*sqrt(a/(a + b))*cos(f*x + e)^2*log(-(a*cos(f*x + e)^2 - 2*(a + b)
*sqrt(a/(a + b))*sin(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) - (2*a - b
)*cos(f*x + e)^2*log(sin(f*x + e) + 1) + (2*a - b)*cos(f*x + e)^2*log(-sin(
f*x + e) + 1) + 2*b*sin(f*x + e))/(b^2*f*cos(f*x + e)^2), -1/4*(4*a*sqrt(-a
/(a + b))*arctan(sqrt(-a/(a + b))*sin(f*x + e))*cos(f*x + e)^2 + (2*a - b)*
cos(f*x + e)^2*log(sin(f*x + e) + 1) - (2*a - b)*cos(f*x + e)^2*log(-sin(f*
x + e) + 1) - 2*b*sin(f*x + e))/(b^2*f*cos(f*x + e)^2)]
```

Sympy [F]

$$\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\sec^5{(e+fx)}}{a+b\sec^2{(e+fx)}}\,dx$$

```
[In] integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sec(e + f*x)**5/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.44

$$\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{2\,a^2\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}b^2} + \frac{(2\,a-b)\log(\sin(fx+e)+1)}{b^2} - \frac{(2\,a-b)\log(\sin(fx+e)-1)}{b^2} + \frac{2\,\sin(fx+e)}{b\sin(fx+e)^2-b}}{4\,f}$$

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*a^2*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((
a + b)*a)))/(sqrt((a + b)*a)*b^2) + (2*a - b)*log(sin(f*x + e) + 1)/b^2 - (
2*a - b)*log(sin(f*x + e) - 1)/b^2 + 2*sin(f*x + e)/(b*sin(f*x + e)^2 - b))
/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.31

$$\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{4\,a^2\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}b^2} + \frac{(2\,a-b)\log(|\sin(fx+e)+1|)}{b^2} - \frac{(2\,a-b)\log(|\sin(fx+e)-1|)}{b^2} + \frac{2\,\sin(fx+e)}{\left(\sin(fx+e)^2-1\right)b}}{4\,f}$$

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/4*(4*a^2*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*b^2)
+ (2*a - b)*log(abs(sin(f*x + e) + 1))/b^2 - (2*a - b)*log(abs(sin(f*x + e)
 - 1))/b^2 + 2*sin(f*x + e)/((sin(f*x + e)^2 - 1)*b))/f
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 591, normalized size of antiderivative = 6.87

$$\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{b\left(a\,\sin\left(e+f\,x\right)-a\,\mathrm{atanh}(\sin\left(e+f\,x\right))+a\,{\sin\left(e+f\,x\right)}^2\,\mathrm{atanh}(\sin\left(e+f\,x\right))\right)+b^2\,\left(\sin\left(e+f\,x\right)+\mathrm{ata}\right.}{}$$

```
[In] int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] (b*(a*sin(e + f*x) - a*atanh(sin(e + f*x)) + a*sin(e + f*x)^2*atanh(sin(e +
 f*x))) + atan((a^5*sin(e + f*x)*(a^3*b + a^4)^(1/2)*8i - b*sin(e + f*x)*(a
^3*b + a^4)^(3/2)*4i - a*sin(e + f*x)*(a^3*b + a^4)^(3/2)*8i - a^2*b^3*sin(
e + f*x)*(a^3*b + a^4)^(1/2)*2i + a^3*b^2*sin(e + f*x)*(a^3*b + a^4)^(1/2)*
1i + a*b^4*sin(e + f*x)*(a^3*b + a^4)^(1/2)*1i + a^4*b*sin(e + f*x)*(a^3*b
+ a^4)^(1/2)*12i)/(a^3*b^4 - a^2*b^5 + 5*a^4*b^3 + 3*a^5*b^2))*(a^3*b + a^4
)^(1/2)*2i + b^2*(sin(e + f*x) + atanh(sin(e + f*x)) - sin(e + f*x)^2*atanh
(sin(e + f*x))) - 2*a^2*atanh(sin(e + f*x)) - atan((a^5*sin(e + f*x)*(a^3*b
 + a^4)^(1/2)*8i - b*sin(e + f*x)*(a^3*b + a^4)^(3/2)*4i - a*sin(e + f*x)*(
a^3*b + a^4)^(3/2)*8i - a^2*b^3*sin(e + f*x)*(a^3*b + a^4)^(1/2)*2i + a^3*b
^2*sin(e + f*x)*(a^3*b + a^4)^(1/2)*1i + a*b^4*sin(e + f*x)*(a^3*b + a^4)^(
1/2)*1i + a^4*b*sin(e + f*x)*(a^3*b + a^4)^(1/2)*12i)/(a^3*b^4 - a^2*b^5 +
5*a^4*b^3 + 3*a^5*b^2))*sin(e + f*x)^2*(a^3*b + a^4)^(1/2)*2i + 2*a^2*sin(e
 + f*x)^2*atanh(sin(e + f*x)))/(f*(2*a*b^2 + 2*b^3 - 2*b^3*sin(e + f*x)^2 -
 2*a*b^2*sin(e + f*x)^2))
```

3.181 $\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 55

$$\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\operatorname{arctanh}(\sin(e+fx))}{bf} - \frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{b\sqrt{a+b}f}$$

```
[Out] arctanh(sin(f*x+e))/b/f-arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))*a^(1/2)/b/f
/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4232, 400, 212, 214}

$$\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\operatorname{arctanh}(\sin(e+fx))}{bf} - \frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{bf\sqrt{a+b}}$$

```
[In] Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ArcTanh[Sin[e + f*x]]/(b*f) - (Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[
a + b]])/(b*Sqrt[a + b]*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+b-ax^2)}\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(e+fx)\right)}{bf} - \frac{a\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{bf}\\
&= \frac{\operatorname{arctanh}(\sin(e+fx))}{bf} - \frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{b\sqrt{a+b}f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\operatorname{arctanh}(\sin(e+fx)) - \frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}}{bf}$$

```
[In] Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
[Out] (ArcTanh[Sin[e + f*x]] - (Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b
]])/Sqrt[a + b])/(b*f)
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{-\frac{a \operatorname{arctanh}\left(\frac{a \sin(fx+e)}{\sqrt{a(a+b)}}\right)}{b\sqrt{a(a+b)}}-\frac{\ln(\sin(fx+e)-1)}{2b}+\frac{\ln(\sin(fx+e)+1)}{2b}}{f}$
default	$\frac{-\frac{a \operatorname{arctanh}\left(\frac{a \sin(fx+e)}{\sqrt{a(a+b)}}\right)}{b\sqrt{a(a+b)}}-\frac{\ln(\sin(fx+e)-1)}{2b}+\frac{\ln(\sin(fx+e)+1)}{2b}}{f}$
risch	$\frac{\ln(e^{i(fx+e)}+i)}{bf}-\frac{\ln(e^{i(fx+e)}-i)}{bf}+\frac{\sqrt{a(a+b)}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{a(a+b)}\,e^{i(fx+e)}}{a}-1\right)}{2(a+b)fb}-\frac{\sqrt{a(a+b)}\,\ln\left(e^{2i(fx+e)}+\frac{2i}{}\right.}{2(a+b)}$

[In] int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)

[Out] 1/f*(-a/b/(a*(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2))-1/2/b*ln(si
n(f*x+e)-1)+1/2/b*ln(sin(f*x+e)+1))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.85

$$\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \left[\frac{\sqrt{\frac{a}{a+b}}\log\left(-\frac{a\cos(fx+e)^2+2\,(a+b)\sqrt{\frac{a}{a+b}}\sin(fx+e)-2\,a-b}{a\cos(fx+e)^2+b}\right)+\log\left(\sin\left(fx+e\right)+1\right)-\log\left(-\sin\left(fx+e\right)+1\right)}{2\,bf},\ \frac{2\,\sqrt{}}{}\right.$$

[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")

[Out] [1/2*(sqrt(a/(a + b))*log(-(a*cos(f*x + e)^2 + 2*(a + b)*sqrt(a/(a + b))*si
n(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + log(sin(f*x + e) + 1) - log
(-sin(f*x + e) + 1))/(b*f), 1/2*(2*sqrt(-a/(a + b))*arctan(sqrt(-a/(a + b))
*sin(f*x + e)) + log(sin(f*x + e) + 1) - log(-sin(f*x + e) + 1))/(b*f)]

Sympy [F]

$$\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\sec^3{(e+fx)}}{a+b\sec^2{(e+fx)}}\,dx$$

```
[In] integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
[Out] Integral(sec(e + f*x)**3/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.51

$$\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{a\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}b} + \frac{\log(\sin(fx+e)+1)}{b} - \frac{\log(\sin(fx+e)-1)}{b}}{2f}$$

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] 1/2*(a*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((a + b
)*a)))/(sqrt((a + b)*a)*b) + log(sin(f*x + e) + 1)/b - log(sin(f*x + e) - 1
)/b)/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.35

$$\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{2\,a\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}b} + \frac{\log(|\sin(fx+e)+1|)}{b} - \frac{\log(|\sin(fx+e)-1|)}{b}}{2f}$$

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] 1/2*(2*a*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*b) + log
(abs(sin(f*x + e) + 1))/b - log(abs(sin(f*x + e) - 1))/b)/f
```

Mupad [B] (verification not implemented)

Time = 18.74 (sec) , antiderivative size = 456, normalized size of antiderivative = 8.29

$$\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\operatorname{atanh}(\sin(e+f\,x))}{b\,f} + \frac{\operatorname{atan}\left(\frac{\frac{\left(2\,a^3\,\sin(e+f\,x)+\frac{\left(2\,a^2\,b^2-\frac{\sin(e+f\,x)\,\left(16\,a^3\,b^2+8\,a^2\,b^3\right)\,\sqrt{a\,(a+b)}}{4\,\left(b^2+a\,b\right)}\right)\,\sqrt{a\,(a+b)}}{2\,\left(b^2+a\,b\right)}\right)\,\sqrt{a\,(a+b)}\,1\mathrm{i}}{b^2+a\,b}+\frac{\left(2\,a^3\,\sin(e+f\,x)-\frac{\left(2\,a^2\,b^2+\frac{\sin(e+f\,x)\,\left(16\,a^3\,b^2\right.}{4\,\left(b^2\right.}\right.}{2\,\left(b^2\right.}\right.}{b^2+a}}{\frac{\left(2\,a^3\,\sin(e+f\,x)+\frac{\left(2\,a^2\,b^2-\frac{\sin(e+f\,x)\,\left(16\,a^3\,b^2+8\,a^2\,b^3\right)\,\sqrt{a\,(a+b)}}{4\,\left(b^2+a\,b\right)}\right)\,\sqrt{a\,(a+b)}}{2\,\left(b^2+a\,b\right)}\right)\,\sqrt{a\,(a+b)}}{b^2+a\,b}-\frac{\left(2\,a^3\,\sin(e+f\,x)-\frac{\left(2\,a^2\,b^2+\frac{\sin(e+f\,x)\,\left(16\,a^3\,b^2\right.}{4\,\left(b^2\right.}\right.}{2\,\left(b^2\right.}\right.}{b^2+a\,b}}\right.}{f\,\left(b^2+a\,b\right)}$$

```
[In] int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] atanh(sin(e + f*x))/(b*f) + (atan((((2*a^3*sin(e + f*x) + ((2*a^2*b^2 - (si
n(e + f*x)*(8*a^2*b^3 + 16*a^3*b^2)*(a*(a + b))^(1/2))/(4*(a*b + b^2)))*(a*
(a + b))^(1/2))/(2*(a*b + b^2)))*(a*(a + b))^(1/2)*1i)/(a*b + b^2) + ((2*a^
3*sin(e + f*x) - ((2*a^2*b^2 + (sin(e + f*x)*(8*a^2*b^3 + 16*a^3*b^2)*(a*(a
 + b))^(1/2))/(4*(a*b + b^2)))*(a*(a + b))^(1/2))/(2*(a*b + b^2)))*(a*(a +
b))^(1/2)*1i)/(a*b + b^2))/(((2*a^3*sin(e + f*x) + ((2*a^2*b^2 - (sin(e + f
*x)*(8*a^2*b^3 + 16*a^3*b^2)*(a*(a + b))^(1/2))/(4*(a*b + b^2)))*(a*(a + b)
)^(1/2))/(2*(a*b + b^2)))*(a*(a + b))^(1/2))/(a*b + b^2) - ((2*a^3*sin(e +
f*x) - ((2*a^2*b^2 + (sin(e + f*x)*(8*a^2*b^3 + 16*a^3*b^2)*(a*(a + b))^(1/
2))/(4*(a*b + b^2)))*(a*(a + b))^(1/2))/(2*(a*b + b^2)))*(a*(a + b))^(1/2))
/(a*b + b^2)))*(a*(a + b))^(1/2)*1i)/(f*(a*b + b^2))
```

3.182 $\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}f}$$

```
[Out] arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/f/a^(1/2)/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4232, 214}

$$\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}f\sqrt{a+b}}$$

```
[In] Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2),x]
[Out] ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b]*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
```

```
n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{f}$$
$$= \frac{\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}f}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}f}$$

```
[In] Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2),x]
[Out] ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b]*f)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\text{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{f\sqrt{a(a+b)}}$	28
default	$\frac{\text{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{f\sqrt{a(a+b)}}$	28
risch	$\frac{\ln\left(\mathrm{e}^{2i(fx+e)}+\frac{2i(a+b)\mathrm{e}^{i(fx+e)}}{\sqrt{a^2+ab}}-1\right)}{2\sqrt{a^2+ab}\,f}-\frac{\ln\left(\mathrm{e}^{2i(fx+e)}-\frac{2i(a+b)\mathrm{e}^{i(fx+e)}}{\sqrt{a^2+ab}}-1\right)}{2\sqrt{a^2+ab}\,f}$	102

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f/(a*(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.25

$$\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx = \left[\frac{\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{a^2+ab}\sin(fx+e)-2a-b}{a\cos(fx+e)^2+b}\right)}{2\sqrt{a^2+ab}f},\ -\frac{\sqrt{-a^2-ab}\arctan\left(\frac{\sqrt{-a^2-ab}\sin(fx+e)}{a+b}\right)}{(a^2+ab)f}\right]$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*
cos(f*x + e)^2 + b))/(sqrt(a^2 + a*b)*f), -sqrt(-a^2 - a*b)*arctan(sqrt(-a^
2 - a*b)*sin(f*x + e)/(a + b))/((a^2 + a*b)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sec(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.39

$$\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{2\sqrt{(a+b)a}f}$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((a + b)*
a)))/(sqrt((a + b)*a)*f)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}f}$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] -arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*f)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)}{\sqrt{a}\,f\,\sqrt{a+b}}$$

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)),x)
[Out] atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))/(a^(1/2)*f*(a + b)^(1/2))
```

3.183 $\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 52

$$\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{3/2}\sqrt{a+b}f}+\frac{\sin(e+fx)}{af}$$

```
[Out] sin(f*x+e)/a/f-b*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(3/2)/f/(a+b)^(1
/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4232, 396, 214}

$$\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\sin(e+fx)}{af}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{3/2}f\sqrt{a+b}}$$

```
[In] Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((b*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a^(3/2)*Sqrt[a + b]*f))
+ Sin[e + f*x]/(a*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1-x^2}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{\sin(e+fx)}{af} - \frac{b\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{af}\\
&= -\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{3/2}\sqrt{a+b}f} + \frac{\sin(e+fx)}{af}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$
\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + \sqrt{a}\sin(e+fx)}{a^{3/2}f}
$$

```
[In] Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2),x]
[Out] (-((b*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/Sqrt[a + b]) + Sqrt[a]*S
in[e + f*x])/(a^(3/2)*f)
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{\frac{\sin(fx+e)}{a}-\frac{b\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{a\sqrt{a(a+b)}}}{f}$	45
default	$\frac{\frac{\sin(fx+e)}{a}-\frac{b\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{a\sqrt{a(a+b)}}}{f}$	45
risch	$-\frac{i\mathrm{e}^{i(fx+e)}}{2af}+\frac{i\mathrm{e}^{-i(fx+e)}}{2af}+\frac{b\ln\left(\mathrm{e}^{2i(fx+e)}-\frac{2i(a+b)\mathrm{e}^{i(fx+e)}}{\sqrt{a^2+ab}}-1\right)}{2\sqrt{a^2+ab}\,fa}-\frac{b\ln\left(\mathrm{e}^{2i(fx+e)}+\frac{2i(a+b)\mathrm{e}^{i(fx+e)}}{\sqrt{a^2+ab}}-1\right)}{2\sqrt{a^2+ab}\,fa}$	146

```
[In] int(cos(f*x+e)/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(sin(f*x+e)/a-b/a/(a*(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2))
)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 164, normalized size of antiderivative = 3.15

$$\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx = \left[\frac{\sqrt{a^2+ab}b\log\left(-\frac{a\cos(fx+e)^2+2\sqrt{a^2+ab}\sin(fx+e)-2a-b}{a\cos(fx+e)^2+b}\right)+2\left(a^2+ab\right)\sin\left(fx+e\right)}{2\left(a^3+a^2b\right)f}, \frac{\sqrt{-a^2-ab}b\arctan\left(\frac{\sqrt{-a}\right.}{}\right.}{}\right.$$

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a^2 + a*b)*b*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(f*x
+ e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(a^2 + a*b)*sin(f*x + e))/((a^3
 + a^2*b)*f), (sqrt(-a^2 - a*b)*b*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a +
 b)) + (a^2 + a*b)*sin(f*x + e))/((a^3 + a^2*b)*f)]
```

Sympy [F]

$$\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\cos{(e+fx)}}{a+b\sec^2{(e+fx)}}\,dx$$

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2),x)
[Out] Integral(cos(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.29

$$\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{b\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}a} + \frac{2\sin(fx+e)}{a}}{2f}$$

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] 1/2*(b*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((a + b
)*a)))/(sqrt((a + b)*a)*a) + 2*sin(f*x + e)/a)/f
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{b\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}a} + \frac{\sin(fx+e)}{a}}{f}$$

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] (b*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*a) + sin(f*x +
 e)/a)/f
```

Mupad [B] (verification not implemented)

Time = 18.39 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx=\frac{\sin\left(e+f\,x\right)}{a\,f}-\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin\left(e+f\,x\right)}{\sqrt{a+b}}\right)}{a^{3/2}\,f\,\sqrt{a+b}}$$

```
[In] int(cos(e + f*x)/(a + b/cos(e + f*x)^2),x)
```

```
[Out] sin(e + f*x)/(a*f) - (b*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2)))/(a^(3/
2)*f*(a + b)^(1/2))
```

3.184 $\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{b^2\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}f} + \frac{(a-b)\sin(e+fx)}{a^2f} - \frac{\sin^3(e+fx)}{3af}$$

```
[Out] (a-b)*sin(f*x+e)/a^2/f-1/3*sin(f*x+e)^3/a/f+b^2*arctanh(sin(f*x+e)*a^(1/2)/
(a+b)^(1/2))/a^(5/2)/f/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4232, 398, 214}

$$\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{b^2\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{5/2}f\sqrt{a+b}} + \frac{(a-b)\sin(e+fx)}{a^2f} - \frac{\sin^3(e+fx)}{3af}$$

```
[In] Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (b^2*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a^(5/2)*Sqrt[a + b]*f) +
 ((a - b)*Sin[e + f*x])/(a^2*f) - Sin[e + f*x]^3/(3*a*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \left(\frac{a-b}{a^2}-\frac{x^2}{a}+\frac{b^2}{a^2(a+b-ax^2)}\right)dx, x, \sin(e+fx)\right)}{f} \\
&= \frac{(a-b)\sin(e+fx)}{a^2f}-\frac{\sin^3(e+fx)}{3af}+\frac{b^2\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{a^2f} \\
&= \frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}f}+\frac{(a-b)\sin(e+fx)}{a^2f}-\frac{\sin^3(e+fx)}{3af}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.38

$$\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{6b^2\left(-\log\left(\sqrt{a+b}-\sqrt{a}\sin(e+fx)\right)+\log\left(\sqrt{a+b}+\sqrt{a}\sin(e+fx)\right)\right)}{\sqrt{a+b}}+3\sqrt{a}(3a-4b)\sin(e+fx)+a^{3/2}\sin(3(e+fx))}{12a^{5/2}f}$$

```
[In] Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((6*b^2*(-Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x]] + Log[Sqrt[a + b] + Sqrt[
a]*Sin[e + f*x]]))/Sqrt[a + b] + 3*Sqrt[a]*(3*a - 4*b)*Sin[e + f*x] + a^(3/
2)*Sin[3*(e + f*x)])/(12*a^(5/2)*f)
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{-\frac{\frac{a\sin(fx+e)^3}{3}-\sin(fx+e)a+\sin(fx+e)b}{a^2}+\frac{b^2\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{a^2\sqrt{a(a+b)}}}{f}$
default	$\frac{-\frac{\frac{a\sin(fx+e)^3}{3}-\sin(fx+e)a+\sin(fx+e)b}{a^2}+\frac{b^2\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{a^2\sqrt{a(a+b)}}}{f}$
risch	$-\frac{3i\mathrm{e}^{i(fx+e)}}{8af}+\frac{i\mathrm{e}^{i(fx+e)}b}{2a^2f}+\frac{3i\mathrm{e}^{-i(fx+e)}}{8af}-\frac{i\mathrm{e}^{-i(fx+e)}b}{2a^2f}+\frac{b^2\ln\left(\mathrm{e}^{2i(fx+e)}+\frac{2i(a+b)\mathrm{e}^{i(fx+e)}}{\sqrt{a^2+ab}}-1\right)}{2\sqrt{a^2+ab}\,f\,a^2}-\frac{b^2\ln\left(\mathrm{e}^{2i(fx\right.}}{}$

```
[In] int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^2*(1/3*a*sin(f*x+e)^3-sin(f*x+e)*a+sin(f*x+e)*b)+b^2/a^2/(a*(a+b)
)^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 230, normalized size of antiderivative = 3.03

$$\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \left[\frac{3\sqrt{a^2+ab}b^2\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{a^2+ab}\sin(fx+e)-2a-b}{a\cos(fx+e)^2+b}\right)+2\left(2a^3-a^2b-3ab^2+(a^3+a^2b)\cos(fx+e)^2\right)}{6(a^4+a^3b)f}\right.$$
$$\left.-\frac{3\sqrt{-a^2-ab}b^2\arctan\left(\frac{\sqrt{-a^2-ab}\sin(fx+e)}{a+b}\right)-\left(2a^3-a^2b-3ab^2+(a^3+a^2b)\cos(fx+e)^2\right)\sin(fx+e)}{3(a^4+a^3b)f}\right]$$

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(a^2 + a*b)*b^2*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(
f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(2*a^3 - a^2*b - 3*a*b^2 +
(a^3 + a^2*b)*cos(f*x + e)^2)*sin(f*x + e))/((a^4 + a^3*b)*f), -1/3*(3*sqrt
(-a^2 - a*b)*b^2*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) - (2*a^3 - a
^2*b - 3*a*b^2 + (a^3 + a^2*b)*cos(f*x + e)^2)*sin(f*x + e))/((a^4 + a^3*b)
*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16

$$\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{3\,b^2\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}a^2}+\frac{2\left(a\sin(fx+e)^3-3\,(a-b)\sin(fx+e)\right)}{a^2}}{6\,f}$$

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/6*(3*b^2*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((
a + b)*a)))/(sqrt((a + b)*a)*a^2) + 2*(a*sin(f*x + e)^3 - 3*(a - b)*sin(f*x
 + e))/a^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.12

$$\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{3\,b^2\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}a^2}+\frac{a^2\sin(fx+e)^3-3\,a^2\sin(fx+e)+3\,ab\sin(fx+e)}{a^3}}{3\,f}$$

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*a^2)
+ (a^2*sin(f*x + e)^3 - 3*a^2*sin(f*x + e) + 3*a*b*sin(f*x + e))/a^3)/f
```

Mupad [B] (verification not implemented)

Time = 18.60 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{b^2\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)}{a^{5/2}\,f\,\sqrt{a+b}} - \frac{\sin\left(e+f\,x\right)^3}{3\,a\,f} - \frac{\sin\left(e+f\,x\right)\,\left(\frac{a+b}{a^2}-\frac{2}{a}\right)}{f}$$

```
[In] int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2),x)
```

```
[Out] (b^2*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2)))/(a^(5/2)*f*(a + b)^(1/2))
 - sin(e + f*x)^3/(3*a*f) - (sin(e + f*x)*((a + b)/a^2 - 2/a))/f
```

3.185 $\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{b^3\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{7/2}\sqrt{a+b}f} + \frac{(a^2-ab+b^2)\sin(e+fx)}{a^3f} - \frac{(2a-b)\sin^3(e+fx)}{3a^2f} + \frac{\sin^5(e+fx)}{5af}$$

```
[Out] (a^2-a*b+b^2)*sin(f*x+e)/a^3/f-1/3*(2*a-b)*sin(f*x+e)^3/a^2/f+1/5*sin(f*x+e
)^5/a/f-b^3*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(7/2)/f/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4232, 398, 214}

$$\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{b^3\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{7/2}f\sqrt{a+b}} - \frac{(2a-b)\sin^3(e+fx)}{3a^2f} + \frac{(a^2-ab+b^2)\sin(e+fx)}{a^3f} + \frac{\sin^5(e+fx)}{5af}$$

```
[In] Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((b^3*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a^(7/2)*Sqrt[a + b]*f)
) + ((a^2 - a*b + b^2)*Sin[e + f*x])/(a^3*f) - ((2*a - b)*Sin[e + f*x]^3)/(
3*a^2*f) + Sin[e + f*x]^5/(5*a*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a^2-ab+b^2}{a^3}-\frac{(2a-b)x^2}{a^2}+\frac{x^4}{a}-\frac{b^3}{a^3(a+b-ax^2)}\right)\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{(a^2-ab+b^2)\sin(e+fx)}{a^3f}-\frac{(2a-b)\sin^3(e+fx)}{3a^2f}\\
&\quad+\frac{\sin^5(e+fx)}{5af}-\frac{b^3\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{a^3f}\\
&= -\frac{b^3\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{7/2}\sqrt{a+b}f}+\frac{(a^2-ab+b^2)\sin(e+fx)}{a^3f}\\
&\quad-\frac{(2a-b)\sin^3(e+fx)}{3a^2f}+\frac{\sin^5(e+fx)}{5af}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.26

$$\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{120b^3\left(\log\left(\sqrt{a+b}-\sqrt{a}\sin(e+fx)\right)-\log\left(\sqrt{a+b}+\sqrt{a}\sin(e+fx)\right)\right)}{\sqrt{a+b}}+30\sqrt{a}(5a^2-6ab+8b^2)\sin(e+fx)+5a^{3/2}(5a-4b)\sin}{240a^{7/2}f}$$

```
[In] Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((120*b^3*(Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x]] - Log[Sqrt[a + b] + Sqrt
[a]*Sin[e + f*x]]))/Sqrt[a + b] + 30*Sqrt[a]*(5*a^2 - 6*a*b + 8*b^2)*Sin[e
+ f*x] + 5*a^(3/2)*(5*a - 4*b)*Sin[3*(e + f*x)] + 3*a^(5/2)*Sin[5*(e + f*x)
])/(240*a^(7/2)*f)
```

Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{\frac{\frac{a^2\sin(fx+e)^5}{5}-\frac{2a^2\sin(fx+e)^3}{3}+\frac{a\sin(fx+e)^3b}{3}+\sin(fx+e)a^2-\sin(fx+e)ab+\sin(fx+e)b^2}{a^3}-\frac{b^3\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{a^3\sqrt{a(a+b)}}}{f}$
default	$\frac{\frac{\frac{a^2\sin(fx+e)^5}{5}-\frac{2a^2\sin(fx+e)^3}{3}+\frac{a\sin(fx+e)^3b}{3}+\sin(fx+e)a^2-\sin(fx+e)ab+\sin(fx+e)b^2}{a^3}-\frac{b^3\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{a^3\sqrt{a(a+b)}}}{f}$
risch	$-\frac{5i e^{i(fx+e)}}{16af}+\frac{3i e^{i(fx+e)}b}{8a^2f}-\frac{i e^{i(fx+e)}b^2}{2a^3f}+\frac{5i e^{-i(fx+e)}}{16af}-\frac{3i e^{-i(fx+e)}b}{8a^2f}+\frac{i e^{-i(fx+e)}b^2}{2a^3f}+\frac{b^3\ln\left(e^{2i(fx+e)}-\right.}{2\sqrt{a^2}}$

```
[In] int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^3*(1/5*a^2*sin(f*x+e)^5-2/3*a^2*sin(f*x+e)^3+1/3*a*sin(f*x+e)^3*b+
sin(f*x+e)*a^2-sin(f*x+e)*a*b+sin(f*x+e)*b^2)-b^3/a^3/(a*(a+b))^(1/2)*arcta
nh(a*sin(f*x+e)/(a*(a+b))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.82

$$\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \left[\frac{15\sqrt{a^2+ab}b^3\log\left(-\frac{a\cos(fx+e)^2+2\sqrt{a^2+ab}\sin(fx+e)-2a-b}{a\cos(fx+e)^2+b}\right)+2\left(3\left(a^4+a^3b\right)\cos\left(fx+e\right)^4+8a^4-2a^3b+5a\right.}{30\left(a^5+a^4b\right)f}\right.$$

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(a^2 + a*b)*b^3*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*si
n(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(3*(a^4 + a^3*b)*cos(f*x
+ e)^4 + 8*a^4 - 2*a^3*b + 5*a^2*b^2 + 15*a*b^3 + (4*a^4 - a^3*b - 5*a^2*b^
2)*cos(f*x + e)^2)*sin(f*x + e))/((a^5 + a^4*b)*f), 1/15*(15*sqrt(-a^2 - a*
b)*b^3*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) + (3*(a^4 + a^3*b)*cos
(f*x + e)^4 + 8*a^4 - 2*a^3*b + 5*a^2*b^2 + 15*a*b^3 + (4*a^4 - a^3*b - 5*a
^2*b^2)*cos(f*x + e)^2)*sin(f*x + e))/((a^5 + a^4*b)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.08

$$\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{15\,b^3\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}a^3}+\frac{2\left(3a^2\sin(fx+e)^5-5\left(2a^2-ab\right)\sin(fx+e)^3+15\left(a^2-ab+b^2\right)\sin(fx+e)\right)}{a^3}}{30f}$$

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/30*(15*b^3*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt(
(a + b)*a)))/(sqrt((a + b)*a)*a^3) + 2*(3*a^2*sin(f*x + e)^5 - 5*(2*a^2 - a
*b)*sin(f*x + e)^3 + 15*(a^2 - a*b + b^2)*sin(f*x + e))/a^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.19

$$\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{15\,b^3\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}a^3} + \frac{3\,a^4\sin(fx+e)^5-10\,a^4\sin(fx+e)^3+5\,a^3b\sin(fx+e)^3+15\,a^4\sin(fx+e)-15\,a^3b\sin(fx+e)+15\,a^2b^2\sin(fx+e)}{a^5}}{15\,f}$$

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(15*b^3*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*a^3)
 + (3*a^4*sin(f*x + e)^5 - 10*a^4*sin(f*x + e)^3 + 5*a^3*b*sin(f*x + e)^3 +
 15*a^4*sin(f*x + e) - 15*a^3*b*sin(f*x + e) + 15*a^2*b^2*sin(f*x + e))/a^5
)/f
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.03

$$\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\sin(e+f\,x)\,\left(\frac{3}{a}+\frac{(a+b)\,\left(\frac{a+b}{a^2}-\frac{3}{a}\right)}{a}\right)}{f} + \frac{\sin(e+f\,x)^5}{5\,a\,f} + \frac{\sin(e+f\,x)^3\,\left(\frac{a+b}{3\,a^2}-\frac{1}{a}\right)}{f} - \frac{b^3\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)}{a^{7/2}\,f\,\sqrt{a+b}}$$

```
[In] int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2),x)
```

```
[Out] (sin(e + f*x)*(3/a + ((a + b)*((a + b)/a^2 - 3/a))/a))/f + sin(e + f*x)^5/(
5*a*f) + (sin(e + f*x)^3*((a + b)/(3*a^2) - 1/a))/f - (b^3*atanh((a^(1/2)*s
in(e + f*x))/(a + b)^(1/2)))/(a^(7/2)*f*(a + b)^(1/2))
```

3.186 $\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx=\frac{a^2\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}f}-\frac{(a-b)\tan(e+fx)}{b^2f}+\frac{\tan^3(e+fx)}{3bf}$$

```
[Out] a^2*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(5/2)/f/(a+b)^(1/2)-(a-b)*tan(
f*x+e)/b^2/f+1/3*tan(f*x+e)^3/b/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4231, 398, 211}

$$\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx=\frac{a^2\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{5/2}f\sqrt{a+b}}-\frac{(a-b)\tan(e+fx)}{b^2f}+\frac{\tan^3(e+fx)}{3bf}$$

```
[In] Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (a^2*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(b^(5/2)*Sqrt[a + b]*f) -
((a - b)*Tan[e + f*x])/(b^2*f) + Tan[e + f*x]^3/(3*b*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{a-b}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2(a+b+bx^2)}\right)dx, x, \tan(e+fx)\right)}{f}\\ &= -\frac{(a-b)\tan(e+fx)}{b^2f}+\frac{\tan^3(e+fx)}{3bf}+\frac{a^2\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{b^2f}\\ &= \frac{a^2\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}f}-\frac{(a-b)\tan(e+fx)}{b^2f}+\frac{\tan^3(e+fx)}{3bf}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.79 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.91

$$\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(-3a^2\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cdots}{6b^2\sqrt{a+b}f\,(a\cdots}$$

```
[In] Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(-3*a^2*ArcTan[(Sec[f*x]*(Co
s[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a
+ b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a + b]*
Sec[e + f*x]*Sqrt[b*(I*Cos[e] + Sin[e])^4]*(Sec[e]*(-3*a + 2*b + b*Sec[e +
f*x]^2)*Sin[f*x] + b*Sec[e + f*x]*Tan[e])))/(6*b^2*Sqrt[a + b]*f*(a + b*Sec
[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{-\frac{-\frac{b\tan(fx+e)^3}{3}+a\tan(fx+e)-b\tan(fx+e)}{b^2}+\frac{a^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^2\sqrt{(a+b)b}}}{f}$
default	$\frac{-\frac{-\frac{b\tan(fx+e)^3}{3}+a\tan(fx+e)-b\tan(fx+e)}{b^2}+\frac{a^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^2\sqrt{(a+b)b}}}{f}$
risch	$-\frac{2i\left(3a\,e^{4i(fx+e)}+6a\,e^{2i(fx+e)}-6b\,e^{2i(fx+e)}+3a-2b\right)}{3f\,b^2\left(e^{2i(fx+e)}+1\right)^3}-\frac{a^2\ln\left(e^{2i(fx+e)}+\frac{2iba+2ib^2+a\sqrt{-ab-b^2}+2b\sqrt{-ab-b^2}}{a\sqrt{-ab-b^2}}\right)}{2\sqrt{-ab-b^2}\,f\,b^2}+\frac{a^2}{}$

[In] int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)

[Out] 1/f*(-1/b^2*(-1/3*b*tan(f*x+e)^3+a*tan(f*x+e)-b*tan(f*x+e))+a^2/b^2/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(67) = 134.

Time = 0.28 (sec) , antiderivative size = 354, normalized size of antiderivative = 4.60

$$\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$=\left[-\frac{3\sqrt{-ab-b^2}a^2\cos(fx+e)^3\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4-2(3ab+4b^2)\cos(fx+e)^2+4\left((a+2b)\cos(fx+e)^3-b\cos(fx+e)\right)}{a^2\cos(fx+e)^4+2ab\cos(fx+e)^2+b^2}\right.}{12(ab^3+b^4)f\cos(fx+}\right.$$
$$-\frac{3\sqrt{ab+b^2}a^2\arctan\left(\frac{(a+2b)\cos(fx+e)^2-b}{2\sqrt{ab+b^2}\cos(fx+e)\sin(fx+e)}\right)\cos(fx+e)^3-2\left(ab^2+b^3-(3a^2b+ab^2-2b^3)\cos(fx\right.}{6(ab^3+b^4)f\cos(fx+e)^3}$$

[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")

[Out] [-1/12*(3*sqrt(-a*b - b^2)*a^2*cos(f*x + e)^3*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 4*(a*b^2 + b^3 - (3*a^2*b + a*b^2 - 2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a*b^3 + b^4)*f*cos(f*x + e)^3), -1/6*(3*sqrt(a*b + b^2)*a^2*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e)^3 - 2*(a*b^2 + b^3 - (3*a^2*b + a*b^2 - 2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a*b^3 + b^4)*f*cos(f*x + e)^3)]

Sympy [F]

$$\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx=\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

```
[In] integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
[Out] Integral(sec(e + f*x)**6/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

$$\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx=\frac{\frac{3\,a^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}b^2}+\frac{b\tan(fx+e)^3-3\,(a-b)\tan(fx+e)}{b^2}}{3\,f}$$

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] 1/3*(3*a^2*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*b^2) + (
b*tan(f*x + e)^3 - 3*(a - b)*tan(f*x + e))/b^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.25

$$\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx=\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)a^2}{\sqrt{ab+b^2}b^2}+\frac{b^2\tan(fx+e)^3-3\,ab\tan(fx+e)+3\,b^2\tan(fx+e)}{b^3}}{3\,f}$$

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] 1/3*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*
b + b^2)))*a^2/(sqrt(a*b + b^2)*b^2) + (b^2*tan(f*x + e)^3 - 3*a*b*tan(f*x
+ e) + 3*b^2*tan(f*x + e))/b^3)/f
```

Mupad [B] (verification not implemented)

Time = 18.55 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx=\frac{\tan(e+f\,x)^3}{3\,b\,f}-\frac{\tan(e+f\,x)\,\left(\frac{a+b}{b^2}-\frac{2}{b}\right)}{f}+\frac{a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(e+f\,x)}{\sqrt{a+b}}\right)}{b^{5/2}\,f\,\sqrt{a+b}}$$

```
[In] int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] tan(e + f*x)^3/(3*b*f) - (tan(e + f*x)*((a + b)/b^2 - 2/b))/f + (a^2*atan((
b^(1/2)*tan(e + f*x))/(a + b)^(1/2)))/(b^(5/2)*f*(a + b)^(1/2))
```

3.187 $\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 52

$$\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx=-\frac{a\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}f}+\frac{\tan(e+fx)}{bf}$$

```
[Out] -a*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(3/2)/f/(a+b)^(1/2)+tan(f*x+e)/
b/f
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4231, 396, 211}

$$\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx=\frac{\tan(e+fx)}{bf}-\frac{a\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{3/2}f\sqrt{a+b}}$$

```
[In] Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((a*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(b^(3/2)*Sqrt[a + b]*f)) +
 Tan[e + f*x]/(b*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1+x^2}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\tan(e+fx)}{bf} - \frac{a\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{bf} \\
&= -\frac{a\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}f} + \frac{\tan(e+fx)}{bf}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.65 (sec) , antiderivative size = 192, normalized size of antiderivative = 3.69

$$\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(a\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e\right.}{2b\sqrt{a+b}f\left(a+b\sec^2(e+fx)\right)\sqrt{b(\cos(e)-}}$$

```
[In] Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(a*ArcTan[(Sec[f*x]*(Cos[2*e
] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*
Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a + b]*Sec[e
]*Sec[e + f*x]*Sqrt[b*(I*Cos[e] + Sin[e])^4]*Sin[f*x]))/(2*b*Sqrt[a + b]*f*
(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)}{b}-\frac{a\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b\sqrt{(a+b)b}}}{f}$
default	$\frac{\frac{\tan(fx+e)}{b}-\frac{a\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b\sqrt{(a+b)b}}}{f}$
risch	$\frac{2i}{fb(e^{2i(fx+e)}+1)}-\frac{a\ln\left(e^{2i(fx+e)}+\frac{-2iba-2ib^2+a\sqrt{-ab-b^2}+2b\sqrt{-ab-b^2}}{a\sqrt{-ab-b^2}}\right)}{2\sqrt{-ab-b^2}\,fb}+\frac{a\ln\left(e^{2i(fx+e)}+\frac{2iba+2ib^2+a\sqrt{-ab-b^2}}{a\sqrt{-ab-b^2}}\right.}{2\sqrt{-ab-b^2}\,fb}$

```
[In] int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(tan(f*x+e)/b-a/b/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(44) = 88.

Time = 0.28 (sec) , antiderivative size = 286, normalized size of antiderivative = 5.50

$$\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$=\left[-\frac{\sqrt{-ab-b^2}a\cos(fx+e)\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4-2(3ab+4b^2)\cos(fx+e)^2-4\left((a+2b)\cos(fx+e)^3-b\cos(fx+e)\right)\sqrt{-}}{a^2\cos(fx+e)^4+2ab\cos(fx+e)^2+b^2}\right.}{4(ab^2+b^3)f\cos(fx+e)}\right.$$

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a*b - b^2)*a*cos(f*x + e)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x +
 e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^3 - b*
cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*
a*b*cos(f*x + e)^2 + b^2)) - 4*(a*b + b^2)*sin(f*x + e))/((a*b^2 + b^3)*f*c
os(f*x + e)), 1/2*(sqrt(a*b + b^2)*a*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 -
 b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e) + 2*(a*b + b^
2)*sin(f*x + e))/((a*b^2 + b^3)*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

```
[In] integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
[Out] Integral(sec(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{a\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}b} - \frac{\tan(fx+e)}{b}}{f}$$

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] -(a*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*b) - tan(f*x +
e)/b)/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.27

$$\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)a}{\sqrt{ab+b^2}b} - \frac{\tan(fx+e)}{b}}{f}$$

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] -((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b
^2)))*a/(sqrt(a*b + b^2)*b) - tan(f*x + e)/b)/f
```

Mupad [B] (verification not implemented)

Time = 18.46 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\tan(e+f\,x)}{b\,f}-\frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(e+f\,x)}{\sqrt{a+b}}\right)}{b^{3/2}\,f\,\sqrt{a+b}}$$

```
[In] int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] tan(e + f*x)/(b*f) - (a*atan((b^(1/2)*tan(e + f*x))/(a + b)^(1/2)))/(b^(3/2
)*f*(a + b)^(1/2))
```

3.188 $\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 36

$$\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}f}$$

```
[Out] arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/f/b^(1/2)/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4231, 211}

$$\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}f\sqrt{a+b}}$$

```
[In] Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
[Out] ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
```

```
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}f}$$

```
[In] Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
[Out] ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*f)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{f\sqrt{(a+b)b}}$	28
default	$\frac{\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{f\sqrt{(a+b)b}}$	28
risch	$-\frac{\ln\left(e^{2i(fx+e)}+\frac{2iba+2ib^2+a\sqrt{-ab-b^2}+2b\sqrt{-ab-b^2}}{a\sqrt{-ab-b^2}}\right)}{2\sqrt{-ab-b^2}\,f}+\frac{\ln\left(e^{2i(fx+e)}+\frac{-2iba-2ib^2+a\sqrt{-ab-b^2}+2b\sqrt{-ab-b^2}}{a\sqrt{-ab-b^2}}\right)}{2\sqrt{-ab-b^2}\,f}$	172

```
[In] int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(28) = 56$.

Time = 0.26 (sec) , antiderivative size = 209, normalized size of antiderivative = 5.81

$$\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[-\frac{\sqrt{-ab-b^2}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\,\left((a+2\,b)\cos(fx+e)^3-b\cos(fx+e)\right)\sqrt{-ab-b^2}\sin(fx+e)+b}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right.}{4\,(ab+b^2)f}\right.$$
$$\left.-\frac{\arctan\left(\frac{(a+2\,b)\cos(fx+e)^2-b}{2\,\sqrt{ab+b^2}\cos(fx+e)\sin(fx+e)}\right)}{2\,\sqrt{ab+b^2}f}\right]$$

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b
 + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sq
rt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)
^2 + b^2))/((a*b + b^2)*f), -1/2*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/
(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e)))/(sqrt(a*b + b^2)*f)]
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

```
[In] integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}f}$$

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*f)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

$$\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx=\frac{\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)}{\sqrt{ab+b^2}f}$$

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] (pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2
)))/(sqrt(a*b + b^2)*f)
```

Mupad [B] (verification not implemented)

Time = 18.86 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx=\frac{\operatorname{atan}\left(\frac{b\tan(e+f\,x)}{\sqrt{b^2+a\,b}}\right)}{f\,\sqrt{b^2+a\,b}}$$

```
[In] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] atan((b*tan(e + f*x))/(a*b + b^2)^(1/2))/(f*(a*b + b^2)^(1/2))
```

3.189 $\int \frac{1}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 45

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx=\frac{x}{a}+\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{a\sqrt{a+b}f}$$

```
[Out] x/a+arctan(cot(f*x+e)*(a+b)^(1/2)/b^(1/2))*b^(1/2)/a/f/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4212, 3260, 211}

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx=\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{af\sqrt{a+b}}+\frac{x}{a}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-1),x]
```

```
[Out] x/a + (Sqrt[b]*ArcTan[(Sqrt[a + b]*Cot[e + f*x])/Sqrt[b]])/(a*Sqrt[a + b]*f
)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
```

```
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 4212

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/a, x
] - Dist[b/a, Int[1/(b + a*Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f}, x
] && NeQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x}{a} - \frac{b\int \frac{1}{b+a\cos^2(e+fx)}\,dx}{a}\\ &= \frac{x}{a} + \frac{b\text{Subst}\left(\int \frac{1}{b+(a+b)x^2}\,dx, x, \cot(e+fx)\right)}{af}\\ &= \frac{x}{a} + \frac{\sqrt{b}\arctan\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{a\sqrt{a+b}f}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.04

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(\sqrt{a+b}fx\sqrt{b(\cos(e)-i\sin(e))^4}+b\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e)}{2\sqrt{a+b}}\right.\right.}{2a\sqrt{a+b}f\left(a+b\sec^2(e+fx)\right)\sqrt{b(\cos(e)-i\sin(e))^4}}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-1),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[a + b]*f*x*Sqrt[b*(Cos
[e] - I*Sin[e])^4] + b*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b
)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^
4])]*(Cos[2*e] - I*Sin[2*e])))/(2*a*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)*Sq
rt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$\frac{-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}+\frac{\arctan(\tan(fx+e))}{a}}{f}$	46
default	$\frac{-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}+\frac{\arctan(\tan(fx+e))}{a}}{f}$	46
risch	$\frac{x}{a}+\frac{\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{2(a+b)fa}-\frac{\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-(a+b)b}-a-2b}{a}\right)}{2(a+b)fa}$	114

```
[In] int(1/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-b/a/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))+1/a*arctan(t
an(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 231, normalized size of antiderivative = 5.13

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[\frac{4\,fx+\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\,\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-\frac{b}{a+b}}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right)}{4\,af}\right.$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*f*x + sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3
- (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f
*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(a*f), 1/2*(2*f*x + sqrt(b/(a + b
))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e
)*sin(f*x + e))))/(a*f)]
```

Sympy [F]

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx=\int \frac{1}{a+b\sec^2{(e+fx)}}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(1/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx=-\frac{\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a}-\frac{fx+e}{a}}{f}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -(b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a) - (f*x + e)/
a)/f
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx=-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{\sqrt{ab+b^2}a}-\frac{fx+e}{a}}{f}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b
^2)))*b/(sqrt(a*b + b^2)*a) - (f*x + e)/a)/f
```

Mupad [B] (verification not implemented)

Time = 20.04 (sec) , antiderivative size = 460, normalized size of antiderivative = 10.22

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \frac{x}{a} - \frac{\operatorname{atan}\left(\frac{\frac{\left(2\,b^3\tan(e+f\,x)-\frac{\left(2\,a^2\,b^2-\frac{\tan(e+f\,x)\,\left(8\,a^3\,b^2+16\,a^2\,b^3\right)\,\sqrt{-b\,(a+b)}}{4\,\left(a^2+b\,a\right)}\right)\sqrt{-b\,(a+b)}}{2\,\left(a^2+b\,a\right)}\right)\sqrt{-b\,(a+b)}\,1\mathrm{i}}{a^2+b\,a}+\frac{\left(2\,b^3\tan(e+f\,x)+\frac{\left(2\,a^2\,b^2+\frac{\tan(e+f\,x)\,\left(8\,a \cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}}{\frac{\left(2\,b^3\tan(e+f\,x)-\frac{\left(2\,a^2\,b^2-\frac{\tan(e+f\,x)\,\left(8\,a^3\,b^2+16\,a^2\,b^3\right)\,\sqrt{-b\,(a+b)}}{4\,\left(a^2+b\,a\right)}\right)\sqrt{-b\,(a+b)}}{2\,\left(a^2+b\,a\right)}\right)\sqrt{-b\,(a+b)}}{a^2+b\,a}-\frac{\left(2\,b^3\tan(e+f\,x)+\frac{\left(2\,a^2\,b^2+\frac{\tan(e+f\,x)\,\left(8\,a \cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}}\right.\cdots}{f\,\left(a^2+b\,a\right)\cdots}$$

```
[In] int(1/(a + b/cos(e + f*x)^2),x)
```

```
[Out] x/a - (atan((((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^2*b^3
 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(
a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(a*b + a^2) + ((2*b^3*tan(e + f*x) + ((
2*a^2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(
a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(a
*b + a^2))/(((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^2*b^3
+ 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a
*b + a^2)))*(-b*(a + b))^(1/2))/(a*b + a^2) - ((2*b^3*tan(e + f*x) + ((2*a^
2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b
+ a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2))/(a*b + a^
2)))*(-b*(a + b))^(1/2)*1i)/(f*(a*b + a^2))
```

3.190 $\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx=\frac{(a-2b)x}{2a^2}+\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}f}+\frac{\cos(e+fx)\sin(e+fx)}{2af}$$

```
[Out] 1/2*(a-2*b)*x/a^2+1/2*cos(f*x+e)*sin(f*x+e)/a/f+b^(3/2)*arctan(b^(1/2)*tan(
f*x+e)/(a+b)^(1/2))/a^2/f/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4231, 425, 536, 209, 211}

$$\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx=\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^2f\sqrt{a+b}}+\frac{x(a-2b)}{2a^2}+\frac{\sin(e+fx)\cos(e+fx)}{2af}$$

```
[In] Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a - 2*b)*x)/(2*a^2) + (b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]]
)/(a^2*Sqrt[a + b]*f) + (Cos[e + f*x]*Sin[e + f*x])/(2*a*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\cos(e+fx)\sin(e+fx)}{2af} - \frac{\text{Subst}\left(\int \frac{-a+b-bx^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{2af}\\
&= \frac{\cos(e+fx)\sin(e+fx)}{2af} + \frac{(a-2b)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{2a^2f}\\
&\quad + \frac{b^2\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{a^2f}\\
&= \frac{(a-2b)x}{2a^2} + \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}f} + \frac{\cos(e+fx)\sin(e+fx)}{2af}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

$$\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{2(a-2b)(e+fx)+\frac{4b^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}+a\sin(2(e+fx))}{4a^2f}$$

```
[In] Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (2*(a - 2*b)*(e + f*x) + (4*b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a +
b]])/Sqrt[a + b] + a*Sin[2*(e + f*x)])/(4*a^2*f)
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{b^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^2\sqrt{(a+b)b}}+\frac{\frac{a\tan(fx+e)}{2+2\tan(fx+e)^2}+\frac{(a-2b)\arctan(\tan(fx+e))}{2}}{a^2}}{f}$
default	$\frac{\frac{b^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^2\sqrt{(a+b)b}}+\frac{\frac{a\tan(fx+e)}{2+2\tan(fx+e)^2}+\frac{(a-2b)\arctan(\tan(fx+e))}{2}}{a^2}}{f}$
risch	$\frac{x}{2a}-\frac{xb}{a^2}-\frac{i\mathrm{e}^{2i(fx+e)}}{8af}+\frac{i\mathrm{e}^{-2i(fx+e)}}{8af}+\frac{\sqrt{-(a+b)b}\,b\ln\left(\mathrm{e}^{2i(fx+e)}-\frac{2i\sqrt{-(a+b)b}-a-2b}{a}\right)}{2(a+b)f\,a^2}-\frac{\sqrt{-(a+b)b}\,b\ln\left(\mathrm{e}^{2i(\ldots}\right.}{2(\ldots}$

```
[In] int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2/a^2/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))+1/a^2*(1/
2*a*tan(f*x+e)/(1+tan(f*x+e)^2)+1/2*(a-2*b)*arctan(tan(f*x+e))))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 272, normalized size of antiderivative = 3.63

$$\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{2\,(a-2\,b)fx+2\,a\cos\left(fx+e\right)\sin\left(fx+e\right)+b\sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos(fx+e)^4-2\,\left(3\,ab+4\,b^2\right)\cos(fx+e)^2-4\ldots}{a^2\cos(fx\ldots}\right.}{4\,a^2f}\right.$$

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a - 2*b)*f*x + 2*a*cos(f*x + e)*sin(f*x + e) + b*sqrt(-b/(a + b))*
log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^
2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqr
t(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^
2 + b^2)))/(a^2*f), 1/2*((a - 2*b)*f*x + a*cos(f*x + e)*sin(f*x + e) - b*sq
rt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*
cos(f*x + e)*sin(f*x + e))))/(a^2*f)]
```

Sympy [F]

$$\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

```
[In] integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cos(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

$$\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{2\,b^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^2} + \frac{(fx+e)(a-2\,b)}{a^2} + \frac{\tan(fx+e)}{a\tan(fx+e)^2+a}}{2\,f}$$

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^2*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^2) + (
f*x + e)*(a - 2*b)/a^2 + tan(f*x + e)/(a*tan(f*x + e)^2 + a))/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.25

$$\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{2\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^2}{\sqrt{ab+b^2}a^2} + \frac{(fx+e)(a-2\,b)}{a^2} + \frac{\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)a}}{2\,f}$$

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*(2*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*
b + b^2)))*b^2/(sqrt(a*b + b^2)*a^2) + (f*x + e)*(a - 2*b)/a^2 + tan(f*x +
e)/((tan(f*x + e)^2 + 1)*a))/f
```

Mupad [B] (verification not implemented)

Time = 20.23 (sec) , antiderivative size = 373, normalized size of antiderivative = 4.97

$$\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx =$$
$$-\frac{2\,b^2\,\mathrm{atan}\left(\frac{\sin(e+f\,x)}{\cos(e+f\,x)}\right)-a\left(\frac{b\,\sin(2\,e+2\,f\,x)}{2}-b\,\mathrm{atan}\left(\frac{\sin(e+f\,x)}{\cos(e+f\,x)}\right)\right)-a^2\left(\frac{\sin(2\,e+2\,f\,x)}{2}+\mathrm{atan}\left(\frac{\sin(e+f\,x)}{\cos(e+f\,x)}\right)\right)+\mathrm{ata}}{}$$

```
[In] int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2),x)
```

```
[Out] -(atan((a*sin(e + f*x)*(- a*b^3 - b^4)^(3/2)*4i + b*sin(e + f*x)*(- a*b^3 -
 b^4)^(3/2)*8i + b^5*sin(e + f*x)*(- a*b^3 - b^4)^(1/2)*8i + a*b^4*sin(e +
f*x)*(- a*b^3 - b^4)^(1/2)*12i + a^4*b*sin(e + f*x)*(- a*b^3 - b^4)^(1/2)*1
i + a^2*b^3*sin(e + f*x)*(- a*b^3 - b^4)^(1/2)*1i - a^3*b^2*sin(e + f*x)*(-
 a*b^3 - b^4)^(1/2)*2i)/(3*a^2*b^5*cos(e + f*x) + 5*a^3*b^4*cos(e + f*x) +
a^4*b^3*cos(e + f*x) - a^5*b^2*cos(e + f*x)))*(- a*b^3 - b^4)^(1/2)*2i + 2*
b^2*atan(sin(e + f*x)/cos(e + f*x)) - a*((b*sin(2*e + 2*f*x))/2 - b*atan(si
n(e + f*x)/cos(e + f*x))) - a^2*(sin(2*e + 2*f*x)/2 + atan(sin(e + f*x)/cos
(e + f*x))))/(f*(2*a^2*b + 2*a^3))
```

3.191 $\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(3a^2-4ab+8b^2)\,x}{8a^3} - \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+b}f} + \frac{(3a-4b)\cos(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af}$$

```
[Out] 1/8*(3*a^2-4*a*b+8*b^2)*x/a^3+1/8*(3*a-4*b)*cos(f*x+e)*sin(f*x+e)/a^2/f+1/4
*cos(f*x+e)^3*sin(f*x+e)/a/f-b^(5/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))
/a^3/f/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4231, 425, 541, 536, 209, 211}

$$\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^3f\sqrt{a+b}} + \frac{(3a-4b)\sin(e+fx)\cos(e+fx)}{8a^2f} + \frac{x(3a^2-4ab+8b^2)}{8a^3} + \frac{\sin(e+fx)\cos^3(e+fx)}{4af}$$

```
[In] Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((3*a^2 - 4*a*b + 8*b^2)*x)/(8*a^3) - (b^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x]
)/Sqrt[a + b]])/(a^3*Sqrt[a + b]*f) + ((3*a - 4*b)*Cos[e + f*x]*Sin[e + f*x
])/(8*a^2*f) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\cos^3(e+fx)\sin(e+fx)}{4af} - \frac{\text{Subst}\left(\int \frac{-3a+b-3bx^2}{(1+x^2)^2(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{4af} \\
&= \frac{(3a-4b)\cos(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af} \\
&\quad + \frac{\text{Subst}\left(\int \frac{3a^2-ab+4b^2+(3a-4b)bx^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{8a^2f} \\
&= \frac{(3a-4b)\cos(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af} \\
&\quad - \frac{b^3\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{a^3f} \\
&\quad + \frac{(3a^2-4ab+8b^2)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{8a^3f} \\
&= \frac{(3a^2-4ab+8b^2)\,x}{8a^3} - \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+b}f} \\
&\quad + \frac{(3a-4b)\cos(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af}
\end{aligned}$$

Mathematica [A] (verified)

Time = 3.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.81

$$\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{4(3a^2-4ab+8b^2)(e+fx) - \frac{32b^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + 8a(a-b)\sin(2(e+fx)) + a^2\sin(4(e+fx))}{32a^3f}$$

```
[In] Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (4*(3*a^2 - 4*a*b + 8*b^2)*(e + f*x) - (32*b^(5/2)*ArcTan[(Sqrt[b]*Tan[e +
f*x])/Sqrt[a + b]])/Sqrt[a + b] + 8*a*(a - b)*Sin[2*(e + f*x)] + a^2*Sin[4*
(e + f*x)])/(32*a^3*f)
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{-\frac{b^{3}\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^{3}\sqrt{(a+b)b}}+\frac{\frac{\left(\frac{3}{8}a^{2}-\frac{1}{2}ab\right)\tan(fx+e)^{3}+\left(-\frac{1}{2}ab+\frac{5}{8}a^{2}\right)\tan(fx+e)}{\left(1+\tan(fx+e)^{2}\right)^{2}}+\frac{\left(3a^{2}-4ab+8b^{2}\right)\arctan(\tan(fx+e))}{8}}{a^{3}}}{f}$
default	$\frac{-\frac{b^{3}\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^{3}\sqrt{(a+b)b}}+\frac{\frac{\left(\frac{3}{8}a^{2}-\frac{1}{2}ab\right)\tan(fx+e)^{3}+\left(-\frac{1}{2}ab+\frac{5}{8}a^{2}\right)\tan(fx+e)}{\left(1+\tan(fx+e)^{2}\right)^{2}}+\frac{\left(3a^{2}-4ab+8b^{2}\right)\arctan(\tan(fx+e))}{8}}{a^{3}}}{f}$
risch	$\frac{3x}{8a}-\frac{xb}{2a^{2}}+\frac{x\,b^{2}}{a^{3}}-\frac{i\mathrm{e}^{2i(fx+e)}}{8af}+\frac{i\mathrm{e}^{2i(fx+e)}b}{8a^{2}f}+\frac{i\mathrm{e}^{-2i(fx+e)}}{8af}-\frac{i\mathrm{e}^{-2i(fx+e)}b}{8a^{2}f}+\frac{\sqrt{-(a+b)b}\,b^{2}\ln\left(\mathrm{e}^{2i(fx+e)}+\right.}{2(a+b)f\,a^{3}}$

```
[In] int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-b^3/a^3/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))+1/a^3*((
(3/8*a^2-1/2*a*b)*tan(f*x+e)^3+(-1/2*a*b+5/8*a^2)*tan(f*x+e))/(1+tan(f*x+e)
^2)^2+1/8*(3*a^2-4*a*b+8*b^2)*arctan(tan(f*x+e))))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.93

$$\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \left[\frac{2\,b^{2}\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^{2}+8\,ab+8\,b^{2})\cos(fx+e)^{4}-2\,(3\,ab+4\,b^{2})\cos(fx+e)^{2}+4\,\left((a^{2}+3\,ab+2\,b^{2})\cos(fx+e)^{3}-(ab+b^{2})\cos(fx+e)\right)\sqrt{-\frac{b}{a+b}}}{a^{2}\cos(fx+e)^{4}+2\,ab\cos(fx+e)^{2}+b^{2}}\right.}{8\,a^{3}f}\right.$$

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*b^2*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*
(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 -
(a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x
 + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + (3*a^2 - 4*a*b + 8*b^2)*f*x + (2*a
^2*cos(f*x + e)^3 + (3*a^2 - 4*a*b)*cos(f*x + e))*sin(f*x + e))/(a^3*f), 1/
8*(4*b^2*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(
a + b))/(b*cos(f*x + e)*sin(f*x + e))) + (3*a^2 - 4*a*b + 8*b^2)*f*x + (2*a
^2*cos(f*x + e)^3 + (3*a^2 - 4*a*b)*cos(f*x + e))*sin(f*x + e))/(a^3*f)]
```

Sympy [F]

$$\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx=\int \frac{\cos^4{(e+fx)}}{a+b\sec^2{(e+fx)}}\,dx$$

```
[In] integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
[Out] Integral(cos(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.08

$$\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{8\,b^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^3}-\frac{(3\,a-4\,b)\tan(fx+e)^3+(5\,a-4\,b)\tan(fx+e)}{a^2\tan(fx+e)^4+2\,a^2\tan(fx+e)^2+a^2}-\frac{(3\,a^2-4\,ab+8\,b^2)(fx+e)}{a^3}}{8\,f}$$

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] -1/8*(8*b^3*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^3) -
((3*a - 4*b)*tan(f*x + e)^3 + (5*a - 4*b)*tan(f*x + e))/(a^2*tan(f*x + e)^4
 + 2*a^2*tan(f*x + e)^2 + a^2) - (3*a^2 - 4*a*b + 8*b^2)*(f*x + e)/a^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.21

$$\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{8\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^3}{\sqrt{ab+b^2}a^3}-\frac{(3\,a^2-4\,ab+8\,b^2)(fx+e)}{a^3}-\frac{3\,a\tan(fx+e)^3-4\,b\tan(fx+e)^3+5\,a\tan(fx+e)-4\,b\tan(fx}{\left(\tan(fx+e)^2+1\right)^2a^2}}{8\,f}$$

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] -1/8*(8*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b + b^2)))*b^3/(sqrt(a*b + b^2)*a^3) - (3*a^2 - 4*a*b + 8*b^2)*(f*x + e)/a
^3 - (3*a*tan(f*x + e)^3 - 4*b*tan(f*x + e)^3 + 5*a*tan(f*x + e) - 4*b*tan(
f*x + e))/((tan(f*x + e)^2 + 1)^2*a^2))/f
```

Mupad [B] (verification not implemented)

Time = 20.26 (sec) , antiderivative size = 1114, normalized size of antiderivative = 9.52

$$\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

```
[In] int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2),x)
```

```
[Out] ((tan(e + f*x)*(5*a - 4*b))/(8*a^2) + (tan(e + f*x)^3*(3*a - 4*b))/(8*a^2))
/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1)) - (atan((((-b^5*(a + b))^(1/2)
*((tan(e + f*x)*(128*b^7 - 64*a*b^6 + 64*a^2*b^5 - 24*a^3*b^4 + 9*a^4*b^3))
/(64*a^4) - ((-b^5*(a + b))^(1/2)*((2*a^6*b^4 - (a^7*b^3)/2 + (3*a^8*b^2)/2
)/(2*a^6) - (tan(e + f*x)*(512*a^6*b^3 + 256*a^7*b^2)*(-b^5*(a + b))^(1/2))
/(128*a^4*(a^3*b + a^4))))/(2*(a^3*b + a^4)))*1i)/(a^3*b + a^4) + ((-b^5*(a
 + b))^(1/2)*((tan(e + f*x)*(128*b^7 - 64*a*b^6 + 64*a^2*b^5 - 24*a^3*b^4 +
 9*a^4*b^3))/(64*a^4) + ((-b^5*(a + b))^(1/2)*((2*a^6*b^4 - (a^7*b^3)/2 + (
3*a^8*b^2)/2)/(2*a^6) + (tan(e + f*x)*(512*a^6*b^3 + 256*a^7*b^2)*(-b^5*(a
+ b))^(1/2))/(128*a^4*(a^3*b + a^4))))/(2*(a^3*b + a^4)))*1i)/(a^3*b + a^4)
)/((((5*a*b^7)/4 - b^8 - (3*a^2*b^6)/4 + (9*a^3*b^5)/32)/a^6 + ((-b^5*(a + b
))^(1/2)*((tan(e + f*x)*(128*b^7 - 64*a*b^6 + 64*a^2*b^5 - 24*a^3*b^4 + 9*a
^4*b^3))/(64*a^4) - ((-b^5*(a + b))^(1/2)*((2*a^6*b^4 - (a^7*b^3)/2 + (3*a^
8*b^2)/2)/(2*a^6) - (tan(e + f*x)*(512*a^6*b^3 + 256*a^7*b^2)*(-b^5*(a + b)
)^(1/2))/(128*a^4*(a^3*b + a^4))))/(2*(a^3*b + a^4))))/(a^3*b + a^4) - ((-b
^5*(a + b))^(1/2)*((tan(e + f*x)*(128*b^7 - 64*a*b^6 + 64*a^2*b^5 - 24*a^3*
b^4 + 9*a^4*b^3))/(64*a^4) + ((-b^5*(a + b))^(1/2)*((2*a^6*b^4 - (a^7*b^3)/
2 + (3*a^8*b^2)/2)/(2*a^6) + (tan(e + f*x)*(512*a^6*b^3 + 256*a^7*b^2)*(-b^
5*(a + b))^(1/2))/(128*a^4*(a^3*b + a^4))))/(2*(a^3*b + a^4))))/(a^3*b + a^
4)))*(-b^5*(a + b))^(1/2)*1i)/(f*(a^3*b + a^4)) - (atan((63*b^4*tan(e + f*x
))/(64*((63*b^4)/64 - (81*a*b^3)/256 + (27*a^2*b^2)/256 - (35*b^5)/(32*a) +
 (5*b^6)/(4*a^2))) - (81*b^3*tan(e + f*x))/(256*((27*a*b^2)/256 - (81*b^3)/
256 + (63*b^4)/(64*a) - (35*b^5)/(32*a^2) + (5*b^6)/(4*a^3))) - (35*b^5*tan
(e + f*x))/(32*((63*a*b^4)/64 - (35*b^5)/32 - (81*a^2*b^3)/256 + (27*a^3*b^
2)/256 + (5*b^6)/(4*a))) + (5*b^6*tan(e + f*x))/(4*((5*b^6)/4 - (35*a*b^5)/
32 + (63*a^2*b^4)/64 - (81*a^3*b^3)/256 + (27*a^4*b^2)/256)) + (27*b^2*tan(
e + f*x))/(256*((27*b^2)/256 - (81*b^3)/(256*a) + (63*b^4)/(64*a^2) - (35*b
^5)/(32*a^3) + (5*b^6)/(4*a^4))))*(a^2*3i - a*b*4i + b^2*8i)*1i)/(8*a^3*f)
```

3.192 $\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 163

$$\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(5a^3-6a^2b+8ab^2-16b^3)\,x}{16a^4} + \frac{b^{7/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^4\sqrt{a+b}f} + \frac{(5a^2-6ab+8b^2)\cos(e+fx)\sin(e+fx)}{16a^3f} + \frac{(5a-6b)\cos^3(e+fx)\sin(e+fx)}{24a^2f} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af}$$

```
[Out] 1/16*(5*a^3-6*a^2*b+8*a*b^2-16*b^3)*x/a^4+1/16*(5*a^2-6*a*b+8*b^2)*cos(f*x+
e)*sin(f*x+e)/a^3/f+1/24*(5*a-6*b)*cos(f*x+e)^3*sin(f*x+e)/a^2/f+1/6*cos(f*
x+e)^5*sin(f*x+e)/a/f+b^(7/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^4/f/
(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4231, 425, 541, 536, 209, 211}

$$\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{b^{7/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^4f\sqrt{a+b}} + \frac{(5a-6b)\sin(e+fx)\cos^3(e+fx)}{24a^2f} + \frac{(5a^2-6ab+8b^2)\sin(e+fx)\cos(e+fx)}{16a^3f} + \frac{x(5a^3-6a^2b+8ab^2-16b^3)}{16a^4} + \frac{\sin(e+fx)\cos^5(e+fx)}{6af}$$

```
[In] Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((5*a^3 - 6*a^2*b + 8*a*b^2 - 16*b^3)*x)/(16*a^4) + (b^(7/2)*ArcTan[(Sqrt[b
]*Tan[e + f*x])/Sqrt[a + b]])/(a^4*Sqrt[a + b]*f) + ((5*a^2 - 6*a*b + 8*b^2
)*Cos[e + f*x]*Sin[e + f*x])/(16*a^3*f) + ((5*a - 6*b)*Cos[e + f*x]^3*Sin[e
 + f*x])/(24*a^2*f) + (Cos[e + f*x]^5*Sin[e + f*x])/(6*a*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
```

```
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^4(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\cos^5(e+fx)\sin(e+fx)}{6af} - \frac{\text{Subst}\left(\int \frac{-5a+b-5bx^2}{(1+x^2)^3(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{6af}\\
&= \frac{(5a-6b)\cos^3(e+fx)\sin(e+fx)}{24a^2f} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af}\\
&\quad + \frac{\text{Subst}\left(\int \frac{3(5a^2-ab+2b^2)+3(5a-6b)bx^2}{(1+x^2)^2(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{24a^2f}\\
&= \frac{(5a^2-6ab+8b^2)\cos(e+fx)\sin(e+fx)}{16a^3f}\\
&\quad + \frac{(5a-6b)\cos^3(e+fx)\sin(e+fx)}{24a^2f} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af}\\
&\quad - \frac{\text{Subst}\left(\int \frac{-3(5a^3-a^2b+2ab^2-8b^3)-3b(5a^2-6ab+8b^2)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{48a^3f}\\
&= \frac{(5a^2-6ab+8b^2)\cos(e+fx)\sin(e+fx)}{16a^3f} + \frac{(5a-6b)\cos^3(e+fx)\sin(e+fx)}{24a^2f}\\
&\quad + \frac{\cos^5(e+fx)\sin(e+fx)}{6af} + \frac{b^4\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{a^4f}\\
&\quad + \frac{(5a^3-6a^2b+8ab^2-16b^3)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{16a^4f}\\
&= \frac{(5a^3-6a^2b+8ab^2-16b^3)\,x}{16a^4} + \frac{b^{7/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^4\sqrt{a+b}f}\\
&\quad + \frac{(5a^2-6ab+8b^2)\cos(e+fx)\sin(e+fx)}{16a^3f}\\
&\quad + \frac{(5a-6b)\cos^3(e+fx)\sin(e+fx)}{24a^2f} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.86 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.82

$$\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{12(5a^3-6a^2b+8ab^2-16b^3)(e+fx)+\frac{192b^{7/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}+3a(15a^2-16ab+16b^2)\sin(2(e+fx))}{192a^4f}$$

[In] Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]

[Out] (12*(5*a^3 - 6*a^2*b + 8*a*b^2 - 16*b^3)*(e + f*x) + (192*b^(7/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/Sqrt[a + b] + 3*a*(15*a^2 - 16*a*b + 16*b^2)*Sin[2*(e + f*x)] + 3*a^2*(3*a - 2*b)*Sin[4*(e + f*x)] + a^3*Sin[6*(e + f*x)])/(192*a^4*f)

Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{\frac{\left(\frac{5}{16}a^3-\frac{3}{8}a^2b+\frac{1}{2}ab^2\right)\tan(fx+e)^5+\left(ab^2+\frac{5}{6}a^3-a^2b\right)\tan(fx+e)^3+\left(-\frac{5}{8}a^2b+\frac{1}{2}ab^2+\frac{11}{16}a^3\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^3}+\frac{(5a^3-6a^2b+8ab^2-16b^3)}{16}}{a^4}}{f}$
default	$\frac{\frac{\frac{\left(\frac{5}{16}a^3-\frac{3}{8}a^2b+\frac{1}{2}ab^2\right)\tan(fx+e)^5+\left(ab^2+\frac{5}{6}a^3-a^2b\right)\tan(fx+e)^3+\left(-\frac{5}{8}a^2b+\frac{1}{2}ab^2+\frac{11}{16}a^3\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^3}+\frac{(5a^3-6a^2b+8ab^2-16b^3)}{16}}{a^4}}{f}$
risch	$\frac{5x}{16a}-\frac{3xb}{8a^2}+\frac{xb^2}{2a^3}-\frac{xb^3}{a^4}-\frac{15ie^{2i(fx+e)}}{128af}+\frac{ie^{2i(fx+e)}b}{8a^2f}-\frac{ie^{2i(fx+e)}b^2}{8a^3f}+\frac{15ie^{-2i(fx+e)}}{128af}-\frac{ie^{-2i(fx+e)}b}{8a^2f}+$

[In] int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)

[Out] 1/f*(1/a^4*(((5/16*a^3-3/8*a^2*b+1/2*a*b^2)*tan(f*x+e)^5+(a*b^2+5/6*a^3-a^2*b)*tan(f*x+e)^3+(-5/8*a^2*b+1/2*a*b^2+11/16*a^3)*tan(f*x+e))/(1+tan(f*x+e)^2)^3+1/16*(5*a^3-6*a^2*b+8*a*b^2-16*b^3)*arctan(tan(f*x+e)))+b^4/a^4/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.60

$$\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \left[\frac{12\,b^3\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2-4\,\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-\frac{b}{a+b}}}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right.}{}\right.$$
$$-\frac{24\,b^3\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)-3\,(5\,a^3-6\,a^2b+8\,ab^2-16\,b^3)fx-\left(8\,a^3\cos(fx+e)^5\right.}{48\,a^4f}$$

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/48*(12*b^3*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3
- (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f
*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 3*(5*a^3 - 6*a^2*b + 8*a*b^2 - 1
6*b^3)*f*x + (8*a^3*cos(f*x + e)^5 + 2*(5*a^3 - 6*a^2*b)*cos(f*x + e)^3 + 3
*(5*a^3 - 6*a^2*b + 8*a*b^2)*cos(f*x + e))*sin(f*x + e))/(a^4*f), -1/48*(24
*b^3*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a +
b))/(b*cos(f*x + e)*sin(f*x + e))) - 3*(5*a^3 - 6*a^2*b + 8*a*b^2 - 16*b^3)
*f*x - (8*a^3*cos(f*x + e)^5 + 2*(5*a^3 - 6*a^2*b)*cos(f*x + e)^3 + 3*(5*a^
3 - 6*a^2*b + 8*a*b^2)*cos(f*x + e))*sin(f*x + e))/(a^4*f)]
```

Sympy [F]

$$\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\cos^6{(e+fx)}}{a+b\sec^2{(e+fx)}}\,dx$$

```
[In] integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cos(e + f*x)**6/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.16

$$\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\frac{48\,b^4\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^4} + \frac{3\,(5\,a^2-6\,ab+8\,b^2)\tan(fx+e)^5+8\,(5\,a^2-6\,ab+6\,b^2)\tan(fx+e)^3+3\,(11\,a^2-10\,ab+8\,b^2)\tan(fx+e)}{a^3\tan(fx+e)^6+3\,a^3\tan(fx+e)^4+3\,a^3\tan(fx+e)^2+a^3} + \frac{3\,(5\,a^3-6}{}}{48\,f}$$

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/48*(48*b^4*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^4) +
 (3*(5*a^2 - 6*a*b + 8*b^2)*tan(f*x + e)^5 + 8*(5*a^2 - 6*a*b + 6*b^2)*tan(
f*x + e)^3 + 3*(11*a^2 - 10*a*b + 8*b^2)*tan(f*x + e))/(a^3*tan(f*x + e)^6
+ 3*a^3*tan(f*x + e)^4 + 3*a^3*tan(f*x + e)^2 + a^3) + 3*(5*a^3 - 6*a^2*b +
 8*a*b^2 - 16*b^3)*(f*x + e)/a^4)/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.33

$$\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\frac{48\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^4}{\sqrt{ab+b^2}a^4} + \frac{3\,(5\,a^3-6\,a^2b+8\,ab^2-16\,b^3)(fx+e)}{a^4} + \frac{15\,a^2\tan(fx+e)^5-18\,ab\tan(fx+e)^5+24\,b^2\tan(}{}}{48\,f}$$

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/48*(48*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(
a*b + b^2)))*b^4/(sqrt(a*b + b^2)*a^4) + 3*(5*a^3 - 6*a^2*b + 8*a*b^2 - 16*
b^3)*(f*x + e)/a^4 + (15*a^2*tan(f*x + e)^5 - 18*a*b*tan(f*x + e)^5 + 24*b^
2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 - 48*a*b*tan(f*x + e)^3 + 48*b^2*t
an(f*x + e)^3 + 33*a^2*tan(f*x + e) - 30*a*b*tan(f*x + e) + 24*b^2*tan(f*x
+ e))/((tan(f*x + e)^2 + 1)^3*a^3))/f
```

Mupad [B] (verification not implemented)

Time = 21.55 (sec) , antiderivative size = 1979, normalized size of antiderivative = 12.14

$$\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

[In] int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2),x)

[Out] ((tan(e + f*x)*(11*a^2 - 10*a*b + 8*b^2))/(16*a^3) + (tan(e + f*x)^3*(5*a^2 - 6*a*b + 6*b^2))/(6*a^3) + (tan(e + f*x)^5*(5*a^2 - 6*a*b + 8*b^2))/(16*a^3))/(f*(3*tan(e + f*x)^2 + 3*tan(e + f*x)^4 + tan(e + f*x)^6 + 1)) + (atan(((((((2*a^8*b^5 - (a^9*b^4)/2 + (a^10*b^3)/4 - (5*a^11*b^2)/4)/a^9 - (tan(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i))/(4096*a^10))*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i))/(32*a^4) - (tan(e + f*x)*(512*b^9 - 256*a*b^8 + 256*a^2*b^7 - 256*a^3*b^6 + 116*a^4*b^5 - 60*a^5*b^4 + 25*a^6*b^3))/(128*a^6))*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i)*1i)/(32*a^4) - (((((2*a^8*b^5 - (a^9*b^4)/2 + (a^10*b^3)/4 - (5*a^11*b^2)/4)/a^9 + (tan(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i))/(4096*a^10))*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i))/(32*a^4) + (tan(e + f*x)*(512*b^9 - 256*a*b^8 + 256*a^2*b^7 - 256*a^3*b^6 + 116*a^4*b^5 - 60*a^5*b^4 + 25*a^6*b^3))/(128*a^6))*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i)*1i)/(32*a^4))/((((((2*a^8*b^5 - (a^9*b^4)/2 + (a^10*b^3)/4 - (5*a^11*b^2)/4)/a^9 - (tan(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i))/(4096*a^10))*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i))/(32*a^4) - (tan(e + f*x)*(512*b^9 - 256*a*b^8 + 256*a^2*b^7 - 256*a^3*b^6 + 116*a^4*b^5 - 60*a^5*b^4 + 25*a^6*b^3))/(128*a^6))*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i))/(32*a^4) - ((5*a*b^10)/4 - b^11 - (11*a^2*b^9)/8 + (29*a^3*b^8)/32 - (15*a^4*b^7)/32 + (25*a^5*b^6)/128)/a^9 + (((((2*a^8*b^5 - (a^9*b^4)/2 + (a^10*b^3)/4 - (5*a^11*b^2)/4)/a^9 + (tan(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i))/(4096*a^10))*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i))/(32*a^4) + (tan(e + f*x)*(512*b^9 - 256*a*b^8 + 256*a^2*b^7 - 256*a^3*b^6 + 116*a^4*b^5 - 60*a^5*b^4 + 25*a^6*b^3))/(128*a^6))*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i))/(32*a^4)))*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i)*1i)/(16*a^4*f) + (atan((((((-b^7*(a + b))^(1/2)*((2*a^8*b^5 - (a^9*b^4)/2 + (a^10*b^3)/4 - (5*a^11*b^2)/4)/(2*a^9) - (tan(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(-b^7*(a + b))^(1/2))/(512*a^6*(a^4*b + a^5))))/(2*(a^4*b + a^5)) - (tan(e + f*x)*(512*b^9 - 256*a*b^8 + 256*a^2*b^7 - 256*a^3*b^6 + 116*a^4*b^5 - 60*a^5*b^4 + 25*a^6*b^3))/(256*a^6))*(-b^7*(a + b))^(1/2)*1i)/(a^4*b + a^5) - ((((-b^7*(a + b))^(1/2)*((2*a^8*b^5 - (a^9*b^4)/2 + (a^10*b^3)/4 - (5*a^11*b^2)/4)/(2*a^9) + (tan(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(-b^7*(a + b))^(1/2))/(512*a^6*(a^4*b + a^5))))/(2*(a^4*b + a^5)) + (tan(e + f*x)*(512*b^9 - 256*a*b^8 + 256*a^2*b^7 - 256*a^3*b^6 + 116*a^4*b^5 - 60*a^5*b^4 + 25*a^6*b^3))/(256*a^6))*(-b^7*(a + b))^(1/2)*1i)/(a^4*b + a^5))/(((((-b^7*(a + b))^(1/2

```
)*((2*a^8*b^5 - (a^9*b^4)/2 + (a^10*b^3)/4 - (5*a^11*b^2)/4)/(2*a^9) - (tan
(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(-b^7*(a + b))^(1/2))/(512*a^6*(a^4
*b + a^5))))/(2*(a^4*b + a^5)) - (tan(e + f*x)*(512*b^9 - 256*a*b^8 + 256*a
^2*b^7 - 256*a^3*b^6 + 116*a^4*b^5 - 60*a^5*b^4 + 25*a^6*b^3))/(256*a^6))*(
-b^7*(a + b))^(1/2))/(a^4*b + a^5) - ((5*a*b^10)/4 - b^11 - (11*a^2*b^9)/8
+ (29*a^3*b^8)/32 - (15*a^4*b^7)/32 + (25*a^5*b^6)/128)/a^9 + ((((-b^7*(a +
 b))^(1/2)*((2*a^8*b^5 - (a^9*b^4)/2 + (a^10*b^3)/4 - (5*a^11*b^2)/4)/(2*a^
9) + (tan(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(-b^7*(a + b))^(1/2))/(512
*a^6*(a^4*b + a^5))))/(2*(a^4*b + a^5)) + (tan(e + f*x)*(512*b^9 - 256*a*b^
8 + 256*a^2*b^7 - 256*a^3*b^6 + 116*a^4*b^5 - 60*a^5*b^4 + 25*a^6*b^3))/(25
6*a^6))*(-b^7*(a + b))^(1/2))/(a^4*b + a^5)))*(-b^7*(a + b))^(1/2)*1i)/(f*(
a^4*b + a^5))
```

3.193 $\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 102

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\operatorname{arctanh}(\sin(e+fx))}{b^2f} - \frac{\sqrt{a}(2a+3b)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2b^2(a+b)^{3/2}f} - \frac{a\sin(e+fx)}{2b(a+b)f\,(a+b-a\sin^2(e+fx))}$$

```
[Out] arctanh(sin(f*x+e))/b^2/f-1/2*a*sin(f*x+e)/b/(a+b)/f/(a+b-a*sin(f*x+e)^2)-1
/2*(2*a+3*b)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))*a^(1/2)/b^2/(a+b)^(3/2
)/f
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4232, 425, 536, 212, 214}

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\sqrt{a}(2a+3b)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2b^2f(a+b)^{3/2}} - \frac{a\sin(e+fx)}{2bf(a+b)\left(-a\sin^2(e+fx)+a+b\right)} + \frac{\operatorname{arctanh}(\sin(e+fx))}{b^2f}$$

```
[In] Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ArcTanh[Sin[e + f*x]]/(b^2*f) - (Sqrt[a]*(2*a + 3*b)*ArcTanh[(Sqrt[a]*Sin[e
 + f*x])/Sqrt[a + b]])/(2*b^2*(a + b)^(3/2)*f) - (a*Sin[e + f*x])/(2*b*(a +
 b)*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+b-ax^2)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{a\sin(e+fx)}{2b(a+b)f\left(a+b-a\sin^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-a-2b-ax^2}{(1-x^2)(a+b-ax^2)}\,dx, x, \sin(e+fx)\right)}{2b(a+b)f}$$

$$= -\frac{a\sin(e+fx)}{2b(a+b)f\left(a+b-a\sin^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(e+fx)\right)}{b^2f} - \frac{(a(2a+3b))\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{2b^2(a+b)f}$$

$$= \frac{\text{arctanh}(\sin(e+fx))}{b^2f} - \frac{\sqrt{a}(2a+3b)\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2b^2(a+b)^{3/2}f} - \frac{a\sin(e+fx)}{2b(a+b)f\left(a+b-a\sin^2(e+fx)\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.73 (sec) , antiderivative size = 980, normalized size of antiderivative = 9.61

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(-2ia(2a+3b)\arctan\left(\frac{2\sin(e)\left(ia+ib+i(a+b)\cos(2e)+\sqrt{a}\sqrt{a+b}\cos(fx)\sqrt{(\mathrm{co}}{i(a+3b)\cos(\mathrm{e}}\right.\right.}{}$$

```
[In] Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*((-2*I)*a*(2*a + 3*b)*ArcTan
[(2*Sin[e]*(I*a + I*b + I*(a + b)*Cos[2*e] + Sqrt[a]*Sqrt[a + b]*Cos[f*x]*S
qrt[(Cos[e] - I*Sin[e])^2] - Sqrt[a]*Sqrt[a + b]*Cos[2*e + f*x]*Sqrt[(Cos[e
] - I*Sin[e])^2] + a*Sin[2*e] + b*Sin[2*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Co
s[e] - I*Sin[e])^2]*Sin[f*x] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e
])^2]*Sin[2*e + f*x]))/(I*(a + 3*b)*Cos[e] + I*(a + b)*Cos[3*e] + I*a*Cos[e
 + 2*f*x] + I*a*Cos[3*e + 2*f*x] + 3*a*Sin[e] + b*Sin[e] + a*Sin[3*e] + b*S
in[3*e] + a*Sin[e + 2*f*x] - a*Sin[3*e + 2*f*x])]*(a + 2*b + a*Cos[2*(e + f
*x)])*Sec[e + f*x]*(Cos[e] - I*Sin[e]) - a*(2*a + 3*b)*(a + 2*b + a*Cos[2*(
e + f*x)])*Log[a + 2*(a + b)*Cos[2*e] - a*Cos[2*(e + f*x)] - (2*I)*a*Sin[2*
e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*S
in[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]]
*Sec[e + f*x]*(Cos[e] - I*Sin[e]) + a*(2*a + 3*b)*(a + 2*b + a*Cos[2*(e + f
*x)])*Log[-a - 2*(a + b)*Cos[2*e] + a*Cos[2*(e + f*x)] + (2*I)*a*Sin[2*e] +
 (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f
*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]]*Sec
[e + f*x]*(Cos[e] - I*Sin[e]) - 8*Sqrt[a]*(a + b)^(3/2)*(a + 2*b + a*Cos[2*
(e + f*x)])*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sec[e + f*x]*Sqrt[(Cos
[e] - I*Sin[e])^2] + 8*Sqrt[a]*(a + b)^(3/2)*(a + 2*b + a*Cos[2*(e + f*x)])
*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sec[e + f*x]*Sqrt[(Cos[e] - I*Sin
```

```
[e])^2] + 2*a*(2*a + 3*b)*ArcTan[((a + b)*Sin[e])/((a + b)*Cos[e] - Sqrt[a]
*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*(Cos[2*e] + I*Sin[2*e])*Sin[e + f*
x])]*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]*(I*Cos[e] + Sin[e]) - 8*a^
(3/2)*b*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[e + f*x]))/(32*Sqrt[a]*
b^2*(a + b)^(3/2)*f*(a + b*Sec[e + f*x]^2)^2*Sqrt[(Cos[e] - I*Sin[e])^2])
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{\frac{\ln(\sin(fx+e)+1)}{2b^2}+\frac{a\left(\frac{b\sin(fx+e)}{2(a+b)\left(a\sin(fx+e)^2-a-b\right)}-\frac{(2a+3b)\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{b^2}-\frac{\ln(\sin(fx+e)-1)}{2b^2}}{f}$
default	$\frac{\frac{\ln(\sin(fx+e)+1)}{2b^2}+\frac{a\left(\frac{b\sin(fx+e)}{2(a+b)\left(a\sin(fx+e)^2-a-b\right)}-\frac{(2a+3b)\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{b^2}-\frac{\ln(\sin(fx+e)-1)}{2b^2}}{f}$
risch	$\frac{ia\left(\mathrm{e}^{3i(fx+e)}-\mathrm{e}^{i(fx+e)}\right)}{b(a+b)f\left(a\,\mathrm{e}^{4i(fx+e)}+2a\,\mathrm{e}^{2i(fx+e)}+4b\,\mathrm{e}^{2i(fx+e)}+a\right)}-\frac{\ln\left(\mathrm{e}^{i(fx+e)}-i\right)}{b^2f}+\frac{\ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{b^2f}+\frac{\sqrt{a(a+b)}\,\ln\left(\mathrm{e}^{2i(fx+e}\right.}{2(a}$

```
[In] int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2/b^2*ln(sin(f*x+e)+1)+a/b^2*(1/2/(a+b)*b*sin(f*x+e)/(a*sin(f*x+e)^2
-a-b)-1/2*(2*a+3*b)/(a+b)/(a*(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1
/2)))-1/2/b^2*ln(sin(f*x+e)-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 392, normalized size of antiderivative = 3.84

$$\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$= \left[-\frac{2\,ab\sin\left(fx+e\right)-\left(\left(2\,a^2+3\,ab\right)\cos\left(fx+e\right)^2+2\,ab+3\,b^2\right)\sqrt{\frac{a}{a+b}}\log\left(-\frac{a\cos(fx+e)^2+2\,(a+b)\sqrt{\frac{a}{a+b}}\sin(fx+}{a\cos(fx+e)^2+b}\right.}{4\left((a^2b\right.}\right.$$

$$-\frac{ab\sin\left(fx+e\right)-\left(\left(2\,a^2+3\,ab\right)\cos\left(fx+e\right)^2+2\,ab+3\,b^2\right)\sqrt{-\frac{a}{a+b}}\arctan\left(\sqrt{-\frac{a}{a+b}}\sin\left(fx+e\right)\right)-\left(\left(\right.\right.}{2\left(\left(a^2b^2+ab^3\right)f\right.}$$

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*sin(f*x + e) - ((2*a^2 + 3*a*b)*cos(f*x + e)^2 + 2*a*b + 3*b^2
)*sqrt(a/(a + b))*log(-(a*cos(f*x + e)^2 + 2*(a + b)*sqrt(a/(a + b))*sin(f*
x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) - 2*((a^2 + a*b)*cos(f*x + e)^2 +
 a*b + b^2)*log(sin(f*x + e) + 1) + 2*((a^2 + a*b)*cos(f*x + e)^2 + a*b + b
^2)*log(-sin(f*x + e) + 1))/((a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a*b^3 +
b^4)*f), -1/2*(a*b*sin(f*x + e) - ((2*a^2 + 3*a*b)*cos(f*x + e)^2 + 2*a*b +
 3*b^2)*sqrt(-a/(a + b))*arctan(sqrt(-a/(a + b))*sin(f*x + e)) - ((a^2 + a*
b)*cos(f*x + e)^2 + a*b + b^2)*log(sin(f*x + e) + 1) + ((a^2 + a*b)*cos(f*x
 + e)^2 + a*b + b^2)*log(-sin(f*x + e) + 1))/((a^2*b^2 + a*b^3)*f*cos(f*x +
 e)^2 + (a*b^3 + b^4)*f)]
```

Sympy [F]

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

```
[In] integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(sec(e + f*x)**5/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.43

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{(2a+3b)a\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(ab^2+b^3)\sqrt{(a+b)a}} - \frac{2a\sin(fx+e)}{a^2b+2ab^2+b^3-(a^2b+ab^2)\sin(fx+e)^2} + \frac{2\log(\sin(fx+e)+1)}{b^2} - \frac{2\log(\sin(fx+e)-1)}{b^2}}{4f}$$

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*a + 3*b)*a*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) +
 sqrt((a + b)*a)))/((a*b^2 + b^3)*sqrt((a + b)*a)) - 2*a*sin(f*x + e)/(a^2*
b + 2*a*b^2 + b^3 - (a^2*b + a*b^2)*sin(f*x + e)^2) + 2*log(sin(f*x + e) +
1)/b^2 - 2*log(sin(f*x + e) - 1)/b^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.24

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{(2\,a^2+3\,ab)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(ab^2+b^3)\sqrt{-a^2-ab}}+\frac{a\sin(fx+e)}{\left(a\sin(fx+e)^2-a-b\right)(ab+b^2)}+\frac{\log(|\sin(fx+e)+1|)}{b^2}-\frac{\log(|\sin(fx+e)-1|)}{b^2}}{2\,f}$$

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((2*a^2 + 3*a*b)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a*b^2 + b^3)
*sqrt(-a^2 - a*b)) + a*sin(f*x + e)/((a*sin(f*x + e)^2 - a - b)*(a*b + b^2)
) + log(abs(sin(f*x + e) + 1))/b^2 - log(abs(sin(f*x + e) - 1))/b^2)/f
```

Mupad [B] (verification not implemented)

Time = 21.33 (sec) , antiderivative size = 2039, normalized size of antiderivative = 19.99

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] (atan((((((4*a^2*b^6 + 6*a^3*b^5 + 2*a^4*b^4)/(2*(2*a*b^4 + b^5 + a^2*b^3))
 - (sin(e + f*x)*(16*a^2*b^7 + 64*a^3*b^6 + 80*a^4*b^5 + 32*a^5*b^4))/(8*b^
2*(2*a*b^3 + b^4 + a^2*b^2)))*1i)/(2*b^2) + (sin(e + f*x)*(20*a^4*b + 8*a^5
 + 13*a^3*b^2)*1i)/(4*(2*a*b^3 + b^4 + a^2*b^2)))/b^2 - ((((4*a^2*b^6 + 6*a
^3*b^5 + 2*a^4*b^4)/(2*(2*a*b^4 + b^5 + a^2*b^3)) + (sin(e + f*x)*(16*a^2*b
^7 + 64*a^3*b^6 + 80*a^4*b^5 + 32*a^5*b^4))/(8*b^2*(2*a*b^3 + b^4 + a^2*b^2
)))*1i)/(2*b^2) - (sin(e + f*x)*(20*a^4*b + 8*a^5 + 13*a^3*b^2)*1i)/(4*(2*a
*b^3 + b^4 + a^2*b^2)))/b^2)/((((4*a^2*b^6 + 6*a^3*b^5 + 2*a^4*b^4)/(2*(2*a
*b^4 + b^5 + a^2*b^3)) - (sin(e + f*x)*(16*a^2*b^7 + 64*a^3*b^6 + 80*a^4*b^
5 + 32*a^5*b^4))/(8*b^2*(2*a*b^3 + b^4 + a^2*b^2)))/(2*b^2) + (sin(e + f*x)
*(20*a^4*b + 8*a^5 + 13*a^3*b^2))/(4*(2*a*b^3 + b^4 + a^2*b^2)))/b^2 + (((4
*a^2*b^6 + 6*a^3*b^5 + 2*a^4*b^4)/(2*(2*a*b^4 + b^5 + a^2*b^3)) + (sin(e +
f*x)*(16*a^2*b^7 + 64*a^3*b^6 + 80*a^4*b^5 + 32*a^5*b^4))/(8*b^2*(2*a*b^3 +
 b^4 + a^2*b^2)))/(2*b^2) - (sin(e + f*x)*(20*a^4*b + 8*a^5 + 13*a^3*b^2))/
(4*(2*a*b^3 + b^4 + a^2*b^2)))/b^2 - ((3*a^3*b)/2 + a^4)/(2*a*b^4 + b^5 + a
^2*b^3)))*1i)/(b^2*f) - (atan(((((sin(e + f*x)*(20*a^4*b + 8*a^5 + 13*a^3*b
^2))/(2*(2*a*b^3 + b^4 + a^2*b^2)) + ((a*(a + b)^3)^(1/2)*((4*a^2*b^6 + 6*a
^3*b^5 + 2*a^4*b^4)/(2*a*b^4 + b^5 + a^2*b^3) - (sin(e + f*x)*(a*(a + b)^3)
```

```
^(1/2)*(2*a + 3*b)*(16*a^2*b^7 + 64*a^3*b^6 + 80*a^4*b^5 + 32*a^5*b^4))/(8*
(2*a*b^3 + b^4 + a^2*b^2)*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))*(2*a + 3*
b))/(4*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))*(a*(a + b)^3)^(1/2)*(2*a + 3
*b)*1i)/(4*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)) + (((sin(e + f*x)*(20*a^4
*b + 8*a^5 + 13*a^3*b^2))/(2*(2*a*b^3 + b^4 + a^2*b^2)) - ((a*(a + b)^3)^(1
/2)*((4*a^2*b^6 + 6*a^3*b^5 + 2*a^4*b^4)/(2*a*b^4 + b^5 + a^2*b^3) + (sin(e
 + f*x)*(a*(a + b)^3)^(1/2)*(2*a + 3*b)*(16*a^2*b^7 + 64*a^3*b^6 + 80*a^4*b
^5 + 32*a^5*b^4))/(8*(2*a*b^3 + b^4 + a^2*b^2)*(3*a*b^4 + b^5 + 3*a^2*b^3 +
 a^3*b^2)))*(2*a + 3*b))/(4*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))*(a*(a +
 b)^3)^(1/2)*(2*a + 3*b)*1i)/(4*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))/(((
3*a^3*b)/2 + a^4)/(2*a*b^4 + b^5 + a^2*b^3) - (((sin(e + f*x)*(20*a^4*b + 8
*a^5 + 13*a^3*b^2))/(2*(2*a*b^3 + b^4 + a^2*b^2)) + ((a*(a + b)^3)^(1/2)*((
4*a^2*b^6 + 6*a^3*b^5 + 2*a^4*b^4)/(2*a*b^4 + b^5 + a^2*b^3) - (sin(e + f*x
)*(a*(a + b)^3)^(1/2)*(2*a + 3*b)*(16*a^2*b^7 + 64*a^3*b^6 + 80*a^4*b^5 + 3
2*a^5*b^4))/(8*(2*a*b^3 + b^4 + a^2*b^2)*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b
^2)))*(2*a + 3*b))/(4*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))*(a*(a + b)^3)
^(1/2)*(2*a + 3*b))/(4*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)) + (((sin(e +
f*x)*(20*a^4*b + 8*a^5 + 13*a^3*b^2))/(2*(2*a*b^3 + b^4 + a^2*b^2)) - ((a*(
a + b)^3)^(1/2)*((4*a^2*b^6 + 6*a^3*b^5 + 2*a^4*b^4)/(2*a*b^4 + b^5 + a^2*b
^3) + (sin(e + f*x)*(a*(a + b)^3)^(1/2)*(2*a + 3*b)*(16*a^2*b^7 + 64*a^3*b^
6 + 80*a^4*b^5 + 32*a^5*b^4))/(8*(2*a*b^3 + b^4 + a^2*b^2)*(3*a*b^4 + b^5 +
 3*a^2*b^3 + a^3*b^2)))*(2*a + 3*b))/(4*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^
2)))*(a*(a + b)^3)^(1/2)*(2*a + 3*b))/(4*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b
^2))))*(a*(a + b)^3)^(1/2)*(2*a + 3*b)*1i)/(2*f*(3*a*b^4 + b^5 + 3*a^2*b^3
+ a^3*b^2)) - (a*sin(e + f*x))/(2*b*f*(a + b)*(a + b - a*sin(e + f*x)^2))
```

3.194 $\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{a}(a+b)^{3/2}f}+\frac{\sin(e+fx)}{2(a+b)f\left(a+b-a\sin^2(e+fx)\right)}$$

```
[Out] 1/2*sin(f*x+e)/(a+b)/f/(a+b-a*sin(f*x+e)^2)+1/2*arctanh(sin(f*x+e)*a^(1/2)/
(a+b)^(1/2))/(a+b)^(3/2)/f/a^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4232, 205, 214}

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{a}f(a+b)^{3/2}}+\frac{\sin(e+fx)}{2f(a+b)\left(-a\sin^2(e+fx)+a+b\right)}$$

```
[In] Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]/(2*Sqrt[a]*(a + b)^(3/2)*f) + S
in[e + f*x]/(2*(a + b)*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
```

```
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(a+b-ax^2)^2}\,dx, x, \sin(e+fx)\right)}{f} \\
&= \frac{\sin(e+fx)}{2(a+b)f\left(a+b-a\sin^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{2(a+b)f} \\
&= \frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{a}(a+b)^{3/2}f} + \frac{\sin(e+fx)}{2(a+b)f\left(a+b-a\sin^2(e+fx)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}(a+b)^{3/2}} + \frac{2\sin(e+fx)}{(a+b)(a+2b+a\cos(2(e+fx)))}}{2f}$$

```
[In] Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]/(Sqrt[a]*(a + b)^(3/2)) + (2*S
in[e + f*x])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])))/(2*f)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{-\frac{\sin(fx+e)}{2(a+b)\left(a\sin(fx+e)^2-a-b\right)}+\frac{\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}}{f}$
default	$\frac{-\frac{\sin(fx+e)}{2(a+b)\left(a\sin(fx+e)^2-a-b\right)}+\frac{\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}}{f}$
risch	$-\frac{i\left(e^{3i(fx+e)}-e^{i(fx+e)}\right)}{f(a+b)\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{\ln\left(e^{2i(fx+e)}+\frac{2i(a+b)e^{i(fx+e)}}{\sqrt{a^2+ab}}-1\right)}{4\sqrt{a^2+ab}\,(a+b)f}-\frac{\ln\left(e^{2i(fx+e)}-\frac{2i(a+}{}\right.}{4\sqrt{a^2+ab}\,(}$

[In] int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/f*(-1/2*sin(f*x+e)/(a+b)/(a*sin(f*x+e)^2-a-b)+1/2/(a+b)/(a*(a+b))^(1/2)*a
rctanh(a*sin(f*x+e)/(a*(a+b))^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.54

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \left[\frac{\left(a\cos(fx+e)^2+b\right)\sqrt{a^2+ab}\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{a^2+ab}\sin(fx+e)-2a-b}{a\cos(fx+e)^2+b}\right)+2\left(a^2+ab\right)\sin(fx+e)}{4\left((a^4+2a^3b+a^2b^2)f\cos(fx+e)^2+(a^3b+2a^2b^2+ab^3)f\right)}, -\frac{\left(a\cos(fx+e)^2+b\right)\sqrt{-a^2-ab}\arctan\left(\frac{\sqrt{-a^2-ab}\sin(fx+e)}{a+b}\right)-\left(a^2+ab\right)\sin(fx+e)}{2\left((a^4+2a^3b+a^2b^2)f\cos(fx+e)^2+(a^3b+2a^2b^2+ab^3)f\right)}\right]$$

[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")

[Out] [1/4*((a*cos(f*x + e)^2 + b)*sqrt(a^2 + a*b)*log(-(a*cos(f*x + e)^2 - 2*sqr
t(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(a^2 + a*b
)*sin(f*x + e))/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b + 2*a^
2*b^2 + a*b^3)*f), -1/2*((a*cos(f*x + e)^2 + b)*sqrt(-a^2 - a*b)*arctan(sqr
t(-a^2 - a*b)*sin(f*x + e)/(a + b)) - (a^2 + a*b)*sin(f*x + e))/((a^4 + 2*a
^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b + 2*a^2*b^2 + a*b^3)*f)]

Sympy [F]

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

```
[In] integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(sec(e + f*x)**3/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.32

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{2\sin(fx+e)}{(a^2+ab)\sin(fx+e)^2-a^2-2ab-b^2}+\frac{\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}(a+b)}}{4f}$$

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*sin(f*x + e)/((a^2 + a*b)*sin(f*x + e)^2 - a^2 - 2*a*b - b^2) + log
((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((a + b)*a)))/(sq
rt((a + b)*a)*(a + b)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}(a+b)}+\frac{\sin(fx+e)}{\left(a\sin(fx+e)^2-a-b\right)(a+b)}}{2f}$$

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*(a + b)) +
sin(f*x + e)/((a*sin(f*x + e)^2 - a - b)*(a + b)))/f
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\sin(e+f\,x)}{2\,f\,(a+b)\,\left(-a\sin(e+f\,x)^2+a+b\right)}+\frac{\operatorname{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)}{2\,\sqrt{a}\,f\,(a+b)^{3/2}}$$

```
[In] int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^2),x)
[Out] sin(e + f*x)/(2*f*(a + b)*(a + b - a*sin(e + f*x)^2)) + atanh((a^(1/2)*sin(
e + f*x))/(a + b)^(1/2))/(2*a^(1/2)*f*(a + b)^(3/2))
```

3.195 $\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{(2a+b)\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}f} - \frac{b\sin(e+fx)}{2a(a+b)f\,(a+b-a\sin^2(e+fx))}$$

```
[Out] 1/2*(2*a+b)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(3/2)/(a+b)^(3/2)/f-1
/2*b*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4232, 393, 214}

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{(2a+b)\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{3/2}f(a+b)^{3/2}} - \frac{b\sin(e+fx)}{2af(a+b)\,(-a\sin^2(e+fx)+a+b)}$$

```
[In] Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((2*a + b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(2*a^(3/2)*(a + b)^
(3/2)*f) - (b*Sin[e + f*x])/(2*a*(a + b)*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1-x^2}{(a+b-ax^2)^2}\,dx, x, \sin(e+fx)\right)}{f}\\
&= -\frac{b\sin(e+fx)}{2a(a+b)f\left(a+b-a\sin^2(e+fx)\right)}+\frac{(2a+b)\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{2a(a+b)f}\\
&= \frac{(2a+b)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}f}-\frac{b\sin(e+fx)}{2a(a+b)f\left(a+b-a\sin^2(e+fx)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

$$\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx=\frac{\frac{(2a+b)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{2\sqrt{a}b\sin(e+fx)}{(a+b)(a+2b+a\cos(2(e+fx)))}}{2a^{3/2}f}$$

```
[In] Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
[Out] (((2*a + b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2) - (2
*Sqrt[a]*b*Sin[e + f*x])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])))/(2*a^(3/
2)*f)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\frac{b\sin(fx+e)}{2a(a+b)\left(a\sin(fx+e)^2-a-b\right)}+\frac{(2a+b)\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2a(a+b)\sqrt{a(a+b)}}}{f}$
default	$\frac{\frac{b\sin(fx+e)}{2a(a+b)\left(a\sin(fx+e)^2-a-b\right)}+\frac{(2a+b)\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2a(a+b)\sqrt{a(a+b)}}}{f}$
risch	$\frac{ib\left(\mathrm{e}^{3i(fx+e)}-\mathrm{e}^{i(fx+e)}\right)}{af(a+b)\left(a\,\mathrm{e}^{4i(fx+e)}+2a\,\mathrm{e}^{2i(fx+e)}+4b\,\mathrm{e}^{2i(fx+e)}+a\right)}+\frac{\ln\left(\mathrm{e}^{2i(fx+e)}+\frac{2i(a+b)\mathrm{e}^{i(fx+e)}}{\sqrt{a^2+ab}}-1\right)}{2\sqrt{a^2+ab}\,(a+b)f}+\frac{\ln\left(\mathrm{e}^{2i(fx+e)}+\frac{2i(a+b)\mathrm{e}}{\sqrt{a^2}}\right.}{4\sqrt{a^2+ab}\,(a+}$

[In] int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/f*(1/2*b/a/(a+b)*sin(f*x+e)/(a*sin(f*x+e)^2-a-b)+1/2*(2*a+b)/a/(a+b)/(a*(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 301, normalized size of antiderivative = 3.63

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \left[\frac{\left((2a^2+ab)\cos(fx+e)^2+2ab+b^2\right)\sqrt{a^2+ab}\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{a^2+ab}\sin(fx+e)-2a-b}{a\cos(fx+e)^2+b}\right)-2(a^2b+ab^2)\,\mathrm{s}}{4\left((a^5+2a^4b+a^3b^2)f\cos(fx+e)^2+(a^4b+2a^3b^2+a^2b^3)f\right)}\right.$$
$$\left.-\frac{\left((2a^2+ab)\cos(fx+e)^2+2ab+b^2\right)\sqrt{-a^2-ab}\arctan\left(\frac{\sqrt{-a^2-ab}\sin(fx+e)}{a+b}\right)+(a^2b+ab^2)\sin(fx+e)}{2\left((a^5+2a^4b+a^3b^2)f\cos(fx+e)^2+(a^4b+2a^3b^2+a^2b^3)f\right)}\right]$$

[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")

[Out] [1/4*(((2*a^2 + a*b)*cos(f*x + e)^2 + 2*a*b + b^2)*sqrt(a^2 + a*b)*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) - 2*(a^2*b + a*b^2)*sin(f*x + e))/((a^5 + 2*a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f), -1/2*(((2*a^2 + a*b)*cos(f*x + e)^2 + 2*a*b + b^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) + (a^2*b + a*b^2)*sin(f*x + e))/((a^5 + 2*a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f)]

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\sec\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
[Out] Integral(sec(e + f*x)/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.34

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{2\,b\sin(fx+e)}{a^3+2\,a^2b+ab^2-(a^3+a^2b)\sin(fx+e)^2}+\frac{(2\,a+b)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}(a^2+ab)}}{4\,f}$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
[Out] -1/4*(2*b*sin(f*x + e)/(a^3 + 2*a^2*b + a*b^2 - (a^3 + a^2*b)*sin(f*x + e)^
2) + (2*a + b)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqr
t((a + b)*a)))/(sqrt((a + b)*a)*(a^2 + a*b)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.10

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{(2\,a+b)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^2+ab)\sqrt{-a^2-ab}}-\frac{b\sin(fx+e)}{\left(a\sin(fx+e)^2-a-b\right)(a^2+ab)}}{2\,f}$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
[Out] -1/2*((2*a + b)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^2 + a*b)*sqrt(-
a^2 - a*b)) - b*sin(f*x + e)/((a*sin(f*x + e)^2 - a - b)*(a^2 + a*b)))/f
```

Mupad [B] (verification not implemented)

Time = 19.74 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.86

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)\,(2\,a+b)}{2\,a^{3/2}\,f\,(a+b)^{3/2}} - \frac{b\,\sin\left(e+f\,x\right)}{2\,a\,f\,\left(a+b\right)\,\left(-a\sin\left(e+f\,x\right)^2+a+b\right)}$$

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] (atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(2*a + b))/(2*a^(3/2)*f*(a + b
)^(3/2)) - (b*sin(e + f*x))/(2*a*f*(a + b)*(a + b - a*sin(e + f*x)^2))
```

3.196 $\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 101

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2} \, dx = -\frac{b(4a+3b)\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{5/2}(a+b)^{3/2}f} + \frac{\sin(e+fx)}{a^2f} + \frac{b^2\sin(e+fx)}{2a^2(a+b)f\left(a+b-a\sin^2(e+fx)\right)}$$

```
[Out] -1/2*b*(4*a+3*b)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(5/2)/(a+b)^(3/2
)/f+sin(f*x+e)/a^2/f+1/2*b^2*sin(f*x+e)/a^2/(a+b)/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4232, 398, 393, 214}

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2} \, dx = -\frac{b(4a+3b)\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{5/2}f(a+b)^{3/2}} + \frac{b^2\sin(e+fx)}{2a^2f(a+b)\left(-a\sin^2(e+fx)+a+b\right)} + \frac{\sin(e+fx)}{a^2f}$$

```
[In] Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/2*(b*(4*a + 3*b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a^(5/2)*(
a + b)^(3/2)*f) + Sin[e + f*x]/(a^2*f) + (b^2*Sin[e + f*x])/(2*a^2*(a + b)*
f*(a + b - a*Sin[e + f*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{(a+b-ax^2)^2}\,dx, x, \sin(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2}-\frac{b(2a+b)-2abx^2}{a^2(a+b-ax^2)^2}\right)\,dx, x, \sin(e+fx)\right)}{f} \\
&= \frac{\sin(e+fx)}{a^2f}-\frac{\text{Subst}\left(\int \frac{b(2a+b)-2abx^2}{(a+b-ax^2)^2}\,dx, x, \sin(e+fx)\right)}{a^2f} \\
&= \frac{\sin(e+fx)}{a^2f}+\frac{b^2\sin(e+fx)}{2a^2(a+b)f\,(a+b-a\sin^2(e+fx))} \\
&\quad -\frac{(b(4a+3b))\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{2a^2(a+b)f} \\
&= -\frac{b(4a+3b)\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{5/2}(a+b)^{3/2}f}+\frac{\sin(e+fx)}{a^2f}+\frac{b^2\sin(e+fx)}{2a^2(a+b)f\,(a+b-a\sin^2(e+fx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{-\frac{b(4a+3b)\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}+\sqrt{a}\sin(e+fx)\left(2+\frac{b^2}{(a+b)(a+b-a\sin^2(e+fx))}\right)}{2a^{5/2}f}$$

```
[In] Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (-((b*(4*a + 3*b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2
)) + Sqrt[a]*Sin[e + f*x]*(2 + b^2/((a + b)*(a + b - a*Sin[e + f*x]^2))))/(
2*a^(5/2)*f)
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{\sin(fx+e)}{a^2}+\frac{b\left(-\frac{b\sin(fx+e)}{2(a+b)\left(a\sin(fx+e)^2-a-b\right)}-\frac{(4a+3b)\,\text{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{a^2}}{f}$
default	$\frac{\frac{\sin(fx+e)}{a^2}+\frac{b\left(-\frac{b\sin(fx+e)}{2(a+b)\left(a\sin(fx+e)^2-a-b\right)}-\frac{(4a+3b)\,\text{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{a^2}}{f}$
risch	$-\frac{i\,e^{i(fx+e)}}{2a^2f}+\frac{i\,e^{-i(fx+e)}}{2a^2f}-\frac{ib^2\left(e^{3i(fx+e)}-e^{i(fx+e)}\right)}{a^2(a+b)f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{\ln\left(e^{2i(fx+e)}-\frac{2i(a+b)e^{i(fx+}}{\sqrt{a^2+ab}}\right.}{\sqrt{a^2+ab}\,(a+b)fa}$

```
[In] int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(sin(f*x+e)/a^2+1/a^2*b*(-1/2/(a+b)*b*sin(f*x+e)/(a*sin(f*x+e)^2-a-b)-1
/2*(4*a+3*b)/(a+b)/(a*(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 391, normalized size of antiderivative = 3.87

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \left[\frac{(4\,ab^2+3\,b^3+(4\,a^2b+3\,ab^2)\cos\left(fx+e\right)^2)\sqrt{a^2+ab}\log\left(-\frac{a\cos(fx+e)^2+2\sqrt{a^2+ab}\sin(fx+e)-2\,a-b}{a\cos(fx+e)^2+b}\right)+2\,(2\,a^3b}{4\,((a^6+2\,a^5b+a^4b^2)f\cos\left(fx+e\right)^2+(a^5b+2\,a^4b}\right.$$

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*((4*a*b^2 + 3*b^3 + (4*a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt(a^2 + a*b
)*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*cos
(f*x + e)^2 + b)) + 2*(2*a^3*b + 5*a^2*b^2 + 3*a*b^3 + 2*(a^4 + 2*a^3*b + a
^2*b^2)*cos(f*x + e)^2)*sin(f*x + e))/((a^6 + 2*a^5*b + a^4*b^2)*f*cos(f*x
+ e)^2 + (a^5*b + 2*a^4*b^2 + a^3*b^3)*f), 1/2*((4*a*b^2 + 3*b^3 + (4*a^2*b
 + 3*a*b^2)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*sin(f*
x + e)/(a + b)) + (2*a^3*b + 5*a^2*b^2 + 3*a*b^3 + 2*(a^4 + 2*a^3*b + a^2*b
^2)*cos(f*x + e)^2)*sin(f*x + e))/((a^6 + 2*a^5*b + a^4*b^2)*f*cos(f*x + e)
^2 + (a^5*b + 2*a^4*b^2 + a^3*b^3)*f)]
```

Sympy [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\cos\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(cos(e + f*x)/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.32

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{2\,b^2\sin(fx+e)}{a^4+2\,a^3b+a^2b^2-(a^4+a^3b)\sin(fx+e)^2}+\frac{(4\,ab+3\,b^2)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(a^3+a^2b)\sqrt{(a+b)a}}+\frac{4\sin(fx+e)}{a^2}}{4\,f}$$

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b^2*sin(f*x + e)/(a^4 + 2*a^3*b + a^2*b^2 - (a^4 + a^3*b)*sin(f*x +
e)^2) + (4*a*b + 3*b^2)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x +
 e) + sqrt((a + b)*a)))/((a^3 + a^2*b)*sqrt((a + b)*a)) + 4*sin(f*x + e)/a^
2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.12

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{b^2\sin(fx+e)}{(a^3+a^2b)\left(a\sin(fx+e)^2-a-b\right)} - \frac{(4\,ab+3\,b^2)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^3+a^2b)\sqrt{-a^2-ab}} - \frac{2\,\sin(fx+e)}{a^2}}{2\,f}$$

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*sin(f*x + e)/((a^3 + a^2*b)*(a*sin(f*x + e)^2 - a - b)) - (4*a*b
+ 3*b^2)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^3 + a^2*b)*sqrt(-a^2 -
 a*b)) - 2*sin(f*x + e)/a^2)/f
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.93

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\sin(e+f\,x)}{a^2\,f} + \frac{b^2\,\sin(e+f\,x)}{2\,f\,(a+b)\,\left(-a^3\sin(e+f\,x)^2+a^3+b\,a^2\right)} - \frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)\,(4\,a+3\,b)}{2\,a^{5/2}\,f\,(a+b)^{3/2}}$$

```
[In] int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] sin(e + f*x)/(a^2*f) + (b^2*sin(e + f*x))/(2*f*(a + b)*(a^2*b + a^3 - a^3*s
in(e + f*x)^2)) - (b*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(4*a + 3*b
))/(2*a^(5/2)*f*(a + b)^(3/2))
```

3.197 $\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 126

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^2} dx = \frac{b^2(6a+5b)\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{7/2}(a+b)^{3/2}f} + \frac{(a-2b)\sin(e+fx)}{a^3f} - \frac{\sin^3(e+fx)}{3a^2f} - \frac{b^3\sin(e+fx)}{2a^3(a+b)f\,(a+b-a\sin^2(e+fx))}$$

```
[Out] 1/2*b^2*(6*a+5*b)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(7/2)/(a+b)^(3/
2)/f+(a-2*b)*sin(f*x+e)/a^3/f-1/3*sin(f*x+e)^3/a^2/f-1/2*b^3*sin(f*x+e)/a^3
/(a+b)/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4232, 398, 393, 214}

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^2} dx = \frac{b^2(6a+5b)\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{7/2}f(a+b)^{3/2}} - \frac{b^3\sin(e+fx)}{2a^3f(a+b)\left(-a\sin^2(e+fx)+a+b\right)} + \frac{(a-2b)\sin(e+fx)}{a^3f} - \frac{\sin^3(e+fx)}{3a^2f}$$

```
[In] Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (b^2*(6*a + 5*b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(2*a^(7/2)*(a
 + b)^(3/2)*f) + ((a - 2*b)*Sin[e + f*x])/(a^3*f) - Sin[e + f*x]^3/(3*a^2*f
) - (b^3*Sin[e + f*x])/(2*a^3*(a + b)*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{(a+b-ax^2)^2}\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a-2b}{a^3}-\frac{x^2}{a^2}+\frac{b^2(3a+2b)-3ab^2x^2}{a^3(a+b-ax^2)^2}\right)\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{(a-2b)\sin(e+fx)}{a^3f}-\frac{\sin^3(e+fx)}{3a^2f}+\frac{\text{Subst}\left(\int \frac{b^2(3a+2b)-3ab^2x^2}{(a+b-ax^2)^2}\,dx, x, \sin(e+fx)\right)}{a^3f}\\
&= \frac{(a-2b)\sin(e+fx)}{a^3f}-\frac{\sin^3(e+fx)}{3a^2f}-\frac{b^3\sin(e+fx)}{2a^3(a+b)f\,(a+b-a\sin^2(e+fx))}\\
&\quad+\frac{(b^2(6a+5b))\,\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{2a^3(a+b)f}
\end{aligned}
$$

$$= \frac{b^2(6a+5b)\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{7/2}(a+b)^{3/2}f} + \frac{(a-2b)\sin(e+fx)}{a^3f} - \frac{\sin^3(e+fx)}{3a^2f} - \frac{b^3\sin(e+fx)}{2a^3(a+b)f\,(a+b-a\sin^2(e+fx))}$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.10

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{-\frac{3b^2(6a+5b)(\log(\sqrt{a+b}-\sqrt{a}\sin(e+fx))-\log(\sqrt{a+b}+\sqrt{a}\sin(e+fx)))}{(a+b)^{3/2}} + 3\sqrt{a}\left(3a-8b-\frac{4b^3}{(a+b)(a+2b+a\cos(2(e+fx)))}\right)\sin(e+fx)}{12a^{7/2}f}$$

[In] Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]

[Out] ((-3*b^2*(6*a + 5*b)*(Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x]] - Log[Sqrt[a + b] + Sqrt[a]*Sin[e + f*x]]))/(a + b)^(3/2) + 3*Sqrt[a]*(3*a - 8*b - (4*b^3)/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])))*Sin[e + f*x] + a^(3/2)*Sin[3*(e + f*x)])/(12*a^(7/2)*f)

Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{-\frac{\frac{a\sin(fx+e)^3}{3}-\sin(fx+e)a+2\sin(fx+e)b}{a^3}-\frac{b^2\left(-\frac{b\sin(fx+e)}{2(a+b)\left(a\sin(fx+e)^2-a-b\right)}-\frac{(6a+5b)\,\text{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{a^3}}{f}$
default	$\frac{-\frac{\frac{a\sin(fx+e)^3}{3}-\sin(fx+e)a+2\sin(fx+e)b}{a^3}-\frac{b^2\left(-\frac{b\sin(fx+e)}{2(a+b)\left(a\sin(fx+e)^2-a-b\right)}-\frac{(6a+5b)\,\text{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{a^3}}{f}$
risch	$-\frac{i\,e^{3i(fx+e)}}{24a^2f}-\frac{3i\,e^{i(fx+e)}}{8a^2f}+\frac{i\,e^{i(fx+e)}b}{a^3f}+\frac{3i\,e^{-i(fx+e)}}{8a^2f}-\frac{i\,e^{-i(fx+e)}b}{a^3f}+\frac{i\,e^{-3i(fx+e)}}{24a^2f}+\frac{ib^3(e^{3i(f}}{a^3(a+b)f(a\,e^{4i(fx+e)}+}$

[In] int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/f*(-1/a^3*(1/3*a*sin(f*x+e)^3-sin(f*x+e)*a+2*sin(f*x+e)*b)-b^2/a^3*(-1/2/(a+b)*b*sin(f*x+e)/(a*sin(f*x+e)^2-a-b)-1/2*(6*a+5*b)/(a+b)/(a*(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(115) = 230$.

Time = 0.31 (sec) , antiderivative size = 490, normalized size of antiderivative = 3.89

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \left[\frac{3\left(6\,ab^3+5\,b^4+(6\,a^2b^2+5\,ab^3)\cos(fx+e)^2\right)\sqrt{a^2+ab}\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{a^2+ab}\sin(fx+e)-2\,a-b}{a\cos(fx+e)^2+b}\right)+2\,(4\cdots}{12\left((a^7+2\,a^6b+a^5\cdots\right.}\right.$$
$$-\frac{3\left(6\,ab^3+5\,b^4+(6\,a^2b^2+5\,ab^3)\cos(fx+e)^2\right)\sqrt{-a^2-ab}\arctan\left(\frac{\sqrt{-a^2-ab}\sin(fx+e)}{a+b}\right)-\left(4\,a^4b-4\,a^3b^2\cdots\right.}{6\left((a^7+2\,a^6b+a^5b^2)f\cos\cdots\right.}$$

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(6*a*b^3 + 5*b^4 + (6*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*sqrt(a^2
+ a*b)*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(
a*cos(f*x + e)^2 + b)) + 2*(4*a^4*b - 4*a^3*b^2 - 23*a^2*b^3 - 15*a*b^4 + 2
*(a^5 + 2*a^4*b + a^3*b^2)*cos(f*x + e)^4 + 2*(2*a^5 - a^4*b - 8*a^3*b^2 -
5*a^2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f
*x + e)^2 + (a^6*b + 2*a^5*b^2 + a^4*b^3)*f), -1/6*(3*(6*a*b^3 + 5*b^4 + (6
*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b
)*sin(f*x + e)/(a + b)) - (4*a^4*b - 4*a^3*b^2 - 23*a^2*b^3 - 15*a*b^4 + 2*
(a^5 + 2*a^4*b + a^3*b^2)*cos(f*x + e)^4 + 2*(2*a^5 - a^4*b - 8*a^3*b^2 - 5
*a^2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*
x + e)^2 + (a^6*b + 2*a^5*b^2 + a^4*b^3)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.22

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{6\,b^3\sin(fx+e)}{a^5+2\,a^4b+a^3b^2-(a^5+a^4b)\sin(fx+e)^2}+\frac{3\,(6\,ab^2+5\,b^3)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(a^4+a^3b)\sqrt{(a+b)a}}+\frac{4\left(a\sin(fx+e)^3-3\,(a-2\,b)\sin(fx+e)\right)}{a^3}}{12\,f}$$

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/12*(6*b^3*sin(f*x + e)/(a^5 + 2*a^4*b + a^3*b^2 - (a^5 + a^4*b)*sin(f*x
+ e)^2) + 3*(6*a*b^2 + 5*b^3)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin
(f*x + e) + sqrt((a + b)*a)))/((a^4 + a^3*b)*sqrt((a + b)*a)) + 4*(a*sin(f*
x + e)^3 - 3*(a - 2*b)*sin(f*x + e))/a^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.16

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{3\,b^3\sin(fx+e)}{(a^4+a^3b)\left(a\sin(fx+e)^2-a-b\right)}-\frac{3\,(6\,ab^2+5\,b^3)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^4+a^3b)\sqrt{-a^2-ab}}-\frac{2\left(a^4\sin(fx+e)^3-3\,a^4\sin(fx+e)+6\,a^3b\sin(fx+e)\right)}{a^6}}{6\,f}$$

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*b^3*sin(f*x + e)/((a^4 + a^3*b)*(a*sin(f*x + e)^2 - a - b)) - 3*(6*a
*b^2 + 5*b^3)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^4 + a^3*b)*sqrt(-
a^2 - a*b)) - 2*(a^4*sin(f*x + e)^3 - 3*a^4*sin(f*x + e) + 6*a^3*b*sin(f*x
+ e))/a^6)/f
```

Mupad [B] (verification not implemented)

Time = 19.79 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{b^2\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)\,(6\,a+5\,b)}{2\,a^{7/2}\,f\,(a+b)^{3/2}} - \frac{\sin\left(e+f\,x\right)^3}{3\,a^2\,f} - \frac{b^3\,\sin\left(e+f\,x\right)}{2\,f\,\left(a+b\right)\,\left(-a^4\sin\left(e+f\,x\right)^2+a^4+b\,a^3\right)} - \frac{\sin\left(e+f\,x\right)\,\left(\frac{2\,(a+b)}{a^3}-\frac{3}{a^2}\right)}{f}$$

```
[In] int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (b^2*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(6*a + 5*b))/(2*a^(7/2)*f*
(a + b)^(3/2)) - sin(e + f*x)^3/(3*a^2*f) - (b^3*sin(e + f*x))/(2*f*(a + b)
*(a^3*b + a^4 - a^4*sin(e + f*x)^2)) - (sin(e + f*x)*((2*(a + b))/a^3 - 3/a
^2))/f
```

3.198 $\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 157

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{b^3(8a+7b)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{9/2}(a+b)^{3/2}f} + \frac{(a^2-2ab+3b^2)\sin(e+fx)}{a^4f} - \frac{2(a-b)\sin^3(e+fx)}{3a^3f} + \frac{\sin^5(e+fx)}{5a^2f} + \frac{b^4\sin(e+fx)}{2a^4(a+b)f\,(a+b-a\sin^2(e+fx))}$$

```
[Out] -1/2*b^3*(8*a+7*b)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(9/2)/(a+b)^(3
/2)/f+(a^2-2*a*b+3*b^2)*sin(f*x+e)/a^4/f-2/3*(a-b)*sin(f*x+e)^3/a^3/f+1/5*s
in(f*x+e)^5/a^2/f+1/2*b^4*sin(f*x+e)/a^4/(a+b)/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4232, 398, 393, 214}

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{b^3(8a+7b)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{9/2}f(a+b)^{3/2}} + \frac{b^4\sin(e+fx)}{2a^4f(a+b)\left(-a\sin^2(e+fx)+a+b\right)} - \frac{2(a-b)\sin^3(e+fx)}{3a^3f} + \frac{\sin^5(e+fx)}{5a^2f} + \frac{(a^2-2ab+3b^2)\sin(e+fx)}{a^4f}$$

```
[In] Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/2*(b^3*(8*a + 7*b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a^(9/2)
*(a + b)^(3/2)*f) + ((a^2 - 2*a*b + 3*b^2)*Sin[e + f*x])/(a^4*f) - (2*(a -
b)*Sin[e + f*x]^3)/(3*a^3*f) + Sin[e + f*x]^5/(5*a^2*f) + (b^4*Sin[e + f*x]
)/(2*a^4*(a + b)*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^4}{(a+b-ax^2)^2}\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a^2-2ab+3b^2}{a^4} - \frac{2(a-b)x^2}{a^3} + \frac{x^4}{a^2} - \frac{b^3(4a+3b)-4ab^3x^2}{a^4(a+b-ax^2)^2}\right)\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{(a^2-2ab+3b^2)\sin(e+fx)}{a^4f} - \frac{2(a-b)\sin^3(e+fx)}{3a^3f}\\
&\quad + \frac{\sin^5(e+fx)}{5a^2f} - \frac{\text{Subst}\left(\int \frac{b^3(4a+3b)-4ab^3x^2}{(a+b-ax^2)^2}\,dx, x, \sin(e+fx)\right)}{a^4f}
\end{aligned}
$$

$$= \frac{(a^2-2ab+3b^2)\sin(e+fx)}{a^4f} - \frac{2(a-b)\sin^3(e+fx)}{3a^3f} + \frac{\sin^5(e+fx)}{5a^2f} + \frac{b^4\sin(e+fx)}{2a^4(a+b)f\left(a+b-a\sin^2(e+fx)\right)} - \frac{(b^3(8a+7b))\,\mathrm{Subst}\left(\int\frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{2a^4(a+b)f}$$

$$= -\frac{b^3(8a+7b)\mathrm{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{9/2}(a+b)^{3/2}f} + \frac{(a^2-2ab+3b^2)\sin(e+fx)}{a^4f} - \frac{2(a-b)\sin^3(e+fx)}{3a^3f} + \frac{\sin^5(e+fx)}{5a^2f} + \frac{b^4\sin(e+fx)}{2a^4(a+b)f\left(a+b-a\sin^2(e+fx)\right)}$$

Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.09

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\frac{60b^3(8a+7b)\left(\log\left(\sqrt{a+b}-\sqrt{a}\sin(e+fx)\right)-\log\left(\sqrt{a+b}+\sqrt{a}\sin(e+fx)\right)\right)}{(a+b)^{3/2}} + 30\sqrt{a}\left(5a^2-12ab+8b^2\left(3+\frac{b^2}{(a+b)(a+2b+a\cos(2(e+fx)))}\right.\right.}{240a^{9/2}f}$$

```
[In] Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((60*b^3*(8*a + 7*b)*(Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x]] - Log[Sqrt[a
+ b] + Sqrt[a]*Sin[e + f*x]]))/(a + b)^(3/2) + 30*Sqrt[a]*(5*a^2 - 12*a*b +
 8*b^2*(3 + b^2/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)]))))*Sin[e + f*x] + 5
*a^(3/2)*(5*a - 8*b)*Sin[3*(e + f*x)] + 3*a^(5/2)*Sin[5*(e + f*x)])/(240*a^
(9/2)*f)
```

Maple [A] (verified)

Time = 4.72 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{\frac{a^2\sin(fx+e)^5}{5}-\frac{2a^2\sin(fx+e)^3}{3}+\frac{2a\sin(fx+e)^3b}{3}+\sin(fx+e)a^2-2\sin(fx+e)ab+3\sin(fx+e)b^2}{a^4}+\frac{b^3\left(-\frac{b\sin(fx+e)}{2(a+b)\left(a\sin(fx+e)^2-a-b\right.}\right.}{}}{f}$
default	$\frac{\frac{\frac{a^2\sin(fx+e)^5}{5}-\frac{2a^2\sin(fx+e)^3}{3}+\frac{2a\sin(fx+e)^3b}{3}+\sin(fx+e)a^2-2\sin(fx+e)ab+3\sin(fx+e)b^2}{a^4}+\frac{b^3\left(-\frac{b\sin(fx+e)}{2(a+b)\left(a\sin(fx+e)^2-a-b\right.}\right.}{}}{f}$
risch	$-\frac{5ie^{3i(fx+e)}}{96a^2f}+\frac{ie^{3i(fx+e)}b}{12a^3f}-\frac{3ie^{-i(fx+e)}b}{4a^3f}+\frac{3ie^{-i(fx+e)}b^2}{2f\,a^4}+\frac{5ie^{-3i(fx+e)}}{96a^2f}-\frac{3ie^{i(fx+e)}b^2}{2f\,a^4}-\frac{5ie^{i(fx+e)}}{16a^2f}+$

```
[In] int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^4*(1/5*a^2*sin(f*x+e)^5-2/3*a^2*sin(f*x+e)^3+2/3*a*sin(f*x+e)^3*b+
sin(f*x+e)*a^2-2*sin(f*x+e)*a*b+3*sin(f*x+e)*b^2)+b^3/a^4*(-1/2/(a+b)*b*sin
(f*x+e)/(a*sin(f*x+e)^2-a-b)-1/2*(8*a+7*b)/(a+b)/(a*(a+b))^(1/2)*arctanh(a*
sin(f*x+e)/(a*(a+b))^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 583, normalized size of antiderivative = 3.71

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$=\left[\frac{15\left(8\,ab^4+7\,b^5+(8\,a^2b^3+7\,ab^4)\cos\left(fx+e\right)^2\right)\sqrt{a^2+ab}\log\left(-\frac{a\cos(fx+e)^2+2\,\sqrt{a^2+ab}\sin(fx+e)-2\,a-b}{a\cos(fx+e)^2+b}\right)+2\,(\right.}{}\right.$$

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(15*(8*a*b^4 + 7*b^5 + (8*a^2*b^3 + 7*a*b^4)*cos(f*x + e)^2)*sqrt(a^2
 + a*b)*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/
(a*cos(f*x + e)^2 + b)) + 2*(6*(a^6 + 2*a^5*b + a^4*b^2)*cos(f*x + e)^6 + 1
6*a^5*b - 8*a^4*b^2 + 26*a^3*b^3 + 155*a^2*b^4 + 105*a*b^5 + 2*(4*a^6 + a^5
*b - 10*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^4 + 2*(8*a^6 + 11*a^4*b^2 + 54*a^
3*b^3 + 35*a^2*b^4)*cos(f*x + e)^2)*sin(f*x + e))/((a^8 + 2*a^7*b + a^6*b^2
)*f*cos(f*x + e)^2 + (a^7*b + 2*a^6*b^2 + a^5*b^3)*f), 1/30*(15*(8*a*b^4 +
7*b^5 + (8*a^2*b^3 + 7*a*b^4)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(
-a^2 - a*b)*sin(f*x + e)/(a + b)) + (6*(a^6 + 2*a^5*b + a^4*b^2)*cos(f*x +
```

```
e)^6 + 16*a^5*b - 8*a^4*b^2 + 26*a^3*b^3 + 155*a^2*b^4 + 105*a*b^5 + 2*(4*a
^6 + a^5*b - 10*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^4 + 2*(8*a^6 + 11*a^4*b^2
 + 54*a^3*b^3 + 35*a^2*b^4)*cos(f*x + e)^2)*sin(f*x + e))/((a^8 + 2*a^7*b +
 a^6*b^2)*f*cos(f*x + e)^2 + (a^7*b + 2*a^6*b^2 + a^5*b^3)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.17

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{30\,b^4\sin(fx+e)}{a^6+2\,a^5b+a^4b^2-(a^6+a^5b)\sin(fx+e)^2} + \frac{15\,(8\,ab^3+7\,b^4)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(a^5+a^4b)\sqrt{(a+b)a}} + \frac{4\left(3\,a^2\sin(fx+e)^5-10\,(a^2-ab)\sin(fx+e)^3+15\,(a^2-2\right.}{a^4}}{60\,f}$$

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
[Out] 1/60*(30*b^4*sin(f*x + e)/(a^6 + 2*a^5*b + a^4*b^2 - (a^6 + a^5*b)*sin(f*x
+ e)^2) + 15*(8*a*b^3 + 7*b^4)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*si
n(f*x + e) + sqrt((a + b)*a)))/((a^5 + a^4*b)*sqrt((a + b)*a)) + 4*(3*a^2*s
in(f*x + e)^5 - 10*(a^2 - a*b)*sin(f*x + e)^3 + 15*(a^2 - 2*a*b + 3*b^2)*si
n(f*x + e))/a^4)/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.20

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{15\,b^4\sin(fx+e)}{(a^5+a^4b)\left(a\sin(fx+e)^2-a-b\right)} - \frac{15\,(8\,ab^3+7\,b^4)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^5+a^4b)\sqrt{-a^2-ab}} - \frac{2\left(3\,a^8\sin(fx+e)^5-10\,a^8\sin(fx+e)^3+10\,a^7b\sin(fx+e)^3+15\,a^8\right.}{a^{10}}}{30\,f}$$

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/30*(15*b^4*sin(f*x + e)/((a^5 + a^4*b)*(a*sin(f*x + e)^2 - a - b)) - 15*
(8*a*b^3 + 7*b^4)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^5 + a^4*b)*sq
rt(-a^2 - a*b)) - 2*(3*a^8*sin(f*x + e)^5 - 10*a^8*sin(f*x + e)^3 + 10*a^7*
b*sin(f*x + e)^3 + 15*a^8*sin(f*x + e) - 30*a^7*b*sin(f*x + e) + 45*a^6*b^2
*sin(f*x + e))/a^10)/f
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.10

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\sin(e+f\,x)^5}{5\,a^2\,f} + \frac{\sin(e+f\,x)^3\,\left(\frac{2\,(a+b)}{3\,a^3}-\frac{4}{3\,a^2}\right)}{f} + \frac{\sin(e+f\,x)\,\left(\frac{6}{a^2}-\frac{(a+b)^2}{a^4}+\frac{2\,(a+b)\,\left(\frac{2\,(a+b)}{a^3}-\frac{4}{a^2}\right)}{a}\right)}{f} + \frac{b^4\,\sin(e+f\,x)}{2\,f\,(a+b)\,\left(-a^5\sin(e+f\,x)^2+a^5+b\,a^4\right)} - \frac{b^3\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)\,(8\,a+7\,b)}{2\,a^{9/2}\,f\,(a+b)^{3/2}}$$

```
[In] int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] sin(e + f*x)^5/(5*a^2*f) + (sin(e + f*x)^3*((2*(a + b))/(3*a^3) - 4/(3*a^2)
))/f + (sin(e + f*x)*(6/a^2 - (a + b)^2/a^4 + (2*(a + b)*((2*(a + b))/a^3 -
 4/a^2))/a))/f + (b^4*sin(e + f*x))/(2*f*(a + b)*(a^4*b + a^5 - a^5*sin(e +
 f*x)^2)) - (b^3*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(8*a + 7*b))/(
2*a^(9/2)*f*(a + b)^(3/2))
```

3.199 $\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{a(3a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{5/2}(a+b)^{3/2}f}+\frac{\tan(e+fx)}{b^2f}+\frac{a^2\tan(e+fx)}{2b^2(a+b)f\,(a+b+b\tan^2(e+fx))}$$

```
[Out] -1/2*a*(3*a+4*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(5/2)/(a+b)^(3/2)
/f+tan(f*x+e)/b^2/f+1/2*a^2*tan(f*x+e)/b^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4231, 398, 393, 211}

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{a^2\tan(e+fx)}{2b^2f(a+b)\,(a+b\tan^2(e+fx)+b)}-\frac{a(3a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{5/2}f(a+b)^{3/2}}+\frac{\tan(e+fx)}{b^2f}$$

```
[In] Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/2*(a*(3*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(b^(5/2)*(a
 + b)^(3/2)*f) + Tan[e + f*x]/(b^2*f) + (a^2*Tan[e + f*x])/(2*b^2*(a + b)*f
*(a + b + b*Tan[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{b^2} - \frac{a(a+2b)+2abx^2}{b^2(a+b+bx^2)^2}\right)\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\tan(e+fx)}{b^2 f} - \frac{\text{Subst}\left(\int \frac{a(a+2b)+2abx^2}{(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{b^2 f} \\
&= \frac{\tan(e+fx)}{b^2 f} + \frac{a^2\tan(e+fx)}{2b^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)} \\
&\quad - \frac{(a(3a+4b))\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{2b^2(a+b)f} \\
&= -\frac{a(3a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{5/2}(a+b)^{3/2}f} + \frac{\tan(e+fx)}{b^2 f} + \frac{a^2\tan(e+fx)}{2b^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.16 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.48

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(\frac{a(3a+4b)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a\cos}{(a+b)^{3/2}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{8b^2f\,(a+}$$

```
[In] Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((a*(3*a + 4*b)*ArcTan[(Sec[
f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2
*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])
*(Cos[2*e] - I*Sin[2*e]))/((a + b)^(3/2)*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + 2
*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e]*Sec[e + f*x]*Sin[f*x] + (a*(-((a + 2
*b)*Sin[2*e]) + a*Sin[2*f*x]))/((a + b)*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])
)))/(8*b^2*f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)}{b^2}-\frac{a\left(-\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(3a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{b^2}}{f}$
default	$\frac{\frac{\tan(fx+e)}{b^2}-\frac{a\left(-\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(3a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{b^2}}{f}$
risch	$\frac{i\left(3a^2e^{4i(fx+e)}+4ab\,e^{4i(fx+e)}+6a^2e^{2i(fx+e)}+14ab\,e^{2i(fx+e)}+8b^2e^{2i(fx+e)}+3a^2+2ab\right)}{(a+b)b^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)\left(e^{2i(fx+e)}+1\right)}-\frac{3a^2\ln\left(e^{2i(fx+e)}-\frac{2iba+2ib^2}{}\right.}{4\sqrt{-ab-b^2}}$

```
[In] int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(tan(f*x+e)/b^2-a/b^2*(-1/2*a/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2
*(3*a+4*b)/(a+b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(88) = 176.

Time = 0.31 (sec) , antiderivative size = 516, normalized size of antiderivative = 5.16

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \left[-\frac{\left((3\,a^3+4\,a^2b)\cos\left(fx+e\right)^3+(3\,a^2b+4\,ab^2)\cos\left(fx+e\right)\right)\sqrt{-ab-b^2}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,a\right.}{8\,\left((a^3b^3+2\,a^2b^4\right.}\right.$$

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(((3*a^3 + 4*a^2*b)*cos(f*x + e)^3 + (3*a^2*b + 4*a*b^2)*cos(f*x + e)
)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4
*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-
a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 +
 b^2)) - 4*(2*a^2*b^2 + 4*a*b^3 + 2*b^4 + (3*a^3*b + 5*a^2*b^2 + 2*a*b^3)*c
os(f*x + e)^2)*sin(f*x + e))/((a^3*b^3 + 2*a^2*b^4 + a*b^5)*f*cos(f*x + e)^
3 + (a^2*b^4 + 2*a*b^5 + b^6)*f*cos(f*x + e)), 1/4*(((3*a^3 + 4*a^2*b)*cos(
f*x + e)^3 + (3*a^2*b + 4*a*b^2)*cos(f*x + e))*sqrt(a*b + b^2)*arctan(1/2*(
(a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e)))
+ 2*(2*a^2*b^2 + 4*a*b^3 + 2*b^4 + (3*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cos(f*x
+ e)^2)*sin(f*x + e))/((a^3*b^3 + 2*a^2*b^4 + a*b^5)*f*cos(f*x + e)^3 + (a^
2*b^4 + 2*a*b^5 + b^6)*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\sec^6\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

```
[In] integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(sec(e + f*x)**6/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.10

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{a^2\tan(fx+e)}{a^2b^2+2\,ab^3+b^4+(ab^3+b^4)\tan(fx+e)^2} - \frac{(3\,a^2+4\,ab)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(ab^2+b^3)\sqrt{(a+b)b}} + \frac{2\,\tan(fx+e)}{b^2}}{2\,f}$$

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2*tan(f*x + e)/(a^2*b^2 + 2*a*b^3 + b^4 + (a*b^3 + b^4)*tan(f*x + e)
^2) - (3*a^2 + 4*a*b)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a*b^2 + b^3)
*sqrt((a + b)*b)) + 2*tan(f*x + e)/b^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.20

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{a^2\tan(fx+e)}{(ab^2+b^3)\left(b\tan(fx+e)^2+a+b\right)} - \frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3\,a^2+4\,ab)}{(ab^2+b^3)\sqrt{ab+b^2}} + \frac{2\,\tan(fx+e)}{b^2}}{2\,f}$$

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*tan(f*x + e)/((a*b^2 + b^3)*(b*tan(f*x + e)^2 + a + b)) - (pi*floo
r((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))*(3*a
^2 + 4*a*b)/((a*b^2 + b^3)*sqrt(a*b + b^2)) + 2*tan(f*x + e)/b^2)/f
```

Mupad [B] (verification not implemented)

Time = 19.57 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.13

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\tan(e+f\,x)}{b^2\,f} + \frac{a^2\tan(e+f\,x)}{2\,f\,(a+b)\,\left(b^3\tan\left(e+f\,x\right)^2+b^3+a\,b^2\right)} - \frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\tan(e+f\,x)\,(3\,a+4\,b)}{\sqrt{a+b}\,(3\,a^2+4\,b\,a)}\right)\,(3\,a+4\,b)}{2\,b^{5/2}\,f\,{(a+b)}^{3/2}}$$

```
[In] int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] tan(e + f*x)/(b^2*f) + (a^2*tan(e + f*x))/(2*f*(a + b)*(a*b^2 + b^3 + b^3*t
an(e + f*x)^2)) - (a*atan((a*b^(1/2)*tan(e + f*x)*(3*a + 4*b))/((a + b)^(1/
2)*(4*a*b + 3*a^2)))*(3*a + 4*b))/(2*b^(5/2)*f*(a + b)^(3/2))
```

3.200 $\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{(a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{3/2}(a+b)^{3/2}f} - \frac{a\tan(e+fx)}{2b(a+b)f\,(a+b+b\tan^2(e+fx))}$$

```
[Out] 1/2*(a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(3/2)/(a+b)^(3/2)/f-1/
2*a*tan(f*x+e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4231, 393, 211}

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{(a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{3/2}f(a+b)^{3/2}} - \frac{a\tan(e+fx)}{2bf(a+b)\,(a+b\tan^2(e+fx)+b)}$$

```
[In] Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*b^(3/2)*(a + b)^(
3/2)*f) - (a*Tan[e + f*x])/(2*b*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1+x^2}{(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan(e+fx)}{2b(a+b)f\,(a+b+b\tan^2(e+fx))} + \frac{(a+2b)\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{2b(a+b)f}$$

$$= \frac{(a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{3/2}(a+b)^{3/2}f} - \frac{a\tan(e+fx)}{2b(a+b)f\,(a+b+b\tan^2(e+fx))}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{(a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{a\sqrt{b}\sin(2(e+fx))}{(a+b)(a+2b+a\cos(2(e+fx)))}}{2b^{3/2}f}$$

```
[In] Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2) - (a*
Sqrt[b]*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])))/(2*b^(3
/2)*f)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{-\frac{a\tan(fx+e)}{2(a+b)b\left(a+b+b\tan(fx+e)^2\right)}+\frac{(a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)b\sqrt{(a+b)b}}}{f}$
default	$\frac{-\frac{a\tan(fx+e)}{2(a+b)b\left(a+b+b\tan(fx+e)^2\right)}+\frac{(a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)b\sqrt{(a+b)b}}}{f}$
risch	$-\frac{i\left(a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a\right)}{bf(a+b)\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}-\frac{a\ln\left(e^{2i(fx+e)}+\frac{2iba+2ib^2+a\sqrt{-ab-b^2}+2b\sqrt{-ab-b^2}}{a\sqrt{-ab-b^2}}\right)}{4\sqrt{-ab-b^2}\,(a+b)fb}-\frac{\ln\left(\right.}{}$

```
[In] int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/2*a/(a+b)/b*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(a+2*b)/(a+b)/b/((a
+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(70) = 140$.

Time = 0.28 (sec) , antiderivative size = 406, normalized size of antiderivative = 4.95

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$=\left[-\frac{4\,(a^2b+ab^2)\cos(fx+e)\sin(fx+e)+\left((a^2+2\,ab)\cos(fx+e)^2+ab+2\,b^2\right)\sqrt{-ab-b^2}\log\left(\frac{(a^2+8\,ab+}{}\right.}{8\left((a^3b^2+2\,a^2b^3+ab^4)f\cos(fx+e)^2+(\right.}\right.$$
$$-\frac{2\,(a^2b+ab^2)\cos(fx+e)\sin(fx+e)+\left((a^2+2\,ab)\cos(fx+e)^2+ab+2\,b^2\right)\sqrt{ab+b^2}\arctan\left(\frac{(a+2}{2\sqrt{ab+b^2}}\right.}{4\left((a^3b^2+2\,a^2b^3+ab^4)f\cos(fx+e)^2+(a^2b^3+2\,ab^4+b^5)f\right)}$$

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(a^2*b + a*b^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + 2*a*b)*cos(f*x
 + e)^2 + a*b + 2*b^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x
+ e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b
*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2
*a*b*cos(f*x + e)^2 + b^2)))/((a^3*b^2 + 2*a^2*b^3 + a*b^4)*f*cos(f*x + e)^
2 + (a^2*b^3 + 2*a*b^4 + b^5)*f), -1/4*(2*(a^2*b + a*b^2)*cos(f*x + e)*sin(
f*x + e) + ((a^2 + 2*a*b)*cos(f*x + e)^2 + a*b + 2*b^2)*sqrt(a*b + b^2)*arc
```

```
tan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*
x + e))))/((a^3*b^2 + 2*a^2*b^3 + a*b^4)*f*cos(f*x + e)^2 + (a^2*b^3 + 2*a*
b^4 + b^5)*f)]
```

Sympy [F]

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

```
[In] integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
[Out] Integral(sec(e + f*x)**4/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{a\tan(fx+e)}{a^2b+2\,ab^2+b^3+(ab^2+b^3)\tan(fx+e)^2} - \frac{(a+2\,b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}(ab+b^2)}}{2\,f}$$

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
[Out] -1/2*(a*tan(f*x + e)/(a^2*b + 2*a*b^2 + b^3 + (a*b^2 + b^3)*tan(f*x + e)^2)
 - (a + 2*b)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*(a*b +
 b^2)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.09

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(a+2\,b)}{(ab+b^2)^{\frac{3}{2}}} - \frac{a\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)(ab+b^2)}}{2\,f}$$

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
[Out] 1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
+ b^2)))*(a + 2*b)/(a*b + b^2)^(3/2) - a*tan(f*x + e)/((b*tan(f*x + e)^2 +
a + b)*(a*b + b^2)))/f
```

Mupad [B] (verification not implemented)

Time = 19.61 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)}{\sqrt{a+b}}\right)\,(a+2\,b)}{2\,b^{3/2}\,f\,(a+b)^{3/2}} - \frac{a\tan(e+f\,x)}{2\,b\,f\,\left(a+b\right)\,\left(b\tan\left(e+f\,x\right)^2+a+b\right)}$$

```
[In] int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^2),x)
[Out] (atan((b^(1/2)*tan(e + f*x))/(a + b)^(1/2))*(a + 2*b))/(2*b^(3/2)*f*(a + b)
^(3/2)) - (a*tan(e + f*x))/(2*b*f*(a + b)*(a + b + b*tan(e + f*x)^2))
```

3.201 $\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 73

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx=\frac{\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{b}(a+b)^{3/2}f}+\frac{\tan(e+fx)}{2(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

```
[Out] 1/2*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/(a+b)^(3/2)/f/b^(1/2)+1/2*tan(f*
x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4231, 205, 211}

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx=\frac{\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{b}f(a+b)^{3/2}}+\frac{\tan(e+fx)}{2f(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

```
[In] Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]]/(2*Sqrt[b]*(a + b)^(3/2)*f) + Ta
n[e + f*x]/(2*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
```

```
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{\tan(e+fx)}{2(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{2(a+b)f}\\ &= \frac{\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{b}(a+b)^{3/2}f} + \frac{\tan(e+fx)}{2(a+b)f\left(a+b+b\tan^2(e+fx)\right)}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.52 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.89

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(-\frac{\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a\cos(2(e+}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{8(a+b)f\left(a+b\sec^2(e+fx)\right)^2}$$

```
[In] Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(-((ArcTan[(Sec[f*x]*(Cos[2*
e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]
*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] -
 I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])) + (-((a + 2*b)*S
in[2*e]) + a*Sin[2*f*x])/(a*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*(a +
b)*f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}}{f}$
default	$\frac{\frac{\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}}{f}$
risch	$\frac{i\left(a\,\mathrm{e}^{2i(fx+e)}+2b\,\mathrm{e}^{2i(fx+e)}+a\right)}{af(a+b)\left(a\,\mathrm{e}^{4i(fx+e)}+2a\,\mathrm{e}^{2i(fx+e)}+4b\,\mathrm{e}^{2i(fx+e)}+a\right)}-\frac{\ln\left(\mathrm{e}^{2i(fx+e)}+\frac{2iba+2ib^2+a\sqrt{-ab-b^2}+2b\sqrt{-ab-b^2}}{a\sqrt{-ab-b^2}}\right)}{4\sqrt{-ab-b^2}\,(a+b)f}+\ln\Big(\mathrm{e}^{2}$

```
[In] int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*tan(f*x+e)/(a+b)/(a+b+b*tan(f*x+e)^2)+1/2/(a+b)/((a+b)*b)^(1/2)*ar
ctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(61) = 122.

Time = 0.28 (sec) , antiderivative size = 368, normalized size of antiderivative = 5.04

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$=\left[\frac{4\left(ab+b^2\right)\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(a\cos\left(fx+e\right)^2+b\right)\sqrt{-ab-b^2}\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos(fx+e)^4-2\,(3\,ab+}{}\right.}{8\left(\left(a^3b+2\,a^2b^2+ab^3\right)f\cos\left(fx+e\right)^2+\left(a^2b^2+2\,ab\right.\right.}\right.$$

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a*b + b^2)*cos(f*x + e)*sin(f*x + e) - (a*cos(f*x + e)^2 + b)*sqrt
(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*
cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b -
b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))
)/((a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^2*b^2 + 2*a*b^3 + b^4)
*f), 1/4*(2*(a*b + b^2)*cos(f*x + e)*sin(f*x + e) - (a*cos(f*x + e)^2 + b)*
sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*
cos(f*x + e)*sin(f*x + e))))/((a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2
+ (a^2*b^2 + 2*a*b^3 + b^4)*f)]
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

```
[In] integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
[Out] Integral(sec(e + f*x)**2/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{\tan(fx+e)}{(ab+b^2)\tan(fx+e)^2+a^2+2\,ab+b^2} + \frac{\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}(a+b)}}{2\,f}$$

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
[Out] 1/2*(tan(f*x + e)/((a*b + b^2)*tan(f*x + e)^2 + a^2 + 2*a*b + b^2) + arctan
(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*(a + b)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)}{\sqrt{ab+b^2}(a+b)} + \frac{\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)(a+b)}}{2\,f}$$

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
[Out] 1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
+ b^2)))/(sqrt(a*b + b^2)*(a + b)) + tan(f*x + e)/((b*tan(f*x + e)^2 + a +
b)*(a + b)))/f
```

Mupad [B] (verification not implemented)

Time = 19.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \frac{\tan(e+f\,x)}{2\,f\,\left(a+b\right)\,\left(b\tan\left(e+f\,x\right)^2+a+b\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)\,(2\,a+2\,b)}{2\,(a+b)^{3/2}}\right)}{2\,\sqrt{b}\,f\,\left(a+b\right)^{3/2}}$$

```
[In] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] tan(e + f*x)/(2*f*(a + b)*(a + b + b*tan(e + f*x)^2)) + atan((b^(1/2)*tan(e
 + f*x)*(2*a + 2*b))/(2*(a + b)^(3/2)))/(2*b^(1/2)*f*(a + b)^(3/2))
```

3.202 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 92

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx=\frac{x}{a^2}-\frac{\sqrt{b}(3a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}f}-\frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

```
[Out] x/a^2-1/2*(3*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/a^2/(a+b
)^(3/2)/f-1/2*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4213, 425, 536, 209, 211}

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx=-\frac{\sqrt{b}(3a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{3/2}}+\frac{x}{a^2}-\frac{b\tan(e+fx)}{2af(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-2),x]
```

```
[Out] x/a^2 - (Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2
*a^2*(a + b)^(3/2)*f) - (b*Tan[e + f*x])/(2*a*(a + b)*f*(a + b + b*Tan[e +
f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a+b-bx^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{2a(a+b)f}\\
&= -\frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^2f}\\
&\quad - \frac{(b(3a+2b))\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{2a^2(a+b)f}
\end{aligned}
$$

$$= \frac{x}{a^2} - \frac{\sqrt{b}(3a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}f} - \frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.61

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(2x(a+2b+a\cos(2(e+fx))) + \frac{b(3a+2b)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))}{2\sqrt{a+b}\sqrt{b}}\right.}{}\right.}{8a^2\left(a+b\sec^2(e+f\right.}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(2*x*(a + 2*b + a*Cos[2*(e +
 f*x)]) + (b*(3*a + 2*b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2
*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e]
)^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/((a + b)^(3/
2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (b*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]
))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*a^2*(a + b*Sec[e +
f*x]^2)^2)
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(3a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^2}+\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
default	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(3a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^2}+\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
risch	$\frac{x}{a^2} - \frac{ib\left(a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a\right)}{a^2(a+b)f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)} + \frac{3\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{4(a+b)^2fa} + \frac{\sqrt{}}{}$

```
[In] int(1/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-b/a^2*(1/2*a/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a+2*b)/(a+b
)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^2*arctan(tan(f*
x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(80) = 160$.

Time = 0.29 (sec) , antiderivative size = 435, normalized size of antiderivative = 4.73

$$\int \frac{1}{(a + b\sec^2(e + fx))^2} dx = \left[\frac{8(a^2 + ab)fx\cos(fx + e)^2 - 4ab\cos(fx + e)\sin(fx + e) + 8(ab + b^2)fx + ((3a^2 + 2ab)\cos(fx + e)}{8((a^4 + a^3} \right.$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 4*a*b*cos(f*x + e)*sin(f*x + e) +
8*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt(-
b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*co
s(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*
x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*co
s(f*x + e)^2 + b^2)))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*b^2)*f
), 1/4*(4*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 2*a*b*cos(f*x + e)*sin(f*x + e)
+ 4*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt
(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*co
s(f*x + e)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*b
^2)*f)]
```

Sympy [F]

$$\int \frac{1}{(a + b\sec^2(e + fx))^2} dx = \int \frac{1}{(a + b\sec^2(e + fx))^2} dx$$

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.15

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{b\tan(fx+e)}{a^3+2\,a^2b+ab^2+(a^2b+ab^2)\tan(fx+e)^2}+\frac{(3\,ab+2\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^3+a^2b)\sqrt{(a+b)b}}-\frac{2\,(fx+e)}{a^2}}{2\,f}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*tan(f*x + e)/(a^3 + 2*a^2*b + a*b^2 + (a^2*b + a*b^2)*tan(f*x + e)^
2) + (3*a*b + 2*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + a^2*b)*
sqrt((a + b)*b)) - 2*(f*x + e)/a^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.24

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3\,ab+2\,b^2)}{(a^3+a^2b)\sqrt{ab+b^2}}+\frac{b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)(a^2+ab)}-\frac{2\,(fx+e)}{a^2}}{2\,f}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
 + b^2)))*(3*a*b + 2*b^2)/((a^3 + a^2*b)*sqrt(a*b + b^2)) + b*tan(f*x + e)/
((b*tan(f*x + e)^2 + a + b)*(a^2 + a*b)) - 2*(f*x + e)/a^2)/f
```

Mupad [B] (verification not implemented)

Time = 21.72 (sec) , antiderivative size = 2056, normalized size of antiderivative = 22.35

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] int(1/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] atan((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^
2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8
*a^2*(2*a^3*b + a^4 + a^2*b^2)))/(2*a^2) + (tan(e + f*x)*(20*a*b^4 + 8*b^5
+ 13*a^2*b^3))/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 - ((((2*a^4*b^4 + 6*a^5*b
^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^2)) + (tan(e + f*x)*(32*a^4*b
^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2
)))/(2*a^2) - (tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(4*(2*a^3*b +
a^4 + a^2*b^2)))/a^2)/((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^
4*b + a^5 + a^3*b^2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3
 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) + (tan(e + f
*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2
+ (((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^2))
 + (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^
2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) - (tan(e + f*x)*(20*a*b^4 + 8*b^5
 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 + ((3*a*b^3)/2 + b^4)
/(2*a^4*b + a^5 + a^3*b^2)))/(a^2*f) + (atan(((((tan(e + f*x)*(20*a*b^4 + 8
*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((-b*(a + b)^3)^(1/2)*(
(2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a^3*b^2) - (tan(e + f*
x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 +
 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3
*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(-b*(a + b)
^3)^(1/2)*(3*a + 2*b)*1i)/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)) + (((ta
n(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) +
 ((-b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5
+ a^3*b^2) + (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 8
0*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b
 + a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 +
3*a^3*b^2)))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*1i)/(4*(3*a^4*b + a^5 + a^2*b
^3 + 3*a^3*b^2)))/((((3*a*b^3)/2 + b^4)/(2*a^4*b + a^5 + a^3*b^2) - (((tan(e
 + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((
-b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a
^3*b^2) - (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a
^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b +
a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a
^3*b^2)))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3
*a^3*b^2)) + (((tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b +
 a^4 + a^2*b^2)) + ((-b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^
2)/(2*a^4*b + a^5 + a^3*b^2) + (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*
b)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 +
 a^2*b^2)*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b
+ a^5 + a^2*b^3 + 3*a^3*b^2)))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b))/(4*(3*a^4*
b + a^5 + a^2*b^3 + 3*a^3*b^2))))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*1i)/(2*f
*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)) - (b*tan(e + f*x))/(2*a*f*(a + b)*(
a + b + b*tan(e + f*x)^2))
```

3.203 $\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 142

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{(a-4b)x}{2a^3}+\frac{b^{3/2}(5a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^3(a+b)^{3/2}f} + \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} + \frac{b(a+2b)\tan(e+fx)}{2a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

```
[Out] 1/2*(a-4*b)*x/a^3+1/2*b^(3/2)*(5*a+4*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/
2))/a^3/(a+b)^(3/2)/f+1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)+1/
2*b*(a+2*b)*tan(f*x+e)/a^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4231, 425, 541, 536, 209, 211}

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{b^{3/2}(5a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^3f(a+b)^{3/2}}+\frac{x(a-4b)}{2a^3} + \frac{b(a+2b)\tan(e+fx)}{2a^2f(a+b)\left(a+b\tan^2(e+fx)+b\right)} + \frac{\sin(e+fx)\cos(e+fx)}{2af\left(a+b\tan^2(e+fx)+b\right)}$$

```
[In] Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a - 4*b)*x)/(2*a^3) + (b^(3/2)*(5*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/
Sqrt[a + b]])/(2*a^3*(a + b)^(3/2)*f) + (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*
(a + b + b*Tan[e + f*x]^2)) + (b*(a + 2*b)*Tan[e + f*x])/(2*a^2*(a + b)*f*(
a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
```

```
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-a+b-3bx^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{2af}\\
&= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} + \frac{b(a+2b)\tan(e+fx)}{2a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)}\\
&\quad - \frac{\text{Subst}\left(\int \frac{-2(a^2-2ab-2b^2)-2b(a+2b)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{4a^2(a+b)f}\\
&= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} + \frac{b(a+2b)\tan(e+fx)}{2a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)}\\
&\quad + \frac{(a-4b)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{2a^3f}\\
&\quad + \frac{\left(b^2(5a+4b)\right)\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{2a^3(a+b)f}\\
&= \frac{(a-4b)x}{2a^3} + \frac{b^{3/2}(5a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^3(a+b)^{3/2}f}\\
&\quad + \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} + \frac{b(a+2b)\tan(e+fx)}{2a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.73

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{2(a-4b)(e+fx) + \frac{2b^{3/2}(5a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} + \left(a + \frac{2ab^2}{(a+b)(a+2b+a\cos(2(e+fx)))}\right)\sin(2(e+fx))}{4a^3f}$$

```
[In] Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (2*(a - 4*b)*(e + f*x) + (2*b^(3/2)*(5*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x
])/Sqrt[a + b]])/(a + b)^(3/2) + (a + (2*a*b^2)/((a + b)*(a + 2*b + a*Cos[2
*(e + f*x)])))*Sin[2*(e + f*x)])/(4*a^3*f)
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{\frac{b^2\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(5a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^3}+\frac{\frac{a\tan(fx+e)}{2+2\tan(fx+e)^2}+\frac{(a-4b)\arctan(\tan(fx+e))}{2}}{a^3}}{f}$
default	$\frac{\frac{b^2\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(5a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^3}+\frac{\frac{a\tan(fx+e)}{2+2\tan(fx+e)^2}+\frac{(a-4b)\arctan(\tan(fx+e))}{2}}{a^3}}{f}$
risch	$\frac{x}{2a^2}-\frac{2xb}{a^3}-\frac{i\mathrm{e}^{2i(fx+e)}}{8a^2f}+\frac{i\mathrm{e}^{-2i(fx+e)}}{8a^2f}+\frac{ib^2\left(a\,\mathrm{e}^{2i(fx+e)}+2b\,\mathrm{e}^{2i(fx+e)}+a\right)}{a^3(a+b)f\left(a\,\mathrm{e}^{4i(fx+e)}+2a\,\mathrm{e}^{2i(fx+e)}+4b\,\mathrm{e}^{2i(fx+e)}+a\right)}+\frac{5\sqrt{-(a+b)b}}{}$

```
[In] int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2/a^3*(1/2*a/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(5*a+4*b)/(a+
b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^3*(1/2*a*tan(f
*x+e)/(1+tan(f*x+e)^2)+1/2*(a-4*b)*arctan(tan(f*x+e))))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 544, normalized size of antiderivative = 3.83

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \left[\frac{4\,(a^3-3\,a^2b-4\,ab^2)fx\cos(fx+e)^2+4\,(a^2b-3\,ab^2-4\,b^3)fx+\left(5\,ab^2+4\,b^3+(5\,a^2b+4\,ab^2)\cos(fx\right.}{}\right.$$

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a^3 - 3*a^2*b - 4*a*b^2)*f*x*cos(f*x + e)^2 + 4*(a^2*b - 3*a*b^2 -
 4*b^3)*f*x + (5*a*b^2 + 4*b^3 + (5*a^2*b + 4*a*b^2)*cos(f*x + e)^2)*sqrt(-
b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*co
s(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*
x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*co
s(f*x + e)^2 + b^2)) + 4*((a^3 + a^2*b)*cos(f*x + e)^3 + (a^2*b + 2*a*b^2)*
cos(f*x + e))*sin(f*x + e))/((a^5 + a^4*b)*f*cos(f*x + e)^2 + (a^4*b + a^3*
b^2)*f), 1/4*(2*(a^3 - 3*a^2*b - 4*a*b^2)*f*x*cos(f*x + e)^2 + 2*(a^2*b - 3
*a*b^2 - 4*b^3)*f*x - (5*a*b^2 + 4*b^3 + (5*a^2*b + 4*a*b^2)*cos(f*x + e)^2
```

```
)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))
/(b*cos(f*x + e)*sin(f*x + e))) + 2*((a^3 + a^2*b)*cos(f*x + e)^3 + (a^2*b
+ 2*a*b^2)*cos(f*x + e))*sin(f*x + e))/((a^5 + a^4*b)*f*cos(f*x + e)^2 + (a
^4*b + a^3*b^2)*f)]
```

Sympy [F]

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

```
[In] integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
[Out] Integral(cos(e + f*x)**2/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.23

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{(5\,ab^2+4\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^4+a^3b)\sqrt{(a+b)b}} + \frac{(ab+2\,b^2)\tan(fx+e)^3+(a^2+2\,ab+2\,b^2)\tan(fx+e)}{(a^3b+a^2b^2)\tan(fx+e)^4+a^4+2\,a^3b+a^2b^2+(a^4+3\,a^3b+2\,a^2b^2)\tan(fx+e)^2} + \frac{(fx+e)(a-4\,b)}{a^3}}{2\,f}$$

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
[Out] 1/2*((5*a*b^2 + 4*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4 + a^3*b
)*sqrt((a + b)*b)) + ((a*b + 2*b^2)*tan(f*x + e)^3 + (a^2 + 2*a*b + 2*b^2)*
tan(f*x + e))/((a^3*b + a^2*b^2)*tan(f*x + e)^4 + a^4 + 2*a^3*b + a^2*b^2 +
 (a^4 + 3*a^3*b + 2*a^2*b^2)*tan(f*x + e)^2) + (f*x + e)*(a - 4*b)/a^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.35

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{(5\,ab^2+4\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^4+a^3b)\sqrt{ab+b^2}} + \frac{ab\tan(fx+e)^3+2\,b^2\tan(fx+e)^3+a^2\tan(fx+e)+2\,ab\tan(fx+e)+2\,b^2\tan(fx+e)}{\left(b\tan(fx+e)^4+a\tan(fx+e)^2+2\,b\tan(fx+e)^2+a+b\right)(a^3+a^2b)}}{2\,f}$$

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((5*a*b^2 + 4*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(
f*x + e)/sqrt(a*b + b^2)))/((a^4 + a^3*b)*sqrt(a*b + b^2)) + (a*b*tan(f*x +
 e)^3 + 2*b^2*tan(f*x + e)^3 + a^2*tan(f*x + e) + 2*a*b*tan(f*x + e) + 2*b^
2*tan(f*x + e))/((b*tan(f*x + e)^4 + a*tan(f*x + e)^2 + 2*b*tan(f*x + e)^2
+ a + b)*(a^3 + a^2*b)) + (f*x + e)*(a - 4*b)/a^3)/f
```

Mupad [B] (verification not implemented)

Time = 22.47 (sec) , antiderivative size = 2401, normalized size of antiderivative = 16.91

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] ((tan(e + f*x)*(2*a*b + a^2 + 2*b^2))/(2*a^2*(a + b)) + (b*tan(e + f*x)^3*(
a + 2*b))/(2*a^2*(a + b)))/(f*(a + b + b*tan(e + f*x)^4 + tan(e + f*x)^2*(a
 + 2*b))) - (atan(((((tan(e + f*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*
b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) - (((4*a^6*b^5 + 8*a^7*b^4 +
2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2) - (tan(e + f*x)*(a*1i - b*
4i)*(32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(8*a^3*(2*a^5*b +
a^6 + a^4*b^2)))*(a*1i - b*4i))/(4*a^3))*(a*1i - b*4i)*1i)/(4*a^3) + (((tan
(e + f*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5
*b + a^6 + a^4*b^2)) + (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2
*a^7*b + a^8 + a^6*b^2) + (tan(e + f*x)*(a*1i - b*4i)*(32*a^6*b^5 + 80*a^7*
b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(8*a^3*(2*a^5*b + a^6 + a^4*b^2)))*(a*1i -
b*4i))/(4*a^3))*(a*1i - b*4i)*1i)/(4*a^3))/((12*a*b^6 + 8*b^7 + (3*a^2*b^5)
/2 - (5*a^3*b^4)/4)/(2*a^7*b + a^8 + a^6*b^2) - (((tan(e + f*x)*(64*a*b^6 +
 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2))
- (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^
2) - (tan(e + f*x)*(a*1i - b*4i)*(32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16
*a^9*b^2))/(8*a^3*(2*a^5*b + a^6 + a^4*b^2)))*(a*1i - b*4i))/(4*a^3))*(a*1i
 - b*4i))/(4*a^3) + (((tan(e + f*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3
*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) + (((4*a^6*b^5 + 8*a^7*b^4 +
 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2) + (tan(e + f*x)*(a*1i - b
*4i)*(32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(8*a^3*(2*a^5*b +
 a^6 + a^4*b^2)))*(a*1i - b*4i))/(4*a^3))*(a*1i - b*4i))/(4*a^3)))*(a*1i -
b*4i)*1i)/(2*a^3*f) - (atan(((((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2)*((tan(e
+ f*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b
+ a^6 + a^4*b^2)) + (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^
7*b + a^8 + a^6*b^2) + (tan(e + f*x)*((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2)*(
32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(2*(2*a^5*b + a^6 + a^4
*b^2)*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))*((5*a)/4 + b)*(-b^3*(a + b)^3
```

)^(1/2))/(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2))*1i)/(3*a^5*b + a^6 + a^3*b^
3 + 3*a^4*b^2) + (((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2)*((tan(e + f*x)*(64*a
*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*
b^2)) - (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 +
a^6*b^2) - (tan(e + f*x)*((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2)*(32*a^6*b^5 +
 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(2*(2*a^5*b + a^6 + a^4*b^2)*(3*a^5
*b + a^6 + a^3*b^3 + 3*a^4*b^2)))*((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2))/(3*
a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2))*1i)/(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^
2))/((12*a*b^6 + 8*b^7 + (3*a^2*b^5)/2 - (5*a^3*b^4)/4)/(2*a^7*b + a^8 + a^
6*b^2) + (((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2)*((tan(e + f*x)*(64*a*b^6 + 3
2*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) +
(((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2)
 + (tan(e + f*x)*((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2)*(32*a^6*b^5 + 80*a^7*
b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(2*(2*a^5*b + a^6 + a^4*b^2)*(3*a^5*b + a^6
 + a^3*b^3 + 3*a^4*b^2)))*((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2))/(3*a^5*b +
a^6 + a^3*b^3 + 3*a^4*b^2)))/(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2) - (((5*a
)/4 + b)*(-b^3*(a + b)^3)^(1/2)*((tan(e + f*x)*(64*a*b^6 + 32*b^7 + 26*a^2*
b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) - (((4*a^6*b^5 +
8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2) - (tan(e + f*x
)*((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2)*(32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^
3 + 16*a^9*b^2))/(2*(2*a^5*b + a^6 + a^4*b^2)*(3*a^5*b + a^6 + a^3*b^3 + 3*
a^4*b^2)))*((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2))/(3*a^5*b + a^6 + a^3*b^3 +
 3*a^4*b^2)))/(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))*((5*a)/4 + b)*(-b^3*(
a + b)^3)^(1/2)*2i)/(f*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2))

3.204 $\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 203

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{(3a^2-8ab+24b^2)\,x}{8a^4} - \frac{b^{5/2}(7a+6b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^4(a+b)^{3/2}f} + \frac{3(a-2b)\cos(e+fx)\sin(e+fx)}{8a^2f\,(a+b+b\tan^2(e+fx))} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))} + \frac{(a-3b)b(3a+4b)\tan(e+fx)}{8a^3(a+b)f\,(a+b+b\tan^2(e+fx))}$$

```
[Out] 1/8*(3*a^2-8*a*b+24*b^2)*x/a^4-1/2*b^(5/2)*(7*a+6*b)*arctan(b^(1/2)*tan(f*x
+e)/(a+b)^(1/2))/a^4/(a+b)^(3/2)/f+3/8*(a-2*b)*cos(f*x+e)*sin(f*x+e)/a^2/f/
(a+b+b*tan(f*x+e)^2)+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)+1
/8*(a-3*b)*b*(3*a+4*b)*tan(f*x+e)/a^3/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used

$= \{4231, 425, 541, 536, 209, 211\}$

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{b^{5/2}(7a+6b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^4f(a+b)^{3/2}} + \frac{b(a-3b)(3a+4b)\tan(e+fx)}{8a^3f(a+b)\left(a+b\tan^2(e+fx)+b\right)} + \frac{3(a-2b)\sin(e+fx)\cos(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)+b\right)} + \frac{x(3a^2-8ab+24b^2)}{8a^4} + \frac{\sin(e+fx)\cos^3(e+fx)}{4af\left(a+b\tan^2(e+fx)+b\right)}$$

```
[In] Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((3*a^2 - 8*a*b + 24*b^2)*x)/(8*a^4) - (b^(5/2)*(7*a + 6*b)*ArcTan[(Sqrt[b]
*Tan[e + f*x])/Sqrt[a + b]])/(2*a^4*(a + b)^(3/2)*f) + (3*(a - 2*b)*Cos[e +
 f*x]*Sin[e + f*x])/(8*a^2*f*(a + b + b*Tan[e + f*x]^2)) + (Cos[e + f*x]^3*
Sin[e + f*x])/(4*a*f*(a + b + b*Tan[e + f*x]^2)) + ((a - 3*b)*b*(3*a + 4*b)
*Tan[e + f*x])/(8*a^3*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))} - \frac{\text{Subst}\left(\int \frac{-3a+b-5bx^2}{(1+x^2)^2(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4af}\\
&= \frac{3(a-2b)\cos(e+fx)\sin(e+fx)}{8a^2f\,(a+b+b\tan^2(e+fx))} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))}\\
&\quad + \frac{\text{Subst}\left(\int \frac{3a^2+ab+6b^2+9(a-2b)bx^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{8a^2f}\\
&= \frac{3(a-2b)\cos(e+fx)\sin(e+fx)}{8a^2f\,(a+b+b\tan^2(e+fx))} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))}\\
&\quad + \frac{(a-3b)b(3a+4b)\tan(e+fx)}{8a^3(a+b)f\,(a+b+b\tan^2(e+fx))}\\
&\quad + \frac{\text{Subst}\left(\int \frac{2(3a^3-2a^2b+11ab^2+12b^3)+2(a-3b)b(3a+4b)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{16a^3(a+b)f}
\end{aligned}
$$

$$= \frac{3(a-2b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)} + \frac{(a-3b)b(3a+4b)\tan(e+fx)}{8a^3(a+b)f\left(a+b+b\tan^2(e+fx)\right)} - \frac{(b^3(7a+6b))\,\text{Subst}\left(\int\frac{1}{a+b+bx^2}\,dx,x,\tan(e+fx)\right)}{2a^4(a+b)f} + \frac{(3a^2-8ab+24b^2)\,\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\tan(e+fx)\right)}{8a^4f}$$

$$= \frac{(3a^2-8ab+24b^2)\,x}{8a^4} - \frac{b^{5/2}(7a+6b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^4(a+b)^{3/2}f} + \frac{3(a-2b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)} + \frac{(a-3b)b(3a+4b)\tan(e+fx)}{8a^3(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.68

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{4(3a^2-8ab+24b^2)(e+fx) - \frac{16b^{5/2}(7a+6b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} + 8a(a-2b)\sin(2(e+fx)) - \frac{16ab^3\sin(2(e+fx))}{(a+b)(a+2b+a\cos(2(e+fx)))} + a^2\sin(4(e+fx))}{32a^4f}$$

```
[In] Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (4*(3*a^2 - 8*a*b + 24*b^2)*(e + f*x) - (16*b^(5/2)*(7*a + 6*b)*ArcTan[(Sqr
t[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2) + 8*a*(a - 2*b)*Sin[2*(e + f
*x)] - (16*a*b^3*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)]))
 + a^2*Sin[4*(e + f*x)])/(32*a^4*f)
```

Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{\frac{\frac{\left(\frac{3}{8}a^2-ab\right)\tan(fx+e)^3+\left(-ab+\frac{5}{8}a^2\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{\left(3a^2-8ab+24b^2\right)\arctan(\tan(fx+e))}{8}}{a^4}-\frac{b^3\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(7a+6}{}\right.}{a^4}}{f}$
default	$\frac{\frac{\frac{\left(\frac{3}{8}a^2-ab\right)\tan(fx+e)^3+\left(-ab+\frac{5}{8}a^2\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{\left(3a^2-8ab+24b^2\right)\arctan(\tan(fx+e))}{8}}{a^4}-\frac{b^3\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(7a+6}{}\right.}{a^4}}{f}$
risch	$\frac{3x}{8a^2}-\frac{xb}{a^3}+\frac{3x\,b^2}{a^4}-\frac{i\mathrm{e}^{4i(fx+e)}}{64a^2f}-\frac{i\mathrm{e}^{2i(fx+e)}}{8a^2f}+\frac{i\mathrm{e}^{2i(fx+e)}b}{4a^3f}+\frac{i\mathrm{e}^{-2i(fx+e)}}{8a^2f}-\frac{i\mathrm{e}^{-2i(fx+e)}b}{4a^3f}+\frac{i\mathrm{e}^{-4i(fx+e)}}{64a^2f}-$

```
[In] int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^4*(((3/8*a^2-a*b)*tan(f*x+e)^3+(-a*b+5/8*a^2)*tan(f*x+e))/(1+tan(f
*x+e)^2)^2+1/8*(3*a^2-8*a*b+24*b^2)*arctan(tan(f*x+e)))-b^3/a^4*(1/2*a/(a+b
)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(7*a+6*b)/(a+b)/((a+b)*b)^(1/2)*arcta
n(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 656, normalized size of antiderivative = 3.23

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$=\left[\frac{(3\,a^4-5\,a^3b+16\,a^2b^2+24\,ab^3)fx\cos\left(fx+e\right)^2+(3\,a^3b-5\,a^2b^2+16\,ab^3+24\,b^4)fx+\left(7\,ab^3+6\,b^4+\right.}{}\right.$$

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*((3*a^4 - 5*a^3*b + 16*a^2*b^2 + 24*a*b^3)*f*x*cos(f*x + e)^2 + (3*a^3
*b - 5*a^2*b^2 + 16*a*b^3 + 24*b^4)*f*x + (7*a*b^3 + 6*b^4 + (7*a^2*b^2 + 6
*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x
 + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f
*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/
(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + (2*(a^4 + a^3*b)*cos(f
*x + e)^5 + 3*(a^4 - a^3*b - 2*a^2*b^2)*cos(f*x + e)^3 + (3*a^3*b - 5*a^2*b
^2 - 12*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^6 + a^5*b)*f*cos(f*x + e)^2
+ (a^5*b + a^4*b^2)*f), 1/8*((3*a^4 - 5*a^3*b + 16*a^2*b^2 + 24*a*b^3)*f*x*
cos(f*x + e)^2 + (3*a^3*b - 5*a^2*b^2 + 16*a*b^3 + 24*b^4)*f*x + 2*(7*a*b^3
 + 6*b^4 + (7*a^2*b^2 + 6*a*b^3)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2
```

```
*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e
))) + (2*(a^4 + a^3*b)*cos(f*x + e)^5 + 3*(a^4 - a^3*b - 2*a^2*b^2)*cos(f*x
 + e)^3 + (3*a^3*b - 5*a^2*b^2 - 12*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^
6 + a^5*b)*f*cos(f*x + e)^2 + (a^5*b + a^4*b^2)*f)]
```

Sympy [F]

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

```
[In] integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(cos(e + f*x)**4/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.32

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{4\,(7\,ab^3+6\,b^4)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^5+a^4b)\sqrt{(a+b)b}} - \frac{(3\,a^2b-5\,ab^2-12\,b^3)\tan(fx+e)^5+(3\,a^3+3\,a^2b-16\,ab^2-24\,b^3)\tan(fx+e)^3+(5\,a^3+2\,a^2b-11\,ab^2-12\,b}{(a^4b+a^3b^2)\tan(fx+e)^6+a^5+2\,a^4b+a^3b^2+(a^5+4\,a^4b+3\,a^3b^2)\tan(fx+e)^4+(2\,a^5+5\,a^4b+3\,a^3b^2)\,\mathrm{ta}}}{8\,f}$$

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(4*(7*a*b^3 + 6*b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^5 + a^
4*b)*sqrt((a + b)*b)) - ((3*a^2*b - 5*a*b^2 - 12*b^3)*tan(f*x + e)^5 + (3*a
^3 + 3*a^2*b - 16*a*b^2 - 24*b^3)*tan(f*x + e)^3 + (5*a^3 + 2*a^2*b - 11*a*
b^2 - 12*b^3)*tan(f*x + e))/((a^4*b + a^3*b^2)*tan(f*x + e)^6 + a^5 + 2*a^4
*b + a^3*b^2 + (a^5 + 4*a^4*b + 3*a^3*b^2)*tan(f*x + e)^4 + (2*a^5 + 5*a^4*
b + 3*a^3*b^2)*tan(f*x + e)^2) - (3*a^2 - 8*a*b + 24*b^2)*(f*x + e)/a^4)/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.96

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{4\,b^3\tan(fx+e)}{(a^4+a^3b)\left(b\tan(fx+e)^2+a+b\right)} + \frac{4\,(7\,ab^3+6\,b^4)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^5+a^4b)\sqrt{ab+b^2}} - \frac{(3\,a^2-8\,ab+24\,b^2)(fx+e)}{a^4} - \frac{3\,a\tan(f}{}}{8\,f}$$

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/8*(4*b^3*tan(f*x + e)/((a^4 + a^3*b)*(b*tan(f*x + e)^2 + a + b)) + 4*(7*
a*b^3 + 6*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)
/sqrt(a*b + b^2)))/((a^5 + a^4*b)*sqrt(a*b + b^2)) - (3*a^2 - 8*a*b + 24*b^
2)*(f*x + e)/a^4 - (3*a*tan(f*x + e)^3 - 8*b*tan(f*x + e)^3 + 5*a*tan(f*x +
 e) - 8*b*tan(f*x + e))/((tan(f*x + e)^2 + 1)^2*a^3))/f
```

Mupad [B] (verification not implemented)

Time = 23.00 (sec) , antiderivative size = 2880, normalized size of antiderivative = 14.19

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (atan(((((tan(e + f*x)*(2112*a*b^8 + 1152*b^9 + 800*a^2*b^7 - 16*a^3*b^6 +
121*a^4*b^5 - 30*a^5*b^4 + 9*a^6*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) - (((
6*a^8*b^6 + (23*a^9*b^5)/2 + (9*a^10*b^4)/2 + (a^11*b^3)/2 + (3*a^12*b^2)/2
)/(2*a^10*b + a^11 + a^9*b^2) - (tan(e + f*x)*(a^2*3i - a*b*8i + b^2*24i)*(
512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2))/(512*a^4*(2*a^7
*b + a^8 + a^6*b^2)))*(a^2*3i - a*b*8i + b^2*24i))/(16*a^4))*(a^2*3i - a*b*
8i + b^2*24i)*1i)/(16*a^4) + (((tan(e + f*x)*(2112*a*b^8 + 1152*b^9 + 800*a
^2*b^7 - 16*a^3*b^6 + 121*a^4*b^5 - 30*a^5*b^4 + 9*a^6*b^3))/(32*(2*a^7*b +
 a^8 + a^6*b^2)) + (((6*a^8*b^6 + (23*a^9*b^5)/2 + (9*a^10*b^4)/2 + (a^11*b
^3)/2 + (3*a^12*b^2)/2)/(2*a^10*b + a^11 + a^9*b^2) + (tan(e + f*x)*(a^2*3i
 - a*b*8i + b^2*24i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11
*b^2))/(512*a^4*(2*a^7*b + a^8 + a^6*b^2)))*(a^2*3i - a*b*8i + b^2*24i))/(1
6*a^4))*(a^2*3i - a*b*8i + b^2*24i)*1i)/(16*a^4))/(((135*a*b^9)/4 + 27*b^10
 - (9*a^2*b^8)/2 - (149*a^3*b^7)/32 + (219*a^4*b^6)/64 - (63*a^5*b^5)/64)/(
2*a^10*b + a^11 + a^9*b^2) - (((tan(e + f*x)*(2112*a*b^8 + 1152*b^9 + 800*a
^2*b^7 - 16*a^3*b^6 + 121*a^4*b^5 - 30*a^5*b^4 + 9*a^6*b^3))/(32*(2*a^7*b +
```

a^8 + a^6*b^2)) - (((6*a^8*b^6 + (23*a^9*b^5)/2 + (9*a^10*b^4)/2 + (a^11*b^3)/2 + (3*a^12*b^2)/2)/(2*a^10*b + a^11 + a^9*b^2) - (tan(e + f*x)*(a^2*3i - a*b*8i + b^2*24i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2))/(512*a^4*(2*a^7*b + a^8 + a^6*b^2)))*(a^2*3i - a*b*8i + b^2*24i))/(16*a^4))*(a^2*3i - a*b*8i + b^2*24i))/(16*a^4) + (((tan(e + f*x)*(2112*a*b^8 + 1152*b^9 + 800*a^2*b^7 - 16*a^3*b^6 + 121*a^4*b^5 - 30*a^5*b^4 + 9*a^6*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) + (((6*a^8*b^6 + (23*a^9*b^5)/2 + (9*a^10*b^4)/2 + (a^11*b^3)/2 + (3*a^12*b^2)/2)/(2*a^10*b + a^11 + a^9*b^2) + (tan(e + f*x)*(a^2*3i - a*b*8i + b^2*24i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2))/(512*a^4*(2*a^7*b + a^8 + a^6*b^2)))*(a^2*3i - a*b*8i + b^2*24i))/(16*a^4))*(a^2*3i - a*b*8i + b^2*24i))/(16*a^4)))*(a^2*3i - a*b*8i + b^2*24i)*1i)/(8*a^4*f) - ((tan(e + f*x)^3*(16*a*b^2 - 3*a^2*b - 3*a^3 + 24*b^3))/(8*a^3*(a + b)) + (tan(e + f*x)*(11*a*b^2 - 2*a^2*b - 5*a^3 + 12*b^3))/(8*a^3*(a + b)) + (tan(e + f*x)^5*(5*a*b^2 - 3*a^2*b + 12*b^3))/(8*a^3*(a + b)))/(f*(a + b + tan(e + f*x)^2*(2*a + 3*b) + b*tan(e + f*x)^6 + tan(e + f*x)^4*(a + 3*b))) + (atan(((((tan(e + f*x)*(2112*a*b^8 + 1152*b^9 + 800*a^2*b^7 - 16*a^3*b^6 + 121*a^4*b^5 - 30*a^5*b^4 + 9*a^6*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) - ((-b^5*(a + b)^3)^(1/2)*((6*a^8*b^6 + (23*a^9*b^5)/2 + (9*a^10*b^4)/2 + (a^11*b^3)/2 + (3*a^12*b^2)/2)/(2*a^10*b + a^11 + a^9*b^2) - (tan(e + f*x)*(-b^5*(a + b)^3)^(1/2)*(7*a + 6*b)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2))/(128*(2*a^7*b + a^8 + a^6*b^2)*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(7*a + 6*b))/(4*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(-b^5*(a + b)^3)^(1/2)*(7*a + 6*b)*1i)/(4*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)) + (((tan(e + f*x)*(2112*a*b^8 + 1152*b^9 + 800*a^2*b^7 - 16*a^3*b^6 + 121*a^4*b^5 - 30*a^5*b^4 + 9*a^6*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) + ((-b^5*(a + b)^3)^(1/2)*((6*a^8*b^6 + (23*a^9*b^5)/2 + (9*a^10*b^4)/2 + (a^11*b^3)/2 + (3*a^12*b^2)/2)/(2*a^10*b + a^11 + a^9*b^2) + (tan(e + f*x)*(-b^5*(a + b)^3)^(1/2)*(7*a + 6*b)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2))/(128*(2*a^7*b + a^8 + a^6*b^2)*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(7*a + 6*b))/(4*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(-b^5*(a + b)^3)^(1/2)*(7*a + 6*b)*1i)/(4*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))/(((135*a*b^9)/4 + 27*b^10 - (9*a^2*b^8)/2 - (149*a^3*b^7)/32 + (219*a^4*b^6)/64 - (63*a^5*b^5)/64)/(2*a^10*b + a^11 + a^9*b^2) - (((tan(e + f*x)*(2112*a*b^8 + 1152*b^9 + 800*a^2*b^7 - 16*a^3*b^6 + 121*a^4*b^5 - 30*a^5*b^4 + 9*a^6*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) - ((-b^5*(a + b)^3)^(1/2)*((6*a^8*b^6 + (23*a^9*b^5)/2 + (9*a^10*b^4)/2 + (a^11*b^3)/2 + (3*a^12*b^2)/2)/(2*a^10*b + a^11 + a^9*b^2) - (tan(e + f*x)*(-b^5*(a + b)^3)^(1/2)*(7*a + 6*b)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2))/(128*(2*a^7*b + a^8 + a^6*b^2)*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(7*a + 6*b))/(4*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(-b^5*(a + b)^3)^(1/2)*(7*a + 6*b))/(4*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)) + (((tan(e + f*x)*(2112*a*b^8 + 1152*b^9 + 800*a^2*b^7 - 16*a^3*b^6 + 121*a^4*b^5 - 30*a^5*b^4 + 9*a^6*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) + ((-b^5*(a + b)^3)^(1/2)*((6*a^8*b^6 + (23*a^9*b^5)/2 + (9*a^10*b^4)/2 + (a^11*b^3)/2 + (3*a^12*b^2)/2)/(2*a^10*b + a^11 + a^9*b^2) + (tan(e + f*x)*(-b^5*(a + b)^3)^(1

```
/2)*(7*a + 6*b)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2)
)/(128*(2*a^7*b + a^8 + a^6*b^2)*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(7
*a + 6*b))/(4*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(-b^5*(a + b)^3)^(1/2
)*(7*a + 6*b))/(4*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2))))*(-b^5*(a + b)^3)
^(1/2)*(7*a + 6*b)*1i)/(2*f*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2))
```

3.205 $\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 278

$$\begin{aligned}\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = {} & \frac{(5a^3-12a^2b+24ab^2-64b^3)\,x}{16a^5} \\ & + \frac{b^{7/2}(9a+8b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^5(a+b)^{3/2}f} \\ & + \frac{(15a^2-26ab+48b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)} \\ & + \frac{(5a-8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)} \\ & + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)} \\ & + \frac{b(5a^3-7a^2b+12ab^2+32b^3)\tan(e+fx)}{16a^4(a+b)f\left(a+b+b\tan^2(e+fx)\right)}\end{aligned}$$

```
[Out] 1/16*(5*a^3-12*a^2*b+24*a*b^2-64*b^3)*x/a^5+1/2*b^(7/2)*(9*a+8*b)*arctan(b^
(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^5/(a+b)^(3/2)/f+1/48*(15*a^2-26*a*b+48*b^2)
*cos(f*x+e)*sin(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)+1/24*(5*a-8*b)*cos(f*x+e)
^3*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)+1/6*cos(f*x+e)^5*sin(f*x+e)/a/f/(a
+b+b*tan(f*x+e)^2)+1/16*b*(5*a^3-7*a^2*b+12*a*b^2+32*b^3)*tan(f*x+e)/a^4/(a
+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4231, 425, 541, 536, 209, 211}

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{b^{7/2}(9a+8b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^5f(a+b)^{3/2}} + \frac{(5a-8b)\sin(e+fx)\cos^3(e+fx)}{24a^2f\,(a+b\tan^2(e+fx)+b)} + \frac{(15a^2-26ab+48b^2)\sin(e+fx)\cos(e+fx)}{48a^3f\,(a+b\tan^2(e+fx)+b)} + \frac{x(5a^3-12a^2b+24ab^2-64b^3)}{16a^5} + \frac{b(5a^3-7a^2b+12ab^2+32b^3)\tan(e+fx)}{16a^4f(a+b)\,(a+b\tan^2(e+fx)+b)} + \frac{\sin(e+fx)\cos^5(e+fx)}{6af\,(a+b\tan^2(e+fx)+b)}$$

```
[In] Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((5*a^3 - 12*a^2*b + 24*a*b^2 - 64*b^3)*x)/(16*a^5) + (b^(7/2)*(9*a + 8*b)*
ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*a^5*(a + b)^(3/2)*f) + ((15*
a^2 - 26*a*b + 48*b^2)*Cos[e + f*x]*Sin[e + f*x])/(48*a^3*f*(a + b + b*Tan[
e + f*x]^2)) + ((5*a - 8*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*a^2*f*(a + b +
 b*Tan[e + f*x]^2)) + (Cos[e + f*x]^5*Sin[e + f*x])/(6*a*f*(a + b + b*Tan[e
 + f*x]^2)) + (b*(5*a^3 - 7*a^2*b + 12*a*b^2 + 32*b^3)*Tan[e + f*x])/(16*a^
4*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
```

```
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^4(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-5a+b-7bx^2}{(1+x^2)^3(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{6af}$$

$$= \frac{(5a-8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{15a^2-ab+8b^2+5(5a-8b)bx^2}{(1+x^2)^2(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{24a^2f}$$

$$= \frac{(15a^2-26ab+48b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(5a-8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-3(5a^3+3a^2b-2ab^2-16b^3)-3b(15a^2-26ab+48b^2)x^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{48a^3f}$$

$$= \frac{(15a^2-26ab+48b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(5a-8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)} + \frac{b(5a^3-7a^2b+12ab^2+32b^3)\tan(e+fx)}{16a^4(a+b)f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-6(5a^4-2a^3b+5a^2b^2-28ab^3-32b^4)-6b(5a^3-7a^2b+12ab^2+32b^3)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{96a^4(a+b)f}$$

$$= \frac{(15a^2-26ab+48b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(5a-8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)} + \frac{b(5a^3-7a^2b+12ab^2+32b^3)\tan(e+fx)}{16a^4(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(b^4(9a+8b))\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{2a^5(a+b)f} + \frac{(5a^3-12a^2b+24ab^2-64b^3)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{16a^5f}$$

$$= \frac{(5a^3-12a^2b+24ab^2-64b^3)\,x}{16a^5} + \frac{b^{7/2}(9a+8b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^5(a+b)^{3/2}f} + \frac{(15a^2-26ab+48b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(5a-8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)} + \frac{b(5a^3-7a^2b+12ab^2+32b^3)\tan(e+fx)}{16a^4(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.75 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.79

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(12(5a^3-12a^2b+24ab^2-64b^3)\,x(a+2b+a\cos(2(e+fx)))-\cdots\right.}{\cdots}$$

```
[In] Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(12*(5*a^3 - 12*a^2*b + 24*a
*b^2 - 64*b^3)*x*(a + 2*b + a*Cos[2*(e + f*x)]) - (96*b^4*(9*a + 8*b)*ArcTa
n[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*
x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e +
 f*x)])*(Cos[2*e] - I*Sin[2*e]))/((a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e]
)^4]) + (3*a*(15*a^2 - 32*a*b + 48*b^2)*Cos[2*f*x]*(a + 2*b + a*Cos[2*(e +
f*x)])*Sin[2*e])/f + (3*a^2*(3*a - 4*b)*Cos[4*f*x]*(a + 2*b + a*Cos[2*(e +
f*x)])*Sin[4*e])/f + (a^3*Cos[6*f*x]*(a + 2*b + a*Cos[2*(e + f*x)])*Sin[6*e
])/f + (3*a*(15*a^2 - 32*a*b + 48*b^2)*Cos[2*e]*(a + 2*b + a*Cos[2*(e + f*x
)])*Sin[2*f*x])/f - (96*b^4*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/((a + b)*f
*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])) + (3*a^2*(3*a - 4*b)*Cos[4*e]*(a + 2*
b + a*Cos[2*(e + f*x)])*Sin[4*f*x])/f + (a^3*Cos[6*e]*(a + 2*b + a*Cos[2*(e
 + f*x)])*Sin[6*f*x])/f))/(768*a^5*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] (verified)

Time = 6.04 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{\frac{b^4\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(9a+8b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^5}+\frac{\left(\frac{5}{16}a^3-\frac{3}{4}a^2b+\frac{3}{2}ab^2\right)\tan(fx+e)^5+\left(3ab^2+\frac{5}{6}a^3-2a^2b\right)\tan(f\cdots}{\left(1+\tan(fx+e)^2\right)^3\cdots}}{f}$
default	$\frac{\frac{b^4\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(9a+8b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^5}+\frac{\left(\frac{5}{16}a^3-\frac{3}{4}a^2b+\frac{3}{2}ab^2\right)\tan(fx+e)^5+\left(3ab^2+\frac{5}{6}a^3-2a^2b\right)\tan(f\cdots}{\left(1+\tan(fx+e)^2\right)^3\cdots}}{f}$
risch	$\frac{5x}{16a^2}-\frac{3xb}{4a^3}+\frac{3xb^2}{2a^4}-\frac{4xb^3}{a^5}+\frac{ie^{4i(fx+e)}b}{32a^3f}+\frac{ib^4\left(a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a\right)}{a^5(a+b)f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}-\frac{15ie^{2i(fx\cdots}}{128a^2\cdots}$

```
[In] int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^4/a^5*(1/2*a/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(9*a+8*b)/(a+
b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^5*(((5/16*a^3-
3/4*a^2*b+3/2*a*b^2)*tan(f*x+e)^5+(3*a*b^2+5/6*a^3-2*a^2*b)*tan(f*x+e)^3+(-
5/4*a^2*b+3/2*a*b^2+11/16*a^3)*tan(f*x+e))/(1+tan(f*x+e)^2)^3+1/16*(5*a^3-1
2*a^2*b+24*a*b^2-64*b^3)*arctan(tan(f*x+e))))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 789, normalized size of antiderivative = 2.84

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \left[\frac{3\,(5\,a^5 - 7\,a^4b + 12\,a^3b^2 - 40\,a^2b^3 - 64\,ab^4)fx\cos{(fx+e)}^2 + 3\,(5\,a^4b - 7\,a^3b^2 + 12\,a^2b^3 - 40\,ab^4 - 64\,b}{} \right.$$

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(5*a^5 - 7*a^4*b + 12*a^3*b^2 - 40*a^2*b^3 - 64*a*b^4)*f*x*cos(f*x
 + e)^2 + 3*(5*a^4*b - 7*a^3*b^2 + 12*a^2*b^3 - 40*a*b^4 - 64*b^5)*f*x + 6*
(9*a*b^4 + 8*b^5 + (9*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^2)*sqrt(-b/(a + b))*l
og(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2
 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt
(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2
 + b^2)) + (8*(a^5 + a^4*b)*cos(f*x + e)^7 + 2*(5*a^5 - 3*a^4*b - 8*a^3*b^2
)*cos(f*x + e)^5 + (15*a^5 - 11*a^4*b + 22*a^3*b^2 + 48*a^2*b^3)*cos(f*x +
e)^3 + 3*(5*a^4*b - 7*a^3*b^2 + 12*a^2*b^3 + 32*a*b^4)*cos(f*x + e))*sin(f*
x + e))/((a^7 + a^6*b)*f*cos(f*x + e)^2 + (a^6*b + a^5*b^2)*f), 1/48*(3*(5*
a^5 - 7*a^4*b + 12*a^3*b^2 - 40*a^2*b^3 - 64*a*b^4)*f*x*cos(f*x + e)^2 + 3*
(5*a^4*b - 7*a^3*b^2 + 12*a^2*b^3 - 40*a*b^4 - 64*b^5)*f*x - 12*(9*a*b^4 +
8*b^5 + (9*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((
a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))
 + (8*(a^5 + a^4*b)*cos(f*x + e)^7 + 2*(5*a^5 - 3*a^4*b - 8*a^3*b^2)*cos(f*
x + e)^5 + (15*a^5 - 11*a^4*b + 22*a^3*b^2 + 48*a^2*b^3)*cos(f*x + e)^3 + 3
*(5*a^4*b - 7*a^3*b^2 + 12*a^2*b^3 + 32*a*b^4)*cos(f*x + e))*sin(f*x + e))/
((a^7 + a^6*b)*f*cos(f*x + e)^2 + (a^6*b + a^5*b^2)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.33

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{24\,(9\,ab^4+8\,b^5)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^6+a^5b)\sqrt{(a+b)b}} + \frac{3\,(5\,a^3b-7\,a^2b^2+12\,ab^3+32\,b^4)\tan(fx+e)^7+(15\,a^4+34\,a^3b-41\,a^2b^2+156\,ab^3+288\,b^4)\tan(fx+e)^5+(40}{(a^5b+a^4b^2)\tan(fx+e)^8+(a^6+5\,a^5b+4\,a^4b^2)\tan(fx+e)^6+a^6+2\,a^5b+a^4b}}{48}$$

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
[Out] 1/48*(24*(9*a*b^4 + 8*b^5)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^6 + a
^5*b)*sqrt((a + b)*b)) + (3*(5*a^3*b - 7*a^2*b^2 + 12*a*b^3 + 32*b^4)*tan(f
*x + e)^7 + (15*a^4 + 34*a^3*b - 41*a^2*b^2 + 156*a*b^3 + 288*b^4)*tan(f*x
+ e)^5 + (40*a^4 + 17*a^3*b - 35*a^2*b^2 + 204*a*b^3 + 288*b^4)*tan(f*x + e
)^3 + 3*(11*a^4 + 2*a^3*b - 5*a^2*b^2 + 28*a*b^3 + 32*b^4)*tan(f*x + e))/((
a^5*b + a^4*b^2)*tan(f*x + e)^8 + (a^6 + 5*a^5*b + 4*a^4*b^2)*tan(f*x + e)^
6 + a^6 + 2*a^5*b + a^4*b^2 + 3*(a^6 + 3*a^5*b + 2*a^4*b^2)*tan(f*x + e)^4
+ (3*a^6 + 7*a^5*b + 4*a^4*b^2)*tan(f*x + e)^2) + 3*(5*a^3 - 12*a^2*b + 24*
a*b^2 - 64*b^3)*(f*x + e)/a^5)/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.97

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{24\,b^4\tan(fx+e)}{(a^5+a^4b)\left(b\tan(fx+e)^2+a+b\right)} + \frac{24\,(9\,ab^4+8\,b^5)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^6+a^5b)\sqrt{ab+b^2}} + \frac{3\,(5\,a^3-12\,a^2b+24\,ab^2-64\,b^3)(fx+e)}{a^5} + 15}{}$$

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/48*(24*b^4*tan(f*x + e)/((a^5 + a^4*b)*(b*tan(f*x + e)^2 + a + b)) + 24*(
9*a*b^4 + 8*b^5)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x +
e)/sqrt(a*b + b^2)))/((a^6 + a^5*b)*sqrt(a*b + b^2)) + 3*(5*a^3 - 12*a^2*b
+ 24*a*b^2 - 64*b^3)*(f*x + e)/a^5 + (15*a^2*tan(f*x + e)^5 - 36*a*b*tan(f*
x + e)^5 + 72*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 - 96*a*b*tan(f*x +
 e)^3 + 144*b^2*tan(f*x + e)^3 + 33*a^2*tan(f*x + e) - 60*a*b*tan(f*x + e)
+ 72*b^2*tan(f*x + e))/((tan(f*x + e)^2 + 1)^3*a^4))/f
```

Mupad [B] (verification not implemented)

Time = 23.68 (sec) , antiderivative size = 3310, normalized size of antiderivative = 11.91

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] ((tan(e + f*x)*(28*a*b^3 + 2*a^3*b + 11*a^4 + 32*b^4 - 5*a^2*b^2))/(16*a^4*
(a + b)) + (tan(e + f*x)^5*(156*a*b^3 + 34*a^3*b + 15*a^4 + 288*b^4 - 41*a^
2*b^2))/(48*a^4*(a + b)) + (tan(e + f*x)^3*(204*a*b^3 + 17*a^3*b + 40*a^4 +
 288*b^4 - 35*a^2*b^2))/(48*a^4*(a + b)) + (b*tan(e + f*x)^7*(12*a*b^2 - 7*
a^2*b + 5*a^3 + 32*b^3))/(16*a^4*(a + b)))/(f*(a + b + tan(e + f*x)^2*(3*a
+ 4*b) + tan(e + f*x)^4*(3*a + 6*b) + b*tan(e + f*x)^8 + tan(e + f*x)^6*(a
+ 4*b))) - (atan(-((((((8*a^10*b^7 + 15*a^11*b^6 + (23*a^12*b^5)/4 - (3*a^1
3*b^4)/4 - (3*a^14*b^3)/4 - (5*a^15*b^2)/4)/(2*a^13*b + a^14 + a^12*b^2) -
(tan(e + f*x)*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i)*(2048*a^10*b^5 + 5
120*a^11*b^4 + 4096*a^12*b^3 + 1024*a^13*b^2))/(4096*a^5*(2*a^9*b + a^10 +
a^8*b^2)))*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i))/(32*a^5) - (tan(e +
f*x)*(14336*a*b^10 + 8192*b^11 + 5248*a^2*b^9 - 64*a^3*b^8 + 64*a^4*b^7 - 5
68*a^5*b^6 + 169*a^6*b^5 - 70*a^7*b^4 + 25*a^8*b^3))/(128*(2*a^9*b + a^10 +
 a^8*b^2)))*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i)*1i)/(32*a^5) - (((((
8*a^10*b^7 + 15*a^11*b^6 + (23*a^12*b^5)/4 - (3*a^13*b^4)/4 - (3*a^14*b^3)/
4 - (5*a^15*b^2)/4)/(2*a^13*b + a^14 + a^12*b^2) + (tan(e + f*x)*(a*b^2*24i
 - a^2*b*12i + a^3*5i - b^3*64i)*(2048*a^10*b^5 + 5120*a^11*b^4 + 4096*a^12
*b^3 + 1024*a^13*b^2))/(4096*a^5*(2*a^9*b + a^10 + a^8*b^2)))*(a*b^2*24i -
a^2*b*12i + a^3*5i - b^3*64i))/(32*a^5) + (tan(e + f*x)*(14336*a*b^10 + 819
2*b^11 + 5248*a^2*b^9 - 64*a^3*b^8 + 64*a^4*b^7 - 568*a^5*b^6 + 169*a^6*b^5
 - 70*a^7*b^4 + 25*a^8*b^3))/(128*(2*a^9*b + a^10 + a^8*b^2)))*(a*b^2*24i -
 a^2*b*12i + a^3*5i - b^3*64i)*1i)/(32*a^5))/((72*a*b^12 + 64*b^13 - 11*a^2
*b^11 + (19*a^3*b^10)/8 + (267*a^4*b^9)/32 - (101*a^5*b^8)/16 + (655*a^6*b^
7)/256 - (225*a^7*b^6)/256)/(2*a^13*b + a^14 + a^12*b^2) + (((((8*a^10*b^7
+ 15*a^11*b^6 + (23*a^12*b^5)/4 - (3*a^13*b^4)/4 - (3*a^14*b^3)/4 - (5*a^15
*b^2)/4)/(2*a^13*b + a^14 + a^12*b^2) - (tan(e + f*x)*(a*b^2*24i - a^2*b*12
```

i + a^3*5i - b^3*64i)*(2048*a^10*b^5 + 5120*a^11*b^4 + 4096*a^12*b^3 + 1024
*a^13*b^2))/(4096*a^5*(2*a^9*b + a^10 + a^8*b^2)))*(a*b^2*24i - a^2*b*12i +
a^3*5i - b^3*64i))/(32*a^5) - (tan(e + f*x)*(14336*a*b^10 + 8192*b^11 + 52
48*a^2*b^9 - 64*a^3*b^8 + 64*a^4*b^7 - 568*a^5*b^6 + 169*a^6*b^5 - 70*a^7*b
^4 + 25*a^8*b^3))/(128*(2*a^9*b + a^10 + a^8*b^2)))*(a*b^2*24i - a^2*b*12i
+ a^3*5i - b^3*64i))/(32*a^5) + (((((8*a^10*b^7 + 15*a^11*b^6 + (23*a^12*b^
5)/4 - (3*a^13*b^4)/4 - (3*a^14*b^3)/4 - (5*a^15*b^2)/4)/(2*a^13*b + a^14 +
a^12*b^2) + (tan(e + f*x)*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i)*(2048
*a^10*b^5 + 5120*a^11*b^4 + 4096*a^12*b^3 + 1024*a^13*b^2))/(4096*a^5*(2*a^
9*b + a^10 + a^8*b^2)))*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i))/(32*a^5
) + (tan(e + f*x)*(14336*a*b^10 + 8192*b^11 + 5248*a^2*b^9 - 64*a^3*b^8 + 6
4*a^4*b^7 - 568*a^5*b^6 + 169*a^6*b^5 - 70*a^7*b^4 + 25*a^8*b^3))/(128*(2*a
^9*b + a^10 + a^8*b^2)))*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i))/(32*a^
5)))*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i)*1i)/(16*a^5*f) - (atan((((-
b^7*(a + b)^3)^(1/2)*((tan(e + f*x)*(14336*a*b^10 + 8192*b^11 + 5248*a^2*b^
9 - 64*a^3*b^8 + 64*a^4*b^7 - 568*a^5*b^6 + 169*a^6*b^5 - 70*a^7*b^4 + 25*a
^8*b^3))/(128*(2*a^9*b + a^10 + a^8*b^2)) - (((8*a^10*b^7 + 15*a^11*b^6 + (
23*a^12*b^5)/4 - (3*a^13*b^4)/4 - (3*a^14*b^3)/4 - (5*a^15*b^2)/4)/(2*a^13*
b + a^14 + a^12*b^2) - (tan(e + f*x)*(-b^7*(a + b)^3)^(1/2)*(9*a + 8*b)*(20
48*a^10*b^5 + 5120*a^11*b^4 + 4096*a^12*b^3 + 1024*a^13*b^2))/(512*(2*a^9*b
+ a^10 + a^8*b^2)*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)))*(-b^7*(a + b)^3)
^(1/2)*(9*a + 8*b))/(4*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)))*(9*a + 8*b)*
1i)/(4*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)) + ((-b^7*(a + b)^3)^(1/2)*((t
an(e + f*x)*(14336*a*b^10 + 8192*b^11 + 5248*a^2*b^9 - 64*a^3*b^8 + 64*a^4*
b^7 - 568*a^5*b^6 + 169*a^6*b^5 - 70*a^7*b^4 + 25*a^8*b^3))/(128*(2*a^9*b +
a^10 + a^8*b^2)) + (((8*a^10*b^7 + 15*a^11*b^6 + (23*a^12*b^5)/4 - (3*a^13
*b^4)/4 - (3*a^14*b^3)/4 - (5*a^15*b^2)/4)/(2*a^13*b + a^14 + a^12*b^2) + (
tan(e + f*x)*(-b^7*(a + b)^3)^(1/2)*(9*a + 8*b)*(2048*a^10*b^5 + 5120*a^11*
b^4 + 4096*a^12*b^3 + 1024*a^13*b^2))/(512*(2*a^9*b + a^10 + a^8*b^2)*(3*a^
7*b + a^8 + a^5*b^3 + 3*a^6*b^2)))*(-b^7*(a + b)^3)^(1/2)*(9*a + 8*b))/(4*(
3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)))*(9*a + 8*b)*1i)/(4*(3*a^7*b + a^8 +
a^5*b^3 + 3*a^6*b^2)))/((72*a*b^12 + 64*b^13 - 11*a^2*b^11 + (19*a^3*b^10)/
8 + (267*a^4*b^9)/32 - (101*a^5*b^8)/16 + (655*a^6*b^7)/256 - (225*a^7*b^6)
/256)/(2*a^13*b + a^14 + a^12*b^2) - ((-b^7*(a + b)^3)^(1/2)*((tan(e + f*x)
*(14336*a*b^10 + 8192*b^11 + 5248*a^2*b^9 - 64*a^3*b^8 + 64*a^4*b^7 - 568*a
^5*b^6 + 169*a^6*b^5 - 70*a^7*b^4 + 25*a^8*b^3))/(128*(2*a^9*b + a^10 + a^8
*b^2)) - (((8*a^10*b^7 + 15*a^11*b^6 + (23*a^12*b^5)/4 - (3*a^13*b^4)/4 - (
3*a^14*b^3)/4 - (5*a^15*b^2)/4)/(2*a^13*b + a^14 + a^12*b^2) - (tan(e + f*x
)*(-b^7*(a + b)^3)^(1/2)*(9*a + 8*b)*(2048*a^10*b^5 + 5120*a^11*b^4 + 4096*
a^12*b^3 + 1024*a^13*b^2))/(512*(2*a^9*b + a^10 + a^8*b^2)*(3*a^7*b + a^8 +
a^5*b^3 + 3*a^6*b^2)))*(-b^7*(a + b)^3)^(1/2)*(9*a + 8*b))/(4*(3*a^7*b + a
^8 + a^5*b^3 + 3*a^6*b^2)))*(9*a + 8*b))/(4*(3*a^7*b + a^8 + a^5*b^3 + 3*a^
6*b^2)) + ((-b^7*(a + b)^3)^(1/2)*((tan(e + f*x)*(14336*a*b^10 + 8192*b^11
+ 5248*a^2*b^9 - 64*a^3*b^8 + 64*a^4*b^7 - 568*a^5*b^6 + 169*a^6*b^5 - 70*a
^7*b^4 + 25*a^8*b^3))/(128*(2*a^9*b + a^10 + a^8*b^2)) + (((8*a^10*b^7 + 15

```
*a^11*b^6 + (23*a^12*b^5)/4 - (3*a^13*b^4)/4 - (3*a^14*b^3)/4 - (5*a^15*b^2
)/4)/(2*a^13*b + a^14 + a^12*b^2) + (tan(e + f*x)*(-b^7*(a + b)^3)^(1/2)*(9
*a + 8*b)*(2048*a^10*b^5 + 5120*a^11*b^4 + 4096*a^12*b^3 + 1024*a^13*b^2))/
(512*(2*a^9*b + a^10 + a^8*b^2)*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)))*(-b
^7*(a + b)^3)^(1/2)*(9*a + 8*b))/(4*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)))
*(9*a + 8*b))/(4*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2))))*(-b^7*(a + b)^3)^
(1/2)*(9*a + 8*b)*1i)/(2*f*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2))
```

3.206 $\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx=\frac{3\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{a}(a+b)^{5/2}f}+\frac{\sin(e+fx)}{4(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2}+\frac{3\sin(e+fx)}{8(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)}$$

```
[Out] 1/4*sin(f*x+e)/(a+b)/f/(a+b-a*sin(f*x+e)^2)^2+3/8*sin(f*x+e)/(a+b)^2/f/(a+b
-a*sin(f*x+e)^2)+3/8*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/f/
a^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4232, 205, 214}

$$\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx=\frac{3\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{a}f(a+b)^{5/2}}+\frac{3\sin(e+fx)}{8f(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\sin(e+fx)}{4f(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}$$

```
[In] Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(8*Sqrt[a]*(a + b)^(5/2)*f)
 + Sin[e + f*x]/(4*(a + b)*f*(a + b - a*Sin[e + f*x]^2)^2) + (3*Sin[e + f*x
])/(8*(a + b)^2*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(a+b-ax^2)^3}\,dx, x, \sin(e+fx)\right)}{f} \\
&= \frac{\sin(e+fx)}{4(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{3\text{Subst}\left(\int \frac{1}{(a+b-ax^2)^2}\,dx, x, \sin(e+fx)\right)}{4(a+b)f} \\
&= \frac{\sin(e+fx)}{4(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{3\sin(e+fx)}{8(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)} \\
&\quad + \frac{3\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{8(a+b)^2f} \\
&= \frac{3\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{a}(a+b)^{5/2}f} + \frac{\sin(e+fx)}{4(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} \\
&\quad + \frac{3\sin(e+fx)}{8(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{3\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}(a+b)^{5/2}} + \frac{2(7a+10b+3a\cos(2(e+fx)))\sin(e+fx)}{(a+b)^2(a+2b+a\cos(2(e+fx)))^2}}{8f}$$

[In] Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]

[Out] ((3*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5/2)) + (2*(7*a + 10*b + 3*a*Cos[2*(e + f*x)])*Sin[e + f*x])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2))/(8*f)

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\frac{\sin(fx+e)}{4(a+b)\left(a\sin(fx+e)^2-a-b\right)^2}+\frac{-\frac{3\sin(fx+e)}{8(a+b)\left(a\sin(fx+e)^2-a-b\right)}+\frac{3\,\text{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{8(a+b)\sqrt{a(a+b)}}}{a+b}}{f}$
default	$\frac{\frac{\sin(fx+e)}{4(a+b)\left(a\sin(fx+e)^2-a-b\right)^2}+\frac{-\frac{3\sin(fx+e)}{8(a+b)\left(a\sin(fx+e)^2-a-b\right)}+\frac{3\,\text{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{8(a+b)\sqrt{a(a+b)}}}{a+b}}{f}$
risch	$-\frac{i\left(3a\,e^{7i(fx+e)}+11a\,e^{5i(fx+e)}+20b\,e^{5i(fx+e)}-11a\,e^{3i(fx+e)}-20b\,e^{3i(fx+e)}-3a\,e^{i(fx+e)}\right)}{4(a+b)^2 f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2}+\frac{3\ln\left(e^{2i(fx+e)}+\frac{2i(a+b)e}{\sqrt{a^2}}\right.}{16\sqrt{a^2+ab}\,(a+b}$

[In] int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)

[Out] 1/f*(1/4*sin(f*x+e)/(a+b)/(a*sin(f*x+e)^2-a-b)^2+3/4/(a+b)*(-1/2*sin(f*x+e)/(a+b)/(a*sin(f*x+e)^2-a-b)+1/2/(a+b)/(a*(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(100) = 200$.

Time = 0.30 (sec) , antiderivative size = 472, normalized size of antiderivative = 4.37

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \left[\frac{3\left(a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2\right)\sqrt{a^2+ab}\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{a^2+ab}\sin(fx+e)-2\,a-b}{a\cos(fx+e)^2+b}\right)+2\left(2\,a^3\right.}{16\left(\left(a^6+3\,a^5b+3\,a^4b^2+a^3b^3\right)f\cos\left(fx+e\right)^4+2\left(a^5b+3\,a^4b^2+3\,a^3b^3+a^2b^4\right)f\cos\left(f\right.\right.}\right.$$
$$-\frac{3\left(a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2\right)\sqrt{-a^2-ab}\arctan\left(\frac{\sqrt{-a^2-ab}\sin(fx+e)}{a+b}\right)-\left(2\,a^3+7\,a^2b+5\,a\right.}{8\left(\left(a^6+3\,a^5b+3\,a^4b^2+a^3b^3\right)f\cos\left(fx+e\right)^4+2\left(a^5b+3\,a^4b^2+3\,a^3b^3+a^2b^4\right)f\cos\left(fx+e\right)^2\right.}$$

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(a^2 + a*b)*
log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*cos(f
*x + e)^2 + b)) + 2*(2*a^3 + 7*a^2*b + 5*a*b^2 + 3*(a^3 + a^2*b)*cos(f*x +
e)^2)*sin(f*x + e))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos(f*x + e)^4
 + 2*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^2 + (a^4*b^2
+ 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*f), -1/8*(3*(a^2*cos(f*x + e)^4 + 2*a*b*co
s(f*x + e)^2 + b^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(
a + b)) - (2*a^3 + 7*a^2*b + 5*a*b^2 + 3*(a^3 + a^2*b)*cos(f*x + e)^2)*sin(
f*x + e))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos(f*x + e)^4 + 2*(a^5*
b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^2 + (a^4*b^2 + 3*a^3*b^
3 + 3*a^2*b^4 + a*b^5)*f)]
```

Sympy [F]

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{\sec^5\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^3}\,dx$$

```
[In] integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral(sec(e + f*x)**5/(a + b*sec(e + f*x)**2)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.66

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{2\left(3\,a\sin(fx+e)^3-5\,(a+b)\sin(fx+e)\right)}{(a^4+2\,a^3b+a^2b^2)\sin(fx+e)^4+a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4-2\,(a^4+3\,a^3b+3\,a^2b^2+ab^3)\sin(fx+e)^2}+\frac{3\,\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}(a^2+2\,ab+b^2)}}{16\,f}$$

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(2*(3*a*sin(f*x + e)^3 - 5*(a + b)*sin(f*x + e))/((a^4 + 2*a^3*b + a^
2*b^2)*sin(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 2*(a^4
+ 3*a^3*b + 3*a^2*b^2 + a*b^3)*sin(f*x + e)^2) + 3*log((a*sin(f*x + e) - sq
rt((a + b)*a))/(a*sin(f*x + e) + sqrt((a + b)*a)))/(sqrt((a + b)*a)*(a^2 +
2*a*b + b^2)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.08

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{3\,\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^2+2\,ab+b^2)\sqrt{-a^2-ab}}+\frac{3\,a\sin(fx+e)^3-5\,a\sin(fx+e)-5\,b\sin(fx+e)}{\left(a\sin(fx+e)^2-a-b\right)^2(a^2+2\,ab+b^2)}}{8\,f}$$

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^2 + 2*a*b + b^2)*sqrt(-
a^2 - a*b)) + (3*a*sin(f*x + e)^3 - 5*a*sin(f*x + e) - 5*b*sin(f*x + e))/((
a*sin(f*x + e)^2 - a - b)^2*(a^2 + 2*a*b + b^2)))/f
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{5\,\sin(e+f\,x)}{8\,(a+b)}-\frac{3\,a\sin(e+f\,x)^3}{8\,(a+b)^2}}{f\,\left(2\,a\,b+a^2+b^2-\sin\left(e+f\,x\right)^2\,\left(2\,a^2+2\,b\,a\right)+a^2\sin\left(e+f\,x\right)^4\right)} + \frac{3\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)}{8\,\sqrt{a}\,f\,(a+b)^{5/2}}$$

```
[In] int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] ((5*sin(e + f*x))/(8*(a + b)) - (3*a*sin(e + f*x)^3)/(8*(a + b)^2))/(f*(2*a
*b + a^2 + b^2 - sin(e + f*x)^2*(2*a*b + 2*a^2) + a^2*sin(e + f*x)^4)) + (3
*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2)))/(8*a^(1/2)*f*(a + b)^(5/2))
```

3.207 $\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 125

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{(4a+b)\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{3/2}(a+b)^{5/2}f} - \frac{b\sin(e+fx)}{4a(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{(4a+b)\sin(e+fx)}{8a(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)}$$

```
[Out] 1/8*(4*a+b)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(3/2)/(a+b)^(5/2)/f-1
/4*b*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*x+e)^2)^2+1/8*(4*a+b)*sin(f*x+e)/a/(
a+b)^2/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4232, 393, 205, 214}

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{(4a+b)\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{3/2}f(a+b)^{5/2}} + \frac{(4a+b)\sin(e+fx)}{8af(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)} - \frac{b\sin(e+fx)}{4af(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}$$

```
[In] Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((4*a + b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(8*a^(3/2)*(a + b)^
(5/2)*f) - (b*Sin[e + f*x])/(4*a*(a + b)*f*(a + b - a*Sin[e + f*x]^2)^2) +
((4*a + b)*Sin[e + f*x])/(8*a*(a + b)^2*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1-x^2}{(a+b-ax^2)^3}\,dx, x, \sin(e+fx)\right)}{f}\\
&= -\frac{b\sin(e+fx)}{4a(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{(4a+b)\text{Subst}\left(\int \frac{1}{(a+b-ax^2)^2}\,dx, x, \sin(e+fx)\right)}{4a(a+b)f}\\
&= -\frac{b\sin(e+fx)}{4a(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{(4a+b)\sin(e+fx)}{8a(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)}\\
&\quad + \frac{(4a+b)\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{8a(a+b)^2f}
\end{aligned}$$

$$= \frac{(4a+b)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{3/2}(a+b)^{5/2}f} - \frac{b\sin(e+fx)}{4a(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{(4a+b)\sin(e+fx)}{8a(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.30

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))^3\sec^6(e+fx)\left(\frac{8\sin(e+fx)}{(a+b-a\sin^2(e+fx))^2} - (4a+b)\left(\frac{3\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}(a+b)^{5/2}} + \frac{4\sin(e+f}{(a+b)^2}\right.\right.}{192af\left(a+b\sec^2(e+fx)\right)^3}$$

```
[In] Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -1/192*((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*((8*Sin[e + f*x])/(
a + b - a*Sin[e + f*x]^2)^2 - (4*a + b)*((3*ArcTanh[(Sqrt[a]*Sin[e + f*x])/
Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5/2)) + (4*Sin[e + f*x]*(5*(a + b) - 3*a*Si
n[e + f*x]^2))/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2))))/(a*f*(a + b*
Sec[e + f*x]^2)^3)
```

Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{\frac{-\frac{(4a+b)\sin(fx+e)^3}{8(a^2+2ab+b^2)}+\frac{(4a-b)\sin(fx+e)}{8a(a+b)}}{(a\sin(fx+e)^2-a-b)^2}+\frac{(4a+b)\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{8(a^2+2ab+b^2)a\sqrt{a(a+b)}}}{f}$
default	$\frac{\frac{-\frac{(4a+b)\sin(fx+e)^3}{8(a^2+2ab+b^2)}+\frac{(4a-b)\sin(fx+e)}{8a(a+b)}}{(a\sin(fx+e)^2-a-b)^2}+\frac{(4a+b)\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{8(a^2+2ab+b^2)a\sqrt{a(a+b)}}}{f}$
risch	$-\frac{i(4a^2e^{7i(fx+e)}+ab\,e^{7i(fx+e)}+4a^2e^{5i(fx+e)}+9ab\,e^{5i(fx+e)}-4b^2e^{5i(fx+e)}-4a^2e^{3i(fx+e)}-9ab\,e^{3i(fx+e)}+4b^2e^{3i(fx+e)}}{4a(a+b)^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2}$

```
[In] int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*((-1/8*(4*a+b)/(a^2+2*a*b+b^2)*sin(f*x+e)^3+1/8*(4*a-b)/a/(a+b)*sin(f*x
+e))/(a*sin(f*x+e)^2-a-b)^2+1/8*(4*a+b)/(a^2+2*a*b+b^2)/a/(a*(a+b))^(1/2)*a
rctanh(a*sin(f*x+e)/(a*(a+b))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(117) = 234$.

Time = 0.30 (sec) , antiderivative size = 544, normalized size of antiderivative = 4.35

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \left[\frac{\left((4\,a^3+a^2b)\cos(fx+e)^4+4\,ab^2+b^3+2\,(4\,a^2b+ab^2)\cos(fx+e)^2\right)\sqrt{a^2+ab}\log\left(-\frac{a\cos(fx+e)^2-2\,\sqrt{a^2\cdots}}{a\cos(fx\cdots}\right.}{16\left((a^7+3\,a^6b+3\,a^5b^2+a^4b^3)f\cos(fx+e)^4+2\,(a^6b+3\,a^5b^2+3\,a^4b^3\cdots\right.}\right.$$
$$-\frac{\left((4\,a^3+a^2b)\cos(fx+e)^4+4\,ab^2+b^3+2\,(4\,a^2b+ab^2)\cos(fx+e)^2\right)\sqrt{-a^2-ab}\arctan\left(\frac{\sqrt{-a^2-ab}\sin(f\cdots}{a+b}\right.}{8\left((a^7+3\,a^6b+3\,a^5b^2+a^4b^3)f\cos(fx+e)^4+2\,(a^6b+3\,a^5b^2+3\,a^4b^3+a^3b\cdots\right.}$$

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(((4*a^3 + a^2*b)*cos(f*x + e)^4 + 4*a*b^2 + b^3 + 2*(4*a^2*b + a*b^2
)*cos(f*x + e)^2)*sqrt(a^2 + a*b)*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b
)*sin(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(2*a^3*b + a^2*b^2 -
a*b^3 + (4*a^4 + 5*a^3*b + a^2*b^2)*cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 3
*a^6*b + 3*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^4 + 2*(a^6*b + 3*a^5*b^2 + 3*a
^4*b^3 + a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2
*b^5)*f), -1/8*(((4*a^3 + a^2*b)*cos(f*x + e)^4 + 4*a*b^2 + b^3 + 2*(4*a^2*
b + a*b^2)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*sin(f*x
 + e)/(a + b)) - (2*a^3*b + a^2*b^2 - a*b^3 + (4*a^4 + 5*a^3*b + a^2*b^2)*c
os(f*x + e)^2)*sin(f*x + e))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*f*cos(f
*x + e)^4 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^2 +
(a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f)]
```

Sympy [F]

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

```
[In] integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral(sec(e + f*x)**3/(a + b*sec(e + f*x)**2)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.70

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{(4a+b)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(a^3+2a^2b+ab^2)\sqrt{(a+b)a}} + \frac{2\left((4a^2+ab)\sin(fx+e)^3-(4a^2+3ab-b^2)\sin(fx+e)\right)}{a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4+(a^5+2a^4b+a^3b^2)\sin(fx+e)^4-2(a^5+3a^4b+3a^3b^2+a^2b^3)\sin(fx+e)^2}}{16f}$$

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((4*a + b)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + s
qrt((a + b)*a)))/((a^3 + 2*a^2*b + a*b^2)*sqrt((a + b)*a)) + 2*((4*a^2 + a*
b)*sin(f*x + e)^3 - (4*a^2 + 3*a*b - b^2)*sin(f*x + e))/(a^5 + 4*a^4*b + 6*
a^3*b^2 + 4*a^2*b^3 + a*b^4 + (a^5 + 2*a^4*b + a^3*b^2)*sin(f*x + e)^4 - 2*
(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sin(f*x + e)^2))/f
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.24

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{(4a+b)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^3+2a^2b+ab^2)\sqrt{-a^2-ab}} + \frac{4a^2\sin(fx+e)^3+ab\sin(fx+e)^3-4a^2\sin(fx+e)-3ab\sin(fx+e)+b^2\sin(fx+e)}{(a^3+2a^2b+ab^2)\left(a\sin(fx+e)^2-a-b\right)^2}}{8f}$$

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((4*a + b)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^3 + 2*a^2*b + a
*b^2)*sqrt(-a^2 - a*b)) + (4*a^2*sin(f*x + e)^3 + a*b*sin(f*x + e)^3 - 4*a^
2*sin(f*x + e) - 3*a*b*sin(f*x + e) + b^2*sin(f*x + e))/((a^3 + 2*a^2*b + a
*b^2)*(a*sin(f*x + e)^2 - a - b)^2))/f
```

Mupad [B] (verification not implemented)

Time = 19.60 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.03

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)\,(4\,a+b)}{8\,a^{3/2}\,f\,(a+b)^{5/2}} - \frac{\frac{\sin(e+f\,x)^3\,(4\,a+b)}{8\,(a+b)^2}-\frac{\sin(e+f\,x)\,(4\,a-b)}{8\,a\,(a+b)}}{f\,\left(2\,a\,b+a^2+b^2-\sin\left(e+f\,x\right)^2\,\left(2\,a^2+2\,b\,a\right)+a^2\sin\left(e+f\,x\right)^4\right)}$$

```
[In] int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] (atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(4*a + b))/(8*a^(3/2)*f*(a + b
)^(5/2)) - ((sin(e + f*x)^3*(4*a + b))/(8*(a + b)^2) - (sin(e + f*x)*(4*a -
 b))/(8*a*(a + b)))/(f*(2*a*b + a^2 + b^2 - sin(e + f*x)^2*(2*a*b + 2*a^2)
+ a^2*sin(e + f*x)^4))
```

3.208 $\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 144

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{(8a^2+8ab+3b^2)\,\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{5/2}(a+b)^{5/2}f} - \frac{b\cos^2(e+fx)\sin(e+fx)}{4a(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} - \frac{3b(2a+b)\sin(e+fx)}{8a^2(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)}$$

```
[Out] 1/8*(8*a^2+8*a*b+3*b^2)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(5/2)/(a+
b)^(5/2)/f-1/4*b*cos(f*x+e)^2*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*x+e)^2)^2-3
/8*b*(2*a+b)*sin(f*x+e)/a^2/(a+b)^2/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4232, 424, 393, 214}

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{3b(2a+b)\sin(e+fx)}{8a^2f(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)} + \frac{(8a^2+8ab+3b^2)\,\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{5/2}f(a+b)^{5/2}} - \frac{b\sin(e+fx)\cos^2(e+fx)}{4af(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}$$

```
[In] Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(8*a^
(5/2)*(a + b)^(5/2)*f) - (b*Cos[e + f*x]^2*Sin[e + f*x])/(4*a*(a + b)*f*(a
+ b - a*Sin[e + f*x]^2)^2) - (3*b*(2*a + b)*Sin[e + f*x])/(8*a^2*(a + b)^2*
f*(a + b - a*Sin[e + f*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{(a+b-ax^2)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{b\cos^2(e+fx)\sin(e+fx)}{4a(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-4a-b+(4a+3b)x^2}{(a+b-ax^2)^2}\,dx, x, \sin(e+fx)\right)}{4a(a+b)f}$$

$$= -\frac{b\cos^2(e+fx)\sin(e+fx)}{4a(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} - \frac{3b(2a+b)\sin(e+fx)}{8a^2(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)} + \frac{(8a^2+8ab+3b^2)\,\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{8a^2(a+b)^2f}$$

$$= \frac{(8a^2+8ab+3b^2)\,\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{5/2}(a+b)^{5/2}f} - \frac{b\cos^2(e+fx)\sin(e+fx)}{4a(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} - \frac{3b(2a+b)\sin(e+fx)}{8a^2(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.45 (sec) , antiderivative size = 927, normalized size of antiderivative = 6.44

$$\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^5(e+fx)\left(-2i(8a^2+8ab+3b^2)\arctan\left(\frac{(a+b)\sin(e)}{(a+b)\cos(e)-\sqrt{a}\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2(c}}\right.\right.}{}$$

```
[In] Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^5*((-2*I)*(8*a^2 + 8*a*b + 3*b
^2)*ArcTan[((a + b)*Sin[e])/((a + b)*Cos[e] - Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos
[e] - I*Sin[e])^2]*(Cos[2*e] + I*Sin[2*e])*Sin[e + f*x])]*(a + 2*b + a*Cos[
2*(e + f*x)])^2*Sec[e + f*x]*(Cos[e] - I*Sin[e]) + (8*a^2 + 8*a*b + 3*b^2)*
(a + 2*b + a*Cos[2*(e + f*x)])^2*Log[a + 2*(a + b)*Cos[2*e] - a*Cos[2*(e +
f*x)] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(C
os[e] - I*Sin[e])^2]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[
e])^2]*Sin[2*e + f*x]]*Sec[e + f*x]*(Cos[e] - I*Sin[e]) - (8*a^2 + 8*a*b +
3*b^2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Log[-a - 2*(a + b)*Cos[2*e] + a*Cos
[2*(e + f*x)] + (2*I)*a*Sin[2*e] + (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]
*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e]
- I*Sin[e])^2]*Sin[2*e + f*x]]*Sec[e + f*x]*(Cos[e] - I*Sin[e]) + 2*(8*a^2
+ 8*a*b + 3*b^2)*ArcTan[(2*Sin[e]*(I*a + I*b + I*(a + b)*Cos[2*e] + Sqrt[a]
*Sqrt[a + b]*Cos[f*x]*Sqrt[(Cos[e] - I*Sin[e])^2] - Sqrt[a]*Sqrt[a + b]*Cos
[2*e + f*x]*Sqrt[(Cos[e] - I*Sin[e])^2] + a*Sin[2*e] + b*Sin[2*e] - I*Sqrt[
a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f*x] - I*Sqrt[a]*Sqrt[a + b]
*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]))/(I*(a + 3*b)*Cos[e] + I*(a +
b)*Cos[3*e] + I*a*Cos[e + 2*f*x] + I*a*Cos[3*e + 2*f*x] + 3*a*Sin[e] + b*Si
n[e] + a*Sin[3*e] + b*Sin[3*e] + a*Sin[e + 2*f*x] - a*Sin[3*e + 2*f*x])]*(a
 + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x]*(I*Cos[e] + Sin[e]) + 32*Sqrt[a
```

]*b^2*(a + b)^(3/2)*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[e + f*x] - 8*Sqrt[a]*b*Sqrt[a + b]*(8*a + 5*b)*(a + 2*b + a*Cos[2*(e + f*x)])*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[e + f*x]))/(256*a^(5/2)*(a + b)^(5/2)*f*(a + b*Sec[e + f*x]^2)^3*Sqrt[(Cos[e] - I*Sin[e])^2])

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{-\frac{-\frac{b(8a+5b)\sin(fx+e)^3}{8a\left(a^2+2ab+b^2\right)}+\frac{(8a+3b)b\sin(fx+e)}{8a^2(a+b)}}{\left(a\sin(fx+e)^2-a-b\right)^2}+\frac{\left(8a^2+8ab+3b^2\right)\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{8\left(a^2+2ab+b^2\right)a^2\sqrt{a(a+b)}}}{f}$
default	$\frac{-\frac{-\frac{b(8a+5b)\sin(fx+e)^3}{8a\left(a^2+2ab+b^2\right)}+\frac{(8a+3b)b\sin(fx+e)}{8a^2(a+b)}}{\left(a\sin(fx+e)^2-a-b\right)^2}+\frac{\left(8a^2+8ab+3b^2\right)\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{8\left(a^2+2ab+b^2\right)a^2\sqrt{a(a+b)}}}{f}$
risch	$\frac{ib\left(8a^2e^{7i(fx+e)}+5ab\,e^{7i(fx+e)}+8a^2e^{5i(fx+e)}+29ab\,e^{5i(fx+e)}+12b^2e^{5i(fx+e)}-8a^2e^{3i(fx+e)}-29ab\,e^{3i(fx+e)}-12b^2e^{3i(\cdots}\right.}{4a^2(a+b)^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2}$

[In] int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)

[Out] 1/f*(-(-1/8*b*(8*a+5*b)/a/(a^2+2*a*b+b^2)*sin(f*x+e)^3+1/8*(8*a+3*b)/a^2*b/(a+b)*sin(f*x+e))/(a*sin(f*x+e)^2-a-b)^2+1/8*(8*a^2+8*a*b+3*b^2)/(a^2+2*a*b+b^2)/a^2/(a*(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(136) = 272.

Time = 0.29 (sec) , antiderivative size = 613, normalized size of antiderivative = 4.26

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \left[\frac{\left((8\,a^4+8\,a^3b+3\,a^2b^2)\cos{(fx+e)}^4+8\,a^2b^2+8\,ab^3+3\,b^4+2\,(8\,a^3b+8\,a^2b^2+3\,ab^3)\cos{(fx+e)}^2\right)\sqrt{\cdots}}{16\left((a^8+3\,a^7b+3\,a^6b^2+a^5b^3)f\cos{(fx+e)}^4+2\,(\cdots\right.}\right.$$
$$-\frac{\left((8\,a^4+8\,a^3b+3\,a^2b^2)\cos{(fx+e)}^4+8\,a^2b^2+8\,ab^3+3\,b^4+2\,(8\,a^3b+8\,a^2b^2+3\,ab^3)\cos{(fx+e)}^2\right)\sqrt{\cdots}}{8\left((a^8+3\,a^7b+3\,a^6b^2+a^5b^3)f\cos{(fx+e)}^4+2\,(a^7b+3\cdots\right.}$$

[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")

[Out] [1/16*(((8*a^4 + 8*a^3*b + 3*a^2*b^2)*cos(f*x + e)^4 + 8*a^2*b^2 + 8*a*b^3 + 3*b^4 + 2*(8*a^3*b + 8*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^2)*sqrt(a^2 + a*b) *log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) - 2*(6*a^3*b^2 + 9*a^2*b^3 + 3*a*b^4 + (8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*f), -1/8*(((8*a^4 + 8*a^3*b + 3*a^2*b^2)*cos(f*x + e)^4 + 8*a^2*b^2 + 8*a*b^3 + 3*b^4 + 2*(8*a^3*b + 8*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) + (6*a^3*b^2 + 9*a^2*b^3 + 3*a*b^4 + (8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*f)]

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)

[Out] Integral(sec(e + f*x)/(a + b*sec(e + f*x)**2)**3, x)

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.62

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{(8a^2+8ab+3b^2)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(a^4+2a^3b+a^2b^2)\sqrt{(a+b)a}} - \frac{2\left((8a^2b+5ab^2)\sin(fx+e)^3-(8a^2b+11ab^2+3b^3)\sin(fx+e)\right)}{a^6+4a^5b+6a^4b^2+4a^3b^3+a^2b^4+(a^6+2a^5b+a^4b^2)\sin(fx+e)^4-2(a^6+3a^5b+3a^4b^2+a^3b^3)\,\mathrm{si}}}{16f}$$

[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")

[Out] -1/16*((8*a^2 + 8*a*b + 3*b^2)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((a + b)*a)))/((a^4 + 2*a^3*b + a^2*b^2)*sqrt((a + b)*a)) - 2*((8*a^2*b + 5*a*b^2)*sin(f*x + e)^3 - (8*a^2*b + 11*a*b^2 + 3*b^3)*sin(f*x + e))/(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4 + (a^6 + 2*a^5*b + a^4*b^2)*sin(f*x + e)^4 - 2*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sin(f*x + e)^2))/f

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.24

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{(8a^2+8ab+3b^2)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^4+2a^3b+a^2b^2)\sqrt{-a^2-ab}} - \frac{8a^2b\sin(fx+e)^3+5ab^2\sin(fx+e)^3-8a^2b\sin(fx+e)-11ab^2\sin(fx+e)-3b^3\sin(fx+e)}{(a^4+2a^3b+a^2b^2)\left(a\sin(fx+e)^2-a-b\right)^2}}{8f}$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((8*a^2 + 8*a*b + 3*b^2)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^4
 + 2*a^3*b + a^2*b^2)*sqrt(-a^2 - a*b)) - (8*a^2*b*sin(f*x + e)^3 + 5*a*b^2
*sin(f*x + e)^3 - 8*a^2*b*sin(f*x + e) - 11*a*b^2*sin(f*x + e) - 3*b^3*sin(
f*x + e))/((a^4 + 2*a^3*b + a^2*b^2)*(a*sin(f*x + e)^2 - a - b)^2))/f
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.03

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{\sin(e+f\,x)^3\,(5\,b^2+8\,a\,b)}{8\,a\,(a+b)^2} - \frac{\sin(e+f\,x)\,(3\,b^2+8\,a\,b)}{8\,a^2\,(a+b)}}{f\,\left(2\,a\,b+a^2+b^2-\sin\left(e+f\,x\right)^2\,\left(2\,a^2+2\,b\,a\right)+a^2\sin\left(e+f\,x\right)^4\right)} + \frac{\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)\,\left(8\,a^2+8\,a\,b+3\,b^2\right)}{8\,a^{5/2}\,f\,(a+b)^{5/2}}$$

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] ((sin(e + f*x)^3*(8*a*b + 5*b^2))/(8*a*(a + b)^2) - (sin(e + f*x)*(8*a*b +
3*b^2))/(8*a^2*(a + b)))/(f*(2*a*b + a^2 + b^2 - sin(e + f*x)^2*(2*a*b + 2*
a^2) + a^2*sin(e + f*x)^4)) + (atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*
(8*a*b + 8*a^2 + 3*b^2))/(8*a^(5/2)*f*(a + b)^(5/2))
```

3.209 $\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^3} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 156

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^3} \, dx = -\frac{3b(4(a+b)^2+(2a+b)^2)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{7/2}(a+b)^{5/2}f} + \frac{\sin(e+fx)}{a^3f} - \frac{b^3\sin(e+fx)}{4a^3(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{3b^2(4a+3b)\sin(e+fx)}{8a^3(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)}$$

```
[Out] -3/8*b*(4*(a+b)^2+(2*a+b)^2)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(7/2
)/(a+b)^(5/2)/f+sin(f*x+e)/a^3/f-1/4*b^3*sin(f*x+e)/a^3/(a+b)/f/(a+b-a*sin(
f*x+e)^2)^2+3/8*b^2*(4*a+3*b)*sin(f*x+e)/a^3/(a+b)^2/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4232, 398, 1171, 393, 214}

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^3} \, dx = -\frac{3b(4(a+b)^2+(2a+b)^2)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{7/2}f(a+b)^{5/2}} - \frac{b^3\sin(e+fx)}{4a^3f(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2} + \frac{3b^2(4a+3b)\sin(e+fx)}{8a^3f(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)} + \frac{\sin(e+fx)}{a^3f}$$

```
[In] Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (-3*b*(4*(a + b)^2 + (2*a + b)^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b
]])/(8*a^(7/2)*(a + b)^(5/2)*f) + Sin[e + f*x]/(a^3*f) - (b^3*Sin[e + f*x])
/(4*a^3*(a + b)*f*(a + b - a*Sin[e + f*x]^2)^2) + (3*b^2*(4*a + 3*b)*Sin[e
+ f*x])/(8*a^3*(a + b)^2*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{(a+b-ax^2)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3} - \frac{b(3a^2+3ab+b^2)-3ab(2a+b)x^2+3a^2bx^4}{a^3(a+b-ax^2)^3}\right)\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sin(e+fx)}{a^3 f} - \frac{\text{Subst}\left(\int \frac{b(3a^2+3ab+b^2)-3ab(2a+b)x^2+3a^2bx^4}{(a+b-ax^2)^3}\,dx, x, \sin(e+fx)\right)}{a^3 f}$$

$$= \frac{\sin(e+fx)}{a^3 f} - \frac{b^3\sin(e+fx)}{4a^3(a+b)f\,(a+b-a\sin^2(e+fx))^2} + \frac{\text{Subst}\left(\int \frac{-3b(2a+b)^2+12ab(a+b)x^2}{(a+b-ax^2)^2}\,dx, x, \sin(e+fx)\right)}{4a^3(a+b)f}$$

$$= \frac{\sin(e+fx)}{a^3 f} - \frac{b^3\sin(e+fx)}{4a^3(a+b)f\,(a+b-a\sin^2(e+fx))^2} + \frac{3b^2(4a+3b)\sin(e+fx)}{8a^3(a+b)^2 f\,(a+b-a\sin^2(e+fx))} - \frac{(3b(4(a+b)^2+(2a+b)^2))\,\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{8a^3(a+b)^2 f}$$

$$= -\frac{3b(4(a+b)^2+(2a+b)^2)\,\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{7/2}(a+b)^{5/2}f} + \frac{\sin(e+fx)}{a^3 f} - \frac{b^3\sin(e+fx)}{4a^3(a+b)f\,(a+b-a\sin^2(e+fx))^2} + \frac{3b^2(4a+3b)\sin(e+fx)}{8a^3(a+b)^2 f\,(a+b-a\sin^2(e+fx))}$$

Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.13

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{-\frac{3b(8a^2+12ab+5b^2)\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + \frac{4\sqrt{a}\sin(e+fx)(8a^4+32a^3b+60a^2b^2+51ab^3+15b^4-a(16a^3+48a^2b+60ab^2+25b^3)\sin^2(e+fx)}{(a+b)^2(a+2b+a\cos(2(e+fx)))^2}}{8a^{7/2}f}$$

```
[In] Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((-3*b*(8*a^2 + 12*a*b + 5*b^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]
)/(a + b)^(5/2) + (4*Sqrt[a]*Sin[e + f*x]*(8*a^4 + 32*a^3*b + 60*a^2*b^2 +
51*a*b^3 + 15*b^4 - a*(16*a^3 + 48*a^2*b + 60*a*b^2 + 25*b^3)*Sin[e + f*x]^
2 + 8*a^2*(a + b)^2*Sin[e + f*x]^4))/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x
)])^2))/(8*a^(7/2)*f)
```

Maple [A] (verified)

Time = 3.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\frac{\sin(fx+e)}{a^3}+\frac{b\left(\frac{-\frac{3ab(4a+3b)\sin(fx+e)^3}{8(a^2+2ab+b^2)}+\frac{(12a+7b)b\sin(fx+e)}{8a+8b}}{\left(a\sin(fx+e)^2-a-b\right)^2}-\frac{3(8a^2+12ab+5b^2)\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{8(a^2+2ab+b^2)\sqrt{a(a+b)}}\right)}{a^3}}{f}$
default	$\frac{\frac{\sin(fx+e)}{a^3}+\frac{b\left(\frac{-\frac{3ab(4a+3b)\sin(fx+e)^3}{8(a^2+2ab+b^2)}+\frac{(12a+7b)b\sin(fx+e)}{8a+8b}}{\left(a\sin(fx+e)^2-a-b\right)^2}-\frac{3(8a^2+12ab+5b^2)\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{8(a^2+2ab+b^2)\sqrt{a(a+b)}}\right)}{a^3}}{f}$
risch	$-\frac{i e^{i(fx+e)}}{2a^3f}+\frac{i e^{-i(fx+e)}}{2a^3f}-\frac{ib^2\left(12a^2e^{7i(fx+e)}+9ab\,e^{7i(fx+e)}+12a^2e^{5i(fx+e)}+49ab\,e^{5i(fx+e)}+28b^2e^{5i(fx+e)}-12a\right.}{4a^3(a+b)^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+\right.}}$

```
[In] int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(sin(f*x+e)/a^3+1/a^3*b*((-3/8*a*b*(4*a+3*b)/(a^2+2*a*b+b^2)*sin(f*x+e)
^3+1/8*(12*a+7*b)*b/(a+b)*sin(f*x+e))/(a*sin(f*x+e)^2-a-b)^2-3/8*(8*a^2+12*
a*b+5*b^2)/(a^2+2*a*b+b^2)/(a*(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(
1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(148) = 296$.

Time = 0.33 (sec) , antiderivative size = 727, normalized size of antiderivative = 4.66

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \left[\frac{3\left(8a^2b^3+12ab^4+5b^5+(8a^4b+12a^3b^2+5a^2b^3)\cos(fx+e)^4+2(8a^3b^2+12a^2b^3+5ab^4)\cos(fx+\right.}{16\left((a^9+3a\right.}\right.$$

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(8*a^2*b^3 + 12*a*b^4 + 5*b^5 + (8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)
*cos(f*x + e)^4 + 2*(8*a^3*b^2 + 12*a^2*b^3 + 5*a*b^4)*cos(f*x + e)^2)*sqrt
(a^2 + a*b)*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a -
 b)/(a*cos(f*x + e)^2 + b)) + 2*(8*a^4*b^2 + 34*a^3*b^3 + 41*a^2*b^4 + 15*a
*b^5 + 8*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cos(f*x + e)^4 + (16*a^5*b +
 60*a^4*b^2 + 69*a^3*b^3 + 25*a^2*b^4)*cos(f*x + e)^2)*sin(f*x + e))/((a^9
+ 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*f*cos(f*x + e)^4 + 2*(a^8*b + 3*a^7*b^2 +
3*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)^2 + (a^7*b^2 + 3*a^6*b^3 + 3*a^5*b^4 +
```

```
a^4*b^5)*f), 1/8*(3*(8*a^2*b^3 + 12*a*b^4 + 5*b^5 + (8*a^4*b + 12*a^3*b^2 +
 5*a^2*b^3)*cos(f*x + e)^4 + 2*(8*a^3*b^2 + 12*a^2*b^3 + 5*a*b^4)*cos(f*x +
 e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) + (8*
a^4*b^2 + 34*a^3*b^3 + 41*a^2*b^4 + 15*a*b^5 + 8*(a^6 + 3*a^5*b + 3*a^4*b^2
 + a^3*b^3)*cos(f*x + e)^4 + (16*a^5*b + 60*a^4*b^2 + 69*a^3*b^3 + 25*a^2*b
^4)*cos(f*x + e)^2)*sin(f*x + e))/((a^9 + 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*f*
cos(f*x + e)^4 + 2*(a^8*b + 3*a^7*b^2 + 3*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)
^2 + (a^7*b^2 + 3*a^6*b^3 + 3*a^5*b^4 + a^4*b^5)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.62

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{3\left(8\,a^2b+12\,ab^2+5\,b^3\right)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(a^5+2\,a^4b+a^3b^2)\sqrt{(a+b)a}} - \frac{2\left(3\left(4\,a^2b^2+3\,ab^3\right)\sin(fx+e)^3-\left(12\,a^2b^2+19\,ab^3+7\,b^4\right)\sin(fx+e)\right)}{a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4+(a^7+2\,a^6b+a^5b^2)\sin(fx+e)^4-2\,(a^7+3\,a^6b+3\,a^5b^2+a^4b^3)}}{16\,f}$$

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*(8*a^2*b + 12*a*b^2 + 5*b^3)*log((a*sin(f*x + e) - sqrt((a + b)*a))
/(a*sin(f*x + e) + sqrt((a + b)*a)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b
)*a)) - 2*(3*(4*a^2*b^2 + 3*a*b^3)*sin(f*x + e)^3 - (12*a^2*b^2 + 19*a*b^3
+ 7*b^4)*sin(f*x + e))/(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4 + (
a^7 + 2*a^6*b + a^5*b^2)*sin(f*x + e)^4 - 2*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^
4*b^3)*sin(f*x + e)^2) + 16*sin(f*x + e)/a^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.26

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{3\left(8a^2b+12ab^2+5b^3\right)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^5+2a^4b+a^3b^2)\sqrt{-a^2-ab}} - \frac{12a^2b^2\sin(fx+e)^3+9ab^3\sin(fx+e)^3-12a^2b^2\sin(fx+e)-19ab^3\sin(fx+e)-7b^4\sin(fx+e)}{(a^5+2a^4b+a^3b^2)\left(a\sin(fx+e)^2-a-b\right)^2} + \cdots}{8f}$$

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(8*a^2*b + 12*a*b^2 + 5*b^3)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))
/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(-a^2 - a*b)) - (12*a^2*b^2*sin(f*x + e)^3
+ 9*a*b^3*sin(f*x + e)^3 - 12*a^2*b^2*sin(f*x + e) - 19*a*b^3*sin(f*x + e)
- 7*b^4*sin(f*x + e))/((a^5 + 2*a^4*b + a^3*b^2)*(a*sin(f*x + e)^2 - a - b)
^2) + 8*sin(f*x + e)/a^3)/f
```

Mupad [B] (verification not implemented)

Time = 19.72 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.12

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\sin(e+f\,x)}{a^3\,f} + \frac{\frac{\sin(e+f\,x)\,\left(7\,b^3+12\,a\,b^2\right)}{8\,(a+b)} - \frac{3\sin(e+f\,x)^3\,\left(4\,a^2\,b^2+3\,a\,b^3\right)}{8\,(a+b)^2}}{f\,\left(2\,a^4\,b-\sin(e+f\,x)^2\,\left(2\,a^5+2\,b\,a^4\right)+a^5+a^3\,b^2+a^5\sin(e+f\,x)^4\right)} - \frac{3\,b\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)\,\left(8\,a^2+12\,a\,b+5\,b^2\right)}{8\,a^{7/2}\,f\,(a+b)^{5/2}}$$

```
[In] int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] sin(e + f*x)/(a^3*f) + ((sin(e + f*x)*(12*a*b^2 + 7*b^3))/(8*(a + b)) - (3*
sin(e + f*x)^3*(3*a*b^3 + 4*a^2*b^2))/(8*(a + b)^2))/(f*(2*a^4*b - sin(e +
f*x)^2*(2*a^4*b + 2*a^5) + a^5 + a^3*b^2 + a^5*sin(e + f*x)^4)) - (3*b*atan
h((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(12*a*b + 8*a^2 + 5*b^2))/(8*a^(7/2
)*f*(a + b)^(5/2))
```

3.210 $\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 181

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^3} dx = \frac{b^2(48a^2+80ab+35b^2)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{9/2}(a+b)^{5/2}f} + \frac{(a-3b)\sin(e+fx)}{a^4f} - \frac{\sin^3(e+fx)}{3a^3f} + \frac{b^4\sin(e+fx)}{4a^4(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} - \frac{b^3(16a+13b)\sin(e+fx)}{8a^4(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)}$$

```
[Out] 1/8*b^2*(48*a^2+80*a*b+35*b^2)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(9
/2)/(a+b)^(5/2)/f+(a-3*b)*sin(f*x+e)/a^4/f-1/3*sin(f*x+e)^3/a^3/f+1/4*b^4*s
in(f*x+e)/a^4/(a+b)/f/(a+b-a*sin(f*x+e)^2)^2-1/8*b^3*(16*a+13*b)*sin(f*x+e)
/a^4/(a+b)^2/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used

= {4232, 398, 1171, 393, 214}

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{b^4\sin(e+fx)}{4a^4f(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2} - \frac{b^3(16a+13b)\sin(e+fx)}{8a^4f(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)} + \frac{(a-3b)\sin(e+fx)}{a^4f} - \frac{\sin^3(e+fx)}{3a^3f} + \frac{b^2(48a^2+80ab+35b^2)\,\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{9/2}f(a+b)^{5/2}}$$

```
[In] Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (b^2*(48*a^2 + 80*a*b + 35*b^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]
)/(8*a^(9/2)*(a + b)^(5/2)*f) + ((a - 3*b)*Sin[e + f*x])/(a^4*f) - Sin[e +
f*x]^3/(3*a^3*f) + (b^4*Sin[e + f*x])/(4*a^4*(a + b)*f*(a + b - a*Sin[e + f
*x]^2)^2) - (b^3*(16*a + 13*b)*Sin[e + f*x])/(8*a^4*(a + b)^2*f*(a + b - a*
Sin[e + f*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
```

```
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^4}{(a+b-ax^2)^3}\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a-3b}{a^4}-\frac{x^2}{a^3}+\frac{b^2(6a^2+8ab+3b^2)-4ab^2(3a+2b)x^2+6a^2b^2x^4}{a^4(a+b-ax^2)^3}\right)\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{(a-3b)\sin(e+fx)}{a^4f}-\frac{\sin^3(e+fx)}{3a^3f}\\
&\quad+\frac{\text{Subst}\left(\int \frac{b^2(6a^2+8ab+3b^2)-4ab^2(3a+2b)x^2+6a^2b^2x^4}{(a+b-ax^2)^3}\,dx, x, \sin(e+fx)\right)}{a^4f}\\
&= \frac{(a-3b)\sin(e+fx)}{a^4f}-\frac{\sin^3(e+fx)}{3a^3f}+\frac{b^4\sin(e+fx)}{4a^4(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2}\\
&\quad-\frac{\text{Subst}\left(\int \frac{-b^2(24a^2+32ab+11b^2)+24ab^2(a+b)x^2}{(a+b-ax^2)^2}\,dx, x, \sin(e+fx)\right)}{4a^4(a+b)f}\\
&= \frac{(a-3b)\sin(e+fx)}{a^4f}-\frac{\sin^3(e+fx)}{3a^3f}+\frac{b^4\sin(e+fx)}{4a^4(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2}\\
&\quad-\frac{b^3(16a+13b)\sin(e+fx)}{8a^4(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)}\\
&\quad+\frac{(b^2(48a^2+80ab+35b^2))\,\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{8a^4(a+b)^2f}\\
&= \frac{b^2(48a^2+80ab+35b^2)\,\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{9/2}(a+b)^{5/2}f}+\frac{(a-3b)\sin(e+fx)}{a^4f}-\frac{\sin^3(e+fx)}{3a^3f}\\
&\quad+\frac{b^4\sin(e+fx)}{4a^4(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2}-\frac{b^3(16a+13b)\sin(e+fx)}{8a^4(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.25 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.07

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{-\frac{3b^2(48a^2+80ab+35b^2)\left(\log\left(\sqrt{a+b}-\sqrt{a}\sin(e+fx)\right)-\log\left(\sqrt{a+b}+\sqrt{a}\sin(e+fx)\right)\right)}{(a+b)^{5/2}}+12\sqrt{a}\left(-12b-\frac{b^4(9a+22b+13a\cos(2(e+fx)))}{(a+b)^2(a+2b+a\cos(2(e+fx)))^2}+\right.}{48a^{9/2}f}$$

```
[In] Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((-3*b^2*(48*a^2 + 80*a*b + 35*b^2)*(Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x]
] - Log[Sqrt[a + b] + Sqrt[a]*Sin[e + f*x]]))/(a + b)^(5/2) + 12*Sqrt[a]*(-
12*b - (b^4*(9*a + 22*b + 13*a*Cos[2*(e + f*x)]))/((a + b)^2*(a + 2*b + a*C
os[2*(e + f*x)])^2) + a*(3 - (16*b^3)/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*
x)])))))*Sin[e + f*x] + 4*a^(3/2)*Sin[3*(e + f*x)])/(48*a^(9/2)*f)
```

Maple [A] (verified)

Time = 5.84 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{\frac{a\sin(fx+e)^3}{3}-\sin(fx+e)a+3\sin(fx+e)b}{a^4}-\frac{b^2\left(\frac{-\frac{ab(16a+13b)\sin(fx+e)^3}{8(a^2+2ab+b^2)}+\frac{(16a+11b)b\sin(fx+e)}{8a+8b}}{(a\sin(fx+e)^2-a-b)^2}-\frac{(48a^2+80ab+35b^2)\operatorname{arctanh}}{8(a^2+2ab+b^2)\sqrt{a}}\right.}{a^4}}{f}$
default	$\frac{-\frac{\frac{a\sin(fx+e)^3}{3}-\sin(fx+e)a+3\sin(fx+e)b}{a^4}-\frac{b^2\left(\frac{-\frac{ab(16a+13b)\sin(fx+e)^3}{8(a^2+2ab+b^2)}+\frac{(16a+11b)b\sin(fx+e)}{8a+8b}}{(a\sin(fx+e)^2-a-b)^2}-\frac{(48a^2+80ab+35b^2)\operatorname{arctanh}}{8(a^2+2ab+b^2)\sqrt{a}}\right.}{a^4}}{f}$
risch	$-\frac{ie^{3i(fx+e)}}{24a^3f}-\frac{3ie^{i(fx+e)}}{8a^3f}+\frac{3ie^{i(fx+e)}b}{2a^4f}+\frac{3ie^{-i(fx+e)}}{8a^3f}-\frac{3ie^{-i(fx+e)}b}{2a^4f}+\frac{ie^{-3i(fx+e)}}{24a^3f}+\frac{ib^3(16a^2e^{7i(fx+e)}+}{}$

```
[In] int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^4*(1/3*a*sin(f*x+e)^3-sin(f*x+e)*a+3*sin(f*x+e)*b)-b^2/a^4*((-1/8
*a*b*(16*a+13*b)/(a^2+2*a*b+b^2)*sin(f*x+e)^3+1/8*(16*a+11*b)*b/(a+b)*sin(f
*x+e))/(a*sin(f*x+e)^2-a-b)^2-1/8*(48*a^2+80*a*b+35*b^2)/(a^2+2*a*b+b^2)/(a
*(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(171) = 342$.

Time = 0.32 (sec) , antiderivative size = 856, normalized size of antiderivative = 4.73

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \left[\frac{3\left(48\,a^2b^4+80\,ab^5+35\,b^6+\left(48\,a^4b^2+80\,a^3b^3+35\,a^2b^4\right)\cos\left(fx+e\right)^4+2\left(48\,a^3b^3+80\,a^2b^4+35\,ab^5\right)\mathrm{co}\right.}{}\right.$$
$$-\frac{3\left(48\,a^2b^4+80\,ab^5+35\,b^6+\left(48\,a^4b^2+80\,a^3b^3+35\,a^2b^4\right)\cos\left(fx+e\right)^4+2\left(48\,a^3b^3+80\,a^2b^4+35\,ab^5\right)\mathrm{c}\right.}{}$$

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(48*a^2*b^4 + 80*a*b^5 + 35*b^6 + (48*a^4*b^2 + 80*a^3*b^3 + 35*a^
2*b^4)*cos(f*x + e)^4 + 2*(48*a^3*b^3 + 80*a^2*b^4 + 35*a*b^5)*cos(f*x + e)
^2)*sqrt(a^2 + a*b)*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e)
 - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(16*a^5*b^2 - 24*a^4*b^3 - 210*a^3*
b^4 - 275*a^2*b^5 - 105*a*b^6 + 8*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*cos
(f*x + e)^6 + 8*(2*a^7 - a^6*b - 15*a^5*b^2 - 19*a^4*b^3 - 7*a^3*b^4)*cos(f
*x + e)^4 + (32*a^6*b - 40*a^5*b^2 - 360*a^4*b^3 - 463*a^3*b^4 - 175*a^2*b^
5)*cos(f*x + e)^2)*sin(f*x + e))/((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*b^3)*f*
cos(f*x + e)^4 + 2*(a^9*b + 3*a^8*b^2 + 3*a^7*b^3 + a^6*b^4)*f*cos(f*x + e)
^2 + (a^8*b^2 + 3*a^7*b^3 + 3*a^6*b^4 + a^5*b^5)*f), -1/24*(3*(48*a^2*b^4 +
 80*a*b^5 + 35*b^6 + (48*a^4*b^2 + 80*a^3*b^3 + 35*a^2*b^4)*cos(f*x + e)^4
+ 2*(48*a^3*b^3 + 80*a^2*b^4 + 35*a*b^5)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*a
rctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) - (16*a^5*b^2 - 24*a^4*b^3 - 2
10*a^3*b^4 - 275*a^2*b^5 - 105*a*b^6 + 8*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b
^3)*cos(f*x + e)^6 + 8*(2*a^7 - a^6*b - 15*a^5*b^2 - 19*a^4*b^3 - 7*a^3*b^4
)*cos(f*x + e)^4 + (32*a^6*b - 40*a^5*b^2 - 360*a^4*b^3 - 463*a^3*b^4 - 175
*a^2*b^5)*cos(f*x + e)^2)*sin(f*x + e))/((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*
b^3)*f*cos(f*x + e)^4 + 2*(a^9*b + 3*a^8*b^2 + 3*a^7*b^3 + a^6*b^4)*f*cos(f
*x + e)^2 + (a^8*b^2 + 3*a^7*b^3 + 3*a^6*b^4 + a^5*b^5)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.50

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{3\,(48\,a^2b^2+80\,ab^3+35\,b^4)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(a^6+2\,a^5b+a^4b^2)\sqrt{(a+b)a}} - \frac{6\left((16\,a^2b^3+13\,ab^4)\sin(fx+e)^3-(16\,a^2b^3+27\,ab^4+11\,b^5)\sin(fx+\right.}{a^8+4\,a^7b+6\,a^6b^2+4\,a^5b^3+a^4b^4+(a^8+2\,a^7b+a^6b^2)\sin(fx+e)^4-2\,(a^8+3\,a^7b+3\,a^6b}}{48\,f}$$

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(3*(48*a^2*b^2 + 80*a*b^3 + 35*b^4)*log((a*sin(f*x + e) - sqrt((a + b
)*a))/(a*sin(f*x + e) + sqrt((a + b)*a)))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt((
a + b)*a)) - 6*((16*a^2*b^3 + 13*a*b^4)*sin(f*x + e)^3 - (16*a^2*b^3 + 27*a
*b^4 + 11*b^5)*sin(f*x + e))/(a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a^4*b
^4 + (a^8 + 2*a^7*b + a^6*b^2)*sin(f*x + e)^4 - 2*(a^8 + 3*a^7*b + 3*a^6*b^
2 + a^5*b^3)*sin(f*x + e)^2) + 16*(a*sin(f*x + e)^3 - 3*(a - 3*b)*sin(f*x +
 e))/a^4)/f
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.27

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{3\,(48\,a^2b^2+80\,ab^3+35\,b^4)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^6+2\,a^5b+a^4b^2)\sqrt{-a^2-ab}} - \frac{3\left(16\,a^2b^3\sin(fx+e)^3+13\,ab^4\sin(fx+e)^3-16\,a^2b^3\sin(fx+e)-27\,ab^4\sin(fx+e)-11\,b^5\sin\right.}{(a^6+2\,a^5b+a^4b^2)\left(a\sin(fx+e)^2-a-b\right)^2}}{24\,f}$$

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/24*(3*(48*a^2*b^2 + 80*a*b^3 + 35*b^4)*arctan(a*sin(f*x + e)/sqrt(-a^2 -
 a*b))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(-a^2 - a*b)) - 3*(16*a^2*b^3*sin(f*x
 + e)^3 + 13*a*b^4*sin(f*x + e)^3 - 16*a^2*b^3*sin(f*x + e) - 27*a*b^4*sin(
f*x + e) - 11*b^5*sin(f*x + e))/((a^6 + 2*a^5*b + a^4*b^2)*(a*sin(f*x + e)^
2 - a - b)^2) + 8*(a^6*sin(f*x + e)^3 - 3*a^6*sin(f*x + e) + 9*a^5*b*sin(f*
x + e))/a^9)/f
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.41

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{b^2\ln\left(\sqrt{a+b}+\sqrt{a}\,\sin(e+f\,x)\right)\,\left(3\,a^2+5\,a\,b+\frac{35\,b^2}{16}\right)}{a^{9/2}\,f\,(a+b)^{5/2}} - \frac{\frac{\sin(e+f\,x)\,\left(11\,b^4+16\,a\,b^3\right)}{8\,(a+b)}-\frac{\sin(e+f\,x)^3\,\left(16\,a^2\,b^3+13\,a\,b^4\right)}{8\,(a+b)^2}}{f\,\left(2\,a^5\,b-\sin(e+f\,x)^2\,\left(2\,a^6+2\,b\,a^5\right)+a^6+a^4\,b^2+a^6\sin(e+f\,x)^4\right)} - \frac{\sin(e+f\,x)^3}{3\,a^3\,f} - \frac{b^2\ln\left(\sqrt{a}\,\sin(e+f\,x)-\sqrt{a+b}\right)\,\left(48\,a^2+80\,a\,b+35\,b^2\right)}{16\,a^{9/2}\,f\,(a+b)^{5/2}} - \frac{\sin(e+f\,x)\,\left(\frac{3\,(a+b)}{a^4}-\frac{4}{a^3}\right)}{f}$$

```
[In] int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] (b^2*log((a + b)^(1/2) + a^(1/2)*sin(e + f*x))*(5*a*b + 3*a^2 + (35*b^2)/16
))/(a^(9/2)*f*(a + b)^(5/2)) - ((sin(e + f*x)*(16*a*b^3 + 11*b^4))/(8*(a +
b)) - (sin(e + f*x)^3*(13*a*b^4 + 16*a^2*b^3))/(8*(a + b)^2))/(f*(2*a^5*b -
 sin(e + f*x)^2*(2*a^5*b + 2*a^6) + a^6 + a^4*b^2 + a^6*sin(e + f*x)^4)) -
sin(e + f*x)^3/(3*a^3*f) - (b^2*log(a^(1/2)*sin(e + f*x) - (a + b)^(1/2))*(
80*a*b + 48*a^2 + 35*b^2))/(16*a^(9/2)*f*(a + b)^(5/2)) - (sin(e + f*x)*((3
*(a + b))/a^4 - 4/a^3))/f
```

3.211 $\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 214

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^3} dx = -\frac{b^3(80a^2+140ab+63b^2)\,\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{11/2}(a+b)^{5/2}f} + \frac{(a^2-3ab+6b^2)\sin(e+fx)}{a^5f} - \frac{(2a-3b)\sin^3(e+fx)}{3a^4f} + \frac{\sin^5(e+fx)}{5a^3f} - \frac{b^5\sin(e+fx)}{4a^5(a+b)f\,(a+b-a\sin^2(e+fx))^2} + \frac{b^4(20a+17b)\sin(e+fx)}{8a^5(a+b)^2f\,(a+b-a\sin^2(e+fx))}$$

```
[Out] -1/8*b^3*(80*a^2+140*a*b+63*b^2)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^
(11/2)/(a+b)^(5/2)/f+(a^2-3*a*b+6*b^2)*sin(f*x+e)/a^5/f-1/3*(2*a-3*b)*sin(f
*x+e)^3/a^4/f+1/5*sin(f*x+e)^5/a^3/f-1/4*b^5*sin(f*x+e)/a^5/(a+b)/f/(a+b-a*
sin(f*x+e)^2)^2+1/8*b^4*(20*a+17*b)*sin(f*x+e)/a^5/(a+b)^2/f/(a+b-a*sin(f*x
+e)^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used

$= \{4232, 398, 1171, 393, 214\}$

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{b^5\sin(e+fx)}{4a^5f(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2} + \frac{b^4(20a+17b)\sin(e+fx)}{8a^5f(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)} - \frac{(2a-3b)\sin^3(e+fx)}{3a^4f} + \frac{\sin^5(e+fx)}{5a^3f} - \frac{b^3(80a^2+140ab+63b^2)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{11/2}f(a+b)^{5/2}} + \frac{(a^2-3ab+6b^2)\sin(e+fx)}{a^5f}$$

```
[In] Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -1/8*(b^3*(80*a^2 + 140*a*b + 63*b^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a
 + b]])/(a^(11/2)*(a + b)^(5/2)*f) + ((a^2 - 3*a*b + 6*b^2)*Sin[e + f*x])/(
a^5*f) - ((2*a - 3*b)*Sin[e + f*x]^3)/(3*a^4*f) + Sin[e + f*x]^5/(5*a^3*f)
- (b^5*Sin[e + f*x])/(4*a^5*(a + b)*f*(a + b - a*Sin[e + f*x]^2)^2) + (b^4*
(20*a + 17*b)*Sin[e + f*x])/(8*a^5*(a + b)^2*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
```

```
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(1-x^2)^5}{(a+b-ax^2)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2-3ab+6b^2}{a^5} - \frac{(2a-3b)x^2}{a^4} + \frac{x^4}{a^3} - \frac{b^3(10a^2+15ab+6b^2)-5ab^3(4a+3b)x^2+10a^2b^3x^4}{a^5(a+b-ax^2)^3}\right) dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a^2-3ab+6b^2)\sin(e+fx)}{a^5f} - \frac{(2a-3b)\sin^3(e+fx)}{3a^4f} + \frac{\sin^5(e+fx)}{5a^3f} - \frac{\text{Subst}\left(\int \frac{b^3(10a^2+15ab+6b^2)-5ab^3(4a+3b)x^2+10a^2b^3x^4}{(a+b-ax^2)^3}\,dx, x, \sin(e+fx)\right)}{a^5f}$$

$$= \frac{(a^2-3ab+6b^2)\sin(e+fx)}{a^5f} - \frac{(2a-3b)\sin^3(e+fx)}{3a^4f} + \frac{\sin^5(e+fx)}{5a^3f} - \frac{b^5\sin(e+fx)}{4a^5(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{-b^3(40a^2+60ab+23b^2)+40ab^3(a+b)x^2}{(a+b-ax^2)^2}\,dx, x, \sin(e+fx)\right)}{4a^5(a+b)f}$$

$$= \frac{(a^2-3ab+6b^2)\sin(e+fx)}{a^5f} - \frac{(2a-3b)\sin^3(e+fx)}{3a^4f} + \frac{\sin^5(e+fx)}{5a^3f} - \frac{b^5\sin(e+fx)}{4a^5(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{b^4(20a+17b)\sin(e+fx)}{8a^5(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)} - \frac{(b^3(80a^2+140ab+63b^2))\,\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{8a^5(a+b)^2f}$$

$$= -\frac{b^3(80a^2+140ab+63b^2)\,\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{11/2}(a+b)^{5/2}f} + \frac{(a^2-3ab+6b^2)\sin(e+fx)}{a^5f} - \frac{(2a-3b)\sin^3(e+fx)}{3a^4f} + \frac{\sin^5(e+fx)}{5a^3f} - \frac{b^5\sin(e+fx)}{4a^5(a+b)f\,(a+b-a\sin^2(e+fx))^2} + \frac{b^4(20a+17b)\sin(e+fx)}{8a^5(a+b)^2f\,(a+b-a\sin^2(e+fx))}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.35 (sec) , antiderivative size = 2670, normalized size of antiderivative = 12.48

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((5*a^2 - 18*a*b + 48*b^2)*Cos[f*x]*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e
+ f*x]^6*Sin[e])/(64*a^5*f*(a + b*Sec[e + f*x]^2)^3) + ((-80*a^2*b^3 - 140*
a*b^4 - 63*b^5)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((I/128)*A
rcTan[((-I)*a*Cos[e] - I*b*Cos[e] + I*a*Cos[3*e] + I*b*Cos[3*e] + a*Sin[e]
+ b*Sin[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] +
 Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + a*Sin[3*e
] + b*Sin[3*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e -
f*x] - (2*I)*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e + f*x] +
 I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])/(a*Cos[e
] + 3*b*Cos[e] + a*Cos[3*e] + b*Cos[3*e] + a*Cos[e + 2*f*x] + a*Cos[3*e + 2
*f*x] - (3*I)*a*Sin[e] - I*b*Sin[e] - I*a*Sin[3*e] - I*b*Sin[3*e] - I*a*Sin
[e + 2*f*x] + I*a*Sin[3*e + 2*f*x])]*Cos[e])/(a^(11/2)*Sqrt[a + b]*f*Sqrt[C
os[2*e] - I*Sin[2*e]]) + (ArcTan[((-I)*a*Cos[e] - I*b*Cos[e] + I*a*Cos[3*e]
 + I*b*Cos[3*e] + a*Sin[e] + b*Sin[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sq
rt[Cos[2*e] - I*Sin[2*e]] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e
] - I*Sin[2*e]] + a*Sin[3*e] + b*Sin[3*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[
2*e] - I*Sin[2*e]]*Sin[e - f*x] - (2*I)*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] -
 I*Sin[2*e]]*Sin[e + f*x] + I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e
]]*Sin[3*e + f*x])/(a*Cos[e] + 3*b*Cos[e] + a*Cos[3*e] + b*Cos[3*e] + a*Cos
[e + 2*f*x] + a*Cos[3*e + 2*f*x] - (3*I)*a*Sin[e] - I*b*Sin[e] - I*a*Sin[3*
e] - I*b*Sin[3*e] - I*a*Sin[e + 2*f*x] + I*a*Sin[3*e + 2*f*x])]*Sin[e])/(12
8*a^(11/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((a + b)^2*(a + b*S
ec[e + f*x]^2)^3) + ((80*a^2*b^3 + 140*a*b^4 + 63*b^5)*(a + 2*b + a*Cos[2*e
 + 2*f*x])^3*Sec[e + f*x]^6*((ArcTanh[(2*(a + b)*Sin[e])/((-2*I)*a*Cos[e] -
 (2*I)*b*Cos[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Sin[2*
e]] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] - I*Sq
```

rt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e - f*x] + I*Sqrt[a]*Sqrt
[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])]*Cos[e])/(128*a^(11/2)*
Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - ((I/128)*ArcTanh[(2*(a + b)*Si
n[e])/((-2*I)*a*Cos[e] - (2*I)*b*Cos[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*
Sqrt[Cos[2*e] - I*Sin[2*e]] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2
*e] - I*Sin[2*e]] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e
 - f*x] + I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])
]*Sin[e])/(a^(11/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((a + b)^2
*(a + b*Sec[e + f*x]^2)^3) + ((-80*a^2*b^3 - 140*a*b^4 - 63*b^5)*(a + 2*b +
 a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((Cos[e]*Log[a + 2*a*Cos[2*e] + 2*b*C
os[2*e] - a*Cos[2*e + 2*f*x] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt
[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b
]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*e + f*x]])/(256*a^(11/2)*Sqrt[a + b]*f*
Sqrt[Cos[2*e] - I*Sin[2*e]]) - ((I/256)*Log[a + 2*a*Cos[2*e] + 2*b*Cos[2*e]
 - a*Cos[2*e + 2*f*x] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqr
t[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[
Cos[2*e] - I*Sin[2*e]]*Sin[2*e + f*x]]*Sin[e])/(a^(11/2)*Sqrt[a + b]*f*Sqrt
[Cos[2*e] - I*Sin[2*e]])))/((a + b)^2*(a + b*Sec[e + f*x]^2)^3) + ((80*a^2*
b^3 + 140*a*b^4 + 63*b^5)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(
(Cos[e]*Log[-a - 2*a*Cos[2*e] - 2*b*Cos[2*e] + a*Cos[2*e + 2*f*x] + (2*I)*a
*Sin[2*e] + (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[
2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*e
+ f*x]])/(256*a^(11/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - ((I/256
)*Log[-a - 2*a*Cos[2*e] - 2*b*Cos[2*e] + a*Cos[2*e + 2*f*x] + (2*I)*a*Sin[2
*e] + (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*
Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*e + f*x]
]*Sin[e])/(a^(11/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((a + b)^2
*(a + b*Sec[e + f*x]^2)^3) + ((5*a - 12*b)*Cos[3*f*x]*(a + 2*b + a*Cos[2*e
+ 2*f*x])^3*Sec[e + f*x]^6*Sin[3*e])/(384*a^4*f*(a + b*Sec[e + f*x]^2)^3) +
 (Cos[5*f*x]*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*Sin[5*e])/(640
*a^3*f*(a + b*Sec[e + f*x]^2)^3) + ((5*a^2 - 18*a*b + 48*b^2)*Cos[e]*(a + 2
*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*Sin[f*x])/(64*a^5*f*(a + b*Sec[e
+ f*x]^2)^3) + ((5*a - 12*b)*Cos[3*e]*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[
e + f*x]^6*Sin[3*f*x])/(384*a^4*f*(a + b*Sec[e + f*x]^2)^3) + (Cos[5*e]*(a
+ 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*Sin[5*f*x])/(640*a^3*f*(a + b*
Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^6*(20*a
*b^4*Sin[e + f*x] + 17*b^5*Sin[e + f*x]))/(32*a^5*(a + b)^2*f*(a + b*Sec[e
+ f*x]^2)^3) - (b^5*(a + 2*b + a*Cos[2*e + 2*f*x])*Sec[e + f*x]^5*Tan[e + f
*x])/(8*a^5*(a + b)*f*(a + b*Sec[e + f*x]^2)^3)

Maple [A] (verified)

Time = 10.27 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\frac{\frac{a^2\sin(fx+e)^5}{5}-\frac{2a^2\sin(fx+e)^3}{3}+a\sin(fx+e)^3b+\sin(fx+e)a^2-3\sin(fx+e)ab+6\sin(fx+e)b^2}{a^5}+b^3\left(\frac{-\frac{ab(20a+17b)\sin(fx+e)^3}{8(a^2+2ab+b^2)}+\frac{5(4a}{}}{(a\sin(fx+e)^2-a-}\right.}{f}$
default	$\frac{\frac{\frac{a^2\sin(fx+e)^5}{5}-\frac{2a^2\sin(fx+e)^3}{3}+a\sin(fx+e)^3b+\sin(fx+e)a^2-3\sin(fx+e)ab+6\sin(fx+e)b^2}{a^5}+b^3\left(\frac{-\frac{ab(20a+17b)\sin(fx+e)^3}{8(a^2+2ab+b^2)}+\frac{5(4a}{}}{(a\sin(fx+e)^2-a-}\right.}{f}$
risch	$-\frac{ie^{-3i(fx+e)}b}{8a^4f}-\frac{5ie^{3i(fx+e)}}{96a^3f}+\frac{3ie^{-i(fx+e)}b^2}{f\,a^5}-\frac{ie^{5i(fx+e)}}{160a^3f}-\frac{9ie^{-i(fx+e)}b}{8a^4f}+\frac{ie^{-5i(fx+e)}}{160a^3f}-\frac{ib^4(20a^2e^{7i(fx+}}{}$

```
[In] int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^5*(1/5*a^2*sin(f*x+e)^5-2/3*a^2*sin(f*x+e)^3+a*sin(f*x+e)^3*b+sin(
f*x+e)*a^2-3*sin(f*x+e)*a*b+6*sin(f*x+e)*b^2)+b^3/a^5*((-1/8*a*b*(20*a+17*b
)/(a^2+2*a*b+b^2)*sin(f*x+e)^3+5/8*(4*a+3*b)*b/(a+b)*sin(f*x+e))/(a*sin(f*x
+e)^2-a-b)^2-1/8*(80*a^2+140*a*b+63*b^2)/(a^2+2*a*b+b^2)/(a*(a+b))^(1/2)*ar
ctanh(a*sin(f*x+e)/(a*(a+b))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 485 vs. 2(202) = 404.

Time = 0.35 (sec) , antiderivative size = 995, normalized size of antiderivative = 4.65

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \left[\frac{15\left(80\,a^2b^5+140\,ab^6+63\,b^7+\left(80\,a^4b^3+140\,a^3b^4+63\,a^2b^5\right)\cos\left(fx+e\right)^4+2\left(80\,a^3b^4+140\,a^2b^5+63\,a\right.}{}\right.$$

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(15*(80*a^2*b^5 + 140*a*b^6 + 63*b^7 + (80*a^4*b^3 + 140*a^3*b^4 + 6
3*a^2*b^5)*cos(f*x + e)^4 + 2*(80*a^3*b^4 + 140*a^2*b^5 + 63*a*b^6)*cos(f*x
 + e)^2)*sqrt(a^2 + a*b)*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(f*x
 + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(24*(a^8 + 3*a^7*b + 3*a^6*b^2
 + a^5*b^3)*cos(f*x + e)^8 + 64*a^6*b^2 - 48*a^5*b^3 + 192*a^4*b^4 + 1774*a
^3*b^5 + 2415*a^2*b^6 + 945*a*b^7 + 8*(4*a^8 + 3*a^7*b - 15*a^6*b^2 - 23*a^
5*b^3 - 9*a^4*b^4)*cos(f*x + e)^6 + 8*(8*a^8 + 2*a^7*b + 21*a^6*b^2 + 131*a
```

```
^5*b^3 + 167*a^4*b^4 + 63*a^3*b^5)*cos(f*x + e)^4 + (128*a^7*b - 64*a^6*b^2
 + 360*a^5*b^3 + 3044*a^4*b^4 + 4067*a^3*b^5 + 1575*a^2*b^6)*cos(f*x + e)^2
)*sin(f*x + e))/((a^11 + 3*a^10*b + 3*a^9*b^2 + a^8*b^3)*f*cos(f*x + e)^4 +
 2*(a^10*b + 3*a^9*b^2 + 3*a^8*b^3 + a^7*b^4)*f*cos(f*x + e)^2 + (a^9*b^2 +
 3*a^8*b^3 + 3*a^7*b^4 + a^6*b^5)*f), 1/120*(15*(80*a^2*b^5 + 140*a*b^6 + 6
3*b^7 + (80*a^4*b^3 + 140*a^3*b^4 + 63*a^2*b^5)*cos(f*x + e)^4 + 2*(80*a^3*
b^4 + 140*a^2*b^5 + 63*a*b^6)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(
-a^2 - a*b)*sin(f*x + e)/(a + b)) + (24*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^
3)*cos(f*x + e)^8 + 64*a^6*b^2 - 48*a^5*b^3 + 192*a^4*b^4 + 1774*a^3*b^5 +
2415*a^2*b^6 + 945*a*b^7 + 8*(4*a^8 + 3*a^7*b - 15*a^6*b^2 - 23*a^5*b^3 - 9
*a^4*b^4)*cos(f*x + e)^6 + 8*(8*a^8 + 2*a^7*b + 21*a^6*b^2 + 131*a^5*b^3 +
167*a^4*b^4 + 63*a^3*b^5)*cos(f*x + e)^4 + (128*a^7*b - 64*a^6*b^2 + 360*a^
5*b^3 + 3044*a^4*b^4 + 4067*a^3*b^5 + 1575*a^2*b^6)*cos(f*x + e)^2)*sin(f*x
 + e))/((a^11 + 3*a^10*b + 3*a^9*b^2 + a^8*b^3)*f*cos(f*x + e)^4 + 2*(a^10*
b + 3*a^9*b^2 + 3*a^8*b^3 + a^7*b^4)*f*cos(f*x + e)^2 + (a^9*b^2 + 3*a^8*b^
3 + 3*a^7*b^4 + a^6*b^5)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.42

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\frac{15\,(80\,a^2b^3+140\,ab^4+63\,b^5)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(a^7+2\,a^6b+a^5b^2)\sqrt{(a+b)a}} - \frac{30\left((20\,a^2b^4+17\,ab^5)\sin(fx+e)^3-5\,(4\,a^2b^4+7\,ab^5+3\,b^6)\sin(fx+e\right.}{a^9+4\,a^8b+6\,a^7b^2+4\,a^6b^3+a^5b^4+(a^9+2\,a^8b+a^7b^2)\sin(fx+e)^4-2\,(a^9+3\,a^8b+3\,a^7b^2}}{240\,f}$$

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
[Out] 1/240*(15*(80*a^2*b^3 + 140*a*b^4 + 63*b^5)*log((a*sin(f*x + e) - sqrt((a +
 b)*a))/(a*sin(f*x + e) + sqrt((a + b)*a)))/((a^7 + 2*a^6*b + a^5*b^2)*sqrt
((a + b)*a)) - 30*((20*a^2*b^4 + 17*a*b^5)*sin(f*x + e)^3 - 5*(4*a^2*b^4 +
7*a*b^5 + 3*b^6)*sin(f*x + e))/(a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5
```

```
*b^4 + (a^9 + 2*a^8*b + a^7*b^2)*sin(f*x + e)^4 - 2*(a^9 + 3*a^8*b + 3*a^7*
b^2 + a^6*b^3)*sin(f*x + e)^2) + 16*(3*a^2*sin(f*x + e)^5 - 5*(2*a^2 - 3*a*
b)*sin(f*x + e)^3 + 15*(a^2 - 3*a*b + 6*b^2)*sin(f*x + e))/a^5)/f
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.27

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{15\,(80\,a^2b^3+140\,ab^4+63\,b^5)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^7+2\,a^6b+a^5b^2)\sqrt{-a^2-ab}} - \frac{15\left(20\,a^2b^4\sin(fx+e)^3+17\,ab^5\sin(fx+e)^3-20\,a^2b^4\sin(fx+e)-35\,ab^5\sin(fx+e)-15\,b^6\sin\right.}{(a^7+2\,a^6b+a^5b^2)\left(a\sin(fx+e)^2-a-b\right)^2}}{120\,f}$$

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/120*(15*(80*a^2*b^3 + 140*a*b^4 + 63*b^5)*arctan(a*sin(f*x + e)/sqrt(-a^2
 - a*b))/((a^7 + 2*a^6*b + a^5*b^2)*sqrt(-a^2 - a*b)) - 15*(20*a^2*b^4*sin(
f*x + e)^3 + 17*a*b^5*sin(f*x + e)^3 - 20*a^2*b^4*sin(f*x + e) - 35*a*b^5*s
in(f*x + e) - 15*b^6*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*(a*sin(f*x +
e)^2 - a - b)^2) + 8*(3*a^12*sin(f*x + e)^5 - 10*a^12*sin(f*x + e)^3 + 15*a
^11*b*sin(f*x + e)^3 + 15*a^12*sin(f*x + e) - 45*a^11*b*sin(f*x + e) + 90*a
^10*b^2*sin(f*x + e))/a^15)/f
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.20

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{5\,\sin(e+f\,x)\,(3\,b^5+4\,a\,b^4)}{8\,(a+b)} - \frac{\sin(e+f\,x)^3\,(20\,a^2\,b^4+17\,a\,b^5)}{8\,(a+b)^2}}{f\,\left(2\,a^6\,b-\sin\left(e+f\,x\right)^2\,\left(2\,a^7+2\,b\,a^6\right)+a^7+a^5\,b^2+a^7\,\sin\left(e+f\,x\right)^4\right)} + \frac{\sin\left(e+f\,x\right)^5}{5\,a^3\,f} + \frac{\sin\left(e+f\,x\right)^3\,\left(\frac{a+b}{a^4}-\frac{5}{3\,a^3}\right)}{f} + \frac{\sin\left(e+f\,x\right)\,\left(\frac{10}{a^3}-\frac{3\,(a+b)^2}{a^5}+\frac{3\,(a+b)\,\left(\frac{3\,(a+b)}{a^4}-\frac{5}{a^3}\right)}{a}\right)}{f} + \frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sin(e+f\,x)\,1{}\mathrm{i}}{\sqrt{a+b}}\right)\,\left(80\,a^2+140\,a\,b+63\,b^2\right)\,1{}\mathrm{i}}{8\,a^{11/2}\,f\,{\left(a+b\right)}^{5/2}}$$

```
[In] int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] ((5*sin(e + f*x)*(4*a*b^4 + 3*b^5))/(8*(a + b)) - (sin(e + f*x)^3*(17*a*b^5
 + 20*a^2*b^4))/(8*(a + b)^2))/(f*(2*a^6*b - sin(e + f*x)^2*(2*a^6*b + 2*a^
7) + a^7 + a^5*b^2 + a^7*sin(e + f*x)^4)) + sin(e + f*x)^5/(5*a^3*f) + (sin
(e + f*x)^3*((a + b)/a^4 - 5/(3*a^3)))/f + (sin(e + f*x)*(10/a^3 - (3*(a +
b)^2)/a^5 + (3*(a + b)*((3*(a + b))/a^4 - 5/a^3))/a))/f + (b^3*atan((a^(1/2
)*sin(e + f*x)*1i)/(a + b)^(1/2))*(140*a*b + 80*a^2 + 63*b^2)*1i)/(8*a^(11/
2)*f*(a + b)^(5/2))
```

3.212 $\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 142

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{(3a^2+8ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8b^{5/2}(a+b)^{5/2}f} - \frac{a\sec^2(e+fx)\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{3a(a+2b)\tan(e+fx)}{8b^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

```
[Out] 1/8*(3*a^2+8*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(5/2)/(a+b
)^(5/2)/f-1/4*a*sec(f*x+e)^2*tan(f*x+e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-3/
8*a*(a+2*b)*tan(f*x+e)/b^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4231, 424, 393, 211}

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{(3a^2+8ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8b^{5/2}f(a+b)^{5/2}} - \frac{3a(a+2b)\tan(e+fx)}{8b^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)} - \frac{a\tan(e+fx)\sec^2(e+fx)}{4bf(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}$$

```
[In] Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((3*a^2 + 8*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*b^(
5/2)*(a + b)^(5/2)*f) - (a*Sec[e + f*x]^2*Tan[e + f*x])/(4*b*(a + b)*f*(a +
 b + b*Tan[e + f*x]^2)^2) - (3*a*(a + 2*b)*Tan[e + f*x])/(8*b^2*(a + b)^2*f
*(a + b + b*Tan[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{a+4b+(3a+4b)x^2}{(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4b(a+b)f}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{3a(a+2b)\tan(e+fx)}{8b^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(3a^2+8ab+8b^2)\,\text{Subst}\left(\int\frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{8b^2(a+b)^2f}$$

$$= \frac{(3a^2+8ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8b^{5/2}(a+b)^{5/2}f} - \frac{a\sec^2(e+fx)\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{3a(a+2b)\tan(e+fx)}{8b^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.88

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{(3a^2+8ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{a\sqrt{b}(3a^2+16ab+16b^2+3a(a+2b)\cos(2(e+fx)))\sin(2(e+fx))}{(a+b)^2(a+2b+a\cos(2(e+fx)))^2}}{8b^{5/2}f}$$

```
[In] Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (((3*a^2 + 8*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a +
b)^(5/2) - (a*Sqrt[b]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f
*x)])*Sin[2*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2))/(8*b^
(5/2)*f)
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\frac{-\frac{a(5a+8b)\tan(fx+e)^3}{8b\left(a^2+2ab+b^2\right)}-\frac{(3a+8b)a\tan(fx+e)}{8b^2(a+b)}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(3a^2+8ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)b^2\sqrt{(a+b)b}}}{f}$
default	$\frac{\frac{-\frac{a(5a+8b)\tan(fx+e)^3}{8b\left(a^2+2ab+b^2\right)}-\frac{(3a+8b)a\tan(fx+e)}{8b^2(a+b)}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(3a^2+8ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)b^2\sqrt{(a+b)b}}}{f}$
risch	$-\frac{i\left(3a^3e^{6i(fx+e)}+8a^2b\,e^{6i(fx+e)}+8a\,b^2e^{6i(fx+e)}+9a^3e^{4i(fx+e)}+42a^2b\,e^{4i(fx+e)}+72a\,b^2e^{4i(fx+e)}+48b^3e^{4i(fx+e)}+9a^3\right.}{4(a+b)^2b^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2}$

```
[In] int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*((-1/8*a*(5*a+8*b)/b/(a^2+2*a*b+b^2)*tan(f*x+e)^3-1/8*(3*a+8*b)*a/b^2/(
a+b)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(3*a^2+8*a*b+8*b^2)/(a^2+2*a*b+
b^2)/b^2/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(128) = 256$.

Time = 0.32 (sec) , antiderivative size = 722, normalized size of antiderivative = 5.08

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \left[-\frac{\left((3a^4+8a^3b+8a^2b^2)\cos(fx+e)^4+3a^2b^2+8ab^3+8b^4+2(3a^3b+8a^2b^2+8ab^3)\cos(fx+e)^2\right)}{32\left((a^5b^3+3a\right.}\right.$$

$$-\frac{\left((3a^4+8a^3b+8a^2b^2)\cos(fx+e)^4+3a^2b^2+8ab^3+8b^4+2(3a^3b+8a^2b^2+8ab^3)\cos(fx+e)^2\right)\sqrt{}}{16\left((a^5b^3+3a^4b^4+3a^3b^5+a^2b^6)f\cos(fx+e\right.}$$

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(((3*a^4 + 8*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 8*a*b^3
 + 8*b^4 + 2*(3*a^3*b + 8*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-a*b - b^
2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f
*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 4*(3*(a
^4*b + 3*a^3*b^2 + 2*a^2*b^3)*cos(f*x + e)^3 + (5*a^3*b^2 + 13*a^2*b^3 + 8*
a*b^4)*cos(f*x + e))*sin(f*x + e))/((a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*
b^6)*f*cos(f*x + e)^4 + 2*(a^4*b^4 + 3*a^3*b^5 + 3*a^2*b^6 + a*b^7)*f*cos(f
*x + e)^2 + (a^3*b^5 + 3*a^2*b^6 + 3*a*b^7 + b^8)*f), -1/16*(((3*a^4 + 8*a^
3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 8*a*b^3 + 8*b^4 + 2*(3*a^3*b
+ 8*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b
)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) + 2*(3*(
a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*cos(f*x + e)^3 + (5*a^3*b^2 + 13*a^2*b^3 + 8
*a*b^4)*cos(f*x + e))*sin(f*x + e))/((a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2
*b^6)*f*cos(f*x + e)^4 + 2*(a^4*b^4 + 3*a^3*b^5 + 3*a^2*b^6 + a*b^7)*f*cos(
f*x + e)^2 + (a^3*b^5 + 3*a^2*b^6 + 3*a*b^7 + b^8)*f)]
```

Sympy [F]

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

[In] integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)

[Out] Integral(sec(e + f*x)**6/(a + b*sec(e + f*x)**2)**3, x)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.48

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \frac{\frac{(3a^2+8ab+8b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^2b^2+2ab^3+b^4)\sqrt{(a+b)b}} - \frac{(5a^2b+8ab^2)\tan(fx+e)^3+(3a^3+11a^2b+8ab^2)\tan(fx+e)}{a^4b^2+4a^3b^3+6a^2b^4+4ab^5+b^6+(a^2b^4+2ab^5+b^6)\tan(fx+e)^4+2(a^3b^3+3a^2b^4+3ab^5+b^6)\tan(fx+e)}}{8f}$$

[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")

[Out] 1/8*((3*a^2 + 8*a*b + 8*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^2*b
^2 + 2*a*b^3 + b^4)*sqrt((a + b)*b)) - ((5*a^2*b + 8*a*b^2)*tan(f*x + e)^3
+ (3*a^3 + 11*a^2*b + 8*a*b^2)*tan(f*x + e))/(a^4*b^2 + 4*a^3*b^3 + 6*a^2*b
^4 + 4*a*b^5 + b^6 + (a^2*b^4 + 2*a*b^5 + b^6)*tan(f*x + e)^4 + 2*(a^3*b^3
+ 3*a^2*b^4 + 3*a*b^5 + b^6)*tan(f*x + e)^2))/f

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.30

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3a^2+8ab+8b^2)}{(a^2b^2+2ab^3+b^4)\sqrt{ab+b^2}} - \frac{5a^2b\tan(fx+e)^3+8ab^2\tan(fx+e)^3+3a^3\tan(fx+e)+11a^2b\tan(fx+e)+8ab}{(a^2b^2+2ab^3+b^4)\left(b\tan(fx+e)^2+a+b\right)^2}}{8f}$$

[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")

[Out] 1/8*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
+ b^2)))*(3*a^2 + 8*a*b + 8*b^2)/((a^2*b^2 + 2*a*b^3 + b^4)*sqrt(a*b + b^2)
) - (5*a^2*b*tan(f*x + e)^3 + 8*a*b^2*tan(f*x + e)^3 + 3*a^3*tan(f*x + e) +
11*a^2*b*tan(f*x + e) + 8*a*b^2*tan(f*x + e))/((a^2*b^2 + 2*a*b^3 + b^4)*(
b*tan(f*x + e)^2 + a + b)^2))/f

Mupad [B] (verification not implemented)

Time = 20.04 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.05

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)}{\sqrt{a+b}}\right)\,\left(3\,a^2+8\,a\,b+8\,b^2\right)}{8\,b^{5/2}\,f\,{(a+b)}^{5/2}} - \frac{\frac{{\tan(e+f\,x)}^3\,\left(5\,a^2+8\,b\,a\right)}{8\,b\,{(a+b)}^2}+\frac{\tan(e+f\,x)\,\left(3\,a^2+8\,b\,a\right)}{8\,b^2\,(a+b)}}{f\,\left(2\,a\,b+a^2+b^2+{\tan\left(e+f\,x\right)}^2\,\left(2\,b^2+2\,a\,b\right)+b^2\,{\tan\left(e+f\,x\right)}^4\right)}$$

```
[In] int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] (atan((b^(1/2)*tan(e + f*x))/(a + b)^(1/2))*(8*a*b + 3*a^2 + 8*b^2))/(8*b^(
5/2)*f*(a + b)^(5/2)) - ((tan(e + f*x)^3*(8*a*b + 5*a^2))/(8*b*(a + b)^2) +
 (tan(e + f*x)*(8*a*b + 3*a^2))/(8*b^2*(a + b)))/(f*(2*a*b + a^2 + b^2 + ta
n(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4))
```

3.213 $\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 123

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3} dx = \frac{(a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8b^{3/2}(a+b)^{5/2}f} - \frac{a\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(a+4b)\tan(e+fx)}{8b(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

```
[Out] 1/8*(a+4*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(3/2)/(a+b)^(5/2)/f-1/
4*a*tan(f*x+e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2+1/8*(a+4*b)*tan(f*x+e)/b/(a
+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4231, 393, 205, 211}

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3} dx = \frac{(a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8b^{3/2}f(a+b)^{5/2}} + \frac{(a+4b)\tan(e+fx)}{8bf(a+b)^2\left(a+b\tan^2(e+fx)+b\right)} - \frac{a\tan(e+fx)}{4bf(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}$$

```
[In] Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*b^(3/2)*(a + b)^(
5/2)*f) - (a*Tan[e + f*x])/(4*b*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) + (
(a + 4*b)*Tan[e + f*x])/(8*b*(a + b)^2*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1+x^2}{(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{a\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}+\frac{(a+4b)\text{Subst}\left(\int \frac{1}{(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4b(a+b)f}\\
&= -\frac{a\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}+\frac{(a+4b)\tan(e+fx)}{8b(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}\\
&\quad+\frac{(a+4b)\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{8b(a+b)^2f}
\end{aligned}
$$

$$= \frac{(a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8b^{3/2}(a+b)^{5/2}f} - \frac{a\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(a+4b)\tan(e+fx)}{8b(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.13 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.30

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(-\frac{(a+4b)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a\cos}{b\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{64(a+b)^2f\,(a+b\,\mathrm{s}}$$

```
[In] Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(-(((a + 4*b)*ArcTan[(Sec[f*
x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*S
qrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2
*(Cos[2*e] - I*Sin[2*e]))/(b*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])) -
(4*(a + b)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(a*(Cos[e] - Sin[e])*(Cos[e
] + Sin[e])) + ((a + 2*b + a*Cos[2*(e + f*x)])*((a + 4*b)*Sin[2*e] - (a - 2
*b)*Sin[2*f*x]))/(b*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(64*(a + b)^2*f*
(a + b*Sec[e + f*x]^2)^3)
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\frac{\frac{(a+4b)\tan(fx+e)^3}{8a^2+16ab+8b^2}-\frac{(a-4b)\tan(fx+e)}{8(a+b)b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{(a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)b\sqrt{(a+b)b}}}{f}$
default	$\frac{\frac{\frac{(a+4b)\tan(fx+e)^3}{8a^2+16ab+8b^2}-\frac{(a-4b)\tan(fx+e)}{8(a+b)b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{(a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)b\sqrt{(a+b)b}}}{f}$
risch	$\frac{i\left(-a^3e^{6i(fx+e)}-4a^2b\,e^{6i(fx+e)}-3a^3e^{4i(fx+e)}-2a^2b\,e^{4i(fx+e)}+8a\,b^2e^{4i(fx+e)}+16b^3e^{4i(fx+e)}-3a^3e^{2i(fx+e)}+4a^2b\,e^{2i(f}\right.}{4a(a+b)^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2b}$

```
[In] int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*((1/8*(a+4*b)/(a^2+2*a*b+b^2)*tan(f*x+e)^3-1/8*(a-4*b)/(a+b)/b*tan(f*x+
e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(a+4*b)/(a^2+2*a*b+b^2)/b/((a+b)*b)^(1/2)*ar
ctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(109) = 218$.

Time = 0.32 (sec) , antiderivative size = 654, normalized size of antiderivative = 5.32

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \left[-\frac{\left((a^3+4\,a^2b)\cos(fx+e)^4+ab^2+4\,b^3+2\,(a^2b+4\,ab^2)\cos(fx+e)^2\right)\sqrt{-ab-b^2}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos}{\ }\right.}{32\,\left((a^5b^2+3\,a^4b^3+3\,a^3b^4+a^2b^5)\right.}\right.$$

$$-\frac{\left((a^3+4\,a^2b)\cos(fx+e)^4+ab^2+4\,b^3+2\,(a^2b+4\,ab^2)\cos(fx+e)^2\right)\sqrt{ab+b^2}\arctan\left(\frac{(a+2\,b)\cos(f}{2\,\sqrt{ab+b^2}\cos(fx+}\right.}{16\,\left((a^5b^2+3\,a^4b^3+3\,a^3b^4+a^2b^5)f\cos(fx+e)^4+2\,(a^4b^3+3\,a^3b\right.}$$

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(((a^3 + 4*a^2*b)*cos(f*x + e)^4 + a*b^2 + 4*b^3 + 2*(a^2*b + 4*a*b^
2)*cos(f*x + e)^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)
^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos
(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b
*cos(f*x + e)^2 + b^2)) + 4*((a^3*b - a^2*b^2 - 2*a*b^3)*cos(f*x + e)^3 - (
a^2*b^2 + 5*a*b^3 + 4*b^4)*cos(f*x + e))*sin(f*x + e))/((a^5*b^2 + 3*a^4*b^
3 + 3*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^4 + 2*(a^4*b^3 + 3*a^3*b^4 + 3*a^2*
b^5 + a*b^6)*f*cos(f*x + e)^2 + (a^3*b^4 + 3*a^2*b^5 + 3*a*b^6 + b^7)*f), -
1/16*(((a^3 + 4*a^2*b)*cos(f*x + e)^4 + a*b^2 + 4*b^3 + 2*(a^2*b + 4*a*b^2)
*cos(f*x + e)^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/
(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) + 2*((a^3*b - a^2*b^2 - 2*a*b^
3)*cos(f*x + e)^3 - (a^2*b^2 + 5*a*b^3 + 4*b^4)*cos(f*x + e))*sin(f*x + e))
/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^4 + 2*(a^4*b^3
 + 3*a^3*b^4 + 3*a^2*b^5 + a*b^6)*f*cos(f*x + e)^2 + (a^3*b^4 + 3*a^2*b^5 +
 3*a*b^6 + b^7)*f)]
```

Sympy [F]

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

```
[In] integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
[Out] Integral(sec(e + f*x)**4/(a + b*sec(e + f*x)**2)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.52

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{(a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^2b+2ab^2+b^3)\sqrt{(a+b)b}} + \frac{(ab+4b^2)\tan(fx+e)^3-(a^2-3ab-4b^2)\tan(fx+e)}{a^4b+4a^3b^2+6a^2b^3+4ab^4+b^5+(a^2b^3+2ab^4+b^5)\tan(fx+e)^4+2(a^3b^2+3a^2b^3+3ab^4+b^5)\tan(fx+e)^2}}{8f}$$

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
[Out] 1/8*((a + 4*b)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^2*b + 2*a*b^2 + b
^3)*sqrt((a + b)*b)) + ((a*b + 4*b^2)*tan(f*x + e)^3 - (a^2 - 3*a*b - 4*b^2
)*tan(f*x + e))/(a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5 + (a^2*b^3 +
 2*a*b^4 + b^5)*tan(f*x + e)^4 + 2*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*ta
n(f*x + e)^2))/f
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.33

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(a+4b)}{(a^2b+2ab^2+b^3)\sqrt{ab+b^2}} + \frac{ab\tan(fx+e)^3+4b^2\tan(fx+e)^3-a^2\tan(fx+e)+3ab\tan(fx+e)+4b^2\tan(fx+e)}{(a^2b+2ab^2+b^3)\left(b\tan(fx+e)^2+a+b\right)^2}}{8f}$$

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
[Out] 1/8*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
+ b^2)))*(a + 4*b)/((a^2*b + 2*a*b^2 + b^3)*sqrt(a*b + b^2)) + (a*b*tan(f*x
 + e)^3 + 4*b^2*tan(f*x + e)^3 - a^2*tan(f*x + e) + 3*a*b*tan(f*x + e) + 4*
b^2*tan(f*x + e))/((a^2*b + 2*a*b^2 + b^3)*(b*tan(f*x + e)^2 + a + b)^2))/f
```

Mupad [B] (verification not implemented)

Time = 19.35 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{\frac{\tan(e+f\,x)^3\,(a+4\,b)}{8\,(a+b)^2}-\frac{\tan(e+f\,x)\,(a-4\,b)}{8\,b\,(a+b)}}{f\,\left(2\,a\,b+a^2+b^2+\tan\left(e+f\,x\right)^2\,\left(2\,b^2+2\,a\,b\right)+b^2\tan\left(e+f\,x\right)^4\right)}$$
$$+\frac{\operatorname{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)}{\sqrt{a+b}}\right)\,\left(a+4\,b\right)}{8\,b^{3/2}\,f\,\left(a+b\right)^{5/2}}$$

```
[In] int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] ((tan(e + f*x)^3*(a + 4*b))/(8*(a + b)^2) - (tan(e + f*x)*(a - 4*b))/(8*b*(
a + b)))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e
 + f*x)^4)) + (atan((b^(1/2)*tan(e + f*x))/(a + b)^(1/2))*(a + 4*b))/(8*b^(
3/2)*f*(a + b)^(5/2))
```

3.214 $\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 106

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{3\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{b}(a+b)^{5/2}f} + \frac{\tan(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{3\tan(e+fx)}{8(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

```
[Out] 3/8*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/(a+b)^(5/2)/f/b^(1/2)+1/4*tan(f*
x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2+3/8*tan(f*x+e)/(a+b)^2/f/(a+b+b*tan(f*x
+e)^2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4231, 205, 211}

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{3\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{b}f(a+b)^{5/2}} + \frac{3\tan(e+fx)}{8f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)} + \frac{\tan(e+fx)}{4f(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}$$

```
[In] Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (3*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*Sqrt[b]*(a + b)^(5/2)*f)
+ Tan[e + f*x]/(4*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) + (3*Tan[e + f*x]
)/(8*(a + b)^2*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\tan(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{3\text{Subst}\left(\int \frac{1}{(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4(a+b)f} \\
&= \frac{\tan(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{3\tan(e+fx)}{8(a+b)^2 f\left(a+b+b\tan^2(e+fx)\right)} \\
&\quad + \frac{3\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{8(a+b)^2 f} \\
&= \frac{3\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{b}(a+b)^{5/2}f} + \frac{\tan(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} \\
&\quad + \frac{3\tan(e+fx)}{8(a+b)^2 f\left(a+b+b\tan^2(e+fx)\right)}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.59 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.50

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(-\frac{3\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a\cos(2(e+}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{64(a+b)}$$

[In] Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]

[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*((-3*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (4*b*(a + b)*Sec[2*e]*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/a^2 + ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[2*e]*(-((5*a^2 + 16*a*b + 8*b^2)*Sin[2*e]) + a*(5*a + 2*b)*Sin[2*f*x]))/a^2))/(64*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3)

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)}{4(a+b)\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\frac{3\tan(fx+e)}{8(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8(a+b)\sqrt{(a+b)b}}}{a+b}}{f}$
default	$\frac{\frac{\tan(fx+e)}{4(a+b)\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\frac{3\tan(fx+e)}{8(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8(a+b)\sqrt{(a+b)b}}}{a+b}}{f}$
risch	$\frac{i\left(5a^3e^{6i(fx+e)}+16a^2b\,e^{6i(fx+e)}+8a\,b^2e^{6i(fx+e)}+15a^3e^{4i(fx+e)}+46a^2b\,e^{4i(fx+e)}+56a\,b^2e^{4i(fx+e)}+16b^3e^{4i(fx+e)}+15a^3\right.}{4a^2(a+b)^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2}$

[In] int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)

[Out] 1/f*(1/4*tan(f*x+e)/(a+b)/(a+b+b*tan(f*x+e)^2)^2+3/4/(a+b)*(1/2*tan(f*x+e)/(a+b)/(a+b+b*tan(f*x+e)^2)+1/2/(a+b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(92) = 184.

Time = 0.30 (sec) , antiderivative size = 580, normalized size of antiderivative = 5.47

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \left[-\frac{3\left(a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2\right)\sqrt{-ab-b^2}\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos(fx+e)^4-2\left(3\,ab+4\,b^2\right)\cos(fx+e)^2}{a^2\cos(fx+e)}\right.}{32\left(\left(a^5b+3\,a^4b^2+3\,a^3b^3+a^2b^4\right)f\cos\left(fx+e\right)^4+\right.}\right.$$
$$-\frac{3\left(a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2\right)\sqrt{ab+b^2}\arctan\left(\frac{(a+2\,b)\cos(fx+e)^2-b}{2\sqrt{ab+b^2}\cos(fx+e)\sin(fx+e)}\right)-2\left(\left(5\,a^2b+7\right.\right.}{16\left(\left(a^5b+3\,a^4b^2+3\,a^3b^3+a^2b^4\right)f\cos\left(fx+e\right)^4+2\left(a^4b^2+3\,a^3b^3+3\,a^2b^4+ab^5\right)f\right.}$$

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a*b - b^2
)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e
)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*
x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 4*((5*a^
2*b + 7*a*b^2 + 2*b^3)*cos(f*x + e)^3 + 3*(a*b^2 + b^3)*cos(f*x + e))*sin(f
*x + e))/((a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^4 + 2*(a
^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*f*cos(f*x + e)^2 + (a^3*b^3 + 3*a^2
*b^4 + 3*a*b^5 + b^6)*f), -1/16*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)
^2 + b^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a
*b + b^2)*cos(f*x + e)*sin(f*x + e))) - 2*((5*a^2*b + 7*a*b^2 + 2*b^3)*cos(
f*x + e)^3 + 3*(a*b^2 + b^3)*cos(f*x + e))*sin(f*x + e))/((a^5*b + 3*a^4*b^
2 + 3*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^4 + 2*(a^4*b^2 + 3*a^3*b^3 + 3*a^2*
b^4 + a*b^5)*f*cos(f*x + e)^2 + (a^3*b^3 + 3*a^2*b^4 + 3*a*b^5 + b^6)*f)]
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

```
[In] integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral(sec(e + f*x)**2/(a + b*sec(e + f*x)**2)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.47

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{3\,b\tan(fx+e)^3+5\,(a+b)\tan(fx+e)}{(a^2b^2+2\,ab^3+b^4)\tan(fx+e)^4+a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4+2\,(a^3b+3\,a^2b^2+3\,ab^3+b^4)\tan(fx+e)^2}+\frac{3\,\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^2+2\,ab+b^2)\sqrt{(a+b)b}}}{8\,f}$$

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*b*tan(f*x + e)^3 + 5*(a + b)*tan(f*x + e))/((a^2*b^2 + 2*a*b^3 + b^
4)*tan(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 + 2*(a^3*b +
3*a^2*b^2 + 3*a*b^3 + b^4)*tan(f*x + e)^2) + 3*arctan(b*tan(f*x + e)/sqrt((
a + b)*b))/((a^2 + 2*a*b + b^2)*sqrt((a + b)*b)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.17

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^2+2\,ab+b^2)\sqrt{ab+b^2}}+\frac{3\,b\tan(fx+e)^3+5\,a\tan(fx+e)+5\,b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^2(a^2+2\,ab+b^2)}}{8\,f}$$

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*
b + b^2)))/((a^2 + 2*a*b + b^2)*sqrt(a*b + b^2)) + (3*b*tan(f*x + e)^3 + 5*
a*tan(f*x + e) + 5*b*tan(f*x + e))/((b*tan(f*x + e)^2 + a + b)^2*(a^2 + 2*a
*b + b^2)))/f
```

Mupad [B] (verification not implemented)

Time = 19.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.06

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{5\tan(e+f\,x)}{8\,(a+b)}+\frac{3\,b\tan(e+f\,x)^3}{8\,(a+b)^2}}{f\,\left(2\,a\,b+a^2+b^2+\tan\left(e+f\,x\right)^2\,\left(2\,b^2+2\,a\,b\right)+b^2\tan\left(e+f\,x\right)^4\right)} + \frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)}{\sqrt{a+b}}\right)}{8\,\sqrt{b}\,f\,(a+b)^{5/2}}$$

```
[In] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] ((5*tan(e + f*x))/(8*(a + b)) + (3*b*tan(e + f*x)^3)/(8*(a + b)^2))/(f*(2*a
*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4)) + (3
*atan((b^(1/2)*tan(e + f*x))/(a + b)^(1/2)))/(8*b^(1/2)*f*(a + b)^(5/2))
```

3.215 $\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 144

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx = \frac{x}{a^3}-\frac{\sqrt{b}(15a^2+20ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}f} - \frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

```
[Out] x/a^3-1/8*(15*a^2+20*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1
/2)/a^3/(a+b)^(5/2)/f-1/4*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/8
*b*(7*a+4*b)*tan(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4213, 425, 541, 536, 209, 211}

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx = \frac{x}{a^3}-\frac{b(7a+4b)\tan(e+fx)}{8a^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)} - \frac{\sqrt{b}(15a^2+20ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{5/2}} - \frac{b\tan(e+fx)}{4af(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-3),x]
```

```
[Out] x/a^3 - (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sq
rt[a + b]])/(8*a^3*(a + b)^(5/2)*f) - (b*Tan[e + f*x])/(4*a*(a + b)*f*(a +
b + b*Tan[e + f*x]^2)^2) - (b*(7*a + 4*b)*Tan[e + f*x])/(8*a^2*(a + b)^2*f*
(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
```

```
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a+b-3bx^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4a(a+b)f}$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{8a^2+9ab+4b^2-b(7a+4b)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{8a^2(a+b)^2f}$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^3f} - \frac{\left(b(15a^2+20ab+8b^2)\right)\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{8a^3(a+b)^2f}$$

$$= \frac{x}{a^3} - \frac{\sqrt{b}(15a^2+20ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}f} - \frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.65 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.31

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(8x(a+2b+a\cos(2(e+fx)))^2 + \frac{b(15a^2+20ab+8b^2)\arctan\left(\frac{\sec(fx)(\cos(2\ldots}{\ldots}\right)}{\ldots}\right.}{\ldots}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-3),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(8*x*(a + 2*b + a*Cos[2*(e +
 f*x)])^2 + (b*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin
[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(C
os[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2*(Cos[2*e] - I*Sin[2
*e]))/((a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) - (4*b^2*((a + 2*b)*S
in[2*e] - a*Sin[2*f*x]))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])) +
(b*(a + 2*b + a*Cos[2*(e + f*x)])*((9*a^2 + 28*a*b + 16*b^2)*Sin[2*e] - 3*a
*(3*a + 2*b)*Sin[2*f*x]))/((a + b)^2*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))
))/(64*a^3*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\frac{ab(7a+4b)\tan(fx+e)^3}{8a^2+16ab+8b^2}+\frac{(9a+4b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}\right)}{a^3}+\frac{\arctan(\tan(fx+e))}{a^3}}{f}$
default	$\frac{-\frac{b\left(\frac{\frac{ab(7a+4b)\tan(fx+e)^3}{8a^2+16ab+8b^2}+\frac{(9a+4b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}\right)}{a^3}+\frac{\arctan(\tan(fx+e))}{a^3}}{f}$
risch	$\frac{x}{a^3}-\frac{ib\left(9a^3e^{6i(fx+e)}+28a^2b\,e^{6i(fx+e)}+16a\,b^2e^{6i(fx+e)}+27a^3e^{4i(fx+e)}+90a^2b\,e^{4i(fx+e)}+120a\,b^2e^{4i(fx+e)}+48b^3e^{4i}\cdots\right.}{4a^3(a+b)^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+}\cdots\right.}$

```
[In] int(1/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-b/a^3*((1/8*a*b*(7*a+4*b)/(a^2+2*a*b+b^2)*tan(f*x+e)^3+1/8*(9*a+4*b)*
a/(a+b)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(15*a^2+20*a*b+8*b^2)/(a^2+2
*a*b+b^2)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^3*arcta
n(tan(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(130) = 260$.

Time = 0.32 (sec) , antiderivative size = 819, normalized size of antiderivative = 5.69

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \left[\frac{32\,(a^4+2\,a^3b+a^2b^2)fx\cos(fx+e)^4+64\,(a^3b+2\,a^2b^2+ab^3)fx\cos(fx+e)^2+32\,(a^2b^2+2\,ab^3+b^4)\cdots}{\cdots}\right.$$

[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")

[Out] [1/32*(32*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 64*(a^3*b + 2*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 32*(a^2*b^2 + 2*a*b^3 + b^4)*f*x + ((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 + 8*b^4 + 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 4*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 + 4*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*f), 1/16*(16*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 32*(a^3*b + 2*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 16*(a^2*b^2 + 2*a*b^3 + b^4)*f*x + ((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 + 8*b^4 + 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e))) - 2*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 + 4*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*f)]

Sympy [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx=\int \frac{1}{\left(a+b\sec^2\left(e+fx\right)\right)^3}\,dx$$

[In] integrate(1/(a+b*sec(f*x+e)**2)**3,x)

[Out] Integral((a + b*sec(e + f*x)**2)**(-3), x)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.60

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{(15\,a^2b+20\,ab^2+8\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^5+2\,a^4b+a^3b^2)\sqrt{(a+b)b}}+\frac{(7\,ab^2+4\,b^3)\tan(fx+e)^3+(9\,a^2b+13\,ab^2+4\,b^3)\tan(fx+e)}{a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4+(a^4b^2+2\,a^3b^3+a^2b^4)\tan(fx+e)^4+2\,(a^5b+3\,a^4b^2+3\,a^3b^3+a^2b^4}}{8\,f}$$

[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")

```
[Out] -1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/
((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)) + ((7*a*b^2 + 4*b^3)*tan(f*x +
e)^3 + (9*a^2*b + 13*a*b^2 + 4*b^3)*tan(f*x + e))/(a^6 + 4*a^5*b + 6*a^4*b^
2 + 4*a^3*b^3 + a^2*b^4 + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*tan(f*x + e)^4 +
2*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*tan(f*x + e)^2) - 8*(f*x + e)/a
^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.36

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\frac{\left(15\,a^2b+20\,ab^2+8\,b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^5+2\,a^4b+a^3b^2\right)\sqrt{ab+b^2}}+\frac{7\,ab^2\tan(fx+e)^3+4\,b^3\tan(fx+e)^3+9\,a^2b\tan(fx+e)+13\,ab^2\tan(fx}{\left(a^4+2\,a^3b+a^2b^2\right)\left(b\tan(fx+e)^2+a+b\right)^2}}{8\,f}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(a*b
 + b^2)) + (7*a*b^2*tan(f*x + e)^3 + 4*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x
 + e) + 13*a*b^2*tan(f*x + e) + 4*b^3*tan(f*x + e))/((a^4 + 2*a^3*b + a^2*b
^2)*(b*tan(f*x + e)^2 + a + b)^2) - 8*(f*x + e)/a^3)/f
```

Mupad [B] (verification not implemented)

Time = 23.59 (sec) , antiderivative size = 3271, normalized size of antiderivative = 22.72

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

```
[In] int(1/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] atan((((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10
*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e +
 f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10
*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6
*b^2)))/(2*a^3) + (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a
^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2
)))/a^3 - ((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*
a^10*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) + (tan
(e + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*
```

a^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/(2*a^3) - (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/a^3)/(((17*a*b^5)/4 + b^6 + (25*a^2*b^4)/4 + (105*a^3*b^3)/32)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(2*a^3) + (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3)*1i)/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/a^3 + (((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) + (tan(e + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(2*a^3) - (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3)*1i)/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/a^3))/(a^3*f) - ((tan(e + f*x)^3*(7*a*b^2 + 4*b^3))/(8*a^2*(a + b)^2) + (tan(e + f*x)*(9*a*b + 4*b^2))/(8*a^2*(a + b)))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4)) + (atan(((((tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) - (((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) - (tan(e + f*x)*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(512*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)*1i)/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)) + (((tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) + (((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (tan(e + f*x)*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(512*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)*1i)/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))/(((17*a*b^5)/4 + b^6 + (25*a^2*b^4)/4 + (105*a^3*b^3)/32)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) - (((tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) - (((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)/(4*a^9*b + a^10 + a^

```
6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) - (tan(e + f*x)*(-b*(a + b)^5)^(1/2)*(20*a*b
 + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^
4 + 1536*a^10*b^3 + 256*a^11*b^2))/(512*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^
3 + 6*a^6*b^2)*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b
^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 +
a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a
*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3
 + 10*a^6*b^2)) + (((tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856
*a^3*b^4 + 289*a^4*b^3))/(32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b
^2)) + (((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10
*b^2)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (tan(e + f*x)*(-
b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 +
4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(512*(4*a^7*b
+ a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b
^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b
^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*
(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^
5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2))))*(-b*(a + b)^5)^(1/2)*(20*a*b +
15*a^2 + 8*b^2)*1i)/(8*f*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3
+ 10*a^6*b^2))
```

3.216 $\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 201

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{(a-6b)x}{2a^4} + \frac{b^{3/2}(35a^2+56ab+24b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{5/2}f} + \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(2a+3b)\tan(e+fx)}{4a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(4a+3b)(a+4b)\tan(e+fx)}{8a^3(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

```
[Out] 1/2*(a-6*b)*x/a^4+1/8*b^(3/2)*(35*a^2+56*a*b+24*b^2)*arctan(b^(1/2)*tan(f*x
+e)/(a+b)^(1/2))/a^4/(a+b)^(5/2)/f+1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan
(f*x+e)^2)^2+1/4*b*(2*a+3*b)*tan(f*x+e)/a^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2+
1/8*b*(4*a+3*b)*(a+4*b)*tan(f*x+e)/a^3/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

= {4231, 425, 541, 536, 209, 211}

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{x(a-6b)}{2a^4}+\frac{b(4a+3b)(a+4b)\tan(e+fx)}{8a^3f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{b(2a+3b)\tan(e+fx)}{4a^2f(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}+\frac{b^{3/2}(35a^2+56ab+24b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^4f(a+b)^{5/2}}+\frac{\sin(e+fx)\cos(e+fx)}{2af\left(a+b\tan^2(e+fx)+b\right)^2}$$

```
[In] Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a - 6*b)*x)/(2*a^4) + (b^(3/2)*(35*a^2 + 56*a*b + 24*b^2)*ArcTan[(Sqrt[b]
*Tan[e + f*x])/Sqrt[a + b]])/(8*a^4*(a + b)^(5/2)*f) + (Cos[e + f*x]*Sin[e
+ f*x])/(2*a*f*(a + b + b*Tan[e + f*x]^2)^2) + (b*(2*a + 3*b)*Tan[e + f*x])
/(4*a^2*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) + (b*(4*a + 3*b)*(a + 4*b)*
Tan[e + f*x])/(8*a^3*(a + b)^2*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-a+b-5bx^2}{(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(2a+3b)\tan(e+fx)}{4a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-2(2a^2-4ab-3b^2)-6b(2a+3b)x^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{8a^2(a+b)f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(2a+3b)\tan(e+fx)}{4a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(4a+3b)(a+4b)\tan(e+fx)}{8a^3(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-2(4a^3-12a^2b-25ab^2-12b^3)-2b(4a+3b)(a+4b)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{16a^3(a+b)^2f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(2a+3b)\tan(e+fx)}{4a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(4a+3b)(a+4b)\tan(e+fx)}{8a^3(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(a-6b)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{2a^4f} + \frac{\left(b^2(35a^2+56ab+24b^2)\right)\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{8a^4(a+b)^2f}$$

$$= \frac{(a-6b)x}{2a^4} + \frac{b^{3/2}(35a^2+56ab+24b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{5/2}f} + \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(2a+3b)\tan(e+fx)}{4a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(4a+3b)(a+4b)\tan(e+fx)}{8a^3(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [A] (verified)

Time = 4.49 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.78

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{4(a-6b)(e+fx) + \frac{b^{3/2}(35a^2+56ab+24b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + a\left(2 + \frac{13ab^2}{(a+b)^2(a+2b+a\cos(2(e+fx)))} + \frac{2b^3(3a+8b+5a\cos}{(a+b)^2(a+2b+a\cos}\right.}{8a^4f}$$

```
[In] Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (4*(a - 6*b)*(e + f*x) + (b^(3/2)*(35*a^2 + 56*a*b + 24*b^2)*ArcTan[(Sqrt[b
]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + a*(2 + (13*a*b^2)/((a + b)^2*
(a + 2*b + a*Cos[2*(e + f*x)])) + (2*b^3*(3*a + 8*b + 5*a*Cos[2*(e + f*x)])
)/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2))*Sin[2*(e + f*x)])/(8*a^4*f)
```

Maple [A] (verified)

Time = 4.40 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{\frac{b^2\left(\frac{\frac{ab(11a+8b)\tan(fx+e)^3}{8a^2+16ab+8b^2}+\frac{(13a+8b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(35a^2+56ab+24b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}\right)}{a^4}+\frac{\frac{a\tan(fx+e)}{2+2\tan(fx+e)^2}+\frac{(a-6b)\arctan}{}}{a^4}}{f}$
default	$\frac{\frac{b^2\left(\frac{\frac{ab(11a+8b)\tan(fx+e)^3}{8a^2+16ab+8b^2}+\frac{(13a+8b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(35a^2+56ab+24b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}\right)}{a^4}+\frac{\frac{a\tan(fx+e)}{2+2\tan(fx+e)^2}+\frac{(a-6b)\arctan}{}}{a^4}}{f}$
risch	$\frac{x}{2a^3}-\frac{3xb}{a^4}-\frac{ie^{2i(fx+e)}}{8a^3f}+\frac{ie^{-2i(fx+e)}}{8a^3f}+\frac{ib^2\left(13a^3e^{6i(fx+e)}+40a^2b\,e^{6i(fx+e)}+24a\,b^2e^{6i(fx+e)}+39a^3e^{4i(fx+e)}+13\right.}{4a^4(a+}$

[In] int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)

[Out] 1/f*(b^2/a^4*((1/8*a*b*(11*a+8*b)/(a^2+2*a*b+b^2)*tan(f*x+e)^3+1/8*(13*a+8*b)*a/(a+b)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(35*a^2+56*a*b+24*b^2)/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^4*(1/2*a*tan(f*x+e)/(1+tan(f*x+e)^2)+1/2*(a-6*b)*arctan(tan(f*x+e))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(183) = 366.

Time = 0.33 (sec) , antiderivative size = 970, normalized size of antiderivative = 4.83

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")

[Out] [1/32*(16*(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*f*x*cos(f*x + e)^4 + 32*(a^4*b - 4*a^3*b^2 - 11*a^2*b^3 - 6*a*b^4)*f*x*cos(f*x + e)^2 + 16*(a^3*b^2 - 4*a^2*b^3 - 11*a*b^4 - 6*b^5)*f*x + (35*a^2*b^3 + 56*a*b^4 + 24*b^5 + (35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cos(f*x + e)^4 + 2*(35*a^3*b^2 + 56*a^2*b^3 + 24*a*b^4)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 4*(4*(a^5 + 2*a^4*b + a^3*b^2)*cos(f*x + e)^5 + (8*a^4*b + 29*a^3*b^2 + 18*a^2*b^3)*cos(f*x + e)^3 + (4*a^3*b^2 + 19*a^2*b^3 + 12*a*b^4)*cos(f*x + e))*sin(f*x + e))/((a^8 + 2*a^7*b + a^6*b^2)*f*cos(f*x + e)^4 + 2*(a^7*b + 2*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^2 + (a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*f), 1/16*(8*(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*f*x*cos(f*x + e)^4 + 16*(a^4*b - 4*a^3*b^2 - 11*a^2*b^3 - 6*a*b^4)*f*x*cos(f*x + e)^2 + 8*(a^3*b^2 - 4*a^2*b^3 - 11*a*b^4 - 6*b^5)*f*x - (35*a^2*b^3 + 56*a*b^4 + 24*b^5 + (35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cos(f*x + e)^4 + 2*(35*a^3*b^2 + 56*a^2*b^3 + 24*a*b^4)*cos

```
(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt
(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e))) + 2*(4*(a^5 + 2*a^4*b + a^3*b^2)
*cos(f*x + e)^5 + (8*a^4*b + 29*a^3*b^2 + 18*a^2*b^3)*cos(f*x + e)^3 + (4*a
^3*b^2 + 19*a^2*b^3 + 12*a*b^4)*cos(f*x + e))*sin(f*x + e))/((a^8 + 2*a^7*b
 + a^6*b^2)*f*cos(f*x + e)^4 + 2*(a^7*b + 2*a^6*b^2 + a^5*b^3)*f*cos(f*x +
e)^2 + (a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.68

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{(35\,a^2b^2+56\,ab^3+24\,b^4)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^6+2\,a^5b+a^4b^2)\sqrt{(a+b)b}} + \frac{(4\,a^2b^2+19\,ab^3+12\,b^4)\tan(fx+e)^5+(8\,a^3b+37\,a^2b^2+56\,ab^3+24\,b^4)\tan(fx+e}{a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4+(a^5b^2+2\,a^4b^3+a^3b^4)\tan(fx+e)^6+(2\,a^6b+7\,a^5b^2+8\,a^4b^3+3}}{8\,f}$$

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((35*a^2*b^2 + 56*a*b^3 + 24*b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b)
)/((a^6 + 2*a^5*b + a^4*b^2)*sqrt((a + b)*b)) + ((4*a^2*b^2 + 19*a*b^3 + 12
*b^4)*tan(f*x + e)^5 + (8*a^3*b + 37*a^2*b^2 + 56*a*b^3 + 24*b^4)*tan(f*x +
 e)^3 + (4*a^4 + 16*a^3*b + 37*a^2*b^2 + 37*a*b^3 + 12*b^4)*tan(f*x + e))/(
a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4 + (a^5*b^2 + 2*a^4*b^3 + a^
3*b^4)*tan(f*x + e)^6 + (2*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 + 3*a^3*b^4)*tan(f
*x + e)^4 + (a^7 + 6*a^6*b + 12*a^5*b^2 + 10*a^4*b^3 + 3*a^3*b^4)*tan(f*x +
 e)^2) + 4*(f*x + e)*(a - 6*b)/a^4)/f
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.13

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{\frac{(35\,a^2b^2+56\,ab^3+24\,b^4)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^6+2\,a^5b+a^4b^2)\sqrt{ab+b^2}} + \frac{11\,ab^3\tan(fx+e)^3+8\,b^4\tan(fx+e)^3+13\,a^2b^2\tan(fx+e)+21\,ab^3\tan(fx}{(a^5+2\,a^4b+a^3b^2)\left(b\tan(fx+e)^2+a+b\right)^2}}{8\,f}$$

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((35*a^2*b^2 + 56*a*b^3 + 24*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b)
+ arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(a
*b + b^2)) + (11*a*b^3*tan(f*x + e)^3 + 8*b^4*tan(f*x + e)^3 + 13*a^2*b^2*t
an(f*x + e) + 21*a*b^3*tan(f*x + e) + 8*b^4*tan(f*x + e))/((a^5 + 2*a^4*b +
 a^3*b^2)*(b*tan(f*x + e)^2 + a + b)^2) + 4*(f*x + e)*(a - 6*b)/a^4 + 4*tan
(f*x + e)/((tan(f*x + e)^2 + 1)*a^3))/f
```

Mupad [B] (verification not implemented)

Time = 24.10 (sec) , antiderivative size = 3708, normalized size of antiderivative = 18.45

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] ((tan(e + f*x)^5*(19*a*b^3 + 12*b^4 + 4*a^2*b^2))/(8*a^3*(a + b)^2) + (tan(
e + f*x)*(25*a*b^2 + 12*a^2*b + 4*a^3 + 12*b^3))/(8*a^3*(a + b)) + (b*tan(e
 + f*x)^3*(56*a*b^2 + 37*a^2*b + 8*a^3 + 24*b^3))/(8*a^3*(a + b)^2))/(f*(2*
a*b + tan(e + f*x)^2*(4*a*b + a^2 + 3*b^2) + a^2 + b^2 + tan(e + f*x)^4*(2*
a*b + 3*b^2) + b^2*tan(e + f*x)^6)) + (atan((((((((6*a^8*b^7 + (49*a^9*b^6)/
2 + 37*a^10*b^5 + (45*a^11*b^4)/2 + 2*a^12*b^3 - 2*a^13*b^2)/(4*a^12*b + a^
13 + a^9*b^4 + 4*a^10*b^3 + 6*a^11*b^2) - (tan(e + f*x)*(a*1i - b*6i)*(512*
a^8*b^7 + 2304*a^9*b^6 + 4096*a^10*b^5 + 3584*a^11*b^4 + 1536*a^12*b^3 + 25
6*a^13*b^2))/(128*a^4*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)))*
(a*1i - b*6i))/(4*a^4) - (tan(e + f*x)*(4800*a*b^8 + 1152*b^9 + 7520*a^2*b^
7 + 5136*a^3*b^6 + 1129*a^4*b^5 - 128*a^5*b^4 + 16*a^6*b^3))/(32*(4*a^9*b +
 a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)))*(a*1i - b*6i)*1i)/(4*a^4) - ((((
(6*a^8*b^7 + (49*a^9*b^6)/2 + 37*a^10*b^5 + (45*a^11*b^4)/2 + 2*a^12*b^3 -
2*a^13*b^2)/(4*a^12*b + a^13 + a^9*b^4 + 4*a^10*b^3 + 6*a^11*b^2) + (tan(e
+ f*x)*(a*1i - b*6i)*(512*a^8*b^7 + 2304*a^9*b^6 + 4096*a^10*b^5 + 3584*a^1
```

1*b^4 + 1536*a^12*b^3 + 256*a^13*b^2))/(128*a^4*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)))*(a*1i - b*6i))/(4*a^4) + (tan(e + f*x)*(4800*a*b^8 + 1152*b^9 + 7520*a^2*b^7 + 5136*a^3*b^6 + 1129*a^4*b^5 - 128*a^5*b^4 + 16*a^6*b^3))/(32*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)))*(a*1i - b*6i)*1i)/(4*a^4))/(((405*a*b^8)/4 + 27*b^9 + (261*a^2*b^7)/2 + (1877*a^3*b^6)/32 - (49*a^4*b^5)/64 - (35*a^5*b^4)/16)/(4*a^12*b + a^13 + a^9*b^4 + 4*a^10*b^3 + 6*a^11*b^2) + (((((6*a^8*b^7 + (49*a^9*b^6)/2 + 37*a^10*b^5 + (45*a^11*b^4)/2 + 2*a^12*b^3 - 2*a^13*b^2)/(4*a^12*b + a^13 + a^9*b^4 + 4*a^10*b^3 + 6*a^11*b^2) - (tan(e + f*x)*(a*1i - b*6i)*(512*a^8*b^7 + 2304*a^9*b^6 + 4096*a^10*b^5 + 3584*a^11*b^4 + 1536*a^12*b^3 + 256*a^13*b^2))/(128*a^4*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)))*(a*1i - b*6i))/(4*a^4) - (tan(e + f*x)*(4800*a*b^8 + 1152*b^9 + 7520*a^2*b^7 + 5136*a^3*b^6 + 1129*a^4*b^5 - 128*a^5*b^4 + 16*a^6*b^3))/(32*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)))*(a*1i - b*6i))/(4*a^4) + (((((6*a^8*b^7 + (49*a^9*b^6)/2 + 37*a^10*b^5 + (45*a^11*b^4)/2 + 2*a^12*b^3 - 2*a^13*b^2)/(4*a^12*b + a^13 + a^9*b^4 + 4*a^10*b^3 + 6*a^11*b^2) + (tan(e + f*x)*(a*1i - b*6i)*(512*a^8*b^7 + 2304*a^9*b^6 + 4096*a^10*b^5 + 3584*a^11*b^4 + 1536*a^12*b^3 + 256*a^13*b^2))/(128*a^4*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)))*(a*1i - b*6i))/(4*a^4) + (tan(e + f*x)*(4800*a*b^8 + 1152*b^9 + 7520*a^2*b^7 + 5136*a^3*b^6 + 1129*a^4*b^5 - 128*a^5*b^4 + 16*a^6*b^3))/(32*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)))*(a*1i - b*6i))/(4*a^4)))*(a*1i - b*6i)*1i)/(2*a^4*f) - (atan(((((tan(e + f*x)*(4800*a*b^8 + 1152*b^9 + 7520*a^2*b^7 + 5136*a^3*b^6 + 1129*a^4*b^5 - 128*a^5*b^4 + 16*a^6*b^3))/(32*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (((6*a^8*b^7 + (49*a^9*b^6)/2 + 37*a^10*b^5 + (45*a^11*b^4)/2 + 2*a^12*b^3 - 2*a^13*b^2)/(4*a^12*b + a^13 + a^9*b^4 + 4*a^10*b^3 + 6*a^11*b^2) - (tan(e + f*x)*(-b^3*(a + b)^5)^(1/2)*(56*a*b + 35*a^2 + 24*b^2)*(512*a^8*b^7 + 2304*a^9*b^6 + 4096*a^10*b^5 + 3584*a^11*b^4 + 1536*a^12*b^3 + 256*a^13*b^2))/(512*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)*(5*a^8*b + a^9 + a^4*b^5 + 5*a^5*b^4 + 10*a^6*b^3 + 10*a^7*b^2)))*(-b^3*(a + b)^5)^(1/2)*(56*a*b + 35*a^2 + 24*b^2))/(16*(5*a^8*b + a^9 + a^4*b^5 + 5*a^5*b^4 + 10*a^6*b^3 + 10*a^7*b^2)))*(-b^3*(a + b)^5)^(1/2)*(56*a*b + 35*a^2 + 24*b^2)*1i)/(16*(5*a^8*b + a^9 + a^4*b^5 + 5*a^5*b^4 + 10*a^6*b^3 + 10*a^7*b^2)) + (((tan(e + f*x)*(4800*a*b^8 + 1152*b^9 + 7520*a^2*b^7 + 5136*a^3*b^6 + 1129*a^4*b^5 - 128*a^5*b^4 + 16*a^6*b^3))/(32*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) + (((6*a^8*b^7 + (49*a^9*b^6)/2 + 37*a^10*b^5 + (45*a^11*b^4)/2 + 2*a^12*b^3 - 2*a^13*b^2)/(4*a^12*b + a^13 + a^9*b^4 + 4*a^10*b^3 + 6*a^11*b^2) + (tan(e + f*x)*(-b^3*(a + b)^5)^(1/2)*(56*a*b + 35*a^2 + 24*b^2)*(512*a^8*b^7 + 2304*a^9*b^6 + 4096*a^10*b^5 + 3584*a^11*b^4 + 1536*a^12*b^3 + 256*a^13*b^2))/(512*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)*(5*a^8*b + a^9 + a^4*b^5 + 5*a^5*b^4 + 10*a^6*b^3 + 10*a^7*b^2)))*(-b^3*(a + b)^5)^(1/2)*(56*a*b + 35*a^2 + 24*b^2))/(16*(5*a^8*b + a^9 + a^4*b^5 + 5*a^5*b^4 + 10*a^6*b^3 + 10*a^7*b^2)))*(-b^3*(a + b)^5)^(1/2)*(56*a*b + 35*a^2 + 24*b^2)*1i)/(16*(5*a^8*b + a^9 + a^4*b^5 + 5*a^5*b^4 + 10*a^6*b^3 + 10*a^7*b^2)))/(((405*a*b^8)/4 + 27*b^9 + (261*a^2*b^7)/2 + (1877*a^3*b^6)/32 - (49*a^4*b^5)/64 - (

35*a^5*b^4)/16)/(4*a^12*b + a^13 + a^9*b^4 + 4*a^10*b^3 + 6*a^11*b^2) - (((tan(e + f*x)*(4800*a*b^8 + 1152*b^9 + 7520*a^2*b^7 + 5136*a^3*b^6 + 1129*a^4*b^5 - 128*a^5*b^4 + 16*a^6*b^3))/(32*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (((6*a^8*b^7 + (49*a^9*b^6)/2 + 37*a^10*b^5 + (45*a^11*b^4)/2 + 2*a^12*b^3 - 2*a^13*b^2)/(4*a^12*b + a^13 + a^9*b^4 + 4*a^10*b^3 + 6*a^11*b^2) - (tan(e + f*x)*(-b^3*(a + b)^5)^(1/2)*(56*a*b + 35*a^2 + 24*b^2)*(512*a^8*b^7 + 2304*a^9*b^6 + 4096*a^10*b^5 + 3584*a^11*b^4 + 1536*a^12*b^3 + 256*a^13*b^2))/(512*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)*(5*a^8*b + a^9 + a^4*b^5 + 5*a^5*b^4 + 10*a^6*b^3 + 10*a^7*b^2)))*(-b^3*(a + b)^5)^(1/2)*(56*a*b + 35*a^2 + 24*b^2))/(16*(5*a^8*b + a^9 + a^4*b^5 + 5*a^5*b^4 + 10*a^6*b^3 + 10*a^7*b^2)))*(-b^3*(a + b)^5)^(1/2)*(56*a*b + 35*a^2 + 24*b^2))/(16*(5*a^8*b + a^9 + a^4*b^5 + 5*a^5*b^4 + 10*a^6*b^3 + 10*a^7*b^2)) + (((tan(e + f*x)*(4800*a*b^8 + 1152*b^9 + 7520*a^2*b^7 + 5136*a^3*b^6 + 1129*a^4*b^5 - 128*a^5*b^4 + 16*a^6*b^3))/(32*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) + (((6*a^8*b^7 + (49*a^9*b^6)/2 + 37*a^10*b^5 + (45*a^11*b^4)/2 + 2*a^12*b^3 - 2*a^13*b^2)/(4*a^12*b + a^13 + a^9*b^4 + 4*a^10*b^3 + 6*a^11*b^2) + (tan(e + f*x)*(-b^3*(a + b)^5)^(1/2)*(56*a*b + 35*a^2 + 24*b^2)*(512*a^8*b^7 + 2304*a^9*b^6 + 4096*a^10*b^5 + 3584*a^11*b^4 + 1536*a^12*b^3 + 256*a^13*b^2))/(512*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)*(5*a^8*b + a^9 + a^4*b^5 + 5*a^5*b^4 + 10*a^6*b^3 + 10*a^7*b^2)))*(-b^3*(a + b)^5)^(1/2)*(56*a*b + 35*a^2 + 24*b^2))/(16*(5*a^8*b + a^9 + a^4*b^5 + 5*a^5*b^4 + 10*a^6*b^3 + 10*a^7*b^2)))*(-b^3*(a + b)^5)^(1/2)*(56*a*b + 35*a^2 + 24*b^2))/(16*(5*a^8*b + a^9 + a^4*b^5 + 5*a^5*b^4 + 10*a^6*b^3 + 10*a^7*b^2))))*(-b^3*(a + b)^5)^(1/2)*(56*a*b + 35*a^2 + 24*b^2)*1i)/(8*f*(5*a^8*b + a^9 + a^4*b^5 + 5*a^5*b^4 + 10*a^6*b^3 + 10*a^7*b^2))

3.217 $\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 269

$$\begin{aligned}\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx &= \frac{3(a^2-4ab+16b^2)\,x}{8a^5} \\ &- \frac{3b^{5/2}(21a^2+36ab+16b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^5(a+b)^{5/2}f} \\ &+ \frac{(3a-8b)\cos(e+fx)\sin(e+fx)}{8a^2f\,(a+b+b\tan^2(e+fx))^2} \\ &+ \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))^2} \\ &+ \frac{b(3a^2-7ab-12b^2)\tan(e+fx)}{8a^3(a+b)f\,(a+b+b\tan^2(e+fx))^2} \\ &+ \frac{3b(a+2b)\,(a^2-4ab-4b^2)\tan(e+fx)}{8a^4(a+b)^2f\,(a+b+b\tan^2(e+fx))}\end{aligned}$$

```
[Out] 3/8*(a^2-4*a*b+16*b^2)*x/a^5-3/8*b^(5/2)*(21*a^2+36*a*b+16*b^2)*arctan(b^(1
/2)*tan(f*x+e)/(a+b)^(1/2))/a^5/(a+b)^(5/2)/f+1/8*(3*a-8*b)*cos(f*x+e)*sin(
f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*
tan(f*x+e)^2)^2+1/8*b*(3*a^2-7*a*b-12*b^2)*tan(f*x+e)/a^3/(a+b)/f/(a+b+b*ta
n(f*x+e)^2)^2+3/8*b*(a+2*b)*(a^2-4*a*b-4*b^2)*tan(f*x+e)/a^4/(a+b)^2/f/(a+b
+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4231, 425, 541, 536, 209, 211}

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{(3a-8b)\sin(e+fx)\cos(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)+b\right)^2} - \frac{3b^{5/2}(21a^2+36ab+16b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^5f(a+b)^{5/2}} + \frac{3x(a^2-4ab+16b^2)}{8a^5} + \frac{3b(a+2b)\left(a^2-4ab-4b^2\right)\tan(e+fx)}{8a^4f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)} + \frac{b(3a^2-7ab-12b^2)\tan(e+fx)}{8a^3f(a+b)\left(a+b\tan^2(e+fx)+b\right)^2} + \frac{\sin(e+fx)\cos^3(e+fx)}{4af\left(a+b\tan^2(e+fx)+b\right)^2}$$

```
[In] Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (3*(a^2 - 4*a*b + 16*b^2)*x)/(8*a^5) - (3*b^(5/2)*(21*a^2 + 36*a*b + 16*b^2
)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*a^5*(a + b)^(5/2)*f) + ((3
*a - 8*b)*Cos[e + f*x]*Sin[e + f*x])/(8*a^2*f*(a + b + b*Tan[e + f*x]^2)^2)
 + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f*(a + b + b*Tan[e + f*x]^2)^2) + (b*
(3*a^2 - 7*a*b - 12*b^2)*Tan[e + f*x])/(8*a^3*(a + b)*f*(a + b + b*Tan[e +
f*x]^2)^2) + (3*b*(a + 2*b)*(a^2 - 4*a*b - 4*b^2)*Tan[e + f*x])/(8*a^4*(a +
 b)^2*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
```

```
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))^2} - \frac{\text{Subst}\left(\int \frac{-3a+b-7bx^2}{(1+x^2)^2(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{4af} \\
&= \frac{(3a-8b)\cos(e+fx)\sin(e+fx)}{8a^2f\,(a+b+b\tan^2(e+fx))^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))^2} \\
&\quad + \frac{\text{Subst}\left(\int \frac{3a^2+3ab+8b^2+5(3a-8b)bx^2}{(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{8a^2f}
\end{aligned}$$

$$= \frac{(3a-8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(3a^2-7ab-12b^2)\tan(e+fx)}{8a^3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{12(a^3+5ab^2+4b^3)+12b(3a^2-7ab-12b^2)x^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{32a^3(a+b)f}$$

$$= \frac{(3a-8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(3a^2-7ab-12b^2)\tan(e+fx)}{8a^3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{3b(a+2b)\left(a^2-4ab-4b^2\right)\tan(e+fx)}{8a^4(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{24(a^4-a^3b+7a^2b^2+16ab^3+8b^4)+24b(a+2b)(a^2-4ab-4b^2)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{64a^4(a+b)^2f}$$

$$= \frac{(3a-8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(3a^2-7ab-12b^2)\tan(e+fx)}{8a^3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{3b(a+2b)\left(a^2-4ab-4b^2\right)\tan(e+fx)}{8a^4(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(3(a^2-4ab+16b^2))\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{8a^5f} - \frac{(3b^3(21a^2+36ab+16b^2))\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{8a^5(a+b)^2f}$$

$$= \frac{3(a^2-4ab+16b^2)\,x}{8a^5} - \frac{3b^{5/2}(21a^2+36ab+16b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^5(a+b)^{5/2}f} + \frac{(3a-8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(3a^2-7ab-12b^2)\tan(e+fx)}{8a^3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{3b(a+2b)\left(a^2-4ab-4b^2\right)\tan(e+fx)}{8a^4(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.96 (sec) , antiderivative size = 1430, normalized size of antiderivative = 5.32

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \frac{(21a^2+36ab+16b^2)\,(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(\frac{3b^3\arctan\left(\sec(fx)\left(\frac{\cos(2e)}{2\sqrt{a+b}\sqrt{b\cos(4e)-ib\sin(4e)}}-\frac{}{2\sqrt{a+b}}\right.\right.}{64a^5\sqrt{a+}}\right.}{}$$

$$+\frac{(a+2b+a\cos(2e+2fx))\sec(2e)\sec^6(e+fx)\,(144a^6fx\cos(2e)+96a^5bfx\cos(2e)+912a^4b^2fx\cos(2}{}$$

```
[In] Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((21*a^2 + 36*a*b + 16*b^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6
*((3*b^3*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin
[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(
-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(64*a^5*Sqrt[a
+ b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - (((3*I)/64)*b^3*ArcTan[Sec[f*x]*(
Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])
/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*
x] + a*Sin[2*e + f*x])]*Sin[2*e])/(a^5*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*
Sin[4*e]])))/((a + b)^2*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e +
 2*f*x])*Sec[2*e]*Sec[e + f*x]^6*(144*a^6*f*x*Cos[2*e] + 96*a^5*b*f*x*Cos[2
*e] + 912*a^4*b^2*f*x*Cos[2*e] + 6720*a^3*b^3*f*x*Cos[2*e] + 16512*a^2*b^4*
f*x*Cos[2*e] + 16896*a*b^5*f*x*Cos[2*e] + 6144*b^6*f*x*Cos[2*e] + 96*a^6*f*
x*Cos[2*f*x] + 480*a^4*b^2*f*x*Cos[2*f*x] + 4416*a^3*b^3*f*x*Cos[2*f*x] + 6
912*a^2*b^4*f*x*Cos[2*f*x] + 3072*a*b^5*f*x*Cos[2*f*x] + 96*a^6*f*x*Cos[4*e
 + 2*f*x] + 480*a^4*b^2*f*x*Cos[4*e + 2*f*x] + 4416*a^3*b^3*f*x*Cos[4*e + 2
*f*x] + 6912*a^2*b^4*f*x*Cos[4*e + 2*f*x] + 3072*a*b^5*f*x*Cos[4*e + 2*f*x]
 + 24*a^6*f*x*Cos[2*e + 4*f*x] - 48*a^5*b*f*x*Cos[2*e + 4*f*x] + 216*a^4*b^
2*f*x*Cos[2*e + 4*f*x] + 672*a^3*b^3*f*x*Cos[2*e + 4*f*x] + 384*a^2*b^4*f*x
*Cos[2*e + 4*f*x] + 24*a^6*f*x*Cos[6*e + 4*f*x] - 48*a^5*b*f*x*Cos[6*e + 4*
f*x] + 216*a^4*b^2*f*x*Cos[6*e + 4*f*x] + 672*a^3*b^3*f*x*Cos[6*e + 4*f*x]
+ 384*a^2*b^4*f*x*Cos[6*e + 4*f*x] + 816*a^3*b^3*Sin[2*e] + 2848*a^2*b^4*Si
n[2*e] + 3968*a*b^5*Sin[2*e] + 1792*b^6*Sin[2*e] + 44*a^6*Sin[2*f*x] + 104*
a^5*b*Sin[2*f*x] - 180*a^4*b^2*Sin[2*f*x] - 1696*a^3*b^3*Sin[2*f*x] - 3264*
a^2*b^4*Sin[2*f*x] - 1664*a*b^5*Sin[2*f*x] + 44*a^6*Sin[4*e + 2*f*x] + 104*
a^5*b*Sin[4*e + 2*f*x] - 180*a^4*b^2*Sin[4*e + 2*f*x] - 608*a^3*b^3*Sin[4*e
 + 2*f*x] - 192*a^2*b^4*Sin[4*e + 2*f*x] + 128*a*b^5*Sin[4*e + 2*f*x] + 38*
a^6*Sin[2*e + 4*f*x] + 60*a^5*b*Sin[2*e + 4*f*x] - 170*a^4*b^2*Sin[2*e + 4*
f*x] - 640*a^3*b^3*Sin[2*e + 4*f*x] - 400*a^2*b^4*Sin[2*e + 4*f*x] + 38*a^6
*Sin[6*e + 4*f*x] + 60*a^5*b*Sin[6*e + 4*f*x] - 170*a^4*b^2*Sin[6*e + 4*f*x
] - 368*a^3*b^3*Sin[6*e + 4*f*x] - 176*a^2*b^4*Sin[6*e + 4*f*x] + 12*a^6*Si
n[4*e + 6*f*x] + 8*a^5*b*Sin[4*e + 6*f*x] - 20*a^4*b^2*Sin[4*e + 6*f*x] - 1
```

```
6*a^3*b^3*Sin[4*e + 6*f*x] + 12*a^6*Sin[8*e + 6*f*x] + 8*a^5*b*Sin[8*e + 6*
f*x] - 20*a^4*b^2*Sin[8*e + 6*f*x] - 16*a^3*b^3*Sin[8*e + 6*f*x] + a^6*Sin[
6*e + 8*f*x] + 2*a^5*b*Sin[6*e + 8*f*x] + a^4*b^2*Sin[6*e + 8*f*x] + a^6*Si
n[10*e + 8*f*x] + 2*a^5*b*Sin[10*e + 8*f*x] + a^4*b^2*Sin[10*e + 8*f*x]))/(
2048*a^5*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A] (verified)

Time = 6.90 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{\frac{\left(\frac{3}{8}a^2-\frac{3}{2}ab\right)\tan(fx+e)^3+\left(-\frac{3}{2}ab+\frac{5}{8}a^2\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{3\left(a^2-4ab+16b^2\right)\arctan(\tan(fx+e))}{8}}{a^5}-\frac{b^3\left(\frac{\frac{3ab(5a+4b)\tan(fx+e)^3}{8\left(a^2+2ab+b^2\right)}+\frac{(17a+12}{}}{\left(a+b+b\tan(fx+e)^2\right)}\right.}{f}$
default	$\frac{\frac{\left(\frac{3}{8}a^2-\frac{3}{2}ab\right)\tan(fx+e)^3+\left(-\frac{3}{2}ab+\frac{5}{8}a^2\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{3\left(a^2-4ab+16b^2\right)\arctan(\tan(fx+e))}{8}}{a^5}-\frac{b^3\left(\frac{\frac{3ab(5a+4b)\tan(fx+e)^3}{8\left(a^2+2ab+b^2\right)}+\frac{(17a+12}{}}{\left(a+b+b\tan(fx+e)^2\right)}\right.}{f}$
risch	$\frac{3x}{8a^3}-\frac{3xb}{2a^4}+\frac{6x\,b^2}{a^5}+\frac{3i\mathrm{e}^{2i(fx+e)}b}{8a^4f}+\frac{i\mathrm{e}^{-2i(fx+e)}}{8a^3f}+\frac{i\mathrm{e}^{-4i(fx+e)}}{64a^3f}-\frac{i\mathrm{e}^{2i(fx+e)}}{8a^3f}-\frac{3i\mathrm{e}^{-2i(fx+e)}b}{8a^4f}-\frac{i\mathrm{e}^{4i(fx+e)}}{64a^3f}$

```
[In] int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^5*(((3/8*a^2-3/2*a*b)*tan(f*x+e)^3+(-3/2*a*b+5/8*a^2)*tan(f*x+e))/
(1+tan(f*x+e)^2)^2+3/8*(a^2-4*a*b+16*b^2)*arctan(tan(f*x+e)))-b^3/a^5*((3/8
*a*b*(5*a+4*b)/(a^2+2*a*b+b^2)*tan(f*x+e)^3+1/8*(17*a+12*b)*a/(a+b)*tan(f*x
+e))/(a+b+b*tan(f*x+e)^2)^2+3/8*(21*a^2+36*a*b+16*b^2)/(a^2+2*a*b+b^2)/((a+
b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. 2(249) = 498.

Time = 0.35 (sec) , antiderivative size = 1129, normalized size of antiderivative = 4.20

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(12*(a^6 - 2*a^5*b + 9*a^4*b^2 + 28*a^3*b^3 + 16*a^2*b^4)*f*x*cos(f*x
 + e)^4 + 24*(a^5*b - 2*a^4*b^2 + 9*a^3*b^3 + 28*a^2*b^4 + 16*a*b^5)*f*x*co
s(f*x + e)^2 + 12*(a^4*b^2 - 2*a^3*b^3 + 9*a^2*b^4 + 28*a*b^5 + 16*b^6)*f*x
 + 3*(21*a^2*b^4 + 36*a*b^5 + 16*b^6 + (21*a^4*b^2 + 36*a^3*b^3 + 16*a^2*b^
```

```
4)*cos(f*x + e)^4 + 2*(21*a^3*b^3 + 36*a^2*b^4 + 16*a*b^5)*cos(f*x + e)^2)*
sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b
^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*
cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*
a*b*cos(f*x + e)^2 + b^2)) + 4*(2*(a^6 + 2*a^5*b + a^4*b^2)*cos(f*x + e)^7
+ (3*a^6 - 2*a^5*b - 13*a^4*b^2 - 8*a^3*b^3)*cos(f*x + e)^5 + (6*a^5*b - 10
*a^4*b^2 - 55*a^3*b^3 - 36*a^2*b^4)*cos(f*x + e)^3 + 3*(a^4*b^2 - 2*a^3*b^3
 - 12*a^2*b^4 - 8*a*b^5)*cos(f*x + e))*sin(f*x + e))/((a^9 + 2*a^8*b + a^7*
b^2)*f*cos(f*x + e)^4 + 2*(a^8*b + 2*a^7*b^2 + a^6*b^3)*f*cos(f*x + e)^2 +
(a^7*b^2 + 2*a^6*b^3 + a^5*b^4)*f), 1/16*(6*(a^6 - 2*a^5*b + 9*a^4*b^2 + 28
*a^3*b^3 + 16*a^2*b^4)*f*x*cos(f*x + e)^4 + 12*(a^5*b - 2*a^4*b^2 + 9*a^3*b
^3 + 28*a^2*b^4 + 16*a*b^5)*f*x*cos(f*x + e)^2 + 6*(a^4*b^2 - 2*a^3*b^3 + 9
*a^2*b^4 + 28*a*b^5 + 16*b^6)*f*x + 3*(21*a^2*b^4 + 36*a*b^5 + 16*b^6 + (21
*a^4*b^2 + 36*a^3*b^3 + 16*a^2*b^4)*cos(f*x + e)^4 + 2*(21*a^3*b^3 + 36*a^2
*b^4 + 16*a*b^5)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(
f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e))) + 2*(2*(a^6
+ 2*a^5*b + a^4*b^2)*cos(f*x + e)^7 + (3*a^6 - 2*a^5*b - 13*a^4*b^2 - 8*a^3
*b^3)*cos(f*x + e)^5 + (6*a^5*b - 10*a^4*b^2 - 55*a^3*b^3 - 36*a^2*b^4)*cos
(f*x + e)^3 + 3*(a^4*b^2 - 2*a^3*b^3 - 12*a^2*b^4 - 8*a*b^5)*cos(f*x + e))*
sin(f*x + e))/((a^9 + 2*a^8*b + a^7*b^2)*f*cos(f*x + e)^4 + 2*(a^8*b + 2*a^
7*b^2 + a^6*b^3)*f*cos(f*x + e)^2 + (a^7*b^2 + 2*a^6*b^3 + a^5*b^4)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.72

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{3\,(21\,a^2b^3+36\,ab^4+16\,b^5)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^7+2\,a^6b+a^5b^2)\sqrt{(a+b)b}} - \frac{3\,(a^3b^2-2\,a^2b^3-12\,ab^4-8\,b^5)\tan(fx+e)^7+(6\,a^4b-a^3b^2-73\,a^2b^3-144\,ab^4-72\,b^5)\tan(f}{(a^6b^2+2\,a^5b^3+a^4b^4)\tan(fx+e)^8+a^8+4\,a^7b+6\,a^6b^2+4\,a^5b^3+a^4b^4+2\,(a^7b+4\,a^6b^2+5\,a}}{}$$

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*(21*a^2*b^3 + 36*a*b^4 + 16*b^5)*arctan(b*tan(f*x + e)/sqrt((a + b)
*b))/((a^7 + 2*a^6*b + a^5*b^2)*sqrt((a + b)*b)) - (3*(a^3*b^2 - 2*a^2*b^3
- 12*a*b^4 - 8*b^5)*tan(f*x + e)^7 + (6*a^4*b - a^3*b^2 - 73*a^2*b^3 - 144*
a*b^4 - 72*b^5)*tan(f*x + e)^5 + (3*a^5 + 10*a^4*b - 24*a^3*b^2 - 136*a^2*b
^3 - 180*a*b^4 - 72*b^5)*tan(f*x + e)^3 + (5*a^5 + 8*a^4*b - 18*a^3*b^2 - 6
9*a^2*b^3 - 72*a*b^4 - 24*b^5)*tan(f*x + e))/((a^6*b^2 + 2*a^5*b^3 + a^4*b^
4)*tan(f*x + e)^8 + a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a^4*b^4 + 2*(a^
7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*tan(f*x + e)^6 + (a^8 + 8*a^7*b +
19*a^6*b^2 + 18*a^5*b^3 + 6*a^4*b^4)*tan(f*x + e)^4 + 2*(a^8 + 5*a^7*b + 9*
a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*tan(f*x + e)^2) - 3*(a^2 - 4*a*b + 16*b^2)
*(f*x + e)/a^5)/f
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.72

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{3\,(21\,a^2b^3+36\,ab^4+16\,b^5)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^7+2\,a^6b+a^5b^2)\sqrt{ab+b^2}} - \frac{3\,a^3b^2\tan(fx+e)^7-6\,a^2b^3\tan(fx+e)^7-36\,ab^4\tan(fx+e)^7-24\,b^5}{}}{}$$

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*(21*a^2*b^3 + 36*a*b^4 + 16*b^5)*(pi*floor((f*x + e)/pi + 1/2)*sgn(
b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^7 + 2*a^6*b + a^5*b^2)*sqr
t(a*b + b^2)) - (3*a^3*b^2*tan(f*x + e)^7 - 6*a^2*b^3*tan(f*x + e)^7 - 36*a
*b^4*tan(f*x + e)^7 - 24*b^5*tan(f*x + e)^7 + 6*a^4*b*tan(f*x + e)^5 - a^3*
b^2*tan(f*x + e)^5 - 73*a^2*b^3*tan(f*x + e)^5 - 144*a*b^4*tan(f*x + e)^5 -
 72*b^5*tan(f*x + e)^5 + 3*a^5*tan(f*x + e)^3 + 10*a^4*b*tan(f*x + e)^3 - 2
4*a^3*b^2*tan(f*x + e)^3 - 136*a^2*b^3*tan(f*x + e)^3 - 180*a*b^4*tan(f*x +
 e)^3 - 72*b^5*tan(f*x + e)^3 + 5*a^5*tan(f*x + e) + 8*a^4*b*tan(f*x + e) -
 18*a^3*b^2*tan(f*x + e) - 69*a^2*b^3*tan(f*x + e) - 72*a*b^4*tan(f*x + e)
- 24*b^5*tan(f*x + e))/((a^6 + 2*a^5*b + a^4*b^2)*(b*tan(f*x + e)^4 + a*tan
(f*x + e)^2 + 2*b*tan(f*x + e)^2 + a + b)^2) - 3*(a^2 - 4*a*b + 16*b^2)*(f*
x + e)/a^5)/f
```

Mupad [B] (verification not implemented)

Time = 24.04 (sec) , antiderivative size = 4158, normalized size of antiderivative = 15.46

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] (atan(((((tan(e + f*x)*(18432*a*b^10 + 4608*b^11 + 27360*a^2*b^9 + 17568*a^
3*b^8 + 3978*a^4*b^7 + 180*a^5*b^6 + 198*a^6*b^5 - 36*a^7*b^4 + 9*a^8*b^3))
/(32*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)) - (3*((12*a^10*b
^8 + 48*a^11*b^7 + (141*a^12*b^6)/2 + (87*a^13*b^5)/2 + 9*a^14*b^4 + (3*a^1
5*b^3)/2 + (3*a^16*b^2)/2)/(4*a^15*b + a^16 + a^12*b^4 + 4*a^13*b^3 + 6*a^1
4*b^2) - (3*tan(e + f*x)*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16*b^2)*
(512*a^10*b^7 + 2304*a^11*b^6 + 4096*a^12*b^5 + 3584*a^13*b^4 + 1536*a^14*b
^3 + 256*a^15*b^2))/(512*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^
2)*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2)))*(-b^5
*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16*b^2))/(16*(5*a^9*b + a^10 + a^5*b^5
 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2)))*(-b^5*(a + b)^5)^(1/2)*(36*a*b +
21*a^2 + 16*b^2)*3i)/(16*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3
 + 10*a^8*b^2)) + (((tan(e + f*x)*(18432*a*b^10 + 4608*b^11 + 27360*a^2*b^9
 + 17568*a^3*b^8 + 3978*a^4*b^7 + 180*a^5*b^6 + 198*a^6*b^5 - 36*a^7*b^4 +
9*a^8*b^3))/(32*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)) + (3*
((12*a^10*b^8 + 48*a^11*b^7 + (141*a^12*b^6)/2 + (87*a^13*b^5)/2 + 9*a^14*b
^4 + (3*a^15*b^3)/2 + (3*a^16*b^2)/2)/(4*a^15*b + a^16 + a^12*b^4 + 4*a^13*
b^3 + 6*a^14*b^2) + (3*tan(e + f*x)*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2
 + 16*b^2)*(512*a^10*b^7 + 2304*a^11*b^6 + 4096*a^12*b^5 + 3584*a^13*b^4 +
1536*a^14*b^3 + 256*a^15*b^2))/(512*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3
+ 6*a^10*b^2)*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b
^2)))*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16*b^2))/(16*(5*a^9*b + a^1
0 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2)))*(-b^5*(a + b)^5)^(1/2)
*(36*a*b + 21*a^2 + 16*b^2)*3i)/(16*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 +
 10*a^7*b^3 + 10*a^8*b^2)))/((756*a*b^11 + 216*b^12 + (1755*a^2*b^10)/2 + (
1215*a^3*b^9)/4 - (1701*a^4*b^8)/32 - (351*a^5*b^7)/64 + (1215*a^6*b^6)/128
 - (567*a^7*b^5)/256)/(4*a^15*b + a^16 + a^12*b^4 + 4*a^13*b^3 + 6*a^14*b^2
) - (3*((tan(e + f*x)*(18432*a*b^10 + 4608*b^11 + 27360*a^2*b^9 + 17568*a^3
*b^8 + 3978*a^4*b^7 + 180*a^5*b^6 + 198*a^6*b^5 - 36*a^7*b^4 + 9*a^8*b^3))/
(32*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)) - (3*((12*a^10*b^
8 + 48*a^11*b^7 + (141*a^12*b^6)/2 + (87*a^13*b^5)/2 + 9*a^14*b^4 + (3*a^15
*b^3)/2 + (3*a^16*b^2)/2)/(4*a^15*b + a^16 + a^12*b^4 + 4*a^13*b^3 + 6*a^14
*b^2) - (3*tan(e + f*x)*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16*b^2)*(
512*a^10*b^7 + 2304*a^11*b^6 + 4096*a^12*b^5 + 3584*a^13*b^4 + 1536*a^14*b^
3 + 256*a^15*b^2))/(512*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2
)*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2)))*(-b^5*
```

(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16*b^2))/(16*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2)))*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16*b^2))/(16*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2)) + (3*((tan(e + f*x)*(18432*a*b^10 + 4608*b^11 + 27360*a^2*b^9 + 17568*a^3*b^8 + 3978*a^4*b^7 + 180*a^5*b^6 + 198*a^6*b^5 - 36*a^7*b^4 + 9*a^8*b^3))/(32*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)) + (3*((12*a^10*b^8 + 48*a^11*b^7 + (141*a^12*b^6)/2 + (87*a^13*b^5)/2 + 9*a^14*b^4 + (3*a^15*b^3)/2 + (3*a^16*b^2)/2)/(4*a^15*b + a^16 + a^12*b^4 + 4*a^13*b^3 + 6*a^14*b^2) + (3*tan(e + f*x)*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16*b^2)*(512*a^10*b^7 + 2304*a^11*b^6 + 4096*a^12*b^5 + 3584*a^13*b^4 + 1536*a^14*b^3 + 256*a^15*b^2))/(512*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2)))*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16*b^2))/(16*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2)))*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16*b^2))/(16*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2))))*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16*b^2)*3i)/(8*f*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2)) - (atan(((((3*((12*a^10*b^8 + 48*a^11*b^7 + (141*a^12*b^6)/2 + (87*a^13*b^5)/2 + 9*a^14*b^4 + (3*a^15*b^3)/2 + (3*a^16*b^2)/2)/(4*a^15*b + a^16 + a^12*b^4 + 4*a^13*b^3 + 6*a^14*b^2) - (3*tan(e + f*x)*(a^2*1i - a*b*4i + b^2*16i)*(512*a^10*b^7 + 2304*a^11*b^6 + 4096*a^12*b^5 + 3584*a^13*b^4 + 1536*a^14*b^3 + 256*a^15*b^2))/(512*a^5*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)))*(a^2*1i - a*b*4i + b^2*16i))/(16*a^5) - (tan(e + f*x)*(18432*a*b^10 + 4608*b^11 + 27360*a^2*b^9 + 17568*a^3*b^8 + 3978*a^4*b^7 + 180*a^5*b^6 + 198*a^6*b^5 - 36*a^7*b^4 + 9*a^8*b^3))/(32*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)))*(a^2*1i - a*b*4i + b^2*16i)*3i)/(16*a^5) - (((3*((12*a^10*b^8 + 48*a^11*b^7 + (141*a^12*b^6)/2 + (87*a^13*b^5)/2 + 9*a^14*b^4 + (3*a^15*b^3)/2 + (3*a^16*b^2)/2)/(4*a^15*b + a^16 + a^12*b^4 + 4*a^13*b^3 + 6*a^14*b^2) + (3*tan(e + f*x)*(a^2*1i - a*b*4i + b^2*16i)*(512*a^10*b^7 + 2304*a^11*b^6 + 4096*a^12*b^5 + 3584*a^13*b^4 + 1536*a^14*b^3 + 256*a^15*b^2))/(512*a^5*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)))*(a^2*1i - a*b*4i + b^2*16i))/(16*a^5) + (tan(e + f*x)*(18432*a*b^10 + 4608*b^11 + 27360*a^2*b^9 + 17568*a^3*b^8 + 3978*a^4*b^7 + 180*a^5*b^6 + 198*a^6*b^5 - 36*a^7*b^4 + 9*a^8*b^3))/(32*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)))*(a^2*1i - a*b*4i + b^2*16i)*3i)/(16*a^5))/((756*a*b^11 + 216*b^12 + (1755*a^2*b^10)/2 + (1215*a^3*b^9)/4 - (1701*a^4*b^8)/32 - (351*a^5*b^7)/64 + (1215*a^6*b^6)/128 - (567*a^7*b^5)/256)/(4*a^15*b + a^16 + a^12*b^4 + 4*a^13*b^3 + 6*a^14*b^2) + (3*((3*((12*a^10*b^8 + 48*a^11*b^7 + (141*a^12*b^6)/2 + (87*a^13*b^5)/2 + 9*a^14*b^4 + (3*a^15*b^3)/2 + (3*a^16*b^2)/2)/(4*a^15*b + a^16 + a^12*b^4 + 4*a^13*b^3 + 6*a^14*b^2) - (3*tan(e + f*x)*(a^2*1i - a*b*4i + b^2*16i)*(512*a^10*b^7 + 2304*a^11*b^6 + 4096*a^12*b^5 + 3584*a^13*b^4 + 1536*a^14*b^3 + 256*a^15*b^2))/(512*a^5*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)))*(a^2*1i - a*b*4i + b^2*16i))/(16*a^5) - (tan(e + f*x)*(18432*a*b^10 + 4608*b^11 + 27360*a^2*b^9 + 17568*a^3*b^8 + 3978*a^4*b^7 + 180*a^5*b^6 + 198*a^6*b^5 - 36*a^7*b^4 + 9*a^8*b^3))/(32*(4

```
*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)))*(a^2*1i - a*b*4i + b^2
*16i))/(16*a^5) + (3*((3*((12*a^10*b^8 + 48*a^11*b^7 + (141*a^12*b^6)/2 + (
87*a^13*b^5)/2 + 9*a^14*b^4 + (3*a^15*b^3)/2 + (3*a^16*b^2)/2)/(4*a^15*b +
a^16 + a^12*b^4 + 4*a^13*b^3 + 6*a^14*b^2) + (3*tan(e + f*x)*(a^2*1i - a*b*
4i + b^2*16i)*(512*a^10*b^7 + 2304*a^11*b^6 + 4096*a^12*b^5 + 3584*a^13*b^4
 + 1536*a^14*b^3 + 256*a^15*b^2))/(512*a^5*(4*a^11*b + a^12 + a^8*b^4 + 4*a
^9*b^3 + 6*a^10*b^2)))*(a^2*1i - a*b*4i + b^2*16i))/(16*a^5) + (tan(e + f*x
)*(18432*a*b^10 + 4608*b^11 + 27360*a^2*b^9 + 17568*a^3*b^8 + 3978*a^4*b^7
+ 180*a^5*b^6 + 198*a^6*b^5 - 36*a^7*b^4 + 9*a^8*b^3))/(32*(4*a^11*b + a^12
 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)))*(a^2*1i - a*b*4i + b^2*16i))/(16*a^5
)))*(a^2*1i - a*b*4i + b^2*16i)*3i)/(8*a^5*f) - ((tan(e + f*x)*(48*a*b^3 -
3*a^3*b - 5*a^4 + 24*b^4 + 21*a^2*b^2))/(8*a^4*(a + b)) + (tan(e + f*x)^5*(
144*a*b^4 - 6*a^4*b + 72*b^5 + 73*a^2*b^3 + a^3*b^2))/(8*a^4*(a + b)^2) + (
tan(e + f*x)^3*(180*a*b^4 - 10*a^4*b - 3*a^5 + 72*b^5 + 136*a^2*b^3 + 24*a^
3*b^2))/(8*a^4*(a + b)^2) + (3*b*tan(e + f*x)^7*(12*a*b^3 - a^3*b + 8*b^4 +
 2*a^2*b^2))/(8*a^4*(a + b)^2))/(f*(2*a*b + tan(e + f*x)^4*(6*a*b + a^2 + 6
*b^2) + a^2 + b^2 + tan(e + f*x)^6*(2*a*b + 4*b^2) + b^2*tan(e + f*x)^8 + t
an(e + f*x)^2*(6*a*b + 2*a^2 + 4*b^2)))
```

3.218 $\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 352

$$\begin{aligned}\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3} dx &= \frac{(5a^3-18a^2b+48ab^2-160b^3)\,x}{16a^6} \\ &+ \frac{b^{7/2}(99a^2+176ab+80b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^6(a+b)^{5/2}f} \\ &+ \frac{(15a^2-34ab+80b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)^2} \\ &+ \frac{5(a-2b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^2} \\ &+ \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^2} \\ &+ \frac{b(15a^3-29a^2b+64ab^2+120b^3)\tan(e+fx)}{48a^4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} \\ &+ \frac{b(5a^4-8a^3b+17a^2b^2+116ab^3+80b^4)\tan(e+fx)}{16a^5(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}\end{aligned}$$

```
[Out] 1/16*(5*a^3-18*a^2*b+48*a*b^2-160*b^3)*x/a^6+1/8*b^(7/2)*(99*a^2+176*a*b+80
*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^6/(a+b)^(5/2)/f+1/48*(15*a^2
-34*a*b+80*b^2)*cos(f*x+e)*sin(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)^2+5/24*(a-
2*b)*cos(f*x+e)^3*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2+1/6*cos(f*x+e)^5*
sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^2+1/48*b*(15*a^3-29*a^2*b+64*a*b^2+120*
b^3)*tan(f*x+e)/a^4/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2+1/16*b*(5*a^4-8*a^3*b+17
*a^2*b^2+116*a*b^3+80*b^4)*tan(f*x+e)/a^5/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4231, 425, 541, 536, 209, 211}

$$\begin{aligned}\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = {}& \frac{5(a-2b)\sin(e+fx)\cos^3(e+fx)}{24a^2f\left(a+b\tan^2(e+fx)+b\right)^2} \\ &+ \frac{b^{7/2}(99a^2+176ab+80b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^6f(a+b)^{5/2}} \\ &+ \frac{(15a^2-34ab+80b^2)\sin(e+fx)\cos(e+fx)}{48a^3f\left(a+b\tan^2(e+fx)+b\right)^2} \\ &+ \frac{x(5a^3-18a^2b+48ab^2-160b^3)}{16a^6} \\ &+ \frac{b(15a^3-29a^2b+64ab^2+120b^3)\tan(e+fx)}{48a^4f(a+b)\left(a+b\tan^2(e+fx)+b\right)^2} \\ &+ \frac{b(5a^4-8a^3b+17a^2b^2+116ab^3+80b^4)\tan(e+fx)}{16a^5f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)} \\ &+ \frac{\sin(e+fx)\cos^5(e+fx)}{6af\left(a+b\tan^2(e+fx)+b\right)^2}\end{aligned}$$

```
[In] Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((5*a^3 - 18*a^2*b + 48*a*b^2 - 160*b^3)*x)/(16*a^6) + (b^(7/2)*(99*a^2 + 1
76*a*b + 80*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*a^6*(a + b)
^(5/2)*f) + ((15*a^2 - 34*a*b + 80*b^2)*Cos[e + f*x]*Sin[e + f*x])/(48*a^3*
f*(a + b + b*Tan[e + f*x]^2)^2) + (5*(a - 2*b)*Cos[e + f*x]^3*Sin[e + f*x])
/(24*a^2*f*(a + b + b*Tan[e + f*x]^2)^2) + (Cos[e + f*x]^5*Sin[e + f*x])/(6
*a*f*(a + b + b*Tan[e + f*x]^2)^2) + (b*(15*a^3 - 29*a^2*b + 64*a*b^2 + 120
*b^3)*Tan[e + f*x])/(48*a^4*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) + (b*(5
*a^4 - 8*a^3*b + 17*a^2*b^2 + 116*a*b^3 + 80*b^4)*Tan[e + f*x])/(16*a^5*(a
+ b)^2*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^4(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-5a+b-9bx^2}{(1+x^2)^3(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{6af}\\
&= \frac{5(a-2b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^2}\\
&\quad + \frac{\text{Subst}\left(\int \frac{15a^2+ab+10b^2+35(a-2b)bx^2}{(1+x^2)^2(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{24a^2f}
\end{aligned}
$$

$$= \frac{(15a^2 - 34ab + 80b^2)\cos(e+fx)\sin(e+fx)}{48a^3 f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{5(a-2b)\cos^3(e+fx)\sin(e+fx)}{24a^2 f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-15a^3-21a^2b+26ab^2+80b^3-5b(15a^2-34ab+80b^2)x^2}{(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{48a^3 f}$$

$$= \frac{(15a^2 - 34ab + 80b^2)\cos(e+fx)\sin(e+fx)}{48a^3 f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{5(a-2b)\cos^3(e+fx)\sin(e+fx)}{24a^2 f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(15a^3-29a^2b+64ab^2+120b^3)\tan(e+fx)}{48a^4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-12(5a^4+2a^3b+a^2b^2-48ab^3-40b^4)-12b(15a^3-29a^2b+64ab^2+120b^3)x^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{192a^4(a+b)f}$$

$$= \frac{(15a^2 - 34ab + 80b^2)\cos(e+fx)\sin(e+fx)}{48a^3 f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{5(a-2b)\cos^3(e+fx)\sin(e+fx)}{24a^2 f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(15a^3-29a^2b+64ab^2+120b^3)\tan(e+fx)}{48a^4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(5a^4-8a^3b+17a^2b^2+116ab^3+80b^4)\tan(e+fx)}{16a^5(a+b)^2 f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-24(5a^5-3a^4b+9a^3b^2-65a^2b^3-156ab^4-80b^5)-24b(5a^4-8a^3b+17a^2b^2+116ab^3+80b^4)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{384a^5(a+b)^2 f}$$

$$= \frac{(15a^2 - 34ab + 80b^2)\cos(e+fx)\sin(e+fx)}{48a^3 f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{5(a-2b)\cos^3(e+fx)\sin(e+fx)}{24a^2 f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(15a^3-29a^2b+64ab^2+120b^3)\tan(e+fx)}{48a^4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(5a^4-8a^3b+17a^2b^2+116ab^3+80b^4)\tan(e+fx)}{16a^5(a+b)^2 f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(b^4(99a^2+176ab+80b^2))\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{8a^6(a+b)^2 f} + \frac{(5a^3-18a^2b+48ab^2-160b^3)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{16a^6 f}$$

$$= \frac{(5a^3 - 18a^2b + 48ab^2 - 160b^3)\,x}{16a^6} + \frac{b^{7/2}(99a^2 + 176ab + 80b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^6(a+b)^{5/2}f}$$
$$+ \frac{(15a^2 - 34ab + 80b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\,(a+b+b\tan^2(e+fx))^2}$$
$$+ \frac{5(a-2b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\,(a+b+b\tan^2(e+fx))^2} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\,(a+b+b\tan^2(e+fx))^2}$$
$$+ \frac{b(15a^3 - 29a^2b + 64ab^2 + 120b^3)\tan(e+fx)}{48a^4(a+b)f\,(a+b+b\tan^2(e+fx))^2}$$
$$+ \frac{b(5a^4 - 8a^3b + 17a^2b^2 + 116ab^3 + 80b^4)\tan(e+fx)}{16a^5(a+b)^2f\,(a+b+b\tan^2(e+fx))}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.12 (sec) , antiderivative size = 1770, normalized size of antiderivative = 5.03

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{(99a^2 + 176ab + 80b^2)\,(a + 2b + a\cos(2e+2fx))^3\sec^6(e+fx)\left(-\frac{b^4\arctan\left(\sec(fx)\left(\frac{\cos(2e)}{2\sqrt{a+b}\sqrt{b\cos(4e)-ib\sin(4e)}} - \frac{}{2\sqrt{a+}}\right.\right.}{64a^6\sqrt{a}}\right.}{}$$
$$+ \frac{(a + 2b + a\cos(2e+2fx))\sec(2e)\sec^6(e+fx)\,(720a^7fx\cos(2e) + 768a^6bfx\cos(2e) + 1296a^5b^2fx\cos(}{}$$

```
[In] Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((99*a^2 + 176*a*b + 80*b^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^
6*(-1/64*(b^4*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*
b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]
]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(a^6*Sqrt[
a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) + ((I/64)*b^4*ArcTan[Sec[f*x]*(Co
s[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(
Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x]
 + a*Sin[2*e + f*x])]*Sin[2*e])/(a^6*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Si
n[4*e]])))/((a + b)^2*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2
*f*x])*Sec[2*e]*Sec[e + f*x]^6*(720*a^7*f*x*Cos[2*e] + 768*a^6*b*f*x*Cos[2*
e] + 1296*a^5*b^2*f*x*Cos[2*e] - 8352*a^4*b^3*f*x*Cos[2*e] - 64128*a^3*b^4*
f*x*Cos[2*e] - 158976*a^2*b^5*f*x*Cos[2*e] - 165888*a*b^6*f*x*Cos[2*e] - 61
440*b^7*f*x*Cos[2*e] + 480*a^7*f*x*Cos[2*f*x] + 192*a^6*b*f*x*Cos[2*f*x] +
96*a^5*b^2*f*x*Cos[2*f*x] - 4608*a^4*b^3*f*x*Cos[2*f*x] - 41856*a^3*b^4*f*x
*Cos[2*f*x] - 67584*a^2*b^5*f*x*Cos[2*f*x] - 30720*a*b^6*f*x*Cos[2*f*x] + 4
```

```
80*a^7*f*x*Cos[4*e + 2*f*x] + 192*a^6*b*f*x*Cos[4*e + 2*f*x] + 96*a^5*b^2*f
*x*Cos[4*e + 2*f*x] - 4608*a^4*b^3*f*x*Cos[4*e + 2*f*x] - 41856*a^3*b^4*f*x
*Cos[4*e + 2*f*x] - 67584*a^2*b^5*f*x*Cos[4*e + 2*f*x] - 30720*a*b^6*f*x*Co
s[4*e + 2*f*x] + 120*a^7*f*x*Cos[2*e + 4*f*x] - 192*a^6*b*f*x*Cos[2*e + 4*f
*x] + 408*a^5*b^2*f*x*Cos[2*e + 4*f*x] - 1968*a^4*b^3*f*x*Cos[2*e + 4*f*x]
- 6528*a^3*b^4*f*x*Cos[2*e + 4*f*x] - 3840*a^2*b^5*f*x*Cos[2*e + 4*f*x] + 1
20*a^7*f*x*Cos[6*e + 4*f*x] - 192*a^6*b*f*x*Cos[6*e + 4*f*x] + 408*a^5*b^2*
f*x*Cos[6*e + 4*f*x] - 1968*a^4*b^3*f*x*Cos[6*e + 4*f*x] - 6528*a^3*b^4*f*x
*Cos[6*e + 4*f*x] - 3840*a^2*b^5*f*x*Cos[6*e + 4*f*x] - 6048*a^3*b^4*Sin[2*
e] - 21312*a^2*b^5*Sin[2*e] - 29952*a*b^6*Sin[2*e] - 13824*b^7*Sin[2*e] + 2
62*a^7*Sin[2*f*x] + 524*a^6*b*Sin[2*f*x] - 26*a^5*b^2*Sin[2*f*x] + 1728*a^4
*b^3*Sin[2*f*x] + 14976*a^3*b^4*Sin[2*f*x] + 28416*a^2*b^5*Sin[2*f*x] + 145
92*a*b^6*Sin[2*f*x] + 262*a^7*Sin[4*e + 2*f*x] + 524*a^6*b*Sin[4*e + 2*f*x]
 - 26*a^5*b^2*Sin[4*e + 2*f*x] + 1728*a^4*b^3*Sin[4*e + 2*f*x] + 6912*a^3*b
^4*Sin[4*e + 2*f*x] + 5376*a^2*b^5*Sin[4*e + 2*f*x] + 768*a*b^6*Sin[4*e + 2
*f*x] + 238*a^7*Sin[2*e + 4*f*x] + 304*a^6*b*Sin[2*e + 4*f*x] - 250*a^5*b^2
*Sin[2*e + 4*f*x] + 1556*a^4*b^3*Sin[2*e + 4*f*x] + 5904*a^3*b^4*Sin[2*e +
4*f*x] + 3744*a^2*b^5*Sin[2*e + 4*f*x] + 238*a^7*Sin[6*e + 4*f*x] + 304*a^6
*b*Sin[6*e + 4*f*x] - 250*a^5*b^2*Sin[6*e + 4*f*x] + 1556*a^4*b^3*Sin[6*e +
 4*f*x] + 3888*a^3*b^4*Sin[6*e + 4*f*x] + 2016*a^2*b^5*Sin[6*e + 4*f*x] + 8
7*a^7*Sin[4*e + 6*f*x] + 46*a^6*b*Sin[4*e + 6*f*x] - 9*a^5*b^2*Sin[4*e + 6*
f*x] + 192*a^4*b^3*Sin[4*e + 6*f*x] + 160*a^3*b^4*Sin[4*e + 6*f*x] + 87*a^7
*Sin[8*e + 6*f*x] + 46*a^6*b*Sin[8*e + 6*f*x] - 9*a^5*b^2*Sin[8*e + 6*f*x]
+ 192*a^4*b^3*Sin[8*e + 6*f*x] + 160*a^3*b^4*Sin[8*e + 6*f*x] + 13*a^7*Sin[
6*e + 8*f*x] + 16*a^6*b*Sin[6*e + 8*f*x] - 7*a^5*b^2*Sin[6*e + 8*f*x] - 10*
a^4*b^3*Sin[6*e + 8*f*x] + 13*a^7*Sin[10*e + 8*f*x] + 16*a^6*b*Sin[10*e + 8
*f*x] - 7*a^5*b^2*Sin[10*e + 8*f*x] - 10*a^4*b^3*Sin[10*e + 8*f*x] + a^7*Si
n[8*e + 10*f*x] + 2*a^6*b*Sin[8*e + 10*f*x] + a^5*b^2*Sin[8*e + 10*f*x] + a
^7*Sin[12*e + 10*f*x] + 2*a^6*b*Sin[12*e + 10*f*x] + a^5*b^2*Sin[12*e + 10*
f*x]))/(12288*a^6*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A] (verified)

Time = 12.97 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{\frac{\frac{\left(\frac{5}{16}a^3-\frac{9}{8}a^2b+3a\,b^2\right)\tan(fx+e)^5+\left(6a\,b^2+\frac{5}{6}a^3-3a^2b\right)\tan(fx+e)^3+\left(-\frac{15}{8}a^2b+3a\,b^2+\frac{11}{16}a^3\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^3}+\frac{\left(5a^3-18a^2b+48a\,b^2-160}{16}}{a^6}}{f}$
default	$\frac{\frac{\frac{\left(\frac{5}{16}a^3-\frac{9}{8}a^2b+3a\,b^2\right)\tan(fx+e)^5+\left(6a\,b^2+\frac{5}{6}a^3-3a^2b\right)\tan(fx+e)^3+\left(-\frac{15}{8}a^2b+3a\,b^2+\frac{11}{16}a^3\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^3}+\frac{\left(5a^3-18a^2b+48a\,b^2-160}{16}}{a^6}}{f}$
risch	$\frac{5x}{16a^3}-\frac{9xb}{8a^4}+\frac{3x\,b^2}{a^5}-\frac{10x\,b^3}{a^6}+\frac{15i\mathrm{e}^{-2i(fx+e)}}{128a^3f}+\frac{3i\mathrm{e}^{-2i(fx+e)}b^2}{4a^5f}-\frac{15i\mathrm{e}^{2i(fx+e)}}{128a^3f}-\frac{3i\mathrm{e}^{2i(fx+e)}b^2}{4a^5f}+\frac{3i\mathrm{e}^{2i(fx}}{8a^4f}$

```
[In] int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^6*(((5/16*a^3-9/8*a^2*b+3*a*b^2)*tan(f*x+e)^5+(6*a*b^2+5/6*a^3-3*a
^2*b)*tan(f*x+e)^3+(-15/8*a^2*b+3*a*b^2+11/16*a^3)*tan(f*x+e))/(1+tan(f*x+e
)^2)^3+1/16*(5*a^3-18*a^2*b+48*a*b^2-160*b^3)*arctan(tan(f*x+e)))+b^4/a^6*(
(1/8*a*b*(19*a+16*b)/(a^2+2*a*b+b^2)*tan(f*x+e)^3+1/8*(21*a+16*b)*a/(a+b)*t
an(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(99*a^2+176*a*b+80*b^2)/(a^2+2*a*b+b^
2)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 1296, normalized size of antiderivative = 3.68

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/96*(6*(5*a^7 - 8*a^6*b + 17*a^5*b^2 - 82*a^4*b^3 - 272*a^3*b^4 - 160*a^2
*b^5)*f*x*cos(f*x + e)^4 + 12*(5*a^6*b - 8*a^5*b^2 + 17*a^4*b^3 - 82*a^3*b^
4 - 272*a^2*b^5 - 160*a*b^6)*f*x*cos(f*x + e)^2 + 6*(5*a^5*b^2 - 8*a^4*b^3
+ 17*a^3*b^4 - 82*a^2*b^5 - 272*a*b^6 - 160*b^7)*f*x + 3*(99*a^2*b^5 + 176*
a*b^6 + 80*b^7 + (99*a^4*b^3 + 176*a^3*b^4 + 80*a^2*b^5)*cos(f*x + e)^4 + 2
*(99*a^3*b^4 + 176*a^2*b^5 + 80*a*b^6)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log
(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 -
 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-
b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 +
 b^2)) + 2*(8*(a^7 + 2*a^6*b + a^5*b^2)*cos(f*x + e)^9 + 10*(a^7 - 3*a^5*b^
2 - 2*a^4*b^3)*cos(f*x + e)^7 + (15*a^7 - 4*a^6*b + 27*a^5*b^2 + 126*a^4*b^
3 + 80*a^3*b^4)*cos(f*x + e)^5 + 2*(15*a^6*b - 19*a^5*b^2 + 43*a^4*b^3 + 26
6*a^3*b^4 + 180*a^2*b^5)*cos(f*x + e)^3 + 3*(5*a^5*b^2 - 8*a^4*b^3 + 17*a^3
*b^4 + 116*a^2*b^5 + 80*a*b^6)*cos(f*x + e))*sin(f*x + e))/((a^10 + 2*a^9*b
```

```
 + a^8*b^2)*f*cos(f*x + e)^4 + 2*(a^9*b + 2*a^8*b^2 + a^7*b^3)*f*cos(f*x +
e)^2 + (a^8*b^2 + 2*a^7*b^3 + a^6*b^4)*f), 1/48*(3*(5*a^7 - 8*a^6*b + 17*a^
5*b^2 - 82*a^4*b^3 - 272*a^3*b^4 - 160*a^2*b^5)*f*x*cos(f*x + e)^4 + 6*(5*a
^6*b - 8*a^5*b^2 + 17*a^4*b^3 - 82*a^3*b^4 - 272*a^2*b^5 - 160*a*b^6)*f*x*c
os(f*x + e)^2 + 3*(5*a^5*b^2 - 8*a^4*b^3 + 17*a^3*b^4 - 82*a^2*b^5 - 272*a*
b^6 - 160*b^7)*f*x - 3*(99*a^2*b^5 + 176*a*b^6 + 80*b^7 + (99*a^4*b^3 + 176
*a^3*b^4 + 80*a^2*b^5)*cos(f*x + e)^4 + 2*(99*a^3*b^4 + 176*a^2*b^5 + 80*a*
b^6)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 -
 b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e))) + (8*(a^7 + 2*a^6*b + a^
5*b^2)*cos(f*x + e)^9 + 10*(a^7 - 3*a^5*b^2 - 2*a^4*b^3)*cos(f*x + e)^7 + (
15*a^7 - 4*a^6*b + 27*a^5*b^2 + 126*a^4*b^3 + 80*a^3*b^4)*cos(f*x + e)^5 +
2*(15*a^6*b - 19*a^5*b^2 + 43*a^4*b^3 + 266*a^3*b^4 + 180*a^2*b^5)*cos(f*x
+ e)^3 + 3*(5*a^5*b^2 - 8*a^4*b^3 + 17*a^3*b^4 + 116*a^2*b^5 + 80*a*b^6)*co
s(f*x + e))*sin(f*x + e))/((a^10 + 2*a^9*b + a^8*b^2)*f*cos(f*x + e)^4 + 2*
(a^9*b + 2*a^8*b^2 + a^7*b^3)*f*cos(f*x + e)^2 + (a^8*b^2 + 2*a^7*b^3 + a^6
*b^4)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 611, normalized size of antiderivative = 1.74

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{6\,(99\,a^2b^4+176\,ab^5+80\,b^6)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^8+2\,a^7b+a^6b^2)\sqrt{(a+b)b}} + \frac{3\,(5\,a^4b^2-8\,a^3b^3+17\,a^2b^4+116\,ab^5+80\,b^6)\tan(fx+e)^9+2\,(15\,a^5b+11\,a^4b^2-5\,a^3b^3+368\,a^2}{(a^7b^2+2\,a^6b^3+a^5b^4)\tan(fx+e)^{10}+a^9+4\,a^8b+6\,a^7b}}{}$$

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
[Out] 1/48*(6*(99*a^2*b^4 + 176*a*b^5 + 80*b^6)*arctan(b*tan(f*x + e)/sqrt((a + b
)*b))/((a^8 + 2*a^7*b + a^6*b^2)*sqrt((a + b)*b)) + (3*(5*a^4*b^2 - 8*a^3*b
^3 + 17*a^2*b^4 + 116*a*b^5 + 80*b^6)*tan(f*x + e)^9 + 2*(15*a^5*b + 11*a^4
*b^2 - 5*a^3*b^3 + 368*a^2*b^4 + 876*a*b^5 + 480*b^6)*tan(f*x + e)^7 + (15*
```

```
a^6 + 86*a^5*b + 3*a^4*b^2 + 240*a^3*b^3 + 1982*a^2*b^4 + 3168*a*b^5 + 1440
*b^6)*tan(f*x + e)^5 + 2*(20*a^6 + 41*a^5*b - 15*a^4*b^2 + 197*a^3*b^3 + 98
0*a^2*b^4 + 1236*a*b^5 + 480*b^6)*tan(f*x + e)^3 + 3*(11*a^6 + 14*a^5*b - 6
*a^4*b^2 + 56*a^3*b^3 + 221*a^2*b^4 + 236*a*b^5 + 80*b^6)*tan(f*x + e))/((a
^7*b^2 + 2*a^6*b^3 + a^5*b^4)*tan(f*x + e)^10 + a^9 + 4*a^8*b + 6*a^7*b^2 +
 4*a^6*b^3 + a^5*b^4 + (2*a^8*b + 9*a^7*b^2 + 12*a^6*b^3 + 5*a^5*b^4)*tan(f
*x + e)^8 + (a^9 + 10*a^8*b + 27*a^7*b^2 + 28*a^6*b^3 + 10*a^5*b^4)*tan(f*x
 + e)^6 + (3*a^9 + 18*a^8*b + 37*a^7*b^2 + 32*a^6*b^3 + 10*a^5*b^4)*tan(f*x
 + e)^4 + (3*a^9 + 14*a^8*b + 24*a^7*b^2 + 18*a^6*b^3 + 5*a^5*b^4)*tan(f*x
+ e)^2) + 3*(5*a^3 - 18*a^2*b + 48*a*b^2 - 160*b^3)*(f*x + e)/a^6)/f
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.99

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\frac{6\,(99\,a^2b^4+176\,ab^5+80\,b^6)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^8+2\,a^7b+a^6b^2)\sqrt{ab+b^2}}+\frac{6\left(19\,ab^5\tan(fx+e)^3+16\,b^6\tan(fx+e)^3+21\,a^2b^4\tan(fx+e)+37\,ab^5\right.}{(a^7+2\,a^6b+a^5b^2)\left(b\tan(fx+e)^2+a+b\right)}}{}$$

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/48*(6*(99*a^2*b^4 + 176*a*b^5 + 80*b^6)*(pi*floor((f*x + e)/pi + 1/2)*sgn
(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^8 + 2*a^7*b + a^6*b^2)*sq
rt(a*b + b^2)) + 6*(19*a*b^5*tan(f*x + e)^3 + 16*b^6*tan(f*x + e)^3 + 21*a^
2*b^4*tan(f*x + e) + 37*a*b^5*tan(f*x + e) + 16*b^6*tan(f*x + e))/((a^7 + 2
*a^6*b + a^5*b^2)*(b*tan(f*x + e)^2 + a + b)^2) + 3*(5*a^3 - 18*a^2*b + 48*
a*b^2 - 160*b^3)*(f*x + e)/a^6 + (15*a^2*tan(f*x + e)^5 - 54*a*b*tan(f*x +
e)^5 + 144*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 - 144*a*b*tan(f*x + e
)^3 + 288*b^2*tan(f*x + e)^3 + 33*a^2*tan(f*x + e) - 90*a*b*tan(f*x + e) +
144*b^2*tan(f*x + e))/((tan(f*x + e)^2 + 1)^3*a^5))/f
```

Mupad [B] (verification not implemented)

Time = 25.07 (sec) , antiderivative size = 4594, normalized size of antiderivative = 13.05

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

```
[In] int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] ((tan(e + f*x)*(156*a*b^4 + 3*a^4*b + 11*a^5 + 80*b^5 + 65*a^2*b^3 - 9*a^3*
b^2))/(16*a^5*(a + b)) + (tan(e + f*x)^7*(876*a*b^5 + 15*a^5*b + 480*b^6 +
368*a^2*b^4 - 5*a^3*b^3 + 11*a^4*b^2))/(24*a^5*(a + b)^2) + (tan(e + f*x)^3
*(1236*a*b^5 + 41*a^5*b + 20*a^6 + 480*b^6 + 980*a^2*b^4 + 197*a^3*b^3 - 15
*a^4*b^2))/(24*a^5*(a + b)^2) + (tan(e + f*x)^5*(3168*a*b^5 + 86*a^5*b + 15
*a^6 + 1440*b^6 + 1982*a^2*b^4 + 240*a^3*b^3 + 3*a^4*b^2))/(48*a^5*(a + b)^
2) + (b*tan(e + f*x)^9*(116*a*b^4 + 5*a^4*b + 80*b^5 + 17*a^2*b^3 - 8*a^3*b
^2))/(16*a^5*(a + b)^2))/(f*(2*a*b + tan(e + f*x)^6*(8*a*b + a^2 + 10*b^2)
+ a^2 + b^2 + tan(e + f*x)^8*(2*a*b + 5*b^2) + b^2*tan(e + f*x)^10 + tan(e
+ f*x)^2*(8*a*b + 3*a^2 + 5*b^2) + tan(e + f*x)^4*(12*a*b + 3*a^2 + 10*b^2)
)) - (atan(-((((((20*a^12*b^9 + 79*a^13*b^8 + (457*a^14*b^7)/4 + (277*a^15*
b^6)/4 + (25*a^16*b^5)/2 - 2*a^17*b^4 - (7*a^18*b^3)/4 - (5*a^19*b^2)/4)/(4
*a^18*b + a^19 + a^15*b^4 + 4*a^16*b^3 + 6*a^17*b^2) - (tan(e + f*x)*(a*b^2
*48i - a^2*b*18i + a^3*5i - b^3*160i)*(2048*a^12*b^7 + 9216*a^13*b^6 + 1638
4*a^14*b^5 + 14336*a^15*b^4 + 6144*a^16*b^3 + 1024*a^17*b^2))/(4096*a^6*(4*
a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)))*(a*b^2*48i - a^2*b*18
i + a^3*5i - b^3*160i))/(32*a^6) - (tan(e + f*x)*(199680*a*b^12 + 51200*b^1
3 + 287488*a^2*b^11 + 178560*a^3*b^10 + 39240*a^4*b^9 - 36*a^5*b^8 - 1119*a
^6*b^7 - 1092*a^7*b^6 + 234*a^8*b^5 - 80*a^9*b^4 + 25*a^10*b^3))/(128*(4*a^
13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)))*(a*b^2*48i - a^2*b*18i
+ a^3*5i - b^3*160i)*1i)/(32*a^6) - (((((20*a^12*b^9 + 79*a^13*b^8 + (457*a
^14*b^7)/4 + (277*a^15*b^6)/4 + (25*a^16*b^5)/2 - 2*a^17*b^4 - (7*a^18*b^3)
/4 - (5*a^19*b^2)/4)/(4*a^18*b + a^19 + a^15*b^4 + 4*a^16*b^3 + 6*a^17*b^2)
 + (tan(e + f*x)*(a*b^2*48i - a^2*b*18i + a^3*5i - b^3*160i)*(2048*a^12*b^7
 + 9216*a^13*b^6 + 16384*a^14*b^5 + 14336*a^15*b^4 + 6144*a^16*b^3 + 1024*a
^17*b^2))/(4096*a^6*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2))
)*(a*b^2*48i - a^2*b*18i + a^3*5i - b^3*160i))/(32*a^6) + (tan(e + f*x)*(19
9680*a*b^12 + 51200*b^13 + 287488*a^2*b^11 + 178560*a^3*b^10 + 39240*a^4*b^
9 - 36*a^5*b^8 - 1119*a^6*b^7 - 1092*a^7*b^6 + 234*a^8*b^5 - 80*a^9*b^4 + 2
5*a^10*b^3))/(128*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)))*
(a*b^2*48i - a^2*b*18i + a^3*5i - b^3*160i)*1i)/(32*a^6))/((3350*a*b^14 + 1
000*b^15 + (7315*a^2*b^13)/2 + (4597*a^3*b^12)/4 - (4325*a^4*b^11)/32 + (10
281*a^5*b^10)/128 + (8973*a^6*b^9)/512 - (25551*a^7*b^8)/1024 + (4235*a^8*b
^7)/512 - (2475*a^9*b^6)/1024)/(4*a^18*b + a^19 + a^15*b^4 + 4*a^16*b^3 + 6
*a^17*b^2) + (((((20*a^12*b^9 + 79*a^13*b^8 + (457*a^14*b^7)/4 + (277*a^15*
b^6)/4 + (25*a^16*b^5)/2 - 2*a^17*b^4 - (7*a^18*b^3)/4 - (5*a^19*b^2)/4)/(4
*a^18*b + a^19 + a^15*b^4 + 4*a^16*b^3 + 6*a^17*b^2) - (tan(e + f*x)*(a*b^2
*48i - a^2*b*18i + a^3*5i - b^3*160i)*(2048*a^12*b^7 + 9216*a^13*b^6 + 1638
4*a^14*b^5 + 14336*a^15*b^4 + 6144*a^16*b^3 + 1024*a^17*b^2))/(4096*a^6*(4*
a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)))*(a*b^2*48i - a^2*b*18
i + a^3*5i - b^3*160i))/(32*a^6) - (tan(e + f*x)*(199680*a*b^12 + 51200*b^1
3 + 287488*a^2*b^11 + 178560*a^3*b^10 + 39240*a^4*b^9 - 36*a^5*b^8 - 1119*a
^6*b^7 - 1092*a^7*b^6 + 234*a^8*b^5 - 80*a^9*b^4 + 25*a^10*b^3))/(128*(4*a^
13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)))*(a*b^2*48i - a^2*b*18i
+ a^3*5i - b^3*160i))/(32*a^6) + (((((20*a^12*b^9 + 79*a^13*b^8 + (457*a^14
```

*b^7)/4 + (277*a^15*b^6)/4 + (25*a^16*b^5)/2 - 2*a^17*b^4 - (7*a^18*b^3)/4
- (5*a^19*b^2)/4)/(4*a^18*b + a^19 + a^15*b^4 + 4*a^16*b^3 + 6*a^17*b^2) +
(tan(e + f*x)*(a*b^2*48i - a^2*b*18i + a^3*5i - b^3*160i)*(2048*a^12*b^7 +
9216*a^13*b^6 + 16384*a^14*b^5 + 14336*a^15*b^4 + 6144*a^16*b^3 + 1024*a^17
*b^2))/(4096*a^6*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)))*(
a*b^2*48i - a^2*b*18i + a^3*5i - b^3*160i))/(32*a^6) + (tan(e + f*x)*(19968
0*a*b^12 + 51200*b^13 + 287488*a^2*b^11 + 178560*a^3*b^10 + 39240*a^4*b^9 -
36*a^5*b^8 - 1119*a^6*b^7 - 1092*a^7*b^6 + 234*a^8*b^5 - 80*a^9*b^4 + 25*a
^10*b^3))/(128*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)))*(a*
b^2*48i - a^2*b*18i + a^3*5i - b^3*160i))/(32*a^6)))*(a*b^2*48i - a^2*b*18i
+ a^3*5i - b^3*160i)*1i)/(16*a^6*f) - (atan((((-b^7*(a + b)^5)^(1/2)*((tan
(e + f*x)*(199680*a*b^12 + 51200*b^13 + 287488*a^2*b^11 + 178560*a^3*b^10 +
39240*a^4*b^9 - 36*a^5*b^8 - 1119*a^6*b^7 - 1092*a^7*b^6 + 234*a^8*b^5 - 8
0*a^9*b^4 + 25*a^10*b^3))/(128*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6
*a^12*b^2)) + (((20*a^12*b^9 + 79*a^13*b^8 + (457*a^14*b^7)/4 + (277*a^15*b
^6)/4 + (25*a^16*b^5)/2 - 2*a^17*b^4 - (7*a^18*b^3)/4 - (5*a^19*b^2)/4)/(4*
a^18*b + a^19 + a^15*b^4 + 4*a^16*b^3 + 6*a^17*b^2) + (tan(e + f*x)*(-b^7*(
a + b)^5)^(1/2)*(11*a*b + (99*a^2)/16 + 5*b^2)*(2048*a^12*b^7 + 9216*a^13*b
^6 + 16384*a^14*b^5 + 14336*a^15*b^4 + 6144*a^16*b^3 + 1024*a^17*b^2))/(128
*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)*(5*a^10*b + a^11 +
a^6*b^5 + 5*a^7*b^4 + 10*a^8*b^3 + 10*a^9*b^2)))*(-b^7*(a + b)^5)^(1/2)*(11
*a*b + (99*a^2)/16 + 5*b^2))/(5*a^10*b + a^11 + a^6*b^5 + 5*a^7*b^4 + 10*a^
8*b^3 + 10*a^9*b^2))*(11*a*b + (99*a^2)/16 + 5*b^2)*1i)/(5*a^10*b + a^11 +
a^6*b^5 + 5*a^7*b^4 + 10*a^8*b^3 + 10*a^9*b^2) + ((-b^7*(a + b)^5)^(1/2)*((
tan(e + f*x)*(199680*a*b^12 + 51200*b^13 + 287488*a^2*b^11 + 178560*a^3*b^1
0 + 39240*a^4*b^9 - 36*a^5*b^8 - 1119*a^6*b^7 - 1092*a^7*b^6 + 234*a^8*b^5
- 80*a^9*b^4 + 25*a^10*b^3))/(128*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3
+ 6*a^12*b^2)) - (((20*a^12*b^9 + 79*a^13*b^8 + (457*a^14*b^7)/4 + (277*a^1
5*b^6)/4 + (25*a^16*b^5)/2 - 2*a^17*b^4 - (7*a^18*b^3)/4 - (5*a^19*b^2)/4)/
(4*a^18*b + a^19 + a^15*b^4 + 4*a^16*b^3 + 6*a^17*b^2) - (tan(e + f*x)*(-b^
7*(a + b)^5)^(1/2)*(11*a*b + (99*a^2)/16 + 5*b^2)*(2048*a^12*b^7 + 9216*a^1
3*b^6 + 16384*a^14*b^5 + 14336*a^15*b^4 + 6144*a^16*b^3 + 1024*a^17*b^2))/(
128*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)*(5*a^10*b + a^11
+ a^6*b^5 + 5*a^7*b^4 + 10*a^8*b^3 + 10*a^9*b^2)))*(-b^7*(a + b)^5)^(1/2)*
(11*a*b + (99*a^2)/16 + 5*b^2))/(5*a^10*b + a^11 + a^6*b^5 + 5*a^7*b^4 + 10
*a^8*b^3 + 10*a^9*b^2))*(11*a*b + (99*a^2)/16 + 5*b^2)*1i)/(5*a^10*b + a^11
+ a^6*b^5 + 5*a^7*b^4 + 10*a^8*b^3 + 10*a^9*b^2))/((3350*a*b^14 + 1000*b^1
5 + (7315*a^2*b^13)/2 + (4597*a^3*b^12)/4 - (4325*a^4*b^11)/32 + (10281*a^5
*b^10)/128 + (8973*a^6*b^9)/512 - (25551*a^7*b^8)/1024 + (4235*a^8*b^7)/512
- (2475*a^9*b^6)/1024)/(4*a^18*b + a^19 + a^15*b^4 + 4*a^16*b^3 + 6*a^17*b
^2) + ((-b^7*(a + b)^5)^(1/2)*((tan(e + f*x)*(199680*a*b^12 + 51200*b^13 +
287488*a^2*b^11 + 178560*a^3*b^10 + 39240*a^4*b^9 - 36*a^5*b^8 - 1119*a^6*b
^7 - 1092*a^7*b^6 + 234*a^8*b^5 - 80*a^9*b^4 + 25*a^10*b^3))/(128*(4*a^13*b
+ a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)) + (((20*a^12*b^9 + 79*a^13*b
^8 + (457*a^14*b^7)/4 + (277*a^15*b^6)/4 + (25*a^16*b^5)/2 - 2*a^17*b^4 - (

```
7*a^18*b^3)/4 - (5*a^19*b^2)/4)/(4*a^18*b + a^19 + a^15*b^4 + 4*a^16*b^3 +
6*a^17*b^2) + (tan(e + f*x)*(-b^7*(a + b)^5)^(1/2)*(11*a*b + (99*a^2)/16 +
5*b^2)*(2048*a^12*b^7 + 9216*a^13*b^6 + 16384*a^14*b^5 + 14336*a^15*b^4 + 6
144*a^16*b^3 + 1024*a^17*b^2))/(128*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^
3 + 6*a^12*b^2)*(5*a^10*b + a^11 + a^6*b^5 + 5*a^7*b^4 + 10*a^8*b^3 + 10*a^
9*b^2)))*(-b^7*(a + b)^5)^(1/2)*(11*a*b + (99*a^2)/16 + 5*b^2))/(5*a^10*b +
 a^11 + a^6*b^5 + 5*a^7*b^4 + 10*a^8*b^3 + 10*a^9*b^2))*(11*a*b + (99*a^2)/
16 + 5*b^2))/(5*a^10*b + a^11 + a^6*b^5 + 5*a^7*b^4 + 10*a^8*b^3 + 10*a^9*b
^2) - ((-b^7*(a + b)^5)^(1/2)*((tan(e + f*x)*(199680*a*b^12 + 51200*b^13 +
287488*a^2*b^11 + 178560*a^3*b^10 + 39240*a^4*b^9 - 36*a^5*b^8 - 1119*a^6*b
^7 - 1092*a^7*b^6 + 234*a^8*b^5 - 80*a^9*b^4 + 25*a^10*b^3))/(128*(4*a^13*b
 + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)) - (((20*a^12*b^9 + 79*a^13*b
^8 + (457*a^14*b^7)/4 + (277*a^15*b^6)/4 + (25*a^16*b^5)/2 - 2*a^17*b^4 - (
7*a^18*b^3)/4 - (5*a^19*b^2)/4)/(4*a^18*b + a^19 + a^15*b^4 + 4*a^16*b^3 +
6*a^17*b^2) - (tan(e + f*x)*(-b^7*(a + b)^5)^(1/2)*(11*a*b + (99*a^2)/16 +
5*b^2)*(2048*a^12*b^7 + 9216*a^13*b^6 + 16384*a^14*b^5 + 14336*a^15*b^4 + 6
144*a^16*b^3 + 1024*a^17*b^2))/(128*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^
3 + 6*a^12*b^2)*(5*a^10*b + a^11 + a^6*b^5 + 5*a^7*b^4 + 10*a^8*b^3 + 10*a^
9*b^2)))*(-b^7*(a + b)^5)^(1/2)*(11*a*b + (99*a^2)/16 + 5*b^2))/(5*a^10*b +
 a^11 + a^6*b^5 + 5*a^7*b^4 + 10*a^8*b^3 + 10*a^9*b^2))*(11*a*b + (99*a^2)/
16 + 5*b^2))/(5*a^10*b + a^11 + a^6*b^5 + 5*a^7*b^4 + 10*a^8*b^3 + 10*a^9*b
^2)))*(-b^7*(a + b)^5)^(1/2)*(11*a*b + (99*a^2)/16 + 5*b^2)*2i)/(f*(5*a^10*
b + a^11 + a^6*b^5 + 5*a^7*b^4 + 10*a^8*b^3 + 10*a^9*b^2))
```

3.219 $\int \frac{1}{(a+b\sec^2(c+dx))^4} dx$

Optimal result

Integrand size = 14, antiderivative size = 204

$$\int \frac{1}{(a+b\sec^2(c+dx))^4} dx = \frac{x}{a^4} - \frac{\sqrt{b}(35a^3+70a^2b+56ab^2+16b^3)\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b}}\right)}{16a^4(a+b)^{7/2}d} - \frac{b\tan(c+dx)}{6a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^3} - \frac{b(11a+6b)\tan(c+dx)}{24a^2(a+b)^2d\left(a+b+b\tan^2(c+dx)\right)^2} - \frac{b(19a^2+22ab+8b^2)\tan(c+dx)}{16a^3(a+b)^3d\left(a+b+b\tan^2(c+dx)\right)}$$

```
[Out] x/a^4-1/16*(35*a^3+70*a^2*b+56*a*b^2+16*b^3)*arctan(b^(1/2)*tan(d*x+c)/(a+b
)^(1/2))*b^(1/2)/a^4/(a+b)^(7/2)/d-1/6*b*tan(d*x+c)/a/(a+b)/d/(a+b+b*tan(d*
x+c)^2)^3-1/24*b*(11*a+6*b)*tan(d*x+c)/a^2/(a+b)^2/d/(a+b+b*tan(d*x+c)^2)^2
-1/16*b*(19*a^2+22*a*b+8*b^2)*tan(d*x+c)/a^3/(a+b)^3/d/(a+b+b*tan(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used

$= \{4213, 425, 541, 536, 209, 211\}$

$$\int \frac{1}{\left(a+b\sec^2(c+dx)\right)^4}\,dx = \frac{x}{a^4} - \frac{b(11a+6b)\tan(c+dx)}{24a^2d(a+b)^2\left(a+b\tan^2(c+dx)+b\right)^2} - \frac{b(19a^2+22ab+8b^2)\tan(c+dx)}{16a^3d(a+b)^3\left(a+b\tan^2(c+dx)+b\right)} - \frac{\sqrt{b}(35a^3+70a^2b+56ab^2+16b^3)\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b}}\right)}{16a^4d(a+b)^{7/2}} - \frac{b\tan(c+dx)}{6ad(a+b)\left(a+b\tan^2(c+dx)+b\right)^3}$$

```
[In] Int[(a + b*Sec[c + d*x]^2)^(-4),x]
```

```
[Out] x/a^4 - (Sqrt[b]*(35*a^3 + 70*a^2*b + 56*a*b^2 + 16*b^3)*ArcTan[(Sqrt[b]*Ta
n[c + d*x])/Sqrt[a + b]])/(16*a^4*(a + b)^(7/2)*d) - (b*Tan[c + d*x])/(6*a*
(a + b)*d*(a + b + b*Tan[c + d*x]^2)^3) - (b*(11*a + 6*b)*Tan[c + d*x])/(24
*a^2*(a + b)^2*d*(a + b + b*Tan[c + d*x]^2)^2) - (b*(19*a^2 + 22*a*b + 8*b^
2)*Tan[c + d*x])/(16*a^3*(a + b)^3*d*(a + b + b*Tan[c + d*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^4}\,dx, x, \tan(c+dx)\right)}{d}\\
&= -\frac{b\tan(c+dx)}{6a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^3}+\frac{\text{Subst}\left(\int \frac{6a+b-5bx^2}{(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(c+dx)\right)}{6a(a+b)d}\\
&= -\frac{b\tan(c+dx)}{6a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^3}-\frac{b(11a+6b)\tan(c+dx)}{24a^2(a+b)^2d\left(a+b+b\tan^2(c+dx)\right)^2}\\
&\quad+\frac{\text{Subst}\left(\int \frac{3(8a^2+5ab+2b^2)-3b(11a+6b)x^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(c+dx)\right)}{24a^2(a+b)^2d}\\
&= -\frac{b\tan(c+dx)}{6a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^3}-\frac{b(11a+6b)\tan(c+dx)}{24a^2(a+b)^2d\left(a+b+b\tan^2(c+dx)\right)^2}\\
&\quad-\frac{b(19a^2+22ab+8b^2)\tan(c+dx)}{16a^3(a+b)^3d\left(a+b+b\tan^2(c+dx)\right)}\\
&\quad+\frac{\text{Subst}\left(\int \frac{3(16a^3+29a^2b+26ab^2+8b^3)-3b(19a^2+22ab+8b^2)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(c+dx)\right)}{48a^3(a+b)^3d}\\
&= -\frac{b\tan(c+dx)}{6a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^3}-\frac{b(11a+6b)\tan(c+dx)}{24a^2(a+b)^2d\left(a+b+b\tan^2(c+dx)\right)^2}\\
&\quad-\frac{b(19a^2+22ab+8b^2)\tan(c+dx)}{16a^3(a+b)^3d\left(a+b+b\tan^2(c+dx)\right)}+\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{a^4d}\\
&\quad-\frac{\left(b(35a^3+70a^2b+56ab^2+16b^3)\right)\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(c+dx)\right)}{16a^4(a+b)^3d}
\end{aligned}
$$

$$= \frac{x}{a^4} - \frac{\sqrt{b}(35a^3 + 70a^2b + 56ab^2 + 16b^3)\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b}}\right)}{16a^4(a+b)^{7/2}d}$$
$$- \frac{b\tan(c+dx)}{6a(a+b)d\,(a+b+b\tan^2(c+dx))^3} - \frac{b(11a+6b)\tan(c+dx)}{24a^2(a+b)^2d\,(a+b+b\tan^2(c+dx))^2}$$
$$- \frac{b(19a^2 + 22ab + 8b^2)\tan(c+dx)}{16a^3(a+b)^3d\,(a+b+b\tan^2(c+dx))}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.44 (sec) , antiderivative size = 1411, normalized size of antiderivative = 6.92

$$\int \frac{1}{(a+b\sec^2(c+dx))^4}\,dx$$
$$= \frac{(35a^3 + 70a^2b + 56ab^2 + 16b^3)\,(a+2b+a\cos(2c+2dx))^4\sec^8(c+dx)\left(\frac{b\arctan\left(\sec(dx)\left(\frac{\cos(2c)}{2\sqrt{a+b}\sqrt{b\cos(4c)-ib\sin(4c)}}\right.\right.}{}\right.}{}$$
$$+ \frac{(a+2b+a\cos(2c+2dx))\sec(2c)\sec^8(c+dx)\,(480a^6dx\cos(2c) + 3168a^5bdx\cos(2c) + 8928a^4b^2dx\cos}{}$$

```
[In] Integrate[(a + b*Sec[c + d*x]^2)^(-4),x]
```

```
[Out] ((35*a^3 + 70*a^2*b + 56*a*b^2 + 16*b^3)*(a + 2*b + a*Cos[2*c + 2*d*x])^4*S
ec[c + d*x]^8*((b*ArcTan[Sec[d*x]*(Cos[2*c]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*c]
- I*b*Sin[4*c]]) - ((I/2)*Sin[2*c])/(Sqrt[a + b]*Sqrt[b*Cos[4*c] - I*b*Sin[
4*c]]))*(-(a*Sin[d*x]) - 2*b*Sin[d*x] + a*Sin[2*c + d*x])]*Cos[2*c])/(256*a
^4*Sqrt[a + b]*d*Sqrt[b*Cos[4*c] - I*b*Sin[4*c]]) - ((I/256)*b*ArcTan[Sec[d
*x]*(Cos[2*c]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*c] - I*b*Sin[4*c]]) - ((I/2)*Sin[
2*c])/(Sqrt[a + b]*Sqrt[b*Cos[4*c] - I*b*Sin[4*c]]))*(-(a*Sin[d*x]) - 2*b*S
in[d*x] + a*Sin[2*c + d*x])]*Sin[2*c])/(a^4*Sqrt[a + b]*d*Sqrt[b*Cos[4*c] -
 I*b*Sin[4*c]])))/((a + b)^3*(a + b*Sec[c + d*x]^2)^4) + ((a + 2*b + a*Cos[
2*c + 2*d*x])*Sec[2*c]*Sec[c + d*x]^8*(480*a^6*d*x*Cos[2*c] + 3168*a^5*b*d*
x*Cos[2*c] + 8928*a^4*b^2*d*x*Cos[2*c] + 14112*a^3*b^3*d*x*Cos[2*c] + 13248
*a^2*b^4*d*x*Cos[2*c] + 6912*a*b^5*d*x*Cos[2*c] + 1536*b^6*d*x*Cos[2*c] + 3
60*a^6*d*x*Cos[2*d*x] + 2232*a^5*b*d*x*Cos[2*d*x] + 5688*a^4*b^2*d*x*Cos[2*
d*x] + 7272*a^3*b^3*d*x*Cos[2*d*x] + 4608*a^2*b^4*d*x*Cos[2*d*x] + 1152*a*b
^5*d*x*Cos[2*d*x] + 360*a^6*d*x*Cos[4*c + 2*d*x] + 2232*a^5*b*d*x*Cos[4*c +
 2*d*x] + 5688*a^4*b^2*d*x*Cos[4*c + 2*d*x] + 7272*a^3*b^3*d*x*Cos[4*c + 2*
d*x] + 4608*a^2*b^4*d*x*Cos[4*c + 2*d*x] + 1152*a*b^5*d*x*Cos[4*c + 2*d*x]
+ 144*a^6*d*x*Cos[2*c + 4*d*x] + 720*a^5*b*d*x*Cos[2*c + 4*d*x] + 1296*a^4*
b^2*d*x*Cos[2*c + 4*d*x] + 1008*a^3*b^3*d*x*Cos[2*c + 4*d*x] + 288*a^2*b^4*
d*x*Cos[2*c + 4*d*x] + 144*a^6*d*x*Cos[6*c + 4*d*x] + 720*a^5*b*d*x*Cos[6*c
```

```
 + 4*d*x] + 1296*a^4*b^2*d*x*Cos[6*c + 4*d*x] + 1008*a^3*b^3*d*x*Cos[6*c +
4*d*x] + 288*a^2*b^4*d*x*Cos[6*c + 4*d*x] + 24*a^6*d*x*Cos[4*c + 6*d*x] + 7
2*a^5*b*d*x*Cos[4*c + 6*d*x] + 72*a^4*b^2*d*x*Cos[4*c + 6*d*x] + 24*a^3*b^3
*d*x*Cos[4*c + 6*d*x] + 24*a^6*d*x*Cos[8*c + 6*d*x] + 72*a^5*b*d*x*Cos[8*c
+ 6*d*x] + 72*a^4*b^2*d*x*Cos[8*c + 6*d*x] + 24*a^3*b^3*d*x*Cos[8*c + 6*d*x
] + 870*a^5*b*Sin[2*c] + 4292*a^4*b^2*Sin[2*c] + 8792*a^3*b^3*Sin[2*c] + 99
36*a^2*b^4*Sin[2*c] + 5824*a*b^5*Sin[2*c] + 1408*b^6*Sin[2*c] - 870*a^5*b*S
in[2*d*x] - 3792*a^4*b^2*Sin[2*d*x] - 6432*a^3*b^3*Sin[2*d*x] - 4608*a^2*b^
4*Sin[2*d*x] - 1248*a*b^5*Sin[2*d*x] + 435*a^5*b*Sin[4*c + 2*d*x] + 2124*a^
4*b^2*Sin[4*c + 2*d*x] + 3972*a^3*b^3*Sin[4*c + 2*d*x] + 3072*a^2*b^4*Sin[4
*c + 2*d*x] + 864*a*b^5*Sin[4*c + 2*d*x] - 435*a^5*b*Sin[2*c + 4*d*x] - 137
4*a^4*b^2*Sin[2*c + 4*d*x] - 1248*a^3*b^3*Sin[2*c + 4*d*x] - 384*a^2*b^4*Si
n[2*c + 4*d*x] + 87*a^5*b*Sin[6*c + 4*d*x] + 366*a^4*b^2*Sin[6*c + 4*d*x] +
 408*a^3*b^3*Sin[6*c + 4*d*x] + 144*a^2*b^4*Sin[6*c + 4*d*x] - 87*a^5*b*Sin
[4*c + 6*d*x] - 116*a^4*b^2*Sin[4*c + 6*d*x] - 44*a^3*b^3*Sin[4*c + 6*d*x])
)/(3072*a^4*(a + b)^3*d*(a + b*Sec[c + d*x]^2)^4)
```

Maple [A] (verified)

Time = 3.25 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(dx+c))}{a^4}-\frac{b\left(\frac{\frac{b^2a\left(19a^2+22ab+8b^2\right)\tan(dx+c)^5}{16a^3+48a^2b+48a\,b^2+16b^3}+\frac{\left(17a^2+18ab+6b^2\right)ab\tan(dx+c)^3}{6a^2+12ab+6b^2}+\frac{\left(29a^2+26ab+8b^2\right)a\tan(dx+c)}{16a+16b}}{\left(a+b+b\tan(dx+c)^2\right)^3}+\frac{(35}{}\right.}{a^4}}{d}$
default	$\frac{\frac{\arctan(\tan(dx+c))}{a^4}-\frac{b\left(\frac{\frac{b^2a\left(19a^2+22ab+8b^2\right)\tan(dx+c)^5}{16a^3+48a^2b+48a\,b^2+16b^3}+\frac{\left(17a^2+18ab+6b^2\right)ab\tan(dx+c)^3}{6a^2+12ab+6b^2}+\frac{\left(29a^2+26ab+8b^2\right)a\tan(dx+c)}{16a+16b}}{\left(a+b+b\tan(dx+c)^2\right)^3}+\frac{(35}{}\right.}{a^4}}{d}$
risch	Expression too large to display

```
[In] int(1/(a+b*sec(d*x+c)^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^4*arctan(tan(d*x+c))-b/a^4*((1/16*b^2*a*(19*a^2+22*a*b+8*b^2)/(a^3
+3*a^2*b+3*a*b^2+b^3)*tan(d*x+c)^5+1/6*(17*a^2+18*a*b+6*b^2)*a*b/(a^2+2*a*b
+b^2)*tan(d*x+c)^3+1/16*(29*a^2+26*a*b+8*b^2)*a/(a+b)*tan(d*x+c))/(a+b+b*ta
n(d*x+c)^2)^3+1/16*(35*a^3+70*a^2*b+56*a*b^2+16*b^3)/(a^3+3*a^2*b+3*a*b^2+b
^3)/((a+b)*b)^(1/2)*arctan(b*tan(d*x+c)/((a+b)*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(188) = 376$.

Time = 0.36 (sec) , antiderivative size = 1323, normalized size of antiderivative = 6.49

$$\int \frac{1}{(a + b\sec^2(c + dx))^4} \, dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*sec(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] [1/192*(192*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*x*cos(d*x + c)^6 + 576*
(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*d*x*cos(d*x + c)^4 + 576*(a^4*b^2
 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*d*x*cos(d*x + c)^2 + 192*(a^3*b^3 + 3*a^2
*b^4 + 3*a*b^5 + b^6)*d*x + 3*((35*a^6 + 70*a^5*b + 56*a^4*b^2 + 16*a^3*b^3
)*cos(d*x + c)^6 + 35*a^3*b^3 + 70*a^2*b^4 + 56*a*b^5 + 16*b^6 + 3*(35*a^5*
b + 70*a^4*b^2 + 56*a^3*b^3 + 16*a^2*b^4)*cos(d*x + c)^4 + 3*(35*a^4*b^2 +
70*a^3*b^3 + 56*a^2*b^4 + 16*a*b^5)*cos(d*x + c)^2)*sqrt(-b/(a + b))*log(((
a^2 + 8*a*b + 8*b^2)*cos(d*x + c)^4 - 2*(3*a*b + 4*b^2)*cos(d*x + c)^2 + 4*
((a^2 + 3*a*b + 2*b^2)*cos(d*x + c)^3 - (a*b + b^2)*cos(d*x + c))*sqrt(-b/(
a + b))*sin(d*x + c) + b^2)/(a^2*cos(d*x + c)^4 + 2*a*b*cos(d*x + c)^2 + b^
2)) - 4*((87*a^5*b + 116*a^4*b^2 + 44*a^3*b^3)*cos(d*x + c)^5 + 2*(68*a^4*b
^2 + 83*a^3*b^3 + 30*a^2*b^4)*cos(d*x + c)^3 + 3*(19*a^3*b^3 + 22*a^2*b^4 +
 8*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*b^
3)*d*cos(d*x + c)^6 + 3*(a^9*b + 3*a^8*b^2 + 3*a^7*b^3 + a^6*b^4)*d*cos(d*x
 + c)^4 + 3*(a^8*b^2 + 3*a^7*b^3 + 3*a^6*b^4 + a^5*b^5)*d*cos(d*x + c)^2 +
(a^7*b^3 + 3*a^6*b^4 + 3*a^5*b^5 + a^4*b^6)*d), 1/96*(96*(a^6 + 3*a^5*b + 3
*a^4*b^2 + a^3*b^3)*d*x*cos(d*x + c)^6 + 288*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3
 + a^2*b^4)*d*x*cos(d*x + c)^4 + 288*(a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b
^5)*d*x*cos(d*x + c)^2 + 96*(a^3*b^3 + 3*a^2*b^4 + 3*a*b^5 + b^6)*d*x + 3*(
(35*a^6 + 70*a^5*b + 56*a^4*b^2 + 16*a^3*b^3)*cos(d*x + c)^6 + 35*a^3*b^3 +
 70*a^2*b^4 + 56*a*b^5 + 16*b^6 + 3*(35*a^5*b + 70*a^4*b^2 + 56*a^3*b^3 + 1
6*a^2*b^4)*cos(d*x + c)^4 + 3*(35*a^4*b^2 + 70*a^3*b^3 + 56*a^2*b^4 + 16*a*
b^5)*cos(d*x + c)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(d*x + c)^2 -
 b)*sqrt(b/(a + b))/(b*cos(d*x + c)*sin(d*x + c))) - 2*((87*a^5*b + 116*a^4
*b^2 + 44*a^3*b^3)*cos(d*x + c)^5 + 2*(68*a^4*b^2 + 83*a^3*b^3 + 30*a^2*b^4
)*cos(d*x + c)^3 + 3*(19*a^3*b^3 + 22*a^2*b^4 + 8*a*b^5)*cos(d*x + c))*sin(
d*x + c))/((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*b^3)*d*cos(d*x + c)^6 + 3*(a^9
*b + 3*a^8*b^2 + 3*a^7*b^3 + a^6*b^4)*d*cos(d*x + c)^4 + 3*(a^8*b^2 + 3*a^7
*b^3 + 3*a^6*b^4 + a^5*b^5)*d*cos(d*x + c)^2 + (a^7*b^3 + 3*a^6*b^4 + 3*a^5
*b^5 + a^4*b^6)*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec^2(c+dx))^4}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*sec(d*x+c)**2)**4,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. 2(188) = 376.

Time = 0.29 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.97

$$\int \frac{1}{(a+b\sec^2(c+dx))^4}\,dx = -\frac{\frac{3\,(35\,a^3b+70\,a^2b^2+56\,ab^3+16\,b^4)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{(a+b)b}}\right)}{(a^7+3\,a^6b+3\,a^5b^2+a^4b^3)\sqrt{(a+b)b}}+\frac{3\,(19\,a^2b^3+22\,ab^4+8\,b^5)\tan(dx+c)^5+8\,(17\,a^3b^2\cdots}{a^9+6\,a^8b+15\,a^7b^2+20\,a^6b^3+15\,a^5b^4+6\,a^4b^5+a^3b^6+(a^6b^3+3\,a^5b^4+3\,a^4b^5+a^3b^6)\,\mathrm{t}\cdots}}{48\,\cdots}$$

```
[In] integrate(1/(a+b*sec(d*x+c)^2)^4,x, algorithm="maxima")
[Out] -1/48*(3*(35*a^3*b + 70*a^2*b^2 + 56*a*b^3 + 16*b^4)*arctan(b*tan(d*x + c)/
sqrt((a + b)*b))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*sqrt((a + b)*b)) +
(3*(19*a^2*b^3 + 22*a*b^4 + 8*b^5)*tan(d*x + c)^5 + 8*(17*a^3*b^2 + 35*a^2*
b^3 + 24*a*b^4 + 6*b^5)*tan(d*x + c)^3 + 3*(29*a^4*b + 84*a^3*b^2 + 89*a^2*
b^3 + 42*a*b^4 + 8*b^5)*tan(d*x + c))/(a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*
b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6 + (a^6*b^3 + 3*a^5*b^4 + 3*a^4*b^5 +
 a^3*b^6)*tan(d*x + c)^6 + 3*(a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 +
 a^3*b^6)*tan(d*x + c)^4 + 3*(a^8*b + 5*a^7*b^2 + 10*a^6*b^3 + 10*a^5*b^4 +
 5*a^4*b^5 + a^3*b^6)*tan(d*x + c)^2) - 48*(d*x + c)/a^4)/d
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.59

$$\int \frac{1}{(a+b\sec^2(c+dx))^4}\,dx = -\frac{\frac{3\,(35\,a^3b+70\,a^2b^2+56\,ab^3+16\,b^4)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab+b^2}}\right)\right)}{(a^7+3\,a^6b+3\,a^5b^2+a^4b^3)\sqrt{ab+b^2}}+\frac{57\,a^2b^3\tan(dx+c)^5+66\,ab^4\tan(dx+c)^5+24\,b^5\tan(dx+c)^5\cdots}{\cdots}}{\cdots}$$

```
[In] integrate(1/(a+b*sec(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] -1/48*(3*(35*a^3*b + 70*a^2*b^2 + 56*a*b^3 + 16*b^4)*(pi*floor((d*x + c)/pi
 + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b + b^2)))/((a^7 + 3*a^6*b +
3*a^5*b^2 + a^4*b^3)*sqrt(a*b + b^2)) + (57*a^2*b^3*tan(d*x + c)^5 + 66*a*b
^4*tan(d*x + c)^5 + 24*b^5*tan(d*x + c)^5 + 136*a^3*b^2*tan(d*x + c)^3 + 28
0*a^2*b^3*tan(d*x + c)^3 + 192*a*b^4*tan(d*x + c)^3 + 48*b^5*tan(d*x + c)^3
 + 87*a^4*b*tan(d*x + c) + 252*a^3*b^2*tan(d*x + c) + 267*a^2*b^3*tan(d*x +
 c) + 126*a*b^4*tan(d*x + c) + 24*b^5*tan(d*x + c))/((a^6 + 3*a^5*b + 3*a^4
*b^2 + a^3*b^3)*(b*tan(d*x + c)^2 + a + b)^3) - 48*(d*x + c)/a^4)/d
```

Mupad [B] (verification not implemented)

Time = 25.03 (sec) , antiderivative size = 4506, normalized size of antiderivative = 22.09

$$\int \frac{1}{\left(a+b\sec^2(c+dx)\right)^4}\,dx = \text{Too large to display}$$

```
[In] int(1/(a + b/cos(c + d*x)^2)^4,x)
```

```
[Out] atan((((((2*a^8*b^8 + (25*a^9*b^7)/2 + (131*a^10*b^6)/4 + (189*a^11*b^5)/4
+ (161*a^12*b^4)/4 + (77*a^13*b^3)/4 + 4*a^14*b^2)*1i)/(2*(6*a^14*b + a^15
+ a^9*b^6 + 6*a^10*b^5 + 15*a^11*b^4 + 20*a^12*b^3 + 15*a^13*b^2)) - (tan(c
 + d*x)*(2048*a^8*b^9 + 13312*a^9*b^8 + 36864*a^10*b^7 + 56320*a^11*b^6 + 5
1200*a^12*b^5 + 27648*a^13*b^4 + 8192*a^14*b^3 + 1024*a^15*b^2))/(512*a^4*(
6*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b
^2)))/(2*a^4) + (tan(c + d*x)*(3328*a*b^8 + 512*b^9 + 9216*a^2*b^7 + 14080*
a^3*b^6 + 12660*a^4*b^5 + 6436*a^5*b^4 + 1481*a^6*b^3))/(256*(6*a^11*b + a^
12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2)))/a^4 - (
(((2*a^8*b^8 + (25*a^9*b^7)/2 + (131*a^10*b^6)/4 + (189*a^11*b^5)/4 + (161*
a^12*b^4)/4 + (77*a^13*b^3)/4 + 4*a^14*b^2)*1i)/(2*(6*a^14*b + a^15 + a^9*b
^6 + 6*a^10*b^5 + 15*a^11*b^4 + 20*a^12*b^3 + 15*a^13*b^2)) + (tan(c + d*x)
*(2048*a^8*b^9 + 13312*a^9*b^8 + 36864*a^10*b^7 + 56320*a^11*b^6 + 51200*a^
12*b^5 + 27648*a^13*b^4 + 8192*a^14*b^3 + 1024*a^15*b^2))/(512*a^4*(6*a^11*
b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2)))/(
2*a^4) - (tan(c + d*x)*(3328*a*b^8 + 512*b^9 + 9216*a^2*b^7 + 14080*a^3*b^6
 + 12660*a^4*b^5 + 6436*a^5*b^4 + 1481*a^6*b^3))/(256*(6*a^11*b + a^12 + a^
6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2)))/a^4)/((((((2*a
^8*b^8 + (25*a^9*b^7)/2 + (131*a^10*b^6)/4 + (189*a^11*b^5)/4 + (161*a^12*b
^4)/4 + (77*a^13*b^3)/4 + 4*a^14*b^2)*1i)/(2*(6*a^14*b + a^15 + a^9*b^6 + 6
*a^10*b^5 + 15*a^11*b^4 + 20*a^12*b^3 + 15*a^13*b^2)) - (tan(c + d*x)*(2048
*a^8*b^9 + 13312*a^9*b^8 + 36864*a^10*b^7 + 56320*a^11*b^6 + 51200*a^12*b^5
 + 27648*a^13*b^4 + 8192*a^14*b^3 + 1024*a^15*b^2))/(512*a^4*(6*a^11*b + a^
12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2)))*1i)/(2*
a^4) + (tan(c + d*x)*(3328*a*b^8 + 512*b^9 + 9216*a^2*b^7 + 14080*a^3*b^6 +
```

12660*a^4*b^5 + 6436*a^5*b^4 + 1481*a^6*b^3)*1i)/(256*(6*a^11*b + a^12 + a
^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2)))/a^4 + (((((2*
a^8*b^8 + (25*a^9*b^7)/2 + (131*a^10*b^6)/4 + (189*a^11*b^5)/4 + (161*a^12*
b^4)/4 + (77*a^13*b^3)/4 + 4*a^14*b^2)*1i)/(2*(6*a^14*b + a^15 + a^9*b^6 +
6*a^10*b^5 + 15*a^11*b^4 + 20*a^12*b^3 + 15*a^13*b^2)) + (tan(c + d*x)*(204
8*a^8*b^9 + 13312*a^9*b^8 + 36864*a^10*b^7 + 56320*a^11*b^6 + 51200*a^12*b^
5 + 27648*a^13*b^4 + 8192*a^14*b^3 + 1024*a^15*b^2))/(512*a^4*(6*a^11*b + a
^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2)))*1i)/(2
*a^4) - (tan(c + d*x)*(3328*a*b^8 + 512*b^9 + 9216*a^2*b^7 + 14080*a^3*b^6
+ 12660*a^4*b^5 + 6436*a^5*b^4 + 1481*a^6*b^3)*1i)/(256*(6*a^11*b + a^12 +
a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2)))/a^4 + ((25*a
*b^7)/4 + b^8 + (131*a^2*b^6)/8 + (721*a^3*b^5)/32 + (525*a^4*b^4)/32 + (66
5*a^5*b^3)/128)/(6*a^14*b + a^15 + a^9*b^6 + 6*a^10*b^5 + 15*a^11*b^4 + 20*
a^12*b^3 + 15*a^13*b^2)))/(a^4*d) - ((tan(c + d*x)^3*(18*a*b^3 + 6*b^4 + 17
*a^2*b^2))/(6*a^3*(a + b)^2) + (tan(c + d*x)^5*(22*a*b^4 + 8*b^5 + 19*a^2*b
^3))/(16*a^3*(a + b)^3) + (tan(c + d*x)*(26*a*b^2 + 29*a^2*b + 8*b^3))/(16*
a^3*(a + b)))/(d*(tan(c + d*x)^4*(3*a*b^2 + 3*b^3) + 3*a*b^2 + 3*a^2*b + ta
n(c + d*x)^2*(6*a*b^2 + 3*a^2*b + 3*b^3) + a^3 + b^3 + b^3*tan(c + d*x)^6))
 + (atan((((-b*(a + b)^7)^(1/2)*((tan(c + d*x)*(3328*a*b^8 + 512*b^9 + 9216
*a^2*b^7 + 14080*a^3*b^6 + 12660*a^4*b^5 + 6436*a^5*b^4 + 1481*a^6*b^3))/(1
28*(6*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^
10*b^2)) - ((-b*(a + b)^7)^(1/2)*((2*a^8*b^8 + (25*a^9*b^7)/2 + (131*a^10*b
^6)/4 + (189*a^11*b^5)/4 + (161*a^12*b^4)/4 + (77*a^13*b^3)/4 + 4*a^14*b^2)
/(6*a^14*b + a^15 + a^9*b^6 + 6*a^10*b^5 + 15*a^11*b^4 + 20*a^12*b^3 + 15*a
^13*b^2) - (tan(c + d*x)*(-b*(a + b)^7)^(1/2)*(56*a*b^2 + 70*a^2*b + 35*a^3
 + 16*b^3)*(2048*a^8*b^9 + 13312*a^9*b^8 + 36864*a^10*b^7 + 56320*a^11*b^6
+ 51200*a^12*b^5 + 27648*a^13*b^4 + 8192*a^14*b^3 + 1024*a^15*b^2))/(4096*(
6*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b
^2)*(7*a^10*b + a^11 + a^4*b^7 + 7*a^5*b^6 + 21*a^6*b^5 + 35*a^7*b^4 + 35*a
^8*b^3 + 21*a^9*b^2)))*(56*a*b^2 + 70*a^2*b + 35*a^3 + 16*b^3))/(32*(7*a^10
*b + a^11 + a^4*b^7 + 7*a^5*b^6 + 21*a^6*b^5 + 35*a^7*b^4 + 35*a^8*b^3 + 21
*a^9*b^2)))*(56*a*b^2 + 70*a^2*b + 35*a^3 + 16*b^3)*1i)/(32*(7*a^10*b + a^1
1 + a^4*b^7 + 7*a^5*b^6 + 21*a^6*b^5 + 35*a^7*b^4 + 35*a^8*b^3 + 21*a^9*b^2
)) + ((-b*(a + b)^7)^(1/2)*((tan(c + d*x)*(3328*a*b^8 + 512*b^9 + 9216*a^2*
b^7 + 14080*a^3*b^6 + 12660*a^4*b^5 + 6436*a^5*b^4 + 1481*a^6*b^3))/(128*(6
*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^
2)) + ((-b*(a + b)^7)^(1/2)*((2*a^8*b^8 + (25*a^9*b^7)/2 + (131*a^10*b^6)/4
 + (189*a^11*b^5)/4 + (161*a^12*b^4)/4 + (77*a^13*b^3)/4 + 4*a^14*b^2)/(6*a
^14*b + a^15 + a^9*b^6 + 6*a^10*b^5 + 15*a^11*b^4 + 20*a^12*b^3 + 15*a^13*b
^2) + (tan(c + d*x)*(-b*(a + b)^7)^(1/2)*(56*a*b^2 + 70*a^2*b + 35*a^3 + 16
b^3)(2048*a^8*b^9 + 13312*a^9*b^8 + 36864*a^10*b^7 + 56320*a^11*b^6 + 512
00*a^12*b^5 + 27648*a^13*b^4 + 8192*a^14*b^3 + 1024*a^15*b^2))/(4096*(6*a^1
1*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2)*(
7*a^10*b + a^11 + a^4*b^7 + 7*a^5*b^6 + 21*a^6*b^5 + 35*a^7*b^4 + 35*a^8*b^
3 + 21*a^9*b^2)))*(56*a*b^2 + 70*a^2*b + 35*a^3 + 16*b^3))/(32*(7*a^10*b +

```
a^11 + a^4*b^7 + 7*a^5*b^6 + 21*a^6*b^5 + 35*a^7*b^4 + 35*a^8*b^3 + 21*a^9*
b^2)))*(56*a*b^2 + 70*a^2*b + 35*a^3 + 16*b^3)*1i)/(32*(7*a^10*b + a^11 + a
^4*b^7 + 7*a^5*b^6 + 21*a^6*b^5 + 35*a^7*b^4 + 35*a^8*b^3 + 21*a^9*b^2)))/(
((25*a*b^7)/4 + b^8 + (131*a^2*b^6)/8 + (721*a^3*b^5)/32 + (525*a^4*b^4)/32
 + (665*a^5*b^3)/128)/(6*a^14*b + a^15 + a^9*b^6 + 6*a^10*b^5 + 15*a^11*b^4
 + 20*a^12*b^3 + 15*a^13*b^2) - ((-b*(a + b)^7)^(1/2)*((tan(c + d*x)*(3328*
a*b^8 + 512*b^9 + 9216*a^2*b^7 + 14080*a^3*b^6 + 12660*a^4*b^5 + 6436*a^5*b
^4 + 1481*a^6*b^3))/(128*(6*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^
4 + 20*a^9*b^3 + 15*a^10*b^2)) - ((-b*(a + b)^7)^(1/2)*((2*a^8*b^8 + (25*a^
9*b^7)/2 + (131*a^10*b^6)/4 + (189*a^11*b^5)/4 + (161*a^12*b^4)/4 + (77*a^1
3*b^3)/4 + 4*a^14*b^2)/(6*a^14*b + a^15 + a^9*b^6 + 6*a^10*b^5 + 15*a^11*b^
4 + 20*a^12*b^3 + 15*a^13*b^2) - (tan(c + d*x)*(-b*(a + b)^7)^(1/2)*(56*a*b
^2 + 70*a^2*b + 35*a^3 + 16*b^3)*(2048*a^8*b^9 + 13312*a^9*b^8 + 36864*a^10
*b^7 + 56320*a^11*b^6 + 51200*a^12*b^5 + 27648*a^13*b^4 + 8192*a^14*b^3 + 1
024*a^15*b^2))/(4096*(6*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 +
20*a^9*b^3 + 15*a^10*b^2)*(7*a^10*b + a^11 + a^4*b^7 + 7*a^5*b^6 + 21*a^6*b
^5 + 35*a^7*b^4 + 35*a^8*b^3 + 21*a^9*b^2)))*(56*a*b^2 + 70*a^2*b + 35*a^3
+ 16*b^3))/(32*(7*a^10*b + a^11 + a^4*b^7 + 7*a^5*b^6 + 21*a^6*b^5 + 35*a^7
*b^4 + 35*a^8*b^3 + 21*a^9*b^2)))*(56*a*b^2 + 70*a^2*b + 35*a^3 + 16*b^3))/
(32*(7*a^10*b + a^11 + a^4*b^7 + 7*a^5*b^6 + 21*a^6*b^5 + 35*a^7*b^4 + 35*a
^8*b^3 + 21*a^9*b^2)) + ((-b*(a + b)^7)^(1/2)*((tan(c + d*x)*(3328*a*b^8 +
512*b^9 + 9216*a^2*b^7 + 14080*a^3*b^6 + 12660*a^4*b^5 + 6436*a^5*b^4 + 148
1*a^6*b^3))/(128*(6*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a
^9*b^3 + 15*a^10*b^2)) + ((-b*(a + b)^7)^(1/2)*((2*a^8*b^8 + (25*a^9*b^7)/2
 + (131*a^10*b^6)/4 + (189*a^11*b^5)/4 + (161*a^12*b^4)/4 + (77*a^13*b^3)/4
 + 4*a^14*b^2)/(6*a^14*b + a^15 + a^9*b^6 + 6*a^10*b^5 + 15*a^11*b^4 + 20*a
^12*b^3 + 15*a^13*b^2) + (tan(c + d*x)*(-b*(a + b)^7)^(1/2)*(56*a*b^2 + 70*
a^2*b + 35*a^3 + 16*b^3)*(2048*a^8*b^9 + 13312*a^9*b^8 + 36864*a^10*b^7 + 5
6320*a^11*b^6 + 51200*a^12*b^5 + 27648*a^13*b^4 + 8192*a^14*b^3 + 1024*a^15
*b^2))/(4096*(6*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b
^3 + 15*a^10*b^2)*(7*a^10*b + a^11 + a^4*b^7 + 7*a^5*b^6 + 21*a^6*b^5 + 35*
a^7*b^4 + 35*a^8*b^3 + 21*a^9*b^2)))*(56*a*b^2 + 70*a^2*b + 35*a^3 + 16*b^3
))/(32*(7*a^10*b + a^11 + a^4*b^7 + 7*a^5*b^6 + 21*a^6*b^5 + 35*a^7*b^4 + 3
5*a^8*b^3 + 21*a^9*b^2)))*(56*a*b^2 + 70*a^2*b + 35*a^3 + 16*b^3))/(32*(7*a
^10*b + a^11 + a^4*b^7 + 7*a^5*b^6 + 21*a^6*b^5 + 35*a^7*b^4 + 35*a^8*b^3 +
 21*a^9*b^2))))*(-b*(a + b)^7)^(1/2)*(56*a*b^2 + 70*a^2*b + 35*a^3 + 16*b^3
)*1i)/(16*d*(7*a^10*b + a^11 + a^4*b^7 + 7*a^5*b^6 + 21*a^6*b^5 + 35*a^7*b^
4 + 35*a^8*b^3 + 21*a^9*b^2))
```

3.220 $\int (a-a\sec^2(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 134

$$\int (a-a\sec^2(c+dx))^{7/2}\,dx = -\frac{a^3\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d} - \frac{a^3\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d} + \frac{a^3\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d} - \frac{a^3\tan^5(c+dx)\sqrt{-a\tan^2(c+dx)}}{6d}$$

```
[Out] -a^3*cot(d*x+c)*ln(cos(d*x+c))*(-a*tan(d*x+c)^2)^(1/2)/d-1/2*a^3*(-a*tan(d*
x+c)^2)^(1/2)*tan(d*x+c)/d+1/4*a^3*(-a*tan(d*x+c)^2)^(1/2)*tan(d*x+c)^3/d-1
/6*a^3*(-a*tan(d*x+c)^2)^(1/2)*tan(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {4206, 3739, 3554, 3556}

$$\int (a-a\sec^2(c+dx))^{7/2}\,dx = -\frac{a^3\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d} - \frac{a^3\tan^5(c+dx)\sqrt{-a\tan^2(c+dx)}}{6d} + \frac{a^3\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d} - \frac{a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\log(\cos(c+dx))}{d}$$

```
[In] Int[(a - a*Sec[c + d*x]^2)^(7/2),x]
```

```
[Out] -((a^3*Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[-(a*Tan[c + d*x]^2)])/d) - (a^3*
Tan[c + d*x]*Sqrt[-(a*Tan[c + d*x]^2)])/(2*d) + (a^3*Tan[c + d*x]^3*Sqrt[-(
a*Tan[c + d*x]^2)])/(4*d) - (a^3*Tan[c + d*x]^5*Sqrt[-(a*Tan[c + d*x]^2)])/
(6*d)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4206

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-a\tan^2(c+dx)\right)^{7/2}\,dx\\
&= -\left(\left(a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)\int \tan^7(c+dx)\,dx\right)\\
&= -\frac{a^3\tan^5(c+dx)\sqrt{-a\tan^2(c+dx)}}{6d}+\left(a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)\int \tan^5(c+dx)\,dx\\
&= \frac{a^3\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d}-\frac{a^3\tan^5(c+dx)\sqrt{-a\tan^2(c+dx)}}{6d}\\
&\quad -\left(a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)\int \tan^3(c+dx)\,dx
\end{aligned}
$$

$$= -\frac{a^3\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d} + \frac{a^3\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d} - \frac{a^3\tan^5(c+dx)\sqrt{-a\tan^2(c+dx)}}{6d} + \left(a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)\int\tan(c+dx)\,dx$$

$$= -\frac{a^3\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d} - \frac{a^3\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d} + \frac{a^3\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d} - \frac{a^3\tan^5(c+dx)\sqrt{-a\tan^2(c+dx)}}{6d}$$

Mathematica [A] (verified)

Time = 2.54 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.52

$$\int(a-a\sec^2(c+dx))^{7/2}\,dx = \frac{\cot^7(c+dx)\left(-a\tan^2(c+dx)\right)^{7/2}\left(12\log(\cos(c+dx))+6\tan^2(c+dx)-3\tan^4(c+dx)+2\,\mathrm{t}\right.}{12d}$$

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(7/2),x]
[Out] (Cot[c + d*x]^7*(-(a*Tan[c + d*x]^2))^(7/2)*(12*Log[Cos[c + d*x]] + 6*Tan[c
 + d*x]^2 - 3*Tan[c + d*x]^4 + 2*Tan[c + d*x]^6))/(12*d)
```

Maple [A] (verified)

Time = 5.16 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.08

method	result
default	$-\frac{\sqrt{-a\tan(dx+c)^2}\,a^3\left(12\cos(dx+c)^6\ln(-\cot(dx+c)+\csc(dx+c)-1)+12\cos(dx+c)^6\ln(-\cot(dx+c)+\csc(dx+c)+1)-12\cos(dx+c)^6\right.}{12d}$
risch	$\frac{a^3\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,x}{e^{2i(dx+c)}-1} - \frac{2a^3\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(dx+c)}{\left(e^{2i(dx+c)}-1\right)d} - \frac{2ia^3\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,\left(9\,e^{10i(dx+c)}+18\right.}{3\left(e^{2i(dx+c)}\right.}$

```
[In] int((a-a*sec(d*x+c)^2)^(7/2),x,method=_RETURNVERBOSE)
[Out] -1/12/d*(-a*tan(d*x+c)^2)^(1/2)*a^3*(12*cos(d*x+c)^6*ln(-cot(d*x+c)+csc(d*x
+c)-1)+12*cos(d*x+c)^6*ln(-cot(d*x+c)+csc(d*x+c)+1)-12*cos(d*x+c)^6*ln(2/(c
os(d*x+c)+1))-11*cos(d*x+c)^6+18*cos(d*x+c)^4-9*cos(d*x+c)^2+2)*sec(d*x+c)^
5*csc(d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.75

$$\int \left(a-a\sec^2(c+dx)\right)^{7/2}\,dx = -\frac{\left(12\,a^3\cos\left(dx+c\right)^6\log\left(-\cos\left(dx+c\right)\right)+18\,a^3\cos\left(dx+c\right)^4-9\,a^3\cos\left(dx+c\right)^2+2\,a^3\right)\sqrt{\frac{a\cos(dx+c)^2-a}{\cos(dx+c)^2}}}{12\,d\cos\left(dx+c\right)^5\sin\left(dx+c\right)}$$

```
[In] integrate((a-a*sec(d*x+c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/12*(12*a^3*cos(d*x + c)^6*log(-cos(d*x + c)) + 18*a^3*cos(d*x + c)^4 - 9
*a^3*cos(d*x + c)^2 + 2*a^3)*sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)/(d
*cos(d*x + c)^5*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \left(a-a\sec^2(c+dx)\right)^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate((a-a*sec(d*x+c)**2)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.60

$$\int \left(a-a\sec^2(c+dx)\right)^{7/2}\,dx = -\frac{2\sqrt{-a}a^3\tan\left(dx+c\right)^6-3\sqrt{-a}a^3\tan\left(dx+c\right)^4+6\sqrt{-a}a^3\tan\left(dx+c\right)^2-6\sqrt{-a}a^3\log\left(\tan\left(dx+c\right)^2+\right.}{12\,d}$$

```
[In] integrate((a-a*sec(d*x+c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/12*(2*sqrt(-a)*a^3*tan(d*x + c)^6 - 3*sqrt(-a)*a^3*tan(d*x + c)^4 + 6*sq
rt(-a)*a^3*tan(d*x + c)^2 - 6*sqrt(-a)*a^3*log(tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.55 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.62

$$\int \left(a - a\sec^2(c + dx)\right)^{7/2} dx =$$

$$-\frac{6\sqrt{-a}a^3\log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + \frac{1}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2} + 2\right) - 6\sqrt{-a}a^3\log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + \frac{1}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2} - 2\right.}{}$$

```
[In] integrate((a-a*sec(d*x+c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/12*(6*sqrt(-a)*a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2
 + 2) - 6*sqrt(-a)*a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^
2 - 2) + (11*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)^3*sqrt(-a)
*a^3 - 90*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)^2*sqrt(-a)*a^
3 + 276*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)*sqrt(-a)*a^3 -
408*sqrt(-a)*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2 - 2)^3
)/d
```

Mupad [F(-1)]

Timed out.

$$\int \left(a - a\sec^2(c + dx)\right)^{7/2} dx = \int \left(a - \frac{a}{\cos\left(c + d\,x\right)^2}\right)^{7/2} dx$$

```
[In] int((a - a/cos(c + d*x)^2)^(7/2),x)
```

```
[Out] int((a - a/cos(c + d*x)^2)^(7/2), x)
```

3.221 $\int (a - a\sec^2(c + dx))^{5/2}\, dx$

Optimal result

Integrand size = 17, antiderivative size = 101

$$\int \left(a - a\sec^2(c + dx)\right)^{5/2}\, dx = -\frac{a^2\cot(c + dx)\log(\cos(c + dx))\sqrt{-a\tan^2(c + dx)}}{d} - \frac{a^2\tan(c + dx)\sqrt{-a\tan^2(c + dx)}}{2d} + \frac{a^2\tan^3(c + dx)\sqrt{-a\tan^2(c + dx)}}{4d}$$

```
[Out] -a^2*cot(d*x+c)*ln(cos(d*x+c))*(-a*tan(d*x+c)^2)^(1/2)/d-1/2*a^2*(-a*tan(d*
x+c)^2)^(1/2)*tan(d*x+c)/d+1/4*a^2*(-a*tan(d*x+c)^2)^(1/2)*tan(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4206, 3739, 3554, 3556}

$$\int \left(a - a\sec^2(c + dx)\right)^{5/2}\, dx = -\frac{a^2\tan(c + dx)\sqrt{-a\tan^2(c + dx)}}{2d} + \frac{a^2\tan^3(c + dx)\sqrt{-a\tan^2(c + dx)}}{4d} - \frac{a^2\cot(c + dx)\sqrt{-a\tan^2(c + dx)}\log(\cos(c + dx))}{d}$$

```
[In] Int[(a - a*Sec[c + d*x]^2)^(5/2),x]
```

```
[Out] -((a^2*Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[-(a*Tan[c + d*x]^2)])/d) - (a^2*
Tan[c + d*x]*Sqrt[-(a*Tan[c + d*x]^2)])/(2*d) + (a^2*Tan[c + d*x]^3*Sqrt[-(
a*Tan[c + d*x]^2)])/(4*d)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4206

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-a\tan^2(c+dx)\right)^{5/2}\,dx\\
&= \left(a^2\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)\int \tan^5(c+dx)\,dx\\
&= \frac{a^2\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d} - \left(a^2\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)\int \tan^3(c+dx)\,dx\\
&= -\frac{a^2\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d} + \frac{a^2\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d}\\
&\quad + \left(a^2\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)\int \tan(c+dx)\,dx\\
&= -\frac{a^2\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d}\\
&\quad - \frac{a^2\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d} + \frac{a^2\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.59

$$\int \left(a-a\sec^2(c+dx)\right)^{5/2}\,dx = -\frac{\cot^5(c+dx)\left(-a\tan^2(c+dx)\right)^{5/2}\left(4\log(\cos(c+dx))+2\tan^2(c+dx)-\tan^4(c+dx)\right)}{4d}$$

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(5/2),x]
[Out] -1/4*(Cot[c + d*x]^5*(-(a*Tan[c + d*x]^2))^(5/2)*(4*Log[Cos[c + d*x]] + 2*T
an[c + d*x]^2 - Tan[c + d*x]^4))/d
```

Maple [A] (verified)

Time = 3.37 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.34

method	result
default	$-\frac{\sqrt{-a\tan(dx+c)^2}\,a^2\left(4\cos(dx+c)^4\ln(-\cot(dx+c)+\csc(dx+c)-1)+4\cos(dx+c)^4\ln(-\cot(dx+c)+\csc(dx+c)+1)-4\cos(dx+c)^4\ln\right.}{4d}$
risch	$\frac{a^2\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,x}{e^{2i(dx+c)}-1}-\frac{2a^2\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(dx+c)}{\left(e^{2i(dx+c)}-1\right)d}-\frac{4ia^2\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\left(e^{6i(dx+c)}+e^{4i}\right.}{\left(e^{2i(dx+c)}-1\right)\left(e^{2i(dx+c)}+\right.}$

```
[In] int((a-a*sec(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/4/d*(-a*tan(d*x+c)^2)^(1/2)*a^2*(4*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c
)-1)+4*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)+1)-4*cos(d*x+c)^4*ln(2/(cos(d
*x+c)+1))-3*cos(d*x+c)^4+4*cos(d*x+c)^2-1)*sec(d*x+c)^3*csc(d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.86

$$\int \left(a-a\sec^2(c+dx)\right)^{5/2}\,dx = -\frac{\left(4\,a^2\cos\left(dx+c\right)^4\log\left(-\cos\left(dx+c\right)\right)+4\,a^2\cos\left(dx+c\right)^2-a^2\right)\sqrt{\frac{a\cos(dx+c)^2-a}{\cos(dx+c)^2}}}{4\,d\cos\left(dx+c\right)^3\sin\left(dx+c\right)}$$

```
[In] integrate((a-a*sec(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a^2*cos(d*x + c)^4*log(-cos(d*x + c)) + 4*a^2*cos(d*x + c)^2 - a^2)
*sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)/(d*cos(d*x + c)^3*sin(d*x + c)
)
```

Sympy [F]

$$\int \left(a - a\sec^2(c + dx)\right)^{5/2} dx = \int \left(-a\sec^2\left(c + dx\right) + a\right)^{\frac{5}{2}} dx$$

```
[In] integrate((a-a*sec(d*x+c)**2)**(5/2),x)
[Out] Integral((-a*sec(c + d*x)**2 + a)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.61

$$\int \left(a - a\sec^2(c + dx)\right)^{5/2} dx = \frac{\sqrt{-a}a^2\tan\left(dx + c\right)^4 - 2\sqrt{-a}a^2\tan\left(dx + c\right)^2 + 2\sqrt{-a}a^2\log\left(\tan\left(dx + c\right)^2 + 1\right)}{4\,d}$$

```
[In] integrate((a-a*sec(d*x+c)^2)^(5/2),x, algorithm="maxima")
[Out] 1/4*(sqrt(-a)*a^2*tan(d*x + c)^4 - 2*sqrt(-a)*a^2*tan(d*x + c)^2 + 2*sqrt(-
a)*a^2*log(tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.80

$$\int \left(a - a\sec^2(c + dx)\right)^{5/2} dx = -\frac{2\sqrt{-a}a^2\log\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + \frac{1}{\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2} + 2\right) - 2\sqrt{-a}a^2\log\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + \frac{1}{\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2} - 2 \right. \cdots}{4\,d}$$

```
[In] integrate((a-a*sec(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/4*(2*sqrt(-a)*a^2*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2
+ 2) - 2*sqrt(-a)*a^2*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2
 - 2) + (3*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)^2*sqrt(-a)*a
^2 - 20*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)*sqrt(-a)*a^2 +
44*sqrt(-a)*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2 - 2)^2)
/d
```

Mupad [F(-1)]

Timed out.

$$\int \left(a-a\sec^2(c+dx)\right)^{5/2}\,dx=\int \left(a-\frac{a}{\cos\left(c+d\,x\right)^2}\right)^{5/2}\,dx$$

```
[In] int((a - a/cos(c + d*x)^2)^(5/2),x)
```

```
[Out] int((a - a/cos(c + d*x)^2)^(5/2), x)
```

3.222 $\int (a - a\sec^2(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 64

$$\int (a - a\sec^2(c+dx))^{3/2}\,dx = -\frac{a\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d} - \frac{a\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}$$

```
[Out] -a*cot(d*x+c)*ln(cos(d*x+c))*(-a*tan(d*x+c)^2)^(1/2)/d-1/2*a*(-a*tan(d*x+c)
^2)^(1/2)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {4206, 3739, 3554, 3556}

$$\int (a - a\sec^2(c+dx))^{3/2}\,dx = -\frac{a\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d} - \frac{a\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\log(\cos(c+dx))}{d}$$

```
[In] Int[(a - a*Sec[c + d*x]^2)^(3/2),x]
```

```
[Out] -((a*Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[-(a*Tan[c + d*x]^2)])/d) - (a*Tan[
c + d*x]*Sqrt[-(a*Tan[c + d*x]^2)])/(2*d)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4206

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-a\tan^2(c+dx)\right)^{3/2}\,dx\\ &= -\left(\left(a\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)\int \tan^3(c+dx)\,dx\right)\\ &= -\frac{a\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}+\left(a\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)\int \tan(c+dx)\,dx\\ &= -\frac{a\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d}-\frac{a\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75

$$\int \left(a-a\sec^2(c+dx)\right)^{3/2}\,dx=\frac{\cot^3(c+dx)\left(-a\tan^2(c+dx)\right)^{3/2}\left(2\log(\cos(c+dx))+\tan^2(c+dx)\right)}{2d}$$

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(3/2),x]
[Out] (Cot[c + d*x]^3*(-(a*Tan[c + d*x]^2))^(3/2)*(2*Log[Cos[c + d*x]] + Tan[c +
d*x]^2))/(2*d)
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.53

method	result
default	$-\frac{\sqrt{-a\tan(dx+c)^2}\,a\left(-2\ln\left(\frac{2}{\cos(dx+c)+1}\right)\cot(dx+c)+2\ln(-\cot(dx+c)+\csc(dx+c)+1)\cot(dx+c)+2\ln(-\cot(dx+c)+\csc(dx+c)-1 \cdots\right.}{2d}$
risch	$-\frac{a\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,\left(ie^{4i(dx+c)}\ln\left(e^{2i(dx+c)}+1\right)+e^{4i(dx+c)}dx+2ie^{2i(dx+c)}\ln\left(e^{2i(dx+c)}+1\right)+2\,e^{4i(dx+c)}c+2\,e^{2i(dx+c)}dx+2ie^{2i(dx \cdots\right.}{\left(e^{2i(dx+c)}-1\right)\left(e^{2i(dx+c)}+1\right)d}$

[In] int((a-a*sec(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/2/d*(-a*tan(d*x+c)^2)^(1/2)*a*(-2*ln(2/(cos(d*x+c)+1))*cot(d*x+c)+2*ln(-cot(d*x+c)+csc(d*x+c)+1)*cot(d*x+c)+2*ln(-cot(d*x+c)+csc(d*x+c)-1)*cot(d*x+c)+tan(d*x+c))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06

$$\int \left(a-a\sec^2(c+dx)\right)^{3/2}\,dx=-\frac{\left(2\,a\cos\left(dx+c\right)^2\log\left(-\cos\left(dx+c\right)\right)+a\right)\sqrt{\frac{a\cos(dx+c)^2-a}{\cos(dx+c)^2}}}{2\,d\cos\left(dx+c\right)\sin\left(dx+c\right)}$$

[In] integrate((a-a*sec(d*x+c)^2)^(3/2),x, algorithm="fricas")

[Out] -1/2*(2*a*cos(d*x + c)^2*log(-cos(d*x + c)) + a)*sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)/(d*cos(d*x + c)*sin(d*x + c))

Sympy [F]

$$\int \left(a-a\sec^2(c+dx)\right)^{3/2}\,dx=\int \left(-a\sec^2\left(c+dx\right)+a\right)^{\frac{3}{2}}\,dx$$

[In] integrate((a-a*sec(d*x+c)**2)**(3/2),x)

[Out] Integral((-a*sec(c + d*x)**2 + a)**(3/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

$$\int \left(a - a\sec^2(c + dx)\right)^{3/2} \, dx = -\frac{\sqrt{-a}a\tan\left(dx + c\right)^2 - \sqrt{-a}a\log\left(\tan\left(dx + c\right)^2 + 1\right)}{2\,d}$$

```
[In] integrate((a-a*sec(d*x+c)^2)^(3/2),x, algorithm="maxima")
[Out] -1/2*(sqrt(-a)*a*tan(d*x + c)^2 - sqrt(-a)*a*log(tan(d*x + c)^2 + 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(58) = 116.

Time = 0.36 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.14

$$\int \left(a - a\sec^2(c + dx)\right)^{3/2} \, dx =$$

$$-\frac{\sqrt{-a}a\log\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + \frac{1}{\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2} + 2\right) - \sqrt{-a}a\log\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + \frac{1}{\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2} - 2\right) + \cdots}{2\,d}$$

```
[In] integrate((a-a*sec(d*x+c)^2)^(3/2),x, algorithm="giac")
[Out] -1/2*(sqrt(-a)*a*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2 + 2)
 - sqrt(-a)*a*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2 - 2) +
((tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)*sqrt(-a)*a - 6*sqrt(-a
)*a)/(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2 - 2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \left(a - a\sec^2(c + dx)\right)^{3/2} \, dx = \int {\left(a - \frac{a}{{\cos\left(c + d\,x\right)}^2}\right)}^{3/2} \,d x$$

```
[In] int((a - a/cos(c + d*x)^2)^(3/2),x)
[Out] int((a - a/cos(c + d*x)^2)^(3/2), x)
```

3.223 $\int \sqrt{a-a\sec^2(c+dx)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 33

$$\int \sqrt{a-a\sec^2(c+dx)}\,dx=-\frac{\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d}$$

```
[Out] -cot(d*x+c)*ln(cos(d*x+c))*(-a*tan(d*x+c)^2)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4206, 3739, 3556}

$$\int \sqrt{a-a\sec^2(c+dx)}\,dx=-\frac{\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\log(\cos(c+dx))}{d}$$

```
[In] Int[Sqrt[a - a*Sec[c + d*x]^2],x]
```

```
[Out] -((Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[-(a*Tan[c + d*x]^2)])/d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
```

```
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4206

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \sqrt{-a\tan^2(c+dx)}\,dx\\ &= \left(\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)\int \tan(c+dx)\,dx\\ &= -\frac{\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \sqrt{a-a\sec^2(c+dx)}\,dx = -\frac{\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d}$$

```
[In] Integrate[Sqrt[a - a*Sec[c + d*x]^2],x]
[Out] -((Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[-(a*Tan[c + d*x]^2)])/d)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

method	result
default	$\frac{\sqrt{-a\tan(dx+c)^2}\,\ln\left(1+\tan(dx+c)^2\right)}{2d\tan(dx+c)}$
risch	$\frac{\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(e^{2i(dx+c)}+1)x}{e^{2i(dx+c)}-1} - \frac{2\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(e^{2i(dx+c)}+1)(dx+c)}{(e^{2i(dx+c)}-1)d} - \frac{i\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(e^{2i(dx+c)}+1)\ln(e^{2i(dx}}{(e^{2i(dx+c)}-1)d}$

```
[In] int((a-a*sec(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/d*(-a*tan(d*x+c)^2)^(1/2)/tan(d*x+c)*ln(1+tan(d*x+c)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.61

$$\int \sqrt{a-a\sec^2(c+dx)}\,dx = -\frac{\sqrt{\frac{a\cos(dx+c)^2-a}{\cos(dx+c)^2}}\cos(dx+c)\log(-\cos(dx+c))}{d\sin(dx+c)}$$

```
[In] integrate((a-a*sec(d*x+c)^2)^(1/2),x, algorithm="fricas")
[Out] -sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)*cos(d*x + c)*log(-cos(d*x + c)
)/(d*sin(d*x + c))
```

Sympy [F]

$$\int \sqrt{a-a\sec^2(c+dx)}\,dx = \int \sqrt{-a\sec^2(c+dx)+a}\,dx$$

```
[In] integrate((a-a*sec(d*x+c)**2)**(1/2),x)
[Out] Integral(sqrt(-a*sec(c + d*x)**2 + a), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.64

$$\int \sqrt{a-a\sec^2(c+dx)}\,dx = \frac{\sqrt{-a}\log\left(\tan(dx+c)^2+1\right)}{2\,d}$$

```
[In] integrate((a-a*sec(d*x+c)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-a)*log(tan(d*x + c)^2 + 1)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(31) = 62.

Time = 0.34 (sec) , antiderivative size = 141, normalized size of antiderivative = 4.27

$$\int \sqrt{a-a\sec^2(c+dx)}\,dx =$$

$$-\frac{\left(\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)\operatorname{sgn}\left(-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{}$$

```
[In] integrate((a-a*sec(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(log(tan(1/2*d*x + 1/2*c)^2 + 1)*sgn(-tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x
 + 1/2*c)) - log(abs(tan(1/2*d*x + 1/2*c) + 1))*sgn(-tan(1/2*d*x + 1/2*c)^3
 - tan(1/2*d*x + 1/2*c)) - log(abs(tan(1/2*d*x + 1/2*c) - 1))*sgn(-tan(1/2*
d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)))*sqrt(-a)*sgn(cos(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a-a\sec^2(c+dx)}\,dx=\int \sqrt{a-\frac{a}{\cos\left(c+d\,x\right)^2}}\,dx$$

```
[In] int((a - a/cos(c + d*x)^2)^(1/2),x)
```

```
[Out] int((a - a/cos(c + d*x)^2)^(1/2), x)
```

3.224 $\int \frac{1}{\sqrt{a-a\sec^2(c+dx)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 32

$$\int \frac{1}{\sqrt{a-a\sec^2(c+dx)}}\,dx = \frac{\log(\sin(c+dx))\tan(c+dx)}{d\sqrt{-a\tan^2(c+dx)}}$$

```
[Out] ln(sin(d*x+c))*tan(d*x+c)/d/(-a*tan(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4206, 3739, 3556}

$$\int \frac{1}{\sqrt{a-a\sec^2(c+dx)}}\,dx = \frac{\tan(c+dx)\log(\sin(c+dx))}{d\sqrt{-a\tan^2(c+dx)}}$$

```
[In] Int[1/Sqrt[a - a*Sec[c + d*x]^2],x]
```

```
[Out] (Log[Sin[c + d*x]]*Tan[c + d*x])/(d*Sqrt[-(a*Tan[c + d*x]^2)])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
```

```
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4206

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{-a\tan^2(c+dx)}}\,dx\\ &= \frac{\tan(c+dx)\int \cot(c+dx)\,dx}{\sqrt{-a\tan^2(c+dx)}}\\ &= \frac{\log(\sin(c+dx))\tan(c+dx)}{d\sqrt{-a\tan^2(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{a-a\sec^2(c+dx)}}\,dx = \frac{(\log(\cos(c+dx))+\log(\tan(c+dx)))\tan(c+dx)}{d\sqrt{-a\tan^2(c+dx)}}$$

```
[In] Integrate[1/Sqrt[a - a*Sec[c + d*x]^2],x]
[Out] ((Log[Cos[c + d*x]] + Log[Tan[c + d*x]])*Tan[c + d*x])/(d*Sqrt[-(a*Tan[c +
d*x]^2)])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.50

method	result	size
default	$\frac{\tan(dx+c)\left(2\ln(\tan(dx+c))-\ln\left(1+\tan(dx+c)^2\right)\right)}{2d\sqrt{-a\tan(dx+c)^2}}$	48
risch	$\frac{\left(e^{2i(dx+c)}-1\right)x}{\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\left(e^{2i(dx+c)}+1\right)}-\frac{2\left(e^{2i(dx+c)}-1\right)(dx+c)}{\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\left(e^{2i(dx+c)}+1\right)d}-\frac{i\left(e^{2i(dx+c)}-1\right)\ln\left(e^{2i(dx+c)}-1\right)}{\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\left(e^{2i(dx+c)}+1\right)d}$	194

```
[In] int(1/(a-a*sec(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*tan(d*x+c)*(2*ln(tan(d*x+c))-ln(1+tan(d*x+c)^2))/(-a*tan(d*x+c)^2)^(1
/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{a-a\sec^2(c+dx)}}\,dx = -\frac{\sqrt{\frac{a\cos(dx+c)^2-a}{\cos(dx+c)^2}}\cos(dx+c)\log\left(\frac{1}{2}\sin(dx+c)\right)}{ad\sin(dx+c)}$$

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)*cos(d*x + c)*log(1/2*sin(d*x +
 c))/(a*d*sin(d*x + c))
```

Sympy [F]

$$\int \frac{1}{\sqrt{a-a\sec^2(c+dx)}}\,dx = \int \frac{1}{\sqrt{-a\sec^2(c+dx)+a}}\,dx$$

```
[In] integrate(1/(a-a*sec(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-a*sec(c + d*x)**2 + a), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{a-a\sec^2(c+dx)}}\,dx = -\frac{\frac{\log\left(\tan(dx+c)^2+1\right)}{\sqrt{-a}} - \frac{2\log(\tan(dx+c))}{\sqrt{-a}}}{2d}$$

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(log(tan(d*x + c)^2 + 1)/sqrt(-a) - 2*log(tan(d*x + c))/sqrt(-a))/d
```

Giac [F]

$$\int \frac{1}{\sqrt{a-a\sec^2(c+dx)}}\,dx=\int \frac{1}{\sqrt{-a\sec\left(dx+c\right)^2+a}}\,dx$$

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-a*sec(d*x + c)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-a\sec^2(c+dx)}}\,dx=\int \frac{1}{\sqrt{a-\frac{a}{\cos(c+d\,x)^2}}}\,dx$$

```
[In] int(1/(a - a/cos(c + d*x)^2)^(1/2),x)
[Out] int(1/(a - a/cos(c + d*x)^2)^(1/2), x)
```

3.225 $\int \frac{1}{(a-a\sec^2(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 67

$$\int \frac{1}{(a-a\sec^2(c+dx))^{3/2}}\,dx=\frac{\cot(c+dx)}{2ad\sqrt{-a\tan^2(c+dx)}}+\frac{\log(\sin(c+dx))\tan(c+dx)}{ad\sqrt{-a\tan^2(c+dx)}}$$

```
[Out] 1/2*cot(d*x+c)/a/d/(-a*tan(d*x+c)^2)^(1/2)+ln(sin(d*x+c))*tan(d*x+c)/a/d/(-
a*tan(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4206, 3739, 3554, 3556}

$$\int \frac{1}{(a-a\sec^2(c+dx))^{3/2}}\,dx=\frac{\cot(c+dx)}{2ad\sqrt{-a\tan^2(c+dx)}}+\frac{\tan(c+dx)\log(\sin(c+dx))}{ad\sqrt{-a\tan^2(c+dx)}}$$

```
[In] Int[(a - a*Sec[c + d*x]^2)^(-3/2),x]
```

```
[Out] Cot[c + d*x]/(2*a*d*Sqrt[-(a*Tan[c + d*x]^2)]) + (Log[Sin[c + d*x]]*Tan[c +
 d*x])/(a*d*Sqrt[-(a*Tan[c + d*x]^2)])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4206

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{\left(-a\tan^2(c+dx)\right)^{3/2}}\,dx \\
&= -\frac{\tan(c+dx)\int \cot^3(c+dx)\,dx}{a\sqrt{-a\tan^2(c+dx)}} \\
&= \frac{\cot(c+dx)}{2ad\sqrt{-a\tan^2(c+dx)}} + \frac{\tan(c+dx)\int \cot(c+dx)\,dx}{a\sqrt{-a\tan^2(c+dx)}} \\
&= \frac{\cot(c+dx)}{2ad\sqrt{-a\tan^2(c+dx)}} + \frac{\log(\sin(c+dx))\tan(c+dx)}{ad\sqrt{-a\tan^2(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$
\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{3/2}}\,dx = -\frac{\left(\cot^2(c+dx)+2\log(\cos(c+dx))+2\log(\tan(c+dx))\right)\tan^3(c+dx)}{2d\left(-a\tan^2(c+dx)\right)^{3/2}}
$$

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(-3/2),x]
[Out] -1/2*((Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[Tan[c + d*x]])*Tan[c +
d*x]^3)/(d*(-(a*Tan[c + d*x]^2))^(3/2))
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.30

method	result
default	$\frac{4\ln(-\cot(dx+c)+\csc(dx+c))\tan(dx+c)-4\ln\left(\frac{2}{\cos(dx+c)+1}\right)\tan(dx+c)+\cot(dx+c)+\sec(dx+c)\csc(dx+c)}{4d\sqrt{-a\tan(dx+c)^{2}}\,a}$
risch	$-\frac{i\,e^{4i(dx+c)}\ln\left(e^{2i(dx+c)}-1\right)+e^{4i(dx+c)}dx-2i\,e^{2i(dx+c)}\ln\left(e^{2i(dx+c)}-1\right)+2\,e^{4i(dx+c)}c-2\,e^{2i(dx+c)}dx-2i\,e^{2i(dx+c)}+i\ln\left(e^{2i(dx+c)}-1\right)}{a\left(e^{2i(dx+c)}-1\right)\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^{2}}{\left(e^{2i(dx+c)}+1\right)^{2}}}\,d}$

[In] int(1/(a-a*sec(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/4/d/(-a*tan(d*x+c)^2)^(1/2)/a*(4*ln(-cot(d*x+c)+csc(d*x+c))*tan(d*x+c)-4*ln(2/(cos(d*x+c)+1))*tan(d*x+c)+cot(d*x+c)+sec(d*x+c)*csc(d*x+c))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.40

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{3/2}}\,dx = -\frac{\left(2\left(\cos\left(dx+c\right)^{3}-\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)^{2}-a}{\cos(dx+c)^{2}}}}{2\left(a^{2}d\cos\left(dx+c\right)^{2}-a^{2}d\right)\sin\left(dx+c\right)}$$

[In] integrate(1/(a-a*sec(d*x+c)^2)^(3/2),x, algorithm="fricas")

[Out] -1/2*(2*(cos(d*x + c)^3 - cos(d*x + c))*log(1/2*sin(d*x + c)) - cos(d*x + c))*sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)/((a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))

Sympy [F]

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(-a\sec^{2}\left(c+dx\right)+a\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/(a-a*sec(d*x+c)**2)**(3/2),x)

[Out] Integral((-a*sec(c + d*x)**2 + a)**(-3/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.90

$$\int \frac{1}{(a - a\sec^2(c+dx))^{3/2}}\,dx = -\frac{\frac{\log\left(\tan(dx+c)^2+1\right)}{\sqrt{-a}a} - \frac{2\log(\tan(dx+c))}{\sqrt{-a}a} + \frac{\sqrt{-a}}{a^2\tan(dx+c)^2}}{2\,d}$$

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(log(tan(d*x + c)^2 + 1)/(sqrt(-a)*a) - 2*log(tan(d*x + c))/(sqrt(-a)*
a) + sqrt(-a)/(a^2*tan(d*x + c)^2))/d
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.61

$$\int \frac{1}{(a - a\sec^2(c+dx))^{3/2}}\,dx = -\frac{\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{\sqrt{-a}a} - \frac{8\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{\sqrt{-a}a} + \frac{4\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)}{\sqrt{-a}a} - \frac{4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1}{\sqrt{-a}a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}{8\,d}$$

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*(tan(1/2*d*x + 1/2*c)^2/(sqrt(-a)*a) - 8*log(tan(1/2*d*x + 1/2*c)^2 +
1)/(sqrt(-a)*a) + 4*log(tan(1/2*d*x + 1/2*c)^2)/(sqrt(-a)*a) - (4*tan(1/2*d
*x + 1/2*c)^2 - 1)/(sqrt(-a)*a*tan(1/2*d*x + 1/2*c)^2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a - a\sec^2(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a - \frac{a}{\cos(c+d\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(1/(a - a/cos(c + d*x)^2)^(3/2),x)
```

```
[Out] int(1/(a - a/cos(c + d*x)^2)^(3/2), x)
```

3.226 $\int \frac{1}{(a-a\sec^2(c+dx))^{5/2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 100

$$\int \frac{1}{(a-a\sec^2(c+dx))^{5/2}} dx = \frac{\cot(c+dx)}{2a^2d\sqrt{-a\tan^2(c+dx)}} - \frac{\cot^3(c+dx)}{4a^2d\sqrt{-a\tan^2(c+dx)}} + \frac{\log(\sin(c+dx))\tan(c+dx)}{a^2d\sqrt{-a\tan^2(c+dx)}}$$

```
[Out] 1/2*cot(d*x+c)/a^2/d/(-a*tan(d*x+c)^2)^(1/2)-1/4*cot(d*x+c)^3/a^2/d/(-a*tan
(d*x+c)^2)^(1/2)+ln(sin(d*x+c))*tan(d*x+c)/a^2/d/(-a*tan(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4206, 3739, 3554, 3556}

$$\int \frac{1}{(a-a\sec^2(c+dx))^{5/2}} dx = -\frac{\cot^3(c+dx)}{4a^2d\sqrt{-a\tan^2(c+dx)}} + \frac{\cot(c+dx)}{2a^2d\sqrt{-a\tan^2(c+dx)}} + \frac{\tan(c+dx)\log(\sin(c+dx))}{a^2d\sqrt{-a\tan^2(c+dx)}}$$

```
[In] Int[(a - a*Sec[c + d*x]^2)^(-5/2),x]
```

```
[Out] Cot[c + d*x]/(2*a^2*d*Sqrt[-(a*Tan[c + d*x]^2)]) - Cot[c + d*x]^3/(4*a^2*d*
Sqrt[-(a*Tan[c + d*x]^2)]) + (Log[Sin[c + d*x]]*Tan[c + d*x])/(a^2*d*Sqrt[-
(a*Tan[c + d*x]^2)])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4206

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{\left(-a\tan^2(c+dx)\right)^{5/2}}\,dx\\
&= \frac{\tan(c+dx)\int \cot^5(c+dx)\,dx}{a^2\sqrt{-a\tan^2(c+dx)}}\\
&= -\frac{\cot^3(c+dx)}{4a^2d\sqrt{-a\tan^2(c+dx)}} - \frac{\tan(c+dx)\int \cot^3(c+dx)\,dx}{a^2\sqrt{-a\tan^2(c+dx)}}\\
&= \frac{\cot(c+dx)}{2a^2d\sqrt{-a\tan^2(c+dx)}} - \frac{\cot^3(c+dx)}{4a^2d\sqrt{-a\tan^2(c+dx)}} + \frac{\tan(c+dx)\int \cot(c+dx)\,dx}{a^2\sqrt{-a\tan^2(c+dx)}}\\
&= \frac{\cot(c+dx)}{2a^2d\sqrt{-a\tan^2(c+dx)}} - \frac{\cot^3(c+dx)}{4a^2d\sqrt{-a\tan^2(c+dx)}} + \frac{\log(\sin(c+dx))\tan(c+dx)}{a^2d\sqrt{-a\tan^2(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.67

$$\int \frac{1}{(a-a\sec^2(c+dx))^{5/2}}\,dx = \frac{2\cot(c+dx)-\cot^3(c+dx)+4(\log(\cos(c+dx))+\log(\tan(c+dx)))\tan(c-}{4a^2d\sqrt{-a\tan^2(c+dx)}}$$

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(-5/2),x]
```

```
[Out] (2*Cot[c + d*x] - Cot[c + d*x]^3 + 4*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]]
)*Tan[c + d*x])/(4*a^2*d*Sqrt[-(a*Tan[c + d*x]^2)])
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.07

method	result
default	$-\frac{\left(-32\ln(-\cot(dx+c)+\csc(dx+c))\sin(dx+c)^4+32\ln\left(\frac{2}{\cos(dx+c)+1}\right)\sin(dx+c)^4+13\cos(dx+c)^4+6\cos(dx+c)^2-11\right)\sec(dx+c)\csc(}{32d\sqrt{-a\tan(dx+c)^2}\,a^2}$
risch	$\frac{(e^{2i(dx+c)}-1)x}{a^2(e^{2i(dx+c)}+1)\sqrt{\frac{a(e^{2i(dx+c)}-1)^2}{(e^{2i(dx+c)}+1)^2}}}-\frac{2(e^{2i(dx+c)}-1)(dx+c)}{a^2(e^{2i(dx+c)}+1)\sqrt{\frac{a(e^{2i(dx+c)}-1)^2}{(e^{2i(dx+c)}+1)^2}}\,d}+\frac{4i(e^{6i(dx+c)}-e^{4i(dx+c)}+e^{2i(dx+c)})}{a^2(e^{2i(dx+c)}-1)^3(e^{2i(dx+c)}+1)\sqrt{\frac{a(e^{2i(dx+c)}-1)}{(e^{2i(dx+c)}+1)}}}$

```
[In] int(1/(a-a*sec(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32/d*(-32*ln(-cot(d*x+c)+csc(d*x+c))*sin(d*x+c)^4+32*ln(2/(cos(d*x+c)+1)
)*sin(d*x+c)^4+13*cos(d*x+c)^4+6*cos(d*x+c)^2-11)/(-a*tan(d*x+c)^2)^(1/2)/a
^2*sec(d*x+c)*csc(d*x+c)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.25

$$\int \frac{1}{(a-a\sec^2(c+dx))^{5/2}}\,dx = \frac{\left(4\cos(dx+c)^3-4\left(\cos(dx+c)^5-2\cos(dx+c)^3+\cos(dx+c)\right)\log\left(\frac{1}{2}\sin\right.\right.}{4\left(a^3d\cos(dx+c)^4-2a^3d\cos(dx+c)^2+a^3d\right.}$$

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*cos(d*x + c)^3 - 4*(cos(d*x + c)^5 - 2*cos(d*x + c)^3 + cos(d*x + c)
)*log(1/2*sin(d*x + c)) - 3*cos(d*x + c))*sqrt((a*cos(d*x + c)^2 - a)/cos(d
*x + c)^2)/((a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)*sin(d*x
 + c))
```

Sympy [F]

$$\int \frac{1}{\left(a - a\sec^2(c + dx)\right)^{5/2}}\,dx = \int \frac{1}{\left(-a\sec^2(c + dx) + a\right)^{\frac{5}{2}}}\,dx$$

[In] integrate(1/(a-a*sec(d*x+c)**2)**(5/2),x)

[Out] Integral((-a*sec(c + d*x)**2 + a)**(-5/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(a - a\sec^2(c + dx)\right)^{5/2}}\,dx = -\frac{\frac{2\log\left(\tan(dx+c)^2+1\right)}{\sqrt{-a}a^2} - \frac{4\log(\tan(dx+c))}{\sqrt{-a}a^2} + \frac{2\sqrt{-a}\tan(dx+c)^2-\sqrt{-a}}{a^3\tan(dx+c)^4}}{4\,d}$$

[In] integrate(1/(a-a*sec(d*x+c)^2)^(5/2),x, algorithm="maxima")

[Out] -1/4*(2*log(tan(d*x + c)^2 + 1)/(sqrt(-a)*a^2) - 4*log(tan(d*x + c))/(sqrt(-a)*a^2) + (2*sqrt(-a)*tan(d*x + c)^2 - sqrt(-a))/(a^3*tan(d*x + c)^4))/d

Giac [A] (verification not implemented)

none

Time = 0.61 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.47

$$\int \frac{1}{\left(a - a\sec^2(c + dx)\right)^{5/2}}\,dx = \frac{\frac{64\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{\sqrt{-a}a^2} - \frac{32\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)}{\sqrt{-a}a^2} - \frac{\sqrt{-a}a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-12\sqrt{-a}a^2\tan(}{a^5}}{64\,d}$$

[In] integrate(1/(a-a*sec(d*x+c)^2)^(5/2),x, algorithm="giac")

[Out] 1/64*(64*log(tan(1/2*d*x + 1/2*c)^2 + 1)/(sqrt(-a)*a^2) - 32*log(tan(1/2*d*x + 1/2*c)^2)/(sqrt(-a)*a^2) - (sqrt(-a)*a^2*tan(1/2*d*x + 1/2*c)^4 - 12*sqrt(-a)*a^2*tan(1/2*d*x + 1/2*c)^2)/a^5 + (48*tan(1/2*d*x + 1/2*c)^4 - 12*tan(1/2*d*x + 1/2*c)^2 + 1)/(sqrt(-a)*a^2*tan(1/2*d*x + 1/2*c)^4))/d

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-a\sec^2(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a-\frac{a}{\cos(c+d\,x)^2}\right)^{5/2}}\,dx$$

```
[In] int(1/(a - a/cos(c + d*x)^2)^(5/2),x)
[Out] int(1/(a - a/cos(c + d*x)^2)^(5/2), x)
```

3.227 $\int \frac{1}{(a-a\sec^2(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 133

$$\int \frac{1}{(a-a\sec^2(c+dx))^{7/2}}\,dx=\frac{\cot(c+dx)}{2a^3d\sqrt{-a\tan^2(c+dx)}}-\frac{\cot^3(c+dx)}{4a^3d\sqrt{-a\tan^2(c+dx)}}+\frac{\cot^5(c+dx)}{6a^3d\sqrt{-a\tan^2(c+dx)}}+\frac{\log(\sin(c+dx))\tan(c+dx)}{a^3d\sqrt{-a\tan^2(c+dx)}}$$

```
[Out] 1/2*cot(d*x+c)/a^3/d/(-a*tan(d*x+c)^2)^(1/2)-1/4*cot(d*x+c)^3/a^3/d/(-a*tan
(d*x+c)^2)^(1/2)+1/6*cot(d*x+c)^5/a^3/d/(-a*tan(d*x+c)^2)^(1/2)+ln(sin(d*x+
c))*tan(d*x+c)/a^3/d/(-a*tan(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4206, 3739, 3554, 3556}

$$\int \frac{1}{(a-a\sec^2(c+dx))^{7/2}}\,dx=\frac{\cot^5(c+dx)}{6a^3d\sqrt{-a\tan^2(c+dx)}}-\frac{\cot^3(c+dx)}{4a^3d\sqrt{-a\tan^2(c+dx)}}+\frac{\cot(c+dx)}{2a^3d\sqrt{-a\tan^2(c+dx)}}+\frac{\tan(c+dx)\log(\sin(c+dx))}{a^3d\sqrt{-a\tan^2(c+dx)}}$$

```
[In] Int[(a - a*Sec[c + d*x]^2)^(-7/2),x]
```

```
[Out] Cot[c + d*x]/(2*a^3*d*Sqrt[-(a*Tan[c + d*x]^2)]) - Cot[c + d*x]^3/(4*a^3*d*
Sqrt[-(a*Tan[c + d*x]^2)]) + Cot[c + d*x]^5/(6*a^3*d*Sqrt[-(a*Tan[c + d*x]^
2)]) + (Log[Sin[c + d*x]]*Tan[c + d*x])/(a^3*d*Sqrt[-(a*Tan[c + d*x]^2)])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4206

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{\left(-a\tan^2(c+dx)\right)^{7/2}}\,dx\\
&= -\frac{\tan(c+dx)\int\cot^7(c+dx)\,dx}{a^3\sqrt{-a\tan^2(c+dx)}}\\
&= \frac{\cot^5(c+dx)}{6a^3d\sqrt{-a\tan^2(c+dx)}}+\frac{\tan(c+dx)\int\cot^5(c+dx)\,dx}{a^3\sqrt{-a\tan^2(c+dx)}}\\
&= -\frac{\cot^3(c+dx)}{4a^3d\sqrt{-a\tan^2(c+dx)}}+\frac{\cot^5(c+dx)}{6a^3d\sqrt{-a\tan^2(c+dx)}}-\frac{\tan(c+dx)\int\cot^3(c+dx)\,dx}{a^3\sqrt{-a\tan^2(c+dx)}}\\
&= \frac{\cot(c+dx)}{2a^3d\sqrt{-a\tan^2(c+dx)}}-\frac{\cot^3(c+dx)}{4a^3d\sqrt{-a\tan^2(c+dx)}}\\
&\quad+\frac{\cot^5(c+dx)}{6a^3d\sqrt{-a\tan^2(c+dx)}}+\frac{\tan(c+dx)\int\cot(c+dx)\,dx}{a^3\sqrt{-a\tan^2(c+dx)}}\\
&= \frac{\cot(c+dx)}{2a^3d\sqrt{-a\tan^2(c+dx)}}-\frac{\cot^3(c+dx)}{4a^3d\sqrt{-a\tan^2(c+dx)}}\\
&\quad+\frac{\cot^5(c+dx)}{6a^3d\sqrt{-a\tan^2(c+dx)}}+\frac{\log(\sin(c+dx))\tan(c+dx)}{a^3d\sqrt{-a\tan^2(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.59

$$\int \frac{1}{(a-a\sec^2(c+dx))^{7/2}}\,dx = -\frac{(6\cot^2(c+dx)-3\cot^4(c+dx)+2\cot^6(c+dx)+12\log(\cos(c+dx))+12\log(\tan(c+dx)))\tan^7(c+d}{12d\left(-a\tan^2(c+dx)\right)^{7/2}}$$

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(-7/2),x]
```

```
[Out] -1/12*((6*Cot[c + d*x]^2 - 3*Cot[c + d*x]^4 + 2*Cot[c + d*x]^6 + 12*Log[Cos
[c + d*x]] + 12*Log[Tan[c + d*x]])*Tan[c + d*x]^7)/(d*(-(a*Tan[c + d*x]^2))
^(7/2))
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.88

method	result
default	$-\frac{\left(48\ln\left(\frac{2}{\cos(dx+c)+1}\right)\sin(dx+c)^6-48\ln(-\cot(dx+c)+\csc(dx+c))\sin(dx+c)^6-25\cos(dx+c)^6+3\cos(dx+c)^4+33\cos(dx+c)^2-19\right)}{48d\sqrt{-a\tan(dx+c)^2}\,a^3}$
risch	$\frac{\left(e^{2i(dx+c)}-1\right)x}{a^3\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}}-\frac{2\left(e^{2i(dx+c)}-1\right)(dx+c)}{a^3\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,d}+\frac{2i\left(9\,e^{10i(dx+c)}-18\,e^{8i(dx+c)}+34\,e^{6i(dx+c)}-18\,e\right.}{3a^3\left(e^{2i(dx+c)}-1\right)^5\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e\right.}{\left(e^{2i}\right.}}}$

```
[In] int(1/(a-a*sec(d*x+c)^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/d*(48*ln(2/(cos(d*x+c)+1))*sin(d*x+c)^6-48*ln(-cot(d*x+c)+csc(d*x+c))
*sin(d*x+c)^6-25*cos(d*x+c)^6+3*cos(d*x+c)^4+33*cos(d*x+c)^2-19)/(-a*tan(d*
x+c)^2)^(1/2)/a^3*sec(d*x+c)*csc(d*x+c)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a-a\sec^2(c+dx))^{7/2}}\,dx = \frac{\left(18\cos(dx+c)^5-27\cos(dx+c)^3-12\left(\cos(dx+c)^7-3\cos(dx+c)^5+\right.\right.}{12\left(a^4d\cos(dx+c)^6-3a^4d\cos(dx\right.}$$

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/12*(18*cos(d*x + c)^5 - 27*cos(d*x + c)^3 - 12*(cos(d*x + c)^7 - 3*cos(d*
x + c)^5 + 3*cos(d*x + c)^3 - cos(d*x + c))*log(1/2*sin(d*x + c)) + 11*cos(
```

```
d*x + c))*sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)/((a^4*d*cos(d*x + c)^
6 - 3*a^4*d*cos(d*x + c)^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{1}{(a-a\sec^2(c+dx))^{7/2}}\,dx=\int \frac{1}{\left(-a\sec^2\left(c+dx\right)+a\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(a-a*sec(d*x+c)**2)**(7/2),x)
[Out] Integral((-a*sec(c + d*x)**2 + a)**(-7/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.71

$$\int \frac{1}{(a-a\sec^2(c+dx))^{7/2}}\,dx=-\frac{\frac{6\,\log\left(\tan(dx+c)^2+1\right)}{\sqrt{-a}a^3}-\frac{12\,\log(\tan(dx+c))}{\sqrt{-a}a^3}+\frac{6\,\sqrt{-a}\tan(dx+c)^4-3\,\sqrt{-a}\tan(dx+c)^2+2\,\sqrt{-a}}{a^4\tan(dx+c)^6}}{12\,d}$$

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(7/2),x, algorithm="maxima")
[Out] -1/12*(6*log(tan(d*x + c)^2 + 1)/(sqrt(-a)*a^3) - 12*log(tan(d*x + c))/(sqr
t(-a)*a^3) + (6*sqrt(-a)*tan(d*x + c)^4 - 3*sqrt(-a)*tan(d*x + c)^2 + 2*sqr
t(-a))/(a^4*tan(d*x + c)^6))/d
```

Giac [A] (verification not implemented)

none

Time = 0.55 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.29

$$\int \frac{1}{(a-a\sec^2(c+dx))^{7/2}}\,dx=\frac{\frac{384\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}{\sqrt{-a}a^3}-\frac{192\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right)}{\sqrt{-a}a^3}+\frac{352\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-87\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a}a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}}{384\,d}$$

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(7/2),x, algorithm="giac")
[Out] 1/384*(384*log(tan(1/2*d*x + 1/2*c)^2 + 1)/(sqrt(-a)*a^3) - 192*log(tan(1/2
*d*x + 1/2*c)^2)/(sqrt(-a)*a^3) + (352*tan(1/2*d*x + 1/2*c)^6 - 87*tan(1/2*
d*x + 1/2*c)^4 + 12*tan(1/2*d*x + 1/2*c)^2 - 1)/(sqrt(-a)*a^3*tan(1/2*d*x +
 1/2*c)^6) - (a^7*tan(1/2*d*x + 1/2*c)^6 - 12*a^7*tan(1/2*d*x + 1/2*c)^4 +
87*a^7*tan(1/2*d*x + 1/2*c)^2)/(sqrt(-a)*a^10))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-a\sec^2(c+dx))^{7/2}}\,dx = \int \frac{1}{\left(a-\frac{a}{\cos(c+d\,x)^2}\right)^{7/2}}\,dx$$

```
[In] int(1/(a - a/cos(c + d*x)^2)^(7/2),x)
```

```
[Out] int(1/(a - a/cos(c + d*x)^2)^(7/2), x)
```

3.228 $\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 372

$$\begin{aligned}
&\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx \\
&= -\frac{(2a^2-3ab-8b^2)\sin(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{15b^2f} \\
&\quad+\frac{(2a^2-3ab-8b^2)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{15b^2f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} \\
&\quad-\frac{(a-8b)(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{15bf\,(a+b-a\sin^2(e+fx))} \\
&\quad+\frac{(a+4b)\sec(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\tan(e+fx)}{15bf} \\
&\quad+\frac{\sec^3(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\tan(e+fx)}{5f}
\end{aligned}$$

```
[Out] -1/15*(2*a^2-3*a*b-8*b^2)*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1
/2)/b^2/f+1/15*(2*a^2-3*a*b-8*b^2)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(c
os(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/b^2/f/(1-a*sin
(f*x+e)^2/(a+b))^(1/2)-1/15*(a-8*b)*(a+b)*EllipticF(sin(f*x+e),(a/(a+b))^(1
/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*(1-a*si
n(f*x+e)^2/(a+b))^(1/2)/b/f/(a+b-a*sin(f*x+e)^2)+1/15*(a+4*b)*sec(f*x+e)*(s
ec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*tan(f*x+e)/b/f+1/5*sec(f*x+e)^3*(se
c(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4233, 1985, 1986, 423, 541, 538, 437, 435, 432, 430}

$$\begin{aligned}
&\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx \\
&= \frac{(2a^2-3ab-8b^2)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}E\left(\arcsin(\sin(e+fx))\left|\frac{a}{a+b}\right.\right)}{15b^2f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} \\
&\quad - \frac{(2a^2-3ab-8b^2)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{15b^2f} \\
&\quad - \frac{(a-8b)(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\,\text{EllipticF}\left(\text{arcs}\right.}{15bf\left(-a\sin^2(e+fx)+a+b\right)} \\
&\quad + \frac{(a+4b)\tan(e+fx)\sec(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{15bf} \\
&\quad + \frac{\tan(e+fx)\sec^3(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{5f}
\end{aligned}$$

```
[In] Int[Sec[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/15*((2*a^2 - 3*a*b - 8*b^2)*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*
Sin[e + f*x]^2)])/(b^2*f) + ((2*a^2 - 3*a*b - 8*b^2)*Sqrt[Cos[e + f*x]^2]*E
llipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Si
n[e + f*x]^2)])/(15*b^2*f*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - ((a - 8*b
)*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*S
qrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/
(a + b)])/(15*b*f*(a + b - a*Sin[e + f*x]^2)) + ((a + 4*b)*Sec[e + f*x]*Sqr
t[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Tan[e + f*x])/(15*b*f) + (Sec[
e + f*x]^3*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Tan[e + f*x])/(5
*f)
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sqrt{a+\frac{b}{1-x^2}}}{(1-x^2)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{\frac{a+b-ax^2}{1-x^2}}}{(1-x^2)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{(1-x^2)^{7/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{\sec^3(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{5f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{-4(a+b)+3ax^2}{(1-x^2)^{5/2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e\right.}{5f\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{(a+4b)\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{15bf} + \frac{\sec^3(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{5f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{(a-8b)(a+b)+a(a+4b)x^2}{(1-x^2)^{3/2}\sqrt{a+b-ax^2}}\,dx, x, \sin\right.}{15bf\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= -\frac{(2a^2-3ab-8b^2)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15b^2f} + \frac{(a+4b)\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{15bf} + \frac{\sec^3(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{5f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{-2a(a-2b)(a+b)+a(2a^2-3ab-8b^2)x^2}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\right.}{15b^2f\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= -\frac{(2a^2-3ab-8b^2)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15b^2f} + \frac{(a+4b)\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{15bf} + \frac{\sec^3(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{5f} - \frac{\left((a-8b)(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\right.}{15bf\sqrt{a+b-a\sin^2(e+fx)}} - \frac{\left((-2a^2+3ab+8b^2)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{\sqrt{1-x^2}}\right.}{15b^2f\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= -\frac{(2a^2-3ab-8b^2)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15b^2f} + \frac{(a+4b)\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{15bf} + \frac{\sec^3(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{5f} - \frac{\left((-2a^2+3ab+8b^2)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{ax^2}{a+b}}}{\sqrt{1-x^2}}\,d\right.}{15b^2f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{\left((a-8b)(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)\text{Subst}}{15bf\left(a+b-a\sin^2(e+fx)\right)}$$

$$= -\frac{(2a^2-3ab-8b^2)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15b^2f}$$
$$+\frac{(2a^2-3ab-8b^2)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e\right.}}{15b^2f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$
$$-\frac{(a-8b)(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin\right.}}{15bf\left(a+b-a\sin^2(e+fx)\right)}$$
$$+\frac{(a+4b)\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{15bf}$$
$$+\frac{\sec^3(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{5f}$$

Mathematica [F]

$$\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

```
[In] Integrate[Sec[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Sec[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 14.83 (sec) , antiderivative size = 8814, normalized size of antiderivative = 23.69

method	result	size
default	Expression too large to display	8814

```
[In] int(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 887, normalized size of antiderivative = 2.38

$$\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{\left(2\left(-2i\,a^3+3i\,a^2b+8i\,ab^2\right)\sqrt{a}\sqrt{\frac{ab+b^2}{a^2}}\cos\left(fx+e\right)^4-\left(-2i\,a^3-i\,a^2b+14i\,ab^2+16i\,b^3\right)\sqrt{a}\cos\left(fx+e\right)^4}{}$$

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*((2*(-2*I*a^3 + 3*I*a^2*b + 8*I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2)*c
os(f*x + e)^4 - (-2*I*a^3 - I*a^2*b + 14*I*a*b^2 + 16*I*b^3)*sqrt(a)*cos(f*
x + e)^4)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(s
qrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e)
)), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2
*(2*I*a^3 - 3*I*a^2*b - 8*I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2)*cos(f*x +
e)^4 - (2*I*a^3 + I*a^2*b - 14*I*a*b^2 - 16*I*b^3)*sqrt(a)*cos(f*x + e)^4)*
sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*s
qrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 +
 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - 4*((I*a^2*b
+ 4*I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^4 + (I*a^3 + I*a^2*
b - 4*I*a*b^2 - 4*I*b^3)*sqrt(a)*cos(f*x + e)^4)*sqrt((2*a*sqrt((a*b + b^2)
/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a -
 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 +
2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - 4*((-I*a^2*b - 4*I*a*b^2)*sqrt(a)*sqrt
((a*b + b^2)/a^2)*cos(f*x + e)^4 + (-I*a^3 - I*a^2*b + 4*I*a*b^2 + 4*I*b^3)
*sqrt(a)*cos(f*x + e)^4)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elli
ptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) -
 I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/
a^2))/a^2) - 2*((2*a^3 - 3*a^2*b - 8*a*b^2)*cos(f*x + e)^4 - 3*a*b^2 - (a^2
*b + 4*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e))/(a*b^2*f*cos(f*x + e)^4)
```

Sympy [F]

$$\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{a+b\sec^2{(e+fx)}}\sec^5{(e+fx)}\,dx$$

```
[In] integrate(sec(f*x+e)**5*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**5, x)
```

Maxima [F]

$$\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec{(fx+e)}^2+a}\sec{(fx+e)}^5\,dx$$

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^5, x)
```

Giac [F]

$$\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec{(fx+e)}^2+a}\sec{(fx+e)}^5\,dx$$

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \frac{\sqrt{a+\frac{b}{{\cos(e+f\,x)}^2}}}{{\cos\left(e+f\,x\right)}^5}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^5,x)
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^5, x)
```

3.229 $\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 288

$$\begin{aligned}
&\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx \\
&= \frac{(a+2b)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3bf} \\
&\quad - \frac{(a+2b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3bf\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} \\
&\quad + \frac{2(a+b)\sqrt{\cos^2(e+fx)}\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-}}{3f\left(a+b-a\sin^2(e+fx)\right)} \\
&\quad + \frac{\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{3f}
\end{aligned}$$

```
[Out] 1/3*(a+2*b)*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/b/f-1/3*(a
+2*b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e
)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/b/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)+2/3*(a+b
)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*
(a+b-a*sin(f*x+e)^2))^(1/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/f/(a+b-a*sin(f*x
+e)^2)+1/3*sec(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*tan(f*x+e)/
f
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4233, 1985, 1986, 423, 541, 538, 437, 435, 432, 430}

$$\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{2(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\,\text{EllipticF}\left(\arcsin(\sin(e\right.}{3f\left(-a\sin^2(e+fx)+a+b\right)}$$
$$-\frac{(a+2b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)}{3bf\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$
$$+\frac{(a+2b)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{3bf}$$
$$+\frac{\tan(e+fx)\sec(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{3f}$$

```
[In] Int[Sec[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((a + 2*b)*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(3
*b*f) - ((a + 2*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(
a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(3*b*f*Sqrt[1 - (a
*Sin[e + f*x]^2)/(a + b)]) + (2*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcS
in[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)
]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*f*(a + b - a*Sin[e + f*x]^2)) +
(Sec[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Tan[e + f*x])
/(3*f)
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
```

```
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sqrt{a+\frac{b}{1-x^2}}}{\left(1-x^2\right)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{\frac{a+b-ax^2}{1-x^2}}}{\left(1-x^2\right)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{\left(1-x^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{3f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{-2(a+b)+ax^2}{\left(1-x^2\right)^{3/2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e}{3f\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{(a+2b)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3bf} + \frac{\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{3f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{a(a+b)-a(a+2b)x^2}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+}{3bf\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{(a+2b)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3bf} + \frac{\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{3f} + \frac{\left(2(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}dx, x, s\right.}{3f\sqrt{a+b-a\sin^2(e+fx)}} - \frac{\left((a+2b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{\sqrt{1-x^2}}dx, x, \sin(e\right.}{3bf\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{(a+2b)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3bf} + \frac{\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{3f} - \frac{\left((a+2b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{ax^2}{a+b}}}{\sqrt{1-x^2}}dx, x, \sin(e+\right.}{3bf\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} + \frac{\left(2(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-}}\right.}{3f\left(a+b-a\sin^2(e+fx)\right)}$$

$$= \frac{(a+2b)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3bf} - \frac{(a+2b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3bf\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} + \frac{2(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+f\right.}}{3f\left(a+b-a\sin^2(e+fx)\right)} + \frac{\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{3f}$$

Mathematica [F]

$$\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

```
[In] Integrate[Sec[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Sec[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.80 (sec) , antiderivative size = 6342, normalized size of antiderivative = 22.02

method	result	size
default	Expression too large to display	6342

```
[In] int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 782, normalized size of antiderivative = 2.72

$$\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\left(2\left(i a^2+2 i ab\right)\sqrt{a}\sqrt{\frac{ab+b^2}{a^2}}\cos\left(fx+e\right)^2-\left(i a^2+4 i ab+4 i b^2\right)\sqrt{a}\cos\left(fx+e\right)^2\right)\sqrt{\frac{2a\sqrt{\frac{ab+b^2}{a^2}}-a-2b}{a}}E(\text{arcs}}{}$$

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] 1/6*((2*(I*a^2 + 2*I*a*b)*sqrt(a)*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^2 - (I
*a^2 + 4*I*a*b + 4*I*b^2)*sqrt(a)*cos(f*x + e)^2)*sqrt((2*a*sqrt((a*b + b^2
)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a
- 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 +
 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*(-I*a^2 - 2*I*a*b)*sqrt(a)*sqrt((a
*b + b^2)/a^2)*cos(f*x + e)^2 - (-I*a^2 - 4*I*a*b - 4*I*b^2)*sqrt(a)*cos(f*
x + e)^2)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(s
qrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e)
)), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - 2*
(2*I*a^(3/2)*b*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^2 + (-I*a^2 - 3*I*a*b - 2
*I*b^2)*sqrt(a)*cos(f*x + e)^2)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/
a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x
 + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b
+ b^2)/a^2))/a^2) - 2*(-2*I*a^(3/2)*b*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^2
+ (I*a^2 + 3*I*a*b + 2*I*b^2)*sqrt(a)*cos(f*x + e)^2)*sqrt((2*a*sqrt((a*b +
 b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2)
- a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a
^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + 2*((a^2 + 2*a*b)*cos(f*x + e)^2 +
 a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*b*f*cos(
f*x + e)^2)
```

Sympy [F]

$$\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{a+b\sec^2{(e+fx)}}\sec^3{(e+fx)}\,dx$$

```
[In] integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**3, x)
```

Maxima [F]

$$\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec{(fx+e)}^2+a}\sec{(fx+e)}^3\,dx$$

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^3, x)
```

Giac [F]

$$\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec{(fx+e)}^2+a}\sec{(fx+e)}^3\,dx$$

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\cos{(e+f\,x)}^3}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^3,x)
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^3, x)
```

3.230 $\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 218

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sin(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{f} - \frac{\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} + \frac{(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-}}{f\,(a+b-a\sin^2(e+fx))}$$

```
[Out] sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f-EllipticE(sin(f*x+e)
,(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^
(1/2)/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)+(a+b)*EllipticF(sin(f*x+e),(a/(a+b))
^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*(1-a
*sin(f*x+e)^2/(a+b))^(1/2)/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules

used = {4233, 1985, 1986, 423, 12, 507, 437, 435, 432, 430}

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx$$
$$-\frac{\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)}{f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$
$$+\frac{\sin(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{f}$$

```
[In] Int[Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/f - (Sqrt[Co
s[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^
2*(a + b - a*Sin[e + f*x]^2)])/(f*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + (
(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqr
t[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a
 + b)])/(f*(a + b - a*Sin[e + f*x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 507

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-b/a, -d/c])
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sqrt{a+\frac{b}{1-x^2}}}{1-x^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{\frac{a+b-ax^2}{1-x^2}}}{1-x^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{(1-x^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{f} + \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{ax^2}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+
$$

$$= \frac{\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{f} + \frac{\left(a\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+
$$

$$= \frac{\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{\sqrt{1-x^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\left((a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx, x, \text{si}
$$

$$= \frac{\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{ax^2}{a+b}}}{\sqrt{1-x^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} + \frac{\left((a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-x
}}\right.}{f\left(a+b-a\sin^2(e+fx)\right)}$$

$$= \frac{\sin(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{f} - \frac{\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} + \frac{(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+f}{f\,(a+b-a\sin^2(e+fx))}$$

Mathematica [F]

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

```
[In] Integrate[Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.34 (sec) , antiderivative size = 4676, normalized size of antiderivative = 21.45

method	result	size
default	Expression too large to display	4676

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/f/(2*I*a^(1/2)*b^(1/2)-a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*((a*(
1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x
+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/((1-cos(f*x+e))^
2*csc(f*x+e)^2-1)^2)^(1/2)*(2*I*a^(3/2)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e))+2*I*a^(1/2)*b^(3/2)*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e))-2*(-(2*I*a^(1/2)*b^(1/2)*(1-c
os(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^
2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc
(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a
+b)/(a+b))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x
+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(
a+b)^2)^(1/2))*a*b+2*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a
*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^
(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^
2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticE(
```

((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3
/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-4*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b*(1-cos(f*x+e))^3*csc(f*x+e)^3+2*(-(2*
I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+
e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2
)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*
x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^
(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*(1-cos(f*x+e))^2*csc(f*x+e)^2-(-(2
*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x
+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/
2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f
*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b
^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*(1-cos(f*x+e))^2*csc(f*x+e)^2-(-(
2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*
x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1
/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(
f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*
b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*
I*a^(3/2)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(1-cos(f*x+e))^5*
csc(f*x+e)^5+2*I*a^(1/2)*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(1
-cos(f*x+e))^5*csc(f*x+e)^5-4*I*a^(3/2)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*(1-cos(f*x+e))^3*csc(f*x+e)^3+4*I*a^(1/2)*b^(3/2)*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*(1-cos(f*x+e))^3*csc(f*x+e)^3+2*I*a^(3/2)*b^(1/
2)*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*
csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2
)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(
1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^
(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))+2*I*a^(1/2)*b^(3/2)*(-(2*I*a^(
1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-
b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-
cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))
^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)
-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2
*(1-cos(f*x+e))^5*csc(f*x+e)^5-2*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*cs
c(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-
a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-
cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2
)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e))
,(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*
a^2+(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2

*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2+(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2*(1-cos(f*x+e))^5*csc(f*x+e)^5+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*(1-cos(f*x+e))^3*csc(f*x+e)^3+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2*(1-cos(f*x+e))^3*csc(f*x+e)^3-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*(csc(f*x+e)-cot(f*x+e))+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2*(csc(f*x+e)-cot(f*x+e))-2*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b*(1-cos(f*x+e))^2*csc(f*x+e)^2-2*I*a^(3/2)*b^(1/2)*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*(1-cos(f*x+e))^2*csc(f*x+e)^2-2*I*a^(1/2)*b^(3/2)*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b*(1-cos(f*x+e))^2*csc(f*x+e)^2)/(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 603, normalized size of antiderivative = 2.77

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{4i\,a^{\frac{3}{2}}\sqrt{\frac{2\,a\sqrt{\frac{ab+b^2}{a^2}}-a-2\,b}{a}}\sqrt{\frac{ab+b^2}{a^2}}F\left(\arcsin\left(\sqrt{\frac{2\,a\sqrt{\frac{ab+b^2}{a^2}}-a-2\,b}{a}}(\cos(fx+e)+i\,\sin(fx+e))\right)\mid\frac{a^2+8\,ab+8\,b^2+4\,(}{a}\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(4*I*a^(3/2)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*sqrt((a*b +
 b^2)/a^2)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*
(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sq
rt((a*b + b^2)/a^2))/a^2) - 4*I*a^(3/2)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a
 - 2*b)/a)*sqrt((a*b + b^2)/a^2)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^
2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^
2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (-2*I*a^(3/2)*sqrt((a*b +
 b^2)/a^2) + sqrt(a)*(I*a + 2*I*b))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2
*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos
(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((
a*b + b^2)/a^2))/a^2) + (2*I*a^(3/2)*sqrt((a*b + b^2)/a^2) + sqrt(a)*(-I*a
- 2*I*b))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(s
qrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e)
)), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - 2*
a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*f)
```

Sympy [F]

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2{(e+fx)}}\sec{(e+fx)}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x), x)
```

Maxima [F]

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)^2+a}\sec\left(fx+e\right)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e), x)
```

Giac [F]

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)^2+a}\sec\left(fx+e\right)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\cos\left(e+f\,x\right)}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x),x)
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x), x)
```

3.231 $\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 80

$$\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$

```
[Out] EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a
+b-a*sin(f*x+e)^2))^(1/2)/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4233, 1985, 1986, 437, 435}

$$\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)}{f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$

```
[In] Int[Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e
 + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(f*Sqrt[1 - (a*Sin[e + f*x]^2)/(a +
b)])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \sqrt{a+\frac{b}{1-x^2}}\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \sqrt{\frac{a+b-ax^2}{1-x^2}}\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{\sqrt{1-x^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a+b-a\sin^2(e+fx)}}\\
&= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{ax^2}{a+b}}}{\sqrt{1-x^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}
\end{aligned}$$

$$= \frac{\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|\frac{a}{a+b}\right.\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

$$\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{2}\cos(e+fx)E\left(e+fx\middle|\frac{a}{a+b}\right)\sqrt{a+b\sec^2(e+fx)}}{f\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}}$$

```
[In] Integrate[Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[2]*Cos[e + f*x]*EllipticE[e + f*x, a/(a + b)]*Sqrt[a + b*Sec[e + f*x]
^2])/(f*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)])
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.87 (sec) , antiderivative size = 4727, normalized size of antiderivative = 59.09

method	result	size
default	Expression too large to display	4727

```
[In] int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/(2*I*a^(1/2)*b^(1/2)-a+b)*((a*
(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*
x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/((1-cos(f*x+e))
^2*csc(f*x+e)^2-1)^2)^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)*(2*I*a^(3/2)*
b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e))+2*I
*a^(1/2)*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*
x+e))+2*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e
))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a
^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^
2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticF(((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4
*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-2*(-(2*I*a^(1/2)*b^(1
/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(
f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e
))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*
x+e)^2+a+b)/(a+b))^(1/2)*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*
b-b^2)/(a+b)^2)^(1/2))*a*b-4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b*(1
```

-cos(f*x+e))^3*csc(f*x+e)^3-(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x
+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/
(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f
*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*Ell
ipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4
*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*(
1-cos(f*x+e))^2*csc(f*x+e)^2-(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*
x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)
/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(
f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*El
lipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(
4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2*
(1-cos(f*x+e))^2*csc(f*x+e)^2+2*I*a^(3/2)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*(1-cos(f*x+e))^5*csc(f*x+e)^5+2*I*a^(1/2)*b^(3/2)*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(1-cos(f*x+e))^5*csc(f*x+e)^5-4*I*a^(3/2)*b^(
1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(1-cos(f*x+e))^3*csc(f*x+e)^3+
4*I*a^(1/2)*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(1-cos(f*x+e))^
3*csc(f*x+e)^3+2*I*a^(3/2)*b^(1/2)*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*
csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^
2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(
1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1
/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e
)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2)
)+2*I*a^(1/2)*b^(3/2)*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+
a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))
^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))
^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticF
(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(
3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))+2*I*a^(3/2)
b^(1/2)(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+
e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*
a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)
^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticF(((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-
4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*(1-cos(f*x+e))^2*csc(f*x
+e)^2+2*I*a^(1/2)*b^(3/2)*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e
)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a
+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x
+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*Ellip
ticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I
*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*(1-cos(
f*x+e))^2*csc(f*x+e)^2-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*(1-cos(f
*x+e))^5*csc(f*x+e)^5-(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+
a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))
^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))

```
^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticE
(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(
3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2-(-(2*I*
a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)
^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*
(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+
e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3
/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*b^2*(1-cos(f*x+e))^5*csc(f*x+e)^5+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a^2*(1-cos(f*x+e))^3*csc(f*x+e)^3+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*b^2*(1-cos(f*x+e))^3*csc(f*x+e)^3-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a^2*(csc(f*x+e)-cot(f*x+e))+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^
2*(csc(f*x+e)-cot(f*x+e))-2*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x
+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/
(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f
*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*Ell
ipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4
*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b*(
1-cos(f*x+e))^2*csc(f*x+e)^2+2*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(
f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-
b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-co
s(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*
EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(
-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^
2*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*c
sc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2
-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1
-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/
2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)
),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))
*a*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^
2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e
)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a
*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^
(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x
+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/
2))*b^2)/(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2
*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/((1
-cos(f*x+e))^2*csc(f*x+e)^2+1)
```

Fricas [F]

$$\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)^2+a}\cos\left(fx+e\right)\,dx$$

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e), x)
```

Sympy [F]

$$\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2\left(e+fx\right)}\cos\left(e+fx\right)dx$$

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cos(e + f*x), x)
```

Maxima [F]

$$\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)^2+a}\cos\left(fx+e\right)\,dx$$

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e), x)
```

Giac [F]

$$\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)^2+a}\cos\left(fx+e\right)\,dx$$

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \cos\left(e+f\,x\right)\,\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}\,dx$$

```
[In] int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.232 $\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 246

$$
\begin{aligned}
&\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx \\
&= \frac{\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3f} \\
&\quad + \frac{(2a+b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|\frac{a}{a+b}\right.\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3af\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} \\
&\quad - \frac{b(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1}}{3af\left(a+b-a\sin^2(e+fx)\right)}
\end{aligned}
$$

```
[Out] 1/3*cos(f*x+e)^2*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f+1/3
*(2*a+b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*
x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/a/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/3*b
*(a+b)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+
e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a/f/(a+b-a*
sin(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used

$= \{4233, 1985, 1986, 428, 538, 437, 435, 432, 430\}$

$$\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx =$$
$$-\frac{b(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\,\text{EllipticF}\left(\arcsin(\sin(e\right.}{3af\left(-a\sin^2(e+fx)+a+b\right)}$$
$$+\frac{(2a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)}{3af\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$
$$+\frac{\sin(e+fx)\cos^2(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{3f}$$

```
[In] Int[Cos[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]^2*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)
])/(3*f) + ((2*a + b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]],
a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(3*a*f*Sqrt[1 -
 (a*Sin[e + f*x]^2)/(a + b)]) - (b*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[A
rcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]
^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*a*f*(a + b - a*Sin[e + f*x]^2
))
```

Rule 428

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*(a + b*x^n)^p*((c + d*x^n)^q/(n*(p + q) + 1)), x] + Dist[n/(n*(p
+ q) + 1), Int[(a + b*x^n)^(p - 1)*(c + d*x^n)^(q - 1)*Simp[a*c*(p + q) + (
q*(b*c - a*d) + a*d*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &
& NeQ[b*c - a*d, 0] && GtQ[q, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, n
, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (1-x^2)\sqrt{a+\frac{b}{1-x^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int (1-x^2)\sqrt{\frac{a+b-ax^2}{1-x^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\right)\text{Subst}\left(\int \sqrt{1-x^2}\sqrt{a+b-ax^2}\,dx, x, \sin(\right.}{f\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{3f} + \frac{\left(2\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\right)\text{Subst}\left(\int \frac{a+b+\frac{1}{2}(-2a-b)x^2}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+\right.}{3f\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{3f} - \frac{\left((-2a-b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{\sqrt{1-x^2}}\,dx, x, \sin(\right.}{3af\sqrt{a+b-a\sin^2(e+fx)}} - \frac{\left(b(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx, x, \text{s}\right.}{3af\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{3f} - \frac{\left((-2a-b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{ax^2}{a+b}}}{\sqrt{1-x^2}}\,dx, x, \sin(e\right.}{3af\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{\left(b(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-}}\right.}{3af\,(a+b-a\sin^2(e+fx))}$$

$$= \frac{\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{3f} + \frac{(2a+b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{3af\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{b(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)), \frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+f}}{3af\,(a+b-a\sin^2(e+fx))}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.40 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.63

$$\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\frac{6\sqrt{2}\sqrt{a+2b+a\cos(2(e+fx))}E\left(e+fx\middle|\frac{a}{a+b}\right)}{\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}}+\frac{\sqrt{-\frac{1}{b}}b\csc^2(2(e+fx))\sec(2(e+fx))\left(4i\sqrt{2}(a^2+\right.}{}\right.}{}$$

```
[In] Integrate[Cos[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*((6*Sqrt[2]*Sqrt[a + 2*b + a*Cos[2
*(e + f*x)]]*EllipticE[e + f*x, a/(a + b)])/Sqrt[(a + 2*b + a*Cos[2*(e + f*
x)])/(a + b)] + (Sqrt[-b^(-1)]*b*Csc[2*(e + f*x)]^2*Sec[2*(e + f*x)]*((4*I)
*Sqrt[2]*(a^2 + 3*a*b + 2*b^2)*Sqrt[-((a*Cos[e + f*x]^2)/b)]*EllipticE[I*Ar
cSinh[(Sqrt[-b^(-1)]*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])/Sqrt[2]], b/(a + b
)]*Sqrt[(a*Sin[e + f*x]^2)/(a + b)] + a*(a*Sqrt[-b^(-1)]*Sqrt[a + 2*b + a*C
os[2*(e + f*x)]]*(-1 + Cos[4*(e + f*x)]) - (4*I)*Sqrt[2]*(a + b)*Sqrt[-((a*
Cos[e + f*x]^2)/b)]*EllipticF[I*ArcSinh[(Sqrt[-b^(-1)]*Sqrt[a + 2*b + a*Cos
[2*(e + f*x)]])/Sqrt[2]], b/(a + b)]*Sqrt[(a*Sin[e + f*x]^2)/(a + b)]))*Sin
[4*(e + f*x)])/(2*a^2)))/(12*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.23 (sec) , antiderivative size = 6300, normalized size of antiderivative = 25.61

method	result	size
default	Expression too large to display	6300

```
[In] int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F]

$$\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec(fx+e)^2+a}\cos(fx+e)^3\,dx$$

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\text{Timed out}$$

```
[In] integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec(fx+e)^2+a}\cos(fx+e)^3\,dx$$

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3, x)
```

Giac [F]

$$\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec(fx+e)^2+a}\cos(fx+e)^3\,dx$$

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \cos\left(e+f\,x\right)^3\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

```
[In] int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.233 $\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 338

$$\begin{aligned}
&\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx \\
&= \frac{2(2a-b)\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15af} \\
&+\frac{\cos^2(e+fx)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{5af} \\
&+\frac{(8a^2+3ab-2b^2)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15a^2f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} \\
&-\frac{2(2a-b)b(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+\ldots\right.}}{15a^2f\left(a+b-a\sin^2(e+fx)\right)}
\end{aligned}$$

```
[Out] 2/15*(2*a-b)*cos(f*x+e)^2*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1
/2)/a/f+1/5*cos(f*x+e)^2*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)*(sec(f*x+e)^2*(a+b
-a*sin(f*x+e)^2))^(1/2)/a/f+1/15*(8*a^2+3*a*b-2*b^2)*EllipticE(sin(f*x+e),(
a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1
/2)/a^2/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-2/15*(2*a-b)*b*(a+b)*EllipticF(sin
(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+
e)^2))^(1/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a^2/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4233, 1985, 1986, 427, 542, 538, 437, 435, 432, 430}

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{(8a^2+3ab-2b^2)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(-a\sin^2(e+fx)+a+b)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)}{15a^2f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$
$$- \frac{2b(2a-b)(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\,(-a\sin^2(e+fx)+a+b)}\,\text{EllipticF}\,(a}{15a^2f\,(-a\sin^2(e+fx)+a+b)}$$
$$+ \frac{2(2a-b)\sin(e+fx)\cos^2(e+fx)\sqrt{\sec^2(e+fx)\,(-a\sin^2(e+fx)+a+b)}}{15af}$$
$$+ \frac{\sin(e+fx)\cos^2(e+fx)\,(-a\sin^2(e+fx)+a+b)\,\sqrt{\sec^2(e+fx)\,(-a\sin^2(e+fx)+a+b)}}{5af}$$

```
[In] Int[Cos[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (2*(2*a - b)*Cos[e + f*x]^2*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin
[e + f*x]^2)])/(15*a*f) + (Cos[e + f*x]^2*Sin[e + f*x]*(a + b - a*Sin[e + f
*x]^2)*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(5*a*f) + ((8*a^2 +
 3*a*b - 2*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a +
 b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(15*a^2*f*Sqrt[1 - (a
*Sin[e + f*x]^2)/(a + b)]) - (2*(2*a - b)*b*(a + b)*Sqrt[Cos[e + f*x]^2]*El
lipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin
[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(15*a^2*f*(a + b - a*Si
n[e + f*x]^2))
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
```

```
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (1-x^2)^2\sqrt{a+\frac{b}{1-x^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int (1-x^2)^2\sqrt{\frac{a+b-ax^2}{1-x^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int (1-x^2)^{3/2}\sqrt{a+b-ax^2}\,dx, x\right.}{f\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{\cos^2(e+fx)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{5af} - \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}\left(-4a+b+2(2a-b)x^2\right)}{\sqrt{1-x^2}}\,dx\right.}{5af\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{2(2a-b)\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15af} + \frac{\cos^2(e+fx)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{5af} + \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{(8a-b)(a+b)+\left(-8a^2-3ab+2b^2\right)x^2}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,d\right.}{15af\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{2(2a-b)\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15af} + \frac{\cos^2(e+fx)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{5af} - \frac{\left(2(2a-b)b(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{a+b-a}}\right.}{15a^2f\sqrt{a+b-a\sin^2(e+fx)}} - \frac{\left((-8a^2-3ab+2b^2)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{\sqrt{1-x^2}}\right.}{15a^2f\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{2(2a-b)\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15af} + \frac{\cos^2(e+fx)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{5af} - \frac{\left((-8a^2-3ab+2b^2)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{ax^2}{a+b}}}{\sqrt{1-x^2}}d\right.}{15a^2f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{\left(2(2a-b)b(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)\text{Sub}}{15a^2f\left(a+b-a\sin^2(e+fx)\right)}$$

$$= \frac{2(2a-b)\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15af} + \frac{\cos^2(e+fx)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{5af} + \frac{(8a^2+3ab-2b^2)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+\right.}}{15a^2f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{2(2a-b)b(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\,\text{si}\right.}}{15a^2f\left(a+b-a\sin^2(e+fx)\right)}$$

Mathematica [F]

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

```
[In] Integrate[Cos[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Cos[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.75 (sec) , antiderivative size = 8763, normalized size of antiderivative = 25.93

method	result	size
default	Expression too large to display	8763

```
[In] int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F]

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec\left(fx+e\right)^2+a}\cos\left(fx+e\right)^5\,dx$$

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\text{Timed out}$$

```
[In] integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec\left(fx+e\right)^2+a}\cos\left(fx+e\right)^5\,dx$$

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5, x)
```

Giac [F]

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cos(fx+e)^5\,dx$$

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \cos(e+f\,x)^5\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,dx$$

```
[In] int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.234 $\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 186

$$\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{(a+b)\left(a^2-2ab+5b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16b^{5/2}f} + \frac{\left(a^2-2ab+5b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16b^2f} - \frac{(3a-5b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{24b^2f} + \frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{6bf}$$

```
[Out] 1/16*(a+b)*(a^2-2*a*b+5*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2
)^(1/2))/b^(5/2)/f+1/16*(a^2-2*a*b+5*b^2)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*
x+e)/b^2/f-1/24*(3*a-5*b)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/b^2/f+1/6*s
ec(f*x+e)^2*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/b/f
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used

$= \{4231, 427, 396, 201, 223, 212\}$

$$\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$=\frac{(a+b)\left(a^2-2ab+5b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16b^{5/2}f}$$
$$+\frac{\left(a^2-2ab+5b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16b^2f}$$
$$-\frac{(3a-5b)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{24b^2f}$$
$$+\frac{\tan(e+fx)\sec^2(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{6bf}$$

```
[In] Int[Sec[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((a + b)*(a^2 - 2*a*b + 5*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b +
b*Tan[e + f*x]^2]])/(16*b^(5/2)*f) + ((a^2 - 2*a*b + 5*b^2)*Tan[e + f*x]*Sq
rt[a + b + b*Tan[e + f*x]^2])/(16*b^2*f) - ((3*a - 5*b)*Tan[e + f*x]*(a + b
 + b*Tan[e + f*x]^2)^(3/2))/(24*b^2*f) + (Sec[e + f*x]^2*Tan[e + f*x]*(a +
b + b*Tan[e + f*x]^2)^(3/2))/(6*b*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (1+x^2)^2 \sqrt{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{6bf} \\
&\quad + \frac{\text{Subst}\left(\int (-a+5b-(3a-5b)x^2)\sqrt{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{6bf} \\
&= -\frac{(3a-5b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{24b^2f} \\
&\quad + \frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{6bf} \\
&\quad + \frac{(a^2-2ab+5b^2)\,\text{Subst}\left(\int \sqrt{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{8b^2f}
\end{aligned}
$$

$$= \frac{(a^2-2ab+5b^2)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16b^2f} - \frac{(3a-5b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{24b^2f} + \frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{6bf} + \frac{((a+b)(a^2-2ab+5b^2))\,\text{Subst}\left(\int\frac{1}{\sqrt{a+b+bx^2}}dx,x,\tan(e+fx)\right)}{16b^2f}$$

$$= \frac{(a^2-2ab+5b^2)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16b^2f} - \frac{(3a-5b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{24b^2f} + \frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{6bf} + \frac{((a+b)(a^2-2ab+5b^2))\,\text{Subst}\left(\int\frac{1}{1-bx^2}dx,x,\frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16b^2f}$$

$$= \frac{(a+b)(a^2-2ab+5b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16b^{5/2}f} + \frac{(a^2-2ab+5b^2)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16b^2f} - \frac{(3a-5b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{24b^2f} + \frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{6bf}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 15.53 (sec) , antiderivative size = 407, normalized size of antiderivative = 2.19

$$\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos(e+fx)\left(-\frac{i\sqrt{b}\left(-1+e^{2i(e+fx)}\right)\left(-3a^2\left(1+e^{2i(e+fx)}\right)^4+4ab\left(1+e^{2i(e+fx)}\right)^2(1+\right.}{}\right.}{}$$

```
[In] Integrate[Sec[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Cos[e + f*x]*(((-I)*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*(-3*a^2*(1 + E
^((2*I)*(e + f*x)))^4 + 4*a*b*(1 + E^((2*I)*(e + f*x)))^2*(1 + 4*E^((2*I)*(
e + f*x)) + E^((4*I)*(e + f*x))) + b^2*(15 + 100*E^((2*I)*(e + f*x)) + 298*
E^((4*I)*(e + f*x)) + 100*E^((6*I)*(e + f*x)) + 15*E^((8*I)*(e + f*x)))))/(
1 + E^((2*I)*(e + f*x)))^6 - (3*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*Log[(-4*Sqr
t[b]*(-1 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*
(1 + E^((2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e + f*x)))])/Sqrt[4*b*E^((2*
I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*Sqrt[a + b*Sec[e + f*x]^2])
/(24*Sqrt[2]*b^(5/2)*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1374 vs. 2(166) = 332.

Time = 17.26 (sec) , antiderivative size = 1375, normalized size of antiderivative = 7.39

method	result	size
default	Expression too large to display	1375

```
[In] int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/96/f/b^(11/2)*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*(30*b^(11/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*sin(f*x+e)+30*b^(11/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*t
an(f*x+e)+8*b^(9/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*sin(f*x+e
)+20*b^(11/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*tan(f*x+e)*sec(f*
x+e)+8*b^(9/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*tan(f*x+e)-6*s
in(f*x+e)*a^2*b^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+20*b^(11/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*tan(f*x+e)*sec(f*x+e)^2+4*b^
(9/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*tan(f*x+e)*sec(f*x+e)-6
*b^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*tan(f*x+e)+16*b^(1
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*tan(f*x+e)*sec(f*x+e)^3+4*
b^(9/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*tan(f*x+e)*sec(f*x+e)
^2+16*b^(11/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*tan(f*x+e)*sec(f
*x+e)^4+3*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x
+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(
sin(f*x+e)+1))*cos(f*x+e)*a^3*b^3-3*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*cos(f*x+e)*a^2*b^4+9*ln(4*(((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*cos(f*x+
e)*a*b^5+15*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f
*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)
```

```
/(sin(f*x+e)+1))*cos(f*x+e)*b^6+3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*cos(f*x+e)*a^3*b^3-3*ln(-4*(((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*cos(f*x+
e)*a^2*b^4+9*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos
(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+
b)/(sin(f*x+e)-1))*cos(f*x+e)*a*b^5+15*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*cos(f*x+e)*b^6)
```

Fricas [A] (verification not implemented)

none

Time = 1.07 (sec) , antiderivative size = 468, normalized size of antiderivative = 2.52

$$\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{3\left(a^3-a^2b+3\,ab^2+5\,b^3\right)\sqrt{b}\cos\left(fx+e\right)^5\log\left(\frac{\left(a^2-6\,ab+b^2\right)\cos(fx+e)^4+8\left(ab-b^2\right)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2\,b\right.}{\cos(fx+e)^4}\right.}{}\right.$$

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*sqrt(b)*cos(f*x + e)^5*log(((a^2
- 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*c
os(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((3*a^2*b - 4*a*b^2 -
15*b^3)*cos(f*x + e)^4 - 8*b^3 - 2*(a*b^2 + 5*b^3)*cos(f*x + e)^2)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^5), 1
/96*(3*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*
x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^5 - 2*((3*a^
2*b - 4*a*b^2 - 15*b^3)*cos(f*x + e)^4 - 8*b^3 - 2*(a*b^2 + 5*b^3)*cos(f*x
+ e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*co
s(f*x + e)^5)]
```

Sympy [F]

$$\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2{(e+fx)}}\sec^6{(e+fx)}\,dx$$

[In] integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2)**(1/2),x)

[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**6, x)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.70

$$\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\frac{8\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}\tan(fx+e)^3}{b} + \frac{3(a+b)^2a\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{5}{2}}} + \frac{3(a+b)^2\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}} - \frac{12(a+b)a\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}}{}$$

[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] 1/48*(8*(b*tan(f*x + e)^2 + a + b)^(3/2)*tan(f*x + e)^3/b + 3*(a + b)^2*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(5/2) + 3*(a + b)^2*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(3/2) - 12*(a + b)*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(3/2) - 12*(a + b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) + 24*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) + 24*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 24*sqrt(b*tan(f*x + e)^2 + a + b)*tan(f*x + e) - 6*(b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)*tan(f*x + e)/b^2 + 3*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2*tan(f*x + e)/b^2 + 24*(b*tan(f*x + e)^2 + a + b)^(3/2)*tan(f*x + e)/b - 12*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(f*x + e)/b)/f

Giac [F]

$$\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec{(fx+e)}^2+a}\sec{(fx+e)}^6\,dx$$

[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^6, x)

Mupad [F(-1)]

Timed out.

$$\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\cos\left(e+f\,x\right)^6}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^6,x)
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^6, x)
```

3.235 $\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 122

$$\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{(a-3b)(a+b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^{3/2}f} - \frac{(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8bf} + \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4bf}$$

```
[Out] -1/8*(a-3*b)*(a+b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b
^(3/2)/f-1/8*(a-3*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f+1/4*tan(f*x+
e)*(a+b+b*tan(f*x+e)^2)^(3/2)/b/f
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4231, 396, 201, 223, 212}

$$\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{(a-3b)(a+b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8b^{3/2}f} + \frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4bf} - \frac{(a-3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8bf}$$

```
[In] Int[Sec[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/8*((a - 3*b)*(a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e
 + f*x]^2]])/(b^(3/2)*f) - ((a - 3*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f
*x]^2])/(8*b*f) + (Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(4*b*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (1+x^2)\sqrt{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4bf} - \frac{(a-3b)\text{Subst}\left(\int \sqrt{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{4bf}$$

$$= -\frac{(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8bf} + \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4bf} - \frac{((a-3b)(a+b))\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{8bf}$$

$$= -\frac{(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8bf} + \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4bf} - \frac{((a-3b)(a+b))\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8bf}$$

$$= -\frac{(a-3b)(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^{3/2}f} - \frac{(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8bf} + \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4bf}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.73 (sec) , antiderivative size = 380, normalized size of antiderivative = 3.11

$$\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx =$$

$$-\frac{b\sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)\tan^3(e+fx)\left(5\arcsin\left(\sqrt{-\frac{b\tan^2(e+fx)}{a+b}}\right)\left(3-\frac{2a\,\mathrm{s}}{}\right.\right.}{}$$

```
[In] Integrate[Sec[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/20*(b*Sec[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2]*(1 - (a*Sin[e + f*x]^2)/
(a + b))*Tan[e + f*x]^3*(5*ArcSin[Sqrt[-((b*Tan[e + f*x]^2)/(a + b))]]*(3 -
 (2*a*Sin[e + f*x]^2)/(a + b)) - (5*Sec[e + f*x]^2*(3*b^2*(1 + Sin[e + f*x]
^2) - 2*a*b*(-3 + Sin[e + f*x]^2 + Sin[e + f*x]^4) + a^2*(3 - 5*Sin[e + f*x
]^2 + 2*Sin[e + f*x]^4))*Sqrt[-((b*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2
```

```
)*Tan[e + f*x]^2)/(a + b)^2)])/(a + b)^2 + 32*Cos[e + f*x]^2*Hypergeometric
2F1[2, 4, 7/2, -((b*Tan[e + f*x]^2)/(a + b))]*(-((b*Sec[e + f*x]^2*(a + b -
 a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(a + b)^2))^(5/2)))/(Sqrt[2]*f*Sqrt[a +
2*b + a*Cos[2*e + 2*f*x]]*Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)]*Sqrt[a + b -
 a*Sin[e + f*x]^2]*(-((b*Tan[e + f*x]^2)/(a + b)))^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 906 vs. 2(106) = 212.

Time = 11.17 (sec) , antiderivative size = 907, normalized size of antiderivative = 7.43

method	result	size
default	Expression too large to display	907

```
[In] int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/f/b^2*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*(3*cos(f*x+e)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^(5/2)+3*cos(f*x+e)*ln(-4*(((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*b^(5/2)+2
*cos(f*x+e)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f
*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)
/(sin(f*x+e)+1))*b^(3/2)*a+2*cos(f*x+e)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*b^(3/2)*a-cos(f*x+e)*ln(4*(((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^(
1/2)*a^2-cos(f*x+e)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1
/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+
e)*a+a+b)/(sin(f*x+e)-1))*b^(1/2)*a^2+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*a*b*sin(f*x+e)+6*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*s
in(f*x+e)+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b*tan(f*x+e)+6*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*tan(f*x+e)+4*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*tan(f*x+e)*sec(f*x+e)+4*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*b^2*tan(f*x+e)*sec(f*x+e)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 390, normalized size of antiderivative = 3.20

$$\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \left[-\frac{(a^2-2\,ab-3\,b^2)\sqrt{b}\cos{(fx+e)}^3\log\left(\frac{(a^2-6\,ab+b^2)\cos(fx+e)^4+8\,(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2\,b\cos(fx\right.}{\cos(fx+e)^4}\right.}{32\,b^2 f\cos\left(f\right.}\right.$$

$$-\frac{(a^2-2\,ab-3\,b^2)\sqrt{-b}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\left(ab\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\cos{(fx+e)}^3-2\left((ab+\right.}{16\,b^2 f\cos{(fx+e)}^3}$$

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((a^2 - 2*a*b - 3*b^2)*sqrt(b)*cos(f*x + e)^3*log(((a^2 - 6*a*b + b^
2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^
3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((a*b + 3*b^2)*cos(f*x + e)^2 + 2*
b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f
*x + e)^3), -1/16*((a^2 - 2*a*b - 3*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(
f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^3 - 2*((a*
b + 3*b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(b^2*f*cos(f*x + e)^3)]
```

Sympy [F]

$$\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2{(e+fx)}}\sec^4{(e+fx)}\,dx$$

```
[In] integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.42

$$\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{(a+b)a\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}+\frac{(a+b)\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-\frac{4\,a\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-4\sqrt{b}\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)-4\sqrt{b\tan\cdots}}{8\,f}$$

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*((a + b)*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(3/2) + (a + b)*a
rcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) - 4*a*arcsinh(b*tan(f*x + e)
/sqrt((a + b)*b))/sqrt(b) - 4*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b
)) - 4*sqrt(b*tan(f*x + e)^2 + a + b)*tan(f*x + e) - 2*(b*tan(f*x + e)^2 +
a + b)^(3/2)*tan(f*x + e)/b + sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(f*
x + e)/b)/f
```

Giac [F]

$$\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\sec(fx+e)^4\,dx$$

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\cos(e+f\,x)^4}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^4,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^4, x)
```

3.236 $\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 76

$$\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{b}f}+\frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

```
[Out] 1/2*(a+b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/b^(1/2)+
1/2*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4231, 201, 223, 212}

$$\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{b}f}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}$$

```
[In] Int[Sec[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2
*Sqrt[b]*f) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \sqrt{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2f}\\
&= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}\\
&= \frac{(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{b}f} + \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 210 vs. $2(76) = 152$.

Time = 1.89 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.76

$$\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}\left(\sqrt{2}(a+b)\text{arctanh}\left(\frac{\sqrt{\frac{b\sin^2(e+fx)}{a+b}}}{\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}\right)\cos^2(e+fx)\sqrt{\frac{a+2b+\cdots}{}}\right.}{\sqrt{2}f(a+2b+a\cos(2(e+fx)))^{3/2}\sqrt{\frac{b\sin^2(\cdots}{a\cdots}}}$$

```
[In] Integrate[Sec[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]*(Sqrt[2]*(a + b)
*ArcTanh[Sqrt[(b*Sin[e + f*x]^2)/(a + b)]/Sqrt[(a + b - a*Sin[e + f*x]^2)/(
a + b)]]*Cos[e + f*x]^2*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)] + (a +
 2*b + a*Cos[2*(e + f*x)])*Sqrt[(b*Sin[e + f*x]^2)/(a + b)])*Tan[e + f*x])/
(Sqrt[2]*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*Sqrt[(b*Sin[e + f*x]^2)/(a
+ b)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. $2(64) = 128$.

Time = 6.68 (sec) , antiderivative size = 537, normalized size of antiderivative = 7.07

method	result
default	$\frac{\sqrt{a+b\sec(fx+e)^2}\left(b^{\frac{3}{2}}\cos(fx+e)\ln\left(-\frac{4\left(-\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{b}\cos(fx+e)+\sin(fx+e)a-\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}+a+b\right)}{\sin(fx+e)+1}\right)+b^{\frac{3}{2}}\cos(fx+e\cdots\right.}{}$

```
[In] int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f/b*(a+b*sec(f*x+e)^2)^(1/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)/(1+cos(f*x+e))*(b^(3/2)*cos(f*x+e)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))+b^(3/2)*cos(f*x+e)*ln(4*(-((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))+b^(1/2
)*cos(f*x+e)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*co
s(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a
+b)/(sin(f*x+e)+1))*a+b^(1/2)*cos(f*x+e)*ln(4*(-((b+a*cos(f*x+e)^2)/(1+cos(
```

```
f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*a+2*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*b*sin(f*x+e)+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*b*tan(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(64) = 128$.

Time = 0.31 (sec) , antiderivative size = 320, normalized size of antiderivative = 4.21

$$\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \left[\frac{(a+b)\sqrt{b}\cos(fx+e)\log\left(\frac{(a^2-6\,ab+b^2)\cos(fx+e)^4+8\,(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cdots}{\cos(fx+e)^4}\right.}{8\,bf\cos(fx+e)}\right.$$

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
+ 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^
2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e))/(b*f*cos(f*x + e)), 1/4*((a + b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(
f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) + 2*b*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b*f*cos(f*x + e))]
```

Sympy [F]

$$\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2{(e+fx)}}\sec^2{(e+fx)}\,dx$$

```
[In] integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

$$\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\frac{a\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}} + \sqrt{b}\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right) + \sqrt{b\tan\left(fx+e\right)^2+a+b}\tan\left(fx+e\right)}{2\,f}$$

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) + sqrt(b)*arcsinh(b*
tan(f*x + e)/sqrt((a + b)*b)) + sqrt(b*tan(f*x + e)^2 + a + b)*tan(f*x + e)
)/f
```

Giac [F]

$$\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)^2+a}\sec\left(fx+e\right)^2\,dx$$

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\cos\left(e+f\,x\right)^2}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^2,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^2, x)
```

3.237 $\int \sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 79

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx=\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f+arctanh(b^(
1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4213, 399, 223, 212, 385, 209}

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx=\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{a\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{a\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} \\
&= \frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 284, normalized size of antiderivative = 3.59

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{i\left(1+e^{2i(e+fx)}\right)\left(2\sqrt{b}\arctan\left(\frac{\sqrt{b}(-1+e^{2i(e+fx)})}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)+\sqrt{a}\operatorname{arctanh}\left(\frac{a+2b+ae^{2i(e+fx)}}{\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)-\right.}{2\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}f}$$

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((-1/2*I)*(1 + E^((2*I)*(e + f*x)))*(2*Sqrt[b]*ArcTan[(Sqrt[b]*(-1 + E^((2*
I)*(e + f*x))))/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^
2]] + Sqrt[a]*ArcTanh[(a + 2*b + a*E^((2*I)*(e + f*x)))/(Sqrt[a]*Sqrt[4*b*E
^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])] - Sqrt[a]*ArcTanh[(a
+ a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)))/(Sqrt[a]*Sqrt[4*b*E^((2*
I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])])*Sqrt[a + b*Sec[e + f*x]^2
])/(Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(67) = 134$.

Time = 2.11 (sec) , antiderivative size = 351, normalized size of antiderivative = 4.44

method	result
default	$\frac{\sqrt{a+b\sec(fx+e)^2}\left(\sqrt{b}\,\ln\left(-\frac{4\left(\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{b}\cos(fx+e)+\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-\sin(fx+e)a+a+b\right)}{\sin(fx+e)-1}\right)\sqrt{-a}+\sqrt{b}\,\ln\left(\frac{4\sqrt{\frac{b+a\cos(fx}{(1+\cos(fx+}}}{}\right.\right.}{2f\sqrt{}}$

```
[In] int((a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f/(-a)^(1/2)*(a+b*sec(f*x+e)^2)^(1/2)*(b^(1/2)*ln(-4*(((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(-a)^(1/2)+b^(1/2)
*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e
)+1))*(-a)^(1/2)+2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*s
in(f*x+e)*a)*a)*cos(f*x+e)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(67) = 134$.

Time = 0.50 (sec) , antiderivative size = 1227, normalized size of antiderivative = 15.53

$$\int \sqrt{a + b\sec^2(e + fx)}\, dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*
a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
 e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*c
os(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e)) + 2*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*co
s(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)
)/f, 1/8*(4*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e)
)*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2
 + b^2)*sin(f*x + e))) + sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a
^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a
^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(
f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b
 + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*sin(f*x + e)))/f, -1/4*(sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5
- 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b
+ a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - sqrt(b)*log(((a^
2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))/f, -1/4*(sqrt(a)*arctan
(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b +
b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*
a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*
x + e))) - 2*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e
))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^
2 + b^2)*sin(f*x + e))))/f]
```

Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2{(e+fx)}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 3227, normalized size of antiderivative = 40.85

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] -1/2*(2*sqrt(a)*b^(3/2)*arctan2(a*sin(2*f*x + 2*e) + (a^2*cos(4*f*x + 4*e)^
2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4
*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*
sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4
*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arcta
n2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) +
2*(a + 2*b)*cos(2*f*x + 2*e) + a)), a*cos(2*f*x + 2*e) + (a^2*cos(4*f*x + 4
*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2
 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b
^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*c
os(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*a
rctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e
) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + a + 2*b) + a^(3/2)*sqrt(b)*arctan2
(2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^
2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b
)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1
/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*
e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f
*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x +
2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b
 + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2
*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos
(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x
 + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 4*a + 4*b) + a*b*log(((a + b
```

)*sqrt((16*b^2*cos(2*f*x + 2*e)^4 + 16*b^2*sin(2*f*x + 2*e)^4 + (a^2 + 2*a*
b + b^2)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^4 - 64*b^2*cos(2*f*x + 2*e)^3 + 96*
b^2*cos(2*f*x + 2*e)^2 - 8*((a*b + b^2)*cos(2*f*x + 2*e)^2 - (a*b + b^2)*si
n(2*f*x + 2*e)^2 + a*b + b^2 - 2*(a*b + b^2)*cos(2*f*x + 2*e))*abs(2*e^(2*I
*f*x + 2*I*e) + 2)^2 - 64*b^2*cos(2*f*x + 2*e) + 32*(b^2*cos(2*f*x + 2*e)^2
 - 2*b^2*cos(2*f*x + 2*e) + b^2)*sin(2*f*x + 2*e)^2 + 16*b^2)/(a^2 + 2*a*b
+ b^2))*cos(1/2*arctan2(8*(b*cos(2*f*x + 2*e) - b)*sin(2*f*x + 2*e)/((a + b
)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2), ((a + b)*abs(2*e^(2*I*f*x + 2*I*e) + 2
)^2 - 4*b*cos(2*f*x + 2*e)^2 + 4*b*sin(2*f*x + 2*e)^2 + 8*b*cos(2*f*x + 2*e
) - 4*b)/((a + b)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2)))^2 + (a + b)*sqrt((16*
b^2*cos(2*f*x + 2*e)^4 + 16*b^2*sin(2*f*x + 2*e)^4 + (a^2 + 2*a*b + b^2)*ab
s(2*e^(2*I*f*x + 2*I*e) + 2)^4 - 64*b^2*cos(2*f*x + 2*e)^3 + 96*b^2*cos(2*f
*x + 2*e)^2 - 8*((a*b + b^2)*cos(2*f*x + 2*e)^2 - (a*b + b^2)*sin(2*f*x + 2
*e)^2 + a*b + b^2 - 2*(a*b + b^2)*cos(2*f*x + 2*e))*abs(2*e^(2*I*f*x + 2*I*
e) + 2)^2 - 64*b^2*cos(2*f*x + 2*e) + 32*(b^2*cos(2*f*x + 2*e)^2 - 2*b^2*co
s(2*f*x + 2*e) + b^2)*sin(2*f*x + 2*e)^2 + 16*b^2)/(a^2 + 2*a*b + b^2))*sin
(1/2*arctan2(8*(b*cos(2*f*x + 2*e) - b)*sin(2*f*x + 2*e)/((a + b)*abs(2*e^(
2*I*f*x + 2*I*e) + 2)^2), ((a + b)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2 - 4*b*c
os(2*f*x + 2*e)^2 + 4*b*sin(2*f*x + 2*e)^2 + 8*b*cos(2*f*x + 2*e) - 4*b)/((
a + b)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2)))^2 + 4*b*cos(2*f*x + 2*e)^2 + 4*b
*sin(2*f*x + 2*e)^2 - 4*sqrt(a*b + b^2)*((16*b^2*cos(2*f*x + 2*e)^4 + 16*b^
2*sin(2*f*x + 2*e)^4 + (a^2 + 2*a*b + b^2)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^4
 - 64*b^2*cos(2*f*x + 2*e)^3 + 96*b^2*cos(2*f*x + 2*e)^2 - 8*((a*b + b^2)*c
os(2*f*x + 2*e)^2 - (a*b + b^2)*sin(2*f*x + 2*e)^2 + a*b + b^2 - 2*(a*b + b
^2)*cos(2*f*x + 2*e))*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2 - 64*b^2*cos(2*f*x +
 2*e) + 32*(b^2*cos(2*f*x + 2*e)^2 - 2*b^2*cos(2*f*x + 2*e) + b^2)*sin(2*f*
x + 2*e)^2 + 16*b^2)/(a^2 + 2*a*b + b^2))^(1/4)*cos(1/2*arctan2(8*(b*cos(2*
f*x + 2*e) - b)*sin(2*f*x + 2*e)/((a + b)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2)
, ((a + b)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2 - 4*b*cos(2*f*x + 2*e)^2 + 4*b*
sin(2*f*x + 2*e)^2 + 8*b*cos(2*f*x + 2*e) - 4*b)/((a + b)*abs(2*e^(2*I*f*x
+ 2*I*e) + 2)^2)))*sin(2*f*x + 2*e) - 4*(sqrt(a*b + b^2)*cos(2*f*x + 2*e) -
 sqrt(a*b + b^2))*((16*b^2*cos(2*f*x + 2*e)^4 + 16*b^2*sin(2*f*x + 2*e)^4 +
 (a^2 + 2*a*b + b^2)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^4 - 64*b^2*cos(2*f*x +
2*e)^3 + 96*b^2*cos(2*f*x + 2*e)^2 - 8*((a*b + b^2)*cos(2*f*x + 2*e)^2 - (a
*b + b^2)*sin(2*f*x + 2*e)^2 + a*b + b^2 - 2*(a*b + b^2)*cos(2*f*x + 2*e))*
abs(2*e^(2*I*f*x + 2*I*e) + 2)^2 - 64*b^2*cos(2*f*x + 2*e) + 32*(b^2*cos(2*
f*x + 2*e)^2 - 2*b^2*cos(2*f*x + 2*e) + b^2)*sin(2*f*x + 2*e)^2 + 16*b^2)/(
a^2 + 2*a*b + b^2))^(1/4)*sin(1/2*arctan2(8*(b*cos(2*f*x + 2*e) - b)*sin(2*
f*x + 2*e)/((a + b)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2), ((a + b)*abs(2*e^(2*
I*f*x + 2*I*e) + 2)^2 - 4*b*cos(2*f*x + 2*e)^2 + 4*b*sin(2*f*x + 2*e)^2 + 8
*b*cos(2*f*x + 2*e) - 4*b)/((a + b)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2))) - 8
*b*cos(2*f*x + 2*e) + 4*b)/((a + b)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2)) - (a
^(3/2) + 2*sqrt(a)*b)*sqrt(b)*arctan2(2*a*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f
*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x +
2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b

```
 + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2
*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin
(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x
 + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*a*cos(2*f*x + 2*e) + 2*(a^2
*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(
2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2
 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2
*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqr
t(a)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*c
os(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 2*a + 4*b))/(a*sqrt(
b)*f)
```

Giac [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec\left(fx+e\right)^2+a}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int((a + b/cos(e + f*x)^2)^(1/2), x)
```

3.238 $\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 82

$$\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{a}f} + \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

```
[Out] 1/2*(a+b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)+1
/2*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4231, 386, 385, 209}

$$\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{a}f} + \frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}$$

```
[In] Int[Cos[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*
Sqrt[a]*f) + (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*
f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}\\
&\quad+\frac{(a+b)\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2f}\\
&= \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}\\
&\quad+\frac{(a+b)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}\\
&= \frac{(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{a}f}+\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.66

$$\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$=\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(2\sqrt{a+b}\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)+\sqrt{2}\sqrt{a}\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}\sin(e+fx)\right)}{2\sqrt{2}\sqrt{a}f\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}}$$

```
[In] Integrate[Cos[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(2*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin
[e + f*x])/Sqrt[a + b]] + Sqrt[2]*Sqrt[a]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)
])/(a + b)]*Sin[e + f*x]))/(2*Sqrt[2]*Sqrt[a]*f*Sqrt[(a + 2*b + a*Cos[2*(e
+ f*x)])/(a + b)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 312 vs. 2(70) = 140.

Time = 2.81 (sec) , antiderivative size = 313, normalized size of antiderivative = 3.82

method	result
default	$\frac{\left(\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{-a}\cos(fx+e)\sin(fx+e)+\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sin(fx+e)+\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\right.\right.}{2f\sqrt{-a}\,(1}$

```
[In] int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f/(-a)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*co
s(f*x+e)*sin(f*x+e)+(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
sin(f*x+e)+ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(
f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e
)*a)*a+ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+
e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)
*b)*(a+b*sec(f*x+e)^2)^(1/2)*cos(f*x+e)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(70) = 140.

Time = 0.42 (sec) , antiderivative size = 499, normalized size of antiderivative = 6.09

$$\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{8\,a\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e) - \sqrt{-a}(a+b)\log\left(128\,a^4\cos(fx+e)^8 - 256\,(a^4-a^3b)\cos(\right.}{} \right.$$

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x
 + e) - sqrt(-a)*(a + b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos
(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a
^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)
*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^
5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^
2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e)))/(a*f), 1/8*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*cos(f*x + e)*sin(f*x + e) - (a + b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)
^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(
a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2
*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))))/(a*f)]
```

Sympy [F]

$$\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2{(e+fx)}}\cos^2{(e+fx)}\,dx$$

```
[In] integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cos(e + f*x)**2, x)
```

Maxima [F]

$$\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)^2+a}\cos\left(fx+e\right)^2\,dx$$

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^2, x)
```

Giac [F]

$$\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)^2+a}\cos\left(fx+e\right)^2\,dx$$

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \cos\left(e+f\,x\right)^2\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

```
[In] int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.239 $\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 140

$$\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{(3a-b)(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{3/2}f} + \frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8af} + \frac{\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4af}$$

```
[Out] 1/8*(3*a-b)*(a+b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(
3/2)/f+1/8*(3*a-b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f+1/4
*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/a/f
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4231, 390, 386, 385, 209}

$$\begin{aligned}&\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx\\&=\frac{(3a-b)(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{3/2}f}\\&\quad+\frac{\sin(e+fx)\cos^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4af}\\&\quad+\frac{(3a-b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8af}\end{aligned}$$

```
[In] Int[Cos[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((3*a - b)*(a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x
]^2]])/(8*a^(3/2)*f) + ((3*a - b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*
Tan[e + f*x]^2])/(8*a*f) + (Cos[e + f*x]^3*Sin[e + f*x]*(a + b + b*Tan[e +
f*x]^2)^(3/2))/(4*a*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{(1+x^2)^3}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4af}\\
&\quad+\frac{(3a-b)\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{4af}\\
&= \frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8af}\\
&\quad+\frac{\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4af}\\
&\quad+\frac{((3a-b)(a+b))\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{8af}\\
&= \frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8af}\\
&\quad+\frac{\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4af}\\
&\quad+\frac{((3a-b)(a+b))\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8af}
\end{aligned}$$

$$= \frac{(3a-b)(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{3/2}f} + \frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8af} + \frac{\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4af}$$

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.09

$$\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\sqrt{2}(3a-b)\sqrt{a+b}\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)+\sqrt{a}(4a+b+a\cos(2(e+fx)\right.}{8a^{3/2}f\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}}$$

```
[In] Integrate[Cos[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(Sqrt[2]*(3*a - b)*Sqrt[a + b]*Arc
Sin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]] + Sqrt[a]*(4*a + b + a*Cos[2*(e + f
*x)])*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(8*a^(3/2
)*f*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(124) = 248$.

Time = 5.10 (sec) , antiderivative size = 587, normalized size of antiderivative = 4.19

method	result
default	$\frac{\left(2\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{-a}\,a\cos(fx+e)^3\sin(fx+e)+2\cos(fx+e)^2\sin(fx+e)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{-a}\,a+3\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{-a}\,a\cos\right.}{}$

```
[In] int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f/a/(-a)^(1/2)*(2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2
)*a*cos(f*x+e)^3*sin(f*x+e)+2*cos(f*x+e)^2*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(-a)^(1/2)*a*cos(f*x+e)*sin(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(-a)^(1/2)*b*cos(f*x+e)*sin(f*x+e)+3*sin(f*x+e)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a+sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+
```

```
cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*b+3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2+2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b-ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)-4*sin(f*x+e)*a)*b^2)*(a+b*sec(f*x+e)^2)^(1/2)*cos(f*x+e)/(1+cos
(f*x+e))/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.55 (sec) , antiderivative size = 567, normalized size of antiderivative = 4.05

$$\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{(3a^2+2ab-b^2)\sqrt{-a}\log\left(128a^4\cos(fx+e)^8-256(a^4-a^3b)\cos(fx+e)^6+32(5a^4-14a^3b+5a^2b\right.}{}\right.$$

$$-\frac{(3a^2+2ab-b^2)\sqrt{a}\arctan\left(\frac{\left(8a^2\cos(fx+e)^5-8(a^2-ab)\cos(fx+e)^3+(a^2-6ab+b^2)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3a^2b)\cos(fx+e)^2\right)\sin(fx+e)}\right)-4(}{32a^2f}$$

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/64*((3*a^2 + 2*a*b - b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4
 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4
 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2
*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*
cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a
^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*sin(f*x + e)) + 8*(2*a^2*cos(f*x + e)^3 + (3*a^2 + a*b)*cos(f
*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f),
 -1/32*((3*a^2 + 2*a*b - b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*
(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*
b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2*a^2*cos(f*x + e
)^3 + (3*a^2 + a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*sin(f*x + e))/(a^2*f)]
```

Sympy [F]

$$\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{a+b\sec^2{(e+fx)}}\cos^4{(e+fx)}\,dx$$

```
[In] integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cos(e + f*x)**4, x)
```

Maxima [F]

$$\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec{(fx+e)}^2+a}\cos{(fx+e)}^4\,dx$$

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^4, x)
```

Giac [F]

$$\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec{(fx+e)}^2+a}\cos{(fx+e)}^4\,dx$$

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int {\cos\left(e+f\,x\right)}^4\,\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}\,dx$$

```
[In] int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.240 $\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 196

$$\begin{aligned}\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx &= \frac{(a+b)\left(5a^2-2ab+b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{5/2}f}\\ &\quad+\frac{(3a-b)(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^2f}\\ &\quad+\frac{(5a+b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24af}\\ &\quad+\frac{\cos^5(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f}\end{aligned}$$

```
[Out] 1/16*(a+b)*(5*a^2-2*a*b+b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)
^(1/2))/a^(5/2)/f+1/48*(3*a-b)*(5*a+3*b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f
*x+e)^2)^(1/2)/a^2/f+1/24*(5*a+b)*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)
^2)^(1/2)/a/f+1/6*cos(f*x+e)^5*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used

$= \{4231, 423, 541, 12, 385, 209\}$

$$\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{(3a-b)(5a+3b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{48a^2f}$$
$$+ \frac{(a+b)\left(5a^2-2ab+b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{5/2}f}$$
$$+ \frac{\sin(e+fx)\cos^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6f}$$
$$+ \frac{(5a+b)\sin(e+fx)\cos^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{24af}$$

```
[In] Int[Cos[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((a + b)*(5*a^2 - 2*a*b + b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b
*Tan[e + f*x]^2]])/(16*a^(5/2)*f) + ((3*a - b)*(5*a + 3*b)*Cos[e + f*x]*Sin
[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(48*a^2*f) + ((5*a + b)*Cos[e + f
*x]^3*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(24*a*f) + (Cos[e + f*x]
^5*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(6*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{(1+x^2)^4}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\cos^5(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f} \\
&\quad - \frac{\text{Subst}\left(\int \frac{-5(a+b)-4bx^2}{(1+x^2)^3\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{6f} \\
&= \frac{(5a+b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24af} \\
&\quad + \frac{\cos^5(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f} \\
&\quad + \frac{\text{Subst}\left(\int \frac{(15a-b)(a+b)+2b(5a+b)x^2}{(1+x^2)^2\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{24af}
\end{aligned}$$

$$= \frac{(3a-b)(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^2f} + \frac{(5a+b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24af} + \frac{\cos^5(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f} - \frac{\text{Subst}\left(\int -\frac{3(a+b)(5a^2-2ab+b^2)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx,x,\tan(e+fx)\right)}{48a^2f}$$

$$= \frac{(3a-b)(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^2f} + \frac{(5a+b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24af} + \frac{\cos^5(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f} + \frac{((a+b)(5a^2-2ab+b^2))\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx,x,\tan(e+fx)\right)}{16a^2f}$$

$$= \frac{(3a-b)(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^2f} + \frac{(5a+b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24af} + \frac{\cos^5(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f} + \frac{((a+b)(5a^2-2ab+b^2))\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx,x,\frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^2f}$$

$$= \frac{(a+b)(5a^2-2ab+b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{5/2}f} + \frac{(3a-b)(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^2f} + \frac{(5a+b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24af} + \frac{\cos^5(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 16.88 (sec) , antiderivative size = 1902, normalized size of antiderivative = 9.70

$$\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{}{f\left(3(a+b)\,\text{AppellF1}\left(\frac{1}{2},-2,-\frac{1}{2},\frac{3}{2},\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)-\left(a\,\text{AppellF1}\left(\frac{3}{2},-2,\frac{1}{2},\frac{5}{2},\sin^2(e+fx),\frac{a\sin\ldots}{}\right.\right.\right.}$$

```
[In] Integrate[Cos[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^10*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sqrt[a + b*Sec[
e + f*x]^2]*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e
+ f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - (a*AppellF1[3/2, -2, 1/2, 5/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 4*(a + b)*AppellF1[3/2, -1, -1/2,
 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*((3*(a +
 b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)
]*Cos[e + f*x]^5*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])/(3*(a + b)*AppellF1[1/
2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - (a*AppellF1
[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 4*(a + b)
*AppellF1[3/2, -1, -1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*
Sin[e + f*x]^2) - (12*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sqrt[a + 2*b + a*Cos[2*(e + f*x)
]]*Sin[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)] - (a*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)] + 4*(a + b)*AppellF1[3/2, -1, -1/2, 5/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2) + (3*(a + b)*Cos[e
 + f*x]^4*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sin[e + f*x]*(-1/3*(a*f*Appell
F1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f
*x]*Sin[e + f*x])/(a + b) - (4*f*AppellF1[3/2, -1, -1/2, 5/2, Sin[e + f*x]^
2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3))/(f*(3*(a + b)
*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] -
 (a*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
 + 4*(a + b)*AppellF1[3/2, -1, -1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[
```

```
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^4*Sqrt[a + 2*b + a*Cos
[2*(e + f*x)]]*Sin[e + f*x]*(-2*f*(a*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*
x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 4*(a + b)*AppellF1[3/2, -1, -1/2, 5/2,
Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*
(a + b)*(-1/3*(a*f*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f
*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (4*f*AppellF1[3/2, -1,
 -1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e
+ f*x])/3) - Sin[e + f*x]^2*(a*((3*a*f*AppellF1[5/2, -2, 3/2, 7/2, Sin[e +
f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b))
- (12*f*AppellF1[5/2, -1, 1/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)]*Cos[e + f*x]*Sin[e + f*x])/5) + 4*(a + b)*((-3*a*f*AppellF1[5/2, -1, 1
/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f
*x])/(5*(a + b)) - (6*f*Cos[e + f*x]*Sin[e + f*x]*(1 - (a*Sin[e + f*x]^2)/(
a + b))^(3/2)*(5/(6*(1 - (a*Sin[e + f*x]^2)/(a + b))) + (5*(a + b)^3*Csc[e
+ f*x]^6*((-2*a*Sin[e + f*x]^2)/(a + b) - (4*a^2*Sin[e + f*x]^4)/(3*(a + b)
^2) + (2*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/(
Sqrt[a + b]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])))/(32*a^3*(1 - (a*Sin[e +
 f*x]^2)/(a + b)))))/5))))/(f*(3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - (a*AppellF1[3/2, -2, 1/2, 5/2, Sin
[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 4*(a + b)*AppellF1[3/2, -1, -1/2
, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)^2) - (3
*a*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^4*Sin[e + f*x]*Sin[2*(e + f*x)])/(Sqrt[a + 2*b + a*Co
s[2*(e + f*x)]]*(3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*
Sin[e + f*x]^2)/(a + b)] - (a*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)] + 4*(a + b)*AppellF1[3/2, -1, -1/2, 5/2, Sin[e +
 f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2))))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(176) = 352$.

Time = 7.74 (sec) , antiderivative size = 970, normalized size of antiderivative = 4.95

method	result	size
default	Expression too large to display	970

```
[In] int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/f/a^2/(-a)^(1/2)*(8*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*a^2*cos(f*x+e)^5*sin(f*x+e)+8*(-a)^(1/2)*sin(f*x+e)*cos(f*x+e)^4*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+10*(-a)^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*a^2*cos(f*x+e)^3*sin(f*x+e)+2*(-a)^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b*cos(f*x+e)^3*sin(f*x+e)+10*(-a)^(1/
2)*sin(f*x+e)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+
2*(-a)^(1/2)*sin(f*x+e)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
```

```
(1/2)*a*b+15*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*cos
(f*x+e)*sin(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*a*b*cos(f*x+e)*sin(f*x+e)-3*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*b^2*cos(f*x+e)*sin(f*x+e)+15*(-a)^(1/2)*sin(f*x+e)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+4*(-a)^(1/2)*sin(f*x+e)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*a*b-3*(-a)^(1/2)*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*b^2+15*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)-4*sin(f*x+e)*a)*a^3+9*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)-4*sin(f*x+e)*a)*a^2*b-3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)-4*sin(f*x+e)*a)*a*b^2+3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)-4*sin(f*x+e)*a)*b^3)*(a+b*sec(f*x+e)^2)^(1/2)*cos(f*x+e)/(1+cos
(f*x+e))/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 1.17 (sec) , antiderivative size = 641, normalized size of antiderivative = 3.27

$$\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \left[-\frac{3\,(5\,a^3+3\,a^2b-ab^2+b^3)\sqrt{-a}\log\left(128\,a^4\cos(fx+e)^8-256\,(a^4-a^3b)\cos(fx+e)^6+32\,(5\,a^4-14\,\right.}{}\right.$$

$$-\frac{3\,(5\,a^3+3\,a^2b-ab^2+b^3)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\,(a^2-ab)\cos(fx+e)^3+(a^2-6\,ab+b^2)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3\,a^2b)\cos(fx+e)^2\right)\sin(fx+e)}\right.}{}$$

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(3*(5*a^3 + 3*a^2*b - a*b^2 + b^3)*sqrt(-a)*log(128*a^4*cos(f*x + e
)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*
cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7
*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*
(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^
```

```
3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(8*a^3*cos(f*x + e)^5 + 2*(5*
a^3 + a^2*b)*cos(f*x + e)^3 + (15*a^3 + 4*a^2*b - 3*a*b^2)*cos(f*x + e))*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*f), -1/192*(3*
(5*a^3 + 3*a^2*b - a*b^2 + b^3)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 -
8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b +
a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(8*a^3*cos(f*x +
 e)^5 + 2*(5*a^3 + a^2*b)*cos(f*x + e)^3 + (15*a^3 + 4*a^2*b - 3*a*b^2)*cos
(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*f
)]
```

Sympy [F(-1)]

Timed out.

$$\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cos(fx+e)^6\,dx$$

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^6, x)
```

Giac [F]

$$\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cos(fx+e)^6\,dx$$

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \cos\left(e+f\,x\right)^6\,\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}\,dx$$

```
[In] int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.241 $\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 450

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx =$$

$$-\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{35b^2f}$$

$$+\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|\frac{a}{a+b}\right.\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+\right.}}{35b^2f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$

$$-\frac{(a+b)\left(a^2-16ab-16b^2\right)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\,\mathrm{s}\right.}}{35bf\left(a+b-a\sin^2(e+fx)\right)}$$

$$+\frac{\left(a^2+11ab+8b^2\right)\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{35bf}$$

$$+\frac{2(4a+3b)\sec^3(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{35f}$$

$$+\frac{b\sec^5(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{7f}$$

```
[Out] -2/35*(a+2*b)*(a^2-4*a*b-4*b^2)*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^
2))^(1/2)/b^2/f+2/35*(a+2*b)*(a^2-4*a*b-4*b^2)*EllipticE(sin(f*x+e),(a/(a+b
))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/b^
2/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/35*(a+b)*(a^2-16*a*b-16*b^2)*EllipticF
(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(
f*x+e)^2))^(1/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/b/f/(a+b-a*sin(f*x+e)^2)+1/
35*(a^2+11*a*b+8*b^2)*sec(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*
tan(f*x+e)/b/f+2/35*(4*a+3*b)*sec(f*x+e)^3*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^
2))^(1/2)*tan(f*x+e)/f+1/7*b*sec(f*x+e)^5*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2
))^(1/2)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4233, 1985, 1986, 424, 541, 538, 437, 435, 432, 430}

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx =$$
$$-\frac{(a+b)\left(a^2-16ab-16b^2\right)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\,\text{Elliptic}}{35bf\left(-a\sin^2(e+fx)+a+b\right)}$$
$$+\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}E\left(\arcsin(\sin(e+fx))\right|}{35b^2f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$
$$-\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{35b^2f}$$
$$+\frac{\left(a^2+11ab+8b^2\right)\tan(e+fx)\sec(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{35bf}$$
$$+\frac{b\tan(e+fx)\sec^5(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{7f}$$
$$+\frac{2(4a+3b)\tan(e+fx)\sec^3(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{35f}$$

```
[In] Int[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*(a + 2*b)*(a^2 - 4*a*b - 4*b^2)*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b
 - a*Sin[e + f*x]^2)])/(35*b^2*f) + (2*(a + 2*b)*(a^2 - 4*a*b - 4*b^2)*Sqrt
[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*
x]^2*(a + b - a*Sin[e + f*x]^2)])/(35*b^2*f*Sqrt[1 - (a*Sin[e + f*x]^2)/(a
+ b)]) - ((a + b)*(a^2 - 16*a*b - 16*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticF[Ar
cSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^
2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(35*b*f*(a + b - a*Sin[e + f*x]^2
)) + ((a^2 + 11*a*b + 8*b^2)*Sec[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Si
n[e + f*x]^2)]*Tan[e + f*x])/(35*b*f) + (2*(4*a + 3*b)*Sec[e + f*x]^3*Sqrt[
Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Tan[e + f*x])/(35*f) + (b*Sec[e
+ f*x]^5*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Tan[e + f*x])/(7*f
)
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
```

```
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\left(a+\frac{b}{1-x^2}\right)^{3/2}}{(1-x^2)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(\frac{a+b-ax^2}{1-x^2}\right)^{3/2}}{(1-x^2)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\right)\text{Subst}\left(\int \frac{(a+b-ax^2)^{3/2}}{(1-x^2)^{9/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{b\sec^5(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\tan(e+fx)}{7f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\right)\text{Subst}\left(\int \frac{-((a+b)(7a+6b))+a(7a+5b)x^2}{(1-x^2)^{7/2}\sqrt{a+b-ax^2}}\,dx, x,\right.}{7f\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{2(4a+3b)\sec^3(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\tan(e+fx)}{35f} + \frac{b\sec^5(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\tan(e+fx)}{7f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\right)\text{Subst}\left(\int \frac{-3b(a+b)(9a+8b)+6ab(4a+3b)x^2}{(1-x^2)^{5/2}\sqrt{a+b-ax^2}}\,dx,\right.}{35bf\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{(a^2+11ab+8b^2)\sec(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\tan(e+fx)}{35bf}$$
$$+ \frac{2(4a+3b)\sec^3(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\tan(e+fx)}{35f}$$
$$+ \frac{b\sec^5(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\tan(e+fx)}{7f}$$
$$- \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\right)\text{Subst}\left(\int \frac{3b(a+b)(a^2-16ab-16b^2)+3ab(a^2+11}{(1-x^2)^{3/2}\sqrt{a+b-ax^2}}\right.}{105b^2f\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= -\frac{2(a+2b)\,(a^2-4ab-4b^2)\sin(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{35b^2f}$$
$$+ \frac{(a^2+11ab+8b^2)\sec(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\tan(e+fx)}{35bf}$$
$$+ \frac{2(4a+3b)\sec^3(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\tan(e+fx)}{35f}$$
$$+ \frac{b\sec^5(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\tan(e+fx)}{7f}$$
$$- \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\right)\text{Subst}\left(\int \frac{-3ab(a+b)(2a^2-5ab-8b^2)+6ab(a+2}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\right.}{105b^3f\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= -\frac{2(a+2b)\,(a^2-4ab-4b^2)\sin(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{35b^2f}$$
$$+ \frac{(a^2+11ab+8b^2)\sec(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\tan(e+fx)}{35bf}$$
$$+ \frac{2(4a+3b)\sec^3(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\tan(e+fx)}{35f}$$
$$+ \frac{b\sec^5(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\tan(e+fx)}{7f}$$
$$- \frac{\left((a+b)\,(a^2-16ab-16b^2)\,\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\right)\text{Subst}\left(\int \right.}{35bf\sqrt{a+b-a\sin^2(e+fx)}}$$
$$+ \frac{\left(2(a+2b)\,(a^2-4ab-4b^2)\,\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\right)\text{Subst}\left(\int \right.}{35b^2f\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= -\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{35b^2f} + \frac{\left(a^2+11ab+8b^2\right)\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{35bf} + \frac{2(4a+3b)\sec^3(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{35f} + \frac{b\sec^5(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{7f} + \frac{\left(2(a+2b)\left(a^2-4ab-4b^2\right)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{\sqrt{}}{\sqrt{}}\right.}{35b^2f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{\left((a+b)\left(a^2-16ab-16b^2\right)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+}{a+b}}\right.}{35bf\left(a+b-a\sin^2(e+fx)\right)}$$

$$= -\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{35b^2f} + \frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\right.}}{35b^2f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{(a+b)\left(a^2-16ab-16b^2\right)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a\right.}}{35bf\left(a+b-a\sin^2(e+fx)\right)} + \frac{\left(a^2+11ab+8b^2\right)\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{35bf} + \frac{2(4a+3b)\sec^3(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{35f} + \frac{b\sec^5(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{7f}$$

Mathematica [F]

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

```
[In] Integrate[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] Integrate[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 16.32 (sec) , antiderivative size = 11007, normalized size of antiderivative = 24.46

method	result	size
default	Expression too large to display	11007

```
[In] int(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 981, normalized size of antiderivative = 2.18

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\text{Too large to display}$$

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] 1/35*((2*(-I*a^4 + 2*I*a^3*b + 12*I*a^2*b^2 + 8*I*a*b^3)*sqrt(a)*sqrt((a*b
+ b^2)/a^2)*cos(f*x + e)^6 - (-I*a^4 + 16*I*a^2*b^2 + 32*I*a*b^3 + 16*I*b^4
)*sqrt(a)*cos(f*x + e)^6)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*ell
iptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e)
+ I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)
/a^2))/a^2) + (2*(I*a^4 - 2*I*a^3*b - 12*I*a^2*b^2 - 8*I*a*b^3)*sqrt(a)*sqr
t((a*b + b^2)/a^2)*cos(f*x + e)^6 - (I*a^4 - 16*I*a^2*b^2 - 32*I*a*b^3 - 16
*I*b^4)*sqrt(a)*cos(f*x + e)^6)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/
a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x
 + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b
+ b^2)/a^2))/a^2) + (2*(-I*a^3*b - 11*I*a^2*b^2 - 8*I*a*b^3)*sqrt(a)*sqrt((
a*b + b^2)/a^2)*cos(f*x + e)^6 - (2*I*a^4 + I*a^3*b - 19*I*a^2*b^2 - 34*I*a
*b^3 - 16*I*b^4)*sqrt(a)*cos(f*x + e)^6)*sqrt((2*a*sqrt((a*b + b^2)/a^2) -
a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)
*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*s
qrt((a*b + b^2)/a^2))/a^2) + (2*(I*a^3*b + 11*I*a^2*b^2 + 8*I*a*b^3)*sqrt(a
)*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^6 - (-2*I*a^4 - I*a^3*b + 19*I*a^2*b^2
 + 34*I*a*b^3 + 16*I*b^4)*sqrt(a)*cos(f*x + e)^6)*sqrt((2*a*sqrt((a*b + b^2
)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a
- 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 +
 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - (2*(a^4 - 2*a^3*b - 12*a^2*b^2 - 8*a*
b^3)*cos(f*x + e)^6 - (a^3*b + 11*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^4 - 5*a*b
^3 - 2*(4*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*sin(f*x + e))/(a*b^2*f*cos(f*x + e)^6)
```

Sympy [F]

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\sec^5\left(e+fx\right)dx$$

```
[In] integrate(sec(f*x+e)**5*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x)**5, x)
```

Maxima [F]

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^5\,dx$$

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^5, x)
```

Giac [F]

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^5\,dx$$

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\cos\left(e+f\,x\right)^5}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^5,x)
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^5, x)
```

3.242 $\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 371

$$\begin{aligned}\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx &= \frac{(3a^2+13ab+8b^2)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15bf} \\ &\quad - \frac{(3a^2+13ab+8b^2)\sqrt{\cos^2(e+fx)}\,E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15bf\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} \\ &\quad + \frac{(a+b)(9a+8b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15f\left(a+b-a\sin^2(e+fx)\right)} \\ &\quad + \frac{2(3a+2b)\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{15f} \\ &\quad + \frac{b\sec^3(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{5f}\end{aligned}$$

```
[Out] 1/15*(3*a^2+13*a*b+8*b^2)*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1
/2)/b/f-1/15*(3*a^2+13*a*b+8*b^2)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(co
s(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/b/f/(1-a*sin(f*
x+e)^2/(a+b))^(1/2)+1/15*(a+b)*(9*a+8*b)*EllipticF(sin(f*x+e),(a/(a+b))^(1/
2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*(1-a*sin
(f*x+e)^2/(a+b))^(1/2)/f/(a+b-a*sin(f*x+e)^2)+2/15*(3*a+2*b)*sec(f*x+e)*(se
c(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*tan(f*x+e)/f+1/5*b*sec(f*x+e)^3*(sec
(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4233, 1985, 1986, 424, 541, 538, 437, 435, 432, 430}

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2} dx =$$
$$-\frac{(3a^2+13ab+8b^2)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}E\left(\arcsin(\sin(e+fx))\left|\frac{a}{a+b}\right.\right)}{15bf\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$
$$+\frac{(3a^2+13ab+8b^2)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{15bf}$$
$$+\frac{(a+b)(9a+8b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\,\text{EllipticF}\left(\arcsin(\right.}{15f\left(-a\sin^2(e+fx)+a+b\right)}$$
$$+\frac{2(3a+2b)\tan(e+fx)\sec(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{15f}$$
$$+\frac{b\tan(e+fx)\sec^3(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{5f}$$

```
[In] Int[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((3*a^2 + 13*a*b + 8*b^2)*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e
 + f*x]^2)])/(15*b*f) - ((3*a^2 + 13*a*b + 8*b^2)*Sqrt[Cos[e + f*x]^2]*Elli
pticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e
 + f*x]^2)])/(15*b*f*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + ((a + b)*(9*a
+ 8*b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt
[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a
+ b)])/(15*f*(a + b - a*Sin[e + f*x]^2)) + (2*(3*a + 2*b)*Sec[e + f*x]*Sqrt
[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Tan[e + f*x])/(15*f) + (b*Sec[e
 + f*x]^3*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Tan[e + f*x])/(5*
f)
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\left(a+\frac{b}{1-x^2}\right)^{3/2}}{(1-x^2)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(\frac{a+b-ax^2}{1-x^2}\right)^{3/2}}{(1-x^2)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{(a+b-ax^2)^{3/2}}{(1-x^2)^{7/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{b\sec^3(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{5f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{-((a+b)(5a+4b))+a(5a+3b)x^2}{(1-x^2)^{5/2}\sqrt{a+b-ax^2}}\,dx, x,\right.}{5f\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{2(3a+2b)\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{15f} + \frac{b\sec^3(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{5f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{-b(a+b)(9a+8b)+2ab(3a+2b)x^2}{(1-x^2)^{3/2}\sqrt{a+b-ax^2}}\,dx, x\right.}{15bf\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{(3a^2+13ab+8b^2)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15bf} + \frac{2(3a+2b)\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{15f} + \frac{b\sec^3(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{5f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{ab(a+b)(3a+4b)-ab(3a^2+13ab+8b^2)}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\right.}{15b^2f\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{(3a^2+13ab+8b^2)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15bf} + \frac{2(3a+2b)\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{15f} + \frac{b\sec^3(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{5f} + \frac{\left((a+b)(9a+8b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{a+b-}}\right.}{15f\sqrt{a+b-a\sin^2(e+fx)}} - \frac{\left((3a^2+13ab+8b^2)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{\sqrt{1-x^2}}\right.}{15bf\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{(3a^2+13ab+8b^2)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15bf} + \frac{2(3a+2b)\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{15f} + \frac{b\sec^3(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{5f} - \frac{\left((3a^2+13ab+8b^2)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{ax^2}{a+b}}}{\sqrt{1-x^2}}\right.}{15bf\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} + \frac{\left((a+b)(9a+8b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)\text{Sub}}{15f\left(a+b-a\sin^2(e+fx)\right)}$$

$$= \frac{(3a^2+13ab+8b^2)\sin(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{15bf}$$
$$- \frac{(3a^2+13ab+8b^2)\,\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,|\,\frac{a}{a+b}\right)\,\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e-}{15bf\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$
$$+ \frac{(a+b)(9a+8b)\sqrt{\cos^2(e+fx)}\,\mathrm{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\,\sqrt{\sec^2(e+fx)\,(a+b-a\sin}{15f\,(a+b-a\sin^2(e+fx))}$$
$$+ \frac{2(3a+2b)\sec(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\tan(e+fx)}{15f}$$
$$+ \frac{b\sec^3(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\tan(e+fx)}{5f}$$

Mathematica [F]

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 13.87 (sec) , antiderivative size = 8946, normalized size of antiderivative = 24.11

method	result	size
default	Expression too large to display	8946

```
[In] int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 890, normalized size of antiderivative = 2.40

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\left(2\left(3i\,a^3+13i\,a^2b+8i\,ab^2\right)\sqrt{a}\sqrt{\frac{ab+b^2}{a^2}}\cos\left(fx+e\right)^4-\left(3i\,a^3+19i\,a^2b+34i\,ab^2+16i\,b^3\right)\sqrt{a}\right.}{}$$

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/30*((2*(3*I*a^3 + 13*I*a^2*b + 8*I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2)*c
os(f*x + e)^4 - (3*I*a^3 + 19*I*a^2*b + 34*I*a*b^2 + 16*I*b^3)*sqrt(a)*cos(
f*x + e)^4)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin
(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x +
e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) +
(2*(-3*I*a^3 - 13*I*a^2*b - 8*I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2)*cos(f*
x + e)^4 - (-3*I*a^3 - 19*I*a^2*b - 34*I*a*b^2 - 16*I*b^3)*sqrt(a)*cos(f*x
+ e)^4)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqr
t((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e)))
, (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - 2*(4
*(3*I*a^2*b + 2*I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^4 + (-3
*I*a^3 - 13*I*a^2*b - 18*I*a*b^2 - 8*I*b^3)*sqrt(a)*cos(f*x + e)^4)*sqrt((2
*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*
b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b
+ 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - 2*(4*(-3*I*a^2*b -
2*I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^4 + (3*I*a^3 + 13*I*a
^2*b + 18*I*a*b^2 + 8*I*b^3)*sqrt(a)*cos(f*x + e)^4)*sqrt((2*a*sqrt((a*b +
b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) -
 a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^
2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + 2*((3*a^3 + 13*a^2*b + 8*a*b^2)*co
s(f*x + e)^4 + 3*a*b^2 + 2*(3*a^2*b + 2*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*b*f*cos(f*x + e)^4)
```

Sympy [F]

$$\int \sec^3(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx = \int \left(a+b\sec^2{(e+fx)}\right)^{\frac{3}{2}} \sec^3{(e+fx)} \, dx$$

```
[In] integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x)**3, x)
```

Maxima [F]

$$\int \sec^3(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx = \int \left(b\sec{(fx+e)}^2+a\right)^{\frac{3}{2}} \sec{(fx+e)}^3 \, dx$$

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^3, x)
```

Giac [F]

$$\int \sec^3(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx = \int \left(b\sec{(fx+e)}^2+a\right)^{\frac{3}{2}} \sec{(fx+e)}^3 \, dx$$

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\cos{(e+f\,x)}^3} \, dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^3,x)
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^3, x)
```

3.243 $\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 290

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{2(2a+b)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3f} - \frac{2(2a+b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|\frac{a}{a+b}\right.\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} + \frac{(a+b)(3a+2b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3f\left(a+b-a\sin^2(e+fx)\right)} + \frac{b\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{3f}$$

```
[Out] 2/3*(2*a+b)*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f-2/3*(2*a
+b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^
2*(a+b-a*sin(f*x+e)^2))^(1/2)/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)+1/3*(a+b)*(3
*a+2*b)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x
+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/f/(a+b-a*s
in(f*x+e)^2)+1/3*b*sec(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*tan
(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4233, 1985, 1986, 424, 541, 538, 437, 435, 432, 430}

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{(a+b)(3a+2b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\,\text{Elli}}{3f\left(-a\sin^2(e+fx)+a+b\right)} - \frac{2(2a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}E\left(\arcsin(\sin(e+fx))\left|\frac{a}{a+b}\right.\right)}{3f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} + \frac{2(2a+b)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{3f} + \frac{b\tan(e+fx)\sec(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{3f}$$

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*(2*a + b)*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/
(3*f) - (2*(2*a + b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a
/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(3*f*Sqrt[1 - (a
*Sin[e + f*x]^2)/(a + b)]) + ((a + b)*(3*a + 2*b)*Sqrt[Cos[e + f*x]^2]*Elli
pticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e
 + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*f*(a + b - a*Sin[e + f
*x]^2)) + (b*Sec[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*T
an[e + f*x])/(3*f)
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
```

```
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\left(a+\frac{b}{1-x^2}\right)^{3/2}}{1-x^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(\frac{a+b-ax^2}{1-x^2}\right)^{3/2}}{1-x^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{(a+b-ax^2)^{3/2}}{(1-x^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{b\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{3f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{-((a+b)(3a+2b))+a(3a+b)x^2}{(1-x^2)^{3/2}\sqrt{a+b-ax^2}}\,dx, x, \text{s}\right.}{3f\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{2(2a+b)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3f} + \frac{b\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{3f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{ab(a+b)-2ab(2a+b)x^2}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e\right.}{3bf\sqrt{a+b-a\sin^2(e+fx)}}$$

$$
\begin{aligned}
&= \frac{2(2a+b)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3f} \\
&+ \frac{b\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{3f} \\
&- \frac{\left(2(2a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{\sqrt{1-x^2}}\,dx, x, \sin\right.}{3f\sqrt{a+b-a\sin^2(e+fx)}} \\
&+ \frac{\left((a+b)(3a+2b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{a+b-}}\right.}{3f\sqrt{a+b-a\sin^2(e+fx)}} \\
&= \frac{2(2a+b)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3f} \\
&+ \frac{b\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{3f} \\
&- \frac{\left(2(2a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{ax^2}{a+b}}}{\sqrt{1-x^2}}\,dx, x, \sin(\right.}{3f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} \\
&+ \frac{\left((a+b)(3a+2b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)\text{Sub}}{3f\left(a+b-a\sin^2(e+fx)\right)} \\
&= \frac{2(2a+b)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3f} \\
&- \frac{2(2a+b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|\frac{a}{a+b}\right.\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} \\
&+ \frac{(a+b)(3a+2b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\,\text{s}\right.}}{3f\left(a+b-a\sin^2(e+fx)\right)} \\
&+ \frac{b\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{3f}
\end{aligned}
$$

Mathematica [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.32 (sec) , antiderivative size = 7055, normalized size of antiderivative = 24.33

method	result	size
default	Expression too large to display	7055

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 798, normalized size of antiderivative = 2.75

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\left(2\left(2i\,a^2+i\,ab\right)\sqrt{a}\sqrt{\frac{ab+b^2}{a^2}}\cos\left(fx+e\right)^2-\left(2i\,a^2+5i\,ab+2i\,b^2\right)\sqrt{a}\cos\left(fx+e\right)^2\right)\sqrt{\frac{2\,a\sqrt{\frac{ab+b^2}{a^2}}}{a}}}{}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] 1/3*((2*(2*I*a^2 + I*a*b)*sqrt(a)*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^2 - (2
*I*a^2 + 5*I*a*b + 2*I*b^2)*sqrt(a)*cos(f*x + e)^2)*sqrt((2*a*sqrt((a*b + b
^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) -
a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2
 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*(-2*I*a^2 - I*a*b)*sqrt(a)*sqrt(
(a*b + b^2)/a^2)*cos(f*x + e)^2 - (-2*I*a^2 - 5*I*a*b - 2*I*b^2)*sqrt(a)*co
s(f*x + e)^2)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcs
in(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x
+ e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2)
+ (2*(-3*I*a^2 - I*a*b)*sqrt(a)*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^2 - (-I*
a^2 - 3*I*a*b - 2*I*b^2)*sqrt(a)*cos(f*x + e)^2)*sqrt((2*a*sqrt((a*b + b^2)
/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a -
 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 +
2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*(3*I*a^2 + I*a*b)*sqrt(a)*sqrt((a*b
 + b^2)/a^2)*cos(f*x + e)^2 - (I*a^2 + 3*I*a*b + 2*I*b^2)*sqrt(a)*cos(f*x +
 e)^2)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt
((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))),
 (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*(2
*a^2 + a*b)*cos(f*x + e)^2 + a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*sin(f*x + e))/(a*f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\sec\left(e+fx\right)dx$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x), x)
```

Maxima [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e), x)
```

Giac [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int\frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\cos\left(e+f\,x\right)}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x),x)
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x), x)
```

3.244 $\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 224

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{b\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{f}+\frac{(a-b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}+\frac{b(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a s}{}}}{f\left(a+b-a\sin^2(e+fx)\right)}$$

```
[Out] b*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f+(a-b)*EllipticE(si
n(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x
+e)^2))^(1/2)/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)+b*(a+b)*EllipticF(sin(f*x+e)
,(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^
(1/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used

$= \{4233, 1985, 1986, 424, 538, 437, 435, 432, 430\}$

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{b(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\,\text{EllipticF}\,(\ldots}{f\left(-a\sin^2(e+fx)+a+b\right)}$$

$$+\frac{(a-b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)}{f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$

$$+\frac{b\sin(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{f}$$

```
[In] Int[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (b*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/f + ((a -
b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec
[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(f*Sqrt[1 - (a*Sin[e + f*x]^2)/(a
+ b)]) + (b*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/
(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e
+ f*x]^2)/(a + b)])/(f*(a + b - a*Sin[e + f*x]^2))
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(a+\frac{b}{1-x^2}\right)^{3/2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a+b-ax^2}{1-x^2}\right)^{3/2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int\frac{(a+b-ax^2)^{3/2}}{(1-x^2)^{3/2}}\,dx,x,\sin(e+fx)\right.}{f\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{b\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int\frac{-a(a+b)+a(a-b)x^2}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx,x,\sin(e\right.}{f\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{b\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{f} - \frac{\left((-a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int\frac{\sqrt{a+b-ax^2}}{\sqrt{1-x^2}}\,dx,x,\sin(\right.}{f\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\left(b(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx,x\right.}{f\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{b\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{f} - \frac{\left((-a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{ax^2}{a+b}}}{\sqrt{1-x^2}}\,dx,x,\sin(e\right.}{f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} + \frac{\left(b(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)\text{Subst}\left(\int\frac{}{\sqrt{1}}\right.}{f\left(a+b-a\sin^2(e+fx)\right)}$$

$$= \frac{b\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{f} + \frac{(a-b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} + \frac{b(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+\right.}}{f\left(a+b-a\sin^2(e+fx)\right)}$$

Mathematica [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

```
[In] Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.35 (sec) , antiderivative size = 5360, normalized size of antiderivative = 23.93

method	result	size
default	Expression too large to display	5360

```
[In] int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)\,dx$$

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] integral((b*cos(f*x + e)*sec(f*x + e)^2 + a*cos(f*x + e))*sqrt(b*sec(f*x +
e)^2 + a), x)
```

Sympy [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\cos\left(e+fx\right)dx$$

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*cos(e + f*x), x)
```

Maxima [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)\,dx$$

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e), x)
```

Giac [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)\,dx$$

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \cos\left(e+f\,x\right)\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}\,dx$$

```
[In] int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.245 $\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 241

$$\int \cos^3(e+fx)\,(a+b\sec^2(e+fx))^{3/2}\,dx = \frac{a\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{3f} + \frac{2(a+2b)\sqrt{\cos^2(e+fx)}\,E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{3f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{b(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a}{}}}{3f\,(a+b-a\sin^2(e+fx))}$$

```
[Out] 1/3*a*cos(f*x+e)^2*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f+2
/3*(a+2*b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(
f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/3*b
*(a+b)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+
e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/f/(a+b-a*si
n(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4233, 1985, 1986, 427, 538, 437, 435, 432, 430}

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx =$$

$$-\frac{b(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\,\text{EllipticF}\left(\arcsin(\sin(e+\right.}{3f\left(-a\sin^2(e+fx)+a+b\right)}$$
$$+\frac{2(a+2b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}E\left(\arcsin(\sin(e+fx))\left|\frac{a}{a+b}\right.\right)}{3f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$
$$+\frac{a\sin(e+fx)\cos^2(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{3f}$$

```
[In] Int[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*Cos[e + f*x]^2*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^
2)])/(3*f) + (2*(a + 2*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x
]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(3*f*Sqrt[1
 - (a*Sin[e + f*x]^2)/(a + b)]) - (b*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF
[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*
x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*f*(a + b - a*Sin[e + f*x]^2
))
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (1-x^2)\left(a+\frac{b}{1-x^2}\right)^{3/2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int (1-x^2)\left(\frac{a+b-ax^2}{1-x^2}\right)^{3/2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\right)\text{Subst}\left(\int \frac{(a+b-ax^2)^{3/2}}{\sqrt{1-x^2}}\,dx, x, \sin(e+fx)\right.}{f\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{a\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{3f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\right)\text{Subst}\left(\int \frac{(a+b)(a-3(a+b))+2a(a+2b)x^2}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx, \right.}{3f\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{a\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{3f} - \frac{\left(b(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx, x\right.}{3f\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\left(2(a+2b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{\sqrt{1-x^2}}\,dx, x, \sin\right.}{3f\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{a\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{3f} + \frac{\left(2(a+2b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{ax^2}{a+b}}}{\sqrt{1-x^2}}\,dx, x, \sin(\right.}{3f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{\left(b(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)\text{Subst}\left(\int \right.}{3f\,(a+b-a\sin^2(e+fx))}$$

$$= \frac{a\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{3f} + \frac{2(a+2b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{3f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{b(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)), \frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+}}{3f\,(a+b-a\sin^2(e+fx))}$$

Mathematica [A] (verified)

Time = 2.74 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.74

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(4\sqrt{2}(a^2+3ab+2b^2)\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}E\left(e+fx\middle|\frac{a}{a+b}\right)-2\right.}{6f(a+2b}$$

[In] Integrate[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]

[Out] (Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(4*Sqrt[2]*(a^2 + 3*a*b + 2*b^2)*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*EllipticE[e + f*x, a/(a + b)] - 2*Sqrt[2]*b*(a + b)*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*EllipticF[e + f*x, a/(a + b)] + a*(a + 2*b + a*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]))/(6*f*(a + 2*b + a*Cos[2*(e + f*x)]))

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.14 (sec) , antiderivative size = 6947, normalized size of antiderivative = 28.83

method	result	size
default	Expression too large to display	6947

[In] int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] result too large to display

Fricas [F]

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}\cos(fx+e)^3\,dx$$

[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] integral((b*cos(f*x + e)^3*sec(f*x + e)^2 + a*cos(f*x + e)^3)*sqrt(b*sec(f*x + e)^2 + a), x)

Sympy [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3\,dx$$

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^3, x)
```

Giac [F]

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3\,dx$$

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \cos\left(e+f\,x\right)^3\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}\,dx$$

```
[In] int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.246 $\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 319

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = -\frac{2(a-3(a+b))\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15f} + \frac{a\cos^4(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{5f} + \frac{(8a^2+13ab+3b^2)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|\frac{a}{a+b}\right.\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15af\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{b(a+b)(4a+3b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15af\left(a+b-a\sin^2(e+fx)\right)}$$

```
[Out] -2/15*(-2*a-3*b)*cos(f*x+e)^2*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2)
)^(1/2)/f+1/5*a*cos(f*x+e)^4*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))
^(1/2)/f+1/15*(8*a^2+13*a*b+3*b^2)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(c
os(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/a/f/(1-a*sin(f
*x+e)^2/(a+b))^(1/2)-1/15*b*(a+b)*(4*a+3*b)*EllipticF(sin(f*x+e),(a/(a+b))^
(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*(1-a*
sin(f*x+e)^2/(a+b))^(1/2)/a/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4233, 1985, 1986, 427, 542, 538, 437, 435, 432, 430}

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx=\frac{(8a^2+13ab+3b^2)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}E\left(\arcsin(\sin(e+\right.}{15af\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a+b)(4a+3b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\,\text{EllipticF}\left(\text{arcs}\right.}{15af\left(-a\sin^2(e+fx)+a+b\right)}+\frac{a\sin(e+fx)\cos^4(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{5f}-\frac{2(a-3(a+b))\sin(e+fx)\cos^2(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{15f}$$

```
[In] Int[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*(a - 3*(a + b))*Cos[e + f*x]^2*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b
- a*Sin[e + f*x]^2)])/(15*f) + (a*Cos[e + f*x]^4*Sin[e + f*x]*Sqrt[Sec[e +
f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(5*f) + ((8*a^2 + 13*a*b + 3*b^2)*Sqrt[
Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x
]^2*(a + b - a*Sin[e + f*x]^2)])/(15*a*f*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b
)]) - (b*(a + b)*(4*a + 3*b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e +
f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 -
(a*Sin[e + f*x]^2)/(a + b)])/(15*a*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
```

```
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (1-x^2)^2 \left(a+\frac{b}{1-x^2}\right)^{3/2} dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int (1-x^2)^2 \left(\frac{a+b-ax^2}{1-x^2}\right)^{3/2} dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \sqrt{1-x^2}(a+b-ax^2)^{3/2} dx, x\right.}{f\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{a\cos^4(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{5f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}((a+b)(a-5(a+b))-2a(a-3(a}{\sqrt{a+b-ax^2}}\right.}{5f\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= -\frac{2(a-3(a+b))\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15f} + \frac{a\cos^4(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{5f} + \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{a(a+b)(8a+9b)-a(8a^2+13ab+3b^2)x^2}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\right.}{15af\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= -\frac{2(a-3(a+b))\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15f} + \frac{a\cos^4(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{5f} - \frac{\left(b(a+b)(4a+3b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{a+b}}\right.}{15af\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\left((8a^2+13ab+3b^2)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{\sqrt{1-x^2}}\right.}{15af\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= -\frac{2(a-3(a+b))\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15f} + \frac{a\cos^4(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{5f} + \frac{\left((8a^2+13ab+3b^2)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{ax^2}{a+b}}}{\sqrt{1-x^2}}dx\right.}{15af\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{\left(b(a+b)(4a+3b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)\text{Sub}}{15af\left(a+b-a\sin^2(e+fx)\right)}$$

$$= -\frac{2(a-3(a+b))\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15f} + \frac{a\cos^4(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{5f} + \frac{(8a^2+13ab+3b^2)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e-\right.}}{15af\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{b(a+b)(4a+3b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin\right.}}{15af\left(a+b-a\sin^2(e+fx)\right)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 18.48 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.12

$$\int\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\cos^3(e+fx)\csc(2(e+fx))\left(a+b\sec^2(e+fx)\right)^{3/2}\left(-8i\sqrt{2}(8a^3+21a^2b+16ab^2+3b^3)\sqrt{-\right.}}{}$$

```
[In] Integrate[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cos[e + f*x]^3*Csc[2*(e + f*x)]*(a + b*Sec[e + f*x]^2)^(3/2)*((-8*I)*Sqrt[
2]*(8*a^3 + 21*a^2*b + 16*a*b^2 + 3*b^3)*Sqrt[-((a*Cos[e + f*x]^2)/b)]*Elli
pticE[I*ArcSinh[(Sqrt[-b^(-1)]*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])/Sqrt[2]]
, b/(a + b)]*Sqrt[(a*Sin[e + f*x]^2)/(a + b)] + a*((8*I)*Sqrt[2]*(8*a^2 + 1
7*a*b + 9*b^2)*Sqrt[-((a*Cos[e + f*x]^2)/b)]*EllipticF[I*ArcSinh[(Sqrt[-b^(
-1)]*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])/Sqrt[2]], b/(a + b)]*Sqrt[(a*Sin[e
 + f*x]^2)/(a + b)] + a*Sqrt[-b^(-1)]*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(1
1*a + 12*b + 3*a*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]^2)))/(30*a^2*Sqrt[-b^(-
1)]*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2))
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.03 (sec) , antiderivative size = 8848, normalized size of antiderivative = 27.74

method	result	size
default	Expression too large to display	8848

```
[In] int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F]

$$\int \cos^5(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \int \left(b \sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \cos\left(fx+e\right)^5 dx$$

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] integral((b*cos(f*x + e)^5*sec(f*x + e)^2 + a*cos(f*x + e)^5)*sqrt(b*sec(f*
x + e)^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \cos^5(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \int \left(b \sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \cos\left(fx+e\right)^5 dx$$

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^5, x)
```

Giac [F]

$$\int \cos^5(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^5 \, dx$$

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^5(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx = \int \cos\left(e+f\,x\right)^5\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2} \,d x$$

```
[In] int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.247 $\int \sec^6(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx$

Optimal result

Integrand size = 25, antiderivative size = 243

$$\int \sec^6(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \frac{(a+b)^2 (3a^2-10ab+35b^2) \operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{128b^{5/2}f} + \frac{(a+b)(3a^2-10ab+35b^2)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{128b^2f} + \frac{(3a^2-10ab+35b^2)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{192b^2f} - \frac{(3a-7b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{48b^2f} + \frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{8bf}$$

```
[Out] 1/128*(a+b)^2*(3*a^2-10*a*b+35*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f
*x+e)^2)^(1/2))/b^(5/2)/f+1/128*(a+b)*(3*a^2-10*a*b+35*b^2)*(a+b+b*tan(f*x+
e)^2)^(1/2)*tan(f*x+e)/b^2/f+1/192*(3*a^2-10*a*b+35*b^2)*tan(f*x+e)*(a+b+b*
tan(f*x+e)^2)^(3/2)/b^2/f-1/48*(3*a-7*b)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(5
/2)/b^2/f+1/8*sec(f*x+e)^2*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(5/2)/b/f
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4231, 427, 396, 201, 223, 212}

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{(a+b)^2\left(3a^2-10ab+35b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{128b^{5/2}f}+\frac{\left(3a^2-10ab+35b^2\right)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{192b^2f}+\frac{(a+b)\left(3a^2-10ab+35b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{128b^2f}-\frac{(3a-7b)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{48b^2f}+\frac{\tan(e+fx)\sec^2(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{8bf}$$

```
[In] Int[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + b)^2*(3*a^2 - 10*a*b + 35*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a
+ b + b*Tan[e + f*x]^2]])/(128*b^(5/2)*f) + ((a + b)*(3*a^2 - 10*a*b + 35*b
^2)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(128*b^2*f) + ((3*a^2 - 10
*a*b + 35*b^2)*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(192*b^2*f) -
 ((3*a - 7*b)*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(5/2))/(48*b^2*f) + (
Sec[e + f*x]^2*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(5/2))/(8*b*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (1+x^2)^2 (a+b+bx^2)^{3/2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{8bf}\\
&\quad+\frac{\text{Subst}\left(\int \left(-a+7b-(3a-7b)x^2\right)(a+b+bx^2)^{3/2}\,dx, x, \tan(e+fx)\right)}{8bf}\\
&= -\frac{(3a-7b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{48b^2f}\\
&\quad+\frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{8bf}\\
&\quad+\frac{(3a^2-10ab+35b^2)\,\text{Subst}\left(\int (a+b+bx^2)^{3/2}\,dx, x, \tan(e+fx)\right)}{48b^2f}
\end{aligned}$$

$$= \frac{(3a^2-10ab+35b^2)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{192b^2f} - \frac{(3a-7b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{48b^2f} + \frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{8bf} + \frac{((a+b)(3a^2-10ab+35b^2))\,\text{Subst}\left(\int\sqrt{a+b+bx^2}\,dx,x,\tan(e+fx)\right)}{64b^2f}$$

$$= \frac{(a+b)(3a^2-10ab+35b^2)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{128b^2f} + \frac{(3a^2-10ab+35b^2)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{192b^2f} - \frac{(3a-7b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{48b^2f} + \frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{8bf} + \frac{((a+b)^2(3a^2-10ab+35b^2))\,\text{Subst}\left(\int\frac{1}{\sqrt{a+b+bx^2}}\,dx,x,\tan(e+fx)\right)}{128b^2f}$$

$$= \frac{(a+b)(3a^2-10ab+35b^2)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{128b^2f} + \frac{(3a^2-10ab+35b^2)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{192b^2f} - \frac{(3a-7b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{48b^2f} + \frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{8bf} + \frac{((a+b)^2(3a^2-10ab+35b^2))\,\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{128b^2f}$$

$$= \frac{(a+b)^2 (3a^2 - 10ab + 35b^2) \operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{128b^{5/2}f} + \frac{(a+b)(3a^2 - 10ab + 35b^2)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{128b^2 f} + \frac{(3a^2 - 10ab + 35b^2)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{192b^2 f} - \frac{(3a-7b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{48b^2 f} + \frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{8bf}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.22 (sec) , antiderivative size = 512, normalized size of antiderivative = 2.11

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2} dx = \frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{i\sqrt{b}\left(-1+e^{2i(e+fx)}\right)\left(-9a^3\left(1+e^{2i(e+fx)}\right)^6+3a\cdots\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Cos[e + f*x]^3*(((-I)*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*(-9*a^3*(1 +
 E^((2*I)*(e + f*x)))^6 + 3*a^2*b*(1 + E^((2*I)*(e + f*x)))^4*(5 + 18*E^((2
*I)*(e + f*x)) + 5*E^((4*I)*(e + f*x))) + a*b^2*(1 + E^((2*I)*(e + f*x)))^2
*(145 + 948*E^((2*I)*(e + f*x)) + 2758*E^((4*I)*(e + f*x)) + 948*E^((6*I)*(
e + f*x)) + 145*E^((8*I)*(e + f*x))) + b^3*(105 + 910*E^((2*I)*(e + f*x)) +
 3591*E^((4*I)*(e + f*x)) + 8644*E^((6*I)*(e + f*x)) + 3591*E^((8*I)*(e + f
*x)) + 910*E^((10*I)*(e + f*x)) + 105*E^((12*I)*(e + f*x)))))/(1 + E^((2*I)
*(e + f*x)))^8 - (3*(a + b)^2*(3*a^2 - 10*a*b + 35*b^2)*Log[(-4*Sqrt[b]*(-1
 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^(
(2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e + f*x)))])/Sqrt[4*b*E^((2*I)*(e +
f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(96*S
qrt[2]*b^(5/2)*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1976 vs. 2(219) = 438.

Time = 21.49 (sec) , antiderivative size = 1977, normalized size of antiderivative = 8.14

method	result	size
default	Expression too large to display	1977

```
[In] int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/768/f/b^(13/2)*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(140*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*b^(15/2)*sin(f*x+e)+105*cos(f*x+e)^3*ln(4*(((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^8+105*cos(f*
x+e)^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e
)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(si
n(f*x+e)-1))*b^8+140*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(15/2)*t
an(f*x+e)+290*b^(13/2)*cos(f*x+e)^2*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*a+290*b^(13/2)*cos(f*x+e)*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*a+30*b^(11/2)*cos(f*x+e)^2*sin(f*x+e)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+30*b^(11/2)*cos(f*x+e)*sin(f*x+e)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2-18*b^(9/2)*cos(f*x+e)^2*sin(f*x+e)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3-18*b^(9/2)*cos(f*x+e)*sin(f
*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3+144*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*b^(13/2)*a*tan(f*x+e)*sec(f*x+e)+144*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(13/2)*a*tan(f*x+e)*sec(f*x+e)^2+12*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(11/2)*a^2*tan(f*x+e)+184*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(13/2)*a*tan(f*x+e)+96*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(15/2)*tan(f*x+e)*sec(f*x+e)^4+112*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(15/2)*tan(f*x+e)*sec(f*x+e)+9*cos(f
*x+e)^3*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e
)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(si
n(f*x+e)+1))*a^4*b^4-12*cos(f*x+e)^3*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a^3*b^5+54*cos(f*x+e)^3*ln(4*(((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a^2*
b^6+180*cos(f*x+e)^3*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1
/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+
e)*a-a-b)/(sin(f*x+e)+1))*a*b^7+9*cos(f*x+e)^3*ln(-4*(((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a^4*b^4-12*cos(f*x+e)^
3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x
```

```
+e)-1))*a^3*b^5+54*cos(f*x+e)^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a^2*b^6+180*cos(f*x+e)^3*ln(-4*(((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a*b^7+
12*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(11/2)*a^2*sin(f*x+e)+184*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(13/2)*a*sin(f*x+e)+210*b^(15
/2)*cos(f*x+e)^2*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+210
*b^(15/2)*cos(f*x+e)*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
+112*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(15/2)*tan(f*x+e)*sec(f*
x+e)^2+96*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(15/2)*tan(f*x+e)*s
ec(f*x+e)^3)
```

Fricas [A] (verification not implemented)

none

Time = 4.13 (sec) , antiderivative size = 566, normalized size of antiderivative = 2.33

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\left[\frac{3\left(3\,a^4-4\,a^3b+18\,a^2b^2+60\,ab^3+35\,b^4\right)\sqrt{b}\cos\left(fx+e\right)^7\log\left(\frac{\left(a^2-6\,ab+b^2\right)\cos(fx+e)^4+8\,(ab-b^2}{}\right.}{}\right.$$

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/1536*(3*(3*a^4 - 4*a^3*b + 18*a^2*b^2 + 60*a*b^3 + 35*b^4)*sqrt(b)*cos(f
*x + e)^7*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x +
 e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((
9*a^3*b - 15*a^2*b^2 - 145*a*b^3 - 105*b^4)*cos(f*x + e)^6 - 2*(3*a^2*b^2 +
 46*a*b^3 + 35*b^4)*cos(f*x + e)^4 - 48*b^4 - 8*(9*a*b^3 + 7*b^4)*cos(f*x +
 e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*cos
(f*x + e)^7), 1/768*(3*(3*a^4 - 4*a^3*b + 18*a^2*b^2 + 60*a*b^3 + 35*b^4)*s
qrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f
*x + e)))*cos(f*x + e)^7 - 2*((9*a^3*b - 15*a^2*b^2 - 145*a*b^3 - 105*b^4)*
cos(f*x + e)^6 - 2*(3*a^2*b^2 + 46*a*b^3 + 35*b^4)*cos(f*x + e)^4 - 48*b^4
- 8*(9*a*b^3 + 7*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^7)]
```

Sympy [F]

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\sec^6\left(e+fx\right)dx$$

[In] integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2)**(3/2),x)

[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x)**6, x)

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.71

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{\frac{48\left(b\tan(fx+e)^2+a+b\right)^{\frac{5}{2}}\tan(fx+e)^3}{b}+\frac{9\,(a+b)^3a\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{5}{2}}}+\frac{9\,(a+b)^3\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}-\frac{48\,(a+b)^2a\,\mathrm{ars}}{}}{}$$

[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] 1/384*(48*(b*tan(f*x + e)^2 + a + b)^(5/2)*tan(f*x + e)^3/b + 9*(a + b)^3*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(5/2) + 9*(a + b)^3*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(3/2) - 48*(a + b)^2*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(3/2) - 48*(a + b)^2*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) + 144*(a + b)*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) + 144*(a + b)*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 96*(b*tan(f*x + e)^2 + a + b)^(3/2)*tan(f*x + e) + 144*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(f*x + e) - 24*(b*tan(f*x + e)^2 + a + b)^(5/2)*(a + b)*tan(f*x + e)/b^2 + 6*(b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2*tan(f*x + e)/b^2 + 9*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3*tan(f*x + e)/b^2 + 128*(b*tan(f*x + e)^2 + a + b)^(5/2)*tan(f*x + e)/b - 32*(b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)*tan(f*x + e)/b - 48*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2*tan(f*x + e)/b)/f

Giac [F]

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^6\,dx$$

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\cos\left(e+f\,x\right)^6}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^6,x)
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^6, x)
```

3.248 $\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 165

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = -\frac{(a-5b)(a+b)^2\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16b^{3/2}f} - \frac{(a-5b)(a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16bf} - \frac{(a-5b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{24bf} + \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{6bf}$$

```
[Out] -1/16*(a-5*b)*(a+b)^2*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2)
)/b^(3/2)/f-1/16*(a-5*b)*(a+b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f-1/
24*(a-5*b)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/b/f+1/6*tan(f*x+e)*(a+b+b*
tan(f*x+e)^2)^(5/2)/b/f
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used

$= \{4231, 396, 201, 223, 212\}$

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = -\frac{(a-5b)(a+b)^2\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16b^{3/2}f} + \frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6bf} - \frac{(a-5b)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{24bf} - \frac{(a-5b)(a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16bf}$$

```
[In] Int[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/16*((a - 5*b)*(a + b)^2*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Ta
n[e + f*x]^2]])/(b^(3/2)*f) - ((a - 5*b)*(a + b)*Tan[e + f*x]*Sqrt[a + b +
b*Tan[e + f*x]^2])/(16*b*f) - ((a - 5*b)*Tan[e + f*x]*(a + b + b*Tan[e + f*
x]^2)^(3/2))/(24*b*f) + (Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(5/2))/(6*
b*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (1+x^2)\left(a+b+bx^2\right)^{3/2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{6bf}\\
&\quad -\frac{(a-5b)\text{Subst}\left(\int \left(a+b+bx^2\right)^{3/2}\,dx, x, \tan(e+fx)\right)}{6bf}\\
&= -\frac{(a-5b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{24bf}\\
&\quad +\frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{6bf}\\
&\quad -\frac{((a-5b)(a+b))\text{Subst}\left(\int \sqrt{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{8bf}\\
&= -\frac{(a-5b)(a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16bf}\\
&\quad -\frac{(a-5b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{24bf}\\
&\quad +\frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{6bf}\\
&\quad -\frac{\left((a-5b)(a+b)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{16bf}
\end{aligned}
$$

$$= -\frac{(a-5b)(a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16bf} - \frac{(a-5b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{24bf} + \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{6bf} - \frac{((a-5b)(a+b)^2)\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16bf}$$

$$= -\frac{(a-5b)(a+b)^2\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16b^{3/2}f} - \frac{(a-5b)(a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16bf} - \frac{(a-5b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{24bf} + \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{6bf}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.97 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.73

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx =$$

$$-\frac{(a+b)^3\left(a+b\sec^2(e+fx)\right)^{3/2}\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^2\tan(e+fx)\left(5\arcsin\left(\sqrt{-\frac{b\tan^2(e+fx)}{a+b}}\right)\left(-9+\frac{6a\sin^2(e-}{a+b}\right.\right.}{}$$

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/60*((a + b)^3*(a + b*Sec[e + f*x]^2)^(3/2)*(1 - (a*Sin[e + f*x]^2)/(a +
b))^2*Tan[e + f*x]*(5*ArcSin[Sqrt[-((b*Tan[e + f*x]^2)/(a + b))]]*(-9 + (6*
a*Sin[e + f*x]^2)/(a + b)) + (5*Sec[e + f*x]^4*(-3*a^3*Cos[e + f*x]^4*(-3 +
 2*Sin[e + f*x]^2) + 3*b^3*(3 + 8*Sin[e + f*x]^2 - 3*Sin[e + f*x]^4) + a*b^
2*(27 + 24*Sin[e + f*x]^2 - 49*Sin[e + f*x]^4 + 6*Sin[e + f*x]^6) + a^2*b*(
27 - 24*Sin[e + f*x]^2 - 19*Sin[e + f*x]^4 + 16*Sin[e + f*x]^6))*Sqrt[-((b*
Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(a + b)^2)])/(a +
 b)^3 - (256*Hypergeometric2F1[2, 5, 7/2, -((b*Tan[e + f*x]^2)/(a + b))]*(a
```

```
 + b - a*Sin[e + f*x]^2)*(-((b*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)*Ta
n[e + f*x]^2)/(a + b)^2))^(5/2))/(a + b)))/(Sqrt[2]*f*(a + 2*b + a*Cos[2*e
+ 2*f*x])^(3/2)*Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)]*(a + b - a*Sin[e + f*x
]^2)^(3/2)*(-((b*Tan[e + f*x]^2)/(a + b)))^(3/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1416 vs. $2(145) = 290$.

Time = 16.72 (sec) , antiderivative size = 1417, normalized size of antiderivative = 8.59

method	result	size
default	Expression too large to display	1417

```
[In] int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/96/f/b^(9/2)*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(-30*b^(11/2)*cos(f*x+e)^2*sin(
f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-30*b^(11/2)*cos(f*x+e)*s
in(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-44*cos(f*x+e)^2*sin(f
*x+e)*a*b^(9/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-20*b^(11/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sin(f*x+e)-44*sin(f*x+e)*cos(f*x+e
)*a*b^(9/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-6*sin(f*x+e)*cos(f*
x+e)^2*a^2*b^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-20*b^(11/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*tan(f*x+e)-28*b^(9/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*sin(f*x+e)-6*sin(f*x+e)*cos(f*x+e)*a^2
*b^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-16*b^(11/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*tan(f*x+e)*sec(f*x+e)-28*b^(9/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*tan(f*x+e)+3*cos(f*x+e)^3*ln(4*(((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a^3*b^
3-9*cos(f*x+e)^3*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*
cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a
-a-b)/(sin(f*x+e)+1))*a^2*b^4-27*cos(f*x+e)^3*ln(4*(((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a*b^5-15*cos(f*x+e)^3*ln
(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1
))*b^6+3*cos(f*x+e)^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^
(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*
x+e)*a+a+b)/(sin(f*x+e)-1))*a^3*b^3-9*cos(f*x+e)^3*ln(-4*(((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a^2*b^4-27*cos(f*x
+e)^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)
+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin
(f*x+e)-1))*a*b^5-15*cos(f*x+e)^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
```

```
^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*b^6-16*b^(11/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*tan(f*x+e)*sec(f*x+e)^2)
```

Fricas [A] (verification not implemented)

none

Time = 1.07 (sec) , antiderivative size = 470, normalized size of antiderivative = 2.85

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left[-\frac{3\left(a^3-3a^2b-9ab^2-5b^3\right)\sqrt{b}\cos(fx+e)^5\log\left(\frac{\left(a^2-6ab+b^2\right)\cos(fx+e)^4+8\left(ab-b^2\right)\cos(fx+e)^2+4}{}\right.}{}\right.$$

$$-\frac{3\left(a^3-3a^2b-9ab^2-5b^3\right)\sqrt{-b}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\left(ab\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\cos(fx+e)^5-}{96b^2f\cos}$$

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*sqrt(b)*cos(f*x + e)^5*log(((a
^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b
)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((3*a^2*b + 22*a*b^
2 + 15*b^3)*cos(f*x + e)^4 + 8*b^3 + 2*(7*a*b^2 + 5*b^3)*cos(f*x + e)^2)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f*x + e)
^5), -1/96*(3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*sqrt(-b)*arctan(-1/2*((a -
b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^5 -
 2*((3*a^2*b + 22*a*b^2 + 15*b^3)*cos(f*x + e)^4 + 8*b^3 + 2*(7*a*b^2 + 5*b
^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))/(b^2*f*cos(f*x + e)^5)]
```

Sympy [F]

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\sec^4\left(e+fx\right)dx$$

```
[In] integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.47

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=$$
$$-\frac{\frac{3\,(a+b)^2a\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}+\frac{3\,(a+b)^2\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-\frac{18\,(a+b)a\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-18\,(a+b)\sqrt{b}\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)\cdots}{\cdots}$$

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] -1/48*(3*(a + b)^2*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(3/2) + 3*(a
 + b)^2*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) - 18*(a + b)*a*arcs
inh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) - 18*(a + b)*sqrt(b)*arcsinh(b*
tan(f*x + e)/sqrt((a + b)*b)) - 12*(b*tan(f*x + e)^2 + a + b)^(3/2)*tan(f*x
 + e) - 18*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(f*x + e) - 8*(b*tan(f
*x + e)^2 + a + b)^(5/2)*tan(f*x + e)/b + 2*(b*tan(f*x + e)^2 + a + b)^(3/2
)*(a + b)*tan(f*x + e)/b + 3*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2*tan(f
*x + e)/b)/f
```

Giac [F]

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^4\,dx$$

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\cos\left(e+f\,x\right)^4}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^4,x)
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^4, x)
```

3.249 $\int \sec^2(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx$

Optimal result

Integrand size = 25, antiderivative size = 111

$$\int \sec^2(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \frac{3(a+b)^2\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{b}f} + \frac{3(a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f}$$

```
[Out] 3/8*(a+b)^2*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/b^(1/2
)+3/8*(a+b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f+1/4*tan(f*x+e)*(a+b+b*t
an(f*x+e)^2)^(3/2)/f
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4231, 201, 223, 212}

$$\int \sec^2(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \frac{3(a+b)^2\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8\sqrt{b}f} + \frac{3(a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8f} + \frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4f}$$

```
[In] Int[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a + b)^2*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
)/(8*Sqrt[b]*f) + (3*(a + b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(
8*f) + (Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(4*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(a+b+bx^2\right)^{3/2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f}+\frac{(3(a+b))\text{Subst}\left(\int \sqrt{a+b+bx^2}\,dx,x,\tan(e+fx)\right)}{4f}\\
&= \frac{3(a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f}\\
&\quad+\frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f}\\
&\quad+\frac{\left(3(a+b)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx,x,\tan(e+fx)\right)}{8f}
\end{aligned}
$$

$$= \frac{3(a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f} + \frac{(3(a+b)^2)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8f}$$

$$= \frac{3(a+b)^2\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{b}f} + \frac{3(a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.46 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2} dx = \frac{(a+b)^2\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},\frac{b\sin^2(e+fx)}{a+b-a\sin^2(e+fx)}\right)\sqrt{a+b\sec^2(e+fx)}\sin(2(e+fx))}{f(a+2b+a\cos(2(e+fx)))}$$

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] ((a + b)^2*Hypergeometric2F1[1/2, 3, 3/2, (b*Sin[e + f*x]^2)/(a + b - a*Sin
[e + f*x]^2)]*Sqrt[a + b*Sec[e + f*x]^2]*Sin[2*(e + f*x)])/(f*(a + 2*b + a*
Cos[2*(e + f*x)]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 950 vs. 2(95) = 190.

Time = 11.80 (sec) , antiderivative size = 951, normalized size of antiderivative = 8.57

method	result	size
default	Expression too large to display	951

```
[In] int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/f/b^(5/2)*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(6*cos(f*x+e)^2*sin(f*x+e)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(7/2)+6*cos(f*x+e)*sin(f*x+e)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(7/2)+10*cos(f*x+e)^2*sin(f*x+e)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(5/2)*a+4*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*b^(7/2)*sin(f*x+e)+10*cos(f*x+e)*sin(f*x+e)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(5/2)*a+3*cos(f*x+e)^3*ln(4*(((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a^2*b^2
+6*cos(f*x+e)^3*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*c
os(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-
a-b)/(sin(f*x+e)+1))*a*b^3+3*cos(f*x+e)^3*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^4+3*cos(f*x+e)^3*ln(-4*(((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a^2
*b^2+6*cos(f*x+e)^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1
/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+
e)*a+a+b)/(sin(f*x+e)-1))*a*b^3+3*cos(f*x+e)^3*ln(-4*(((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*b^4+4*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*b^(7/2)*tan(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 390, normalized size of antiderivative = 3.51

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left[\frac{3\left(a^2+2\,ab+b^2\right)\sqrt{b}\cos\left(fx+e\right)^3\log\left(\frac{\left(a^2-6\,ab+b^2\right)\cos(fx+e)^4+8\left(ab-b^2\right)\cos(fx+e)^2+4\left((a-b)\cos(fx+\right.}{\cos(fx+e)^4}\right.}{}\right.$$

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(a^2 + 2*a*b + b^2)*sqrt(b)*cos(f*x + e)^3*log(((a^2 - 6*a*b + b^2
)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3
 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*si
n(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*((5*a*b + 3*b^2)*cos(f*x + e)^2 + 2
*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b*f*cos(f*
```

```
x + e)^3), 1/16*(3*(a^2 + 2*a*b + b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*
x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^3 + 2*((5*a*
b + 3*b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(b*f*cos(f*x + e)^3)]
```

Sympy [F]

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\sec^2\left(e+fx\right)dx$$

```
[In] integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.94

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{\frac{3\,(a+b)a\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}+3\,(a+b)\sqrt{b}\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)+2\left(b\tan\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}}\tan\left(f\right.}{8\,f}$$

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] 1/8*(3*(a + b)*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) + 3*(a + b
)*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 2*(b*tan(f*x + e)^2 + a
 + b)^(3/2)*tan(f*x + e) + 3*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(f*x
 + e))/f
```

Giac [F]

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^2\,dx$$

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\cos\left(e+f\,x\right)^2}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^2,x)
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^2, x)
```

3.250 $\int (a+b\sec^2(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 118

$$\int (a+b\sec^2(e+fx))^{3/2}\,dx=\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}(3a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}+\frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

```
[Out] a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/2*(3*a+b)
*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f+1/2*b*(a+
b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4213, 427, 537, 223, 212, 385, 209}

$$\int (a+b\sec^2(e+fx))^{3/2}\,dx=\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\sqrt{b}(3a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}+\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*(3*a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f
*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{(a+b)(2a+b)+b(3a+b)x^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{a^2\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} + \frac{(b(3a+b))\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{a^2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(b(3a+b))\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}$$

$$= \frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}(3a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.14 (sec) , antiderivative size = 527, normalized size of antiderivative = 4.47

$$\int (a+b\sec^2(e+fx))^{3/2}\,dx = \frac{\sqrt{2}e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{ib\left(-1+e^{2i(e+fx)}\right)}{\left(1+e^{2i(e+fx)}\right)^2} + \frac{2a^{3/2}fx-ia^{3/2}\log\left(a\right.}{}\right.}{}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*(((-I)*b*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((
2*I)*(e + f*x)))^2 + (2*a^(3/2)*f*x - I*a^(3/2)*Log[a + 2*b + a*E^((2*I)*(e
 + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)
))^2]] + I*a^(3/2)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x))
+ Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] -
3*a*Sqrt[b]*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (2*I)*Sqrt[4*b*E
^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(b*(3*a + b)*(1 + E^
((2*I)*(e + f*x))))] - b^(3/2)*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f
 + (2*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(
b*(3*a + b)*(1 + E^((2*I)*(e + f*x))))])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(
1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(f*(a + 2*b + a*
Cos[2*e + 2*f*x])^(3/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 708 vs. $2(100) = 200$.

Time = 7.76 (sec) , antiderivative size = 709, normalized size of antiderivative = 6.01

method	result
default	$\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}\left(\cos(fx+e)^3 b^{\frac{5}{2}}\ln\left(\frac{-4\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{b}\cos(fx+e)+4\sin(fx+e)a-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4a-4b}{\sin(fx+e)-1}\right)\sqrt{-a}+\cos(f\right.}{}$

```
[In] int((a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f/(-a)^(1/2)/b*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(cos(f*x+e)^3*b^(5/2)*ln(4*(
-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*
a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*
(-a)^(1/2)+cos(f*x+e)^3*b^(5/2)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*(-a)^(1/2)+3*cos(f*x+e)^3*b^(3/2)*ln
(4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x
+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-
1))*(-a)^(1/2)*a+3*cos(f*x+e)^3*b^(3/2)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*(-a)^(1/2)*a+2*sin(f*x+e)*co
s(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*b^2+4*ln(
4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^
(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*cos(f*x+e
)^3*a^2*b+2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*cos(
f*x+e)*sin(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(100) = 200.

Time = 0.60 (sec) , antiderivative size = 1457, normalized size of antiderivative = 12.35

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] [1/8*(sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b
)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a
*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
+ e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7
*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e)) + (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^
2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^
3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e)), 1/8*(2*(3*a + b)*sqrt(-b)*arcta
n(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos
(f*x + e) + sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 -
 a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 +
 a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b
^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*co
s(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2
*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e)) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e))/(f*cos(f*x + e)), -1/8*(2*a^(3/2)*arctan(1/4*(8*a^2*cos(f*x
+ e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*s
qrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 -
 a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e
) - (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
+ 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^
2)/cos(f*x + e)^4) - 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e))/(f*cos(f*x + e)), -1/4*(a^(3/2)*arctan(1/4*(8*a^2*cos(f*x + e)^5 -
8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b +
a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) - (3*a
+ b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(
-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)
*sin(f*x + e)))*cos(f*x + e) - 2*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(f*cos(f*x + e))]
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int((a + b/cos(e + f*x)^2)^(3/2), x)
```

3.251 $\int \cos^2(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx$

Optimal result

Integrand size = 25, antiderivative size = 124

$$\int \cos^2(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \frac{\sqrt{a}(a+3b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{a\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

```
[Out] b^(3/2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/2*(a+3*b
)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f+1/2*a*cos
(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4231, 424, 537, 223, 212, 385, 209}

$$\int \cos^2(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \frac{\sqrt{a}(a+3b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f} + \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f} + \frac{a\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}$$

```
[In] Int[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a]*(a + 3*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x
]^2]])/(2*f) + (b^(3/2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e
```

```
 + f*x]^2]])/f + (a*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2
])/(2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
```

```
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^{3/2}}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{(a+b)(a+2b)+2b^2x^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2f}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{b^2\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} + \frac{(a(a+3b))\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2f}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{b^2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(a(a+3b))\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}$$

$$= \frac{\sqrt{a}(a+3b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{a\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.32 (sec) , antiderivative size = 466, normalized size of antiderivative = 3.76

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{e^{-i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-ia\left(-1+e^{2i(e+fx)}\right)+\frac{2e^{2i(e+fx)}\left(2a^3\right.}{}\right.}{}$$

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*Cos[e + f*
x]^3*((-I)*a*(-1 + E^((2*I)*(e + f*x))) + (2*E^((2*I)*(e + f*x))*(2*a^(3/2)
*f*x + 6*Sqrt[a]*b*f*x - I*Sqrt[a]*(a + 3*b)*Log[(a + 2*b + a*E^((2*I)*(e +
 f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))
^2])/E^((2*I)*e)] + I*Sqrt[a]*(a + 3*b)*Log[(a + a*E^((2*I)*(e + f*x)) + 2*
b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2
*I)*(e + f*x)))^2])/E^((2*I)*e)] - 4*b^(3/2)*Log[-1/2*(E^((3*I)*e)*(Sqrt[b]
*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2
*I)*(e + f*x)))^2])*f)/(b^2*(1 + E^((2*I)*(e + f*x))))]))/Sqrt[4*b*E^((2*I)
*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))
/(2*Sqrt[2]*E^(I*(e + f*x))*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. $2(106) = 212$.

Time = 9.31 (sec) , antiderivative size = 537, normalized size of antiderivative = 4.33

method	result
default	$\frac{\left(\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{-a}\,a\cos(fx+e)\sin(fx+e)+b^{\frac{3}{2}}\ln\left(-\frac{4\left(-\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{b}\cos(fx+e)+\sin(fx+e)a-\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}+a+b\right.}{\sin(fx+e)+1}\right.\right.}{}$

```
[In] int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f/(-a)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a*
cos(f*x+e)*sin(f*x+e)+b^(3/2)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
```

```
*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*(-a)^(1/2)+b^(3/2)*ln(4*(-((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*(-a)^(1/2)+si
n(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a+ln(4*(-a)
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2+3*ln(4*(-a)
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b)*(a+b*sec(f
*x+e)^2)^(3/2)*cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/(b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(106) = 212$.

Time = 0.61 (sec) , antiderivative size = 1403, normalized size of antiderivative = 11.31

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x
 + e) + sqrt(-a)*(a + 3*b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*c
os(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28
*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^
3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e
)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*
b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*sin(f*x + e)) + 4*b^(3/2)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a
*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sq
rt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos
(f*x + e)^4))/f, 1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(
f*x + e)*sin(f*x + e) + 8*sqrt(-b)*b*arctan(-1/2*((a - b)*cos(f*x + e)^3 +
2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*
b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) + sqrt(-a)*(a + 3*b)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/f, 1/8*(4*a*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) - (a + 3*b)*sqrt(
a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 -
 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^
```

```
2)*sin(f*x + e))) + 2*b^(3/2)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(
a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*s
qrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/co
s(f*x + e)^4))/f, 1/8*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(
f*x + e)*sin(f*x + e) - (a + 3*b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5
- 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b
+ a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*sqrt(-b)*b*arc
tan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))))/
f]
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^2\,dx$$

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^2, x)
```

Giac [F]

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^2\,dx$$

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \cos\left(e+f\,x\right)^2\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

```
[In] int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.252 $\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 125

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{3(a+b)^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{a}f}+\frac{3(a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f}+\frac{\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f}$$

```
[Out] 3/8*(a+b)^2*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)
+3/8*(a+b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f+1/4*cos(f*x+e
)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/f
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4231, 386, 385, 209}

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{3(a+b)^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8\sqrt{a}f}+\frac{\sin(e+fx)\cos^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4f}+\frac{3(a+b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8f}$$

```
[In] Int[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a + b)^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])
/(8*Sqrt[a]*f) + (3*(a + b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e
+ f*x]^2])/(8*f) + (Cos[e + f*x]^3*Sin[e + f*x]*(a + b + b*Tan[e + f*x]^2)^
(3/2))/(4*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^{3/2}}{(1+x^2)^3}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f} \\
&\quad + \frac{(3(a+b))\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{4f}
\end{aligned}
$$

$$= \frac{3(a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f} + \frac{(3(a+b)^2)\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{8f}$$

$$= \frac{3(a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f} + \frac{(3(a+b)^2)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8f}$$

$$= \frac{3(a+b)^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{a}f} + \frac{3(a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f}$$

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.53

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\cos(e+fx)\left(b+a\cos^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}\left(3(a+b)^{3/2}\text{ar}\right.}{2\sqrt{a}f(a+2b+a\cos(2(e+fx))}$$

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cos[e + f*x]*(b + a*Cos[e + f*x]^2)*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b
- a*Sin[e + f*x]^2]*(3*(a + b)^(3/2)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a +
 b]] + Sqrt[a]*(4*a + 5*b + a*Cos[2*(e + f*x)])*Sin[e + f*x]*Sqrt[(a + b -
a*Sin[e + f*x]^2)/(a + b)]))/(2*Sqrt[a]*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3
/2)*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. 2(109) = 218.

Time = 3.58 (sec) , antiderivative size = 602, normalized size of antiderivative = 4.82

method	result
default	$\frac{\left(2\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\sqrt{-a}\,a\cos(fx+e)^3\sin(fx+e)+2\cos(fx+e)^2\sin(fx+e)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\sqrt{-a}\,a+3\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\sqrt{-a}\,a\cos(f\right.}{}$

```
[In] int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f/(-a)^(1/2)*(2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*
a*cos(f*x+e)^3*sin(f*x+e)+2*cos(f*x+e)^2*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(-a)^(1/2)*a*cos(f*x+e)*sin(f*x+e)+5*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(-a)^(1/2)*b*cos(f*x+e)*sin(f*x+e)+3*sin(f*x+e)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a+5*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*b+3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2+6*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b+3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^2)*(a+b*sec(f*x+e)^2)^(3/2)*cos(f*x+e)^3/
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(109) = 218.

Time = 0.57 (sec) , antiderivative size = 563, normalized size of antiderivative = 4.50

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\left[-\frac{3\left(a^2+2ab+b^2\right)\sqrt{-a}\log\left(128a^4\cos(fx+e)^8-256\left(a^4-a^3b\right)\cos(fx+e)^6+32\left(5a^4\right.\right.}{}\right.$$

$$-\frac{3\left(a^2+2ab+b^2\right)\sqrt{a}\arctan\left(\frac{\left(8a^2\cos(fx+e)^5-8\left(a^2-ab\right)\cos(fx+e)^3+\left(a^2-6ab+b^2\right)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2a^3\cos(fx+e)^4-a^2b+ab^2-\left(a^3-3a^2b\right)\cos(fx+e)^2\right)\sin(fx+e)}\right)-4\,(2}{32\,af}$$

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(3*(a^2 + 2*a*b + b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^
4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^
2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)
*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*
a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e)) - 8*(2*a^2*cos(f*x + e)^3 + (3*a^2 + 5*a*b)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*f)
, -1/32*(3*(a^2 + 2*a*b + b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8
*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a
*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2*a^2*cos(f*x +
e)^3 + (3*a^2 + 5*a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e))/(a*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^4\,dx$$

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^4, x)
```

Giac [F]

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^4\,dx$$

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int\cos\left(e+f\,x\right)^4\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

```
[In] int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.253 $\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 193

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{(5a-b)(a+b)^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{3/2}f} + \frac{(5a-b)(a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16af} + \frac{(5a-b)\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{24af} + \frac{\cos^5(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{6af}$$

```
[Out] 1/16*(5*a-b)*(a+b)^2*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/
a^(3/2)/f+1/16*(5*a-b)*(a+b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/
2)/a/f+1/24*(5*a-b)*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/a/f+
1/6*cos(f*x+e)^5*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(5/2)/a/f
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{4231, 390, 386, 385, 209\}$

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{(5a-b)(a+b)^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{3/2}f} + \frac{\sin(e+fx)\cos^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6af} + \frac{(5a-b)\sin(e+fx)\cos^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{24af} + \frac{(5a-b)(a+b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16af}$$

```
[In] Int[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((5*a - b)*(a + b)^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f
*x]^2]])/(16*a^(3/2)*f) + ((5*a - b)*(a + b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt
[a + b + b*Tan[e + f*x]^2])/(16*a*f) + ((5*a - b)*Cos[e + f*x]^3*Sin[e + f*
x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(24*a*f) + (Cos[e + f*x]^5*Sin[e + f*x
]*(a + b + b*Tan[e + f*x]^2)^(5/2))/(6*a*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^{3/2}}{(1+x^2)^4} \, dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{6af} + \frac{(5a-b)\text{Subst}\left(\int \frac{(a+b+bx^2)^{3/2}}{(1+x^2)^3} \, dx, x, \tan(e+fx)\right)}{6af}$$

$$= \frac{(5a-b)\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{24af} + \frac{\cos^5(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{6af} + \frac{((5a-b)(a+b))\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{(1+x^2)^2} \, dx, x, \tan(e+fx)\right)}{8af}$$

$$= \frac{(5a-b)(a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16af} + \frac{(5a-b)\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{24af} + \frac{\cos^5(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{6af} + \frac{\left((5a-b)(a+b)^2\right)\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}} \, dx, x, \tan(e+fx)\right)}{16af}$$

$$=\frac{(5a-b)(a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16af}$$
$$+\frac{(5a-b)\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{24af}$$
$$+\frac{\cos^5(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{6af}$$
$$+\frac{((5a-b)(a+b)^2)\,\text{Subst}\left(\int\frac{1}{1+ax^2}\,dx,x,\frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16af}$$

$$=\frac{(5a-b)(a+b)^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{3/2}f}$$
$$+\frac{(5a-b)(a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16af}$$
$$+\frac{(5a-b)\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{24af}$$
$$+\frac{\cos^5(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{6af}$$

Mathematica [A] (verified)

Time = 2.22 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.85

$$\int\cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\frac{3\sqrt{2}\sqrt{a+b}(5a^2+4ab-b^2)\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}}+\sqrt{a}(23a^2+29ab+3b^2\right.}{48a^{3/2}f}$$

```
[In] Integrate[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*((3*Sqrt[2]*Sqrt[a + b]*(5*a^2 + 4
*a*b - b^2)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/Sqrt[(a + 2*b + a*C
os[2*(e + f*x)])/(a + b)] + Sqrt[a]*(23*a^2 + 29*a*b + 3*b^2 + a*(9*a + 7*b
)*Cos[2*(e + f*x)] + a^2*Cos[4*(e + f*x)])*Sin[e + f*x]))/(48*a^(3/2)*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 985 vs. $2(173) = 346$.

Time = 6.04 (sec) , antiderivative size = 986, normalized size of antiderivative = 5.11

method	result	size
default	Expression too large to display	986

```
[In] int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/f/a/(-a)^(1/2)*(8*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*a^2*cos(f*x+e)^5*sin(f*x+e)+8*(-a)^(1/2)*sin(f*x+e)*cos(f*x+e)^4*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+10*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*a^2*cos(f*x+e)^3*sin(f*x+e)+14*(-a)^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b*cos(f*x+e)^3*sin(f*x+e)+10*(-a)^(1/2
)*sin(f*x+e)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+1
4*(-a)^(1/2)*sin(f*x+e)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*a*b+15*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*cos
(f*x+e)*sin(f*x+e)+22*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*a*b*cos(f*x+e)*sin(f*x+e)+3*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*b^2*cos(f*x+e)*sin(f*x+e)+15*(-a)^(1/2)*sin(f*x+e)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+22*(-a)^(1/2)*sin(f*x+e)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*a*b+3*(-a)^(1/2)*sin(f*x+e)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*b^2+15*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)-4*sin(f*x+e)*a)*a^3+27*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)-4*sin(f*x+e)*a)*a^2*b+9*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^2-3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^3)*(a+b*sec(f*x+e)^2)^(3/2)*cos(f*x+e)^3/(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)
```

Fricas [A] (verification not implemented)

none

Time = 1.19 (sec) , antiderivative size = 647, normalized size of antiderivative = 3.35

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left[\frac{3\,(5\,a^3+9\,a^2b+3\,ab^2-b^3)\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8-256\,(a^4-a^3b)\cos\left(fx+e\right)^6+3\right.}{}\right.$$

$$-\frac{3\,(5\,a^3+9\,a^2b+3\,ab^2-b^3)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\,(a^2-ab)\cos(fx+e)^3+(a^2-6\,ab+b^2)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3\,a^2b)\cos(fx+e)^2\right)\sin(fx+e)}\right.}{}$$

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/384*(3*(5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*sqrt(-a)*log(128*a^4*cos(f*x +
e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)
*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 -
7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24
*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)
^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^3*cos(f*x + e)^5 + 2*(5
*a^3 + 7*a^2*b)*cos(f*x + e)^3 + (15*a^3 + 22*a^2*b + 3*a*b^2)*cos(f*x + e)
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f), -1/192
*(3*(5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e
)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt
(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^
2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(8*a^3*cos
(f*x + e)^5 + 2*(5*a^3 + 7*a^2*b)*cos(f*x + e)^3 + (15*a^3 + 22*a^2*b + 3*a
*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))/(a^2*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \cos^6(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \cos^6(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \int \left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}} \cos(fx+e)^6 dx$$

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^6, x)
```

Giac [F]

$$\int \cos^6(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \int \left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}} \cos(fx+e)^6 dx$$

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^6(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \int \cos(e+f\,x)^6 \left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2} dx$$

```
[In] int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.254 $\int \left(a+b\sec^2(c+dx)\right)^{5/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 166

$$\int \left(a+b\sec^2(c+dx)\right)^{5/2}\,dx = \frac{a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+b+b\tan^2(c+dx)}}\right)}{d} + \frac{\sqrt{b}(15a^2+10ab+3b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b+b\tan^2(c+dx)}}\right)}{8d} + \frac{b(7a+3b)\tan(c+dx)\sqrt{a+b+b\tan^2(c+dx)}}{8d} + \frac{b\tan(c+dx)\left(a+b+b\tan^2(c+dx)\right)^{3/2}}{4d}$$

```
[Out] a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+b+b*tan(d*x+c)^2)^(1/2))/d+1/8*(15*a^2
+10*a*b+3*b^2)*arctanh(b^(1/2)*tan(d*x+c)/(a+b+b*tan(d*x+c)^2)^(1/2))*b^(1/
2)/d+1/8*b*(7*a+3*b)*(a+b+b*tan(d*x+c)^2)^(1/2)*tan(d*x+c)/d+1/4*b*tan(d*x+
c)*(a+b+b*tan(d*x+c)^2)^(3/2)/d
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used

$= \{4213, 427, 542, 537, 223, 212, 385, 209\}$

$$\int \left(a+b\sec^2(c+dx)\right)^{5/2} dx = \frac{a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)+b}}\right)}{d} + \frac{\sqrt{b}(15a^2+10ab+3b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)+b}}\right)}{8d} + \frac{b\tan(c+dx)\left(a+b\tan^2(c+dx)+b\right)^{3/2}}{4d} + \frac{b(7a+3b)\tan(c+dx)\sqrt{a+b\tan^2(c+dx)+b}}{8d}$$

```
[In] Int[(a + b*Sec[c + d*x]^2)^(5/2),x]
```

```
[Out] (a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + b + b*Tan[c + d*x]^2]])/d +
 (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTanh[(Sqrt[b]*Tan[c + d*x])/Sqrt[a +
 b + b*Tan[c + d*x]^2]])/(8*d) + (b*(7*a + 3*b)*Tan[c + d*x]*Sqrt[a + b + b
*Tan[c + d*x]^2])/(8*d) + (b*Tan[c + d*x]*(a + b + b*Tan[c + d*x]^2)^(3/2))
/(4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
```

```
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^{5/2}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}\\
&= \frac{b\tan(c+dx)\left(a+b+b\tan^2(c+dx)\right)^{3/2}}{4d}\\
&\quad+\frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}\left((a+b)(4a+3b)+b(7a+3b)x^2\right)}{1+x^2}\,dx, x, \tan(c+dx)\right)}{4d}\\
&= \frac{b(7a+3b)\tan(c+dx)\sqrt{a+b+b\tan^2(c+dx)}}{8d}\\
&\quad+\frac{b\tan(c+dx)\left(a+b+b\tan^2(c+dx)\right)^{3/2}}{4d}\\
&\quad+\frac{\text{Subst}\left(\int \frac{(a+b)(8a^2+7ab+3b^2)+b(15a^2+10ab+3b^2)x^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(c+dx)\right)}{8d}
\end{aligned}$$

$$= \frac{b(7a+3b)\tan(c+dx)\sqrt{a+b+b\tan^2(c+dx)}}{8d} + \frac{b\tan(c+dx)\left(a+b+b\tan^2(c+dx)\right)^{3/2}}{4d} + \frac{a^3\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(c+dx)\right)}{d} + \frac{(b(15a^2+10ab+3b^2))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(c+dx)\right)}{8d}$$

$$= \frac{b(7a+3b)\tan(c+dx)\sqrt{a+b+b\tan^2(c+dx)}}{8d} + \frac{b\tan(c+dx)\left(a+b+b\tan^2(c+dx)\right)^{3/2}}{4d} + \frac{a^3\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(c+dx)}{\sqrt{a+b+b\tan^2(c+dx)}}\right)}{d} + \frac{(b(15a^2+10ab+3b^2))\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(c+dx)}{\sqrt{a+b+b\tan^2(c+dx)}}\right)}{8d}$$

$$= \frac{a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+b+b\tan^2(c+dx)}}\right)}{d} + \frac{\sqrt{b}(15a^2+10ab+3b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b+b\tan^2(c+dx)}}\right)}{8d} + \frac{b(7a+3b)\tan(c+dx)\sqrt{a+b+b\tan^2(c+dx)}}{8d} + \frac{b\tan(c+dx)\left(a+b+b\tan^2(c+dx)\right)^{3/2}}{4d}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.62 (sec) , antiderivative size = 706, normalized size of antiderivative = 4.25

$$\int \left(a+b\sec^2(c+dx)\right)^{5/2}\,dx = \frac{e^{i(c+dx)}\sqrt{4b+ae^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^2}\cos^5(c+dx)\left(-\frac{ib\left(-1+e^{2i(c+dx)}\right)\left(9a\left(1+e^{2i(c+dx)}\right)^2+b(3+14e^{\cdots}\right.}{\left(1+e^{2i(c+dx)}\right)^4}\right.}{\cdots}$$

```
[In] Integrate[(a + b*Sec[c + d*x]^2)^(5/2),x]
```

```
[Out] (E^(I*(c + d*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(c + d*x)))^2)/E^((2*I)*(c + d
*x))]*Cos[c + d*x]^5*(((-I)*b*(-1 + E^((2*I)*(c + d*x)))*(9*a*(1 + E^((2*I)
```

```
*(c + d*x)))^2 + b*(3 + 14*E^((2*I)*(c + d*x)) + 3*E^((4*I)*(c + d*x)))))/(
1 + E^((2*I)*(c + d*x)))^4 + (8*a^(5/2)*d*x - (4*I)*a^(5/2)*Log[a + 2*b + a
*E^((2*I)*(c + d*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(c + d*x)) + a*(1 + E^((2*
I)*(c + d*x)))^2]] + (4*I)*a^(5/2)*Log[a + a*E^((2*I)*(c + d*x)) + 2*b*E^((
2*I)*(c + d*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(c + d*x)) + a*(1 + E^((2*I)*(c
 + d*x)))^2]] - 15*a^2*Sqrt[b]*Log[(-4*Sqrt[b]*d*(-1 + E^((2*I)*(c + d*x)))
 + (4*I)*d*Sqrt[4*b*E^((2*I)*(c + d*x)) + a*(1 + E^((2*I)*(c + d*x)))^2])/(
b*(15*a^2 + 10*a*b + 3*b^2)*(1 + E^((2*I)*(c + d*x))))] - 10*a*b^(3/2)*Log[
(-4*Sqrt[b]*d*(-1 + E^((2*I)*(c + d*x))) + (4*I)*d*Sqrt[4*b*E^((2*I)*(c + d
*x)) + a*(1 + E^((2*I)*(c + d*x)))^2])/(b*(15*a^2 + 10*a*b + 3*b^2)*(1 + E^
((2*I)*(c + d*x))))] - 3*b^(5/2)*Log[(-4*Sqrt[b]*d*(-1 + E^((2*I)*(c + d*x)
)) + (4*I)*d*Sqrt[4*b*E^((2*I)*(c + d*x)) + a*(1 + E^((2*I)*(c + d*x)))^2])
/(b*(15*a^2 + 10*a*b + 3*b^2)*(1 + E^((2*I)*(c + d*x))))])/Sqrt[4*b*E^((2*I
)*(c + d*x)) + a*(1 + E^((2*I)*(c + d*x)))^2])*(a + b*Sec[c + d*x]^2)^(5/2)
)/(Sqrt[2]*d*(a + 2*b + a*Cos[2*c + 2*d*x])^(5/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1136 vs. 2(144) = 288.

Time = 21.08 (sec) , antiderivative size = 1137, normalized size of antiderivative = 6.85

method	result	size
default	Expression too large to display	1137

```
[In] int((a+b*sec(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d/(-a)^(1/2)/b^2*(a+b*sec(d*x+c)^2)^(5/2)/((a*cos(d*x+c)^2+b)/(cos(d*x
+c)+1)^2)^(1/2)/(a*cos(d*x+c)^2+b)^2/(cos(d*x+c)+1)*(3*b^(9/2)*(-a)^(1/2)*c
os(d*x+c)^5*ln(4*(-b^(1/2)*((a*cos(d*x+c)^2+b)/(cos(d*x+c)+1)^2)^(1/2)*cos(
d*x+c)+sin(d*x+c)*a-b^(1/2)*((a*cos(d*x+c)^2+b)/(cos(d*x+c)+1)^2)^(1/2)-a-b
)/(sin(d*x+c)-1))+3*b^(9/2)*(-a)^(1/2)*cos(d*x+c)^5*ln(-4*(-b^(1/2)*((a*cos
(d*x+c)^2+b)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)+sin(d*x+c)*a-b^(1/2)*((a*co
s(d*x+c)^2+b)/(cos(d*x+c)+1)^2)^(1/2)+a+b)/(sin(d*x+c)+1))+10*b^(7/2)*(-a)^
(1/2)*cos(d*x+c)^5*ln(4*(-b^(1/2)*((a*cos(d*x+c)^2+b)/(cos(d*x+c)+1)^2)^(1/
2)*cos(d*x+c)+sin(d*x+c)*a-b^(1/2)*((a*cos(d*x+c)^2+b)/(cos(d*x+c)+1)^2)^(1
/2)-a-b)/(sin(d*x+c)-1))*a+10*b^(7/2)*(-a)^(1/2)*cos(d*x+c)^5*ln(-4*(-b^(1/
2)*((a*cos(d*x+c)^2+b)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)+sin(d*x+c)*a-b^(1
/2)*((a*cos(d*x+c)^2+b)/(cos(d*x+c)+1)^2)^(1/2)+a+b)/(sin(d*x+c)+1))*a+15*b
^(5/2)*(-a)^(1/2)*cos(d*x+c)^5*ln(4*(-b^(1/2)*((a*cos(d*x+c)^2+b)/(cos(d*x+
c)+1)^2)^(1/2)*cos(d*x+c)+sin(d*x+c)*a-b^(1/2)*((a*cos(d*x+c)^2+b)/(cos(d*x
+c)+1)^2)^(1/2)-a-b)/(sin(d*x+c)-1))*a^2+15*b^(5/2)*(-a)^(1/2)*cos(d*x+c)^5
*ln(-4*(-b^(1/2)*((a*cos(d*x+c)^2+b)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)+sin
(d*x+c)*a-b^(1/2)*((a*cos(d*x+c)^2+b)/(cos(d*x+c)+1)^2)^(1/2)+a+b)/(sin(d*x
+c)+1))*a^2+18*(-a)^(1/2)*cos(d*x+c)^4*sin(d*x+c)*((a*cos(d*x+c)^2+b)/(cos(
d*x+c)+1)^2)^(1/2)*a*b^3+6*(-a)^(1/2)*cos(d*x+c)^4*sin(d*x+c)*((a*cos(d*x+c
```

```
)^2+b)/(cos(d*x+c)+1)^2)^(1/2)*b^4+16*cos(d*x+c)^5*ln(4*(-a)^(1/2)*((a*cos(
d*x+c)^2+b)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)+4*(-a)^(1/2)*((a*cos(d*x+c)^
2+b)/(cos(d*x+c)+1)^2)^(1/2)-4*sin(d*x+c)*a)*a^3*b^2+18*(-a)^(1/2)*cos(d*x+
c)^3*sin(d*x+c)*((a*cos(d*x+c)^2+b)/(cos(d*x+c)+1)^2)^(1/2)*a*b^3+6*(-a)^(1
/2)*cos(d*x+c)^3*sin(d*x+c)*((a*cos(d*x+c)^2+b)/(cos(d*x+c)+1)^2)^(1/2)*b^4
+4*(-a)^(1/2)*cos(d*x+c)^2*sin(d*x+c)*((a*cos(d*x+c)^2+b)/(cos(d*x+c)+1)^2)
^(1/2)*b^4+4*(-a)^(1/2)*cos(d*x+c)*sin(d*x+c)*((a*cos(d*x+c)^2+b)/(cos(d*x+
c)+1)^2)^(1/2)*b^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(144) = 288$.

Time = 1.29 (sec) , antiderivative size = 1611, normalized size of antiderivative = 9.70

$$\int \left(a+b\sec^2(c+dx)\right)^{5/2}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/32*(4*sqrt(-a)*a^2*cos(d*x + c)^3*log(128*a^4*cos(d*x + c)^8 - 256*(a^4
- a^3*b)*cos(d*x + c)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(d*x + c)^4
+ a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*
b^2 - a*b^3)*cos(d*x + c)^2 - 8*(16*a^3*cos(d*x + c)^7 - 24*(a^3 - a^2*b)*c
os(d*x + c)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(d*x + c)^3 - (a^3 - 7*a^
2*b + 7*a*b^2 - b^3)*cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c)^2 + b)/cos
(d*x + c)^2)*sin(d*x + c)) + (15*a^2 + 10*a*b + 3*b^2)*sqrt(b)*cos(d*x + c)
^3*log(((a^2 - 6*a*b + b^2)*cos(d*x + c)^4 + 8*(a*b - b^2)*cos(d*x + c)^2 +
 4*((a - b)*cos(d*x + c)^3 + 2*b*cos(d*x + c))*sqrt(b)*sqrt((a*cos(d*x + c)
^2 + b)/cos(d*x + c)^2)*sin(d*x + c) + 8*b^2)/cos(d*x + c)^4) + 4*(3*(3*a*b
 + b^2)*cos(d*x + c)^2 + 2*b^2)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)
*sin(d*x + c))/(d*cos(d*x + c)^3), 1/16*(2*sqrt(-a)*a^2*cos(d*x + c)^3*log(
128*a^4*cos(d*x + c)^8 - 256*(a^4 - a^3*b)*cos(d*x + c)^6 + 32*(5*a^4 - 14*
a^3*b + 5*a^2*b^2)*cos(d*x + c)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3
+ b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(d*x + c)^2 - 8*(16*a^3*c
os(d*x + c)^7 - 24*(a^3 - a^2*b)*cos(d*x + c)^5 + 2*(5*a^3 - 14*a^2*b + 5*a
*b^2)*cos(d*x + c)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(d*x + c))*sqrt(-
a)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c)) + (15*a^2 + 10
*a*b + 3*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(d*x + c)^3 + 2*b*cos(d*x +
c))*sqrt(-b)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)/((a*b*cos(d*x + c)
^2 + b^2)*sin(d*x + c)))*cos(d*x + c)^3 + 2*(3*(3*a*b + b^2)*cos(d*x + c)^2
 + 2*b^2)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c))/(d*cos(
d*x + c)^3), -1/32*(8*a^(5/2)*arctan(1/4*(8*a^2*cos(d*x + c)^5 - 8*(a^2 - a
*b)*cos(d*x + c)^3 + (a^2 - 6*a*b + b^2)*cos(d*x + c))*sqrt(a)*sqrt((a*cos(
d*x + c)^2 + b)/cos(d*x + c)^2)/((2*a^3*cos(d*x + c)^4 - a^2*b + a*b^2 - (a
^3 - 3*a^2*b)*cos(d*x + c)^2)*sin(d*x + c)))*cos(d*x + c)^3 - (15*a^2 + 10*
```

```
a*b + 3*b^2)*sqrt(b)*cos(d*x + c)^3*log(((a^2 - 6*a*b + b^2)*cos(d*x + c)^4
 + 8*(a*b - b^2)*cos(d*x + c)^2 + 4*((a - b)*cos(d*x + c)^3 + 2*b*cos(d*x +
 c))*sqrt(b)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c) + 8*b
^2)/cos(d*x + c)^4) - 4*(3*(3*a*b + b^2)*cos(d*x + c)^2 + 2*b^2)*sqrt((a*co
s(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^3), -1/16*(
4*a^(5/2)*arctan(1/4*(8*a^2*cos(d*x + c)^5 - 8*(a^2 - a*b)*cos(d*x + c)^3 +
 (a^2 - 6*a*b + b^2)*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c)^2 + b)/cos(
d*x + c)^2)/((2*a^3*cos(d*x + c)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(d*
x + c)^2)*sin(d*x + c)))*cos(d*x + c)^3 - (15*a^2 + 10*a*b + 3*b^2)*sqrt(-b
)*arctan(-1/2*((a - b)*cos(d*x + c)^3 + 2*b*cos(d*x + c))*sqrt(-b)*sqrt((a*
cos(d*x + c)^2 + b)/cos(d*x + c)^2)/((a*b*cos(d*x + c)^2 + b^2)*sin(d*x + c
)))*cos(d*x + c)^3 - 2*(3*(3*a*b + b^2)*cos(d*x + c)^2 + 2*b^2)*sqrt((a*cos
(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^3)]
```

Sympy [F]

$$\int \left(a+b\sec^2(c+dx)\right)^{5/2}\,dx=\int \left(a+b\sec^2{\left(c+dx\right)}\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*sec(d*x+c)**2)**(5/2),x)
[Out] Integral((a + b*sec(c + d*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \left(a+b\sec^2(c+dx)\right)^{5/2}\,dx=\int \left(b\sec\left(dx+c\right)^2+a\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*sec(d*x+c)^2)^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \left(a+b\sec^2(c+dx)\right)^{5/2}\,dx=\int \left(b\sec\left(dx+c\right)^2+a\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*sec(d*x+c)^2)^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(c+dx)\right)^{5/2}\,dx=\int \left(a+\frac{b}{\cos\left(c+d\,x\right)^2}\right)^{5/2}\,dx$$

```
[In] int((a + b/cos(c + d*x)^2)^(5/2),x)
[Out] int((a + b/cos(c + d*x)^2)^(5/2), x)
```

3.255 $\int \left(1+\sec^2(x)\right)^{3/2} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 42

$$\int \left(1+\sec^2(x)\right)^{3/2} \, dx = 2\text{arcsinh}\left(\frac{\tan(x)}{\sqrt{2}}\right) + \arctan\left(\frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right) + \frac{1}{2}\tan(x)\sqrt{2+\tan^2(x)}$$

```
[Out] 2*arcsinh(1/2*2^(1/2)*tan(x))+arctan(tan(x)/(2+tan(x)^2)^(1/2))+1/2*(2+tan(
x)^2)^(1/2)*tan(x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {4213, 427, 537, 221, 385, 209}

$$\int \left(1+\sec^2(x)\right)^{3/2} \, dx = 2\text{arcsinh}\left(\frac{\tan(x)}{\sqrt{2}}\right) + \arctan\left(\frac{\tan(x)}{\sqrt{\tan^2(x)+2}}\right) + \frac{1}{2}\tan(x)\sqrt{\tan^2(x)+2}$$

```
[In] Int[(1 + Sec[x]^2)^(3/2),x]
```

```
[Out] 2*ArcSinh[Tan[x]/Sqrt[2]] + ArcTan[Tan[x]/Sqrt[2 + Tan[x]^2]] + (Tan[x]*Sqr
t[2 + Tan[x]^2])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{(2+x^2)^{3/2}}{1+x^2}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\tan(x)\sqrt{2+\tan^2(x)}+\frac{1}{2}\text{Subst}\left(\int \frac{6+4x^2}{(1+x^2)\sqrt{2+x^2}}\,dx, x, \tan(x)\right)
\end{aligned}
$$

$$= \frac{1}{2}\tan(x)\sqrt{2+\tan^2(x)} + 2\text{Subst}\left(\int \frac{1}{\sqrt{2+x^2}}\,dx, x, \tan(x)\right) + \text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{2+x^2}}\,dx, x, \tan(x)\right)$$

$$= 2\text{arcsinh}\left(\frac{\tan(x)}{\sqrt{2}}\right) + \frac{1}{2}\tan(x)\sqrt{2+\tan^2(x)} + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right)$$

$$= 2\text{arcsinh}\left(\frac{\tan(x)}{\sqrt{2}}\right) + \arctan\left(\frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right) + \frac{1}{2}\tan(x)\sqrt{2+\tan^2(x)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.60

$$\int (1+\sec^2(x))^{3/2}\,dx = \frac{(1+\cos^2(x))\sec(x)\sqrt{1+\sec^2(x)}\left(4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sin(x)}{\sqrt{3+\cos(2x)}}\right)\cos^2(x) - 2i\sqrt{2}\cos^2(x)\log\left(\sqrt{\right.\right.}{(3+\cos(2x))^{3/2}}$$

```
[In] Integrate[(1 + Sec[x]^2)^(3/2),x]
```

```
[Out] ((1 + Cos[x]^2)*Sec[x]*Sqrt[1 + Sec[x]^2]*(4*Sqrt[2]*ArcTanh[(Sqrt[2]*Sin[x
])/Sqrt[3 + Cos[2*x]]]*Cos[x]^2 - (2*I)*Sqrt[2]*Cos[x]^2*Log[Sqrt[3 + Cos[2
*x]] + I*Sqrt[2]*Sin[x]] + Sqrt[3 + Cos[2*x]]*Sin[x]))/(3 + Cos[2*x])^(3/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(35) = 70.

Time = 8.05 (sec) , antiderivative size = 189, normalized size of antiderivative = 4.50

method	result
default	$\frac{\sqrt{2}\cos(x)\left(\sin(x)\cos(x)\sqrt{\frac{\cos(x)^2+1}{(\cos(x)+1)^2}}+2\cos(x)^2\arctan\left(\frac{\sin(x)}{(\cos(x)+1)\sqrt{\frac{\cos(x)^2+1}{(\cos(x)+1)^2}}}\right)-2\cos(x)^2\text{arctanh}\left(\frac{\sin(x)-2}{(\cos(x)+1)\sqrt{\frac{\cos(x)^2+1}{(\cos(x)+1)^2}}}\right)-2\right.}{4\sqrt{\frac{\cos(x)^2+1}{(\cos(x)+1)^2}}(\cos(x)+1)\left(\cos(x)^2+1\right.}$

```
[In] int((1+sec(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*2^(1/2)*cos(x)*(sin(x)*cos(x)*((cos(x)^2+1)/(cos(x)+1)^2)^(1/2)+2*cos(x
)^2*arctan(sin(x)/(cos(x)+1)/((cos(x)^2+1)/(cos(x)+1)^2)^(1/2))-2*cos(x)^2*
arctanh((sin(x)-2)/(cos(x)+1)/((cos(x)^2+1)/(cos(x)+1)^2)^(1/2))-2*cos(x)^2
```

```
*arctanh((sin(x)+2)/(cos(x)+1)/((cos(x)^2+1)/(cos(x)+1)^2)^(1/2))+((cos(x)^
2+1)/(cos(x)+1)^2)^(1/2)*sin(x))*(2+2*sec(x)^2)^(1/2)*(1+sec(x)^2)/((cos(x)
^2+1)/(cos(x)+1)^2)^(1/2)/(cos(x)+1)/(cos(x)^2+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(35) = 70.

Time = 0.26 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.81

$$\int \left(1+\sec^2(x)\right)^{3/2} dx = \frac{\arctan\left(\frac{\sqrt{\frac{\cos(x)^2+1}{\cos(x)^2}}\cos(x)^3\sin(x)+\cos(x)\sin(x)}{\cos(x)^4+\cos(x)^2-1}\right)\cos(x)-\arctan\left(\frac{\sin(x)}{\cos(x)}\right)\cos(x)+2\cos(x)\log}{}$$

```
[In] integrate((1+sec(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(arctan((sqrt((cos(x)^2 + 1)/cos(x)^2)*cos(x)^3*sin(x) + cos(x)*sin(x))
/(cos(x)^4 + cos(x)^2 - 1))*cos(x) - arctan(sin(x)/cos(x))*cos(x) + 2*cos(x
)*log(cos(x)^2 + cos(x)*sin(x) + (cos(x)^2 + cos(x)*sin(x))*sqrt((cos(x)^2
+ 1)/cos(x)^2) + 1) - 2*cos(x)*log(cos(x)^2 - cos(x)*sin(x) + (cos(x)^2 - c
os(x)*sin(x))*sqrt((cos(x)^2 + 1)/cos(x)^2) + 1) + sqrt((cos(x)^2 + 1)/cos(
x)^2)*sin(x))/cos(x)
```

Sympy [F]

$$\int \left(1+\sec^2(x)\right)^{3/2} dx = \int \left(\sec^2(x)+1\right)^{\frac{3}{2}} dx$$

```
[In] integrate((1+sec(x)**2)**(3/2),x)
```

```
[Out] Integral((sec(x)**2 + 1)**(3/2), x)
```

Maxima [F]

$$\int \left(1+\sec^2(x)\right)^{3/2} dx = \int \left(\sec(x)^2+1\right)^{\frac{3}{2}} dx$$

```
[In] integrate((1+sec(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((sec(x)^2 + 1)^(3/2), x)
```

Giac [F]

$$\int \left(1+\sec^2(x)\right)^{3/2}\,dx=\int \left(\sec\left(x\right)^2+1\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((1+sec(x)^2)^(3/2),x, algorithm="giac")
[Out] integrate((sec(x)^2 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1+\sec^2(x)\right)^{3/2}\,dx=\int \left(\frac{1}{\cos\left(x\right)^2}+1\right)^{3/2}\,dx$$

```
[In] int((1/cos(x)^2 + 1)^(3/2),x)
[Out] int((1/cos(x)^2 + 1)^(3/2), x)
```

3.256 $\int \sqrt{1+\sec^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 24

$$\int \sqrt{1+\sec^2(x)}\,dx = \operatorname{arcsinh}\left(\frac{\tan(x)}{\sqrt{2}}\right) + \arctan\left(\frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right)$$

```
[Out] arcsinh(1/2*2^(1/2)*tan(x))+arctan(tan(x)/(2+tan(x)^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4213, 399, 221, 385, 209}

$$\int \sqrt{1+\sec^2(x)}\,dx = \operatorname{arcsinh}\left(\frac{\tan(x)}{\sqrt{2}}\right) + \arctan\left(\frac{\tan(x)}{\sqrt{\tan^2(x)+2}}\right)$$

```
[In] Int[Sqrt[1 + Sec[x]^2],x]
```

```
[Out] ArcSinh[Tan[x]/Sqrt[2]] + ArcTan[Tan[x]/Sqrt[2 + Tan[x]^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\sqrt{2+x^2}}{1+x^2}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{\sqrt{2+x^2}}\,dx, x, \tan(x)\right) + \text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{2+x^2}}\,dx, x, \tan(x)\right)\\
&= \operatorname{arcsinh}\left(\frac{\tan(x)}{\sqrt{2}}\right) + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right)\\
&= \operatorname{arcsinh}\left(\frac{\tan(x)}{\sqrt{2}}\right) + \arctan\left(\frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 57 vs. $2(24) = 48$.

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.38

$$\int \sqrt{1+\sec^2(x)}\,dx = \frac{\sqrt{2}\left(\arcsin\left(\frac{\sin(x)}{\sqrt{2}}\right) + \operatorname{arctanh}\left(\frac{\sqrt{2}\sin(x)}{\sqrt{3+\cos(2x)}}\right)\right)\cos(x)\sqrt{1+\sec^2(x)}}{\sqrt{3+\cos(2x)}}$$

```
[In] Integrate[Sqrt[1 + Sec[x]^2],x]
[Out] (Sqrt[2]*(ArcSin[Sin[x]/Sqrt[2]] + ArcTanh[(Sqrt[2]*Sin[x])/Sqrt[3 + Cos[2*
x]]])*Cos[x]*Sqrt[1 + Sec[x]^2])/Sqrt[3 + Cos[2*x]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(21) = 42$.

Time = 5.98 (sec) , antiderivative size = 120, normalized size of antiderivative = 5.00

method	result
default	$\frac{\sqrt{1+\sec(x)^2}\left(2\arctan\left(\frac{\sin(x)}{(\cos(x)+1)\sqrt{\frac{\cos(x)^2+1}{(\cos(x)+1)^2}}}\right)-\operatorname{arctanh}\left(\frac{\sin(x)+2}{(\cos(x)+1)\sqrt{\frac{\cos(x)^2+1}{(\cos(x)+1)^2}}}\right)-\operatorname{arctanh}\left(\frac{\sin(x)-2}{(\cos(x)+1)\sqrt{\frac{\cos(x)^2+1}{(\cos(x)+1)^2}}}\right)\right)\cos}{2(\cos(x)+1)\sqrt{\frac{\cos(x)^2+1}{(\cos(x)+1)^2}}}$

```
[In] int((1+sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(1+sec(x)^2)^(1/2)*(2*arctan(sin(x)/(cos(x)+1)/((cos(x)^2+1)/(cos(x)+1)
^2)^(1/2))-arctanh((sin(x)+2)/(cos(x)+1)/((cos(x)^2+1)/(cos(x)+1)^2)^(1/2))
-arctanh((sin(x)-2)/(cos(x)+1)/((cos(x)^2+1)/(cos(x)+1)^2)^(1/2)))*cos(x)/(
cos(x)+1)/((cos(x)^2+1)/(cos(x)+1)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(21) = 42$.

Time = 0.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 5.46

$$\int \sqrt{1+\sec^2(x)}\,dx = \frac{1}{2}\arctan\left(\frac{\sqrt{\frac{\cos(x)^2+1}{\cos(x)^2}}\cos(x)^3\sin(x)+\cos(x)\sin(x)}{\cos(x)^4+\cos(x)^2-1}\right) - \frac{1}{2}\arctan\left(\frac{\sin(x)}{\cos(x)}\right)+\frac{1}{2}\log\left(\cos(x)^2+\cos(x)\sin(x) + (\cos(x)^2+\cos(x)\sin(x))\sqrt{\frac{\cos(x)^2+1}{\cos(x)^2}}+1\right)-\frac{1}{2}\log\left(\cos(x)^2 - \cos(x)\sin(x)+(\cos(x)^2-\cos(x)\sin(x))\sqrt{\frac{\cos(x)^2+1}{\cos(x)^2}}+1\right)$$

```
[In] integrate((1+sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*arctan((sqrt((cos(x)^2 + 1)/cos(x)^2)*cos(x)^3*sin(x) + cos(x)*sin(x))/
(cos(x)^4 + cos(x)^2 - 1)) - 1/2*arctan(sin(x)/cos(x)) + 1/2*log(cos(x)^2 +
 cos(x)*sin(x) + (cos(x)^2 + cos(x)*sin(x))*sqrt((cos(x)^2 + 1)/cos(x)^2) +
 1) - 1/2*log(cos(x)^2 - cos(x)*sin(x) + (cos(x)^2 - cos(x)*sin(x))*sqrt((c
os(x)^2 + 1)/cos(x)^2) + 1)
```

Sympy [F]

$$\int \sqrt{1+\sec^2(x)}\,dx = \int \sqrt{\sec^2(x)+1}\,dx$$

```
[In] integrate((1+sec(x)**2)**(1/2),x)
[Out] Integral(sqrt(sec(x)**2 + 1), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 1391, normalized size of antiderivative = 57.96

$$\int \sqrt{1+\sec^2(x)}\,dx = \text{Too large to display}$$

```
[In] integrate((1+sec(x)^2)^(1/2),x, algorithm="maxima")
[Out] -1/2*arctan2(2*(2*(6*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 +
sin(4*x)^2 + 12*sin(4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*
sin(1/2*arctan2(sin(4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)), 2*(2*(6
*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12*sin(
4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*cos(1/2*arctan2(sin(
4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + 8) + 3/2*arctan2(2*(2*(6*c
os(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12*sin(4*
x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*sin(1/2*arctan2(sin(4*
x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + 2*sin(2*x), 2*(2*(6*cos(2*x)
 + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12*sin(4*x)*sin(
2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x) + 6*
sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + 2*cos(2*x) + 6) - arctan2((2*(6*cos
(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12*sin(4*x)
*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*sin(1/2*arctan2(sin(4*x)
 + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + sin(2*x), (2*(6*cos(2*x) + 1)*
cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12*sin(4*x)*sin(2*x) +
 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x) + 6*sin(2*
x), cos(4*x) + 6*cos(2*x) + 1)) + cos(2*x) + 3) - 1/2*log(-(2*sqrt(2)*(abs(
2*e^(2*I*x) + 2)^4 + 4*cos(2*x)^4 + 4*sin(2*x)^4 - 4*(cos(2*x)^2 - sin(2*x)
^2 - 2*cos(2*x) + 1)*abs(2*e^(2*I*x) + 2)^2 - 16*cos(2*x)^3 + 8*(cos(2*x)^2
 - 2*cos(2*x) + 1)*sin(2*x)^2 + 24*cos(2*x)^2 - 16*cos(2*x) + 4)^(1/4)*cos(
1/2*arctan2(4*(cos(2*x) - 1)*sin(2*x)/abs(2*e^(2*I*x) + 2)^2, (abs(2*e^(2*I
*x) + 2)^2 - 2*cos(2*x)^2 + 2*sin(2*x)^2 + 4*cos(2*x) - 2)/abs(2*e^(2*I*x)
+ 2)^2))*sin(2*x) - sqrt(abs(2*e^(2*I*x) + 2)^4 + 4*cos(2*x)^4 + 4*sin(2*x)
^4 - 4*(cos(2*x)^2 - sin(2*x)^2 - 2*cos(2*x) + 1)*abs(2*e^(2*I*x) + 2)^2 -
```

```
16*cos(2*x)^3 + 8*(cos(2*x)^2 - 2*cos(2*x) + 1)*sin(2*x)^2 + 24*cos(2*x)^2
- 16*cos(2*x) + 4)*cos(1/2*arctan2(4*(cos(2*x) - 1)*sin(2*x)/abs(2*e^(2*I*x
) + 2)^2, (abs(2*e^(2*I*x) + 2)^2 - 2*cos(2*x)^2 + 2*sin(2*x)^2 + 4*cos(2*x
) - 2)/abs(2*e^(2*I*x) + 2)^2))^2 - sqrt(abs(2*e^(2*I*x) + 2)^4 + 4*cos(2*x
)^4 + 4*sin(2*x)^4 - 4*(cos(2*x)^2 - sin(2*x)^2 - 2*cos(2*x) + 1)*abs(2*e^(
2*I*x) + 2)^2 - 16*cos(2*x)^3 + 8*(cos(2*x)^2 - 2*cos(2*x) + 1)*sin(2*x)^2
+ 24*cos(2*x)^2 - 16*cos(2*x) + 4)*sin(1/2*arctan2(4*(cos(2*x) - 1)*sin(2*x
)/abs(2*e^(2*I*x) + 2)^2, (abs(2*e^(2*I*x) + 2)^2 - 2*cos(2*x)^2 + 2*sin(2*
x)^2 + 4*cos(2*x) - 2)/abs(2*e^(2*I*x) + 2)^2))^2 + 2*(abs(2*e^(2*I*x) + 2)
^4 + 4*cos(2*x)^4 + 4*sin(2*x)^4 - 4*(cos(2*x)^2 - sin(2*x)^2 - 2*cos(2*x)
+ 1)*abs(2*e^(2*I*x) + 2)^2 - 16*cos(2*x)^3 + 8*(cos(2*x)^2 - 2*cos(2*x) +
1)*sin(2*x)^2 + 24*cos(2*x)^2 - 16*cos(2*x) + 4)^(1/4)*(sqrt(2)*cos(2*x) -
sqrt(2))*sin(1/2*arctan2(4*(cos(2*x) - 1)*sin(2*x)/abs(2*e^(2*I*x) + 2)^2,
(abs(2*e^(2*I*x) + 2)^2 - 2*cos(2*x)^2 + 2*sin(2*x)^2 + 4*cos(2*x) - 2)/abs
(2*e^(2*I*x) + 2)^2)) - 2*cos(2*x)^2 - 2*sin(2*x)^2 + 4*cos(2*x) - 2)/abs(2
*e^(2*I*x) + 2)^2)
```

Giac [F]

$$\int \sqrt{1+\sec^2(x)}\,dx = \int \sqrt{\sec(x)^2+1}\,dx$$

```
[In] integrate((1+sec(x)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sec(x)^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+\sec^2(x)}\,dx = \int \sqrt{\frac{1}{\cos(x)^2}+1}\,dx$$

```
[In] int((1/cos(x)^2 + 1)^(1/2),x)
[Out] int((1/cos(x)^2 + 1)^(1/2), x)
```

3.257 $\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 330

$$\begin{aligned}
&\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx \\
&= \frac{2(a-b)E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\left(a+b-a\sin^2(e+fx)\right)}{3b^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} \\
&\quad - \frac{(a-2b)\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{3bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} \\
&\quad - \frac{2(a-b)\sec(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{3b^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} \\
&\quad + \frac{\sec^3(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{3bf\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}
\end{aligned}$$

```
[Out] 2/3*(a-b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/b^2/f/
(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x
+e)^2/(a+b))^(1/2)-1/3*(a-2*b)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*s
in(f*x+e)^2/(a+b))^(1/2)/b/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(
f*x+e)^2))^(1/2)-2/3*(a-b)*sec(f*x+e)*(a+b-a*sin(f*x+e)^2)*tan(f*x+e)/b^2/f
/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+1/3*sec(f*x+e)^3*(a+b-a*sin(f*x+
e)^2)*tan(f*x+e)/b/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4233, 1985, 1986, 425, 541, 538, 437, 435, 432, 430}

$$\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{2(a-b)\left(-a\sin^2(e+fx)+a+b\right)E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{3b^2f\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}} - \frac{(a-2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{3bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}} - \frac{2(a-b)\tan(e+fx)\sec(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{3b^2f\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}} + \frac{\tan(e+fx)\sec^3(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{3bf\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

```
[In] Int[Sec[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (2*(a - b)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a + b - a*Sin[e + f*
x]^2))/(3*b^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e +
 f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - ((a - 2*b)*EllipticF[ArcS
in[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*b*f*S
qrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) - (2*(
a - b)*Sec[e + f*x]*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x])/(3*b^2*f*Sqrt[
Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) + (Sec[e + f*x]^3*(a + b - a*Si
n[e + f*x]^2)*Tan[e + f*x])/(3*b*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f
*x]^2)])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
```

```
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3\sqrt{a+\frac{b}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3\sqrt{\frac{a+b-ax^2}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(1-x^2)^{5/2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= \frac{\sec^3(e+fx)\,(a+b-a\sin^2(e+fx))\tan(e+fx)}{3bf\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{-a+2b-ax^2}{(1-x^2)^{3/2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{3bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= -\frac{2(a-b)\sec(e+fx)\,(a+b-a\sin^2(e+fx))\tan(e+fx)}{3b^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{\sec^3(e+fx)\,(a+b-a\sin^2(e+fx))\tan(e+fx)}{3bf\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{a(2a-b)-2a(a-b)x^2}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{3b^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= -\frac{2(a-b)\sec(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{3b^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{\sec^3(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{3bf\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{\left(2(a-b)\sqrt{a+b-a\sin^2(e+fx)}\right)\text{Subst}\left(\int\frac{\sqrt{a+b-ax^2}}{\sqrt{1-x^2}}\,dx,x,\sin(e+fx)\right)}{3b^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{\left((a-2b)\sqrt{a+b-a\sin^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx,x,\sin(e+fx)\right)}{3bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

$$= -\frac{2(a-b)\sec(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{3b^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{\sec^3(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{3bf\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{\left(2(a-b)\left(a+b-a\sin^2(e+fx)\right)\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{ax^2}{a+b}}}{\sqrt{1-x^2}}\,dx,x,\sin(e+fx)\right)}{3b^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{\left((a-2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{ax^2}{a+b}}}\,dx,x,\sin(e+fx)\right)}{3bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

$$= \frac{2(a-b)E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\left(a+b-a\sin^2(e+fx)\right)}{3b^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{(a-2b)\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{3bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{2(a-b)\sec(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{3b^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{\sec^3(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{3bf\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

Mathematica [F]

$$\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

```
[In] Integrate[Sec[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Sec[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 12.32 (sec) , antiderivative size = 6274, normalized size of antiderivative = 19.01

method	result	size
default	Expression too large to display	6274

```
[In] int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 784, normalized size of antiderivative = 2.38

$$\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$
$$=\frac{\left(2\left(-i a^2+i ab\right)\sqrt{a}\sqrt{\frac{ab+b^2}{a^2}}\cos(fx+e)^2-\left(-i a^2-i ab+2i b^2\right)\sqrt{a}\cos(fx+e)^2\right)\sqrt{\frac{2a\sqrt{\frac{ab+b^2}{a^2}}-a-2b}{a}}E(\text{ar}}{}$$

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*((2*(-I*a^2 + I*a*b)*sqrt(a)*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^2 - (-I
*a^2 - I*a*b + 2*I*b^2)*sqrt(a)*cos(f*x + e)^2)*sqrt((2*a*sqrt((a*b + b^2)/
a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a -
2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2
*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*(I*a^2 - I*a*b)*sqrt(a)*sqrt((a*b +
b^2)/a^2)*cos(f*x + e)^2 - (I*a^2 + I*a*b - 2*I*b^2)*sqrt(a)*cos(f*x + e)^2
)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a
*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2
```

```
 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - (2*I*a^(3/
2)*b*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^2 + (2*I*a^2 + 3*I*a*b - 2*I*b^2)*s
qrt(a)*cos(f*x + e)^2)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*ellipt
ic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I
*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^
2))/a^2) - (-2*I*a^(3/2)*b*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^2 + (-2*I*a^2
 - 3*I*a*b + 2*I*b^2)*sqrt(a)*cos(f*x + e)^2)*sqrt((2*a*sqrt((a*b + b^2)/a^
2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*
b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a
*b)*sqrt((a*b + b^2)/a^2))/a^2) - (2*(a^2 - a*b)*cos(f*x + e)^2 - a*b)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*b^2*f*cos(f*x + e)
^2)
```

Sympy [F]

$$\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sec^5{(e+fx)}}{\sqrt{a+b\sec^2{(e+fx)}}}\,dx$$

```
[In] integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sec(e + f*x)**5/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sec{(fx+e)}^5}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sec{(fx+e)}^5}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{1}{\cos\left(e+f\,x\right)^5\,\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}}\,dx$$

```
[In] int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2)),x)
[Out] int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

3.258 $\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 170

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\sqrt{a}\sqrt{a+b}E\left(\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)|\frac{a+b}{a}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{\sec(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{bf\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

```
[Out] -EllipticE(sin(f*x+e)*a^(1/2)/(a+b)^(1/2),((a+b)/a)^(1/2))*a^(1/2)*(a+b)^(1
/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/b/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(
a+b-a*sin(f*x+e)^2))^(1/2)+sec(f*x+e)*(a+b-a*sin(f*x+e)^2)*tan(f*x+e)/b/f/(
sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4233, 1985, 1986, 425, 21, 438, 435}

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\tan(e+fx)\sec(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{bf\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}} - \frac{\sqrt{a}\sqrt{a+b}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}E\left(\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)|\frac{a+b}{a}\right)}{bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

```
[In] Int[Sec[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*Sqrt[a + b]*EllipticE[ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]
, (a + b)/a]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(b*f*Sqrt[Cos[e + f*x]^2
```

```
]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])) + (Sec[e + f*x]*(a + b
- a*Sin[e + f*x]^2)*Tan[e + f*x])/(b*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e
 + f*x]^2)])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2\sqrt{a+\frac{b}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2\sqrt{\frac{a+b-ax^2}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\\
&= \frac{\sec(e+fx)\,(a+b-a\sin^2(e+fx))\tan(e+fx)}{bf\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\\
&\quad+\frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{-a+ax^2}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\\
&= \frac{\sec(e+fx)\,(a+b-a\sin^2(e+fx))\tan(e+fx)}{bf\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\\
&\quad-\frac{\left(a\sqrt{a+b-a\sin^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\\
&= \frac{\sec(e+fx)\,(a+b-a\sin^2(e+fx))\tan(e+fx)}{bf\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\\
&\quad-\frac{\left(a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\sqrt{1-\frac{ax^2}{a+b}}}\,dx, x, \sin(e+fx)\right)}{bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\\
&= -\frac{\sqrt{a}\sqrt{a+b}E\left(\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\\
&\quad+\frac{\sec(e+fx)\,(a+b-a\sin^2(e+fx))\tan(e+fx)}{bf\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}
\end{aligned}
$$

Mathematica [F]

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

```
[In] Integrate[Sec[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Sec[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.89 (sec) , antiderivative size = 4103, normalized size of antiderivative = 24.14

method	result	size
default	Expression too large to display	4103

```
[In] int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/b/(2*I*a^(1/2)*b^(1/2)-a+b)*(2*
I*a^(3/2)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f
*x+e))+2*I*a^(1/2)*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x
+e)-cot(f*x+e))-4*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1
-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/
2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*c
sc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticF(((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)
*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b+2*(-(2*I*a^
(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2
-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1
-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e)
)^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2
)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*a*b*(1-cos(f*x+e))^3*csc(f*x+e)^3-(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))
^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+
e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-
a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))
^(1/2)*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*
x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1
/2))*a^2*(1-cos(f*x+e))^2*csc(f*x+e)^2-(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e)
)^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x
+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2
```

```
-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b)
)^(1/2)*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f
*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(
1/2))*b^2*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*I*a^(3/2)*b^(1/2)*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*(1-cos(f*x+e))^5*csc(f*x+e)^5+2*I*a^(1/2)*b^(3/2)*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(1-cos(f*x+e))^5*csc(f*x+e)^5-4*I*a
^(3/2)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(1-cos(f*x+e))^3*csc
(f*x+e)^3+4*I*a^(1/2)*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(1-co
s(f*x+e))^3*csc(f*x+e)^3+2*I*a^(1/2)*b^(3/2)*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(
f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*c
sc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*
x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)
/(a+b))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)
-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b
)^2)^(1/2))+2*I*a^(3/2)*b^(1/2)*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc
(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a
-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-c
os(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)
*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),
(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*(
1-cos(f*x+e))^2*csc(f*x+e)^2-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*(1
-cos(f*x+e))^5*csc(f*x+e)^5+(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x
+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/
(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f
*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*Ell
ipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4
*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2+(
-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(
f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^
(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-co
s(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticE(((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2
)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*b^2*(1-cos(f*x+e))^5*csc(f*x+e)^5+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*a^2*(1-cos(f*x+e))^3*csc(f*x+e)^3+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*b^2*(1-cos(f*x+e))^3*csc(f*x+e)^3-((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*a^2*(csc(f*x+e)-cot(f*x+e))+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*b^2*(csc(f*x+e)-cot(f*x+e))-2*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*c
sc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2
-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1
-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/
2)*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)
),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))
*a*b*(1-cos(f*x+e))^2*csc(f*x+e)^2-2*I*a^(1/2)*b^(3/2)*(-(2*I*a^(1/2)*b^(1/
2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f
```

```
*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e)
)^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x
+e)^2+a+b)/(a+b))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(
csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b
-b^2)/(a+b)^2)^(1/2))*(1-cos(f*x+e))^2*csc(f*x+e)^2-2*I*a^(3/2)*b^(1/2)*(-(
2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*
x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1
/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(
f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*
b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))+4*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+
e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f
*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)
^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+
b))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot
(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)
^(1/2))*a*b*(1-cos(f*x+e))^2*csc(f*x+e)^2)/((a*(1-cos(f*x+e))^4*csc(f*x+e)^
4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-
cos(f*x+e))^2*csc(f*x+e)^2+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^2)^(1/2)/
((1-cos(f*x+e))^2*csc(f*x+e)^2-1)/(csc(f*x+e)-cot(f*x+e)+1)/(csc(f*x+e)-cot
(f*x+e)-1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 594, normalized size of antiderivative = 3.49

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{2\sqrt{a}(-ia-2ib)\sqrt{\frac{2a\sqrt{\frac{ab+b^2}{a^2}}-a-2b}{a}}F(\arcsin\left(\sqrt{\frac{2a\sqrt{\frac{ab+b^2}{a^2}}-a-2b}{a}}(\cos(fx+e)+i\sin(fx+e))\right)\mid\frac{a^2+8ab}{}}{}$$

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] -1/2*(2*sqrt(a)*(-I*a - 2*I*b)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a
)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x
+ e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b +
 b^2)/a^2))/a^2) + 2*sqrt(a)*(I*a + 2*I*b)*sqrt((2*a*sqrt((a*b + b^2)/a^2)
- a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/
a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)
*sqrt((a*b + b^2)/a^2))/a^2) + (-2*I*a^(3/2)*sqrt((a*b + b^2)/a^2) + sqrt(a
)*(I*a + 2*I*b))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(a
```

```
rcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f
*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^
2) + (2*I*a^(3/2)*sqrt((a*b + b^2)/a^2) + sqrt(a)*(-I*a - 2*I*b))*sqrt((2*a
*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b
+ b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b +
8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - 2*a*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*b*f)
```

Sympy [F]

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sec^3{(e+fx)}}{\sqrt{a+b\sec^2{(e+fx)}}}\,dx$$

```
[In] integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sec(e + f*x)**3/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sec{(fx+e)}^3}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sec{(fx+e)}^3}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{1}{\cos\left(e+f\,x\right)^3\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

```
[In] int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2)),x)
[Out] int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

3.259 $\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 80

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

```
[Out] EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/f/(cos
(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4233, 1985, 1986, 432, 430}

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(-a\sin^2(e+fx)+a+b)}}$$

```
[In] Int[Sec[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a
+ b)])/(f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^
2)])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
```

```
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] && !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+\frac{b}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{\frac{a+b-ax^2}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= \frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{ax^2}{a+b}}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= \frac{\text{EllipticF}\left(\arcsin(\sin(e+fx)), \frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}\,\text{EllipticF}\left(e+fx, \frac{a}{a+b}\right)\sec(e+fx)}{\sqrt{2}f\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*EllipticF[e + f*x, a/(a + b)]
*Sec[e + f*x])/(Sqrt[2]*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.49 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.04

method	result
default	$-\frac{2\sqrt{-\frac{i\left(i\cos(fx+e)a-\sqrt{a}\,\sqrt{b}\,\cos(fx+e)+ib+\sqrt{a}\,\sqrt{b}\right)}{(a+b)(1+\cos(fx+e))}}\,\sqrt{-\frac{i\left(\sqrt{a}\,\sqrt{b}\,\cos(fx+e)+i\cos(fx+e)a-\sqrt{a}\,\sqrt{b}+ib\right)}{(a+b)(1+\cos(fx+e))}}\,\text{EllipticF}\left(\sqrt{\frac{2i\sqrt{a}\,\sqrt{b}+a-b}{a+b}}\,(\cot(fx\right.}{f\sqrt{\frac{2i\sqrt{a}\,\sqrt{b}+a-b}{a+b}}\,\sqrt{a+b\sec(fx+e)^2}}$

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(-I/(a+b)*(I*cos(f*x+e)*a-a^(1
/2)*b^(1/2)*cos(f*x+e)+I*b+a^(1/2)*b^(1/2))/(1+cos(f*x+e)))^(1/2)*(-I/(a+b)
*(a^(1/2)*b^(1/2)*cos(f*x+e)+I*cos(f*x+e)*a-a^(1/2)*b^(1/2)+I*b)/(1+cos(f*x
+e)))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(cot(f*x+e)-c
sc(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^
2)^(1/2))/(a+b*sec(f*x+e)^2)^(1/2)*(sec(f*x+e)+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.79

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\left(2i\,a^{\frac{3}{2}}\sqrt{\frac{ab+b^2}{a^2}}+\sqrt{a}(i\,a+2i\,b)\right)\sqrt{\frac{2\,a\sqrt{\frac{ab+b^2}{a^2}}-a-2\,b}{a}}F\left(\arcsin\left(\sqrt{\frac{2\,a\sqrt{\frac{ab+b^2}{a^2}}-a-2\,b}{a}}(\cos(fx+e)+i\,\sin(fx\right.\right.}{}$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -((2*I*a^(3/2)*sqrt((a*b + b^2)/a^2) + sqrt(a)*(I*a + 2*I*b))*sqrt((2*a*sqr
t((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^
2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^
2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (-2*I*a^(3/2)*sqrt((a*b +
 b^2)/a^2) + sqrt(a)*(-I*a - 2*I*b))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a -
2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(co
s(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt(
(a*b + b^2)/a^2))/a^2))/(a^2*f)
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec(fx+e)}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sec(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}}\,dx$$

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2)),x)
[Out] int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

3.260 $\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 105

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\sqrt{a+b}E\left(\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)|\frac{a+b}{a}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{a}f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

```
[Out] EllipticE(sin(f*x+e)*a^(1/2)/(a+b)^(1/2),((a+b)/a)^(1/2))*(a+b)^(1/2)*(1-a*
sin(f*x+e)^2/(a+b))^(1/2)/f/a^(1/2)/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b
-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4233, 1985, 1986, 438, 435}

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\sqrt{a+b}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}E\left(\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)|\frac{a+b}{a}\right)}{\sqrt{a}f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

```
[In] Int[Cos[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + b]*EllipticE[ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]], (a + b)/
a]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(Sqrt[a]*f*Sqrt[Cos[e + f*x]^2]*Sq
rt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{b}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\frac{a+b-ax^2}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f} \\
&= \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\
&= \frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\sqrt{1-\frac{ax^2}{a+b}}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}
\end{aligned}$$

$$= \frac{\sqrt{a+b}E\left(\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)|\frac{a+b}{a}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{a}f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.87 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.61

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \frac{\csc(2(e+fx))\sin(e+fx)\left(a^2\sqrt{-\frac{1}{b}}\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}\,\text{EllipticF}\left(e+fx,\frac{a}{a+b}\right)-2i\sqrt{-\frac{a\cos^2(e+fx)}{b}}\sqrt{a+\right.}{}$$

```
[In] Integrate[Cos[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Csc[2*(e + f*x)]*Sin[e + f*x]*(a^2*Sqrt[-b^(-1)]*Sqrt[(a + 2*b + a*Cos[2*(
e + f*x)])/(a + b)]*EllipticF[e + f*x, a/(a + b)] - (2*I)*Sqrt[-((a*Cos[e +
 f*x]^2)/b)]*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Csc[2*(e + f*x)]*(2*(a + b)
*EllipticE[I*ArcSinh[(Sqrt[-b^(-1)]*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])/Sqr
t[2]], b/(a + b)] - a*EllipticF[I*ArcSinh[(Sqrt[-b^(-1)]*Sqrt[a + 2*b + a*C
os[2*(e + f*x)]])/Sqrt[2]], b/(a + b)])*Sqrt[(a*Sin[e + f*x]^2)/(a + b)]))/
(Sqrt[2]*a^2*Sqrt[-b^(-1)]*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.41 (sec) , antiderivative size = 4095, normalized size of antiderivative = 39.00

method	result	size
default	Expression too large to display	4095

```
[In] int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a/(2*I*a^(1/2)*b^(1/2)-a+b)*(2
*I*a^(3/2)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(
f*x+e))+2*I*a^(1/2)*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*
x+e)-cot(f*x+e))+4*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(
1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1
/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*
csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticF(((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2
```

```
)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-2*(-(2*I*a
^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^
2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(
1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e
))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/
2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a*b*(1-cos(f*x+e))^3*csc(f*x+e)^3-(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e)
)^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x
+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2
-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b)
)^(1/2)*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f
*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(
1/2))*a^2*(1-cos(f*x+e))^2*csc(f*x+e)^2-(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e
))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*
x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^
2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b
))^(1/2)*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(
f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^
(1/2))*b^2*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*I*a^(3/2)*b^(1/2)*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)*(1-cos(f*x+e))^5*csc(f*x+e)^5+2*I*a^(1/2)*b^(3/2)
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(1-cos(f*x+e))^5*csc(f*x+e)^5-4*I*
a^(3/2)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(1-cos(f*x+e))^3*cs
c(f*x+e)^3+4*I*a^(1/2)*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(1-c
os(f*x+e))^3*csc(f*x+e)^3+2*I*a^(3/2)*b^(1/2)*(-(2*I*a^(1/2)*b^(1/2)*(1-cos
(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*
csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f
*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b
)/(a+b))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e
)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+
b)^2)^(1/2))+2*I*a^(3/2)*b^(1/2)*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*cs
c(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-
a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-
cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2
)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e))
,(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*
(1-cos(f*x+e))^2*csc(f*x+e)^2-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*(
1-cos(f*x+e))^5*csc(f*x+e)^5-(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*
x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)
/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(
f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*El
lipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(
4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2-
(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc
(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b
```

^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-c
os(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticE(((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/
2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2+((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*b^2*(1-cos(f*x+e))^5*csc(f*x+e)^5+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*a^2*(1-cos(f*x+e))^3*csc(f*x+e)^3+2*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*b^2*(1-cos(f*x+e))^3*csc(f*x+e)^3-2*I*a^(1/2)*b^(3/2)*(-(2*I*a
^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^
2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(
1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e
))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/
2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a
^2*(csc(f*x+e)-cot(f*x+e))+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2*(csc
(f*x+e)-cot(f*x+e))-2*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+
a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))
^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))
^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticE
(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(
3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b*(1-cos(
f*x+e))^2*csc(f*x+e)^2-2*I*a^(1/2)*b^(3/2)*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*
x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc
(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+
e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(
a+b))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-c
ot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^
2)^(1/2))*(1-cos(f*x+e))^2*csc(f*x+e)^2+4*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x
+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(
f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e
)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a
+b))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-co
t(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2
)^(1/2))*a*b*(1-cos(f*x+e))^2*csc(f*x+e)^2)/((a*(1-cos(f*x+e))^4*csc(f*x+e)
^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1
-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^2)^(1/2)
/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)/((1-cos(f*x+e))^2*csc(f*x+e)^2+1)

Fricas [F]

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] integral(cos(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Sympy [F]

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(cos(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos(e+f\,x)}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

```
[In] int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.261 $\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 255

$$\begin{aligned}\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx &= \frac{\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{3af\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} \\ &\quad+\frac{2(a-b)E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\left(a+b-a\sin^2(e+fx)\right)}{3a^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} \\ &\quad-\frac{(a-2b)b\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{3a^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}\end{aligned}$$

```
[Out] 1/3*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))
^(1/2)+2/3*(a-b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)
/a^2/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*
sin(f*x+e)^2/(a+b))^(1/2)-1/3*(a-2*b)*b*EllipticF(sin(f*x+e),(a/(a+b))^(1/2
))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a^2/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*
(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4233, 1985, 1986, 427, 538, 437, 435, 432, 430}

$$\begin{aligned}&\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx\\&=-\frac{b(a-2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{3a^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}\\&\quad+\frac{2(a-b)\left(-a\sin^2(e+fx)+a+b\right)E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)}{3a^2f\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}\\&\quad+\frac{\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{3af\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}\end{aligned}$$

```
[In] Int[Cos[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sin[e + f*x]*(a + b - a*Sin[e + f*x]^2))/(3*a*f*Sqrt[Sec[e + f*x]^2*(a + b
 - a*Sin[e + f*x]^2)]) + (2*(a - b)*EllipticE[ArcSin[Sin[e + f*x]], a/(a +
b)]*(a + b - a*Sin[e + f*x]^2))/(3*a^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e +
f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) -
((a - 2*b)*b*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e +
 f*x]^2)/(a + b)])/(3*a^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b
 - a*Sin[e + f*x]^2)])
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1-x^2}{\sqrt{a+\frac{b}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1-x^2}{\sqrt{\frac{a+b-ax^2}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{3/2}}{\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= \frac{\sin(e+fx)\,(a+b-a\sin^2(e+fx))}{3af\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} - \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{-2a+b+2(a-b)x^2}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{3af\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= \frac{\sin(e+fx)\,(a+b-a\sin^2(e+fx))}{3af\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{\left(2(a-b)\sqrt{a+b-a\sin^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{\sqrt{1-x^2}}\,dx, x, \sin(e+fx)\right)}{3a^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} - \frac{\left((a-2b)b\sqrt{a+b-a\sin^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{3a^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= \frac{\sin(e+fx)\,(a+b-a\sin^2(e+fx))}{3af\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{(2(a-b)\,(a+b-a\sin^2(e+fx)))\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{ax^2}{a+b}}}{\sqrt{1-x^2}}\,dx, x, \sin(e+fx)\right)}{3a^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{\left((a-2b)b\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{ax^2}{a+b}}}\,dx, x, \sin(e+fx)\right)}{3a^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= \frac{\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{3af\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{2(a-b)E\left(\arcsin(\sin(e+fx))\left|\frac{a}{a+b}\right.\right)\left(a+b-a\sin^2(e+fx)\right)}{3a^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{(a-2b)b\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{3a^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

Mathematica [F]

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

```
[In] Integrate[Cos[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Cos[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.02 (sec) , antiderivative size = 6273, normalized size of antiderivative = 24.60

method	result	size
default	Expression too large to display	6273

```
[In] int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F]

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos\left(fx+e\right)^3}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] integral(cos(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos\left(fx+e\right)^3}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos\left(fx+e\right)^3}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos\left(e+f\,x\right)^3}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

```
[In] int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.262 $\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 345

$$\begin{aligned}\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx &= \frac{4(a-b)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{15a^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} \\ &\quad+\frac{\cos^2(e+fx)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{5af\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} \\ &\quad+\frac{(8a^2-7ab+8b^2)\,E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\left(a+b-a\sin^2(e+fx)\right)}{15a^3f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} \\ &\quad-\frac{b(4a^2-3ab+8b^2)\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{15a^3f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}\end{aligned}$$

```
[Out] 4/15*(a-b)*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a^2/f/(sec(f*x+e)^2*(a+b-a*sin(f
*x+e)^2))^(1/2)+1/5*cos(f*x+e)^2*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a/f/(sec(f
*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+1/15*(8*a^2-7*a*b+8*b^2)*EllipticE(sin(
f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a^3/f/(cos(f*x+e)^2)^(1/2)/(se
c(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/15*
b*(4*a^2-3*a*b+8*b^2)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)
^2/(a+b))^(1/2)/a^3/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^
2))^(1/2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4233, 1985, 1986, 427, 542, 538, 437, 435, 432, 430}

$$\begin{aligned}&\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx\\&=\frac{4(a-b)\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{15a^2f\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}\\&-\frac{b(4a^2-3ab+8b^2)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{15a^3f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}\\&+\frac{(8a^2-7ab+8b^2)\left(-a\sin^2(e+fx)+a+b\right)E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)}{15a^3f\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}\\&+\frac{\sin(e+fx)\cos^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{5af\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}\end{aligned}$$

```
[In] Int[Cos[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (4*(a - b)*Sin[e + f*x]*(a + b - a*Sin[e + f*x]^2))/(15*a^2*f*Sqrt[Sec[e +
f*x]^2*(a + b - a*Sin[e + f*x]^2)]) + (Cos[e + f*x]^2*Sin[e + f*x]*(a + b -
 a*Sin[e + f*x]^2))/(5*a*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
 + ((8*a^2 - 7*a*b + 8*b^2)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a +
 b - a*Sin[e + f*x]^2))/(15*a^3*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*
(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (b*(4*a
^2 - 3*a*b + 8*b^2)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*
Sin[e + f*x]^2)/(a + b)])/(15*a^3*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^
2*(a + b - a*Sin[e + f*x]^2)])
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
```

```
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{\sqrt{a+\frac{b}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{\sqrt{\frac{a+b-ax^2}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{5/2}}{\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= \frac{\cos^2(e+fx)\sin(e+fx)\,(a+b-a\sin^2(e+fx))}{5af\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} - \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{\sqrt{1-x^2}\,(-4a+b+4(a-b)x^2)}{\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{5af\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= \frac{4(a-b)\sin(e+fx)\,(a+b-a\sin^2(e+fx))}{15a^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{\cos^2(e+fx)\sin(e+fx)\,(a+b-a\sin^2(e+fx))}{5af\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{8a^2-3ab+4b^2+(-8a^2+7ab-8b^2)x^2}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{15a^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= \frac{4(a-b)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{15a^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{\cos^2(e+fx)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{5af\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{\left((-8a^2+7ab-8b^2)\sqrt{a+b-a\sin^2(e+fx)}\right)\text{Subst}\left(\int\frac{\sqrt{a+b-ax^2}}{\sqrt{1-x^2}}\,dx,x,\sin(e+fx)\right)}{15a^3f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{\left((-((a+b)(-8a^2+7ab-8b^2))-a(8a^2-3ab+4b^2))\sqrt{a+b-a\sin^2(e+fx)}\right)\text{Subst}\left(\int\frac{}{\sqrt{1-x}}\right.}{15a^3f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

$$= \frac{4(a-b)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{15a^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{\cos^2(e+fx)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{5af\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{\left((-8a^2+7ab-8b^2)\left(a+b-a\sin^2(e+fx)\right)\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{ax^2}{a+b}}}{\sqrt{1-x^2}}\,dx,x,\sin(e+fx)\right)}{15a^3f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{\left((-((a+b)(-8a^2+7ab-8b^2))-a(8a^2-3ab+4b^2))\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{ax}{a+}}}\right.}{15a^3f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

$$= \frac{4(a-b)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{15a^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{\cos^2(e+fx)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{5af\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{(8a^2-7ab+8b^2)\,E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\left(a+b-a\sin^2(e+fx)\right)}{15a^3f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{b(4a^2-3ab+8b^2)\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{15a^3f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

Mathematica [F]

$$\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

```
[In] Integrate[Cos[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Cos[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.26 (sec) , antiderivative size = 8756, normalized size of antiderivative = 25.38

method	result	size
default	Expression too large to display	8756

```
[In] int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F]

$$\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cos(fx+e)^5}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] integral(cos(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\text{Timed out}$$

```
[In] integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos(fx+e)^5}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos(fx+e)^5}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos(e+f\,x)^5}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

```
[In] int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.263 $\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 137

$$\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{(3a^2-2ab+3b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^{5/2}f} - \frac{3(a-b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8b^2f} + \frac{\sec^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4bf}$$

```
[Out] 1/8*(3*a^2-2*a*b+3*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/
2))/b^(5/2)/f-3/8*(a-b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b^2/f+1/4*sec
(f*x+e)^2*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4231, 427, 396, 223, 212}

$$\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{(3a^2-2ab+3b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8b^{5/2}f} - \frac{3(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8b^2f} + \frac{\tan(e+fx)\sec^2(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4bf}$$

```
[In] Int[Sec[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((3*a^2 - 2*a*b + 3*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[
e + f*x]^2]])/(8*b^(5/2)*f) - (3*(a - b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e
+ f*x]^2])/(8*b^2*f) + (Sec[e + f*x]^2*Tan[e + f*x]*Sqrt[a + b + b*Tan[e +
f*x]^2])/(4*b*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\sec^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4bf} + \frac{\text{Subst}\left(\int \frac{-a+3b-3(a-b)x^2}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{4bf}$$

$$= -\frac{3(a-b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8b^2f} + \frac{\sec^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4bf} + \frac{(3a^2-2ab+3b^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{8b^2f}$$

$$= -\frac{3(a-b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8b^2f} + \frac{\sec^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4bf} + \frac{(3a^2-2ab+3b^2)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^2f}$$

$$= \frac{(3a^2-2ab+3b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^{5/2}f} - \frac{3(a-b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8b^2f} + \frac{\sec^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4bf}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.55 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.38

$$\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\sqrt{a+2b+a\cos(2e+2fx)}\left(-\frac{i\sqrt{b}\left(-1+e^{2i(e+fx)}\right)\left(-3a\left(1+e^{2i(e+fx)}\right)^2+b(}{\left(1+e^{2i(e+fx)}\right)^4}\right.}{8\sqrt{2}b^{5/2}f\sqrt{a+b\,s}}$$

```
[In] Integrate[Sec[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*(((-I)*Sqrt[b]*(-1 + E^((2*I)*(e +
 f*x)))*(-3*a*(1 + E^((2*I)*(e + f*x)))^2 + b*(3 + 14*E^((2*I)*(e + f*x)) +
 3*E^((4*I)*(e + f*x)))))/(1 + E^((2*I)*(e + f*x)))^4 - ((3*a^2 - 2*a*b + 3
*b^2)*Log[(-4*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b*E^((2*I
)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e + f*x)))]
)/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*Sec[e + f*
x])/(8*Sqrt[2]*b^(5/2)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1611 vs. $2(121) = 242$.

Time = 16.83 (sec) , antiderivative size = 1612, normalized size of antiderivative = 11.77

method	result	size
default	Expression too large to display	1612

```
[In] int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/f/b^(9/2)/(a+b*sec(f*x+e)^2)^(1/2)*(6*b^(7/2)*a*tan(f*x+e)+6*b^(9/2)*t
an(f*x+e)*sec(f*x+e)^2-6*b^(5/2)*a^2*tan(f*x+e)-2*b^(7/2)*a*tan(f*x+e)*sec(
f*x+e)^2+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a^2*b^2-2*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a*b^3+3*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b
^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f
*x+e)*a+a+b)/(sin(f*x+e)-1))*b^4+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^
(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*
x+e)+1))*a^2*b^2-2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a*b^3+3
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^4+4*b^(9/2)*tan(f*x+e)*
sec(f*x+e)^4+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a^2*b^2*se
c(f*x+e)-2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a*b^3*sec(f*x+
e)+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)
```

```
/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*b^4*sec(f*x+e)+3*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a^2*b^2*sec(f*x+e)-2*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a*b^3*sec(f*x+e)+3*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-
sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^4*sec(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.89

$$\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \left[\frac{(3a^2-2ab+3b^2)\sqrt{b}\cos(fx+e)^3\log\left(\frac{(a^2-6ab+b^2)\cos(fx+e)^4+8(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2b\cos(fx\cdots\right.}{\cos(fx+e)^4}\right.}{32b^3f\cos(fx\cdots}\right.$$

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((3*a^2 - 2*a*b + 3*b^2)*sqrt(b)*cos(f*x + e)^3*log(((a^2 - 6*a*b + b
^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)
^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*(3*(a*b - b^2)*cos(f*x + e)^2 - 2
*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*cos(
f*x + e)^3), 1/16*((3*a^2 - 2*a*b + 3*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*co
s(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^3 - 2*(3
*(a*b - b^2)*cos(f*x + e)^2 - 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^3)]
```

Sympy [F]

$$\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

```
[In] integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sec(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.17

$$\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\frac{2\sqrt{b\tan(fx+e)^2+a+b}\tan(fx+e)^3}{b}+\frac{3(a+b)a\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{5}{2}}}+\frac{3(a+b)\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}-\frac{8a\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}-3\sqrt{b\tan\cdots}}{8f}$$

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] 1/8*(2*sqrt(b*tan(f*x + e)^2 + a + b)*tan(f*x + e)^3/b + 3*(a + b)*a*arcsin
h(b*tan(f*x + e)/sqrt((a + b)*b))/b^(5/2) + 3*(a + b)*arcsinh(b*tan(f*x + e
)/sqrt((a + b)*b))/b^(3/2) - 8*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^
(3/2) - 3*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(f*x + e)/b^2 + 8*sqrt(
b*tan(f*x + e)^2 + a + b)*tan(f*x + e)/b)/f
```

Giac [F]

$$\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sec(fx+e)^6}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\cos{(e+f\,x)}^6\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

```
[In] int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2)),x)
[Out] int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

3.264 $\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 81

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(a-b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{3/2}f} + \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf}$$

```
[Out] -1/2*(a-b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(3/2)/f
+1/2*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4231, 396, 223, 212}

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2bf} - \frac{(a-b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{3/2}f}$$

```
[In] Int[Sec[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/2*((a - b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]
])/(b^(3/2)*f) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*b*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1+x^2}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf} - \frac{(a-b)\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2bf}\\
&= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf} - \frac{(a-b)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2bf}\\
&= -\frac{(a-b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{3/2}f} + \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.00 (sec) , antiderivative size = 326, normalized size of antiderivative = 4.02

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \frac{\sqrt{a+2b+a\cos(2e+2fx)}\sec^4(e+fx)\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)\tan(e+fx)\left(\frac{16b^2(b+a\cos^2(e+fx))\,\text{Hypergeometric2F1}\left(2\right.}{}\right.}{30\sqrt{2}f\sqrt{a+b\sec^2(e+f}}$$

```
[In] Integrate[Sec[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x]^4*(1 - (a*Sin[e + f*x]^2)/
(a + b))*Tan[e + f*x]*((16*b^2*(b + a*Cos[e + f*x]^2)*Hypergeometric2F1[2,
3, 7/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^4*Sqrt[-((b*Sec[e + f*x
]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(a + b)^2)])/(a + b)^3 + (15
*(3*b + a*(3 - 2*Sin[e + f*x]^2))*(ArcSin[Sqrt[-((b*Tan[e + f*x]^2)/(a + b)
)]] - Sqrt[-((b*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(
a + b)^2)]))/(a + b)))/(30*Sqrt[2]*f*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[(a + b
*Sec[e + f*x]^2)/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2]*(-((b*Tan[e + f*x]
^2)/(a + b)))^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1038 vs. 2(69) = 138.

Time = 11.29 (sec) , antiderivative size = 1039, normalized size of antiderivative = 12.83

method	result	size
default	Expression too large to display	1039

```
[In] int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f/b^(5/2)/(a+b*sec(f*x+e)^2)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x
+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(
sin(f*x+e)+1))*a*b-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^2+((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
```

```
e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a*b-((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1
/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+
e)*a+a+b)/(sin(f*x+e)-1))*b^2-2*b^(3/2)*a*tan(f*x+e)+((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(
1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x
+e)*a-a-b)/(sin(f*x+e)+1))*a*b*sec(f*x+e)-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*
x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/
(sin(f*x+e)+1))*b^2*sec(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e
)-1))*a*b*sec(f*x+e)-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*b^2*
sec(f*x+e)-2*b^(5/2)*tan(f*x+e)*sec(f*x+e)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(69) = 138$.

Time = 0.32 (sec) , antiderivative size = 324, normalized size of antiderivative = 4.00

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[-\frac{(a-b)\sqrt{b}\cos(fx+e)\log\left(\frac{(a^2-6\,ab+b^2)\cos(fx+e)^4+8\,(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos(\cdots}{\cos\cdots}}}{\cos(fx+e)^4}\right.}{8\,b^2f\cos(fx+e)}\right.$$

$$-\frac{(a-b)\sqrt{-b}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\left(ab\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\cos(fx+e)-2\,b\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\,\mathrm{s}\cdots}{4\,b^2f\cos(fx+e)}$$

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((a - b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
 e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b
^2)/cos(f*x + e)^4) - 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f
*x + e))/(b^2*f*cos(f*x + e)), -1/4*((a - b)*sqrt(-b)*arctan(-1/2*((a - b)*
```

```
cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) - 2*b*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f*x +
e))]
```

Sympy [F]

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sec^4{(e+fx)}}{\sqrt{a+b\sec^2{(e+fx)}}}\,dx$$

```
[In] integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sec(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\frac{a\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}-\frac{\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-\frac{\sqrt{b\tan(fx+e)^2+a+b}\,\tan(fx+e)}{b}}{2\,f}$$

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] -1/2*(a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(3/2) - arcsinh(b*tan(f*x
 + e)/sqrt((a + b)*b))/sqrt(b) - sqrt(b*tan(f*x + e)^2 + a + b)*tan(f*x + e
)/b)/f
```

Giac [F]

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sec{(fx+e)}^4}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{1}{\cos\left(e+f\,x\right)^4\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

```
[In] int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2)),x)
[Out] int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

3.265 $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 39

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{b}f}$$

```
[Out] arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/b^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {4231, 223, 212}

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}f}$$

```
[In] Int[Sec[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[b]*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

$$= \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{b}f}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 87 vs. $2(39) = 78$.

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.23

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)}{\sqrt{2}\sqrt{b}f\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2*
b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[b]*f*Sqrt[a + b*Sec[e +
 f*x]^2])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(33) = 66$.

Time = 7.88 (sec) , antiderivative size = 241, normalized size of antiderivative = 6.18

method	result
default	$\frac{\left(\ln\left(-\frac{4\left(-\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\sqrt{b}\,\cos(fx+e)+\sin(fx+e)a-\sqrt{b}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}+a+b\right)}{\sin(fx+e)+1}\right)+\ln\left(\frac{-4\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\sqrt{b}\,\cos(fx+e)+4\sin(fx+e}{\sin(fx+e)-1}\right.\right.}{2f\sqrt{b}\,\sqrt{a+b\sec(fx+e)^2}}$

```
[In] int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f/b^(1/2)*(ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*
cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
+a+b)/(sin(f*x+e)+1))+ln(4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^
(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)-a-b)/(sin(f*x+e)-1)))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/(a
+b*sec(f*x+e)^2)^(1/2)*(sec(f*x+e)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(33) = 66$.

Time = 0.30 (sec) , antiderivative size = 215, normalized size of antiderivative = 5.51

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \left[\frac{\log\left(\frac{(a^2-6\,ab+b^2)\cos(fx+e)^4+8\,(ab-b^2)\cos(fx+e)^2+4\,\left((a-b)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)+8\,b^2}{\cos(fx+e)^4}\right)}{4\sqrt{b}f}, \frac{\sqrt{-}}{}\right.$$

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2
 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)/(sqrt(b)*f)
, 1/2*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt
(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2
)*sin(f*x + e)))/(b*f)]
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

```
[In] integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sec(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.59

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}f}$$

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt(b)*f)
```

Giac [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec(fx+e)^2}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\cos(e+f\,x)^2\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

```
[In] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2)),x)
[Out] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

3.266 $\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4213, 385, 209}

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}$$

```
[In] Int[1/Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} \\
&= \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(39) = 78.

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.23

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)}{\sqrt{2}\sqrt{a}f\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Integrate[1/Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] (ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2*b
 + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[e +
f*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(33) = 66$.

Time = 3.02 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.54

method	result	size
default	$\frac{\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)(\sec(fx+e)+1)}{f\sqrt{-a}\sqrt{a+b\sec(fx+e)^2}}$	138

```
[In] int(1/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)/(a+b*sec(f*x+e)^2)^(1/
2)*(sec(f*x+e)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(33) = 66$.

Time = 0.37 (sec) , antiderivative size = 408, normalized size of antiderivative = 10.46

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[-\frac{\sqrt{-a}\log\left(128\,a^4\cos(fx+e)^8-256\,(a^4-a^3b)\cos(fx+e)^6+32\,(5\,a^4-14\,a^3b+5\,a^2b^2)\cos(fx+e)^4\right.}{}\right.$$

$$\left.-\frac{\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\,(a^2-ab)\cos(fx+e)^3+(a^2-6\,ab+b^2)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3\,a^2b)\cos(fx+e)^2\right)\sin(fx+e)}\right)}{4\,\sqrt{a}f}\right]$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*
a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
 e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*c
```

```
os(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e))/(a*f), -1/4*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x +
 e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)
*cos(f*x + e)^2)*sin(f*x + e)))/(sqrt(a)*f)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(1/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. 2(33) = 66.

Time = 0.40 (sec) , antiderivative size = 992, normalized size of antiderivative = 25.44

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*(arctan2(2*a*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f
*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*
sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e
)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4
*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x
+ 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos
(2*f*x + 2*e) + a)), 2*a*cos(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2
*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 +
 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f
*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x +
4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*si
n(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a +
2*b)*cos(2*f*x + 2*e) + a)) + 2*a + 4*b) - arctan2(2*(a^2*cos(4*f*x + 4*e)^
2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4
*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*
sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4
*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arcta
n2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) +
```

```
2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f
*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*
sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e
)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4
*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x
+ 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos
(2*f*x + 2*e) + a)) + 4*a + 4*b))/(sqrt(a)*f)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{1}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

```
[In] int(1/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(1/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.267 $\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 87

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{(a-b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{3/2}f} + \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af}$$

```
[Out] 1/2*(a-b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f+1
/2*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4231, 390, 385, 209}

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{(a-b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{3/2}f} + \frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2af}$$

```
[In] Int[Cos[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((a - b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*
a^(3/2)*f) + (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*
a*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af} \\
&\quad + \frac{(a-b)\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2af} \\
&= \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af} \\
&\quad + \frac{(a-b)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2af} \\
&= \frac{(a-b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{3/2}f} + \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.45

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\sqrt{a+2b+a\cos(2(e+fx))}\left((a-b)\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\sec(e+fx)+\sqrt{a}\sqrt{a+b-a\sin^2(e+fx)}\right.}{2\sqrt{2}a^{3/2}f\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Integrate[Cos[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*((a - b)*ArcTan[(Sqrt[a]*Sin[e + f*x])/
Sqrt[a + b - a*Sin[e + f*x]^2]]*Sec[e + f*x] + Sqrt[a]*Sqrt[a + b - a*Sin[e
 + f*x]^2]*Tan[e + f*x]))/(2*Sqrt[2]*a^(3/2)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 506 vs. 2(75) = 150.

Time = 5.12 (sec) , antiderivative size = 507, normalized size of antiderivative = 5.83

method	result
default	$\frac{\sin(fx+e)\cos(fx+e)\sqrt{-a}\,a+\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)a}{}$

```
[In] int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f/(-a)^(1/2)/a/(a+b*sec(f*x+e)^2)^(1/2)*(sin(f*x+e)*cos(f*x+e)*(-a)^(1/
2)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a-((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*c
os(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*
x+e)*a)*b+(-a)^(1/2)*b*tan(f*x+e)+sec(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*c
os(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*
x+e)*a)*a-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b*sec(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(75) = 150$.

Time = 0.40 (sec) , antiderivative size = 502, normalized size of antiderivative = 5.77

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[\frac{8\,a\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e)+\sqrt{-a}(a-b)\log\left(128\,a^4\cos(fx+e)^8-256\,(a^4-a^3b)\cos(\right.}{}\right.$$

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x
 + e) + sqrt(-a)*(a - b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos
(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a
^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)
*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^
5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^
2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e)))/(a^2*f), 1/8*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*cos(f*x + e)*sin(f*x + e) - (a - b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x +
e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqr
t(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a
^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))))/(a^2*f)]
```

Sympy [F]

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos^2{(e+fx)}}{\sqrt{a+b\sec^2{(e+fx)}}}\,dx$$

```
[In] integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cos(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cos\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cos\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cos\left(e+f\,x\right)^2}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

```
[In] int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.268 $\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 143

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{(3a^2-2ab+3b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{5/2}f} + \frac{3(a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4af}$$

```
[Out] 1/8*(3*a^2-2*a*b+3*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2
))/a^(5/2)/f+3/8*(a-b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a^2
/f+1/4*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4231, 425, 541, 12, 385, 209}

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{3(a-b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8a^2f} + \frac{(3a^2-2ab+3b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{5/2}f} + \frac{\sin(e+fx)\cos^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4af}$$

```
[In] Int[Cos[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((3*a^2 - 2*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e
 + f*x]^2]])/(8*a^(5/2)*f) + (3*(a - b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a +
b + b*Tan[e + f*x]^2])/(8*a^2*f) + (Cos[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b
+ b*Tan[e + f*x]^2])/(4*a*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
```

```
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4af} \\
&\quad - \frac{\text{Subst}\left(\int \frac{-3a+b-2bx^2}{(1+x^2)^2\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{4af} \\
&= \frac{3(a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f} \\
&\quad + \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4af} \\
&\quad + \frac{\text{Subst}\left(\int \frac{3a^2-2ab+3b^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{8a^2f} \\
&= \frac{3(a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f} \\
&\quad + \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4af} \\
&\quad + \frac{(3a^2-2ab+3b^2)\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{8a^2f} \\
&= \frac{3(a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f} \\
&\quad + \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4af} \\
&\quad + \frac{(3a^2-2ab+3b^2)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^2f} \\
&= \frac{(3a^2-2ab+3b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{5/2}f} \\
&\quad + \frac{3(a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f} \\
&\quad + \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4af}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 16.22 (sec) , antiderivative size = 1840, normalized size of antiderivative = 12.87

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \frac{}{f\sqrt{a+2b+a\cos(2(e+fx))}\sqrt{a+b\sec^2(e+fx)}\left(3(a+b)\operatorname{AppellF1}\left(\frac{1}{2},-2,\frac{1}{2},\frac{3}{2},\sin^2(e+fx),\frac{a\sin^2(e+f}{a+b}\right.\right.}$$

```
[In] Integrate[Cos[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^8*Sin[e + f*x])/(f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*
Sqrt[a + b*Sec[e + f*x]^2]*(3*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f
*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -2, 3/2, 5/2, Sin[e +
 f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 1/2, 5/2
, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*((3*(a + b)*
AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos
[e + f*x]^5)/(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b)*AppellF1[1/2, -
2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2,
 -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Appe
llF1[3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e
+ f*x]^2)) - (12*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin
[e + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt[a + 2*b + a*Cos[
2*(e + f*x)]]*(3*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin
[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*S
in[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 1/2, 5/2, Sin[e + f*x
]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos[e + f*x
]^4*Sin[e + f*x]*((a*f*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - (4*f*AppellF1[
3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]
*Sin[e + f*x])/3))/(f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b)*AppellF
1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*Appel
lF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a +
 b)*AppellF1[3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
)*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2,
```

```
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^4*Sin[e + f*x]*(2*f*(a*AppellF1[3
/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*A
ppellF1[3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Cos
[e + f*x]*Sin[e + f*x] + 3*(a + b)*((a*f*AppellF1[3/2, -2, 3/2, 5/2, Sin[e
+ f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)
) - (4*f*AppellF1[3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
+ b)]*Cos[e + f*x]*Sin[e + f*x])/3) + Sin[e + f*x]^2*(a*((9*a*f*AppellF1[5/
2, -2, 5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*S
in[e + f*x])/(5*(a + b)) - (12*f*AppellF1[5/2, -1, 3/2, 7/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/5) - 4*(a + b)*((3
*a*f*AppellF1[5/2, -1, 3/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)
]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b)) - (9*(a + b)^3*f*Cot[e + f*x]*Csc[
e + f*x]^4*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]*((-2*a*Sin[e + f*x]^2)/(a +
 b) - (4*a^2*Sin[e + f*x]^4)/(3*(a + b)^2) + (2*Sqrt[a]*ArcSin[(Sqrt[a]*Sin
[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/(Sqrt[a + b]*Sqrt[1 - (a*Sin[e + f*x]
^2)/(a + b)])))/(8*a^3)))))/(f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b
)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] +
 (a*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
 - 4*(a + b)*AppellF1[3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)
/(a + b)])*Sin[e + f*x]^2)^2) + (3*a*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Si
n[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^4*Sin[e + f*x]*Sin[2
*(e + f*x)])/((a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(3*(a + b)*AppellF1[1/2,
 -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/
2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Ap
pellF1[3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[
e + f*x]^2))))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(127) = 254$.

Time = 7.14 (sec) , antiderivative size = 809, normalized size of antiderivative = 5.66

method	result
default	$\frac{2\sin(fx+e)\cos(fx+e)^3\sqrt{-a}\,a^2+3\sin(fx+e)\cos(fx+e)\sqrt{-a}\,a^2-\cos(fx+e)\sqrt{-a}\,\sin(fx+e)ab+3\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a}{(1+c}}\right.}{}$

```
[In] int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f/(-a)^(1/2)/a^2/(a+b*sec(f*x+e)^2)^(1/2)*(2*sin(f*x+e)*cos(f*x+e)^3*(-
a)^(1/2)*a^2+3*sin(f*x+e)*cos(f*x+e)*(-a)^(1/2)*a^2-cos(f*x+e)*(-a)^(1/2)*s
in(f*x+e)*a*b+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2-2*ln(4*(-a)^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*
```

```
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*a*b+3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2+3*se
c(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2-2*ln(4*(-a)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*a*b*sec(f*x+e)+3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*
sec(f*x+e)+3*(-a)^(1/2)*a*b*tan(f*x+e)-3*(-a)^(1/2)*b^2*tan(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.53 (sec) , antiderivative size = 567, normalized size of antiderivative = 3.97

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[-\frac{(3a^2-2ab+3b^2)\sqrt{-a}\log\left(128a^4\cos(fx+e)^8-256(a^4-a^3b)\cos(fx+e)^6+32(5a^4-14a^3b+\cdots\right)}{\cdots}\right.$$

$$-\frac{(3a^2-2ab+3b^2)\sqrt{a}\arctan\left(\frac{\left(8a^2\cos(fx+e)^5-8(a^2-ab)\cos(fx+e)^3+(a^2-6ab+b^2)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3a^2b)\cos(fx+e)^2\right)\sin(fx+e)}\right)-\cdots}{32a^3f}$$

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/64*((3*a^2 - 2*a*b + 3*b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(
a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e
)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*
a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*
b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 -
7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e)) - 8*(2*a^2*cos(f*x + e)^3 + 3*(a^2 - a*b)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*
```

```
f), -1/32*((3*a^2 - 2*a*b + 3*b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5
 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b
 + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2*a^2*cos(f*
x + e)^3 + 3*(a^2 - a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e))/(a^3*f)]
```

Sympy [F]

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos^4{(e+fx)}}{\sqrt{a+b\sec^2{(e+fx)}}}\,dx$$

```
[In] integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(cos(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos{(fx+e)}^4}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos{(fx+e)}^4}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cos\left(e+f\,x\right)^4}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

```
[In] int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.269 $\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 204

$$
\begin{aligned}
\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx
&= \frac{(a-b)\left(5a^2+2ab+5b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{7/2}f} \\
&\quad+ \frac{\left(15a^2-14ab+15b^2\right)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} \\
&\quad+ \frac{5(a-b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24a^2f} \\
&\quad+ \frac{\cos^5(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6af}
\end{aligned}
$$

```
[Out] 1/16*(a-b)*(5*a^2+2*a*b+5*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^
2)^(1/2))/a^(7/2)/f+1/48*(15*a^2-14*a*b+15*b^2)*cos(f*x+e)*sin(f*x+e)*(a+b+
b*tan(f*x+e)^2)^(1/2)/a^3/f+5/24*(a-b)*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f
*x+e)^2)^(1/2)/a^2/f+1/6*cos(f*x+e)^5*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)
/a/f
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4231, 425, 541, 12, 385, 209}

$$\begin{aligned}&\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx\\&=\frac{5(a-b)\sin(e+fx)\cos^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{24a^2f}\\&+\frac{(a-b)\left(5a^2+2ab+5b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{7/2}f}\\&+\frac{\left(15a^2-14ab+15b^2\right)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{48a^3f}\\&+\frac{\sin(e+fx)\cos^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6af}\end{aligned}$$

```
[In] Int[Cos[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((a - b)*(5*a^2 + 2*a*b + 5*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b +
 b*Tan[e + f*x]^2]])/(16*a^(7/2)*f) + ((15*a^2 - 14*a*b + 15*b^2)*Cos[e + f
*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(48*a^3*f) + (5*(a - b)*Co
s[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(24*a^2*f) + (Cos
[e + f*x]^5*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(6*a*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^4\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\cos^5(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6af}\\
&\quad - \frac{\text{Subst}\left(\int \frac{-5a+b-4bx^2}{(1+x^2)^3\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{6af}\\
&= \frac{5(a-b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24a^2f}\\
&\quad + \frac{\cos^5(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6af}\\
&\quad + \frac{\text{Subst}\left(\int \frac{15a^2-4ab+5b^2+10(a-b)bx^2}{(1+x^2)^2\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{24a^2f}
\end{aligned}
$$

$$= \frac{(15a^2-14ab+15b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} + \frac{5(a-b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24a^2f} + \frac{\cos^5(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6af} - \frac{\text{Subst}\left(\int -\frac{3(a-b)(5a^2+2ab+5b^2)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{48a^3f}$$

$$= \frac{(15a^2-14ab+15b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} + \frac{5(a-b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24a^2f} + \frac{\cos^5(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6af} + \frac{((a-b)(5a^2+2ab+5b^2))\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{16a^3f}$$

$$= \frac{(15a^2-14ab+15b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} + \frac{5(a-b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24a^2f} + \frac{\cos^5(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6af} + \frac{((a-b)(5a^2+2ab+5b^2))\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^3f}$$

$$= \frac{(a-b)(5a^2+2ab+5b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{7/2}f} + \frac{(15a^2-14ab+15b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} + \frac{5(a-b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24a^2f} + \frac{\cos^5(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6af}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 15.44 (sec) , antiderivative size = 1739, normalized size of antiderivative = 8.52

$$\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \frac{}{f\sqrt{a+2b+a\cos(2(e+fx))}\sqrt{a+b\sec^2(e+fx)}\left(3(a+b)\,\text{AppellF1}\left(\frac{1}{2},-3,\frac{1}{2},\frac{3}{2},\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right.\right.}$$

```
[In] Integrate[Cos[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^12*Sin[e + f*x])/(f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]
*Sqrt[a + b*Sec[e + f*x]^2]*(3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e +
f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 3/2, 5/2, Sin[e
+ f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 1/2, 5/
2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*((3*(a + b)
*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Co
s[e + f*x]^7)/(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b)*AppellF1[1/2,
-3, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2
, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*App
ellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e
 + f*x]^2)) - (18*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Si
n[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(Sqrt[a + 2*b + a*Cos
[2*(e + f*x)]]*(3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Si
n[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*
Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*
x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos[e + f*
x]^6*Sin[e + f*x]*((a*f*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[
e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - 2*f*AppellF1[
3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]
*Sin[e + f*x]))/(f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b)*AppellF1[1
/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1
[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)
*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*S
in[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a
*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^6*Sin[e + f*x]*(2*f*(a*AppellF1[3/2,
 -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*Appe
llF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Cos[e
+ f*x]*Sin[e + f*x] + 3*(a + b)*((a*f*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f
```

```
*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) -
 2*f*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)
]*Cos[e + f*x]*Sin[e + f*x]) + Sin[e + f*x]^2*(a*((9*a*f*AppellF1[5/2, -3,
5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e +
f*x])/(5*(a + b)) - (18*f*AppellF1[5/2, -2, 3/2, 7/2, Sin[e + f*x]^2, (a*Si
n[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/5) - 6*(a + b)*((3*a*f*Ap
pellF1[5/2, -2, 3/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e
 + f*x]*Sin[e + f*x])/(5*(a + b)) - (12*f*AppellF1[5/2, -1, 1/2, 7/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/5))))/(f*
Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, S
in[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 3/2, 5/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2,
1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)^2) +
 (3*a*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)]*Cos[e + f*x]^6*Sin[e + f*x]*Sin[2*(e + f*x)])/((a + 2*b + a*Cos[
2*(e + f*x)])^(3/2)*(3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 1/2, 5/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2))))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1135 vs. 2(184) = 368.

Time = 9.68 (sec) , antiderivative size = 1136, normalized size of antiderivative = 5.57

method	result	size
default	Expression too large to display	1136

```
[In] int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/f/(-a)^(1/2)/a^3/(a+b*sec(f*x+e)^2)^(1/2)*(8*cos(f*x+e)^5*sin(f*x+e)*(
-a)^(1/2)*a^3+10*cos(f*x+e)^3*sin(f*x+e)*(-a)^(1/2)*a^3-2*cos(f*x+e)^3*sin(
f*x+e)*(-a)^(1/2)*a^2*b+15*cos(f*x+e)*sin(f*x+e)*(-a)^(1/2)*a^3-4*cos(f*x+e
)*sin(f*x+e)*(-a)^(1/2)*a^2*b+5*cos(f*x+e)*sin(f*x+e)*(-a)^(1/2)*a*b^2+15*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3-9*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos
(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+
e)*a)*a^2*b+9*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^2-15*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)-4*sin(f*x+e)*a)*b^3+15*(-a)^(1/2)*a^2*b*tan(f*x+e)-14*(-a)^(1/2)*a*
```

```
b^2*tan(f*x+e)+15*(-a)^(1/2)*b^3*tan(f*x+e)+15*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(
f*x+e)*a)*a^3*sec(f*x+e)-9*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4
*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2*b*sec(
f*x+e)+9*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^2*sec(f*x+e)-15*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^3*sec(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 1.14 (sec) , antiderivative size = 643, normalized size of antiderivative = 3.15

$$\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[\frac{3\left(5a^3-3a^2b+3ab^2-5b^3\right)\sqrt{-a}\log\left(128a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6+32\left(5a^4-14\right.\right.}{}\right.$$

$$-\frac{3\left(5a^3-3a^2b+3ab^2-5b^3\right)\sqrt{a}\arctan\left(\frac{\left(8a^2\cos(fx+e)^5-8\left(a^2-ab\right)\cos(fx+e)^3+\left(a^2-6ab+b^2\right)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+}{\cos(fx+}}}{4\left(2a^3\cos(fx+e)^4-a^2b+ab^2-\left(a^3-3a^2b\right)\cos(fx+e)^2\right)\sin(fx+e)}\right.}{}$$

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(3*(5*a^3 - 3*a^2*b + 3*a*b^2 - 5*b^3)*sqrt(-a)*log(128*a^4*cos(f*x
+ e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^
2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4
- 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 -
24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x +
e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^3*cos(f*x + e)^5 + 10
*(a^3 - a^2*b)*cos(f*x + e)^3 + (15*a^3 - 14*a^2*b + 15*a*b^2)*cos(f*x + e)
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^4*f), -1/192
```

```
*(3*(5*a^3 - 3*a^2*b + 3*a*b^2 - 5*b^3)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x +
 e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sq
rt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 -
a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(8*a^3*c
os(f*x + e)^5 + 10*(a^3 - a^2*b)*cos(f*x + e)^3 + (15*a^3 - 14*a^2*b + 15*a
*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))/(a^4*f)]
```

Sympy [F]

$$\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cos^6{(e+fx)}}{\sqrt{a+b\sec^2{(e+fx)}}}\,dx$$

```
[In] integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(cos(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cos{(fx+e)}^6}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cos{(fx+e)}^6}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos(e+f\,x)^6}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

```
[In] int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.270 $\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 289

$$\begin{aligned}\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx &= \frac{a(2a+b)\sin(e+fx)}{b^2(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\ &- \frac{(2a+b)E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{b^2(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} \\ &+ \frac{\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\ &+ \frac{\sec(e+fx)\tan(e+fx)}{bf\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\end{aligned}$$

```
[Out] a*(2*a+b)*sin(f*x+e)/b^2/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)-
(2*a+b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/b^2/(a+b
)/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin
(f*x+e)^2/(a+b))^(1/2)+EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e
)^2/(a+b))^(1/2)/b/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2
))^(1/2)+sec(f*x+e)*tan(f*x+e)/b/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2
)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4233, 1985, 1986, 425, 541, 538, 437, 435, 432, 430}

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(2a+b)\left(-a\sin^2(e+fx)+a+b\right)E\left(\arcsin(\sin(e+fx))\left|\frac{a}{a+b}\right.\right)}{b^2f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}} + \frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}} + \frac{a(2a+b)\sin(e+fx)}{b^2f(a+b)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}} + \frac{\tan(e+fx)\sec(e+fx)}{bf\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

```
[In] Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*(2*a + b)*Sin[e + f*x])/(b^2*(a + b)*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Si
n[e + f*x]^2)]) - ((2*a + b)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a
+ b - a*Sin[e + f*x]^2))/(b^2*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f
*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + (
EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a +
 b)])/(b*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]
^2)]) + (Sec[e + f*x]*Tan[e + f*x])/(b*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin
[e + f*x]^2)])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
```

```
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3\left(a+\frac{b}{1-x^2}\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3\left(\frac{a+b-ax^2}{1-x^2}\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}(a+b-ax^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}\\
&= \frac{\sec(e+fx)\tan(e+fx)}{bf\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}\\
&\quad+\frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{-a-ax^2}{\sqrt{1-x^2}(a+b-ax^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}\\
&= \frac{a(2a+b)\sin(e+fx)}{b^2(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}\\
&\quad+\frac{\sec(e+fx)\tan(e+fx)}{bf\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}\\
&\quad-\frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{2a(a+b)-a(2a+b)x^2}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{b^2(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}
\end{aligned}$$

$$= \frac{a(2a+b)\sin(e+fx)}{b^2(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{\sec(e+fx)\tan(e+fx)}{bf\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{\sqrt{a+b-a\sin^2(e+fx)}\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx,x,\sin(e+fx)\right)}{bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{\left((2a+b)\sqrt{a+b-a\sin^2(e+fx)}\right)\text{Subst}\left(\int\frac{\sqrt{a+b-ax^2}}{\sqrt{1-x^2}}\,dx,x,\sin(e+fx)\right)}{b^2(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

$$= \frac{a(2a+b)\sin(e+fx)}{b^2(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{\sec(e+fx)\tan(e+fx)}{bf\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{\left((2a+b)\left(a+b-a\sin^2(e+fx)\right)\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{ax^2}{a+b}}}{\sqrt{1-x^2}}\,dx,x,\sin(e+fx)\right)}{b^2(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} + \frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{ax^2}{a+b}}}\,dx,x,\sin(e+fx)\right)}{bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

$$= \frac{a(2a+b)\sin(e+fx)}{b^2(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{(2a+b)E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)\left(a+b-a\sin^2(e+fx)\right)}{b^2(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} + \frac{\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{\sec(e+fx)\tan(e+fx)}{bf\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

Mathematica [F]

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

```
[In] Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.40 (sec) , antiderivative size = 10329, normalized size of antiderivative = 35.74

method	result	size
default	Expression too large to display	10329

```
[In] int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 960, normalized size of antiderivative = 3.32

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] 1/2*((2*(2*I*a^2*b + I*a*b^2 + (2*I*a^3 + I*a^2*b)*cos(f*x + e)^2)*sqrt(a)*
sqrt((a*b + b^2)/a^2) - (2*I*a^2*b + 5*I*a*b^2 + 2*I*b^3 + (2*I*a^3 + 5*I*a
^2*b + 2*I*a*b^2)*cos(f*x + e)^2)*sqrt(a))*sqrt((2*a*sqrt((a*b + b^2)/a^2)
- a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/
a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)
*sqrt((a*b + b^2)/a^2))/a^2) + (2*(-2*I*a^2*b - I*a*b^2 + (-2*I*a^3 - I*a^2
*b)*cos(f*x + e)^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) - (-2*I*a^2*b - 5*I*a*b^2
 - 2*I*b^3 + (-2*I*a^3 - 5*I*a^2*b - 2*I*a*b^2)*cos(f*x + e)^2)*sqrt(a))*sq
rt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqr
t((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8
*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - 4*((-I*a^2*b*c
os(f*x + e)^2 - I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) + (-I*a^2*b - 3*I*a*
```

```
b^2 - 2*I*b^3 + (-I*a^3 - 3*I*a^2*b - 2*I*a*b^2)*cos(f*x + e)^2)*sqrt(a))*s
qrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sq
rt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 +
8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - 4*((I*a^2*b*c
os(f*x + e)^2 + I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) + (I*a^2*b + 3*I*a*b
^2 + 2*I*b^3 + (I*a^3 + 3*I*a^2*b + 2*I*a*b^2)*cos(f*x + e)^2)*sqrt(a))*sqr
t((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt
((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*
a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + 2*(a^2*b + a*b^
2 + (2*a^3 + a^2*b)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*sin(f*x + e))/((a^3*b^2 + a^2*b^3)*f*cos(f*x + e)^2 + (a^2*b^3 + a*b^4
)*f)
```

Sympy [F]

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(sec(e + f*x)**5/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)^5}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)^5}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)^5\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}}\,dx$$

```
[In] int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2)),x)
[Out] int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

3.271 $\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 150

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = -\frac{a\sin(e+fx)}{b(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\left(a+b-a\sin^2(e+fx)\right)}{b(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$

```
[Out] -a*sin(f*x+e)/b/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+EllipticE
(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/b/(a+b)/f/(cos(f*x+e)^2)^
(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1
/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4233, 1985, 1986, 425, 21, 437, 435}

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(-a\sin^2(e+fx)+a+b\right)E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)}{bf(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+\right.}} - \frac{a\sin(e+fx)}{bf(a+b)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

```
[In] Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a*Sin[e + f*x])/(b*(a + b)*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]
^2)])) + (EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a + b - a*Sin[e + f*x
```

```
]^2))/(b*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[
e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2\left(a+\frac{b}{1-x^2}\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2\left(\frac{a+b-ax^2}{1-x^2}\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}(a+b-ax^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= -\frac{a\sin(e+fx)}{b(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} - \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{-a-b+ax^2}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{b(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= -\frac{a\sin(e+fx)}{b(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{\sqrt{1-x^2}}\,dx, x, \sin(e+fx)\right)}{b(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= -\frac{a\sin(e+fx)}{b(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{(a+b-a\sin^2(e+fx))\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{ax^2}{a+b}}}{\sqrt{1-x^2}}\,dx, x, \sin(e+fx)\right)}{b(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$

$$= -\frac{a\sin(e+fx)}{b(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{b(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$

Mathematica [A] (verified)

Time = 3.66 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(\sqrt{2}(a+b)\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}E(e\right.}{4b(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

```
[In] Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(Sqrt[2]*(a + b)*Sqrt[(a + 2
*b + a*Cos[2*(e + f*x)])/(a + b)]*EllipticE[e + f*x, a/(a + b)] - a*Sin[2*(
e + f*x)]))/(4*b*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.14 (sec) , antiderivative size = 2535, normalized size of antiderivative = 16.90

method	result	size
default	Expression too large to display	2535

```
[In] int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f/(2*I*a^(1/2)*b^(1/2)-a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/(a+b
)/b*(2*I*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(3/2)*b^(1/2)*(1-cos(f*x
+e))^3*csc(f*x+e)^3+2*I*a^(1/2)*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*(csc(f*x+e)-cot(f*x+e))-2*I*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^
(3/2)*b^(1/2)*(csc(f*x+e)-cot(f*x+e))+2*I*a^(3/2)*b^(1/2)*(-(2*I*a^(1/2)*b^
(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-co
s(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x
+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(
f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*
a*b-b^2)/(a+b)^2)^(1/2))+2*I*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(1/2
)*b^(3/2)*(1-cos(f*x+e))^3*csc(f*x+e)^3+2*I*a^(1/2)*b^(3/2)*(-(2*I*a^(1/2)*
b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-
cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f
*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*cs
c(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+
6*a*b-b^2)/(a+b)^2)^(1/2))-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*(1-c
os(f*x+e))^3*csc(f*x+e)^3+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2*(1-co
s(f*x+e))^3*csc(f*x+e)^3+2*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+
```

e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(
a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*
x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*Elli
pticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*
I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b+2*
(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc
(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b
^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-c
os(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/
2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2-(-(2*I*a^(1/2)*b^(1/2)*(1-cos
(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*
csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f
x+e)^2-a(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b
)/(a+b))^(1/2)*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e
)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+
b)^2)^(1/2))*a^2-2*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(
1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1
/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*
csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticE(((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2
)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-(-(2*I*a^(
1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-
b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-
cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))
^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)
-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
a^2(csc(f*x+e)-cot(f*x+e))-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b*
(csc(f*x+e)-cot(f*x+e))+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2*(csc(f*
x+e)-cot(f*x+e)))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f
*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2
+a+b)/((a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a
*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/((1-c
os(f*x+e))^2*csc(f*x+e)^2-1)^2)^(3/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^3

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 894, normalized size of antiderivative = 5.96

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx =$$

$$-\frac{2\,a^3\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos\left(fx+e\right)^2\sin\left(fx+e\right)-\left(2\left(-i\,a^3\cos\left(fx+e\right)^2-i\,a^2b\right)\sqrt{a}\sqrt{\frac{ab+b^2}{a^2}}-\left(-i\,a^2b-2i\,a\right.\right.}{}$$

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2*sin(
f*x + e) - (2*(-I*a^3*cos(f*x + e)^2 - I*a^2*b)*sqrt(a)*sqrt((a*b + b^2)/a^
2) - (-I*a^2*b - 2*I*a*b^2 + (-I*a^3 - 2*I*a^2*b)*cos(f*x + e)^2)*sqrt(a))*
sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*s
qrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 +
 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - (2*(I*a^3*co
s(f*x + e)^2 + I*a^2*b)*sqrt(a)*sqrt((a*b + b^2)/a^2) - (I*a^2*b + 2*I*a*b^
2 + (I*a^3 + 2*I*a^2*b)*cos(f*x + e)^2)*sqrt(a))*sqrt((2*a*sqrt((a*b + b^2)
/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a -
 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 +
2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + 2*(2*(I*a^2*b*cos(f*x + e)^2 + I*a*b^2
)*sqrt(a)*sqrt((a*b + b^2)/a^2) + (I*a^2*b + 3*I*a*b^2 + 2*I*b^3 + (I*a^3 +
 3*I*a^2*b + 2*I*a*b^2)*cos(f*x + e)^2)*sqrt(a))*sqrt((2*a*sqrt((a*b + b^2)
/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a -
 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 +
2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + 2*(2*(-I*a^2*b*cos(f*x + e)^2 - I*a*b^
2)*sqrt(a)*sqrt((a*b + b^2)/a^2) + (-I*a^2*b - 3*I*a*b^2 - 2*I*b^3 + (-I*a^
3 - 3*I*a^2*b - 2*I*a*b^2)*cos(f*x + e)^2)*sqrt(a))*sqrt((2*a*sqrt((a*b + b
^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) -
a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2
 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2))/((a^4*b + a^3*b^2)*f*cos(f*x + e)^2
+ (a^3*b^2 + a^2*b^3)*f)
```

Sympy [F]

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(sec(e + f*x)**3/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)^3}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)^3}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\cos(e+f\,x)^3\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2)),x)
[Out] int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

3.272 $\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 229

$$\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\sin(e+fx)}{(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)\left(a+b-a\sin^2(e+fx)\right)}{a(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} + \frac{\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{af\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

```
[Out] sin(f*x+e)/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)-EllipticE(sin(
f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a/(a+b)/f/(cos(f*x+e)^2)^(1/2)
/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)+E
llipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a/f/(co
s(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used

$= \{4233, 1985, 1986, 423, 507, 437, 435, 432, 430\}$

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{af\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}} - \frac{\left(-a\sin^2(e+fx)+a+b\right)E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)}{af(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}} + \frac{\sin(e+fx)}{f(a+b)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

```
[In] Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Sin[e + f*x]/((a + b)*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) -
(EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a + b - a*Sin[e + f*x]^2))/(a*
(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^
2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + (EllipticF[ArcSin[Sin[e + f*x]]
, a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(a*f*Sqrt[Cos[e + f*x]^2
]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 507

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-b/a, -d/c])
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a+\frac{b}{1-x^2}\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(\frac{a+b-ax^2}{1-x^2}\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{a+b-a\sin^2(e+fx)}\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{(a+b-ax^2)^{3/2}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= \frac{\sin(e+fx)}{(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{\sqrt{a+b-a\sin^2(e+fx)}\text{Subst}\left(\int \frac{x^2}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx,x,\sin(e+fx)\right)}{(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= \frac{\sin(e+fx)}{(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{\sqrt{a+b-a\sin^2(e+fx)}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx,x,\sin(e+fx)\right)}{af\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} - \frac{\sqrt{a+b-a\sin^2(e+fx)}\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{\sqrt{1-x^2}}\,dx,x,\sin(e+fx)\right)}{a(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= \frac{\sin(e+fx)}{(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} - \frac{(a+b-a\sin^2(e+fx))\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{ax^2}{a+b}}}{\sqrt{1-x^2}}\,dx,x,\sin(e+fx)\right)}{a(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} + \frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{ax^2}{a+b}}}\,dx,x,\sin(e+fx)\right)}{af\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= \frac{\sin(e+fx)}{(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} - \frac{E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{a(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} + \frac{\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{af\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 18.39 (sec) , antiderivative size = 905, normalized size of antiderivative = 3.95

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a+2b+a\cos(2e+2fx))^{3/2}\sec^3(e+fx)\left(\frac{\sqrt{-\frac{1}{b}}\left(\left(-\frac{1}{b}\right)^{3/2}b(-a+a\cos(2e+2fx))(a+a\right.}{}\right.}{}$$

```
[In] Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3*((Sqrt[-b^(-1)]*((-b^(
-1))^(3/2)*b*(-a + a*Cos[2*e + 2*f*x])*(a + a*Cos[2*e + 2*f*x]) + I*(a + b)
*Sqrt[(a - a*Cos[2*e + 2*f*x])/(a + b)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*
Sqrt[4 - (2*(a + 2*b + a*Cos[2*e + 2*f*x]))/b]*EllipticE[I*ArcSinh[(Sqrt[-b
^(-1)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], b/(a + b)] - I*(a + b)
*Sqrt[(a - a*Cos[2*e + 2*f*x])/(a + b)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*
Sqrt[4 - (2*(a + 2*b + a*Cos[2*e + 2*f*x]))/b]*EllipticF[I*ArcSinh[(Sqrt[-b
^(-1)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], b/(a + b)])*Sin[2*e +
2*f*x])/(4*a*(a + b)*f*Sqrt[((a - a*Cos[2*e + 2*f*x])*(a + a*Cos[2*e + 2*f*
x]))/a^2]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sqrt[1 - Cos[2*e + 2*f*x]^2])
- (Sqrt[-b^(-1)]*Cos[2*(e + f*x)]*((-b^(-1))^(3/2)*b*(a + 2*b)*(-a + a*Cos[
2*e + 2*f*x])*(a + a*Cos[2*e + 2*f*x]) + I*(a^2 + 3*a*b + 2*b^2)*Sqrt[(a -
a*Cos[2*e + 2*f*x])/(a + b)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sqrt[4 - (2
*(a + 2*b + a*Cos[2*e + 2*f*x]))/b]*EllipticE[I*ArcSinh[(Sqrt[-b^(-1)]*Sqrt
[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], b/(a + b)] - I*a*(a + b)*Sqrt[a +
 2*b + a*Cos[2*e + 2*f*x]]*Sqrt[(4*a + 4*b - 2*(a + 2*b + a*Cos[2*e + 2*f*x
]))/(a + b)]*Sqrt[2 - (a + 2*b + a*Cos[2*e + 2*f*x])/b]*EllipticF[I*ArcSinh
[(Sqrt[-b^(-1)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], b/(a + b)])*S
ec[2*(e + (-2*e + ArcCos[Cos[2*e + 2*f*x]])/2)]*Sin[2*e + 2*f*x])/(4*a^2*(a
 + b)*f*Sqrt[((a - a*Cos[2*e + 2*f*x])*(a + a*Cos[2*e + 2*f*x]))/a^2]*Sqrt[
a + 2*b + a*Cos[2*e + 2*f*x]]*Sqrt[1 - Cos[2*e + 2*f*x]^2])))/(2*(a + b*Sec
[e + f*x]^2)^(3/2))
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.98 (sec) , antiderivative size = 2534, normalized size of antiderivative = 11.07

method	result	size
default	Expression too large to display	2534

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/f/(2*I*a^(1/2)*b^(1/2)-a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/(a+b)
/a*(-2*I*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(3/2)*b^(1/2)*(csc(f*x+e
)-cot(f*x+e))-2*I*a^(1/2)*b^(3/2)*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*c
sc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2
-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1
-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/
2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)
),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))
-2*I*a^(3/2)*b^(1/2)*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a
*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^
(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^
2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticF(
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3
/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))+2*I*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(1/2)*b^(3/2)*(1-cos(f*x+e))^3*csc(f*x+e)
^3+2*I*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(3/2)*b^(1/2)*(1-cos(f*x+e
))^3*csc(f*x+e)^3+2*I*a^(1/2)*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)*(csc(f*x+e)-cot(f*x+e))-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*(1-c
os(f*x+e))^3*csc(f*x+e)^3+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2*(1-co
s(f*x+e))^3*csc(f*x+e)^3+2*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+
e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(
a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*
x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*Elli
pticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*
I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2+2*
(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc
(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b
^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-c
os(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/
2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-(-(2*I*a^(1/2)*b^(1/2)*(1-cos
(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*
csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f
*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b
)/(a+b))^(1/2)*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e
)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+
b)^2)^(1/2))*a^2-2*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(
1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1
/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*
csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticE(((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2
)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-(-(2*I*a^(
1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-
```

```
b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-
cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))
^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)
-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
*a^2*(csc(f*x+e)-cot(f*x+e))-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b*
(csc(f*x+e)-cot(f*x+e))+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2*(csc(f*
x+e)-cot(f*x+e)))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f
*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2
+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^3/((a*(1-cos(f*x+e))^4*csc(f*x+e)^4
+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-c
os(f*x+e))^2*csc(f*x+e)^2+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^2)^(3/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 762, normalized size of antiderivative = 3.33

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{2\,a^2\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)^2\sin(fx+e) - 4\left(i\,a^2\cos(fx+e)^2 + i\,ab\right)\sqrt{a}}{}$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2*sin(f
*x + e) - 4*(I*a^2*cos(f*x + e)^2 + I*a*b)*sqrt(a)*sqrt((2*a*sqrt((a*b + b^
2)/a^2) - a - 2*b)/a)*sqrt((a*b + b^2)/a^2)*elliptic_f(arcsin(sqrt((2*a*sqr
t((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8
*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - 4*(-I*a^2*cos(
f*x + e)^2 - I*a*b)*sqrt(a)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*s
qrt((a*b + b^2)/a^2)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a
- 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 +
 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*(I*a^2*cos(f*x + e)^2 + I*a*b)*sqr
t(a)*sqrt((a*b + b^2)/a^2) - ((I*a^2 + 2*I*a*b)*cos(f*x + e)^2 + I*a*b + 2*
I*b^2)*sqrt(a))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(ar
csin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*
x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2
) + (2*(-I*a^2*cos(f*x + e)^2 - I*a*b)*sqrt(a)*sqrt((a*b + b^2)/a^2) - ((-I
*a^2 - 2*I*a*b)*cos(f*x + e)^2 - I*a*b - 2*I*b^2)*sqrt(a))*sqrt((2*a*sqrt((
a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/
a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 +
 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2))/((a^4 + a^3*b)*f*cos(f*x + e)
^2 + (a^3*b + a^2*b^2)*f)
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(sec(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(sec(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate(sec(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\cos(e+f\,x)\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2)),x)
[Out] int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

3.273 $\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 240

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{b\sin(e+fx)}{a(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{(a+2b)E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{a^2(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{2b\,\mathrm{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{a^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

```
[Out] -b*sin(f*x+e)/a/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+(a+2*b)*E
llipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a^2/(a+b)/f/(cos(
f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2
/(a+b))^(1/2)-2*b*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(
a+b))^(1/2)/a^2/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^
(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used

$= \{4233, 1985, 1986, 424, 538, 437, 435, 432, 430\}$

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{2b\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{a^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}} + \frac{(a+2b)\left(-a\sin^2(e+fx)+a+b\right)E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{a^2f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}} - \frac{b\sin(e+fx)}{af(a+b)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

```
[In] Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((b*Sin[e + f*x])/(a*(a + b)*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]
^2)])) + ((a + 2*b)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a + b - a*S
in[e + f*x]^2))/(a^2*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a
+ b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (2*b*Ellip
ticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])
/(a^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)
])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{\left(a+\frac{b}{1-x^2}\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$=\frac{\text{Subst}\left(\int \frac{1}{\left(\frac{a+b-ax^2}{1-x^2}\right)^{3/2}}dx,x,\sin(e+fx)\right)}{f}$$

$$=\frac{\sqrt{a+b-a\sin^2(e+fx)}\text{Subst}\left(\int \frac{(1-x^2)^{3/2}}{(a+b-ax^2)^{3/2}}dx,x,\sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$=-\frac{b\sin(e+fx)}{a(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}-\frac{\sqrt{a+b-a\sin^2(e+fx)}\text{Subst}\left(\int \frac{-a-b+(a+2b)x^2}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}dx,x,\sin(e+fx)\right)}{a(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$=-\frac{b\sin(e+fx)}{a(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}-\frac{\left(2b\sqrt{a+b-a\sin^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}dx,x,\sin(e+fx)\right)}{a^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}+\frac{\left((a+2b)\sqrt{a+b-a\sin^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{\sqrt{1-x^2}}dx,x,\sin(e+fx)\right)}{a^2(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$=-\frac{b\sin(e+fx)}{a(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}+\frac{((a+2b)\,(a+b-a\sin^2(e+fx)))\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{ax^2}{a+b}}}{\sqrt{1-x^2}}dx,x,\sin(e+fx)\right)}{a^2(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{\left(2b\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{ax^2}{a+b}}}dx,x,\sin(e+fx)\right)}{a^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$=-\frac{b\sin(e+fx)}{a(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}+\frac{(a+2b)E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{a^2(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{2b\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{a^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

Mathematica [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

```
[In] Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.97 (sec) , antiderivative size = 6126, normalized size of antiderivative = 25.52

method	result	size
default	Expression too large to display	6126

```
[In] int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)/(b^2*sec(f*x + e)^4 + 2*a*
b*sec(f*x + e)^2 + a^2), x)
```

Sympy [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(cos(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(e+f\,x)}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.274 $\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 335

$$\begin{aligned}\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = &-\frac{b\cos^2(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\\
&+\frac{(a+4b)\sin(e+fx)\,(a+b-a\sin^2(e+fx))}{3a^2(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\\
&+\frac{(2a^2-3ab-8b^2)\,E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{3a^3(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}\\
&-\frac{(a-8b)b\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{3a^3f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\end{aligned}$$

```
[Out] -b*cos(f*x+e)^2*sin(f*x+e)/a/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1
/2)+1/3*(a+4*b)*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a^2/(a+b)/f/(sec(f*x+e)^2*(
a+b-a*sin(f*x+e)^2))^(1/2)+1/3*(2*a^2-3*a*b-8*b^2)*EllipticE(sin(f*x+e),(a/
(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a^3/(a+b)/f/(cos(f*x+e)^2)^(1/2)/(sec(f*
x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/3*(a-8*
b)*b*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a
^3/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4233, 1985, 1986, 424, 542, 538, 437, 435, 432, 430}

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{b(a-8b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{3a^3f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{(a+4b)\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{3a^2f(a+b)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{(2a^2-3ab-8b^2)\left(-a\sin^2(e+fx)+a+b\right)E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{3a^3f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{b\sin(e+fx)\cos^2(e+fx)}{af(a+b)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

```
[In] Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((b*Cos[e + f*x]^2*Sin[e + f*x])/(a*(a + b)*f*Sqrt[Sec[e + f*x]^2*(a + b -
 a*Sin[e + f*x]^2)])) + ((a + 4*b)*Sin[e + f*x]*(a + b - a*Sin[e + f*x]^2))
/(3*a^2*(a + b)*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) + ((2*a^
2 - 3*a*b - 8*b^2)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a + b - a*Si
n[e + f*x]^2))/(3*a^3*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a
 + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - ((a - 8*b
)*b*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/
(a + b)])/(3*a^3*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[
e + f*x]^2)])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
```

```
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a+\frac{b}{1-x^2}\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1-x^2}{\left(\frac{a+b-ax^2}{1-x^2}\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{5/2}}{(a+b-ax^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= -\frac{b\cos^2(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} - \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{\sqrt{1-x^2}\,(-a-b+(a+4b)x^2)}{\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{a(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= -\frac{b\cos^2(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{(a+4b)\sin(e+fx)\,(a+b-a\sin^2(e+fx))}{3a^2(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{2(a-2b)(a+b)+(-2a^2+3ab+8b^2)x^2}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{3a^2(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= -\frac{b\cos^2(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{(a+4b)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{3a^2(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{\left((a-8b)b\sqrt{a+b-a\sin^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx,x,\sin(e+fx)\right)}{3a^3f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{\left((-2a^2+3ab+8b^2)\sqrt{a+b-a\sin^2(e+fx)}\right)\text{Subst}\left(\int\frac{\sqrt{a+b-ax^2}}{\sqrt{1-x^2}}\,dx,x,\sin(e+fx)\right)}{3a^3(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

$$= -\frac{b\cos^2(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{(a+4b)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{3a^2(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{\left((-2a^2+3ab+8b^2)\left(a+b-a\sin^2(e+fx)\right)\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{ax^2}{a+b}}}{\sqrt{1-x^2}}\,dx,x,\sin(e+fx)\right)}{3a^3(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{\left((a-8b)b\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{ax^2}{a+b}}}\,dx,x,\sin(e+fx)\right)}{3a^3f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

$$= -\frac{b\cos^2(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{(a+4b)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{3a^2(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{(2a^2-3ab-8b^2)\,E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\left(a+b-a\sin^2(e+fx)\right)}{3a^3(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{(a-8b)b\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{3a^3f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

Mathematica [F]

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

```
[In] Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.67 (sec) , antiderivative size = 15843, normalized size of antiderivative = 47.29

method	result	size
default	Expression too large to display	15843

```
[In] int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F]

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3/(b^2*sec(f*x + e)^4 + 2*
a*b*sec(f*x + e)^2 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)^3}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)^3}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(e+f\,x)^3}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.275 $\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 436

$$\begin{aligned}\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx &= -\frac{b\cos^4(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\\ &+\frac{(4a^2-5ab-24b^2)\sin(e+fx)\,(a+b-a\sin^2(e+fx))}{15a^3(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\\ &+\frac{(a+6b)\cos^2(e+fx)\sin(e+fx)\,(a+b-a\sin^2(e+fx))}{5a^2(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\\ &+\frac{(8a^3-9a^2b+16ab^2+48b^3)\,E\left(\arcsin(\sin(e+fx))\left|\frac{a}{a+b}\right.\right)(a+b-a\sin^2(e+fx))}{15a^4(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}\\ &-\frac{4b(a^2-2ab+12b^2)\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{15a^4f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\end{aligned}$$

```
[Out] -b*cos(f*x+e)^4*sin(f*x+e)/a/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1
/2)+1/15*(4*a^2-5*a*b-24*b^2)*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a^3/(a+b)/f/(
sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+1/5*(a+6*b)*cos(f*x+e)^2*sin(f*x+e
)*(a+b-a*sin(f*x+e)^2)/a^2/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2
)+1/15*(8*a^3-9*a^2*b+16*a*b^2+48*b^3)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2)
)*(a+b-a*sin(f*x+e)^2)/a^4/(a+b)/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-
a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-4/15*b*(a^2-2*a*b+12*
b^2)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a
^4/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4233, 1985, 1986, 424, 542, 538, 437, 435, 432, 430}

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a+6b)\sin(e+fx)\cos^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{5a^2f(a+b)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$
$$-\frac{4b(a^2-2ab+12b^2)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{15a^4f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$
$$+\frac{(4a^2-5ab-24b^2)\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{15a^3f(a+b)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$
$$+\frac{(8a^3-9a^2b+16ab^2+48b^3)\left(-a\sin^2(e+fx)+a+b\right)E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{15a^4f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$
$$-\frac{b\sin(e+fx)\cos^4(e+fx)}{af(a+b)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

```
[In] Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((b*Cos[e + f*x]^4*Sin[e + f*x])/(a*(a + b)*f*Sqrt[Sec[e + f*x]^2*(a + b -
 a*Sin[e + f*x]^2)])) + ((4*a^2 - 5*a*b - 24*b^2)*Sin[e + f*x]*(a + b - a*S
in[e + f*x]^2))/(15*a^3*(a + b)*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*
x]^2)]) + ((a + 6*b)*Cos[e + f*x]^2*Sin[e + f*x]*(a + b - a*Sin[e + f*x]^2)
)/(5*a^2*(a + b)*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) + ((8*a
^3 - 9*a^2*b + 16*a*b^2 + 48*b^3)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)
]*(a + b - a*Sin[e + f*x]^2))/(15*a^4*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[S
ec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a +
b)]) - (4*b*(a^2 - 2*a*b + 12*b^2)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b
)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(15*a^4*f*Sqrt[Cos[e + f*x]^2]*Sqr
t[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{\left(a+\frac{b}{1-x^2}\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{\left(\frac{a+b-ax^2}{1-x^2}\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{7/2}}{(a+b-ax^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= -\frac{b\cos^4(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} - \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{3/2}(-a-b+(a+6b)x^2)}{\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{a(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= -\frac{b\cos^4(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{(a+6b)\cos^2(e+fx)\sin(e+fx)\,(a+b-a\sin^2(e+fx))}{5a^2(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{\sqrt{1-x^2}\,(2(2a-3b)(a+b)+(-4a^2+5ab+24b^2)x^2)}{\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{5a^2(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= -\frac{b\cos^4(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{(4a^2-5ab-24b^2)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{15a^3(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{(a+6b)\cos^2(e+fx)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{5a^2(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{-((a+b)(8a^2-13ab+24b^2))+(8a^3-9a^2b+16ab^2+48b^3)x^2}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{15a^3(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

$$= -\frac{b\cos^4(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{(4a^2-5ab-24b^2)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{15a^3(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{(a+6b)\cos^2(e+fx)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{5a^2(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{\left(4b(a^2-2ab+12b^2)\sqrt{a+b-a\sin^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{15a^4f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{\left((8a^3-9a^2b+16ab^2+48b^3)\sqrt{a+b-a\sin^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{\sqrt{1-x^2}}\,dx, x, \sin(e+fx)\right)}{15a^4(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

$$= -\frac{b\cos^4(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{(4a^2-5ab-24b^2)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{15a^3(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{(a+6b)\cos^2(e+fx)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{5a^2(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{\left((8a^3-9a^2b+16ab^2+48b^3)\left(a+b-a\sin^2(e+fx)\right)\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{ax^2}{a+b}}}{\sqrt{1-x^2}}\,dx, x, \sin(e+fx)\right)}{15a^4(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{\left(4b(a^2-2ab+12b^2)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{ax^2}{a+b}}}\,dx, x, \sin(e+fx)\right)}{15a^4f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

$$
\begin{aligned}
&= -\frac{b\cos^4(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\
&\quad+\frac{(4a^2-5ab-24b^2)\sin(e+fx)\,(a+b-a\sin^2(e+fx))}{15a^3(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\
&\quad+\frac{(a+6b)\cos^2(e+fx)\sin(e+fx)\,(a+b-a\sin^2(e+fx))}{5a^2(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\
&\quad+\frac{(8a^3-9a^2b+16ab^2+48b^3)\,E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{15a^4(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} \\
&\quad-\frac{4b(a^2-2ab+12b^2)\,\mathrm{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{15a^4f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}
\end{aligned}
$$

Mathematica [F]

$$
\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx
$$

```
[In] Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 14.01 (sec) , antiderivative size = 18649, normalized size of antiderivative = 42.77

method	result	size
default	Expression too large to display	18649

```
[In] int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F]

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)^5}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5/(b^2*sec(f*x + e)^4 + 2*
a*b*sec(f*x + e)^2 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)^5}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)^5}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos\left(e+f\,x\right)^5}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.276 $\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 138

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(3a-b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{5/2}f} - \frac{a\sec^2(e+fx)\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2b^2(a+b)f}$$

```
[Out] -1/2*(3*a-b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(5/2)
/f-a*sec(f*x+e)^2*tan(f*x+e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/2*(3*a+
b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b^2/(a+b)/f
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4231, 424, 396, 223, 212}

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(3a-b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{5/2}f} + \frac{(3a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b^2f(a+b)} - \frac{a\tan(e+fx)\sec^2(e+fx)}{bf(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[In] Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*((3*a - b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^
2]])/(b^(5/2)*f) - (a*Sec[e + f*x]^2*Tan[e + f*x])/(b*(a + b)*f*Sqrt[a + b
```

```
+ b*Tan[e + f*x]^2]) + ((3*a + b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^
2])/(2*b^2*(a + b)*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a+b+(3a+b)x^2}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{b(a+b)f}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2b^2(a+b)f} - \frac{(3a-b)\text{Subst}\left(\int\frac{1}{\sqrt{a+b+bx^2}}\,dx,x,\tan(e+fx)\right)}{2b^2f}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2b^2(a+b)f} - \frac{(3a-b)\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^2f}$$

$$= -\frac{(3a-b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{5/2}f} - \frac{a\sec^2(e+fx)\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2b^2(a+b)f}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.39 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.88

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx =$$

$$-\frac{(a+2b+a\cos(2(e+fx)))\sec^8(e+fx)\left(-7(a+b)\cos^2(e+fx)\left(15b^2+10ab(2+\cos(2(e+fx)))+a^2(\right.\right.}{}$$

```
[In] Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/210*((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^8*(-7*(a + b)*Cos[e + f
*x]^2*(15*b^2 + 10*a*b*(2 + Cos[2*(e + f*x)]) + a^2*(8 + 6*Cos[2*(e + f*x)]
 + Cos[4*(e + f*x)]))*Hypergeometric2F1[1, 2, 7/2, -((b*Tan[e + f*x]^2)/(a
+ b))] + 4*b*(a + 2*b + a*Cos[2*(e + f*x)])^2*HypergeometricPFQ[{2, 2, 3},
{1, 9/2}, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^2 + 16*b*Hypergeometr
ic2F1[2, 3, 9/2, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^2*(4*(a + b)^2
 - 7*a*(a + b)*Sin[e + f*x]^2 + 3*a^2*Sin[e + f*x]^4))*Tan[e + f*x])/((a +
b)^4*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3209 vs. 2(124) = 248.

Time = 12.25 (sec) , antiderivative size = 3210, normalized size of antiderivative = 23.26

method	result	size
default	Expression too large to display	3210

```
[In] int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f/(a+b)/b^(7/2)/(a+b*sec(f*x+e)^2)^(3/2)*(-2*b^(7/2)*a*tan(f*x+e)*sec(
f*x+e)^4+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a^2*b^2*sec(f*
x+e)^2+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a*b^3*sec(f*x+e)
^2+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a^2*b^2*sec(f*x+e)^2+
2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a*b^3*sec(f*x+e)^2+3*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a^2*b^2*sec(f*x+e)^3+2*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a*b^3*sec(f*x+e)^3+3*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a^2*b^2*sec(f*x+e)^3+2*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a*b^3*sec(f*x+e)^3+3*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-s
in(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a^3*b*sec(f*x+e)+3*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1
/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+
e)*a-a-b)/(sin(f*x+e)+1))*a^3*b*sec(f*x+e)-2*b^(5/2)*a^2*tan(f*x+e)-2*b^(9/
2)*tan(f*x+e)*sec(f*x+e)^4+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln
(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-
1))*a^2*b^2*sec(f*x+e)-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*((
```

```
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a*
b^3*sec(f*x+e)+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a^2*b^2*s
ec(f*x+e)-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a*b^3*sec(f*x+e)
-8*b^(5/2)*a^2*tan(f*x+e)*sec(f*x+e)^2-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+
e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(s
in(f*x+e)-1))*b^4*sec(f*x+e)^2-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)
+1))*b^4*sec(f*x+e)^2-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*b^4
*sec(f*x+e)^3-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^4*sec(f*x+
e)^3+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a^3*b+3*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a^3*b-4*b^(7/2)*a*tan(f*x+e)*sec(f*x
+e)^2+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a^2*b^2-((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a*b^3+2*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)
*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*
a-a-b)/(sin(f*x+e)+1))*a^2*b^2-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)
+1))*a*b^3-6*b^(3/2)*a^3*tan(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 524, normalized size of antiderivative = 3.80

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[-\frac{\left(\left(3\,a^3+2\,a^2b-ab^2\right)\cos\left(fx+e\right)^3+\left(3\,a^2b+2\,ab^2-b^3\right)\cos\left(fx+e\right)\right)\sqrt{b}\,\mathrm{l}\ldots}{\ldots}\right.$$

$$-\frac{\left(\left(3\,a^3+2\,a^2b-ab^2\right)\cos\left(fx+e\right)^3+\left(3\,a^2b+2\,ab^2-b^3\right)\cos\left(fx+e\right)\right)\sqrt{-b}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^3+2\,b\cos(\ldots\right.}{2\left(ab\cos(fx+e\ldots\right.}\right.}{4\left(\left(a^2b^3+ab^4\right)f\cos\left(fx+e\right)^3+\left(a\ldots\right.\right.}$$

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((3*a^3 + 2*a^2*b - a*b^2)*cos(f*x + e)^3 + (3*a^2*b + 2*a*b^2 - b^3
)*cos(f*x + e))*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b -
b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x
+ e)^4) - 4*(a*b^2 + b^3 + (3*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^2*b^3 + a*b^4)*f*cos(f*x +
e)^3 + (a*b^4 + b^5)*f*cos(f*x + e)), -1/4*(((3*a^3 + 2*a^2*b - a*b^2)*cos(
f*x + e)^3 + (3*a^2*b + 2*a*b^2 - b^3)*cos(f*x + e))*sqrt(-b)*arctan(-1/2*(
(a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) - 2*(a*b^2
+ b^3 + (3*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*sin(f*x + e))/((a^2*b^3 + a*b^4)*f*cos(f*x + e)^3 + (a*b^4 + b^5
)*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec^6\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.17

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\frac{\tan(fx+e)^3}{\sqrt{b\tan(fx+e)^2+a+b}b} - \frac{3\,(a+b)\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{5}{2}}} + \frac{4\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}} + \frac{2\,\tan(f}{\sqrt{b\tan(fx+e)^2}}}{2\,f}$$

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(tan(f*x + e)^3/(sqrt(b*tan(f*x + e)^2 + a + b)*b) - 3*(a + b)*arcsinh(
b*tan(f*x + e)/sqrt((a + b)*b))/b^(5/2) + 4*arcsinh(b*tan(f*x + e)/sqrt((a
+ b)*b))/b^(3/2) + 2*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b))
+ 3*(a + b)*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*b^2) - 4*tan(f*x +
 e)/(sqrt(b*tan(f*x + e)^2 + a + b)*b))/f
```

Giac [F]

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec\left(fx+e\right)^6}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)^6\,{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)}^{3/2}}\,dx$$

```
[In] int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

```
[Out] int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

3.277 $\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{3/2}f} - \frac{a\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(3/2)/f-a*tan(f*x+
e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4231, 393, 223, 212}

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{b^{3/2}f} - \frac{a\tan(e+fx)}{bf(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[In] Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(b^(3/2)*f)
- (a*Tan[e + f*x])/(b*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1+x^2}{(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{a\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{bf} \\
&= -\frac{a\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{bf} \\
&= \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{3/2}f} - \frac{a\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.31 (sec) , antiderivative size = 405, normalized size of antiderivative = 5.26

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a+2b+a\cos(2e+2fx))^{3/2}\sec^4(e+fx)\sqrt{1-\frac{2a\sin^2(e+fx)}{2a+2b}}\tan(e+fx)\left(15\arcsin\left(\sqrt{-\frac{b\tan^2(e+fx)}{a+b}}\right)\sec^2(\right.}{}$$

[In] Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]

[Out] -1/15*((a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^4*Sqrt[1 - (2*a*Si
n[e + f*x]^2)/(2*a + 2*b)]*Tan[e + f*x]*(15*ArcSin[Sqrt[-((b*Tan[e + f*x]^2
)/(a + b))]]*Sec[e + f*x]^2*(3*b^2 + a*b*(6 - 5*Sin[e + f*x]^2) + a^2*(3 -
5*Sin[e + f*x]^2 + 2*Sin[e + f*x]^4)) + 15*(a + b)*(-3*b + a*(-3 + 2*Sin[e
+ f*x]^2))*Sqrt[-((b*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]
^2)/(a + b)^2)] + 4*b*(a + b)*Hypergeometric2F1[2, 2, 7/2, -((b*Tan[e + f*x
]^2)/(a + b))]*Sin[e + f*x]^2*(-((b*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^
2)*Tan[e + f*x]^2)/(a + b)^2))^(3/2)))/((a + b)^2*(2*a + 2*b)*f*(a + b*Sec[
e + f*x]^2)^(3/2)*Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)]*Sqrt[2*a + 2*b - 2*a
*Sin[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]*(-((b*Tan[e + f*x]^2)
/(a + b)))^(3/2))

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1512 vs. 2(69) = 138.

Time = 8.13 (sec) , antiderivative size = 1513, normalized size of antiderivative = 19.65

method	result	size
default	Expression too large to display	1513

[In] int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/2/f/(a+b)/b^(5/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*cs
c(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e
)^2+a+b)*(-4*b^(3/2)*a*(csc(f*x+e)-cot(f*x+e))+ln(4*(a*(1-cos(f*x+e))^2*csc
(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+b^(1/2)*(a*(1-cos(f*x+e))^4*csc(f
x+e)^4+b(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2
b(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e))+a+b
)/((1-cos(f*x+e))^2*csc(f*x+e)^2-2*csc(f*x+e)+2*cot(f*x+e)+1))*(a*(1-cos(f*
x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*c

```
sc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*b*a+ln(4*(a*(1-cos
(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+b^(1/2)*(a*(1-cos(f
*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*
csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-c
ot(f*x+e))+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-2*csc(f*x+e)+2*cot(f*x+e)+1)
)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-c
os(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*b^2+
ln(4*(-a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2+b^(1
/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1
-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*
a*(csc(f*x+e)-cot(f*x+e))-a-b)/((1-cos(f*x+e))^2*csc(f*x+e)^2+2*csc(f*x+e)-
2*cot(f*x+e)+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*
x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+
a+b)^(1/2)*b*a+ln(4*(-a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*cs
c(f*x+e)^2+b^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(
f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^
2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e))-a-b)/((1-cos(f*x+e))^2*csc(f*x+e)^
2+2*csc(f*x+e)-2*cot(f*x+e)+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f
*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^
2*csc(f*x+e)^2+a+b)^(1/2)*b^2)/((a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f
*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^
2*csc(f*x+e)^2+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^2)^(3/2)/((1-cos(f*x+
e))^2*csc(f*x+e)^2-1)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(69) = 138$.

Time = 0.33 (sec) , antiderivative size = 410, normalized size of antiderivative = 5.32

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \left[-\frac{4\,ab\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e) - \left((a^2+ab)\cos(fx+e)^2 + \cdots\right.}{4\,(\cdots}\right.$$

$$-\frac{2\,ab\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e) - \left((a^2+ab)\cos(fx+e)^2+ab+b^2\right)\sqrt{-b}\arctan\left(-\frac{\left((a-b)\cos\cdots\right.}{\cdots}\right.}{2\left((a^2b^2+ab^3)f\cos(fx+e)^2+(ab^3+b^4)f\right)}$$

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f
*x + e) - ((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*sqrt(b)*log(((a^2 - 6*a*
b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x
 + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))/((a^2*b^2 + a*b^3)*f*cos(f*x +
e)^2 + (a*b^3 + b^4)*f), -1/2*(2*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*cos(f*x + e)*sin(f*x + e) - ((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*
sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(
f*x + e))))/((a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a*b^3 + b^4)*f)]
```

Sympy [F]

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\frac{\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}} + \frac{\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)} - \frac{\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}b}}{f}$$

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(3/2) + tan(f*x + e)/(sqrt(b*tan
(f*x + e)^2 + a + b)*(a + b)) - tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b
)*b))/f
```

Giac [F]

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)^4}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\cos(e+f\,x)^4\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2)),x)
[Out] int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

3.278 $\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 32

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=\frac{\tan(e+fx)}{(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] tan(f*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {4231, 197}

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=\frac{\tan(e+fx)}{f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[In] Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] Tan[e + f*x]/((a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
```

```
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.78

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\tan(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

```
[In] Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*Tan[e + f*x])/(2*(a + b)*f*(
a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.53

method	result	size
default	$\frac{a\tan(fx+e)+b\tan(fx+e)\sec(fx+e)^2}{f(a+b)\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}$	49

```
[In] int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/f/(a+b)/(a+b*sec(f*x+e)^2)^(3/2)*(a*tan(f*x+e)+b*tan(f*x+e)*sec(f*x+e)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(30) = 60$.

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.03

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e)}{(a^2+ab)f\cos(fx+e)^2+(ab+b^2)f}$$

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e)/((a^2
 + a*b)*f*cos(f*x + e)^2 + (a*b + b^2)*f)
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(sec(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)f}$$

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*f)
```

Giac [F]

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{\sec\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 20.52 (sec) , antiderivative size = 199, normalized size of antiderivative = 6.22

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\frac{\sqrt{\frac{a+2\,b+a\,\cos\left(2\,e+2\,f\,x\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(5\,a\,\sin\left(2\,e+2\,f\,x\right)+4\,a\,\sin\left(4\right.\right.}{f\,\left(a+b\right)\,\left(24\,a\,b+10\,a^2+16\,b^2+15\,a^2\,\cos\left(2\,e+2\,f\,x\right)+6\,a^2\,\cos\left(4\,e+4\right.\right.}$$

```
[In] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

```
[Out] (((a + 2*b + a*cos(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2)*(5*a*sin(2*e
 + 2*f*x) + 4*a*sin(4*e + 4*f*x) + a*sin(6*e + 6*f*x) + 8*b*sin(2*e + 2*f*x
) + 4*b*sin(4*e + 4*f*x)))/(f*(a + b)*(24*a*b + 10*a^2 + 16*b^2 + 15*a^2*co
s(2*e + 2*f*x) + 6*a^2*cos(4*e + 4*f*x) + a^2*cos(6*e + 6*f*x) + 16*b^2*cos
(2*e + 2*f*x) + 32*a*b*cos(2*e + 2*f*x) + 8*a*b*cos(4*e + 4*f*x)))
```

3.279 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 77

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f}-\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*tan(f*x+e
)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4213, 390, 385, 209}

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{b\tan(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f) -
 (b*Tan[e + f*x])/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{af}$$

$$= -\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{af}$$

$$= \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 168 vs. 2(77) = 154.

Time = 1.54 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.18

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(\sqrt{a+b}\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a+2b \ldots\right.}{4a^{3/2}(a+b)f\,(a+b\sec^2(e+fx)) \ldots}$$

[In] Integrate[(a + b*Sec[e + f*x]^2)^(-3/2),x]

[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - Sqrt[2]*Sqrt[a]*b*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(4*a^(3/2)*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. 2(69) = 138.

Time = 2.54 (sec) , antiderivative size = 515, normalized size of antiderivative = 6.69

method	result
default	$-\frac{\left(b+a\cos(fx+e)^2\right)\left(-\cos(fx+e)\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)\sqrt{\frac{b+a\cos(fx+e \ldots}{(1+\cos(fx+e) \ldots}} \ldots}{\ldots}$

[In] int(1/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/f/(a+b)/a/(-a)^(1/2)*(b+a*cos(f*x+e)^2)*(-cos(f*x+e)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b*cos(f*x+e)+(-a)^(1/2)*b*sin(f*x+e)-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b)/(a+b*sec(f*x+e)^2)^(3/2)*sec(f*x+e)^3

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(69) = 138$.

Time = 0.45 (sec) , antiderivative size = 601, normalized size of antiderivative = 7.81

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[-\frac{8\,ab\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e)+\left((a^2+ab)\cos(fx+e)^2+\right.}{}\right.$$

$$-\frac{4\,ab\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e)+\left((a^2+ab)\cos(fx+e)^2+ab+b^2\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e\right.}{4\,(}\right.}{4\,\left((a^4+a^3b)f\cos(fx+e)^2+(a^3b+a^2b^2)f\right)}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(8*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f
*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*sqrt(-a)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/((a^4 + a^3*b)*f*cos(f*x
 + e)^2 + (a^3*b + a^2*b^2)*f), -1/4*(4*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b
+ b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e
)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*c
os(f*x + e)^2)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a
^2*b^2)*f)]
```

Sympy [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2055 vs. $2(69) = 138$.

Time = 0.54 (sec) , antiderivative size = 2055, normalized size of antiderivative = 26.69

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*b*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*
e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))*sin(2*f*x + 2*e
) - 2*(a^2 + a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*
x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^3 - 2*(a*
b*cos(2*f*x + 2*e) + (a^2 + a*b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a
+ 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e)
+ a))^2 - a^2 - 2*a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin
(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) - (a
^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*co
s(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a
^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos
(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*(
((a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e),
 a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a + b)*sin(1/
2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x +
4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*a*sin(2*f*x + 2*e) +
 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^
2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b
)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1
/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*
e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*a*cos(2*f*x
+ 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*
b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x
+ 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2
 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x +
2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*
f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 2*a +
 4*b) - ((a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x
 + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a + b
)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(
4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*(a^2*cos(4*f
*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x +
2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b
 + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2
```

```
*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin
(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x
 + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f*x + 4*e)^2 + a
^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2
 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2
*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x
+ 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*
sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a
+ 2*b)*cos(2*f*x + 2*e) + a)) + 4*a + 4*b))*sqrt(a))/((a^2*cos(4*f*x + 4*e)
^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 +
4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)
*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(
4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*((a^3 + a^2*b)*cos(1
/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x +
 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a^3 + a^2*b)*sin(1/2*arctan
2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2
*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*f)
```

Giac [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(1/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(1/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.280 $\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=\frac{(a-3b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{5/2}f}+\frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}}+\frac{b(a+3b)\tan(e+fx)}{2a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] 1/2*(a-3*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f
+1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/2*b*(a+3*b)*tan
(f*x+e)/a^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4231, 425, 541, 12, 385, 209}

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=\frac{(a-3b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{5/2}f}+\frac{b(a+3b)\tan(e+fx)}{2a^2f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\sin(e+fx)\cos(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[In] Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - 3*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(
2*a^(5/2)*f) + (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*Sqrt[a + b + b*Tan[e + f*
```

```
x]^2]) + (b*(a + 3*b)*Tan[e + f*x])/(2*a^2*(a + b)*f*Sqrt[a + b + b*Tan[e +
 f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{-a+b-2bx^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{b(a+3b)\tan(e+fx)}{2a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{(a-3b)(a+b)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2a^2(a+b)f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{b(a+3b)\tan(e+fx)}{2a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a-3b)\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2a^2f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{b(a+3b)\tan(e+fx)}{2a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a-3b)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^2f}$$

$$= \frac{(a-3b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{5/2}f} + \frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{b(a+3b)\tan(e+fx)}{2a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 15.39 (sec) , antiderivative size = 2059, normalized size of antiderivative = 15.72

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] (3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^6*Sin[e + f*x])/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]
]*(a + b*Sec[e + f*x]^2)^(3/2)*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*Appell
```

F1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*Ap
pellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(
a + b)*AppellF1[3/2, -1, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
b)])*Sin[e + f*x]^2)*((3*a*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]
^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(Sqrt[a + 2*
b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)^2*(3*(a + b)*AppellF1[1/
2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF
1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b
)*AppellF1[3/2, -1, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*
Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5)/(2*Sqrt[a + 2*b + a*Cos[2*(e + f
x)]](a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin
[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -2, 5/2, 5/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1,
3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (
6*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
 + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(
a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*
x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -2, 5/2, 5/2, Sin[e
+ f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 3/2, 5/
2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a +
b)*Cos[e + f*x]^4*Sin[e + f*x]*((a*f*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*
x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (4*f
*AppellF1[3/2, -1, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Co
s[e + f*x]*Sin[e + f*x])/3))/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b
 - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x
]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 3/2, 5/2, Si
n[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*Ap
pellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e
 + f*x]^4*Sin[e + f*x]*(2*f*(3*a*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 3/2, 5/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a +
b)*((a*f*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
+ b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (4*f*AppellF1[3/2, -1, 3/2, 5/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3)
+ Sin[e + f*x]^2*(3*a*((3*a*f*AppellF1[5/2, -2, 7/2, 7/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (12*f*Appel
lF1[5/2, -1, 5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e +
f*x]*Sin[e + f*x])/5) - 4*(a + b)*((9*a*f*AppellF1[5/2, -1, 5/2, 7/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b
)) - (6*f*Cos[e + f*x]*Hypergeometric2F1[3/2, 5/2, 7/2, (a*Sin[e + f*x]^2)/
(a + b)]*Sin[e + f*x])/5))))/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b
 - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x

```
]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 3/2, 5/2, Si
n[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)^2) + (3*a*(a + b
)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*C
os[e + f*x]^4*Sin[e + f*x]*Sin[2*(e + f*x)])/(2*(a + 2*b + a*Cos[2*(e + f*x
)])^(3/2)*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -2, 5/2,
5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2,
-1, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2))
))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(115) = 230$.

Time = 4.13 (sec) , antiderivative size = 809, normalized size of antiderivative = 6.18

method	result
default	$\frac{\left(b+a\cos(fx+e)^2\right)\left(\cos(fx+e)^2\sin(fx+e)\sqrt{-a}\,a^2+\sqrt{-a}\,ab\cos(fx+e)^2\sin(fx+e)+\cos(fx+e)\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+\right.\right.}{}$

```
[In] int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f/(a+b)/a^2/(-a)^(1/2)*(b+a*cos(f*x+e)^2)*(cos(f*x+e)^2*sin(f*x+e)*(-a)
^(1/2)*a^2+(-a)^(1/2)*a*b*cos(f*x+e)^2*sin(f*x+e)+cos(f*x+e)*ln(4*(-a)^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*a^2-2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b*cos
(f*x+e)-3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f
*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)
*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*cos(f*x+e)+(-a)^(1/2)*a
*b*sin(f*x+e)+3*(-a)^(1/2)*b^2*sin(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(
f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e
)*a)*a^2-2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(
f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e
)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b-3*ln(4*(-a)^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*b^2)/(a+b*sec(f*x+e)^2)^(3/2)*sec(f*x+e)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(115) = 230$.

Time = 0.63 (sec) , antiderivative size = 699, normalized size of antiderivative = 5.34

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[\frac{(a^2b-2ab^2-3b^3+(a^3-2a^2b-3ab^2)\cos(fx+e)^2)\sqrt{-a}\log\left(128a^4\cos\ldots\right.}{\ldots}\right.$$

$$-\frac{(a^2b-2ab^2-3b^3+(a^3-2a^2b-3ab^2)\cos(fx+e)^2)\sqrt{a}\arctan\left(\frac{(8a^2\cos(fx+e)^5-8(a^2-ab)\cos(fx+e)^3+(a^2-6\ldots}{4(2a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3\ldots}\right.}{8((a^5+a^4b)f\cos\ldots}$$

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*((a^2*b - 2*a*b^2 - 3*b^3 + (a^3 - 2*a^2*b - 3*a*b^2)*cos(f*x + e)^2)
*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 3
2*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b
^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2
 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 -
14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*
x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))
+ 8*((a^3 + a^2*b)*cos(f*x + e)^3 + (a^2*b + 3*a*b^2)*cos(f*x + e))*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^5 + a^4*b)*f*cos(f*x
 + e)^2 + (a^4*b + a^3*b^2)*f), -1/8*((a^2*b - 2*a*b^2 - 3*b^3 + (a^3 - 2*a
^2*b - 3*a*b^2)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 -
8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b +
a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*((a^3 + a^2*b)*c
os(f*x + e)^3 + (a^2*b + 3*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e))/((a^5 + a^4*b)*f*cos(f*x + e)^2 + (a^4*b + a
^3*b^2)*f)]
```

Sympy [F]

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos^2{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(cos(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos{(fx+e)}^2}{(b\sec{(fx+e)}^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos{(fx+e)}^2}{(b\sec{(fx+e)}^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos{(e+f\,x)}^2}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.281 $\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 194

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{3(a^2-2ab+5b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{7/2}f} + \frac{(3a-5b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a-3b)b(3a+5b)\tan(e+fx)}{8a^3(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] 3/8*(a^2-2*a*b+5*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))
/a^(7/2)/f+1/8*(3*a-5*b)*cos(f*x+e)*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^(
1/2)+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/8*(a-3*b)
*b*(3*a+5*b)*tan(f*x+e)/a^3/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4231, 425, 541, 12, 385, 209}

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{b(a-3b)(3a+5b)\tan(e+fx)}{8a^3f(a+b)\sqrt{a+b\tan^2(e+fx)+b}} + \frac{(3a-5b)\sin(e+fx)\cos(e+fx)}{8a^2f\sqrt{a+b\tan^2(e+fx)+b}} + \frac{3(a^2-2ab+5b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{7/2}f} + \frac{\sin(e+fx)\cos^3(e+fx)}{4af\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[In] Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a^2 - 2*a*b + 5*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e
 + f*x]^2]])/(8*a^(7/2)*f) + ((3*a - 5*b)*Cos[e + f*x]*Sin[e + f*x])/(8*a^2
*f*Sqrt[a + b + b*Tan[e + f*x]^2]) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f*S
qrt[a + b + b*Tan[e + f*x]^2]) + ((a - 3*b)*b*(3*a + 5*b)*Tan[e + f*x])/(8*
a^3*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
```

```
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{-3a+b-4bx^2}{(1+x^2)^2(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= \frac{(3a-5b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{3a^2+5b^2+2(3a-5b)bx^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{8a^2f}$$

$$= \frac{(3a-5b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a-3b)b(3a+5b)\tan(e+fx)}{8a^3(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{3(a+b)(a^2-2ab+5b^2)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{8a^3(a+b)f}$$

$$= \frac{(3a-5b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a-3b)b(3a+5b)\tan(e+fx)}{8a^3(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3(a^2-2ab+5b^2))\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{8a^3f}$$

$$= \frac{(3a-5b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a-3b)b(3a+5b)\tan(e+fx)}{8a^3(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3(a^2-2ab+5b^2))\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^3f}$$

$$= \frac{3(a^2-2ab+5b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{7/2}f} + \frac{(3a-5b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a-3b)b(3a+5b)\tan(e+fx)}{8a^3(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 16.18 (sec) , antiderivative size = 2046, normalized size of antiderivative = 10.55

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
+ b)]*Cos[e + f*x]^10*Sin[e + f*x])/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]
*(a + b*Sec[e + f*x]^2)^(3/2)*(a + b - a*Sin[e + f*x]^2)*((a + b)*AppellF1[
1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF
1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b
)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*
Sin[e + f*x]^2)*((a*(a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*
Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^7*Sin[e + f*x]^2)/(Sqrt[a + 2*b + a*C
os[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)^2*((a + b)*AppellF1[1/2, -3, 3/
2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3,
5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[
3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x
]^2)) + ((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x
]^2)/(a + b)]*Cos[e + f*x]^7)/(2*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b
- a*Sin[e + f*x]^2)*((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a
*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e +
 f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF
1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*
x]^5*Sin[e + f*x]^2)/(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e +
 f*x]^2)*((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*
x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e +
f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a
*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + ((a + b)*Cos[e + f*x]^6*Sin[e
 + f*x]*((a*f*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - 2*f*AppellF1[3/2, -2, 3/2,
5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])
)/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)*((a +
```

```
b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
+ (a*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)
] - 2*(a + b)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)])*Sin[e + f*x]^2)) - ((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e +
 f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^6*Sin[e + f*x]*(2*f*(a*Ap
pellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(
a + b)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
b)])*Cos[e + f*x]*Sin[e + f*x] + (a + b)*((a*f*AppellF1[3/2, -3, 5/2, 5/2,
Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a +
 b) - 2*f*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
 + b)]*Cos[e + f*x]*Sin[e + f*x]) + Sin[e + f*x]^2*(a*((3*a*f*AppellF1[5/2,
 -3, 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin
[e + f*x])/(a + b) - (18*f*AppellF1[5/2, -2, 5/2, 7/2, Sin[e + f*x]^2, (a*S
in[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/5) - 2*(a + b)*((9*a*f*A
ppellF1[5/2, -2, 5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[
e + f*x]*Sin[e + f*x])/(5*(a + b)) - (12*f*AppellF1[5/2, -1, 3/2, 7/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/5))))/(2
*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)*((a + b)*A
ppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a
*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] -
2*(a + b)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
 + b)])*Sin[e + f*x]^2)^2) + (a*(a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e +
 f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^6*Sin[e + f*x]*Sin[2*(e +
 f*x)])/(2*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(a + b - a*Sin[e + f*x]^2)*
((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
+ b)] + (a*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)] - 2*(a + b)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e +
f*x]^2)/(a + b)])*Sin[e + f*x]^2))))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1142 vs. $2(174) = 348$.

Time = 6.40 (sec) , antiderivative size = 1143, normalized size of antiderivative = 5.89

method	result	size
default	Expression too large to display	1143

```
[In] int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f/(a+b)/a^3/(-a)^(1/2)*(b+a*cos(f*x+e)^2)*(2*(-a)^(1/2)*a^3*cos(f*x+e)^
4*sin(f*x+e)+2*(-a)^(1/2)*a^2*b*cos(f*x+e)^4*sin(f*x+e)+3*(-a)^(1/2)*a^3*co
s(f*x+e)^2*sin(f*x+e)-2*(-a)^(1/2)*a^2*b*cos(f*x+e)^2*sin(f*x+e)-5*(-a)^(1/
2)*a*b^2*cos(f*x+e)^2*sin(f*x+e)+3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^
```

```
3*cos(f*x+e)-3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f
*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b*cos(f*x+e)+9*ln(
4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^
(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^2*cos(f*x+e)+15*ln(4*(-a)^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*b^3*cos(f*x+e)+3*(-a)^(1/2)*a^2*b*sin(f*x+e)-4*(-a)^(1/2)
*a*b^2*sin(f*x+e)-15*(-a)^(1/2)*b^3*sin(f*x+e)+3*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*si
n(f*x+e)*a)*a^3-3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2*b+9*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)-4*sin(f*x+e)*a)*a*b^2+15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)
+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b
^3)/(a+b*sec(f*x+e)^2)^(3/2)*sec(f*x+e)^3
```

Fricas [A] (verification not implemented)

none

Time = 1.48 (sec) , antiderivative size = 811, normalized size of antiderivative = 4.18

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[-\frac{3\left(a^3b-a^2b^2+3\,ab^3+5\,b^4+\left(a^4-a^3b+3\,a^2b^2+5\,ab^3\right)\cos\left(fx+e\right)^2\right)\sqrt{-}}{}\right.$$

$$-\frac{3\left(a^3b-a^2b^2+3\,ab^3+5\,b^4+\left(a^4-a^3b+3\,a^2b^2+5\,ab^3\right)\cos\left(fx+e\right)^2\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\,(a^2-ab)\,c\right.}{4\left(2\,a^3\cos(fx+e)^4\right.}\right.}{}$$

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(3*(a^3*b - a^2*b^2 + 3*a*b^3 + 5*b^4 + (a^4 - a^3*b + 3*a^2*b^2 + 5
*a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^
```

3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(2*(a^4 + a^3*b)*cos(f*x + e)^5 + (3*a^4 - 2*a^3*b - 5*a^2*b^2)*cos(f*x + e)^3 + (3*a^3*b - 4*a^2*b^2 - 15*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^6 + a^5*b)*f*cos(f*x + e)^2 + (a^5*b + a^4*b^2)*f), -1/32*(3*(a^3*b - a^2*b^2 + 3*a*b^3 + 5*b^4 + (a^4 - a^3*b + 3*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2*(a^4 + a^3*b)*cos(f*x + e)^5 + (3*a^4 - 2*a^3*b - 5*a^2*b^2)*cos(f*x + e)^3 + (3*a^3*b - 4*a^2*b^2 - 15*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^6 + a^5*b)*f*cos(f*x + e)^2 + (a^5*b + a^4*b^2)*f)]

Sympy [F]

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

[In] integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)

[Out] Integral(cos(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)

Maxima [F]

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)^4}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate(cos(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)

Giac [F]

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{\cos(fx+e)^4}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{\cos(e+f\,x)^4}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.282 $\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 271

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(5a^3-9a^2b+15ab^2-35b^3)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{9/2}f} + \frac{(15a^2-22ab+35b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5a-7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{b(15a^3-17a^2b+25ab^2+105b^3)\tan(e+fx)}{48a^4(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] 1/16*(5*a^3-9*a^2*b+15*a*b^2-35*b^3)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f
*x+e)^2)^(1/2))/a^(9/2)/f+1/48*(15*a^2-22*a*b+35*b^2)*cos(f*x+e)*sin(f*x+e)
/a^3/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/24*(5*a-7*b)*cos(f*x+e)^3*sin(f*x+e)/a^
2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/6*cos(f*x+e)^5*sin(f*x+e)/a/f/(a+b+b*tan(f
*x+e)^2)^(1/2)+1/48*b*(15*a^3-17*a^2*b+25*a*b^2+105*b^3)*tan(f*x+e)/a^4/(a+
b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used

$= \{4231, 425, 541, 12, 385, 209\}$

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(5a-7b)\sin(e+fx)\cos^3(e+fx)}{24a^2f\sqrt{a+b\tan^2(e+fx)+b}} + \frac{(15a^2-22ab+35b^2)\sin(e+fx)\cos(e+fx)}{48a^3f\sqrt{a+b\tan^2(e+fx)+b}} + \frac{(5a^3-9a^2b+15ab^2-35b^3)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{9/2}f} + \frac{b(15a^3-17a^2b+25ab^2+105b^3)\tan(e+fx)}{48a^4f(a+b)\sqrt{a+b\tan^2(e+fx)+b}} + \frac{\sin(e+fx)\cos^5(e+fx)}{6af\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[In] Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((5*a^3 - 9*a^2*b + 15*a*b^2 - 35*b^3)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a
 + b + b*Tan[e + f*x]^2]])/(16*a^(9/2)*f) + ((15*a^2 - 22*a*b + 35*b^2)*Cos
[e + f*x]*Sin[e + f*x])/(48*a^3*f*Sqrt[a + b + b*Tan[e + f*x]^2]) + ((5*a -
 7*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*a^2*f*Sqrt[a + b + b*Tan[e + f*x]^2]
) + (Cos[e + f*x]^5*Sin[e + f*x])/(6*a*f*Sqrt[a + b + b*Tan[e + f*x]^2]) +
(b*(15*a^3 - 17*a^2*b + 25*a*b^2 + 105*b^3)*Tan[e + f*x])/(48*a^4*(a + b)*f
*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^4(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\cos^5(e+fx)\sin(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{-5a+b-6bx^2}{(1+x^2)^3(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{6af} \\
&= \frac{(5a-7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} \\
&\quad + \frac{\text{Subst}\left(\int \frac{15a^2-2ab+7b^2+4(5a-7b)bx^2}{(1+x^2)^2(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{24a^2f} \\
&= \frac{(15a^2-22ab+35b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} \\
&\quad + \frac{(5a-7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} \\
&\quad - \frac{\text{Subst}\left(\int \frac{-15a^3-3a^2b-ab^2+35b^3-2b(15a^2-22ab+35b^2)x^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{48a^3f}
\end{aligned}$$

$$= \frac{(15a^2 - 22ab + 35b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5a-7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{b(15a^3 - 17a^2b + 25ab^2 + 105b^3)\tan(e+fx)}{48a^4(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int -\frac{3(a+b)(5a^3-9a^2b+15ab^2-35b^3)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{48a^4(a+b)f}$$

$$= \frac{(15a^2 - 22ab + 35b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5a-7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{b(15a^3 - 17a^2b + 25ab^2 + 105b^3)\tan(e+fx)}{48a^4(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5a^3 - 9a^2b + 15ab^2 - 35b^3)\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{16a^4f}$$

$$= \frac{(15a^2 - 22ab + 35b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5a-7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{b(15a^3 - 17a^2b + 25ab^2 + 105b^3)\tan(e+fx)}{48a^4(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5a^3 - 9a^2b + 15ab^2 - 35b^3)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^4f}$$

$$= \frac{(5a^3 - 9a^2b + 15ab^2 - 35b^3)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{9/2}f} + \frac{(15a^2 - 22ab + 35b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5a-7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{b(15a^3 - 17a^2b + 25ab^2 + 105b^3)\tan(e+fx)}{48a^4(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 17.72 (sec) , antiderivative size = 2068, normalized size of antiderivative = 7.63

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^14*Sin[e + f*x])/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)
]]*(a + b*Sec[e + f*x]^2)^(3/2)*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*Appel
lF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*A
ppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*
(a + b)*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)])*Sin[e + f*x]^2)*((3*a*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x
]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^9*Sin[e + f*x]^2)/(Sqrt[a + 2
*b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)^2*(3*(a + b)*AppellF1[1
/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*Appell
F1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a +
b)*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])
*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^9)/(2*Sqrt[a + 2*b + a*Cos[2*(e +
f*x)]]*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Si
n[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -4, 5/2, 5/2
, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3,
 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) -
(12*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^7*Sin[e + f*x]^2)/(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]
*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e +
f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -4, 5/2, 5/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 3/2,
5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a
+ b)*Cos[e + f*x]^8*Sin[e + f*x]*((a*f*AppellF1[3/2, -4, 5/2, 5/2, Sin[e +
f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (8
*f*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*
Cos[e + f*x]*Sin[e + f*x])/3))/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a +
 b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^
2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f
*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 3/2, 5/2,
Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*
AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos
[e + f*x]^8*Sin[e + f*x]*(2*f*(3*a*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]
^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 3/2, 5/2, Sin
[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a
+ b)*((a*f*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (8*f*AppellF1[3/2, -3, 3/2, 5/
2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3
) + Sin[e + f*x]^2*(3*a*((3*a*f*AppellF1[5/2, -4, 7/2, 7/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (24*f*App
ellF1[5/2, -3, 5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e
+ f*x]*Sin[e + f*x])/5) - 8*(a + b)*((9*a*f*AppellF1[5/2, -3, 5/2, 7/2, Sin
[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a +
 b)) - (18*f*AppellF1[5/2, -2, 3/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)
```

```
/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/5))))/(2*f*Sqrt[a + 2*b + a*Cos[2*(e +
 f*x)]]*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, S
in[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -4, 5/2, 5/
2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3
, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)^2)
 + (3*a*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]
^2)/(a + b)]*Cos[e + f*x]^8*Sin[e + f*x]*Sin[2*(e + f*x)])/(2*(a + 2*b + a*
Cos[2*(e + f*x)])^(3/2)*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2,
 -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[
3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*
AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Si
n[e + f*x]^2))))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1498 vs. $2(247) = 494$.

Time = 9.78 (sec) , antiderivative size = 1499, normalized size of antiderivative = 5.53

method	result	size
default	Expression too large to display	1499

```
[In] int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/f/a^4/(-a)^(1/2)/(a+b)*(b+a*cos(f*x+e)^2)*(8*(-a)^(1/2)*a^4*cos(f*x+e)
^6*sin(f*x+e)+8*(-a)^(1/2)*a^3*b*cos(f*x+e)^6*sin(f*x+e)+10*(-a)^(1/2)*a^4*
cos(f*x+e)^4*sin(f*x+e)-4*(-a)^(1/2)*a^3*b*cos(f*x+e)^4*sin(f*x+e)-14*(-a)^
(1/2)*a^2*b^2*cos(f*x+e)^4*sin(f*x+e)+15*(-a)^(1/2)*a^4*cos(f*x+e)^2*sin(f*
x+e)-7*(-a)^(1/2)*a^3*b*cos(f*x+e)^2*sin(f*x+e)+13*(-a)^(1/2)*a^2*b^2*cos(f
*x+e)^2*sin(f*x+e)+35*(-a)^(1/2)*a*b^3*cos(f*x+e)^2*sin(f*x+e)+15*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)-4*sin(f*x+e)*a)*a^4*cos(f*x+e)-12*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(
f*x+e)*a)*a^3*b*cos(f*x+e)+18*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*l
n(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a
)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2*b^2
*cos(f*x+e)-60*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^3*cos(f*x+e)-105*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^4*cos(f*x+e)+15*(-a)^(1/2)*a^3*b*sin
(f*x+e)-17*(-a)^(1/2)*a^2*b^2*sin(f*x+e)+25*(-a)^(1/2)*a*b^3*sin(f*x+e)+105
*(-a)^(1/2)*b^4*sin(f*x+e)+15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*l
```

```
n(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a
)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^4-12*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3*b+18*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(
f*x+e)*a)*a^2*b^2-60*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^
(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^3-105*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^4)/(a+b*sec(f*x+e)^2)^(3/2)*sec(f*x+e)^3
```

Fricas [A] (verification not implemented)

none

Time = 4.39 (sec) , antiderivative size = 941, normalized size of antiderivative = 3.47

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/384*(3*(5*a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 20*a*b^4 - 35*b^5 + (5*a^5 - 4
*a^4*b + 6*a^3*b^2 - 20*a^2*b^3 - 35*a*b^4)*cos(f*x + e)^2)*sqrt(-a)*log(12
8*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos
(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b
^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*(a^5 + a^
4*b)*cos(f*x + e)^7 + 2*(5*a^5 - 2*a^4*b - 7*a^3*b^2)*cos(f*x + e)^5 + (15*
a^5 - 7*a^4*b + 13*a^3*b^2 + 35*a^2*b^3)*cos(f*x + e)^3 + (15*a^4*b - 17*a^
3*b^2 + 25*a^2*b^3 + 105*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e))/((a^7 + a^6*b)*f*cos(f*x + e)^2 + (a^6*b + a^5
*b^2)*f), -1/192*(3*(5*a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 20*a*b^4 - 35*b^5 +
(5*a^5 - 4*a^4*b + 6*a^3*b^2 - 20*a^2*b^3 - 35*a*b^4)*cos(f*x + e)^2)*sqrt(
a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 -
 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^
2)*sin(f*x + e))) - 4*(8*(a^5 + a^4*b)*cos(f*x + e)^7 + 2*(5*a^5 - 2*a^4*b
- 7*a^3*b^2)*cos(f*x + e)^5 + (15*a^5 - 7*a^4*b + 13*a^3*b^2 + 35*a^2*b^3)*
cos(f*x + e)^3 + (15*a^4*b - 17*a^3*b^2 + 25*a^2*b^3 + 105*a*b^4)*cos(f*x +
```

```
e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 + a^6*
b)*f*cos(f*x + e)^2 + (a^6*b + a^5*b^2)*f)]
```

Sympy [F]

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos^6{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(cos(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos{(fx+e)}^6}{(b\sec{(fx+e)}^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos{(fx+e)}^6}{(b\sec{(fx+e)}^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos{(e+f\,x)}^6}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.283 $\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 321

$$\begin{aligned}\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx &= -\frac{2a(a+2b)\sin(e+fx)}{3b^2(a+b)^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\ &- \frac{a\sin(e+fx)}{3b(a+b)f\,(a+b-a\sin^2(e+fx))\,\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\ &+ \frac{2(a+2b)E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{3b^2(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} \\ &- \frac{\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{3b(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\end{aligned}$$

```
[Out] -2/3*a*(a+2*b)*sin(f*x+e)/b^2/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))
^(1/2)-1/3*a*sin(f*x+e)/b/(a+b)/f/(a+b-a*sin(f*x+e)^2)/(sec(f*x+e)^2*(a+b-a
*sin(f*x+e)^2))^(1/2)+2/3*(a+2*b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+
b-a*sin(f*x+e)^2)/b^2/(a+b)^2/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*s
in(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/3*EllipticF(sin(f*x+e)
,(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/b/(a+b)/f/(cos(f*x+e)^2)^(
1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4233, 1985, 1986, 425, 541, 538, 437, 435, 432, 430}

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{2(a+2b)\left(-a\sin^2(e+fx)+a+b\right)E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{3b^2f(a+b)^2\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)\right.}}$$

$$-\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{3bf(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

$$-\frac{2a(a+2b)\sin(e+fx)}{3b^2f(a+b)^2\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

$$-\frac{a\sin(e+fx)}{3bf(a+b)\left(-a\sin^2(e+fx)+a+b\right)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

```
[In] Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (-2*a*(a + 2*b)*Sin[e + f*x])/(3*b^2*(a + b)^2*f*Sqrt[Sec[e + f*x]^2*(a + b
 - a*Sin[e + f*x]^2)]) - (a*Sin[e + f*x])/(3*b*(a + b)*f*(a + b - a*Sin[e +
 f*x]^2)*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) + (2*(a + 2*b)*El
lipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a + b - a*Sin[e + f*x]^2))/(3*b^2
*(a + b)^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*
x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (EllipticF[ArcSin[Sin[e + f*
x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*b*(a + b)*f*Sqrt[C
os[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3\left(a+\frac{b}{1-x^2}\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3\left(\frac{a+b-ax^2}{1-x^2}\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}(a+b-ax^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= -\frac{a\sin(e+fx)}{3b(a+b)f\,(a+b-a\sin^2(e+fx))\,\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} - \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{-a-3b-ax^2}{\sqrt{1-x^2}(a+b-ax^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{3b(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= -\frac{2a(a+2b)\sin(e+fx)}{3b^2(a+b)^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} - \frac{a\sin(e+fx)}{3b(a+b)f\,(a+b-a\sin^2(e+fx))\,\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{(a+b)(2a+3b)-2a(a+2b)x^2}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{3b^2(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= -\frac{2a(a+2b)\sin(e+fx)}{3b^2(a+b)^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} - \frac{a\sin(e+fx)}{3b(a+b)f\,(a+b-a\sin^2(e+fx))\,\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} - \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{3b(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{\left(2(a+2b)\sqrt{a+b-a\sin^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{\sqrt{1-x^2}}\,dx, x, \sin(e+fx)\right)}{3b^2(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$
\begin{aligned}
&= -\frac{2a(a+2b)\sin(e+fx)}{3b^2(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} \\
&- \frac{a\sin(e+fx)}{3b(a+b)f\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} \\
&+ \frac{\left(2(a+2b)\left(a+b-a\sin^2(e+fx)\right)\right)\mathrm{Subst}\left(\int\frac{\sqrt{1-\frac{ax^2}{a+b}}}{\sqrt{1-x^2}}\,dx,x,\sin(e+fx)\right)}{3b^2(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} \\
&- \frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\mathrm{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{ax^2}{a+b}}}\,dx,x,\sin(e+fx)\right)}{3b(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} \\
&= -\frac{2a(a+2b)\sin(e+fx)}{3b^2(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} \\
&- \frac{a\sin(e+fx)}{3b(a+b)f\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} \\
&+ \frac{2(a+2b)E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\left(a+b-a\sin^2(e+fx)\right)}{3b^2(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} \\
&- \frac{\mathrm{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{3b(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 4.18 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.52

$$
\int\frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^5(e+fx)\left(\sqrt{2}(a+b)^2\left(\frac{a+2b+a\cos(2(e+fx))}{a+b}\right)^{3/2}\right.}{}
$$

```
[In] Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^5*(Sqrt[2]*(a + b)^2*((a + 2*b
 + a*Cos[2*(e + f*x)])/(a + b))^(3/2)*(2*(a + 2*b)*EllipticE[e + f*x, a/(a
+ b)] - b*EllipticF[e + f*x, a/(a + b)]) - 2*a*(a^2 + 5*a*b + 5*b^2 + a*(a
+ 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]))/(24*b^2*(a + b)^2*f*(a + b*Sec[
e + f*x]^2)^(5/2))
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.01 (sec) , antiderivative size = 16966, normalized size of antiderivative = 52.85

method	result	size
default	Expression too large to display	16966

```
[In] int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 1374, normalized size of antiderivative = 4.28

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
[Out] 1/3*((2*(-I*a^3*b^2 - 2*I*a^2*b^3 + (-I*a^5 - 2*I*a^4*b)*cos(f*x + e)^4 - 2
*(I*a^4*b + 2*I*a^3*b^2)*cos(f*x + e)^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) - (-
I*a^3*b^2 - 4*I*a^2*b^3 - 4*I*a*b^4 + (-I*a^5 - 4*I*a^4*b - 4*I*a^3*b^2)*co
s(f*x + e)^4 + 2*(-I*a^4*b - 4*I*a^3*b^2 - 4*I*a^2*b^3)*cos(f*x + e)^2)*sqr
t(a))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt(
(2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))),
(a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*(I*
a^3*b^2 + 2*I*a^2*b^3 + (I*a^5 + 2*I*a^4*b)*cos(f*x + e)^4 - 2*(-I*a^4*b -
2*I*a^3*b^2)*cos(f*x + e)^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) - (I*a^3*b^2 + 4
*I*a^2*b^3 + 4*I*a*b^4 + (I*a^5 + 4*I*a^4*b + 4*I*a^3*b^2)*cos(f*x + e)^4 +
 2*(I*a^4*b + 4*I*a^3*b^2 + 4*I*a^2*b^3)*cos(f*x + e)^2)*sqrt(a))*sqrt((2*a
*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b
+ b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b +
8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*(-I*a^2*b^3 - 3*I*
a*b^4 + (-I*a^4*b - 3*I*a^3*b^2)*cos(f*x + e)^4 - 2*(I*a^3*b^2 + 3*I*a^2*b^
3)*cos(f*x + e)^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) - (2*I*a^3*b^2 + 9*I*a^2*b
^3 + 13*I*a*b^4 + 6*I*b^5 + (2*I*a^5 + 9*I*a^4*b + 13*I*a^3*b^2 + 6*I*a^2*b
^3)*cos(f*x + e)^4 + 2*(2*I*a^4*b + 9*I*a^3*b^2 + 13*I*a^2*b^3 + 6*I*a*b^4)
*cos(f*x + e)^2)*sqrt(a))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*ell
iptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e)
+ I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)
/a^2))/a^2) + (2*(I*a^2*b^3 + 3*I*a*b^4 + (I*a^4*b + 3*I*a^3*b^2)*cos(f*x +
 e)^4 - 2*(-I*a^3*b^2 - 3*I*a^2*b^3)*cos(f*x + e)^2)*sqrt(a)*sqrt((a*b + b^
```

```
2)/a^2) - (-2*I*a^3*b^2 - 9*I*a^2*b^3 - 13*I*a*b^4 - 6*I*b^5 + (-2*I*a^5 -
9*I*a^4*b - 13*I*a^3*b^2 - 6*I*a^2*b^3)*cos(f*x + e)^4 + 2*(-2*I*a^4*b - 9*
I*a^3*b^2 - 13*I*a^2*b^3 - 6*I*a*b^4)*cos(f*x + e)^2)*sqrt(a))*sqrt((2*a*sq
rt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b
^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b
^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - (2*(a^5 + 2*a^4*b)*cos(f
*x + e)^4 + (3*a^4*b + 5*a^3*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*sin(f*x + e))/((a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*f*cos(f*x
 + e)^4 + 2*(a^5*b^3 + 2*a^4*b^4 + a^3*b^5)*f*cos(f*x + e)^2 + (a^4*b^4 + 2
*a^3*b^5 + a^2*b^6)*f)
```

Sympy [F]

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\sec^5{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(sec(e + f*x)**5/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\sec{(fx+e)}^5}{(b\sec{(fx+e)}^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\sec{(fx+e)}^5}{(b\sec{(fx+e)}^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\cos(e+f\,x)^5\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

```
[In] int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(5/2)),x)
[Out] int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

3.284 $\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 319

$$\begin{aligned}\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx &= -\frac{(a-b)\sin(e+fx)}{3b(a+b)^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\\ &+\frac{\sin(e+fx)}{3(a+b)f\,(a+b-a\sin^2(e+fx))\,\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\\ &+\frac{(a-b)E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{3ab(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}\\ &+\frac{\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{3a(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\end{aligned}$$

```
[Out] -1/3*(a-b)*sin(f*x+e)/b/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
+1/3*sin(f*x+e)/(a+b)/f/(a+b-a*sin(f*x+e)^2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e
)^2))^(1/2)+1/3*(a-b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+
e)^2)/a/b/(a+b)^2/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2)
)^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)+1/3*EllipticF(sin(f*x+e),(a/(a+b))^(
1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a/(a+b)/f/(cos(f*x+e)^2)^(1/2)/(sec(f*
x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4233, 1985, 1986, 423, 541, 538, 437, 435, 432, 430}

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{3af(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$
$$+\frac{(a-b)\left(-a\sin^2(e+fx)+a+b\right)E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{3abf(a+b)^2\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$
$$-\frac{(a-b)\sin(e+fx)}{3bf(a+b)^2\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$
$$+\frac{\sin(e+fx)}{3f(a+b)\left(-a\sin^2(e+fx)+a+b\right)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

```
[In] Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/3*((a - b)*Sin[e + f*x])/(b*(a + b)^2*f*Sqrt[Sec[e + f*x]^2*(a + b - a*S
in[e + f*x]^2)]) + Sin[e + f*x]/(3*(a + b)*f*(a + b - a*Sin[e + f*x]^2)*Sqr
t[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) + ((a - b)*EllipticE[ArcSin[S
in[e + f*x]], a/(a + b)]*(a + b - a*Sin[e + f*x]^2))/(3*a*b*(a + b)^2*f*Sqr
t[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 -
(a*Sin[e + f*x]^2)/(a + b)]) + (EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*
Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*a*(a + b)*f*Sqrt[Cos[e + f*x]^2]*S
qrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2\left(a+\frac{b}{1-x^2}\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2\left(\frac{a+b-ax^2}{1-x^2}\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{a+b-a\sin^2(e+fx)}\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{(a+b-ax^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= \frac{\sin(e+fx)}{3(a+b)f\,(a+b-a\sin^2(e+fx))\,\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} - \frac{\sqrt{a+b-a\sin^2(e+fx)}\text{Subst}\left(\int \frac{-2+x^2}{\sqrt{1-x^2}(a+b-ax^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{3(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= -\frac{(a-b)\sin(e+fx)}{3b(a+b)^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{\sin(e+fx)}{3(a+b)f\,(a+b-a\sin^2(e+fx))\,\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{\sqrt{a+b-a\sin^2(e+fx)}\text{Subst}\left(\int \frac{a+b+(-a+b)x^2}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{3b(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= -\frac{(a-b)\sin(e+fx)}{3b(a+b)^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{\sin(e+fx)}{3(a+b)f\,(a+b-a\sin^2(e+fx))\,\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} - \frac{\left((-a+b)\sqrt{a+b-a\sin^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{\sqrt{1-x^2}}\,dx, x, \sin(e+fx)\right)}{3ab(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{\sqrt{a+b-a\sin^2(e+fx)}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{3a(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= -\frac{(a-b)\sin(e+fx)}{3b(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{\sin(e+fx)}{3(a+b)f\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{\left((-a+b)\left(a+b-a\sin^2(e+fx)\right)\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{ax^2}{a+b}}}{\sqrt{1-x^2}}\,dx, x, \sin(e+fx)\right)}{3ab(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} + \frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{ax^2}{a+b}}}\,dx, x, \sin(e+fx)\right)}{3a(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

$$= -\frac{(a-b)\sin(e+fx)}{3b(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{\sin(e+fx)}{3(a+b)f\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{(a-b)E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)\left(a+b-a\sin^2(e+fx)\right)}{3ab(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} + \frac{\text{EllipticF}\left(\arcsin(\sin(e+fx)), \frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{3a(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 22.18 (sec) , antiderivative size = 1204, normalized size of antiderivative = 3.77

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{(a+2b+a\cos(2e+2fx))^{5/2}\sec^5(e+fx)\left(-\frac{\left(-2\sqrt{-\frac{1}{b}}(-a-a\cos(2e+2fx))(2a^2(a-\right.}{}\right.}{}$$

```
[In] Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5*(-1/24*((-2*Sqrt[-b^(-
1)]*(-a - a*Cos[2*e + 2*f*x])*(2*a^2*(a + 3*b + a*Cos[2*e + 2*f*x]) + b*(2*
b^2 + 3*b*(a + 2*b + a*Cos[2*e + 2*f*x]) - 2*(a + 2*b + a*Cos[2*e + 2*f*x])
^2) + a*(4*b^2 + 5*b*(a + 2*b + a*Cos[2*e + 2*f*x]) - (a + 2*b + a*Cos[2*e
+ 2*f*x])^2)) + (2*I)*(a^2 + 3*a*b + 2*b^2)*Sqrt[(a - a*Cos[2*e + 2*f*x])/(
a + b)]*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sqrt[4 - (2*(a + 2*b + a*Cos[2
*e + 2*f*x]))/b]*EllipticE[I*ArcSinh[(Sqrt[-b^(-1)]*Sqrt[a + 2*b + a*Cos[2*
e + 2*f*x]])/Sqrt[2]], b/(a + b)] - I*(2*a^2 + 5*a*b + 3*b^2)*(a + 2*b + a*
```

```
Cos[2*e + 2*f*x])^(3/2)*Sqrt[(4*a + 4*b - 2*(a + 2*b + a*Cos[2*e + 2*f*x]))
/(a + b)]*Sqrt[2 - (a + 2*b + a*Cos[2*e + 2*f*x])/b]*EllipticF[I*ArcSinh[(S
qrt[-b^(-1)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], b/(a + b)])*Sin[
2*e + 2*f*x])/(a*Sqrt[-b^(-1)]*b^2*(a + b)^2*f*Sqrt[((a - a*Cos[2*e + 2*f*x
])*(a + a*Cos[2*e + 2*f*x]))/a^2]*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sqrt
[1 - Cos[2*e + 2*f*x]^2]) + (Cos[2*(e + f*x)]*(-2*Sqrt[-b^(-1)]*(-a - a*Cos
[2*e + 2*f*x])*(4*b^4 - b^2*(a + 2*b + a*Cos[2*e + 2*f*x])^2 + 2*a^3*(a + 3
*b + a*Cos[2*e + 2*f*x]) + a*b*(10*b^2 + b*(a + 2*b + a*Cos[2*e + 2*f*x]) -
 (a + 2*b + a*Cos[2*e + 2*f*x])^2) + a^2*(8*b^2 + 3*b*(a + 2*b + a*Cos[2*e
+ 2*f*x]) - (a + 2*b + a*Cos[2*e + 2*f*x])^2)) + (2*I)*(a^3 + 2*a^2*b + 2*a
*b^2 + b^3)*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sqrt[(4*a + 4*b - 2*(a + 2
*b + a*Cos[2*e + 2*f*x]))/(a + b)]*Sqrt[2 - (a + 2*b + a*Cos[2*e + 2*f*x])/
b]*EllipticE[I*ArcSinh[(Sqrt[-b^(-1)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])/S
qrt[2]], b/(a + b)] - I*a*(2*a^2 + 3*a*b + b^2)*(a + 2*b + a*Cos[2*e + 2*f*
x])^(3/2)*Sqrt[(4*a + 4*b - 2*(a + 2*b + a*Cos[2*e + 2*f*x]))/(a + b)]*Sqrt
[2 - (a + 2*b + a*Cos[2*e + 2*f*x])/b]*EllipticF[I*ArcSinh[(Sqrt[-b^(-1)]*S
qrt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], b/(a + b)])*Sec[2*(e + (-2*e +
 ArcCos[Cos[2*e + 2*f*x]])/2)]*Sin[2*e + 2*f*x])/(24*a^2*Sqrt[-b^(-1)]*b^2*
(a + b)^2*f*Sqrt[((a - a*Cos[2*e + 2*f*x])*(a + a*Cos[2*e + 2*f*x]))/a^2]*(
a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sqrt[1 - Cos[2*e + 2*f*x]^2])))/(2*(a +
 b*Sec[e + f*x]^2)^(5/2))
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.09 (sec) , antiderivative size = 12063, normalized size of antiderivative = 37.82

method	result	size
default	Expression too large to display	12063

```
[In] int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 1244, normalized size of antiderivative = 3.90

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*((2*((-I*a^4 + I*a^3*b)*cos(f*x + e)^4 - I*a^2*b^2 + I*a*b^3 - 2*(I*a^3
*b - I*a^2*b^2)*cos(f*x + e)^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) - ((-I*a^4 -
I*a^3*b + 2*I*a^2*b^2)*cos(f*x + e)^4 - I*a^2*b^2 - I*a*b^3 + 2*I*b^4 + 2*(
-I*a^3*b - I*a^2*b^2 + 2*I*a*b^3)*cos(f*x + e)^2)*sqrt(a))*sqrt((2*a*sqrt((
a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/
a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 +
 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*((I*a^4 - I*a^3*b)*cos(f*
x + e)^4 + I*a^2*b^2 - I*a*b^3 - 2*(-I*a^3*b + I*a^2*b^2)*cos(f*x + e)^2)*s
qrt(a)*sqrt((a*b + b^2)/a^2) - ((I*a^4 + I*a^3*b - 2*I*a^2*b^2)*cos(f*x + e
)^4 + I*a^2*b^2 + I*a*b^3 - 2*I*b^4 + 2*(I*a^3*b + I*a^2*b^2 - 2*I*a*b^3)*c
os(f*x + e)^2)*sqrt(a))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*ellip
tic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) -
I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a
^2))/a^2) - 2*(4*(I*a^3*b*cos(f*x + e)^4 + 2*I*a^2*b^2*cos(f*x + e)^2 + I*a
*b^3)*sqrt(a)*sqrt((a*b + b^2)/a^2) + ((I*a^4 + 3*I*a^3*b + 2*I*a^2*b^2)*co
s(f*x + e)^4 + I*a^2*b^2 + 3*I*a*b^3 + 2*I*b^4 + 2*(I*a^3*b + 3*I*a^2*b^2 +
 2*I*a*b^3)*cos(f*x + e)^2)*sqrt(a))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a -
2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(co
s(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt(
(a*b + b^2)/a^2))/a^2) - 2*(4*(-I*a^3*b*cos(f*x + e)^4 - 2*I*a^2*b^2*cos(f*
x + e)^2 - I*a*b^3)*sqrt(a)*sqrt((a*b + b^2)/a^2) + ((-I*a^4 - 3*I*a^3*b -
2*I*a^2*b^2)*cos(f*x + e)^4 - I*a^2*b^2 - 3*I*a*b^3 - 2*I*b^4 + 2*(-I*a^3*b
 - 3*I*a^2*b^2 - 2*I*a*b^3)*cos(f*x + e)^2)*sqrt(a))*sqrt((2*a*sqrt((a*b +
b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) -
 a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^
2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + 2*(2*a^2*b^2*cos(f*x + e)^2 - (a^4
 - a^3*b)*cos(f*x + e)^4)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f
*x + e))/((a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^4 + 2*(a^5*b^2 + 2*a
^4*b^3 + a^3*b^4)*f*cos(f*x + e)^2 + (a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*f)
```

Sympy [F]

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\sec^3{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)**3/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec(fx+e)^3}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec(fx+e)^3}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\cos(e+f\,x)^3\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

```
[In] int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(5/2)),x)
[Out] int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

3.285 $\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 327

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{2(2a+b)\sin(e+fx)}{3a(a+b)^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} - \frac{b\sin(e+fx)}{3a(a+b)f\,(a+b-a\sin^2(e+fx))\,\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} - \frac{2(2a+b)E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{3a^2(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} + \frac{(3a+2b)\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{3a^2(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

```
[Out] 2/3*(2*a+b)*sin(f*x+e)/a/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2
)-1/3*b*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*x+e)^2)/(sec(f*x+e)^2*(a+b-a*sin(
f*x+e)^2))^(1/2)-2/3*(2*a+b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*s
in(f*x+e)^2)/a^2/(a+b)^2/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*
x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)+1/3*(3*a+2*b)*EllipticF(sin(f
*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a^2/(a+b)/f/(cos(f*x+
e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4233, 1985, 1986, 424, 541, 538, 437, 435, 432, 430}

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{(3a+2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{3a^2f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}} - \frac{2(2a+b)\left(-a\sin^2(e+fx)+a+b\right)E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{3a^2f(a+b)^2\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}} + \frac{2(2a+b)\sin(e+fx)}{3af(a+b)^2\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}} - \frac{b\sin(e+fx)}{3af(a+b)\left(-a\sin^2(e+fx)+a+b\right)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

```
[In] Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (2*(2*a + b)*Sin[e + f*x])/(3*a*(a + b)^2*f*Sqrt[Sec[e + f*x]^2*(a + b - a*
Sin[e + f*x]^2)]) - (b*Sin[e + f*x])/(3*a*(a + b)*f*(a + b - a*Sin[e + f*x]
^2)*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) - (2*(2*a + b)*Ellipti
cE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a + b - a*Sin[e + f*x]^2))/(3*a^2*(a +
 b)^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)
]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + ((3*a + 2*b)*EllipticF[ArcSin[Sin
[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*a^2*(a + b)
*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a+\frac{b}{1-x^2}\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(\frac{a+b-ax^2}{1-x^2}\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{3/2}}{(a+b-ax^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\\
&= -\frac{b\sin(e+fx)}{3a(a+b)f\,(a+b-a\sin^2(e+fx))\,\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\\
&\quad -\frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{-3a-b+(3a+2b)x^2}{\sqrt{1-x^2}(a+b-ax^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{3a(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\\
&= \frac{2(2a+b)\sin(e+fx)}{3a(a+b)^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\\
&\quad -\frac{b\sin(e+fx)}{3a(a+b)f\,(a+b-a\sin^2(e+fx))\,\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\\
&\quad +\frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{-b(a+b)+2b(2a+b)x^2}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{3ab(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\\
&= \frac{2(2a+b)\sin(e+fx)}{3a(a+b)^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\\
&\quad -\frac{b\sin(e+fx)}{3a(a+b)f\,(a+b-a\sin^2(e+fx))\,\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\\
&\quad -\frac{\left(2(2a+b)\sqrt{a+b-a\sin^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{\sqrt{1-x^2}}\,dx, x, \sin(e+fx)\right)}{3a^2(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\\
&\quad +\frac{\left((3a+2b)\sqrt{a+b-a\sin^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{3a^2(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2(2a+b)\sin(e+fx)}{3a(a+b)^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\
&- \frac{b\sin(e+fx)}{3a(a+b)f\,(a+b-a\sin^2(e+fx))\,\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\
&- \frac{(2(2a+b)\,(a+b-a\sin^2(e+fx)))\,\text{Subst}\left(\int\frac{\sqrt{1-\frac{ax^2}{a+b}}}{\sqrt{1-x^2}}\,dx,x,\sin(e+fx)\right)}{3a^2(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} \\
&+ \frac{\left((3a+2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{ax^2}{a+b}}}\,dx,x,\sin(e+fx)\right)}{3a^2(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\
&= \frac{2(2a+b)\sin(e+fx)}{3a(a+b)^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\
&- \frac{b\sin(e+fx)}{3a(a+b)f\,(a+b-a\sin^2(e+fx))\,\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\
&- \frac{2(2a+b)E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{3a^2(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} \\
&+ \frac{(3a+2b)\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{3a^2(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}
\end{aligned}
$$

Mathematica [F]

$$
\int\frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int\frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx
$$

```
[In] Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
[Out] Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.02 (sec) , antiderivative size = 16966, normalized size of antiderivative = 51.88

method	result	size
default	Expression too large to display	16966

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

[Out] result too large to display

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 1293, normalized size of antiderivative = 3.95

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((2*((2*I*a^4 + I*a^3*b)*cos(f*x + e)^4 + 2*I*a^2*b^2 + I*a*b^3 - 2*(-2
*I*a^3*b - I*a^2*b^2)*cos(f*x + e)^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) - ((2*I
*a^4 + 5*I*a^3*b + 2*I*a^2*b^2)*cos(f*x + e)^4 + 2*I*a^2*b^2 + 5*I*a*b^3 +
2*I*b^4 + 2*(2*I*a^3*b + 5*I*a^2*b^2 + 2*I*a*b^3)*cos(f*x + e)^2)*sqrt(a))*
sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*s
qrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 +
 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*((-2*I*a^
4 - I*a^3*b)*cos(f*x + e)^4 - 2*I*a^2*b^2 - I*a*b^3 - 2*(2*I*a^3*b + I*a^2*
b^2)*cos(f*x + e)^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) - ((-2*I*a^4 - 5*I*a^3*b
 - 2*I*a^2*b^2)*cos(f*x + e)^4 - 2*I*a^2*b^2 - 5*I*a*b^3 - 2*I*b^4 + 2*(-2*
I*a^3*b - 5*I*a^2*b^2 - 2*I*a*b^3)*cos(f*x + e)^2)*sqrt(a))*sqrt((2*a*sqrt(
(a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)
/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2
+ 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*((-3*I*a^4 - I*a^3*b)*co
s(f*x + e)^4 - 3*I*a^2*b^2 - I*a*b^3 - 2*(3*I*a^3*b + I*a^2*b^2)*cos(f*x +
e)^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) - ((-I*a^4 - 3*I*a^3*b - 2*I*a^2*b^2)*c
os(f*x + e)^4 - I*a^2*b^2 - 3*I*a*b^3 - 2*I*b^4 + 2*(-I*a^3*b - 3*I*a^2*b^2
 - 2*I*a*b^3)*cos(f*x + e)^2)*sqrt(a))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a
- 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(
cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqr
t((a*b + b^2)/a^2))/a^2) + (2*((3*I*a^4 + I*a^3*b)*cos(f*x + e)^4 + 3*I*a^2
*b^2 + I*a*b^3 - 2*(-3*I*a^3*b - I*a^2*b^2)*cos(f*x + e)^2)*sqrt(a)*sqrt((a
*b + b^2)/a^2) - ((I*a^4 + 3*I*a^3*b + 2*I*a^2*b^2)*cos(f*x + e)^4 + I*a^2*
b^2 + 3*I*a*b^3 + 2*I*b^4 + 2*(I*a^3*b + 3*I*a^2*b^2 + 2*I*a*b^3)*cos(f*x +
 e)^2)*sqrt(a))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(ar
csin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*
x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2
) + (2*(2*a^4 + a^3*b)*cos(f*x + e)^4 + (3*a^3*b + a^2*b^2)*cos(f*x + e)^2)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b
+ a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e
)^2 + (a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*f)
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(sec(e + f*x)/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(sec(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate(sec(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\cos(e+f\,x)\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(5/2)),x)
[Out] int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

3.286 $\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 349

$$\begin{aligned}\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = &-\frac{2b(3a+2b)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\\&-\frac{b\cos^2(e+fx)\sin(e+fx)}{3a(a+b)f\,(a+b-a\sin^2(e+fx))\,\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\\&+\frac{(3a^2+13ab+8b^2)\,E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{3a^3(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}\\&-\frac{b(9a+8b)\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{3a^3(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}\end{aligned}$$

```
[Out] -2/3*b*(3*a+2*b)*sin(f*x+e)/a^2/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2
))^(1/2)-1/3*b*cos(f*x+e)^2*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*x+e)^2)/(sec(
f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+1/3*(3*a^2+13*a*b+8*b^2)*EllipticE(sin
(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a^3/(a+b)^2/f/(cos(f*x+e)^2)^
(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1
/2)-1/3*b*(9*a+8*b)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2
/(a+b))^(1/2)/a^3/(a+b)/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x
+e)^2))^(1/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4233, 1985, 1986, 424, 540, 538, 437, 435, 432, 430}

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{b(9a+8b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{3a^3f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}} - \frac{2b(3a+2b)\sin(e+fx)}{3a^2f(a+b)^2\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}} + \frac{(3a^2+13ab+8b^2)\left(-a\sin^2(e+fx)+a+b\right)E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{3a^3f(a+b)^2\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}} - \frac{b\sin(e+fx)\cos^2(e+fx)}{3af(a+b)\left(-a\sin^2(e+fx)+a+b\right)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

```
[In] Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (-2*b*(3*a + 2*b)*Sin[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[Sec[e + f*x]^2*(a +
 b - a*Sin[e + f*x]^2)]) - (b*Cos[e + f*x]^2*Sin[e + f*x])/(3*a*(a + b)*f*(
a + b - a*Sin[e + f*x]^2)*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
+ ((3*a^2 + 13*a*b + 8*b^2)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a +
 b - a*Sin[e + f*x]^2))/(3*a^3*(a + b)^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e
+ f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])
- (b*(9*a + 8*b)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin
[e + f*x]^2)/(a + b)])/(3*a^3*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f
*x]^2*(a + b - a*Sin[e + f*x]^2)])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
```

```
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{\left(a+\frac{b}{1-x^2}\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\left(\frac{a+b-ax^2}{1-x^2}\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{5/2}}{(a+b-ax^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= -\frac{b\cos^2(e+fx)\sin(e+fx)}{3a(a+b)f\,(a+b-a\sin^2(e+fx))\,\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} - \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{\sqrt{1-x^2}\,(-3a-b+(3a+4b)x^2)}{(a+b-ax^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{3a(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= -\frac{2b(3a+2b)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} - \frac{b\cos^2(e+fx)\sin(e+fx)}{3a(a+b)f\,(a+b-a\sin^2(e+fx))\,\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} - \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{-((a+b)(3a+4b))+(3a^2+13ab+8b^2)x^2}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{3a^2(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= -\frac{2b(3a+2b)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{b\cos^2(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{\left(b(9a+8b)\sqrt{a+b-a\sin^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}dx,x,\sin(e+fx)\right)}{3a^3(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{\left((3a^2+13ab+8b^2)\sqrt{a+b-a\sin^2(e+fx)}\right)\text{Subst}\left(\int\frac{\sqrt{a+b-ax^2}}{\sqrt{1-x^2}}dx,x,\sin(e+fx)\right)}{3a^3(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

$$= -\frac{2b(3a+2b)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{b\cos^2(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{\left((3a^2+13ab+8b^2)\left(a+b-a\sin^2(e+fx)\right)\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{ax^2}{a+b}}}{\sqrt{1-x^2}}dx,x,\sin(e+fx)\right)}{3a^3(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{\left(b(9a+8b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{ax^2}{a+b}}}dx,x,\sin(e+fx)\right)}{3a^3(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

$$= -\frac{2b(3a+2b)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{b\cos^2(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{(3a^2+13ab+8b^2)E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)\left(a+b-a\sin^2(e+fx)\right)}{3a^3(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{b(9a+8b)\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{3a^3(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

Mathematica [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

```
[In] Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
[Out] Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.61 (sec) , antiderivative size = 20403, normalized size of antiderivative = 58.46

method	result	size
default	Expression too large to display	20403

```
[In] int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)/(b^3*sec(f*x + e)^6 + 3*a*
b^2*sec(f*x + e)^4 + 3*a^2*b*sec(f*x + e)^2 + a^3), x)
```

Sympy [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(cos(e + f*x)/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(e+f\,x)}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

```
[In] int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.287 $\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 441

$$\begin{aligned}
\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx &= -\frac{2b(4a+3b)\cos^2(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\
&- \frac{b\cos^4(e+fx)\sin(e+fx)}{3a(a+b)f\,(a+b-a\sin^2(e+fx))\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\
&+ \frac{(a^2+11ab+8b^2)\sin(e+fx)\,(a+b-a\sin^2(e+fx))}{3a^3(a+b)^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\
&+ \frac{2(a+2b)\,(a^2-4ab-4b^2)\,E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{3a^4(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} \\
&- \frac{b(a^2-16ab-16b^2)\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{3a^4(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}
\end{aligned}$$

```
[Out] -2/3*b*(4*a+3*b)*cos(f*x+e)^2*sin(f*x+e)/a^2/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a
*sin(f*x+e)^2))^(1/2)-1/3*b*cos(f*x+e)^4*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*
x+e)^2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+1/3*(a^2+11*a*b+8*b^2)*si
n(f*x+e)*(a+b-a*sin(f*x+e)^2)/a^3/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)
^2))^(1/2)+2/3*(a+2*b)*(a^2-4*a*b-4*b^2)*EllipticE(sin(f*x+e),(a/(a+b))^(1/
2))*(a+b-a*sin(f*x+e)^2)/a^4/(a+b)^2/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(
a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/3*b*(a^2-16*a*b
-16*b^2)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/
2)/a^4/(a+b)/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/
2)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {4233, 1985, 1986, 424, 540, 542, 538, 437, 435, 432, 430}

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{2b(4a+3b)\sin(e+fx)\cos^2(e+fx)}{3a^2f(a+b)^2\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}} - \frac{b(a^2-16ab-16b^2)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{3a^4f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}} + \frac{2(a+2b)\left(a^2-4ab-4b^2\right)\left(-a\sin^2(e+fx)+a+b\right)E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{3a^4f(a+b)^2\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}} + \frac{(a^2+11ab+8b^2)\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{3a^3f(a+b)^2\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}} - \frac{b\sin(e+fx)\cos^4(e+fx)}{3af(a+b)\left(-a\sin^2(e+fx)+a+b\right)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

```
[In] Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (-2*b*(4*a + 3*b)*Cos[e + f*x]^2*Sin[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[Sec[
e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) - (b*Cos[e + f*x]^4*Sin[e + f*x])/(
3*a*(a + b)*f*(a + b - a*Sin[e + f*x]^2)*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin
[e + f*x]^2)]) + ((a^2 + 11*a*b + 8*b^2)*Sin[e + f*x]*(a + b - a*Sin[e + f*
x]^2))/(3*a^3*(a + b)^2*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
+ (2*(a + 2*b)*(a^2 - 4*a*b - 4*b^2)*EllipticE[ArcSin[Sin[e + f*x]], a/(a +
 b)]*(a + b - a*Sin[e + f*x]^2))/(3*a^4*(a + b)^2*f*Sqrt[Cos[e + f*x]^2]*Sq
rt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(
a + b)]) - (b*(a^2 - 16*a*b - 16*b^2)*EllipticF[ArcSin[Sin[e + f*x]], a/(a
+ b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*a^4*(a + b)*f*Sqrt[Cos[e + f
*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
```

```
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a+\frac{b}{1-x^2}\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1-x^2}{\left(\frac{a+b-ax^2}{1-x^2}\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{7/2}}{(a+b-ax^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

$$= -\frac{b\cos^4(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{3/2}(-3a-b+3(a+2b)x^2)}{(a+b-ax^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{3a(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

$$=-\frac{2b(4a+3b)\cos^2(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$
$$-\frac{b\cos^4(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$
$$-\frac{\sqrt{a+b-a\sin^2(e+fx)}\text{Subst}\left(\int\frac{\sqrt{1-x^2}\left(-3(a+b)(a+2b)+3\left(a^2+11ab+8b^2\right)x^2\right)}{\sqrt{a+b-ax^2}}\,dx,x,\sin(e+fx)\right)}{3a^2(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

$$=-\frac{2b(4a+3b)\cos^2(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$
$$-\frac{b\cos^4(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$
$$+\frac{\left(a^2+11ab+8b^2\right)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{3a^3(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$
$$+\frac{\sqrt{a+b-a\sin^2(e+fx)}\text{Subst}\left(\int\frac{3(a+b)\left(2a^2-5ab-8b^2\right)-6(a+2b)\left(a^2-4ab-4b^2\right)x^2}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx,x,\sin(e+fx)\right)}{9a^3(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

$$=-\frac{2b(4a+3b)\cos^2(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$
$$-\frac{b\cos^4(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$
$$+\frac{\left(a^2+11ab+8b^2\right)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{3a^3(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$
$$-\frac{\left(b\left(a^2-16ab-16b^2\right)\sqrt{a+b-a\sin^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx,x,\sin(e+fx)\right)}{3a^4(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$
$$+\frac{\left(2(a+2b)\left(a^2-4ab-4b^2\right)\sqrt{a+b-a\sin^2(e+fx)}\right)\text{Subst}\left(\int\frac{\sqrt{a+b-ax^2}}{\sqrt{1-x^2}}\,dx,x,\sin(e+fx)\right)}{3a^4(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

$$= -\frac{2b(4a+3b)\cos^2(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} - \frac{b\cos^4(e+fx)\sin(e+fx)}{3a(a+b)f\,(a+b-a\sin^2(e+fx))\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{(a^2+11ab+8b^2)\sin(e+fx)\,(a+b-a\sin^2(e+fx))}{3a^3(a+b)^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{(2(a+2b)\,(a^2-4ab-4b^2)\,(a+b-a\sin^2(e+fx)))\,\text{Subst}\left(\int\frac{\sqrt{1-\frac{ax^2}{a+b}}}{\sqrt{1-x^2}}\,dx,x,\sin(e+fx)\right)}{3a^4(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{\left(b(a^2-16ab-16b^2)\,\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{ax^2}{a+b}}}\,dx,x,\sin(e+fx)\right)}{3a^4(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= -\frac{2b(4a+3b)\cos^2(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} - \frac{b\cos^4(e+fx)\sin(e+fx)}{3a(a+b)f\,(a+b-a\sin^2(e+fx))\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{(a^2+11ab+8b^2)\sin(e+fx)\,(a+b-a\sin^2(e+fx))}{3a^3(a+b)^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{2(a+2b)\,(a^2-4ab-4b^2)\,E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{3a^4(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{b(a^2-16ab-16b^2)\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{3a^4(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

Mathematica [F]

$$\int\frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int\frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

```
[In] Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
[Out] Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.93 (sec) , antiderivative size = 24449, normalized size of antiderivative = 55.44

method	result	size
default	Expression too large to display	24449

```
[In] int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F]

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\cos\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3/(b^3*sec(f*x + e)^6 + 3*
a*b^2*sec(f*x + e)^4 + 3*a^2*b*sec(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\cos\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)^3}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(e+f\,x)^3}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

```
[In] int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.288 $\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 559

$$\begin{aligned}
\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx &= -\frac{2b(5a+4b)\cos^4(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} \\
&- \frac{b\cos^6(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} \\
&+ \frac{2(2a^3-3a^2b-42ab^2-32b^3)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{15a^4(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} \\
&+ \frac{(3a^2+61ab+48b^2)\cos^2(e+fx)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{15a^3(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} \\
&+ \frac{(8a^4-11a^3b+27a^2b^2+184ab^3+128b^4)\,E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\left(a+b-a\sin^2(e+fx)\right)}{15a^5(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} \\
&- \frac{b(4a^3-9a^2b+120ab^2+128b^3)\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{15a^5(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}
\end{aligned}$$

```
[Out] -2/3*b*(5*a+4*b)*cos(f*x+e)^4*sin(f*x+e)/a^2/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a
*sin(f*x+e)^2))^(1/2)-1/3*b*cos(f*x+e)^6*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*
x+e)^2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+2/15*(2*a^3-3*a^2*b-42*a*
b^2-32*b^3)*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a^4/(a+b)^2/f/(sec(f*x+e)^2*(a+
b-a*sin(f*x+e)^2))^(1/2)+1/15*(3*a^2+61*a*b+48*b^2)*cos(f*x+e)^2*sin(f*x+e)
*(a+b-a*sin(f*x+e)^2)/a^3/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/
2)+1/15*(8*a^4-11*a^3*b+27*a^2*b^2+184*a*b^3+128*b^4)*EllipticE(sin(f*x+e),
(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a^5/(a+b)^2/f/(cos(f*x+e)^2)^(1/2)/(s
ec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/15
*b*(4*a^3-9*a^2*b+120*a*b^2+128*b^3)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*
```

```
(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a^5/(a+b)/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)
^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {4233, 1985, 1986, 424, 540, 542, 538, 437, 435, 432, 430}

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{2b(5a+4b)\sin(e+fx)\cos^4(e+fx)}{3a^2f(a+b)^2\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$
$$+\frac{(3a^2+61ab+48b^2)\sin(e+fx)\cos^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{15a^3f(a+b)^2\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$
$$-\frac{b(4a^3-9a^2b+120ab^2+128b^3)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{15a^5f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$
$$+\frac{2(2a^3-3a^2b-42ab^2-32b^3)\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{15a^4f(a+b)^2\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$
$$+\frac{(8a^4-11a^3b+27a^2b^2+184ab^3+128b^4)\left(-a\sin^2(e+fx)+a+b\right)E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{15a^5f(a+b)^2\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$
$$-\frac{b\sin(e+fx)\cos^6(e+fx)}{3af(a+b)\left(-a\sin^2(e+fx)+a+b\right)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

```
[In] Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (-2*b*(5*a + 4*b)*Cos[e + f*x]^4*Sin[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[Sec[
e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) - (b*Cos[e + f*x]^6*Sin[e + f*x])/(
3*a*(a + b)*f*(a + b - a*Sin[e + f*x]^2)*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin
[e + f*x]^2)]) + (2*(2*a^3 - 3*a^2*b - 42*a*b^2 - 32*b^3)*Sin[e + f*x]*(a +
 b - a*Sin[e + f*x]^2))/(15*a^4*(a + b)^2*f*Sqrt[Sec[e + f*x]^2*(a + b - a*
Sin[e + f*x]^2)]) + ((3*a^2 + 61*a*b + 48*b^2)*Cos[e + f*x]^2*Sin[e + f*x]*
(a + b - a*Sin[e + f*x]^2))/(15*a^3*(a + b)^2*f*Sqrt[Sec[e + f*x]^2*(a + b
- a*Sin[e + f*x]^2)]) + ((8*a^4 - 11*a^3*b + 27*a^2*b^2 + 184*a*b^3 + 128*b
^4)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a + b - a*Sin[e + f*x]^2))/
(15*a^5*(a + b)^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin
[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (b*(4*a^3 - 9*a^2*b +
 120*a*b^2 + 128*b^3)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (
a*Sin[e + f*x]^2)/(a + b)])/(15*a^5*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec
[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
```

```
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
```

```
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{\left(a+\frac{b}{1-x^2}\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{\left(\frac{a+b-ax^2}{1-x^2}\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{9/2}}{(a+b-ax^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

$$= -\frac{b\cos^6(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{5/2}(-3a-b+(3a+8b)x^2)}{(a+b-ax^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{3a(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

$$= -\frac{2b(5a+4b)\cos^4(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{b\cos^6(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{3/2}(-((a+b)(3a+8b))+(3a^2+61ab+48b^2)x^2)}{\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{3a^2(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

$$= -\frac{2b(5a+4b)\cos^4(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{b\cos^6(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{(3a^2+61ab+48b^2)\cos^2(e+fx)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{15a^3(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{\sqrt{1-x^2}(3(a+b)(4a^2-7ab-16b^2)-6(2a^3-3a^2b-42ab^2-32b^3)x^2)}{\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{15a^3(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

$$= -\frac{2b(5a+4b)\cos^4(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{b\cos^6(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{2(2a^3-3a^2b-42ab^2-32b^3)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{15a^4(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{(3a^2+61ab+48b^2)\cos^2(e+fx)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{15a^3(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{\sqrt{a+b-a\sin^2(e+fx)}\,\text{Subst}\left(\int \frac{-3(a+b)(8a^3-15a^2b+36ab^2+64b^3)+3(8a^4-11a^3b+27a^2b^2+184ab^3+128b^4)x^2}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx\right.}{45a^4(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

$$= -\frac{2b(5a+4b)\cos^4(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{b\cos^6(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{2(2a^3-3a^2b-42ab^2-32b^3)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{15a^4(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{(3a^2+61ab+48b^2)\cos^2(e+fx)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{15a^3(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{\left(b(4a^3-9a^2b+120ab^2+128b^3)\sqrt{a+b-a\sin^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}dx,x,\sin(e+fx)\right)}{15a^5(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{\left((8a^4-11a^3b+27a^2b^2+184ab^3+128b^4)\sqrt{a+b-a\sin^2(e+fx)}\right)\text{Subst}\left(\int\frac{\sqrt{a+b-ax^2}}{\sqrt{1-x^2}}dx,x,\sin(e+fx)\right)}{15a^5(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

$$= -\frac{2b(5a+4b)\cos^4(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{b\cos^6(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{2(2a^3-3a^2b-42ab^2-32b^3)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{15a^4(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{(3a^2+61ab+48b^2)\cos^2(e+fx)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{15a^3(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{\left((8a^4-11a^3b+27a^2b^2+184ab^3+128b^4)\left(a+b-a\sin^2(e+fx)\right)\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{ax^2}{a+b}}}{\sqrt{1-x^2}}dx,x,\sin(e+fx)\right)}{15a^5(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{\left(b(4a^3-9a^2b+120ab^2+128b^3)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{ax^2}{a+b}}}dx,x,\sin(e+fx)\right)}{15a^5(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

$$
\begin{aligned}
&= -\frac{2b(5a+4b)\cos^4(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\
&\quad -\frac{b\cos^6(e+fx)\sin(e+fx)}{3a(a+b)f\,(a+b-a\sin^2(e+fx))\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\
&\quad +\frac{2(2a^3-3a^2b-42ab^2-32b^3)\sin(e+fx)\,(a+b-a\sin^2(e+fx))}{15a^4(a+b)^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\
&\quad +\frac{(3a^2+61ab+48b^2)\cos^2(e+fx)\sin(e+fx)\,(a+b-a\sin^2(e+fx))}{15a^3(a+b)^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\
&\quad +\frac{(8a^4-11a^3b+27a^2b^2+184ab^3+128b^4)\,E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{15a^5(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} \\
&\quad -\frac{b(4a^3-9a^2b+120ab^2+128b^3)\,\mathrm{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{15a^5(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}
\end{aligned}
$$

Mathematica [F]

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

```
[In] Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
[Out] Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.44 (sec) , antiderivative size = 31577, normalized size of antiderivative = 56.49

method	result	size
default	Expression too large to display	31577

```
[In] int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F]

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)^5}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5/(b^3*sec(f*x + e)^6 + 3*
a*b^2*sec(f*x + e)^4 + 3*a^2*b*sec(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)^5}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)^5}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(e+f\,x)^5}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

```
[In] int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.289 $\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 133

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{5/2}f} - \frac{a\sec^2(e+fx)\tan(e+fx)}{3b(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{a(3a+5b)\tan(e+fx)}{3b^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(5/2)/f-1/3*a*(3*a
+5*b)*tan(f*x+e)/b^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*a*sec(f*x+e)^
2*tan(f*x+e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4231, 424, 393, 223, 212}

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{b^{5/2}f} - \frac{a(3a+5b)\tan(e+fx)}{3b^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}} - \frac{a\tan(e+fx)\sec^2(e+fx)}{3bf(a+b)\,(a+b\tan^2(e+fx)+b)^{3/2}}$$

```
[In] Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(b^(5/2)*f)
- (a*Sec[e + f*x]^2*Tan[e + f*x])/(3*b*(a + b)*f*(a + b + b*Tan[e + f*x]^2)
```

```
^(3/2)) - (a*(3*a + 5*b)*Tan[e + f*x])/(3*b^2*(a + b)^2*f*Sqrt[a + b + b*Ta
n[e + f*x]^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{3b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a+3b+3(a+b)x^2}{(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3b(a+b)f}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{3b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{a(3a+5b)\tan(e+fx)}{3b^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{b^2f}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{3b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{a(3a+5b)\tan(e+fx)}{3b^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^2f}$$

$$= \frac{\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{5/2}f} - \frac{a\sec^2(e+fx)\tan(e+fx)}{3b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{a(3a+5b)\tan(e+fx)}{3b^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.35 (sec) , antiderivative size = 592, normalized size of antiderivative = 4.45

$$\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{(a+2b+a\cos(2e+2fx))^{5/2}\sec^6(e+fx)\sqrt{1-\frac{2a\sin^2(e+fx)}{2a+2b}}\tan(e+fx)\left(-\cdots\right.}{\cdots}$$

```
[In] Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^6*Sqrt[1 - (2*a*Sin[e +
f*x]^2)/(2*a + 2*b)]*Tan[e + f*x]*((-24*b*Cos[e + f*x]^2*HypergeometricPFQ[
{2, 2, 2}, {1, 9/2}, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^2*(-((b*Se
c[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(a + b)^2))^(5/2))/
(a + b) - (Sec[e + f*x]^6*(24*b^3*Hypergeometric2F1[2, 2, 9/2, -((b*Tan[e +
 f*x]^2)/(a + b))]*Sin[e + f*x]^6*(4*b^2 + a*b*(8 - 7*Sin[e + f*x]^2) + a^2
*(4 - 7*Sin[e + f*x]^2 + 3*Sin[e + f*x]^4))*Sqrt[-((b*Sec[e + f*x]^2*(a + b
 - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(a + b)^2)] + 35*(a + b)*Cos[e + f*x]^
2*(15*b^2 + 10*a*b*(3 - 2*Sin[e + f*x]^2) + a^2*(15 - 20*Sin[e + f*x]^2 + 8
*Sin[e + f*x]^4))*(-3*ArcSin[Sqrt[-((b*Tan[e + f*x]^2)/(a + b))]]*(a + b -
a*Sin[e + f*x]^2)^2 - (a + b)*Cos[e + f*x]^2*(-3*a*Cos[e + f*x]^2 - b*(3 +
Sin[e + f*x]^2))*Sqrt[-((b*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e
+ f*x]^2)/(a + b)^2)])))/(a + b)^5))/(315*(2*a + 2*b)^2*f*(a + b*Sec[e + f*
```

```
x]^2)^(5/2)*Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)]*Sqrt[2*a + 2*b - 2*a*Sin[e
 + f*x]^2]*(1 - (a*Sin[e + f*x]^2)/(a + b))^(3/2)*(-((b*Tan[e + f*x]^2)/(a
+ b)))^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2354 vs. 2(119) = 238.

Time = 6.33 (sec) , antiderivative size = 2355, normalized size of antiderivative = 17.71

method	result	size
default	Expression too large to display	2355

```
[In] int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/f/(a+b)^2/b^(7/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*
csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x
+e)^2+a+b)*(-36*b^(5/2)*a^2*(1-cos(f*x+e))^5*csc(f*x+e)^5-24*b^(7/2)*a*(1-c
os(f*x+e))^5*csc(f*x+e)^5-12*b^(3/2)*a^3*(1-cos(f*x+e))^5*csc(f*x+e)^5+8*b^
(5/2)*a^2*(1-cos(f*x+e))^3*csc(f*x+e)^3-48*(1-cos(f*x+e))^3*b^(7/2)*a*csc(f
*x+e)^3+24*b^(3/2)*a^3*(1-cos(f*x+e))^3*csc(f*x+e)^3-36*b^(5/2)*a^2*(csc(f*
x+e)-cot(f*x+e))-24*b^(7/2)*a*(csc(f*x+e)-cot(f*x+e))+3*ln(4*(-a*(1-cos(f*x
+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2+b^(1/2)*(a*(1-cos(f*x+e
))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(
f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f
*x+e))-a-b)/((1-cos(f*x+e))^2*csc(f*x+e)^2+2*csc(f*x+e)-2*cot(f*x+e)+1))*(a
*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f
*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(3/2)*a^2*b+6*
ln(4*(-a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2+b^(1
/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1
-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*
a*(csc(f*x+e)-cot(f*x+e))-a-b)/((1-cos(f*x+e))^2*csc(f*x+e)^2+2*csc(f*x+e)-
2*cot(f*x+e)+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*
x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+
a+b)^(3/2)*a*b^2+3*ln(4*(-a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^
2*csc(f*x+e)^2+b^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*
csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x
+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e))-a-b)/((1-cos(f*x+e))^2*csc(f*x
+e)^2+2*csc(f*x+e)-2*cot(f*x+e)+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-c
os(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+
e))^2*csc(f*x+e)^2+a+b)^(3/2)*b^3+3*ln(4*(a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b
*(1-cos(f*x+e))^2*csc(f*x+e)^2+b^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(
1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f
*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e))+a+b)/((1-cos(f
*x+e))^2*csc(f*x+e)^2-2*csc(f*x+e)+2*cot(f*x+e)+1))*(a*(1-cos(f*x+e))^4*csc
(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2
```

```
+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(3/2)*a^2*b+6*ln(4*(a*(1-cos(f*x+e)
)^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+b^(1/2)*(a*(1-cos(f*x+e))^
4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x
+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+
e))+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-2*csc(f*x+e)+2*cot(f*x+e)+1))*(a*(1
-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+
e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(3/2)*a*b^2+3*ln(
4*(a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+b^(1/2)*
(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos
(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(c
sc(f*x+e)-cot(f*x+e))+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-2*csc(f*x+e)+2*co
t(f*x+e)+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)
^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)
^(3/2)*b^3-12*b^(3/2)*a^3*(csc(f*x+e)-cot(f*x+e)))/((a*(1-cos(f*x+e))^4*csc
(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2
+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^2
)^(5/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(119) = 238$.

Time = 0.50 (sec) , antiderivative size = 688, normalized size of antiderivative = 5.17

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \left[\frac{3\left((a^4+2\,a^3b+a^2b^2)\cos{(fx+e)}^4+a^2b^2+2\,ab^3+b^4+2\,(a^3b+2\,a^2b^2+a\right.}{}\right.$$

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 2*a*b^3 + b^
4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sqrt(b)*log(((a^2 - 6*a*b
 + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x
+ e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((3*a^3*b + 5*a^2*b^2)*cos(f*
x + e)^3 + 2*(2*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b
)/cos(f*x + e)^2)*sin(f*x + e))/((a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*f*cos(f*x
+ e)^4 + 2*(a^3*b^4 + 2*a^2*b^5 + a*b^6)*f*cos(f*x + e)^2 + (a^2*b^5 + 2*a*
b^6 + b^7)*f), 1/6*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 +
 2*a*b^3 + b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sqrt(-b)*arc
tan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) -
```

```
 2*((3*a^3*b + 5*a^2*b^2)*cos(f*x + e)^3 + 2*(2*a^2*b^2 + 3*a*b^3)*cos(f*x
+ e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^4*b^3 +
 2*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^4 + 2*(a^3*b^4 + 2*a^2*b^5 + a*b^6)*f*
cos(f*x + e)^2 + (a^2*b^5 + 2*a*b^6 + b^7)*f)]
```

Sympy [F]

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)**6/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(119) = 238$.

Time = 0.20 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.07

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\left(\frac{3\tan(fx+e)^2}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}b}+\frac{2a}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}b^2}+\frac{2}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}b}\right)\tan(fx+e)-\frac{3\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{5}{2}}}-\frac{\cdots}{\sqrt{b\cdots}}\cdots}{\cdots}$$

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*((3*tan(f*x + e)^2/((b*tan(f*x + e)^2 + a + b)^(3/2)*b) + 2*a/((b*tan(
f*x + e)^2 + a + b)^(3/2)*b^2) + 2/((b*tan(f*x + e)^2 + a + b)^(3/2)*b))*ta
n(f*x + e) - 3*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(5/2) - 2*tan(f*x
+ e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) - tan(f*x + e)/((b*tan(f*x
+ e)^2 + a + b)^(3/2)*(a + b)) + 3*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a
+ b)*b^2) - 2*a*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*b^2) +
 2*tan(f*x + e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*b) - 4*tan(f*x + e)/(sqrt
(b*tan(f*x + e)^2 + a + b)*(a + b)*b))/f
```

Giac [F]

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec(fx+e)^6}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\cos(e+f\,x)^6\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

```
[In] int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(5/2)),x)
[Out] int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

3.290 $\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 79

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\sec^2(e+fx)\tan(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{2\tan(e+fx)}{3(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] 2/3*tan(f*x+e)/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/3*sec(f*x+e)^2*tan(f*
x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {4231, 386, 197}

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{2\tan(e+fx)}{3f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}} + \frac{\tan(e+fx)\sec^2(e+fx)}{3f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[In] Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)
) + (2*Tan[e + f*x])/(3*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1+x^2}{(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{\sec^2(e+fx)\tan(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{2\text{Subst}\left(\int \frac{1}{(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3(a+b)f}\\ &= \frac{\sec^2(e+fx)\tan(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{2\tan(e+fx)}{3(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 5.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.94

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))(2a+3b+a\cos(2(e+fx)))\sec^4(e+fx)\tan(e+fx)}{6(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

```
[In] Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*(2*a + 3*b + a*Cos[2*(e + f*x)])*Sec[e + f*
x]^4*Tan[e + f*x])/(6*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [A] (verified)

Time = 3.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\left(b+a\cos(fx+e)^2\right)\left(2a\cos(fx+e)^2+a+3b\right)\tan(fx+e)\sec(fx+e)^4}{3f(a^2+2ab+b^2)\left(a+b\sec(fx+e)^2\right)^{\frac{5}{2}}}$	75

```
[In] int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f/(a^2+2*a*b+b^2)*(b+a*cos(f*x+e)^2)*(2*a*cos(f*x+e)^2+a+3*b)/(a+b*sec(
f*x+e)^2)^(5/2)*tan(f*x+e)*sec(f*x+e)^4
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.70

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\left(2\,a\cos\left(fx+e\right)^3+(a+3\,b)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin}{3\left((a^4+2\,a^3b+a^2b^2)f\cos\left(fx+e\right)^4+2\,(a^3b+2\,a^2b^2+ab^3)f\cos\left(fx+e\right)^2\right.}$$

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*a*cos(f*x + e)^3 + (a + 3*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*sin(f*x + e)/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^4
 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^2*b^2 + 2*a*b^3 + b^
4)*f)
```

Sympy [F]

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec^4\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.48

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\frac{2\,\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^2} + \frac{\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)} - \frac{\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}b} + \frac{\cdots}{\sqrt{b\tan(\cdots}}}{3\,f}$$

[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")

[Out] 1/3*(2*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) + tan(f*x + e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)) - tan(f*x + e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*b) + tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*b))/f

Giac [F]

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec(fx+e)^4}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")

[Out] sage0*x

Mupad [B] (verification not implemented)

Time = 29.44 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.94

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{2\left(e^{e\,4i+f\,x\,4i}-1\right)\sqrt{a+\frac{b}{\left(\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}\right)^2}}\left(a\,1i+a\,e^{e\,2i+f\,x\,2i}\,4i+a\,e^{e\,4i+f\,x\,4i}\,1i+b\,e^{e\,2i+f\,x\,2i}\,6i\right)}{3\,f\,(a+b)^2\left(a+2\,a\,e^{e\,2i+f\,x\,2i}+a\,e^{e\,4i+f\,x\,4i}+4\,b\,e^{e\,2i+f\,x\,2i}\right)^2}$$

[In] int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(5/2)),x)

[Out] -(2*(exp(e*4i + f*x*4i) - 1)*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*(a*1i + a*exp(e*2i + f*x*2i)*4i + a*exp(e*4i + f*x*4i)*1i + b*exp(e*2i + f*x*2i)*6i))/(3*f*(a + b)^2*(a + 2*a*exp(e*2i + f*x*2i) + a*exp(e*4i + f*x*4i) + 4*b*exp(e*2i + f*x*2i))^2)

3.291 $\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 71

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\tan(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{2\tan(e+fx)}{3(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] 2/3*tan(f*x+e)/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/3*tan(f*x+e)/(a+b)/f/
(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {4231, 198, 197}

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{2\tan(e+fx)}{3f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}} + \frac{\tan(e+fx)}{3f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[In] Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] Tan[e + f*x]/(3*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (2*Tan[e + f*
x])/(3*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{\tan(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{2\text{Subst}\left(\int \frac{1}{(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3(a+b)f}\\ &= \frac{\tan(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{2\tan(e+fx)}{3(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 215 vs. $2(71) = 142$.

Time = 6.14 (sec) , antiderivative size = 215, normalized size of antiderivative = 3.03

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{(3a+b)(a+2b+a\cos(2e+2fx))^{5/2}\sec^5(e+fx)\left(\frac{\sqrt{2}\sin(e+fx)}{(a+b)\left(a+b-a\sin^2(e+fx)\right)^{3/2}} + \right.}{48af\left(a+b\sec^2(e+fx)\right)^{5/2}}$$
$$- \frac{(a+2b+a\cos(2e+2fx))^{5/2}\sec^4(e+fx)\tan(e+fx)}{8\sqrt{2}af\left(a+b\sec^2(e+fx)\right)^{5/2}\left(a+b-a\sin^2(e+fx)\right)^{3/2}}$$

```
[In] Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((3*a + b)*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5*((Sqrt[2]*Si
n[e + f*x])/((a + b)*(a + b - a*Sin[e + f*x]^2)^(3/2)) + (2*Sqrt[2]*Sin[e +
 f*x])/((a + b)^2*Sqrt[a + b - a*Sin[e + f*x]^2])))/(48*a*f*(a + b*Sec[e +
f*x]^2)^(5/2)) - ((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^4*Tan[e
 + f*x])/(8*Sqrt[2]*a*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a + b - a*Sin[e + f*x
]^2)^(3/2))
```

Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.18

method	result	size
default	$\frac{\left(3a\cos\left(fx+e\right)^{2}+\cos\left(fx+e\right)^{2}b+2b\right)\left(b+a\cos\left(fx+e\right)^{2}\right)\tan\left(fx+e\right)\sec\left(fx+e\right)^{4}}{3f\left(a^{2}+2ab+b^{2}\right)\left(a+b\sec\left(fx+e\right)^{2}\right)^{\frac{5}{2}}}$	84

```
[In] int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f/(a^2+2*a*b+b^2)*(3*a*cos(f*x+e)^2+cos(f*x+e)^2*b+2*b)*(b+a*cos(f*x+e)
^2)/(a+b*sec(f*x+e)^2)^(5/2)*tan(f*x+e)*sec(f*x+e)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(63) = 126$.

Time = 0.38 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.89

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\left((3\,a+b)\cos\left(fx+e\right)^3+2\,b\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\,\sin}{3\,\left((a^4+2\,a^3b+a^2b^2)f\cos\left(fx+e\right)^4+2\,(a^3b+2\,a^2b^2+ab^3)f\cos\left(fx+e\right)^2\right.}$$

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((3*a + b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*sin(f*x + e)/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^4
 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^2*b^2 + 2*a*b^3 + b^
4)*f)
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(sec(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\frac{2\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^2} + \frac{\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)}}{3f}$$

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] 1/3*(2*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) + tan(f*x +
e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)))/f
```

Giac [F]

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec(fx+e)^2}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 30.12 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.42

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$

$$-\frac{\left(\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}-1\right)\sqrt{a+\frac{b}{\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}\right)^2}}\,\left(a\,3\mathrm{i}+b\,1\mathrm{i}+a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,6\mathrm{i}+a\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,3\mathrm{i}+b\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,10\mathrm{i}+b}{3\,f\,(a+b)^2\,\left(a+2\,a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+a\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}+4\,b\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\right)^2}$$

```
[In] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

```
[Out] -((exp(e*4i + f*x*4i) - 1)*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*
1i)/2)^2)^(1/2)*(a*3i + b*1i + a*exp(e*2i + f*x*2i)*6i + a*exp(e*4i + f*x*4
i)*3i + b*exp(e*2i + f*x*2i)*10i + b*exp(e*4i + f*x*4i)*1i))/(3*f*(a + b)^2
*(a + 2*a*exp(e*2i + f*x*2i) + a*exp(e*4i + f*x*4i) + 4*b*exp(e*2i + f*x*2i
))^2)
```

3.292 $\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 125

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\tan(e+fx)}{3a(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*(5*a+
3*b)*tan(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*b*tan(f*x+e)/a
/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4213, 425, 541, 12, 385, 209}

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f} - \frac{b(5a+3b)\tan(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}} - \frac{b\tan(e+fx)}{3af(a+b)\,(a+b\tan^2(e+fx)+b)^{3/2}}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f) -
 (b*Tan[e + f*x])/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*(5*
a + 3*b)*Tan[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$=-\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}+\frac{\text{Subst}\left(\int\frac{3a+b-2bx^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx,x,\tan(e+fx)\right)}{3a(a+b)f}$$

$$=-\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}-\frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}+\frac{\text{Subst}\left(\int\frac{3(a+b)^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx,x,\tan(e+fx)\right)}{3a^2(a+b)^2f}$$

$$=-\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}-\frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}+\frac{\text{Subst}\left(\int\frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx,x,\tan(e+fx)\right)}{a^2f}$$

$$=-\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}-\frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}+\frac{\text{Subst}\left(\int\frac{1}{1+ax^2}\,dx,x,\frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^2f}$$

$$=\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f}-\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}-\frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 6.50 (sec) , antiderivative size = 1927, normalized size of antiderivative = 15.42

$$\int\frac{1}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=\frac{\quad}{4\sqrt{2}f\left(a+b\sec^2(e+fx)\right)^{5/2}\left(a+b-a\sin^2(e+fx)\right)^{5/2}\left(3(a+b)\,\text{AppellF1}\left(\right.\right.}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

[Out] (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^4*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*((15*a*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos[e + f*x]^4*Sin[e + f*x]*((5*a*f*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - (4*f*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^4*Sin[e + f*x]*(2*f*(5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a + b)*((5*a*f*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - (4*f*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3) + Sin[e + f*x]^2*(5*a*((21*a*f*AppellF1[5/2, -2, 9/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b)) - (12*f*AppellF1[5/2, -1, 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/5) - 4*(a + b)*((3*a*f*AppellF1[5/2, -1, 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (6*(a + b)^3*f*Cot[e + f*x]*Csc[e + f*x]^4*(-1 + (a*Sin[e + f*x]^2)/(a + b))^2*((Sqrt[a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/(Sqrt[a + b]*Sq

```
rt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + (a^2*Sin[e + f*x]^4)/(3*(a + b)^2*(-1
 + (a*Sin[e + f*x]^2)/(a + b))^2) + (a*Sin[e + f*x]^2)/((a + b)*(-1 + (a*Si
n[e + f*x]^2)/(a + b)))))/(a^3*(1 - (a*Sin[e + f*x]^2)/(a + b))^(3/2))))))/
(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2,
5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
-2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Appel
lF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e +
 f*x]^2)^2)))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1262 vs. $2(111) = 222$.

Time = 3.76 (sec) , antiderivative size = 1263, normalized size of antiderivative = 10.10

method	result	size
default	Expression too large to display	1263

```
[In] int(1/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f/(a+b)^2/a^2/(-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+
e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*c
sc(f*x+e)^2+a+b)*(12*(-a)^(1/2)*a^2*b*(1-cos(f*x+e))^5*csc(f*x+e)^5+18*(-a)
^(1/2)*a*b^2*(1-cos(f*x+e))^5*csc(f*x+e)^5+6*(-a)^(1/2)*b^3*(1-cos(f*x+e))^
5*csc(f*x+e)^5-24*(1-cos(f*x+e))^3*a^2*(-a)^(1/2)*b*csc(f*x+e)^3+4*(-a)^(1/
2)*a*b^2*(1-cos(f*x+e))^3*csc(f*x+e)^3+12*(-a)^(1/2)*b^3*(1-cos(f*x+e))^3*c
sc(f*x+e)^3-3*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e
)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b
)^(3/2)*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^
4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f
*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e
)^2+1))*a^2-6*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e
)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b
)^(3/2)*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^
4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f
*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e
)^2+1))*a*b-3*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e
)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b
)^(3/2)*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^
4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f
*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e
)^2+1))*b^2+12*(-a)^(1/2)*a^2*b*(csc(f*x+e)-cot(f*x+e))+18*(-a)^(1/2)*a*b^2
*(csc(f*x+e)-cot(f*x+e))+6*(-a)^(1/2)*b^3*(csc(f*x+e)-cot(f*x+e)))/((a*(1-c
os(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e)
)^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/((1-cos(f*x+e))^2*c
sc(f*x+e)^2-1)^2)^(5/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(111) = 222$.

Time = 0.73 (sec) , antiderivative size = 881, normalized size of antiderivative = 7.05

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \left[-\frac{3\left(\left(a^4+2a^3b+a^2b^2\right)\cos\left(fx+e\right)^4+a^2b^2+2ab^3+b^4+2\left(a^3b+2a^2b^2\right.\right.}{}\right.$$

$$-\frac{3\left(\left(a^4+2a^3b+a^2b^2\right)\cos\left(fx+e\right)^4+a^2b^2+2ab^3+b^4+2\left(a^3b+2a^2b^2+ab^3\right)\cos\left(fx+e\right)^2\right)\sqrt{a}\arctan\left(\right.}{12\left(\left(a^7+2a^6b+a^5b^2\right)\right.}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 2*a*b^3 + b
^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*(3*a^3*b + 2*a^2*b
^2)*cos(f*x + e)^3 + (5*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*
x + e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*
a^4*b^3 + a^3*b^4)*f), -1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 +
 a^2*b^2 + 2*a*b^3 + b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sq
rt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^
2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x +
e)^2)*sin(f*x + e))) + 4*(2*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (5*a^2*b
^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b + 2*a^5*b
^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*f)]
```

Sympy [F]

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=\int \frac{1}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral((a + b*sec(e + f*x)**2)**(-5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=\int \frac{1}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=\int \frac{1}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}}\,dx$$

```
[In] int(1/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(1/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.293 $\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 187

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\frac{(a-5b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{7/2}f}+\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}}+\frac{b(3a+5b)\tan(e+fx)}{6a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}+\frac{b(3a^2+22ab+15b^2)\tan(e+fx)}{6a^3(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] 1/2*(a-5*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(7/2)/f
+1/6*b*(3*a^2+22*a*b+15*b^2)*tan(f*x+e)/a^3/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^
(1/2)+1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(3/2)+1/6*b*(3*a+5
*b)*tan(f*x+e)/a^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4231, 425, 541, 12, 385, 209}

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\frac{(a-5b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{7/2}f}+\frac{b(3a+5b)\tan(e+fx)}{6a^2f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}+\frac{b(3a^2+22ab+15b^2)\tan(e+fx)}{6a^3f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\sin(e+fx)\cos(e+fx)}{2af\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[In] Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a - 5*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(
2*a^(7/2)*f) + (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*(a + b + b*Tan[e + f*x]^2
)^(3/2)) + (b*(3*a + 5*b)*Tan[e + f*x])/(6*a^2*(a + b)*f*(a + b + b*Tan[e +
 f*x]^2)^(3/2)) + (b*(3*a^2 + 22*a*b + 15*b^2)*Tan[e + f*x])/(6*a^3*(a + b)
^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
```

```
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-a+b-4bx^2}{(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(3a+5b)\tan(e+fx)}{6a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-3a^2+6ab+5b^2-2b(3a+5b)x^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{6a^2(a+b)f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(3a+5b)\tan(e+fx)}{6a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(3a^2+22ab+15b^2)\tan(e+fx)}{6a^3(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int -\frac{3(a-5b)(a+b)^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{6a^3(a+b)^2f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(3a+5b)\tan(e+fx)}{6a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(3a^2+22ab+15b^2)\tan(e+fx)}{6a^3(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a-5b)\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2a^3f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(3a+5b)\tan(e+fx)}{6a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(3a^2+22ab+15b^2)\tan(e+fx)}{6a^3(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a-5b)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^3f}$$

$$= \frac{(a-5b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{7/2}f} + \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(3a+5b)\tan(e+fx)}{6a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(3a^2+22ab+15b^2)\tan(e+fx)}{6a^3(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 16.74 (sec) , antiderivative size = 1775, normalized size of antiderivative = 9.49

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{}{4\sqrt{2}f\left(a+b\sec^2(e+fx)\right)^{5/2}\left(a+b-a\sin^2(e+fx)\right)^{5/2}\left(3(a+b)\,\text{AppellF1}\left(\right.\right.}$$

```
[In] Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^8*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5
/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -3, 5/2, 3/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -3, 7/2,
5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2,
-2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*
((15*a*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^
2)/(a + b)]*Cos[e + f*x]^7*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Sin[e + f*
x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*x]^2, (a*S
in[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x
]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1/
2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^7
)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -3,
5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
-3, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*Appel
lF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e +
 f*x]^2)) - (9*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(2*Sqrt[2]*(a + b - a*Si
n[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*x]
^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 5/2, 5/2, Sin
[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos
[e + f*x]^6*Sin[e + f*x]*((5*a*f*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - 2*f*
AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos
[e + f*x]*Sin[e + f*x]))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(
a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
b)] + (5*a*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)] - 6*(a + b)*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e +
f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -3, 5/2, 3/2,
```

```
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^6*Sin[e + f*x]*(2
*f*(5*a*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)] - 6*(a + b)*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x
]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a + b)*((5*a*f*AppellF1[3/2,
-3, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[
e + f*x])/(3*(a + b)) - 2*f*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*
Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x]) + Sin[e + f*x]^2*(5*a*(
(21*a*f*AppellF1[5/2, -3, 9/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b)) - (18*f*AppellF1[5/2, -2, 7/2,
7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])
/5) - 6*(a + b)*((3*a*f*AppellF1[5/2, -2, 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[
e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (12*f*AppellF1[5/
2, -1, 5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*S
in[e + f*x])/5))))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)
*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] +
(5*a*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)
] - 6*(a + b)*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)])*Sin[e + f*x]^2)^2)))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2142 vs. $2(167) = 334$.

Time = 5.35 (sec) , antiderivative size = 2143, normalized size of antiderivative = 11.46

method	result	size
default	Expression too large to display	2143

```
[In] int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/f/(a+b)^2/a^3/(-a)^(1/2)*(b+a*cos(f*x+e)^2)*(-15*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-
4*sin(f*x+e)*a)*a*b^3*cos(f*x+e)^2+6*(-a)^(1/2)*a^3*b*cos(f*x+e)^2*sin(f*x+
e)+30*(-a)^(1/2)*a^2*b^2*cos(f*x+e)^2*sin(f*x+e)+20*(-a)^(1/2)*a*b^3*cos(f*
x+e)^2*sin(f*x+e)+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3*b*cos(f*x+e)-
9*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2*b^2*cos(f*x+e)-27*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)-4*sin(f*x+e)*a)*a*b^3*cos(f*x+e)+6*(-a)^(1/2)*a^3*b*cos(f*x+e)^4*s
in(f*x+e)+3*(-a)^(1/2)*a^2*b^2*cos(f*x+e)^4*sin(f*x+e)-9*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
```

```
)^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)-4*sin(f*x+e)*a)*a^3*b*cos(f*x+e)^3-27*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(
f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e
)*a)*a^2*b^2*cos(f*x+e)^3-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln
(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^3*co
s(f*x+e)^3-9*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3*b*cos(f*x+e)^2+15*(-
a)^(1/2)*b^4*sin(f*x+e)-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4
*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^4+3*(-a)
^(1/2)*a^2*b^2*sin(f*x+e)+22*(-a)^(1/2)*a*b^3*sin(f*x+e)+3*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)-4*sin(f*x+e)*a)*a^4*cos(f*x+e)^3+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f
*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)
*a)*a^4*cos(f*x+e)^2-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-
a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^4*cos(f*x+e
)+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3*b-9*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*s
in(f*x+e)*a)*a^2*b^2-27*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-
a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^3+3*(-a)^
(1/2)*a^4*cos(f*x+e)^4*sin(f*x+e)-27*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e
)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*
a^2*b^2*cos(f*x+e)^2)/(a+b*sec(f*x+e)^2)^(5/2)*sec(f*x+e)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(167) = 334$.

Time = 1.78 (sec) , antiderivative size = 1023, normalized size of antiderivative = 5.47

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(a^3*b^2 - 3*a^2*b^3 - 9*a*b^4 - 5*b^5 + (a^5 - 3*a^4*b - 9*a^3*b^
2 - 5*a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 - 9*a^2*b^3 - 5*a*b^4)
*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*co
s(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*
a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3
)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)
^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b
^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*sin(f*x + e)) + 8*(3*(a^5 + 2*a^4*b + a^3*b^2)*cos(f*x + e)^5 + 2*(3*a^4*
b + 15*a^3*b^2 + 10*a^2*b^3)*cos(f*x + e)^3 + (3*a^3*b^2 + 22*a^2*b^3 + 15*
a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
e))/((a^8 + 2*a^7*b + a^6*b^2)*f*cos(f*x + e)^4 + 2*(a^7*b + 2*a^6*b^2 + a^
5*b^3)*f*cos(f*x + e)^2 + (a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*f), -1/24*(3*(a^3
*b^2 - 3*a^2*b^3 - 9*a*b^4 - 5*b^5 + (a^5 - 3*a^4*b - 9*a^3*b^2 - 5*a^2*b^3
)*cos(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 - 9*a^2*b^3 - 5*a*b^4)*cos(f*x + e)
^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3
 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(
f*x + e)^2)*sin(f*x + e))) - 4*(3*(a^5 + 2*a^4*b + a^3*b^2)*cos(f*x + e)^5
+ 2*(3*a^4*b + 15*a^3*b^2 + 10*a^2*b^3)*cos(f*x + e)^3 + (3*a^3*b^2 + 22*a^
2*b^3 + 15*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e))/((a^8 + 2*a^7*b + a^6*b^2)*f*cos(f*x + e)^4 + 2*(a^7*b + 2*a
^6*b^2 + a^5*b^3)*f*cos(f*x + e)^2 + (a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*f)]
```

Sympy [F]

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos^2\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(cos(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)^2}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(e+f\,x)^2}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

```
[In] int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.294 $\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 261

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{(3a^2-10ab+35b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{9/2}f} + \frac{(3a-7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(9a^2-18ab-35b^2)\tan(e+fx)}{24a^3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(9a^3-15a^2b-145ab^2-105b^3)\tan(e+fx)}{24a^4(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] 1/8*(3*a^2-10*a*b+35*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1
/2))/a^(9/2)/f+1/24*b*(9*a^3-15*a^2*b-145*a*b^2-105*b^3)*tan(f*x+e)/a^4/(a+
b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/8*(3*a-7*b)*cos(f*x+e)*sin(f*x+e)/a^2/f
/(a+b+b*tan(f*x+e)^2)^(3/2)+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(f*x+
e)^2)^(3/2)+1/24*b*(9*a^2-18*a*b-35*b^2)*tan(f*x+e)/a^3/(a+b)/f/(a+b+b*tan(
f*x+e)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4231, 425, 541, 12, 385, 209}

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{(3a-7b)\sin(e+fx)\cos(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)+b\right)^{3/2}} + \frac{(3a^2-10ab+35b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{9/2}f} + \frac{b(9a^2-18ab-35b^2)\tan(e+fx)}{24a^3f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}} + \frac{b(9a^3-15a^2b-145ab^2-105b^3)\tan(e+fx)}{24a^4f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}} + \frac{\sin(e+fx)\cos^3(e+fx)}{4af\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[In] Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((3*a^2 - 10*a*b + 35*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan
[e + f*x]^2]])/(8*a^(9/2)*f) + ((3*a - 7*b)*Cos[e + f*x]*Sin[e + f*x])/(8*a
^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a
*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (b*(9*a^2 - 18*a*b - 35*b^2)*Tan[e +
 f*x])/(24*a^3*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (b*(9*a^3 - 15
*a^2*b - 145*a*b^2 - 105*b^3)*Tan[e + f*x])/(24*a^4*(a + b)^2*f*Sqrt[a + b
+ b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-3a+b-6bx^2}{(1+x^2)^2(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{4af} \\
&= \frac{(3a-7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} \\
&\quad + \frac{\text{Subst}\left(\int \frac{3a^2+2ab+7b^2+4(3a-7b)bx^2}{(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{8a^2f} \\
&= \frac{(3a-7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} \\
&\quad + \frac{b(9a^2-18ab-35b^2)\tan(e+fx)}{24a^3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} \\
&\quad + \frac{\text{Subst}\left(\int \frac{9a^3-3a^2b+39ab^2+35b^3+2b(9a^2-18ab-35b^2)x^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{24a^3(a+b)f}
\end{aligned}$$

$$= \frac{(3a-7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(9a^2-18ab-35b^2)\tan(e+fx)}{24a^3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(9a^3-15a^2b-145ab^2-105b^3)\tan(e+fx)}{24a^4(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{3(a+b)^2(3a^2-10ab+35b^2)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{24a^4(a+b)^2f}$$

$$= \frac{(3a-7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(9a^2-18ab-35b^2)\tan(e+fx)}{24a^3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(9a^3-15a^2b-145ab^2-105b^3)\tan(e+fx)}{24a^4(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a^2-10ab+35b^2)\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{8a^4f}$$

$$= \frac{(3a-7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(9a^2-18ab-35b^2)\tan(e+fx)}{24a^3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(9a^3-15a^2b-145ab^2-105b^3)\tan(e+fx)}{24a^4(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a^2-10ab+35b^2)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^4f}$$

$$= \frac{(3a^2-10ab+35b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{9/2}f} + \frac{(3a-7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(9a^2-18ab-35b^2)\tan(e+fx)}{24a^3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(9a^3-15a^2b-145ab^2-105b^3)\tan(e+fx)}{24a^4(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 18.42 (sec) , antiderivative size = 1777, normalized size of antiderivative = 6.81

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{}{4\sqrt{2}f\,(a+b\sec^2(e+fx))^{5/2}\,(a+b-a\sin^2(e+fx))^{5/2}\,\Big(3(a+b)\,\text{AppellF1}}$$

```
[In] Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^12*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(
5/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -4, 5/2, 3/2
, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -4, 7/2,
 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2,
 -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)
*((15*a*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]
^2)/(a + b)]*Cos[e + f*x]^9*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Sin[e + f
*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[
e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -4, 7/2, 5/2, Sin[e + f*x]^2, (a*
Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*
x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1
/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^
9)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -4,
 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
 -4, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*Appe
llF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e
+ f*x]^2)) - (3*Sqrt[2]*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^7*Sin[e + f*x]^2)/((a + b - a*Sin
[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -4, 7/2, 5/2, Sin[e + f*x]^
2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 5/2, 5/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos[
e + f*x]^8*Sin[e + f*x]*((5*a*f*AppellF1[3/2, -4, 7/2, 5/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - (8*f*
AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos
[e + f*x]*Sin[e + f*x])/3))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(
3*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
 + b)] + (5*a*AppellF1[3/2, -4, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -4, 5/2, 3
```

```
/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^8*Sin[e + f*x]
*(2*f*(5*a*AppellF1[3/2, -4, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)] - 8*(a + b)*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e +
f*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a + b)*((5*a*f*AppellF1[3/
2, -4, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*S
in[e + f*x])/(3*(a + b)) - (8*f*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3) + Sin[e + f*x]^2
*(5*a*((21*a*f*AppellF1[5/2, -4, 9/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^
2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b)) - (24*f*AppellF1[5/2, -3
, 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e
+ f*x])/5) - 8*(a + b)*((3*a*f*AppellF1[5/2, -3, 7/2, 7/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (18*f*Appe
llF1[5/2, -2, 5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e +
 f*x]*Sin[e + f*x])/5))))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*
(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)] + (5*a*AppellF1[3/2, -4, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e +
 f*x]^2)/(a + b)])*Sin[e + f*x]^2)^2)))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2769 vs. $2(237) = 474$.

Time = 8.14 (sec) , antiderivative size = 2770, normalized size of antiderivative = 10.61

method	result	size
default	Expression too large to display	2770

```
[In] int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/f/a^4/(-a)^(1/2)/(a+b)^2*(b+a*cos(f*x+e)^2)*(-105*(-a)^(1/2)*b^5*sin(f
*x+e)+180*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f
*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)
*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b^3*cos(f*x+e)^2+105*ln
(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^4*cos(f*x+e)^2+18*(-a)^(1/2)*a^4*b*
cos(f*x+e)^2*sin(f*x+e)-24*(-a)^(1/2)*a^3*b^2*cos(f*x+e)^2*sin(f*x+e)-198*(
-a)^(1/2)*a^2*b^3*cos(f*x+e)^2*sin(f*x+e)-140*(-a)^(1/2)*a*b^4*cos(f*x+e)^2
*sin(f*x+e)+9*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*c
os(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*
x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^4*b*cos(f*x+e)-12*ln(
4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^
(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b^2*cos(f*x+e)+54*ln(4*(-a)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos
```

(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b^3*cos(f*x+e)+180*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^4*cos(f*x+e)+12*(-a)^(1/2)*a^4*b*cos(f*x+e)^6*sin(f*x+e)+6*(-a)^(1/2)*a^3*b^2*cos(f*x+e)^6*sin(f*x+e)-3*(-a)^(1/2)*a^4*b*cos(f*x+e)^4*sin(f*x+e)-33*(-a)^(1/2)*a^3*b^2*cos(f*x+e)^4*sin(f*x+e)-21*(-a)^(1/2)*a^2*b^3*cos(f*x+e)^4*sin(f*x+e)-12*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^4*b*cos(f*x+e)^3+54*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b^2*cos(f*x+e)^3+180*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b^3*cos(f*x+e)^3+105*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^4*cos(f*x+e)^3-12*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^4*b*cos(f*x+e)^2+54*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b^2*cos(f*x+e)^2+9*(-a)^(1/2)*a^3*b^2*sin(f*x+e)-15*(-a)^(1/2)*a^2*b^3*sin(f*x+e)-145*(-a)^(1/2)*a*b^4*sin(f*x+e)+6*(-a)^(1/2)*a^5*cos(f*x+e)^6*sin(f*x+e)+9*(-a)^(1/2)*a^5*cos(f*x+e)^4*sin(f*x+e)+9*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^5*cos(f*x+e)^3+9*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^5*cos(f*x+e)^2+105*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^5*cos(f*x+e)+9*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^4*b-12*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b^2+54*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b^3+180*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2

```
)/(1+cos(f*x+e))^2)^(1/2)*a*b^4+105*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b
^5)/(a+b*sec(f*x+e)^2)^(5/2)*sec(f*x+e)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(237) = 474$.

Time = 5.42 (sec) , antiderivative size = 1187, normalized size of antiderivative = 4.55

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
[Out] [-1/192*(3*(3*a^4*b^2 - 4*a^3*b^3 + 18*a^2*b^4 + 60*a*b^5 + 35*b^6 + (3*a^6
 - 4*a^5*b + 18*a^4*b^2 + 60*a^3*b^3 + 35*a^2*b^4)*cos(f*x + e)^4 + 2*(3*a^
5*b - 4*a^4*b^2 + 18*a^3*b^3 + 60*a^2*b^4 + 35*a*b^5)*cos(f*x + e)^2)*sqrt(
-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a
^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 2
8*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(
16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2
*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e)
)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(6
*(a^6 + 2*a^5*b + a^4*b^2)*cos(f*x + e)^7 + 3*(3*a^6 - a^5*b - 11*a^4*b^2 -
 7*a^3*b^3)*cos(f*x + e)^5 + 2*(9*a^5*b - 12*a^4*b^2 - 99*a^3*b^3 - 70*a^2*
b^4)*cos(f*x + e)^3 + (9*a^4*b^2 - 15*a^3*b^3 - 145*a^2*b^4 - 105*a*b^5)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^9
 + 2*a^8*b + a^7*b^2)*f*cos(f*x + e)^4 + 2*(a^8*b + 2*a^7*b^2 + a^6*b^3)*f*
cos(f*x + e)^2 + (a^7*b^2 + 2*a^6*b^3 + a^5*b^4)*f), -1/96*(3*(3*a^4*b^2 -
4*a^3*b^3 + 18*a^2*b^4 + 60*a*b^5 + 35*b^6 + (3*a^6 - 4*a^5*b + 18*a^4*b^2
+ 60*a^3*b^3 + 35*a^2*b^4)*cos(f*x + e)^4 + 2*(3*a^5*b - 4*a^4*b^2 + 18*a^3
*b^3 + 60*a^2*b^4 + 35*a*b^5)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos
(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x +
e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e
)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(6
*(a^6 + 2*a^5*b + a^4*b^2)*cos(f*x + e)^7 + 3*(3*a^6 - a^5*b - 11*a^4*b^2 -
 7*a^3*b^3)*cos(f*x + e)^5 + 2*(9*a^5*b - 12*a^4*b^2 - 99*a^3*b^3 - 70*a^2*
b^4)*cos(f*x + e)^3 + (9*a^4*b^2 - 15*a^3*b^3 - 145*a^2*b^4 - 105*a*b^5)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^9
 + 2*a^8*b + a^7*b^2)*f*cos(f*x + e)^4 + 2*(a^8*b + 2*a^7*b^2 + a^6*b^3)*f*
cos(f*x + e)^2 + (a^7*b^2 + 2*a^6*b^3 + a^5*b^4)*f)]
```

Sympy [F]

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(cos(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)^4}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)^4}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(e+f\,x)^4}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

```
[In] int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.295 $\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 332

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{5(a-3b)(a^2+7b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{11/2}f} + \frac{(5a^2-10ab+21b^2)\cos(e+fx)\sin(e+fx)}{16a^3f\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{(5a-9b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{b(15a^3-25a^2b+49ab^2+105b^3)\tan(e+fx)}{48a^4(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{b(15a^4-20a^3b+38a^2b^2+420ab^3+315b^4)\tan(e+fx)}{48a^5(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] 5/16*(a-3*b)*(a^2+7*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/
2))/a^(11/2)/f+1/48*b*(15*a^4-20*a^3*b+38*a^2*b^2+420*a*b^3+315*b^4)*tan(f*
x+e)/a^5/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/16*(5*a^2-10*a*b+21*b^2)*co
s(f*x+e)*sin(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)^(3/2)+1/24*(5*a-9*b)*cos(f*x
+e)^3*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^(3/2)+1/6*cos(f*x+e)^5*sin(f*x+
e)/a/f/(a+b+b*tan(f*x+e)^2)^(3/2)+1/48*b*(15*a^3-25*a^2*b+49*a*b^2+105*b^3)
*tan(f*x+e)/a^4/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4231, 425, 541, 12, 385, 209}

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{(5a-9b)\sin(e+fx)\cos^3(e+fx)}{24a^2f\left(a+b\tan^2(e+fx)+b\right)^{3/2}} + \frac{5(a-3b)\left(a^2+7b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{11/2}f} + \frac{(5a^2-10ab+21b^2)\sin(e+fx)\cos(e+fx)}{16a^3f\left(a+b\tan^2(e+fx)+b\right)^{3/2}} + \frac{b(15a^3-25a^2b+49ab^2+105b^3)\tan(e+fx)}{48a^4f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}} + \frac{b(15a^4-20a^3b+38a^2b^2+420ab^3+315b^4)\tan(e+fx)}{48a^5f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}} + \frac{\sin(e+fx)\cos^5(e+fx)}{6af\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[In] Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (5*(a - 3*b)*(a^2 + 7*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan
[e + f*x]^2]])/(16*a^(11/2)*f) + ((5*a^2 - 10*a*b + 21*b^2)*Cos[e + f*x]*Si
n[e + f*x])/(16*a^3*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((5*a - 9*b)*Cos[
e + f*x]^3*Sin[e + f*x])/(24*a^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (Cos
[e + f*x]^5*Sin[e + f*x])/(6*a*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (b*(15
*a^3 - 25*a^2*b + 49*a*b^2 + 105*b^3)*Tan[e + f*x])/(48*a^4*(a + b)*f*(a +
b + b*Tan[e + f*x]^2)^(3/2)) + (b*(15*a^4 - 20*a^3*b + 38*a^2*b^2 + 420*a*b
^3 + 315*b^4)*Tan[e + f*x])/(48*a^5*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]
^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^4 (a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-5a+b-8bx^2}{(1+x^2)^3 (a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{6af}$$

$$= \frac{(5a-9b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3(5a^2+3b^2)+6(5a-9b)bx^2}{(1+x^2)^2 (a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{24a^2f}$$

$$= \frac{(5a^2-10ab+21b^2)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(5a-9b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-3(5a^3+5a^2b-5ab^2-21b^3)-12b(5a^2-10ab+21b^2)x^2}{(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{48a^3f}$$

$$= \frac{(5a^2-10ab+21b^2)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(5a-9b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(15a^3-25a^2b+49ab^2+105b^3)\tan(e+fx)}{48a^4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-3(15a^4+10a^2b^2-112ab^3-105b^4)-6b(15a^3-25a^2b+49ab^2+105b^3)x^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{144a^4(a+b)f}$$

$$= \frac{(5a^2-10ab+21b^2)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(5a-9b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(15a^3-25a^2b+49ab^2+105b^3)\tan(e+fx)}{48a^4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(15a^4-20a^3b+38a^2b^2+420ab^3+315b^4)\tan(e+fx)}{48a^5(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int -\frac{45(a-3b)(a+b)^2(a^2+7b^2)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{144a^5(a+b)^2f}$$

$$= \frac{(5a^2-10ab+21b^2)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(5a-9b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(15a^3-25a^2b+49ab^2+105b^3)\tan(e+fx)}{48a^4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(15a^4-20a^3b+38a^2b^2+420ab^3+315b^4)\tan(e+fx)}{48a^5(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5(a-3b)(a^2+7b^2))\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{16a^5f}$$

$$= \frac{(5a^2-10ab+21b^2)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(5a-9b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(15a^3-25a^2b+49ab^2+105b^3)\tan(e+fx)}{48a^4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(15a^4-20a^3b+38a^2b^2+420ab^3+315b^4)\tan(e+fx)}{48a^5(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5(a-3b)(a^2+7b^2))\,\mathrm{Subst}\left(\int\frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^5f}$$

$$= \frac{5(a-3b)(a^2+7b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{11/2}f} + \frac{(5a^2-10ab+21b^2)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(5a-9b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(15a^3-25a^2b+49ab^2+105b^3)\tan(e+fx)}{48a^4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(15a^4-20a^3b+38a^2b^2+420ab^3+315b^4)\tan(e+fx)}{48a^5(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 21.31 (sec) , antiderivative size = 1776, normalized size of antiderivative = 5.35

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{}{4\sqrt{2}f\left(a+b\sec^2(e+fx)\right)^{5/2}\left(a+b-a\sin^2(e+fx)\right)^{5/2}\left(3(a+b)\,\mathrm{AppellF1}\left(\right.\right.}$$

```
[In] Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^16*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(
5/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2
, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 5*(a*AppellF1[3/2, -5, 7/2,
 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2,
 -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)
*((15*a*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]
```

^2)/(a + b)]*Cos[e + f*x]^11*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 5*(a*AppellF1[3/2, -5, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^11)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 5*(a*AppellF1[3/2, -5, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (15*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^9*Sin[e + f*x]^2)/(2*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 5*(a*AppellF1[3/2, -5, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos[e + f*x]^10*Sin[e + f*x]*((5*a*f*AppellF1[3/2, -5, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - (10*f*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 5*(a*AppellF1[3/2, -5, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^10*Sin[e + f*x]*(10*f*(a*AppellF1[3/2, -5, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a + b)*((5*a*f*AppellF1[3/2, -5, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - (10*f*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3) + 5*Sin[e + f*x]^2*(a*((21*a*f*AppellF1[5/2, -5, 9/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b)) - 6*f*AppellF1[5/2, -4, 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x]) - 2*(a + b)*((3*a*f*AppellF1[5/2, -4, 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (24*f*AppellF1[5/2, -3, 5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/5))))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 5*(a*AppellF1[3/2, -5, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)^2)))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3398 vs. $2(304) = 608$.

Time = 10.81 (sec) , antiderivative size = 3399, normalized size of antiderivative = 10.24

method	result	size
default	Expression too large to display	3399

```
[In] int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/f/a^5/(-a)^(1/2)/(a+b)^2*(b+a*cos(f*x+e)^2)*(315*(-a)^(1/2)*b^6*sin(f*
x+e)-315*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^6+420*(-a)^(1/2)*a*b^5*cos
(f*x+e)^2*sin(f*x+e)-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-
a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^5*b*cos(f*x
+e)^2+30*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^4*b^2*cos(f*x+e)^2-150*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3*b^3*cos(f*x+e)^2-525*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)-4*sin(f*x+e)*a)*a^2*b^4*cos(f*x+e)^2-315*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin
(f*x+e)*a)*a*b^5*cos(f*x+e)^2+15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*
(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^5*
b*cos(f*x+e)+16*(-a)^(1/2)*a^5*b*cos(f*x+e)^8*sin(f*x+e)+8*(-a)^(1/2)*a^4*b
^2*cos(f*x+e)^8*sin(f*x+e)+2*(-a)^(1/2)*a^5*b*cos(f*x+e)^6*sin(f*x+e)-26*(-
a)^(1/2)*a^4*b^2*cos(f*x+e)^6*sin(f*x+e)-18*(-a)^(1/2)*a^3*b^3*cos(f*x+e)^6
*sin(f*x+e)+18*(-a)^(1/2)*a^4*b^2*cos(f*x+e)^4*sin(f*x+e)+96*(-a)^(1/2)*a^3
*b^3*cos(f*x+e)^4*sin(f*x+e)-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(
-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^4*b
^2*cos(f*x+e)+30*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3*b^3*cos(f*x+e)-1
50*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2*b^4*cos(f*x+e)-525*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
```

(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^5*cos(f*x+e)+63*(-a)^(1/2)*a^2*b^4*cos(f*x+e)^4*sin(f*x+e)-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^5*b*cos(f*x+e)^3+30*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^4*b^2*cos(f*x+e)^3-150*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3*b^3*cos(f*x+e)^3-525*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2*b^4*cos(f*x+e)^3-315*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^5*cos(f*x+e)^3+30*(-a)^(1/2)*a^5*b*cos(f*x+e)^2*sin(f*x+e)-30*(-a)^(1/2)*a^4*b^2*cos(f*x+e)^2*sin(f*x+e)+62*(-a)^(1/2)*a^3*b^3*cos(f*x+e)^2*sin(f*x+e)+574*(-a)^(1/2)*a^2*b^4*cos(f*x+e)^2*sin(f*x+e)+15*(-a)^(1/2)*a^4*b^2*sin(f*x+e)-20*(-a)^(1/2)*a^3*b^3*sin(f*x+e)+38*(-a)^(1/2)*a^2*b^4*sin(f*x+e)+420*(-a)^(1/2)*a*b^5*sin(f*x+e)+8*(-a)^(1/2)*a^6*cos(f*x+e)^8*sin(f*x+e)+10*(-a)^(1/2)*a^6*cos(f*x+e)^6*sin(f*x+e)+15*(-a)^(1/2)*a^6*cos(f*x+e)^4*sin(f*x+e)+15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^6*cos(f*x+e)^3+15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^6*cos(f*x+e)^2-315*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^6*cos(f*x+e)+15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^5*b-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^4*b^2+30*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3*b^3-150*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2*b^4-525*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^5)/(a+b*sec(f*x+e)^2)^(5/2)*sec(f*x+e)^5

Fricas [A] (verification not implemented)

none

Time = 16.21 (sec) , antiderivative size = 1337, normalized size of antiderivative = 4.03

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/384*(15*(a^5*b^2 - a^4*b^3 + 2*a^3*b^4 - 10*a^2*b^5 - 35*a*b^6 - 21*b^7
+ (a^7 - a^6*b + 2*a^5*b^2 - 10*a^4*b^3 - 35*a^3*b^4 - 21*a^2*b^5)*cos(f*x
+ e)^4 + 2*(a^6*b - a^5*b^2 + 2*a^4*b^3 - 10*a^3*b^4 - 35*a^2*b^5 - 21*a*b^
6)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*
cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 2
8*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b
^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x +
e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a
*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e)) + 8*(8*(a^7 + 2*a^6*b + a^5*b^2)*cos(f*x + e)^9 + 2*(5*a^
7 + a^6*b - 13*a^5*b^2 - 9*a^4*b^3)*cos(f*x + e)^7 + 3*(5*a^7 + 6*a^5*b^2 +
 32*a^4*b^3 + 21*a^3*b^4)*cos(f*x + e)^5 + 2*(15*a^6*b - 15*a^5*b^2 + 31*a^
4*b^3 + 287*a^3*b^4 + 210*a^2*b^5)*cos(f*x + e)^3 + (15*a^5*b^2 - 20*a^4*b^
3 + 38*a^3*b^4 + 420*a^2*b^5 + 315*a*b^6)*cos(f*x + e))*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^10 + 2*a^9*b + a^8*b^2)*f*cos(f*
x + e)^4 + 2*(a^9*b + 2*a^8*b^2 + a^7*b^3)*f*cos(f*x + e)^2 + (a^8*b^2 + 2*
a^7*b^3 + a^6*b^4)*f), -1/192*(15*(a^5*b^2 - a^4*b^3 + 2*a^3*b^4 - 10*a^2*b
^5 - 35*a*b^6 - 21*b^7 + (a^7 - a^6*b + 2*a^5*b^2 - 10*a^4*b^3 - 35*a^3*b^4
 - 21*a^2*b^5)*cos(f*x + e)^4 + 2*(a^6*b - a^5*b^2 + 2*a^4*b^3 - 10*a^3*b^4
 - 35*a^2*b^5 - 21*a*b^6)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x
 + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*
sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4
- a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(8*(a^
7 + 2*a^6*b + a^5*b^2)*cos(f*x + e)^9 + 2*(5*a^7 + a^6*b - 13*a^5*b^2 - 9*a
^4*b^3)*cos(f*x + e)^7 + 3*(5*a^7 + 6*a^5*b^2 + 32*a^4*b^3 + 21*a^3*b^4)*co
s(f*x + e)^5 + 2*(15*a^6*b - 15*a^5*b^2 + 31*a^4*b^3 + 287*a^3*b^4 + 210*a^
2*b^5)*cos(f*x + e)^3 + (15*a^5*b^2 - 20*a^4*b^3 + 38*a^3*b^4 + 420*a^2*b^5
 + 315*a*b^6)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))/((a^10 + 2*a^9*b + a^8*b^2)*f*cos(f*x + e)^4 + 2*(a^9*b + 2*a^8*
b^2 + a^7*b^3)*f*cos(f*x + e)^2 + (a^8*b^2 + 2*a^7*b^3 + a^6*b^4)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos\left(fx+e\right)^6}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos\left(fx+e\right)^6}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos\left(e+f\,x\right)^6}{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{5/2}}\,dx$$

```
[In] int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.296 $\int \frac{1}{\left(a+b\sec^2(c+dx)\right)^{7/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 179

$$\int \frac{1}{(a+b\sec^2(c+dx))^{7/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+b+b\tan^2(c+dx)}}\right)}{a^{7/2}d} - \frac{b\tan(c+dx)}{5a(a+b)d\,(a+b+b\tan^2(c+dx))^{5/2}} - \frac{b(9a+5b)\tan(c+dx)}{15a^2(a+b)^2d\,(a+b+b\tan^2(c+dx))^{3/2}} - \frac{b(33a^2+40ab+15b^2)\tan(c+dx)}{15a^3(a+b)^3d\sqrt{a+b+b\tan^2(c+dx)}}$$

```
[Out] arctan(a^(1/2)*tan(d*x+c)/(a+b+b*tan(d*x+c)^2)^(1/2))/a^(7/2)/d-1/15*b*(33*
a^2+40*a*b+15*b^2)*tan(d*x+c)/a^3/(a+b)^3/d/(a+b+b*tan(d*x+c)^2)^(1/2)-1/5*
b*tan(d*x+c)/a/(a+b)/d/(a+b+b*tan(d*x+c)^2)^(5/2)-1/15*b*(9*a+5*b)*tan(d*x+
c)/a^2/(a+b)^2/d/(a+b+b*tan(d*x+c)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4213, 425, 541, 12, 385, 209}

$$\int \frac{1}{(a+b\sec^2(c+dx))^{7/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)+b}}\right)}{a^{7/2}d} - \frac{b(9a+5b)\tan(c+dx)}{15a^2d(a+b)^2\,(a+b\tan^2(c+dx)+b)^{3/2}} - \frac{b(33a^2+40ab+15b^2)\tan(c+dx)}{15a^3d(a+b)^3\sqrt{a+b\tan^2(c+dx)+b}} - \frac{b\tan(c+dx)}{5ad(a+b)\,(a+b\tan^2(c+dx)+b)^{5/2}}$$

```
[In] Int[(a + b*Sec[c + d*x]^2)^(-7/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + b + b*Tan[c + d*x]^2]]/(a^(7/2)*d) -
 (b*Tan[c + d*x])/(5*a*(a + b)*d*(a + b + b*Tan[c + d*x]^2)^(5/2)) - (b*(9*
a + 5*b)*Tan[c + d*x])/(15*a^2*(a + b)^2*d*(a + b + b*Tan[c + d*x]^2)^(3/2)
) - (b*(33*a^2 + 40*a*b + 15*b^2)*Tan[c + d*x])/(15*a^3*(a + b)^3*d*Sqrt[a
+ b + b*Tan[c + d*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
```

```
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^{7/2}}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{b\tan(c+dx)}{5a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{5a+b-4bx^2}{(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(c+dx)\right)}{5a(a+b)d}$$

$$= -\frac{b\tan(c+dx)}{5a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^{5/2}} - \frac{b(9a+5b)\tan(c+dx)}{15a^2(a+b)^2d\left(a+b+b\tan^2(c+dx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{15a^2+12ab+5b^2-2b(9a+5b)x^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(c+dx)\right)}{15a^2(a+b)^2d}$$

$$= -\frac{b\tan(c+dx)}{5a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^{5/2}} - \frac{b(9a+5b)\tan(c+dx)}{15a^2(a+b)^2d\left(a+b+b\tan^2(c+dx)\right)^{3/2}} - \frac{b(33a^2+40ab+15b^2)\tan(c+dx)}{15a^3(a+b)^3d\sqrt{a+b+b\tan^2(c+dx)}} + \frac{\text{Subst}\left(\int \frac{15(a+b)^3}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(c+dx)\right)}{15a^3(a+b)^3d}$$

$$= -\frac{b\tan(c+dx)}{5a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^{5/2}} - \frac{b(9a+5b)\tan(c+dx)}{15a^2(a+b)^2d\left(a+b+b\tan^2(c+dx)\right)^{3/2}} - \frac{b(33a^2+40ab+15b^2)\tan(c+dx)}{15a^3(a+b)^3d\sqrt{a+b+b\tan^2(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(c+dx)\right)}{a^3d}$$

$$= -\frac{b\tan(c+dx)}{5a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^{5/2}} - \frac{b(9a+5b)\tan(c+dx)}{15a^2(a+b)^2d\left(a+b+b\tan^2(c+dx)\right)^{3/2}} - \frac{b(33a^2+40ab+15b^2)\tan(c+dx)}{15a^3(a+b)^3d\sqrt{a+b+b\tan^2(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(c+dx)}{\sqrt{a+b+b\tan^2(c+dx)}}\right)}{a^3d}$$

$$= \frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+b+b\tan^2(c+dx)}}\right)}{a^{7/2}d} - \frac{b\tan(c+dx)}{5a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^{5/2}} - \frac{b(9a+5b)\tan(c+dx)}{15a^2(a+b)^2d\left(a+b+b\tan^2(c+dx)\right)^{3/2}} - \frac{b(33a^2+40ab+15b^2)\tan(c+dx)}{15a^3(a+b)^3d\sqrt{a+b+b\tan^2(c+dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 16.81 (sec) , antiderivative size = 1777, normalized size of antiderivative = 9.93

$$\int \frac{1}{(a+b\sec^2(c+dx))^{7/2}}\,dx = \frac{}{8\sqrt{2}d\,(a+b\sec^2(c+dx))^{7/2}\,(a+b-a\sin^2(c+dx))^{7/2}\left(3(a+b)\,\text{AppellF1}\right.}$$

```
[In] Integrate[(a + b*Sec[c + d*x]^2)^(-7/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(
a + b)]*Cos[c + d*x]^6*Sin[c + d*x])/(8*Sqrt[2]*d*(a + b*Sec[c + d*x]^2)^(7
/2)*(a + b - a*Sin[c + d*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -3, 7/2, 3/2,
 Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)] + (7*a*AppellF1[3/2, -3, 9/2,
5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2,
-2, 7/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)])*Sin[c + d*x]^2)*
((21*a*(a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^
2)/(a + b)]*Cos[c + d*x]^7*Sin[c + d*x]^2)/(8*Sqrt[2]*(a + b - a*Sin[c + d*
x]^2)^(9/2)*(3*(a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c
 + d*x]^2)/(a + b)] + (7*a*AppellF1[3/2, -3, 9/2, 5/2, Sin[c + d*x]^2, (a*S
in[c + d*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 7/2, 5/2, Sin[c + d*x
]^2, (a*Sin[c + d*x]^2)/(a + b)])*Sin[c + d*x]^2)) + (3*(a + b)*AppellF1[1/
2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)]*Cos[c + d*x]^7
)/(8*Sqrt[2]*(a + b - a*Sin[c + d*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -3,
7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)] + (7*a*AppellF1[3/2,
-3, 9/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)] - 6*(a + b)*Appel
lF1[3/2, -2, 7/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)])*Sin[c +
 d*x]^2)) - (9*(a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c
 + d*x]^2)/(a + b)]*Cos[c + d*x]^5*Sin[c + d*x]^2)/(4*Sqrt[2]*(a + b - a*Si
n[c + d*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2,
(a*Sin[c + d*x]^2)/(a + b)] + (7*a*AppellF1[3/2, -3, 9/2, 5/2, Sin[c + d*x]
^2, (a*Sin[c + d*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 7/2, 5/2, Sin
[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)])*Sin[c + d*x]^2)) + (3*(a + b)*Cos
[c + d*x]^6*Sin[c + d*x]*((7*a*d*AppellF1[3/2, -3, 9/2, 5/2, Sin[c + d*x]^2
, (a*Sin[c + d*x]^2)/(a + b)]*Cos[c + d*x]*Sin[c + d*x])/(3*(a + b)) - 2*d*
AppellF1[3/2, -2, 7/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)]*Cos
[c + d*x]*Sin[c + d*x]))/(8*Sqrt[2]*d*(a + b - a*Sin[c + d*x]^2)^(7/2)*(3*(
a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a +
b)] + (7*a*AppellF1[3/2, -3, 9/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(
a + b)] - 6*(a + b)*AppellF1[3/2, -2, 7/2, 5/2, Sin[c + d*x]^2, (a*Sin[c +
d*x]^2)/(a + b)])*Sin[c + d*x]^2)) - (3*(a + b)*AppellF1[1/2, -3, 7/2, 3/2,
```

```
 Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)]*Cos[c + d*x]^6*Sin[c + d*x]*(2
*d*(7*a*AppellF1[3/2, -3, 9/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a +
 b)] - 6*(a + b)*AppellF1[3/2, -2, 7/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x
]^2)/(a + b)])*Cos[c + d*x]*Sin[c + d*x] + 3*(a + b)*((7*a*d*AppellF1[3/2,
-3, 9/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)]*Cos[c + d*x]*Sin[
c + d*x])/(3*(a + b)) - 2*d*AppellF1[3/2, -2, 7/2, 5/2, Sin[c + d*x]^2, (a*
Sin[c + d*x]^2)/(a + b)]*Cos[c + d*x]*Sin[c + d*x]) + Sin[c + d*x]^2*(7*a*(
(27*a*d*AppellF1[5/2, -3, 11/2, 7/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a
+ b)]*Cos[c + d*x]*Sin[c + d*x])/(5*(a + b)) - (18*d*AppellF1[5/2, -2, 9/2,
 7/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)]*Cos[c + d*x]*Sin[c + d*x]
)/5) - 6*(a + b)*((21*a*d*AppellF1[5/2, -2, 9/2, 7/2, Sin[c + d*x]^2, (a*Si
n[c + d*x]^2)/(a + b)]*Cos[c + d*x]*Sin[c + d*x])/(5*(a + b)) - (12*d*Appel
lF1[5/2, -1, 7/2, 7/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)]*Cos[c +
d*x]*Sin[c + d*x])/5))))/(8*Sqrt[2]*d*(a + b - a*Sin[c + d*x]^2)^(7/2)*(3*(
a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a +
b)] + (7*a*AppellF1[3/2, -3, 9/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(
a + b)] - 6*(a + b)*AppellF1[3/2, -2, 7/2, 5/2, Sin[c + d*x]^2, (a*Sin[c +
d*x]^2)/(a + b)])*Sin[c + d*x]^2)^2)))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2017 vs. $2(161) = 322$.

Time = 5.46 (sec) , antiderivative size = 2018, normalized size of antiderivative = 11.27

method	result	size
default	Expression too large to display	2018

```
[In] int(1/(a+b*sec(d*x+c)^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/d/(a+b)^3/a^3/(-a)^(1/2)*(a*(1-cos(d*x+c))^4*csc(d*x+c)^4+b*(1-cos(d*
x+c))^4*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+2*b*(1-cos(d*x+c))^2
*csc(d*x+c)^2+a+b)*(-30*(-a)^(1/2)*b^5*(-cot(d*x+c)+csc(d*x+c))+15*ln(4*((-
a)^(1/2)*(a*(1-cos(d*x+c))^4*csc(d*x+c)^4+b*(1-cos(d*x+c))^4*csc(d*x+c)^4-2
*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+2*b*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/
2)-2*a*(-cot(d*x+c)+csc(d*x+c)))/((1-cos(d*x+c))^2*csc(d*x+c)^2+1))*(a*(1-c
os(d*x+c))^4*csc(d*x+c)^4+b*(1-cos(d*x+c))^4*csc(d*x+c)^4-2*a*(1-cos(d*x+c)
)^2*csc(d*x+c)^2+2*b*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(5/2)*a^3+15*ln(4*(
(-a)^(1/2)*(a*(1-cos(d*x+c))^4*csc(d*x+c)^4+b*(1-cos(d*x+c))^4*csc(d*x+c)^4
-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+2*b*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(
1/2)-2*a*(-cot(d*x+c)+csc(d*x+c)))/((1-cos(d*x+c))^2*csc(d*x+c)^2+1))*(a*(1
-cos(d*x+c))^4*csc(d*x+c)^4+b*(1-cos(d*x+c))^4*csc(d*x+c)^4-2*a*(1-cos(d*x+
c))^2*csc(d*x+c)^2+2*b*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(5/2)*b^3-80*(-a)
^(1/2)*a^2*b^3*(1-cos(d*x+c))^7*csc(d*x+c)^7-320*(-a)^(1/2)*a*b^4*(1-cos(d*
x+c))^7*csc(d*x+c)^7-340*(-a)^(1/2)*a*b^4*(1-cos(d*x+c))^5*csc(d*x+c)^5+480
*(-a)^(1/2)*a^3*b^2*(1-cos(d*x+c))^3*csc(d*x+c)^3-80*(-a)^(1/2)*a^2*b^3*(1-
```

```
cos(d*x+c))^3*csc(d*x+c)^3+360*(-a)^(1/2)*a^4*b*(1-cos(d*x+c))^3*csc(d*x+c)
^3-320*(-a)^(1/2)*a*b^4*(1-cos(d*x+c))^3*csc(d*x+c)^3-300*(-a)^(1/2)*a^2*b^
3*(1-cos(d*x+c))^9*csc(d*x+c)^9-150*(-a)^(1/2)*a*b^4*(1-cos(d*x+c))^9*csc(d
*x+c)^9+480*(-a)^(1/2)*a^3*b^2*(1-cos(d*x+c))^7*csc(d*x+c)^7-90*(-a)^(1/2)*
a^4*b*(1-cos(d*x+c))^9*csc(d*x+c)^9-270*(-a)^(1/2)*a^3*b^2*(1-cos(d*x+c))^9
*csc(d*x+c)^9+360*(-a)^(1/2)*a^4*b*(1-cos(d*x+c))^7*csc(d*x+c)^7-540*(1-cos
(d*x+c))^5*a^4*(-a)^(1/2)*b*csc(d*x+c)^5-420*(-a)^(1/2)*a^3*b^2*(1-cos(d*x+
c))^5*csc(d*x+c)^5-296*(-a)^(1/2)*a^2*b^3*(1-cos(d*x+c))^5*csc(d*x+c)^5-180
*(-a)^(1/2)*b^5*(1-cos(d*x+c))^5*csc(d*x+c)^5-120*(-a)^(1/2)*b^5*(1-cos(d*x
+c))^3*csc(d*x+c)^3-270*(-a)^(1/2)*a^3*b^2*(-cot(d*x+c)+csc(d*x+c))+45*ln(4
*((-a)^(1/2)*(a*(1-cos(d*x+c))^4*csc(d*x+c)^4+b*(1-cos(d*x+c))^4*csc(d*x+c)
^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+2*b*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)
^(1/2)-2*a*(-cot(d*x+c)+csc(d*x+c)))/((1-cos(d*x+c))^2*csc(d*x+c)^2+1))*(a*
(1-cos(d*x+c))^4*csc(d*x+c)^4+b*(1-cos(d*x+c))^4*csc(d*x+c)^4-2*a*(1-cos(d*
x+c))^2*csc(d*x+c)^2+2*b*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(5/2)*a^2*b+45*
ln(4*((-a)^(1/2)*(a*(1-cos(d*x+c))^4*csc(d*x+c)^4+b*(1-cos(d*x+c))^4*csc(d*
x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+2*b*(1-cos(d*x+c))^2*csc(d*x+c)^2+
a+b)^(1/2)-2*a*(-cot(d*x+c)+csc(d*x+c)))/((1-cos(d*x+c))^2*csc(d*x+c)^2+1))
*(a*(1-cos(d*x+c))^4*csc(d*x+c)^4+b*(1-cos(d*x+c))^4*csc(d*x+c)^4-2*a*(1-co
s(d*x+c))^2*csc(d*x+c)^2+2*b*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(5/2)*a*b^2
-30*(-a)^(1/2)*b^5*(1-cos(d*x+c))^9*csc(d*x+c)^9-120*(-a)^(1/2)*b^5*(1-cos(
d*x+c))^7*csc(d*x+c)^7-300*(-a)^(1/2)*a^2*b^3*(-cot(d*x+c)+csc(d*x+c))-150*
(-a)^(1/2)*a*b^4*(-cot(d*x+c)+csc(d*x+c))-90*(-a)^(1/2)*a^4*b*(-cot(d*x+c)+
csc(d*x+c)))/((a*(1-cos(d*x+c))^4*csc(d*x+c)^4+b*(1-cos(d*x+c))^4*csc(d*x+c
)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+2*b*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b
)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^2)^(7/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2
-1)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 560 vs. $2(161) = 322$.

Time = 2.12 (sec) , antiderivative size = 1241, normalized size of antiderivative = 6.93

$$\int \frac{1}{\left(a+b\sec^2(c+dx)\right)^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*sec(d*x+c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/120*(15*((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cos(d*x + c)^6 + a^3*b^3
 + 3*a^2*b^4 + 3*a*b^5 + b^6 + 3*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*
cos(d*x + c)^4 + 3*(a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*cos(d*x + c)^2
)*sqrt(-a)*log(128*a^4*cos(d*x + c)^8 - 256*(a^4 - a^3*b)*cos(d*x + c)^6 +
32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(d*x + c)^4 + a^4 - 28*a^3*b + 70*a^2*
b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(d*x + c)^
```

```
2 + 8*(16*a^3*cos(d*x + c)^7 - 24*(a^3 - a^2*b)*cos(d*x + c)^5 + 2*(5*a^3 -
 14*a^2*b + 5*a*b^2)*cos(d*x + c)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(d
*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c))
 + 8*((45*a^5*b + 60*a^4*b^2 + 23*a^3*b^3)*cos(d*x + c)^5 + (75*a^4*b^2 + 9
4*a^3*b^3 + 35*a^2*b^4)*cos(d*x + c)^3 + (33*a^3*b^3 + 40*a^2*b^4 + 15*a*b^
5)*cos(d*x + c))*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c))/
((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*b^3)*d*cos(d*x + c)^6 + 3*(a^9*b + 3*a^8
*b^2 + 3*a^7*b^3 + a^6*b^4)*d*cos(d*x + c)^4 + 3*(a^8*b^2 + 3*a^7*b^3 + 3*a
^6*b^4 + a^5*b^5)*d*cos(d*x + c)^2 + (a^7*b^3 + 3*a^6*b^4 + 3*a^5*b^5 + a^4
*b^6)*d), -1/60*(15*((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cos(d*x + c)^6 +
 a^3*b^3 + 3*a^2*b^4 + 3*a*b^5 + b^6 + 3*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a
^2*b^4)*cos(d*x + c)^4 + 3*(a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*cos(d*
x + c)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(d*x + c)^5 - 8*(a^2 - a*b)*cos(d*x
+ c)^3 + (a^2 - 6*a*b + b^2)*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c)^2 +
 b)/cos(d*x + c)^2)/((2*a^3*cos(d*x + c)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b
)*cos(d*x + c)^2)*sin(d*x + c))) + 4*((45*a^5*b + 60*a^4*b^2 + 23*a^3*b^3)*
cos(d*x + c)^5 + (75*a^4*b^2 + 94*a^3*b^3 + 35*a^2*b^4)*cos(d*x + c)^3 + (3
3*a^3*b^3 + 40*a^2*b^4 + 15*a*b^5)*cos(d*x + c))*sqrt((a*cos(d*x + c)^2 + b
)/cos(d*x + c)^2)*sin(d*x + c))/((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*b^3)*d*c
os(d*x + c)^6 + 3*(a^9*b + 3*a^8*b^2 + 3*a^7*b^3 + a^6*b^4)*d*cos(d*x + c)^
4 + 3*(a^8*b^2 + 3*a^7*b^3 + 3*a^6*b^4 + a^5*b^5)*d*cos(d*x + c)^2 + (a^7*b
^3 + 3*a^6*b^4 + 3*a^5*b^5 + a^4*b^6)*d)]
```

Sympy [F]

$$\int \frac{1}{(a+b\sec^2(c+dx))^{7/2}}\,dx=\int \frac{1}{\left(a+b\sec^2\left(c+dx\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c)**2)**(7/2),x)
[Out] Integral((a + b*sec(c + d*x)**2)**(-7/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec^2(c+dx))^{7/2}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a+b*sec(d*x+c)^2)^(7/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{1}{\left(a+b\sec^2(c+dx)\right)^{7/2}}\,dx=\int \frac{1}{\left(b\sec\left(dx+c\right)^2+a\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c)^2)^(7/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c)^2 + a)^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sec^2(c+dx)\right)^{7/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)^2}\right)^{7/2}}\,dx$$

```
[In] int(1/(a + b/cos(c + d*x)^2)^(7/2),x)
[Out] int(1/(a + b/cos(c + d*x)^2)^(7/2), x)
```

3.297 $\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 14

$$\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx = \arctan\left(\frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right)$$

```
[Out] arctan(tan(x)/(2+tan(x)^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4213, 385, 209}

$$\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx = \arctan\left(\frac{\tan(x)}{\sqrt{\tan^2(x)+2}}\right)$$

```
[In] Int[1/Sqrt[1 + Sec[x]^2],x]
[Out] ArcTan[Tan[x]/Sqrt[2 + Tan[x]^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{2+x^2}}\,dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right)\\ &= \arctan\left(\frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(14) = 28.

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.64

$$\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx = \frac{\arcsin\left(\frac{\sin(x)}{\sqrt{2}}\right)\sqrt{3+\cos(2x)}\sec(x)}{\sqrt{2}\sqrt{1+\sec^2(x)}}$$

```
[In] Integrate[1/Sqrt[1 + Sec[x]^2],x]
[Out] (ArcSin[Sin[x]/Sqrt[2]]*Sqrt[3 + Cos[2*x]]*Sec[x])/(Sqrt[2]*Sqrt[1 + Sec[x]
^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(12) = 24.

Time = 0.87 (sec) , antiderivative size = 59, normalized size of antiderivative = 4.21

method	result	size
default	$\frac{\sqrt{2}\, \sqrt{\frac{\cos(x)^2+1}{(\cos(x)+1)^2}}\, \arctan\left(\frac{\sin(x)}{(\cos(x)+1)\sqrt{\frac{\cos(x)^2+1}{(\cos(x)+1)^2}}}\right)(1+\sec(x))}{\sqrt{2+2\sec(x)^2}}$	59

[In] int(1/(1+sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2^(1/2)*((cos(x)^2+1)/(cos(x)+1)^2)^(1/2)*arctan(sin(x)/(cos(x)+1)/((cos(x)^2+1)/(cos(x)+1)^2)^(1/2))/(2+2*sec(x)^2)^(1/2)*(1+sec(x))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.79

$$\int \frac{1}{\sqrt{1+\sec^2(x)}}\, dx = \frac{1}{2} \arctan\left(\frac{\sqrt{\frac{\cos(x)^2+1}{\cos(x)^2}}\cos(x)^3\sin(x)+\cos(x)\sin(x)}{\cos(x)^4+\cos(x)^2-1}\right) - \frac{1}{2}\arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

[In] integrate(1/(1+sec(x)^2)^(1/2),x, algorithm="fricas")

[Out] 1/2*arctan((sqrt((cos(x)^2 + 1)/cos(x)^2)*cos(x)^3*sin(x) + cos(x)*sin(x))/(cos(x)^4 + cos(x)^2 - 1)) - 1/2*arctan(sin(x)/cos(x))

Sympy [F]

$$\int \frac{1}{\sqrt{1+\sec^2(x)}}\, dx = \int \frac{1}{\sqrt{\sec^2(x)+1}}\, dx$$

[In] integrate(1/(1+sec(x)**2)**(1/2),x)

[Out] Integral(1/sqrt(sec(x)**2 + 1), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(12) = 24$.

Time = 0.38 (sec) , antiderivative size = 388, normalized size of antiderivative = 27.71

$$\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx = -\frac{1}{2}\arctan\left(2\left(2\left(6\cos(2x)+1\right)\cos(4x)+\cos(4x)^2+36\cos(2x)^2+\sin(4x)^2+12\sin(4x)\sin(2x)+8\right)+\frac{1}{2}\arctan\left(2\left(2\left(6\cos(2x)+1\right)\cos(4x)+\cos(4x)^2+36\cos(2x)^2+\sin(4x)^2+12\sin(4x)\sin(2x)+2\sin(2x),2\left(2\left(6\cos(2x)+1\right)\cos(4x)+\cos(4x)^2+36\cos(2x)^2+\sin(4x)^2+12\sin(4x)\sin(2x)+2\cos(2x)+6\right)\right.\right.$$

```
[In] integrate(1/(1+sec(x)^2)^(1/2),x, algorithm="maxima")
[Out] -1/2*arctan2(2*(2*(6*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 +
sin(4*x)^2 + 12*sin(4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*
sin(1/2*arctan2(sin(4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)), 2*(2*(6
*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12*sin(
4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*cos(1/2*arctan2(sin(
4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + 8) + 1/2*arctan2(2*(2*(6*c
os(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12*sin(4*
x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*sin(1/2*arctan2(sin(4*
x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + 2*sin(2*x), 2*(2*(6*cos(2*x)
 + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12*sin(4*x)*sin(
2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x) + 6*
sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + 2*cos(2*x) + 6)
```

Giac [F]

$$\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx = \int \frac{1}{\sqrt{\sec(x)^2+1}}\,dx$$

```
[In] integrate(1/(1+sec(x)^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(sec(x)^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx=\int \frac{1}{\sqrt{\frac{1}{\cos(x)^2}+1}}\,dx$$

```
[In] int(1/(1/cos(x)^2 + 1)^(1/2),x)
[Out] int(1/(1/cos(x)^2 + 1)^(1/2), x)
```

3.298 $\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p \,dx$

Optimal result

Integrand size = 25, antiderivative size = 111

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p \,dx$$
$$= \frac{\operatorname{AppellF1}\left(\frac{m}{2},\frac{1}{2},-p,\frac{2+m}{2},\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)\cot(e+fx)(d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p \left(\right.}{fm}$$

```
[Out] AppellF1(1/2*m,1/2,-p,1+1/2*m,sec(f*x+e)^2,-b*sec(f*x+e)^2/a)*cos(f*x+e)*(d
*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p*(-tan(f*x+e)^2)^(1/2)/f/m/((1+b*sec(f*x
+e)^2/a)^p)/sin(f*x+e)
```

Rubi [F]

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p \,dx = \int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p \,dx$$

```
[In] Int[(d*Sec[e + f*x])^m*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] Defer[Int][(d*Sec[e + f*x])^m*(a + b*Sec[e + f*x]^2)^p, x]
```

Rubi steps

$$\text{integral} = \int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p \,dx$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2195 vs. $2(111) = 222$.

Time = 20.08 (sec) , antiderivative size = 2195, normalized size of antiderivative = 19.77

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p \, dx = \text{Result too large to show}$$

```
[In] Integrate[(d*Sec[e + f*x])^m*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(d*Sec[e + f*x])^m*(Sec[e
+ f*x]^2)^(-1 + m/2 + p)*(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x])/(f*(3*(a +
b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
 + b))] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*T
an[e + f*x]^2)/(a + b))] + (a + b)*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2,
 -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)*((3*(a +
b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
 + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(m/2 + p))/(3*(a
+ b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b
*Tan[e + f*x]^2)/(a + b))] + (a + b)*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/
2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (6*a*
(a + b)*p*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x
]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^(-1
 + m/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*AppellF1[1/2, 1 - m/2
, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*AppellF
1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
 + (a + b)*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*T
an[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (6*(a + b)*(-1 + m/2 + p)*Appel
lF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-1 + m/2 + p)*Tan[e + f*
x]^2)/(3*(a + b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e +
f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (a + b)*(-2 + m)*AppellF1[3/2, 2 -
 m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x
]^2) + (3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-1 + m
/2 + p)*Tan[e + f*x]*((2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b
)) - (2*(1 - m/2)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan
[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*AppellF1
[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (
2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))] + (a + b)*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f
*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (3*(a + b)*AppellF
```

```
1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a
 + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-1 + m/2 + p)*Tan[e + f*x]
*(2*(2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e +
 f*x]^2)/(a + b))] + (a + b)*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] + 3
*(a + b)*((2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*T
an[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*(1 -
 m/2)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3) + Tan[e + f*x]^2*(2*b*p*((-6*b*(
1 - p)*AppellF1[5/2, 1 - m/2, 2 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x
]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (6*(1 - m/2)*Appe
llF1[5/2, 2 - m/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b
))]*Sec[e + f*x]^2*Tan[e + f*x])/5) + (a + b)*(-2 + m)*((6*b*p*AppellF1[5/2
, 2 - m/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[
e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (6*(2 - m/2)*AppellF1[5/2, 3 - m/2,
-p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan
[e + f*x])/5))))/(3*(a + b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2
, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2
, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (a + b)*(-2 + m)*Appell
F1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*
Tan[e + f*x]^2)^2))
```

Maple [F]

$$\int (d\sec(fx+e))^m \left(a+b\sec(fx+e)^2\right)^p dx$$

```
[In] int((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x)
[Out] int((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p dx = \int \left(b\sec(fx+e)^2+a\right)^p (d\sec(fx+e))^m dx$$

```
[In] integrate((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

Sympy [F]

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p\,dx = \int (d\sec{(e+fx)})^m \left(a+b\sec^2{(e+fx)}\right)^p\,dx$$

```
[In] integrate((d*sec(f*x+e))**m*(a+b*sec(f*x+e)**2)**p,x)
[Out] Integral((d*sec(e + f*x))**m*(a + b*sec(e + f*x)**2)**p, x)
```

Maxima [F]

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec{(fx+e)}^2+a\right)^p (d\sec{(fx+e)})^m\,dx$$

```
[In] integrate((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

Giac [F]

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec{(fx+e)}^2+a\right)^p (d\sec{(fx+e)})^m\,dx$$

```
[In] integrate((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p \left(\frac{d}{\cos\left(e+f\,x\right)}\right)^m\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^p*(d/cos(e + f*x))^m,x)
[Out] int((a + b/cos(e + f*x)^2)^p*(d/cos(e + f*x))^m, x)
```

3.299 $\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 103

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$=\frac{\text{AppellF1}\left(\frac{1}{2},2+p,-p,\frac{3}{2},\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\sin(e+fx)\left(\sec^2(e+fx)\left(a+b-a\sin\right.\right.}{f}$$

```
[Out] AppellF1(1/2,2+p,-p,3/2,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(cos(f*x+e)^2)^p
*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^p/f/((1-a*sin(f*x+e)^2/(a+b
))^p)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4233, 1985, 1986, 441, 440}

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$=\frac{\sin(e+fx)\cos^2(e+fx)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},p+2,-p,\frac{3}{2},\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\left(\sec^2(e\right.}{f}$$

```
[In] Int[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 2 + p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*
(Cos[e + f*x]^2)^p*Sin[e + f*x]*(Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2))
^p)/(f*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\left(a+\frac{b}{1-x^2}\right)^p}{(1-x^2)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(\frac{a+b-ax^2}{1-x^2}\right)^p}{(1-x^2)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\,(a+b-a\sin^2(e+fx))^{-p}\,(\sec^2(e+fx)\,(a+b-a\sin^2(e+fx)))^p\right)\text{Subst}\left(\int (1-\right.}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)\right)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\right)\mathrm{Subst}\left(\int\left(1-x^2\right)^{-2}\right.}{f}$$

$$= \frac{\mathrm{AppellF1}\left(\frac{1}{2},2+p,-p,\frac{3}{2},\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\sin(e+fx)\left(\sec^2(e+fx)\left(a+\right.\right.}{f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1989 vs. $2(103) = 206$.

Time = 17.99 (sec) , antiderivative size = 1989, normalized size of antiderivative = 19.31

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$= \frac{}{f\left(3(a+b)\,\mathrm{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)+\left(2bp\,\mathrm{AppellF1}\left(\frac{3}{2},-\frac{1}{2},1-p,\frac{5}{2},-\mathrm{ta}\right.\right.\right.}$$

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*Sec[e + f*x]^3*(Sec[e + f*x]^
2)^(1/2 + p)*(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x])/(f*(3*(a + b)*AppellF1[
1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*
p*AppellF1[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
+ b))] + (a + b)*AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e +
f*x]^2)/(a + b))])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, -1/2, -p, 3/2,
 -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*
x)])^p*(Sec[e + f*x]^2)^(3/2 + p))/(3*(a + b)*AppellF1[1/2, -1/2, -p, 3/2,
-Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*AppellF1[3/2, -1/2
, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (a + b)*App
ellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*T
an[e + f*x]^2) - (6*a*(a + b)*p*AppellF1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^
2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(
Sec[e + f*x]^2)^(1/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*AppellF
1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*
b*p*AppellF1[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))] + (a + b)*AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
+ f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (6*(a + b)*(1/2 + p)*AppellF1[1/2, -
1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*
```

```
Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(1/2 + p)*Tan[e + f*x]^2)/(3*(a + b)*A
ppellF1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
 + (2*b*p*AppellF1[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x
]^2)/(a + b))] + (a + b)*AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*x]^2, -((b*
Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (3*(a + b)*(a + 2*b + a*Cos[2*
(e + f*x)])^p*(Sec[e + f*x]^2)^(1/2 + p)*Tan[e + f*x]*((2*b*p*AppellF1[3/2,
 -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e +
f*x]^2*Tan[e + f*x])/(3*(a + b)) + (AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a
 + b)*AppellF1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
 + b))] + (2*b*p*AppellF1[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))] + (a + b)*AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*x]^2
, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (3*(a + b)*AppellF1[1/2
, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b +
 a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(1/2 + p)*Tan[e + f*x]*(2*(2*b*p*Ap
pellF1[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
)] + (a + b)*AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2
, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e +
 f*x]^2*Tan[e + f*x])/(3*(a + b)) + (AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f
*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3) + Tan
[e + f*x]^2*(2*b*p*((-6*b*(1 - p)*AppellF1[5/2, -1/2, 2 - p, 7/2, -Tan[e +
f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a +
 b)) + (3*AppellF1[5/2, 1/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5) + (a + b)*((6*b*p*AppellF1[5/
2, 1/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e +
 f*x]^2*Tan[e + f*x])/(5*(a + b)) - (3*AppellF1[5/2, 3/2, -p, 7/2, -Tan[e +
 f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/
(3*(a + b)*AppellF1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))] + (2*b*p*AppellF1[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b
*Tan[e + f*x]^2)/(a + b))] + (a + b)*AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f
*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2))
```

Maple [F]

$$\int \sec (fx+e)^3 \left(a+b\sec (fx+e)^2\right)^p dx$$

```
[In] int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \sec^3(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \sec\left(fx+e\right)^3 \, dx$$

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^3, x)
```

Sympy [F]

$$\int \sec^3(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^p \sec^3\left(e+fx\right) dx$$

```
[In] integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2)**p,x)
[Out] Integral((a + b*sec(e + f*x)**2)**p*sec(e + f*x)**3, x)
```

Maxima [F]

$$\int \sec^3(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \sec\left(fx+e\right)^3 \, dx$$

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^3, x)
```

Giac [F]

$$\int \sec^3(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \sec\left(fx+e\right)^3 \, dx$$

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\cos\left(e+f\,x\right)^3}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^3,x)
[Out] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^3, x)
```

3.300 $\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\operatorname{AppellF1}\left(\frac{1}{2},1+p,-p,\frac{3}{2},\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\sin(e+fx)\left(\sec^2(e+fx)\left(a+b-a\sin\cdots\right.\right.}{f}$$

```
[Out] AppellF1(1/2,p+1,-p,3/2,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(cos(f*x+e)^2)^p
*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^p/f/((1-a*sin(f*x+e)^2/(a+b
))^p)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4233, 1985, 1986, 441, 440}

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\sin(e+fx)\cos^2(e+fx)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},p+1,-p,\frac{3}{2},\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\left(\sec^2(e\cdots\right.}{f}$$

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1 + p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*
(Cos[e + f*x]^2)^p*Sin[e + f*x]*(Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2))
^p)/(f*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\left(a+\frac{b}{1-x^2}\right)^p}{1-x^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(\frac{a+b-ax^2}{1-x^2}\right)^p}{1-x^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\,(a+b-a\sin^2(e+fx))^{-p}\,(\sec^2(e+fx)\,(a+b-a\sin^2(e+fx)))^p\right)\text{Subst}\left(\int (1-\right.}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)\right)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\right)\mathrm{Subst}\left(\int\left(1-x^2\right)^{-1}\right.}{f}$$

$$= \frac{\mathrm{AppellF1}\left(\frac{1}{2},1+p,-p,\frac{3}{2},\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\sin(e+fx)\left(\sec^2(e+fx)\left(a+\right.\right.}{f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1995 vs. $2(103) = 206$.

Time = 17.42 (sec) , antiderivative size = 1995, normalized size of antiderivative = 19.37

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$= \frac{}{f\left(3(a+b)\,\mathrm{AppellF1}\left(\frac{1}{2},\frac{1}{2},-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)+\left(2bp\,\mathrm{AppellF1}\left(\frac{3}{2},\frac{1}{2},1-p,\frac{5}{2},-\tan^2(\right.\right.\right.}$$

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*Sec[e + f*x]*(Sec[e + f*x]^2)^
(-1/2 + p)*(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x])/(f*(3*(a + b)*AppellF1[1/
2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*A
ppellF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
)] - (a + b)*AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, 1/2, -p, 3/2, -Tan
[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^
p*(Sec[e + f*x]^2)^(1/2 + p))/(3*(a + b)*AppellF1[1/2, 1/2, -p, 3/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*AppellF1[3/2, 1/2, 1 - p
, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - (a + b)*AppellF1[3
/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e +
f*x]^2) - (6*a*(a + b)*p*AppellF1[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*
Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e +
f*x]^2)^(-1/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*AppellF1[1/2,
1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*Appe
llF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
- (a + b)*AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/(a + b))])*Tan[e + f*x]^2) + (6*(a + b)*(-1/2 + p)*AppellF1[1/2, 1/2, -p,
3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e
```

```
+ f*x)])^p*(Sec[e + f*x]^2)^(-1/2 + p)*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[
1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p
*AppellF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
b))] - (a + b)*AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))])*Tan[e + f*x]^2) + (3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)]
)^p*(Sec[e + f*x]^2)^(-1/2 + p)*Tan[e + f*x]*((2*b*p*AppellF1[3/2, 1/2, 1 -
 p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan
[e + f*x])/(3*(a + b)) - (AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b
*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*Appe
llF1[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (
2*b*p*AppellF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))] - (a + b)*AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
 + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (3*(a + b)*AppellF1[1/2, 1/2, -p, 3
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e +
 f*x)])^p*(Sec[e + f*x]^2)^(-1/2 + p)*Tan[e + f*x]*(2*(2*b*p*AppellF1[3/2,
1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - (a + b)*
AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
)*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1/2, 1 - p,
 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e
+ f*x])/(3*(a + b)) - (AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Ta
n[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3) + Tan[e + f*x]^2*(2
*b*p*((-6*b*(1 - p)*AppellF1[5/2, 1/2, 2 - p, 7/2, -Tan[e + f*x]^2, -((b*Ta
n[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (3*Appel
lF1[5/2, 3/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*S
ec[e + f*x]^2*Tan[e + f*x])/5) - (a + b)*((6*b*p*AppellF1[5/2, 3/2, 1 - p,
7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e +
 f*x])/(5*(a + b)) - (9*AppellF1[5/2, 5/2, -p, 7/2, -Tan[e + f*x]^2, -((b*T
an[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*Appe
llF1[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (
2*b*p*AppellF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))] - (a + b)*AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
 + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2))
```

Maple [F]

$$\int \sec(fx+e)\left(a+b\sec(fx+e)^2\right)^p\,dx$$

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int\left(b\sec\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e), x)
```

Sympy [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int\left(a+b\sec^2\left(e+fx\right)\right)^p\sec\left(e+fx\right)dx$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2)**p,x)
[Out] Integral((a + b*sec(e + f*x)**2)**p*sec(e + f*x), x)
```

Maxima [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int\left(b\sec\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e), x)
```

Giac [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int\left(b\sec\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\cos\left(e+f\,x\right)}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x),x)
[Out] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x), x)
```

3.301 $\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 101

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{AppellF1}\left(\frac{1}{2},p,-p,\frac{3}{2},\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\sin(e+fx)\left(\sec^2(e+fx)\left(a+b-a\sin^2(e\right.\right.}{f}$$

```
[Out] AppellF1(1/2,p,-p,3/2,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(cos(f*x+e)^2)^p*s
in(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^p/f/((1-a*sin(f*x+e)^2/(a+b))
^p)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4233, 1985, 1986, 441, 440}

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\sin(e+fx)\cos^2(e+fx)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},p,-p,\frac{3}{2},\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\left(\sec^2(e+f\right.}{f}$$

```
[In] Int[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*(Cos
[e + f*x]^2)^p*Sin[e + f*x]*(Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2))^p)/
(f*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(a+\frac{b}{1-x^2}\right)^p dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a+b-ax^2}{1-x^2}\right)^p dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p \left(a+b-a\sin^2(e+fx)\right)^{-p} \left(\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)\right)^p\right) \text{Subst}\left(\int (1-$$

$$= \frac{\left(\cos^2(e+fx)^p \left(\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)\right)^p \left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\right) \text{Subst}\left(\int (1-x^2)^{-p} \left(\right.$$

$$= \frac{\text{AppellF1}\left(\frac{1}{2}, p, -p, \frac{3}{2}, \sin^2(e+fx), \frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\sin(e+fx)\left(\sec^2(e+fx)\left(a+b-\right.\right.}{f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1983 vs. $2(101) = 202$.

Time = 17.08 (sec) , antiderivative size = 1983, normalized size of antiderivative = 19.63

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx =$$

$$-\frac{}{f\left(-3(a+b)\,\text{AppellF1}\left(\frac{1}{2},\frac{3}{2},-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)+\left(-2bp\,\text{AppellF1}\left(\frac{3}{2},\frac{3}{2},1-p,\frac{5}{2},-\right.\right.\right.}$$

```
[In] Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (-3*(a + b)*AppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-3/2 + p)*(
a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(-3*(a + b)*AppellF1[1/2, 3/2, -p,
 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/
2, 3/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 3*(a
+ b)*AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
 b))])*Tan[e + f*x]^2)*((-3*(a + b)*AppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[
e + f*x]^2)^(-1/2 + p))/(-3*(a + b)*AppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/2, 3/2, 1 - p, 5/
2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 3*(a + b)*AppellF1[3/2
, 5/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*
x]^2) + (6*a*(a + b)*p*AppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Ta
n[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*
x]^2)^(-3/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(-3*(a + b)*AppellF1[1/2, 3
/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*Appe
llF1[3/2, 3/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
+ 3*(a + b)*AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))])*Tan[e + f*x]^2) - (6*(a + b)*(-3/2 + p)*AppellF1[1/2, 3/2, -p
, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(
e + f*x)])^p*(Sec[e + f*x]^2)^(-3/2 + p)*Tan[e + f*x]^2)/(-3*(a + b)*Appell
F1[1/2, 3/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2
*b*p*AppellF1[3/2, 3/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))] + 3*(a + b)*AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[
```

```
e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (3*(a + b)*(a + 2*b + a*Cos[2*(e +
 f*x)])^p*(Sec[e + f*x]^2)^(-3/2 + p)*Tan[e + f*x]*((2*b*p*AppellF1[3/2, 3/
2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]
^2*Tan[e + f*x])/(3*(a + b)) - AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2,
 -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]))/(-3*(a + b)*A
ppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
+ (-2*b*p*AppellF1[3/2, 3/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))] + 3*(a + b)*AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2, -((b
*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (3*(a + b)*AppellF1[1/2, 3/2,
 -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[
2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-3/2 + p)*Tan[e + f*x]*(2*(-2*b*p*AppellF
1[3/2, 3/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 3
*(a + b)*AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] - 3*(a + b)*((2*b*p*AppellF1[3/2, 3/
2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]
^2*Tan[e + f*x])/(3*(a + b)) - AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2,
 -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]) + Tan[e + f*x]
^2*(-2*b*p*((-6*b*(1 - p)*AppellF1[5/2, 3/2, 2 - p, 7/2, -Tan[e + f*x]^2, -
((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (9
*AppellF1[5/2, 5/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
b))]*Sec[e + f*x]^2*Tan[e + f*x])/5) + 3*(a + b)*((6*b*p*AppellF1[5/2, 5/2,
 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2
*Tan[e + f*x])/(5*(a + b)) - 3*AppellF1[5/2, 7/2, -p, 7/2, -Tan[e + f*x]^2,
 -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]))))/(-3*(a + b)
*AppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))
] + (-2*b*p*AppellF1[3/2, 3/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))] + 3*(a + b)*AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2, -(
(b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2))
```

Maple [F]

$$\int \cos(fx+e)\left(a+b\sec(fx+e)^2\right)^p\,dx$$

[In] int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)

[Out] int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)

Fricas [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)\,dx$$

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e), x)
```

Sympy [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^p\cos\left(e+fx\right)dx$$

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2)**p,x)
[Out] Integral((a + b*sec(e + f*x)**2)**p*cos(e + f*x), x)
```

Maxima [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)\,dx$$

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e), x)
```

Giac [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)\,dx$$

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \cos\left(e+f\,x\right)\,\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)^p\,dx$$

```
[In] int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^p, x)
```

3.302 $\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 103

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$=\frac{\operatorname{AppellF1}\left(\frac{1}{2},-1+p,-p,\frac{3}{2},\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\sin(e+fx)\left(\sec^2(e+fx)\left(a+b-a\,s\right.\right.}{f}$$

```
[Out] AppellF1(1/2,-1+p,-p,3/2,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(cos(f*x+e)^2)^
p*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^p/f/((1-a*sin(f*x+e)^2/(a+
b))^p)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4233, 1985, 1986, 441, 440}

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$=\frac{\sin(e+fx)\cos^2(e+fx)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},p-1,-p,\frac{3}{2},\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\left(\sec^2(e\right.}{f}$$

```
[In] Int[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, -1 + p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
*(Cos[e + f*x]^2)^p*Sin[e + f*x]*(Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)
)^p)/(f*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (1-x^2)\left(a+\frac{b}{1-x^2}\right)^p\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int (1-x^2)\left(\frac{a+b-ax^2}{1-x^2}\right)^p\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(a+b-a\sin^2(e+fx)\right)^{-p}\left(\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)\right)^p\right)\text{Subst}\left(\int (1-\right.}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)\right)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int (1-x^2)^{1-p}\right.}{f}$$

$$= \frac{\text{AppellF1}\left(\frac{1}{2}, -1+p, -p, \frac{3}{2}, \sin^2(e+fx), \frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\sin(e+fx)\left(\sec^2(e+fx)\left(a-\right.\right.}{f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1987 vs. $2(103) = 206$.

Time = 17.43 (sec) , antiderivative size = 1987, normalized size of antiderivative = 19.29

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx =$$

$$-\frac{}{f\left(-3(a+b)\,\text{AppellF1}\left(\frac{1}{2}, \frac{5}{2}, -p, \frac{3}{2}, -\tan^2(e+fx), -\frac{b\tan^2(e+fx)}{a+b}\right)+\left(-2bp\,\text{AppellF1}\left(\frac{3}{2}, \frac{5}{2}, 1-p, \frac{5}{2}, -\right.\right.\right.}$$

```
[In] Integrate[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (-3*(a + b)*AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-7/2 + p)*(
a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(-3*(a + b)*AppellF1[1/2, 5/2, -p,
 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/
2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 5*(a
+ b)*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
 b))])*Tan[e + f*x]^2)*((-3*(a + b)*AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[
e + f*x]^2)^(-3/2 + p))/(-3*(a + b)*AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/2, 5/2, 1 - p, 5/
2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 5*(a + b)*AppellF1[3/2
, 7/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*
x]^2) + (6*a*(a + b)*p*AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Ta
n[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*
x]^2)^(-5/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(-3*(a + b)*AppellF1[1/2, 5
/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*Appe
llF1[3/2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
+ 5*(a + b)*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))])*Tan[e + f*x]^2) - (6*(a + b)*(-5/2 + p)*AppellF1[1/2, 5/2, -p
, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(
e + f*x)])^p*(Sec[e + f*x]^2)^(-5/2 + p)*Tan[e + f*x]^2)/(-3*(a + b)*Appell
F1[1/2, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2
*b*p*AppellF1[3/2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))] + 5*(a + b)*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[
```

```
e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (3*(a + b)*(a + 2*b + a*Cos[2*(e +
 f*x)])^p*(Sec[e + f*x]^2)^(-5/2 + p)*Tan[e + f*x]*((2*b*p*AppellF1[3/2, 5/
2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]
^2*Tan[e + f*x])/(3*(a + b)) - (5*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]
^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(-3*(a
+ b)*AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
 b))] + (-2*b*p*AppellF1[3/2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
+ f*x]^2)/(a + b))] + 5*(a + b)*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]^2
, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (3*(a + b)*AppellF1[1/2
, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b +
a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-5/2 + p)*Tan[e + f*x]*(2*(-2*b*p*A
ppellF1[3/2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
)] + 5*(a + b)*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] - 3*(a + b)*((2*b*p*AppellF1[3
/2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e
+ f*x]^2*Tan[e + f*x])/(3*(a + b)) - (5*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e
+ f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3) +
Tan[e + f*x]^2*(-2*b*p*((-6*b*(1 - p)*AppellF1[5/2, 5/2, 2 - p, 7/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(
a + b)) - 3*AppellF1[5/2, 7/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]) + 5*(a + b)*((6*b*p*AppellF1[5
/2, 7/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e
+ f*x]^2*Tan[e + f*x])/(5*(a + b)) - (21*AppellF1[5/2, 9/2, -p, 7/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5)))
)/(-3*(a + b)*AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x
]^2)/(a + b))] + (-2*b*p*AppellF1[3/2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -(
(b*Tan[e + f*x]^2)/(a + b))] + 5*(a + b)*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2))
```

Maple [F]

$$\int \cos(fx+e)^3 \left(a+b\sec(fx+e)^2\right)^p dx$$

[In] int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)

[Out] int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)

Fricas [F]

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^3\,dx$$

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2)**p,x)
[Out] Timed out
```

Maxima [F]

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^3\,dx$$

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^3, x)
```

Giac [F]

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^3\,dx$$

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \cos\left(e+f\,x\right)^3\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

```
[In] int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^p, x)
```

3.303 $\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 103

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$=\frac{\operatorname{AppellF1}\left(\frac{1}{2},-2+p,-p,\frac{3}{2},\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\sin(e+fx)\left(\sec^2(e+fx)\left(a+b-a\,\mathrm{s}\right.\right.}{f}$$

```
[Out] AppellF1(1/2,-2+p,-p,3/2,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(cos(f*x+e)^2)^
p*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^p/f/((1-a*sin(f*x+e)^2/(a+
b))^p)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4233, 1985, 1986, 441, 440}

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$=\frac{\sin(e+fx)\cos^2(e+fx)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},p-2,-p,\frac{3}{2},\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\left(\sec^2(e\right.}{f}$$

```
[In] Int[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, -2 + p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
*(Cos[e + f*x]^2)^p*Sin[e + f*x]*(Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)
)^p)/(f*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (1-x^2)^2 \left(a+\frac{b}{1-x^2}\right)^p dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int (1-x^2)^2 \left(\frac{a+b-ax^2}{1-x^2}\right)^p dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p \left(a+b-a\sin^2(e+fx)\right)^{-p} \left(\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)\right)^p\right) \text{Subst}\left(\int (1-\right.}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p \left(\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)\right)^p \left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\right) \text{Subst}\left(\int (1-x^2)^{2-p}\right.}{f}$$

$$= \frac{\text{AppellF1}\left(\frac{1}{2}, -2+p, -p, \frac{3}{2}, \sin^2(e+fx), \frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\sin(e+fx)\left(\sec^2(e+fx)\left(a\right.\right.}{f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1997 vs. $2(103) = 206$.

Time = 17.60 (sec) , antiderivative size = 1997, normalized size of antiderivative = 19.39

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx =$$

$$-\frac{}{f\left(-3(a+b)\,\text{AppellF1}\left(\frac{1}{2}, \frac{7}{2}, -p, \frac{3}{2}, -\tan^2(e+fx), -\frac{b\tan^2(e+fx)}{a+b}\right) + \left(-2bp\,\text{AppellF1}\left(\frac{3}{2}, \frac{7}{2}, 1-p, \frac{5}{2}, -\right.\right.\right.}$$

```
[In] Integrate[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (-3*(a + b)*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))]*Cos[e + f*x]^4*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^
2)^(-7/2 + p)*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(-3*(a + b)*AppellF
1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*
b*p*AppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
 + b))] + 7*(a + b)*AppellF1[3/2, 9/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
 + f*x]^2)/(a + b))])*Tan[e + f*x]^2)*((-3*(a + b)*AppellF1[1/2, 7/2, -p, 3
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e +
 f*x)])^p*(Sec[e + f*x]^2)^(-5/2 + p))/(-3*(a + b)*AppellF1[1/2, 7/2, -p, 3
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/2,
 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 7*(a +
b)*AppellF1[3/2, 9/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b
))])*Tan[e + f*x]^2) + (6*a*(a + b)*p*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e +
f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 +
 p)*(Sec[e + f*x]^2)^(-7/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(-3*(a + b)*
AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
 + (-2*b*p*AppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x
]^2)/(a + b))] + 7*(a + b)*AppellF1[3/2, 9/2, -p, 5/2, -Tan[e + f*x]^2, -((
b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (6*(a + b)*(-7/2 + p)*Appell
F1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a +
2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-7/2 + p)*Tan[e + f*x]^2)/(-3
*(a + b)*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))] + (-2*b*p*AppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Ta
n[e + f*x]^2)/(a + b))] + 7*(a + b)*AppellF1[3/2, 9/2, -p, 5/2, -Tan[e + f*
```

```
x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (3*(a + b)*(a + 2*b
 + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-7/2 + p)*Tan[e + f*x]*((2*b*p*A
ppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
)]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (7*AppellF1[3/2, 9/2, -p, 5/2
, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*
x])/3))/(-3*(a + b)*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e
 + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]
^2, -((b*Tan[e + f*x]^2)/(a + b))] + 7*(a + b)*AppellF1[3/2, 9/2, -p, 5/2,
-Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (3*(a +
b)*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b
))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-7/2 + p)*Tan[e + f*
x]*(2*(-2*b*p*AppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e +
f*x]^2)/(a + b))] + 7*(a + b)*AppellF1[3/2, 9/2, -p, 5/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] - 3*(a + b)*((2
*b*p*AppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (7*AppellF1[3/2, 9/2, -
p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[
e + f*x])/3) + Tan[e + f*x]^2*(-2*b*p*((-6*b*(1 - p)*AppellF1[5/2, 7/2, 2 -
 p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan
[e + f*x])/(5*(a + b)) - (21*AppellF1[5/2, 9/2, 1 - p, 7/2, -Tan[e + f*x]^2
, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5) + 7*(a + b
)*((6*b*p*AppellF1[5/2, 9/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (27*AppellF1[5/2,
11/2, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]
^2*Tan[e + f*x])/5))))/(-3*(a + b)*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x
]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/2, 7/2, 1 - p, 5/2
, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 7*(a + b)*AppellF1[3/2,
 9/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x
]^2)^2))
```

Maple [F]

$$\int \cos(fx+e)^5 \left(a+b\sec(fx+e)^2\right)^p dx$$

```
[In] int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \cos^5(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \cos\left(fx+e\right)^5 \, dx$$

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2)**p,x)
[Out] Timed out
```

Maxima [F]

$$\int \cos^5(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \cos\left(fx+e\right)^5 \, dx$$

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^5, x)
```

Giac [F]

$$\int \cos^5(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \cos\left(fx+e\right)^5 \, dx$$

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \cos\left(e+f\,x\right)^5\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

```
[In] int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^p, x)
```

3.304 $\int \sec^6(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 216

$$\int \sec^6(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx$$
$$= -\frac{(3a-2b(2+p))\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)}$$
$$+\frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{bf(5+2p)}$$
$$+\frac{(3a^2-4ab(1+p)+4b^2(2+3p+p^2))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\,(a+b}{b^2f(3+2p)(5+2p)}$$

```
[Out] -(3*a-2*b*(2+p))*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(p+1)/b^2/f/(4*p^2+16*p+15
)+sec(f*x+e)^2*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(p+1)/b/f/(5+2*p)+(3*a^2-4*a
*b*(p+1)+4*b^2*(p^2+3*p+2))*hypergeom([1/2, -p],[3/2],-b*tan(f*x+e)^2/(a+b)
)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^p/b^2/f/(4*p^2+16*p+15)/((1+b*tan(f*x+e)^
2/(a+b))^p)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used

$= \{4231, 427, 396, 252, 251\}$

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$= \frac{(3a^2-4ab(p+1)+4b^2(p^2+3p+2))\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{Hypergeor}}{b^2f(2p+3)(2p+5)}$$
$$-\frac{(3a-2b(p+2))\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{b^2f(2p+3)(2p+5)}$$
$$+\frac{\tan(e+fx)\sec^2(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{bf(2p+5)}$$

```
[In] Int[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -(((3*a - 2*b*(2 + p))*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(b^
2*f*(3 + 2*p)*(5 + 2*p))) + (Sec[e + f*x]^2*Tan[e + f*x]*(a + b + b*Tan[e +
 f*x]^2)^(1 + p))/(b*f*(5 + 2*p)) + ((3*a^2 - 4*a*b*(1 + p) + 4*b^2*(2 + 3*
p + p^2))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*Ta
n[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(b^2*f*(3 + 2*p)*(5 + 2*p)*(1 + (b
*Tan[e + f*x]^2)/(a + b))^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
```

```
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (1+x^2)^2 (a+b+bx^2)^p \, dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{bf(5+2p)} + \frac{\text{Subst}\left(\int (a+b+bx^2)^p \left(-a+2b(2+p)-(3a-2b(2+p))x^2\right) dx, x, \tan(e+fx)\right)}{bf(5+2p)}$$

$$= -\frac{(3a-2b(2+p))\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} + \frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{bf(5+2p)} + \frac{(3a^2-4ab(1+p)+4b^2(2+3p+p^2))\,\text{Subst}\left(\int (a+b+bx^2)^p \, dx, x, \tan(e+fx)\right)}{b^2f(3+2p)(5+2p)}$$

$$= -\frac{(3a-2b(2+p))\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} + \frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{bf(5+2p)} + \frac{\left((3a^2-4ab(1+p)+4b^2(2+3p+p^2))\left(a+b+b\tan^2(e+fx)\right)^p \left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\Big(}{b^2f(3+2p)(5+2p)}$$

$$= -\frac{(3a-2b(2+p))\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} + \frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{bf(5+2p)} + \frac{(3a^2-4ab(1+p)+4b^2(2+3p+p^2))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\,(}{b^2f(3+2p)(5+2p)}$$

Mathematica [A] (verified)

Time = 2.16 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.69

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\left(a+b\sec^2(e+fx)\right)^p\tan(e+fx)\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\left(15\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)+1\right.}{}$$

```
[In] Integrate[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] ((a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]*(15*Hypergeometric2F1[1/2, -p, 3/2,
-((b*Tan[e + f*x]^2)/(a + b))] + 10*Hypergeometric2F1[3/2, -p, 5/2, -((b*Ta
n[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2 + 3*Hypergeometric2F1[5/2, -p, 7/2,
-((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^4))/(15*f*(1 + (b*Tan[e + f*x]^
2)/(a + b))^p)
```

Maple [F]

$$\int \sec(fx+e)^6\left(a+b\sec(fx+e)^2\right)^p dx$$

```
[In] int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec(fx+e)^2+a\right)^p\sec(fx+e)^6\,dx$$

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^6, x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2)**p,x)
[Out] Timed out
```

Maxima [F]

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^6\,dx$$

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^6, x)
```

Giac [F]

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^6\,dx$$

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\cos\left(e+f\,x\right)^6}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^6,x)
[Out] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^6, x)
```

3.305 $\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 129

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{bf(3+2p)}-\frac{(a-2b(1+p))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\right.}{bf(3+2p)}$$

```
[Out] tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(p+1)/b/f/(3+2*p)-(a-2*b*(p+1))*hypergeom([
1/2, -p],[3/2],-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^p/b/f
/(3+2*p)/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4231, 396, 252, 251}

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{bf(2p+3)}-\frac{(a-2b(p+1))\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\right.}{bf(2p+3)}$$

```
[In] Int[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(b*f*(3 + 2*p)) - ((a - 2
*b*(1 + p))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(b*f*(3 + 2*p)*(1 + (b*Tan[e + f
*x]^2)/(a + b))^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (1+x^2)\left(a+b+bx^2\right)^p\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{bf(3+2p)}\\
&\quad - \frac{(a-2b(1+p))\text{Subst}\left(\int \left(a+b+bx^2\right)^p\,dx, x, \tan(e+fx)\right)}{bf(3+2p)}\\
&= \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{bf(3+2p)}\\
&\quad - \frac{\left((a-2b(1+p))\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \left(1+\frac{bx^2}{a+b}\right)^p\,dx, x, \tan\right.}{bf(3+2p)}
\end{aligned}
$$

$$= \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{bf(3+2p)} - \frac{(a-2b(1+p))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)}{bf(3+2p)}$$

Mathematica [A] (verified)

Time = 2.14 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\left(a+b\sec^2(e+fx)\right)^p\tan(e+fx)\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\left((-a+2b(1+p))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b}{\cdots}\right)\cdots\right)}{bf(3+2p)}$$

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] ((a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]*((-a + 2*b*(1 + p))*Hypergeometric2F
1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))] + (a + b + b*Tan[e + f*x]^2)
*(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(b*f*(3 + 2*p)*(1 + (b*Tan[e + f*x]^2
)/(a + b))^p)
```

Maple [F]

$$\int \sec\left(fx+e\right)^4\left(a+b\sec\left(fx+e\right)^2\right)^p\,dx$$

```
[In] int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^4\,dx$$

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^4(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2)**p,x)
[Out] Timed out
```

Maxima [F]

$$\int \sec^4(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \sec\left(fx+e\right)^4 \, dx$$

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^4, x)
```

Giac [F]

$$\int \sec^4(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \sec\left(fx+e\right)^4 \, dx$$

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^4(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\cos\left(e+f\,x\right)^4} \, dx$$

```
[In] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^4,x)
[Out] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^4, x)
```

3.306 $\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{f}$$

```
[Out] hypergeom([1/2, -p],[3/2],-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b+b*tan(f*x+
e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4231, 252, 251}

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

```
[In] Int[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x
]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
```

```
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (a+b+bx^2)^p \, dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((a+b+b\tan^2(e+fx))^p \left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right) \text{Subst}\left(\int \left(1+\frac{bx^2}{a+b}\right)^p \, dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Hypergeometric2F1}\left(\frac{1}{2}, -p, \frac{3}{2}, -\frac{b\tan^2(e+fx)}{a+b}\right) \tan(e+fx)\,(a+b+b\tan^2(e+fx))^p \left(1+\frac{b\tan^2(e+f}{a+b}\right.}{f}$$

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \sec^2(e+fx)\,(a+b\sec^2(e+fx))^p \, dx$$
$$= \frac{\text{Hypergeometric2F1}\left(\frac{1}{2}, -p, \frac{3}{2}, -\frac{b\tan^2(e+fx)}{a+b}\right)(a+b\sec^2(e+fx))^p \tan(e+fx)\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{f}$$

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + b*Sec[
e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p)
```

Maple [F]

$$\int \sec(fx+e)^2 \left(a+b\sec(fx+e)^2\right)^p dx$$

```
[In] int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec(fx+e)^2+a\right)^p \sec(fx+e)^2\,dx$$

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^2, x)
```

Sympy [F]

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(a+b\sec^2(e+fx)\right)^p \sec^2(e+fx)\,dx$$

```
[In] integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2)**p,x)
[Out] Integral((a + b*sec(e + f*x)**2)**p*sec(e + f*x)**2, x)
```

Maxima [F]

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec(fx+e)^2+a\right)^p \sec(fx+e)^2\,dx$$

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^2, x)
```

Giac [F]

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^2\,dx$$

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\cos\left(e+f\,x\right)^2}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^2,x)
[Out] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^2, x)
```

3.307 $\int (a+b\sec^2(e+fx))^p \, dx$

Optimal result

Integrand size = 14, antiderivative size = 83

$$\int (a+b\sec^2(e+fx))^p \, dx = \frac{\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\,(a+b+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right.}{f}$$

```
[Out] AppellF1(1/2,1,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b+
b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4213, 441, 440}

$$\int (a+b\sec^2(e+fx))^p \, dx = \frac{\tan(e+fx)\,(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+}{a+b}\right.}{f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((a+b+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b}{\ }\right.}{f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2137 vs. 2(83) = 166.

Time = 6.25 (sec) , antiderivative size = 2137, normalized size of antiderivative = 25.75

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*
(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, -p, 1, 3
```

/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-1 + p))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (6*(a + b)*p*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (6*a*(a + b)*p*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^p*Sin[e + f*x]*Sin[2*(e + f*x)])/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (3*(a + b)*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]*(4*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5 - (6*b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -(

```
(b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/
(5*(a + b))) - (a + b)*((6*b*p*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*
x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) -
 (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]
^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2,
 -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 -
 p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*Appel
lF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e
 + f*x]^2)^2))
```

Maple [F]

$$\int \left(a+b\sec \left(fx+e\right)^{2}\right)^{p}dx$$

[In] int((a+b*sec(f*x+e)^2)^p,x)

[Out] int((a+b*sec(f*x+e)^2)^p,x)

Fricas [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec \left(fx+e\right)^{2}+a\right)^{p}dx$$

[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")

[Out] integral((b*sec(f*x + e)^2 + a)^p, x)

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(a+b\sec^{2}\left(e+fx\right)\right)^{p}\,dx$$

[In] integrate((a+b*sec(f*x+e)**2)**p,x)

[Out] Integral((a + b*sec(e + f*x)**2)**p, x)

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\,dx$$

[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e)^2 + a)^p, x)

Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\,dx$$

[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="giac")

[Out] integrate((b*sec(f*x + e)^2 + a)^p, x)

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

[In] int((a + b/cos(e + f*x)^2)^p,x)

[Out] int((a + b/cos(e + f*x)^2)^p, x)

3.308 $\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\operatorname{AppellF1}\left(\frac{1}{2},2,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+f\ldots}{a+b}\right.}{f}$$

```
[Out] AppellF1(1/2,2,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b+
b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4231, 441, 440}

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},2,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e\ldots}{a+b\ldots}\right.}{f}$$

```
[In] Int[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^p}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((a+b+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right)^p}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{AppellF1}\left(\frac{1}{2}, 2, -p, \frac{3}{2}, -\tan^2(e+fx), -\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\,(a+b+b\tan^2(e+fx))^p\left(1+\frac{b\,\text{ta}}{}\right.}{f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1914 vs. 2(83) = 166.

Time = 16.55 (sec) , antiderivative size = 1914, normalized size of antiderivative = 23.06

$$\int \cos^2(e+fx)\,(a+b\sec^2(e+fx))^p\,dx$$

$$= \frac{}{f\left(3(a+b)\,\text{AppellF1}\left(\frac{1}{2}, 2, -p, \frac{3}{2}, -\tan^2(e+fx), -\frac{b\tan^2(e+fx)}{a+b}\right)+2\left(bp\,\text{AppellF1}\left(\frac{3}{2}, 2, 1-p, \frac{5}{2}, -\tan^2(e\right.\right.}$$

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-
2 + p)*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, 2
, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF
1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 2*(
a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
 b))])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^
2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e +
 f*x]^2)^(-1 + p))/(3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -(
(b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e +
 f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 2*(a + b)*AppellF1[3/2, 3, -p, 5/
2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (6*a*
(a + b)*p*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^(-2 + p)*
Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e
+ f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 2, 1 - p, 5
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 2*(a + b)*AppellF1[3/
2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x
]^2) + (6*(a + b)*(-2 + p)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*
Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)
^(-2 + p)*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x
]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -
Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 2*(a + b)*AppellF1[3/2, 3,
 -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)
+ (3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-2 + p)*Tan
[e + f*x]*((2*b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
+ f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (4*AppellF1[
3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*
x]^2*Tan[e + f*x])/3))/(3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2
, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan
[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 2*(a + b)*AppellF1[3/2, 3, -p
, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (
3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-2 + p)*Tan[e +
 f*x]*(4*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))] - 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b
*Tan[e + f*x]^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p
*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
)]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (4*AppellF1[3/2, 3, -p, 5/2,
-Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]
)/3) + 2*Tan[e + f*x]^2*(b*p*((-6*b*(1 - p)*AppellF1[5/2, 2, 2 - p, 7/2, -T
an[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/
(5*(a + b)) - (12*AppellF1[5/2, 3, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e
```

```
+ f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5) - 2*(a + b)*((6*b*p*App
ellF1[5/2, 3, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*S
ec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (18*AppellF1[5/2, 4, -p, 7/2, -Ta
n[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5
))))/(3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x
]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*
Tan[e + f*x]^2)/(a + b))] - 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2))
```

Maple [F]

$$\int \cos\left(fx+e\right)^{2}\left(a+b\sec\left(fx+e\right)^{2}\right)^{p}dx$$

```
[In] int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^2\,dx$$

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^2\,dx$$

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^2, x)
```

Giac [F]

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^2\,dx$$

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \cos\left(e+f\,x\right)^2\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

```
[In] int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^p, x)
```

3.309 $\int \cos^4(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \cos^4(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \frac{\text{AppellF1}\left(\frac{1}{2},3,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right.}{f}$$

```
[Out] AppellF1(1/2,3,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b+
b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4231, 441, 440}

$$\int \cos^4(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},3,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+}{a+b}\right.}{f}$$

```
[In] Int[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^p}{(1+x^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((a+b+b\tan^2(e+fx))^p \left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right)^p}{(1+x^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{AppellF1}\left(\frac{1}{2}, 3, -p, \frac{3}{2}, -\tan^2(e+fx), -\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\,(a+b+b\tan^2(e+fx))^p \left(1+\frac{b}{}\right.}{f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1912 vs. 2(83) = 166.

Time = 16.25 (sec) , antiderivative size = 1912, normalized size of antiderivative = 23.04

$$\int \cos^4(e+fx)\,(a+b\sec^2(e+fx))^p\,dx$$

$$= \frac{}{f\left(3(a+b)\,\text{AppellF1}\left(\frac{1}{2}, 3, -p, \frac{3}{2}, -\tan^2(e+fx), -\frac{b\tan^2(e+fx)}{a+b}\right) + 2\left(bp\,\text{AppellF1}\left(\frac{3}{2}, 3, 1-p, \frac{5}{2}, -\tan^2(\right.\right.}$$

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]

[Out] (3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))]*Cos[e + f*x]^3*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^
(-3 + p)*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2,
 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*Appel
lF1[3/2, 3, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 3
*(a + b)*AppellF1[3/2, 4, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
 + b))])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x
]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e
 + f*x]^2)^(-2 + p))/(3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 3, 1 - p, 5/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 3*(a + b)*AppellF1[3/2, 4, -p,
5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (6*
a*(a + b)*p*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^(-3 + p
)*Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 3, 1 - p,
 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 3*(a + b)*AppellF1[
3/2, 4, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f
*x]^2) + (6*(a + b)*(-3 + p)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((
b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^
2)^(-3 + p)*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f
*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 3, 1 - p, 5/2,
 -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 3*(a + b)*AppellF1[3/2,
4, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2
) + (3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-3 + p)*T
an[e + f*x]*((2*b*p*AppellF1[3/2, 3, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - 2*AppellF1
[3/2, 4, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f
*x]^2*Tan[e + f*x]))/(3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 3, 1 - p, 5/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 3*(a + b)*AppellF1[3/2, 4, -p,
5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (3*
(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
+ b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-3 + p)*Tan[e + f
*x]*(4*(b*p*AppellF1[3/2, 3, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))] - 3*(a + b)*AppellF1[3/2, 4, -p, 5/2, -Tan[e + f*x]^2, -((b*T
an[e + f*x]^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*A
ppellF1[3/2, 3, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - 2*AppellF1[3/2, 4, -p, 5/2, -Ta
n[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]) +
 2*Tan[e + f*x]^2*(b*p*((-6*b*(1 - p)*AppellF1[5/2, 3, 2 - p, 7/2, -Tan[e +
 f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a
+ b)) - (18*AppellF1[5/2, 4, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
```

```
^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5) - 3*(a + b)*((6*b*p*AppellF1[
5/2, 4, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e +
 f*x]^2*Tan[e + f*x])/(5*(a + b)) - (24*AppellF1[5/2, 5, -p, 7/2, -Tan[e +
f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(
3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))] + 2*(b*p*AppellF1[3/2, 3, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
+ f*x]^2)/(a + b))] - 3*(a + b)*AppellF1[3/2, 4, -p, 5/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2))
```

Maple [F]

$$\int \cos(fx+e)^4 \left(a+b\sec(fx+e)^2\right)^p dx$$

[In] `int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)`

[Out] `int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)`

Fricas [F]

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec(fx+e)^2+a\right)^p \cos(fx+e)^4\,dx$$

[In] `integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")`

[Out] `integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^4, x)`

Sympy [F(-1)]

Timed out.

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Timed out}$$

[In] `integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2)**p,x)`

[Out] `Timed out`

Maxima [F]

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^4\,dx$$

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^4, x)
```

Giac [F]

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^4\,dx$$

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \cos\left(e+f\,x\right)^4\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

```
[In] int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^p, x)
```

3.310 $\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$=\frac{\text{AppellF1}\left(\frac{1}{2},4,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{f}$$

```
[Out] AppellF1(1/2,4,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b+
b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4231, 441, 440}

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$=\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},4,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

```
[In] Int[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^p}{(1+x^2)^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((a+b+b\tan^2(e+fx))^p \left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right)^p}{(1+x^2)^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{AppellF1}\left(\frac{1}{2}, 4, -p, \frac{3}{2}, -\tan^2(e+fx), -\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\,(a+b+b\tan^2(e+fx))^p \left(1+\frac{b\,\text{ta}}{\right.}\right.}{f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1914 vs. 2(83) = 166.

Time = 16.86 (sec) , antiderivative size = 1914, normalized size of antiderivative = 23.06

$$\int \cos^6(e+fx)\,(a+b\sec^2(e+fx))^p\,dx$$

$$= \frac{}{f\left(3(a+b)\,\text{AppellF1}\left(\frac{1}{2}, 4, -p, \frac{3}{2}, -\tan^2(e+fx), -\frac{b\tan^2(e+fx)}{a+b}\right) + 2\left(bp\,\text{AppellF1}\left(\frac{3}{2}, 4, 1-p, \frac{5}{2}, -\tan^2(e\right.\right.\right.}$$

[In] Integrate[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]

[Out] (3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Cos[e + f*x]^5*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-4 + p)*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 4, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 4*(a + b)*AppellF1[3/2, 5, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-3 + p))/(3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 4, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 4*(a + b)*AppellF1[3/2, 5, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (6*a*(a + b)*p*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^(-4 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 4, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 4*(a + b)*AppellF1[3/2, 5, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (6*(a + b)*(-4 + p)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-4 + p)*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 4, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 4*(a + b)*AppellF1[3/2, 5, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-4 + p)*Tan[e + f*x]*((2*b*p*AppellF1[3/2, 4, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (8*AppellF1[3/2, 5, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 4, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 4*(a + b)*AppellF1[3/2, 5, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-4 + p)*Tan[e + f*x]*(4*(b*p*AppellF1[3/2, 4, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 4*(a + b)*AppellF1[3/2, 5, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 4, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (8*AppellF1[3/2, 5, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*b*(1 - p)*AppellF1[5/2, 4, 2 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (24*AppellF1[5/2, 5, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[

```
e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5) - 4*(a + b)*((6*b*p*A
ppellF1[5/2, 5, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - 6*AppellF1[5/2, 6, -p, 7/2, -Ta
n[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])))
)/(3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/(a + b))] + 2*(b*p*AppellF1[3/2, 4, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan
[e + f*x]^2)/(a + b))] - 4*(a + b)*AppellF1[3/2, 5, -p, 5/2, -Tan[e + f*x]^
2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2))
```

Maple [F]

$$\int \cos(fx+e)^6 \left(a+b\sec(fx+e)^2\right)^p dx$$

```
[In] int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \int \left(b\sec(fx+e)^2+a\right)^p \cos(fx+e)^6 dx$$

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^6, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2)**p,x)
[Out] Timed out
```

Maxima [F]

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^6\,dx$$

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^6, x)
```

Giac [F]

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^6\,dx$$

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \cos\left(e+f\,x\right)^6\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

```
[In] int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^p, x)
```

3.311 $\int (a+b\sec^2(e+fx))\tan^5(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 72

$$\int (a+b\sec^2(e+fx))\tan^5(e+fx)\,dx = -\frac{a\log(\cos(e+fx))}{f} - \frac{(2a-b)\sec^2(e+fx)}{2f} + \frac{(a-2b)\sec^4(e+fx)}{4f} + \frac{b\sec^6(e+fx)}{6f}$$

```
[Out] -a*ln(cos(f*x+e))/f-1/2*(2*a-b)*sec(f*x+e)^2/f+1/4*(a-2*b)*sec(f*x+e)^4/f+1
/6*b*sec(f*x+e)^6/f
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4223, 457, 77}

$$\int (a+b\sec^2(e+fx))\tan^5(e+fx)\,dx = \frac{(a-2b)\sec^4(e+fx)}{4f} - \frac{(2a-b)\sec^2(e+fx)}{2f} - \frac{a\log(\cos(e+fx))}{f} + \frac{b\sec^6(e+fx)}{6f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^5,x]
```

```
[Out] -((a*Log[Cos[e + f*x]])/f) - ((2*a - b)*Sec[e + f*x]^2)/(2*f) + ((a - 2*b)*
Sec[e + f*x]^4)/(4*f) + (b*Sec[e + f*x]^6)/(6*f)
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
```

```
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2(b+ax^2)}{x^7}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1-x)^2(b+ax)}{x^4}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{b}{x^4}+\frac{a-2b}{x^3}+\frac{-2a+b}{x^2}+\frac{a}{x}\right)\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{a\log(\cos(e+fx))}{f}-\frac{(2a-b)\sec^2(e+fx)}{2f}+\frac{(a-2b)\sec^4(e+fx)}{4f}+\frac{b\sec^6(e+fx)}{6f}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

$$\int (a+b\sec^2(e+fx))\tan^5(e+fx)\,dx = \frac{b\tan^6(e+fx)}{6f}-\frac{a(4\log(\cos(e+fx))+2\tan^2(e+fx)-\tan^4(e+fx))}{4f}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^5,x]
[Out] (b*Tan[e + f*x]^6)/(6*f) - (a*(4*Log[Cos[e + f*x]] + 2*Tan[e + f*x]^2 - Tan
[e + f*x]^4))/(4*f)
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

method	result
parts	$\frac{a\left(\frac{\tan(fx+e)^4}{4}-\frac{\tan(fx+e)^2}{2}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}+\frac{b\tan(fx+e)^6}{6f}$
derivativedivides	$\frac{\frac{\sec(fx+e)^6 b}{6}+\frac{\sec(fx+e)^4 a}{4}-\frac{\sec(fx+e)^4 b}{2}-\sec(fx+e)^2 a+\frac{b\sec(fx+e)^2}{2}+a\ln(\sec(fx+e))}{f}$
default	$\frac{\frac{\sec(fx+e)^6 b}{6}+\frac{\sec(fx+e)^4 a}{4}-\frac{\sec(fx+e)^4 b}{2}-\sec(fx+e)^2 a+\frac{b\sec(fx+e)^2}{2}+a\ln(\sec(fx+e))}{f}$
risch	$iax+\frac{2iae}{f}+\frac{-4a\,\mathrm{e}^{10i(fx+e)}+2b\,\mathrm{e}^{10i(fx+e)}-12a\,\mathrm{e}^{8i(fx+e)}-16a\,\mathrm{e}^{6i(fx+e)}+\frac{20b\,\mathrm{e}^{6i(fx+e)}}{3}-12a\,\mathrm{e}^{4i(fx+e)}-4a\,\mathrm{e}^{2i(fx+e}}{f\left(\mathrm{e}^{2i(fx+e)}+1\right)^6}$

```
[In] int((a+b*sec(f*x+e)^2)*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
[Out] a/f*(1/4*tan(f*x+e)^4-1/2*tan(f*x+e)^2+1/2*ln(1+tan(f*x+e)^2))+1/6*b/f*tan(
f*x+e)^6
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \left(a+b\sec^2(e+fx)\right)\tan^5(e+fx)\,dx = -\frac{12\,a\cos\left(fx+e\right)^6\log\left(-\cos\left(fx+e\right)\right)+6\left(2\,a-b\right)\cos\left(fx+e\right)^4-3\left(a-2\,b\right)\cos\left(fx+e\right)^2-2\,b}{12\,f\cos\left(fx+e\right)^6}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^5,x, algorithm="fricas")
[Out] -1/12*(12*a*cos(f*x + e)^6*log(-cos(f*x + e)) + 6*(2*a - b)*cos(f*x + e)^4
 - 3*(a - 2*b)*cos(f*x + e)^2 - 2*b)/(f*cos(f*x + e)^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(56) = 112.

Time = 0.55 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.61

$$\int \left(a+b\sec^2(e+fx)\right)\tan^5(e+fx)\,dx = \begin{cases} \frac{a\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a\tan^4\left(e+fx\right)}{4f}-\frac{a\tan^2\left(e+fx\right)}{2f}+\frac{b\tan^4\left(e+fx\right)\sec^2\left(e+fx\right)}{6f}-\frac{b\tan^2\left(e+fx\right)\sec^2\left(e+fx\right)}{6f}+\frac{b\sec^2\left(e+fx\right)}{6f} \\ x\left(a+b\sec^2\left(e\right)\right)\tan^5\left(e\right) \end{cases}$$

```
[In] integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**5,x)
```

```
[Out] Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**4/(4*f) - a*t
an(e + f*x)**2/(2*f) + b*tan(e + f*x)**4*sec(e + f*x)**2/(6*f) - b*tan(e +
f*x)**2*sec(e + f*x)**2/(6*f) + b*sec(e + f*x)**2/(6*f), Ne(f, 0)), (x*(a +
 b*sec(e)**2)*tan(e)**5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.32

$$\int (a+b\sec^2(e+fx))\tan^5(e+fx)\,dx = -\frac{6\,a\log\left(\sin\left(fx+e\right)^2-1\right)-\frac{6\,(2\,a-b)\sin(fx+e)^4-3\,(7\,a-2\,b)\sin(fx+e)^2+9\,a-2\,b}{\sin(fx+e)^6-3\,\sin(fx+e)^4+3\,\sin(fx+e)^2-1}}{12\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(6*a*log(sin(f*x + e)^2 - 1) - (6*(2*a - b)*sin(f*x + e)^4 - 3*(7*a -
 2*b)*sin(f*x + e)^2 + 9*a - 2*b)/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*si
n(f*x + e)^2 - 1))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(66) = 132.

Time = 1.59 (sec) , antiderivative size = 280, normalized size of antiderivative = 3.89

$$\int (a+b\sec^2(e+fx))\tan^5(e+fx)\,dx = \frac{6\,a\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right)-6\,a\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-2\right|\right)+\frac{11\,a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(}{\cos(}\right.}{}}{12\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] 1/12*(6*a*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) -
1)/(cos(f*x + e) + 1) + 2)) - 6*a*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e)
 - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2)) + (11*a*((cos(f*x + e)
+ 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1))^3 + 90*a*(
(cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) +
1))^2 + 276*a*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(
cos(f*x + e) + 1)) + 280*a - 128*b)/((cos(f*x + e) + 1)/(cos(f*x + e) - 1)
+ (cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2)^3)/f
```

Mupad [B] (verification not implemented)

Time = 19.69 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.72

$$\int \left(a+b\sec^2(e+fx)\right)\tan^5(e+fx)\,dx$$
$$=\frac{\frac{a\ \ln\left(\tan(e+f\,x)^2+1\right)}{2}-\frac{a\tan(e+f\,x)^2}{2}+\frac{a\tan(e+f\,x)^4}{4}+\frac{b\tan(e+f\,x)^6}{6}}{f}$$

```
[In] int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2),x)
```

```
[Out] ((a*log(tan(e + f*x)^2 + 1))/2 - (a*tan(e + f*x)^2)/2 + (a*tan(e + f*x)^4)/
4 + (b*tan(e + f*x)^6)/6)/f
```

3.312 $\int (a+b\sec^2(e+fx))\tan^3(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int (a+b\sec^2(e+fx))\tan^3(e+fx)\,dx=\frac{a\log(\cos(e+fx))}{f}+\frac{(a-b)\sec^2(e+fx)}{2f}+\frac{b\sec^4(e+fx)}{4f}$$

```
[Out] a*ln(cos(f*x+e))/f+1/2*(a-b)*sec(f*x+e)^2/f+1/4*b*sec(f*x+e)^4/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4223, 457, 77}

$$\int (a+b\sec^2(e+fx))\tan^3(e+fx)\,dx=\frac{(a-b)\sec^2(e+fx)}{2f}+\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^4(e+fx)}{4f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^3,x]
```

```
[Out] (a*Log[Cos[e + f*x]])/f + ((a - b)*Sec[e + f*x]^2)/(2*f) + (b*Sec[e + f*x]^
4)/(4*f)
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
```

```
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)(b+ax^2)}{x^5}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \frac{(1-x)(b+ax)}{x^3}\,dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{b}{x^3}+\frac{a-b}{x^2}-\frac{a}{x}\right)\,dx, x, \cos^2(e+fx)\right)}{2f}\\
&= \frac{a\log(\cos(e+fx))}{f}+\frac{(a-b)\sec^2(e+fx)}{2f}+\frac{b\sec^4(e+fx)}{4f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$
\int \left(a+b\sec^2(e+fx)\right)\tan^3(e+fx)\,dx = \frac{b\tan^4(e+fx)}{4f} + \frac{a(2\log(\cos(e+fx))+\tan^2(e+fx))}{2f}
$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^3,x]
[Out] (b*Tan[e + f*x]^4)/(4*f) + (a*(2*Log[Cos[e + f*x]] + Tan[e + f*x]^2))/(2*f)
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

method	result	siz
parts	$\frac{a\left(\frac{\tan(fx+e)^2}{2}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}+\frac{b\tan(fx+e)^4}{4f}$	45
derivativedivides	$\frac{\frac{\sec(fx+e)^4 b}{4}+\frac{\sec(fx+e)^2 a}{2}-\frac{b\sec(fx+e)^2}{2}-a\ln(\sec(fx+e))}{f}$	49
default	$\frac{\frac{\sec(fx+e)^4 b}{4}+\frac{\sec(fx+e)^2 a}{2}-\frac{b\sec(fx+e)^2}{2}-a\ln(\sec(fx+e))}{f}$	49
risch	$-iax-\frac{2iae}{f}-\frac{2\left(-a\,e^{6i(fx+e)}+b\,e^{6i(fx+e)}-2a\,e^{4i(fx+e)}-a\,e^{2i(fx+e)}+b\,e^{2i(fx+e)}\right)}{f\left(e^{2i(fx+e)}+1\right)^4}+\frac{a\ln\left(e^{2i(fx+e)}+1\right)}{f}$	10

```
[In] int((a+b*sec(f*x+e)^2)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
[Out] a/f*(1/2*tan(f*x+e)^2-1/2*ln(1+tan(f*x+e)^2))+1/4*b/f*tan(f*x+e)^4
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \left(a+b\sec^2(e+fx)\right)\tan^3(e+fx)\,dx = \frac{4\,a\cos\left(fx+e\right)^4\log\left(-\cos\left(fx+e\right)\right)+2\left(a-b\right)\cos\left(fx+e\right)^2+b}{4\,f\cos\left(fx+e\right)^4}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^3,x, algorithm="fricas")
[Out] 1/4*(4*a*cos(f*x + e)^4*log(-cos(f*x + e)) + 2*(a - b)*cos(f*x + e)^2 + b)/
(f*cos(f*x + e)^4)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.63

$$\int \left(a+b\sec^2(e+fx)\right)\tan^3(e+fx)\,dx = \begin{cases} -\frac{a\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a\tan^2\left(e+fx\right)}{2f}+\frac{b\tan^2\left(e+fx\right)\sec^2\left(e+fx\right)}{4f}-\frac{b\sec^2\left(e+fx\right)}{4f} & \text{for } f\neq 0 \\ x\left(a+b\sec^2\left(e\right)\right)\tan^3\left(e\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**3,x)
```

```
[Out] Piecewise((-a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**2/(2*f) + b*
tan(e + f*x)**2*sec(e + f*x)**2/(4*f) - b*sec(e + f*x)**2/(4*f), Ne(f, 0)),
 (x*(a + b*sec(e)**2)*tan(e)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.31

$$\int \left(a+b\sec^2(e+fx)\right)\tan^3(e+fx)\,dx = \frac{2\,a\log\left(\sin\left(fx+e\right)^2-1\right)-\frac{2\,(a-b)\sin(fx+e)^2-2\,a+b}{\sin(fx+e)^4-2\,\sin(fx+e)^2+1}}{4\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*a*log(sin(f*x + e)^2 - 1) - (2*(a - b)*sin(f*x + e)^2 - 2*a + b)/(si
n(f*x + e)^4 - 2*sin(f*x + e)^2 + 1))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(45) = 90.

Time = 0.66 (sec) , antiderivative size = 236, normalized size of antiderivative = 4.82

$$\int \left(a+b\sec^2(e+fx)\right)\tan^3(e+fx)\,dx =$$

$$-\frac{2\,a\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right)-2\,a\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-2\right|\right)+\frac{3\,a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(}{\cos(}\right.}{}}{4\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*a*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) -
1)/(cos(f*x + e) + 1) + 2)) - 2*a*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e)
 - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2)) + (3*a*((cos(f*x + e) +
 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1))^2 + 20*a*((
cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1
)) + 28*a - 16*b)/((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) -
1)/(cos(f*x + e) + 1) + 2)^2)/f
```

Mupad [B] (verification not implemented)

Time = 19.54 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \left(a+b\sec^2(e+fx)\right)\tan^3(e+fx)\,dx=\frac{a\tan(e+f\,x)^2}{2\,f}-\frac{a\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f}+\frac{b\tan(e+f\,x)^4}{4\,f}$$

```
[In] int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2),x)
[Out] (a*tan(e + f*x)^2)/(2*f) - (a*log(tan(e + f*x)^2 + 1))/(2*f) + (b*tan(e + f
*x)^4)/(4*f)
```

3.313 $\int (a+b\sec^2(e+fx))\tan(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 30

$$\int (a+b\sec^2(e+fx))\tan(e+fx)\,dx=-\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^2(e+fx)}{2f}$$

```
[Out] -a*ln(cos(f*x+e))/f+1/2*b*sec(f*x+e)^2/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4223, 14}

$$\int (a+b\sec^2(e+fx))\tan(e+fx)\,dx=\frac{b\sec^2(e+fx)}{2f}-\frac{a\log(\cos(e+fx))}{f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x],x]
```

```
[Out] -((a*Log[Cos[e + f*x]])/f) + (b*Sec[e + f*x]^2)/(2*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
```

```
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\text{Subst}\left(\int \frac{b+ax^2}{x^3}\,dx, x, \cos(e+fx)\right)}{f} \\ &= -\frac{\text{Subst}\left(\int \left(\frac{b}{x^3}+\frac{a}{x}\right)\,dx, x, \cos(e+fx)\right)}{f} \\ &= -\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^2(e+fx)}{2f} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \left(a+b\sec^2(e+fx)\right)\tan(e+fx)\,dx = -\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^2(e+fx)}{2f}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x],x]
[Out] -((a*Log[Cos[e + f*x]])/f) + (b*Sec[e + f*x]^2)/(2*f)
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{\frac{b\sec(fx+e)^2}{2}+a\ln(\sec(fx+e))}{f}$	26
default	$\frac{\frac{b\sec(fx+e)^2}{2}+a\ln(\sec(fx+e))}{f}$	26
parts	$\frac{a\ln\left(1+\tan(fx+e)^2\right)}{2f}+\frac{b\sec(fx+e)^2}{2f}$	33
risch	$iax+\frac{2iae}{f}+\frac{2b\,\mathrm{e}^{2i(fx+e)}}{f\left(\mathrm{e}^{2i(fx+e)}+1\right)^2}-\frac{a\ln\left(\mathrm{e}^{2i(fx+e)}+1\right)}{f}$	61

```
[In] int((a+b*sec(f*x+e)^2)*tan(f*x+e),x,method=_RETURNVERBOSE)
[Out] 1/f*(1/2*b*sec(f*x+e)^2+a*ln(sec(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int (a+b\sec^2(e+fx))\tan(e+fx)\,dx = -\frac{2\,a\cos{(fx+e)}^2\log{(-\cos{(fx+e)})}-b}{2\,f\cos{(fx+e)}^2}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e),x, algorithm="fricas")
[Out] -1/2*(2*a*cos(f*x + e)^2*log(-cos(f*x + e)) - b)/(f*cos(f*x + e)^2)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int (a+b\sec^2(e+fx))\tan(e+fx)\,dx = \begin{cases} \frac{a\log{(\tan^2{(e+fx)}+1)}}{2f}+\frac{b\sec^2{(e+fx)}}{2f} & \text{for } f\neq 0 \\ x(a+b\sec^2{(e)})\tan{(e)} & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*sec(f*x+e)**2)*tan(f*x+e),x)
[Out] Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) + b*sec(e + f*x)**2/(2*f), Ne(f
, 0)), (x*(a + b*sec(e)**2)*tan(e), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

$$\int (a+b\sec^2(e+fx))\tan(e+fx)\,dx = -\frac{a\log\left(\sin{(fx+e)}^2-1\right)+\frac{b}{\sin(fx+e)^2-1}}{2\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e),x, algorithm="maxima")
[Out] -1/2*(a*log(sin(f*x + e)^2 - 1) + b/(sin(f*x + e)^2 - 1))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(28) = 56$.

Time = 0.33 (sec) , antiderivative size = 190, normalized size of antiderivative = 6.33

$$\int \left(a + b\sec^2(e + fx)\right)\tan(e + fx)\,dx$$

$$= \frac{a\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 2\right|\right) - a\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} - 2\right|\right) + \frac{a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1} + \frac{\cos(fx+e)-}{\cos(fx+e)+}\right.}{\frac{\cos(fx+e)+1}{\cos(fx+e)-1} + \frac{\cos(fx+e}{\cos(fx+e}}}{2f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] 1/2*(a*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/
(cos(f*x + e) + 1) + 2)) - a*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1)
 - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2)) + (a*((cos(f*x + e) + 1)/(co
s(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 2*a - 4*b)/((cos
(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1) +
 2))/f
```

Mupad [B] (verification not implemented)

Time = 18.98 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \left(a + b\sec^2(e + fx)\right)\tan(e + fx)\,dx = \frac{a\,\ln\left(\tan(e + f\,x)^2 + 1\right)}{2\,f} + \frac{b\tan(e + f\,x)^2}{2\,f}$$

```
[In] int(tan(e + f*x)*(a + b/cos(e + f*x)^2),x)
```

```
[Out] (a*log(tan(e + f*x)^2 + 1))/(2*f) + (b*tan(e + f*x)^2)/(2*f)
```

3.314 $\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{b\log(\cos(e+fx))}{f}+\frac{(a+b)\log(\sin(e+fx))}{f}$$

```
[Out] -b*ln(cos(f*x+e))/f+(a+b)*ln(sin(f*x+e))/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4223, 457, 78}

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(a+b)\log(\sin(e+fx))}{f}-\frac{b\log(\cos(e+fx))}{f}$$

```
[In] Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((b*Log[Cos[e + f*x]])/f) + ((a + b)*Log[Sin[e + f*x]])/f
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{b+ax^2}{x(1-x^2)}\,dx, x, \cos(e+fx)\right)}{f} \\
&= -\frac{\text{Subst}\left(\int \frac{b+ax}{(1-x)x}\,dx, x, \cos^2(e+fx)\right)}{2f} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{-a-b}{-1+x}+\frac{b}{x}\right)\,dx, x, \cos^2(e+fx)\right)}{2f} \\
&= -\frac{b\log(\cos(e+fx))}{f}+\frac{(a+b)\log(\sin(e+fx))}{f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$
\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{a\log(\cos(e+fx))}{f}-\frac{b\log(\cos(e+fx))}{f}+\frac{b\log(\sin(e+fx))}{f}+\frac{a\log(\tan(e+fx))}{f}
$$

```
[In] Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^2),x]
[Out] (a*Log[Cos[e + f*x]])/f - (b*Log[Cos[e + f*x]])/f + (b*Log[Sin[e + f*x]])/f
 + (a*Log[Tan[e + f*x]])/f
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{a\ln(\sin(fx+e))+b\ln(\tan(fx+e))}{f}$	24
default	$\frac{a\ln(\sin(fx+e))+b\ln(\tan(fx+e))}{f}$	24
risch	$-iax-\frac{2iae}{f}+\frac{\ln(\mathrm{e}^{2i(fx+e)}-1)a}{f}+\frac{\ln(\mathrm{e}^{2i(fx+e)}-1)b}{f}-\frac{b\ln(\mathrm{e}^{2i(fx+e)}+1)}{f}$	67

```
[In] int(cot(f*x+e)*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(a*ln(sin(f*x+e))+b*ln(tan(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=-\frac{b\log\left(\cos\left(fx+e\right)^2\right)-(a+b)\log\left(-\frac{1}{4}\,\cos\left(fx+e\right)^2+\frac{1}{4}\right)}{2\,f}$$

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
[Out] -1/2*(b*log(cos(f*x + e)^2) - (a + b)*log(-1/4*cos(f*x + e)^2 + 1/4))/f
```

Sympy [F]

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\int\left(a+b\sec^2\left(e+fx\right)\right)\cot\left(e+fx\right)dx$$

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2),x)
[Out] Integral((a + b*sec(e + f*x)**2)*cot(e + f*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{b\log\left(\sin\left(fx+e\right)^2-1\right)-(a+b)\log\left(\sin\left(fx+e\right)^2\right)}{2\,f}$$

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] -1/2*(b*log(sin(f*x + e)^2 - 1) - (a + b)*log(sin(f*x + e)^2))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(28) = 56.

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 3.50

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{a\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right)+b\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-2\right|\right)}{2\,f}$$

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] -1/2*(a*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)
/(cos(f*x + e) + 1) + 2)) + b*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1
) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2)))/f
```

Mupad [B] (verification not implemented)

Time = 19.68 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\ln\left(\tan(e+f\,x)\right)\,(a+b)}{f}-\frac{a\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f}$$

```
[In] int(cot(e + f*x)*(a + b/cos(e + f*x)^2),x)
[Out] (log(tan(e + f*x))*(a + b))/f - (a*log(tan(e + f*x)^2 + 1))/(2*f)
```

3.315 $\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 32

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{(a+b)\csc^2(e+fx)}{2f}-\frac{a\log(\sin(e+fx))}{f}$$

```
[Out] -1/2*(a+b)*csc(f*x+e)^2/f-a*ln(sin(f*x+e))/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4223, 455, 45}

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{(a+b)\csc^2(e+fx)}{2f}-\frac{a\log(\sin(e+fx))}{f}$$

```
[In] Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
[Out] -1/2*((a + b)*Csc[e + f*x]^2)/f - (a*Log[Sin[e + f*x]])/f
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x(b+ax^2)}{(1-x^2)^2}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \frac{b+ax}{(1-x)^2}\,dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a+b}{(-1+x)^2}+\frac{a}{-1+x}\right)\,dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{(a+b)\csc^2(e+fx)}{2f}-\frac{a\log(\sin(e+fx))}{f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.62

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= -\frac{b\csc^2(e+fx)}{2f}-\frac{a(\cot^2(e+fx)+2\log(\cos(e+fx))+2\log(\tan(e+fx)))}{2f}$$

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
[Out] -1/2*(b*Csc[e + f*x]^2)/f - (a*(Cot[e + f*x]^2 + 2*Log[Cos[e + f*x]] + 2*Lo
g[Tan[e + f*x]]))/(2*f)
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

method	result	size
derivativedivides	$\frac{a\left(-\frac{\cot(fx+e)^2}{2}-\ln(\sin(fx+e))\right)-\frac{b}{2\sin(fx+e)^2}}{f}$	39
default	$\frac{a\left(-\frac{\cot(fx+e)^2}{2}-\ln(\sin(fx+e))\right)-\frac{b}{2\sin(fx+e)^2}}{f}$	39
risch	$iax+\frac{2iae}{f}+\frac{2(a+b)\,e^{2i(fx+e)}}{f\left(e^{2i(fx+e)}-1\right)^2}-\frac{\ln\left(e^{2i(fx+e)}-1\right)a}{f}$	63

```
[In] int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(a*(-1/2*cot(f*x+e)^2-ln(sin(f*x+e)))-1/2*b/sin(f*x+e)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.56

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{2\left(a\cos\left(fx+e\right)^2-a\right)\log\left(\frac{1}{2}\,\sin\left(fx+e\right)\right)-a-b}{2\left(f\cos\left(fx+e\right)^2-f\right)}$$

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
[Out] -1/2*(2*(a*cos(f*x + e)^2 - a)*log(1/2*sin(f*x + e)) - a - b)/(f*cos(f*x +
e)^2 - f)
```

Sympy [F]

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\int\left(a+b\sec^2\left(e+fx\right)\right)\cot^3\left(e+fx\right)dx$$

```
[In] integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2),x)
[Out] Integral((a + b*sec(e + f*x)**2)*cot(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{a\log\left(\sin\left(fx+e\right)^2\right)+\frac{a+b}{\sin(fx+e)^2}}{2\,f}$$

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(a*log(sin(f*x + e)^2) + (a + b)/sin(f*x + e)^2)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(30) = 60.

Time = 0.31 (sec) , antiderivative size = 146, normalized size of antiderivative = 4.56

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=$$

$$-\frac{4\,a\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)-8\,a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)-\frac{\left(a+b+\frac{4\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)(\cos(fx+e)+1)}{\cos(fx+e)-1}-\frac{a(\cos(fx+e)-1)}{\cos(fx+e)+1}}{8\,f}$$

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/8*(4*a*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - 8*a*log(abs(-
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)) - (a + b + 4*a*(cos(f*x + e) -
1)/(cos(f*x + e) + 1))*(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - a*(cos(f*x +
 e) - 1)/(cos(f*x + e) + 1) - b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/f
```

Mupad [B] (verification not implemented)

Time = 19.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.59

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{a\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f}-\frac{a\,\ln\left(\tan(e+f\,x)\right)}{f}-\frac{\cot(e+f\,x)^2\,\left(\frac{a}{2}+\frac{b}{2}\right)}{f}$$

```
[In] int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2),x)
```

```
[Out] (a*log(tan(e + f*x)^2 + 1))/(2*f) - (a*log(tan(e + f*x)))/f - (cot(e + f*x)
^2*(a/2 + b/2))/f
```

3.316 $\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(2a+b)\csc^2(e+fx)}{2f}-\frac{(a+b)\csc^4(e+fx)}{4f}+\frac{a\log(\sin(e+fx))}{f}$$

```
[Out] 1/2*(2*a+b)*csc(f*x+e)^2/f-1/4*(a+b)*csc(f*x+e)^4/f+a*ln(sin(f*x+e))/f
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4223, 457, 78}

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{(a+b)\csc^4(e+fx)}{4f}+\frac{(2a+b)\csc^2(e+fx)}{2f}+\frac{a\log(\sin(e+fx))}{f}$$

```
[In] Int[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((2*a + b)*Csc[e + f*x]^2)/(2*f) - ((a + b)*Csc[e + f*x]^4)/(4*f) + (a*Log[
Sin[e + f*x]])/f
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
```

```
&& ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^3(b+ax^2)}{(1-x^2)^3}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \frac{x(b+ax)}{(1-x)^3}\,dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{-a-b}{(-1+x)^3}+\frac{-2a-b}{(-1+x)^2}-\frac{a}{-1+x}\right)dx, x, \cos^2(e+fx)\right)}{2f}\\
&= \frac{(2a+b)\csc^2(e+fx)}{2f}-\frac{(a+b)\csc^4(e+fx)}{4f}+\frac{a\log(\sin(e+fx))}{f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.43

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{a\cot^2(e+fx)}{2f}-\frac{a\cot^4(e+fx)}{4f}-\frac{b\cot^4(e+fx)}{4f}+\frac{a\log(\cos(e+fx))}{f}+\frac{a\log(\tan(e+fx))}{f}$$

```
[In] Integrate[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
[Out] (a*Cot[e + f*x]^2)/(2*f) - (a*Cot[e + f*x]^4)/(4*f) - (b*Cot[e + f*x]^4)/(4
*f) + (a*Log[Cos[e + f*x]])/f + (a*Log[Tan[e + f*x]])/f
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{a\left(-\frac{\cot(fx+e)^4}{4}+\frac{\cot(fx+e)^2}{2}+\ln(\sin(fx+e))\right)-\frac{b\cos(fx+e)^4}{4\sin(fx+e)^4}}{f}$	55
default	$\frac{a\left(-\frac{\cot(fx+e)^4}{4}+\frac{\cot(fx+e)^2}{2}+\ln(\sin(fx+e))\right)-\frac{b\cos(fx+e)^4}{4\sin(fx+e)^4}}{f}$	55
risch	$-iax-\frac{2iae}{f}-\frac{2\left(2a\,e^{6i(fx+e)}+b\,e^{6i(fx+e)}-2a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+b\,e^{2i(fx+e)}\right)}{f\left(e^{2i(fx+e)}-1\right)^4}+\frac{\ln\left(e^{2i(fx+e)}-1\right)a}{f}$	109

[In] int(cot(f*x+e)^5*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)

[Out] 1/f*(a*(-1/4*cot(f*x+e)^4+1/2*cot(f*x+e)^2+ln(sin(f*x+e)))-1/4*b/sin(f*x+e)^4*cos(f*x+e)^4)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.63

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{2\,(2\,a+b)\cos\left(fx+e\right)^2-4\left(a\cos\left(fx+e\right)^4-2\,a\cos\left(fx+e\right)^2+a\right)\log\left(\frac{1}{2}\sin\left(fx+e\right)\right)-3\,a-b}{4\left(f\cos\left(fx+e\right)^4-2\,f\cos\left(fx+e\right)^2+f\right)}$$

[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="fricas")

[Out] -1/4*(2*(2*a + b)*cos(f*x + e)^2 - 4*(a*cos(f*x + e)^4 - 2*a*cos(f*x + e)^2 + a)*log(1/2*sin(f*x + e)) - 3*a - b)/(f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 + f)

Sympy [F]

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)\cot^5\left(e+fx\right)dx$$

[In] integrate(cot(f*x+e)**5*(a+b*sec(f*x+e)**2),x)

[Out] Integral((a + b*sec(e + f*x)**2)*cot(e + f*x)**5, x)

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{2\,a\log\left(\sin\left(fx+e\right)^2\right)+\frac{2\,(2\,a+b)\sin(fx+e)^2-a-b}{\sin(fx+e)^4}}{4\,f}$$

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] 1/4*(2*a*log(sin(f*x + e)^2) + (2*(2*a + b)*sin(f*x + e)^2 - a - b)/sin(f*x
 + e)^4)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(47) = 94.

Time = 0.35 (sec) , antiderivative size = 238, normalized size of antiderivative = 4.67

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$=\frac{32\,a\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)-64\,a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)-\frac{\left(a+b+\frac{12\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{4\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{48\,a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right.}{(\cos(fx+e)-1)^2}}{64\,f}$$

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] 1/64*(32*a*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - 64*a*log(abs
(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)) - (a + b + 12*a*(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) + 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 48*a
*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(cos(f*x +
 e) - 1)^2 - 12*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) - a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - b*
(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/f
```

Mupad [B] (verification not implemented)

Time = 19.88 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{a\,\ln\left(\tan(e+f\,x)\right)}{f}-\frac{a\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f}-\frac{-\frac{a\tan(e+f\,x)^2}{2}+\frac{a}{4}+\frac{b}{4}}{f\tan\left(e+f\,x\right)^4}$$

```
[In] int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2),x)
[Out] (a*log(tan(e + f*x)))/f - (a*log(tan(e + f*x)^2 + 1))/(2*f) - (a/4 + b/4 -
(a*tan(e + f*x)^2)/2)/(f*tan(e + f*x)^4)
```

3.317 $\int (a+b\sec^2(e+fx))\tan^6(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 64

$$\int (a+b\sec^2(e+fx))\tan^6(e+fx)\,dx=-ax+\frac{a\tan(e+fx)}{f}-\frac{a\tan^3(e+fx)}{3f}+\frac{a\tan^5(e+fx)}{5f}+\frac{b\tan^7(e+fx)}{7f}$$

```
[Out] -a*x+a*tan(f*x+e)/f-1/3*a*tan(f*x+e)^3/f+1/5*a*tan(f*x+e)^5/f+1/7*b*tan(f*x
+e)^7/f
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4226, 1816, 209}

$$\int (a+b\sec^2(e+fx))\tan^6(e+fx)\,dx=\frac{a\tan^5(e+fx)}{5f}-\frac{a\tan^3(e+fx)}{3f}+\frac{a\tan(e+fx)}{f}-ax+\frac{b\tan^7(e+fx)}{7f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^6,x]
```

```
[Out] -(a*x) + (a*Tan[e + f*x])/f - (a*Tan[e + f*x]^3)/(3*f) + (a*Tan[e + f*x]^5)
/(5*f) + (b*Tan[e + f*x]^7)/(7*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^6(a+b(1+x^2))}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(a-ax^2+ax^4+bx^6-\frac{a}{1+x^2}\right)\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{a\tan(e+fx)}{f}-\frac{a\tan^3(e+fx)}{3f}+\frac{a\tan^5(e+fx)}{5f}\\
&\quad+\frac{b\tan^7(e+fx)}{7f}-\frac{a\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -ax+\frac{a\tan(e+fx)}{f}-\frac{a\tan^3(e+fx)}{3f}+\frac{a\tan^5(e+fx)}{5f}+\frac{b\tan^7(e+fx)}{7f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

$$\int \left(a+b\sec^2(e+fx)\right)\tan^6(e+fx)\,dx = -\frac{a\arctan(\tan(e+fx))}{f}+\frac{a\tan(e+fx)}{f}-\frac{a\tan^3(e+fx)}{3f}+\frac{a\tan^5(e+fx)}{5f}+\frac{b\tan^7(e+fx)}{7f}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^6,x]
[Out] -((a*ArcTan[Tan[e + f*x]])/f) + (a*Tan[e + f*x])/f - (a*Tan[e + f*x]^3)/(3*
f) + (a*Tan[e + f*x]^5)/(5*f) + (b*Tan[e + f*x]^7)/(7*f)
```

Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

method	result
parts	$\frac{a\left(\frac{\tan(fx+e)^5}{5}-\frac{\tan(fx+e)^3}{3}+\tan(fx+e)-\arctan(\tan(fx+e))\right)}{f}+\frac{b\tan(fx+e)^7}{7f}$
derivativedivides	$\frac{a\left(\frac{\tan(fx+e)^5}{5}-\frac{\tan(fx+e)^3}{3}+\tan(fx+e)-fx-e\right)+\frac{b\sin(fx+e)^7}{7\cos(fx+e)^7}}{f}$
default	$\frac{a\left(\frac{\tan(fx+e)^5}{5}-\frac{\tan(fx+e)^3}{3}+\tan(fx+e)-fx-e\right)+\frac{b\sin(fx+e)^7}{7\cos(fx+e)^7}}{f}$
risch	$-ax-\frac{2i(-315a\,\mathrm{e}^{12i(fx+e)}+105b\,\mathrm{e}^{12i(fx+e)}-1260a\,\mathrm{e}^{10i(fx+e)}-2555a\,\mathrm{e}^{8i(fx+e)}+525b\,\mathrm{e}^{8i(fx+e)}-3080a\,\mathrm{e}^{6i(fx+e)}-2}{105f\left(\mathrm{e}^{2i(fx+e)}+1\right)^7}$

```
[In] int((a+b*sec(f*x+e)^2)*tan(f*x+e)^6,x,method=_RETURNVERBOSE)
```

```
[Out] a/f*(1/5*tan(f*x+e)^5-1/3*tan(f*x+e)^3+tan(f*x+e)-arctan(tan(f*x+e)))+1/7*b
*tan(f*x+e)^7/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.39

$$\int \left(a+b\sec^2(e+fx)\right)\tan^6(e+fx)\,dx =$$
$$-\frac{105\,afx\cos\left(fx+e\right)^7-\left((161\,a-15\,b)\cos\left(fx+e\right)^6-(77\,a-45\,b)\cos\left(fx+e\right)^4+3\,(7\,a-15\,b)\cos\left(f\right.\right.}{105\,f\cos\left(fx+e\right)^7}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] -1/105*(105*a*f*x*cos(f*x + e)^7 - ((161*a - 15*b)*cos(f*x + e)^6 - (77*a -
 45*b)*cos(f*x + e)^4 + 3*(7*a - 15*b)*cos(f*x + e)^2 + 15*b)*sin(f*x + e))
/(f*cos(f*x + e)^7)
```

Sympy [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int \left(a+b\sec^2(e+fx)\right)\tan^6(e+fx)\,dx = a\left(\begin{cases} -x+\frac{\tan^5(e+fx)}{5f}-\frac{\tan^3(e+fx)}{3f}+\frac{\tan(e+fx)}{f} & \text{for } f\neq 0 \\ x\tan^6(e) & \text{otherwise} \end{cases}\right) + b\left(\begin{cases} x\tan^6(e)\sec^2(e) & \text{for } f=0 \\ \frac{\tan^7(e+fx)}{7f} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**6,x)
[Out] a*Piecewise((-x + tan(e + f*x)**5/(5*f) - tan(e + f*x)**3/(3*f) + tan(e + f
*x)/f, Ne(f, 0)), (x*tan(e)**6, True)) + b*Piecewise((x*tan(e)**6*sec(e)**2
, Eq(f, 0)), (tan(e + f*x)**7/(7*f), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \left(a+b\sec^2(e+fx)\right)\tan^6(e+fx)\,dx = \frac{15\,b\tan(fx+e)^7+21\,a\tan(fx+e)^5-35\,a\tan(fx+e)^3-105\,(fx+e)a+105\,a\tan(fx+e)}{105\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^6,x, algorithm="maxima")
[Out] 1/105*(15*b*tan(f*x + e)^7 + 21*a*tan(f*x + e)^5 - 35*a*tan(f*x + e)^3 - 10
5*(f*x + e)*a + 105*a*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 2.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \left(a+b\sec^2(e+fx)\right)\tan^6(e+fx)\,dx = \frac{15\,b\tan(fx+e)^7+21\,a\tan(fx+e)^5-35\,a\tan(fx+e)^3-105\,(fx+e)a+105\,a\tan(fx+e)}{105\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^6,x, algorithm="giac")
[Out] 1/105*(15*b*tan(f*x + e)^7 + 21*a*tan(f*x + e)^5 - 35*a*tan(f*x + e)^3 - 10
5*(f*x + e)*a + 105*a*tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 18.99 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80

$$\int \left(a+b\sec^2(e+fx)\right)\tan^6(e+fx)\,dx = \frac{\frac{b\tan(e+f\,x)^7}{7}+\frac{a\tan(e+f\,x)^5}{5}-\frac{a\tan(e+f\,x)^3}{3}+a\tan(e+f\,x)-a\,f\,x}{f}$$

```
[In] int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2),x)
```

```
[Out] (a*tan(e + f*x) - (a*tan(e + f*x)^3)/3 + (a*tan(e + f*x)^5)/5 + (b*tan(e +
f*x)^7)/7 - a*f*x)/f
```

3.318 $\int (a+b\sec^2(e+fx))\tan^4(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 48

$$\int (a+b\sec^2(e+fx))\tan^4(e+fx)\,dx = ax-\frac{a\tan(e+fx)}{f}+\frac{a\tan^3(e+fx)}{3f}+\frac{b\tan^5(e+fx)}{5f}$$

```
[Out] a*x-a*tan(f*x+e)/f+1/3*a*tan(f*x+e)^3/f+1/5*b*tan(f*x+e)^5/f
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4226, 1816, 209}

$$\int (a+b\sec^2(e+fx))\tan^4(e+fx)\,dx = \frac{a\tan^3(e+fx)}{3f}-\frac{a\tan(e+fx)}{f}+ax+\frac{b\tan^5(e+fx)}{5f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^4,x]
```

```
[Out] a*x - (a*Tan[e + f*x])/f + (a*Tan[e + f*x]^3)/(3*f) + (b*Tan[e + f*x]^5)/(5
*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4(a+b(1+x^2))}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \left(-a+ax^2+bx^4+\frac{a}{1+x^2}\right)\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{a\tan(e+fx)}{f}+\frac{a\tan^3(e+fx)}{3f}+\frac{b\tan^5(e+fx)}{5f}+\frac{a\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= ax-\frac{a\tan(e+fx)}{f}+\frac{a\tan^3(e+fx)}{3f}+\frac{b\tan^5(e+fx)}{5f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \left(a+b\sec^2(e+fx)\right)\tan^4(e+fx)\,dx = \frac{a\arctan(\tan(e+fx))}{f}-\frac{a\tan(e+fx)}{f}+\frac{a\tan^3(e+fx)}{3f}+\frac{b\tan^5(e+fx)}{5f}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^4,x]
[Out] (a*ArcTan[Tan[e + f*x]])/f - (a*Tan[e + f*x])/f + (a*Tan[e + f*x]^3)/(3*f)
+ (b*Tan[e + f*x]^5)/(5*f)
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

method	result	si
parts	$\frac{a\left(\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+\arctan(\tan(fx+e))\right)}{f}+\frac{b\tan(fx+e)^5}{5f}$	4
derivativedivides	$\frac{a\left(\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+fx+e\right)+\frac{b\sin(fx+e)^5}{5\cos(fx+e)^5}}{f}$	5
default	$\frac{a\left(\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+fx+e\right)+\frac{b\sin(fx+e)^5}{5\cos(fx+e)^5}}{f}$	5
risch	$ax+\frac{2i\left(-30a\,e^{8i(fx+e)}+15b\,e^{8i(fx+e)}-90a\,e^{6i(fx+e)}-110a\,e^{4i(fx+e)}+30b\,e^{4i(fx+e)}-70a\,e^{2i(fx+e)}-20a+3b\right)}{15f\left(e^{2i(fx+e)}+1\right)^5}$	1

```
[In] int((a+b*sec(f*x+e)^2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
[Out] a/f*(1/3*tan(f*x+e)^3-tan(f*x+e)+arctan(tan(f*x+e)))+1/5*b*tan(f*x+e)^5/f
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.50

$$\int (a+b\sec^2(e+fx))\tan^4(e+fx)\,dx = \frac{15\,afx\cos(fx+e)^5-\left((20\,a-3\,b)\cos(fx+e)^4-(5\,a-6\,b)\cos(fx+e)^2-3\,b\right)\sin(fx+e)}{15\,f\cos(fx+e)^5}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^4,x, algorithm="fricas")
[Out] 1/15*(15*a*f*x*cos(f*x + e)^5 - ((20*a - 3*b)*cos(f*x + e)^4 - (5*a - 6*b)*
cos(f*x + e)^2 - 3*b)*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int (a+b\sec^2(e+fx))\tan^4(e+fx)\,dx = a\left(\begin{cases} x+\frac{\tan^3(e+fx)}{3f}-\frac{\tan(e+fx)}{f} & \text{for } f\neq 0 \\ x\tan^4(e) & \text{otherwise}\end{cases}\right) + b\left(\begin{cases} x\tan^4(e)\sec^2(e) & \text{for } f=0 \\ \frac{\tan^5(e+fx)}{5f} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**4,x)
```

```
[Out] a*Piecewise((x + tan(e + f*x)**3/(3*f) - tan(e + f*x)/f, Ne(f, 0)), (x*tan(
e)**4, True)) + b*Piecewise((x*tan(e)**4*sec(e)**2, Eq(f, 0)), (tan(e + f*x
)**5/(5*f), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int (a+b\sec^2(e+fx))\tan^4(e+fx)\,dx = \frac{3\,b\tan(fx+e)^5+5\,a\tan(fx+e)^3+15\,(fx+e)a-15\,a\tan(fx+e)}{15\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] 1/15*(3*b*tan(f*x + e)^5 + 5*a*tan(f*x + e)^3 + 15*(f*x + e)*a - 15*a*tan(f
*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.94 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int (a+b\sec^2(e+fx))\tan^4(e+fx)\,dx = \frac{3\,b\tan(fx+e)^5+5\,a\tan(fx+e)^3+15\,(fx+e)a-15\,a\tan(fx+e)}{15\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] 1/15*(3*b*tan(f*x + e)^5 + 5*a*tan(f*x + e)^3 + 15*(f*x + e)*a - 15*a*tan(f
*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 19.46 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \left(a+b\sec^2(e+fx)\right)\tan^4(e+fx)\,dx=\frac{\frac{b\tan(e+f\,x)^5}{5}+\frac{a\tan(e+f\,x)^3}{3}-a\tan(e+f\,x)+a\,f\,x}{f}$$

```
[In] int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2),x)
[Out] ((a*tan(e + f*x)^3)/3 - a*tan(e + f*x) + (b*tan(e + f*x)^5)/5 + a*f*x)/f
```

3.319 $\int (a+b\sec^2(e+fx))\tan^2(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 32

$$\int (a+b\sec^2(e+fx))\tan^2(e+fx)\,dx=-ax+\frac{a\tan(e+fx)}{f}+\frac{b\tan^3(e+fx)}{3f}$$

```
[Out] -a*x+a*tan(f*x+e)/f+1/3*b*tan(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4226, 1816, 209}

$$\int (a+b\sec^2(e+fx))\tan^2(e+fx)\,dx=\frac{a\tan(e+fx)}{f}-ax+\frac{b\tan^3(e+fx)}{3f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^2,x]
```

```
[Out] -(a*x) + (a*Tan[e + f*x])/f + (b*Tan[e + f*x]^3)/(3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2(a+b(1+x^2))}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(a+bx^2-\frac{a}{1+x^2}\right)\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{a\tan(e+fx)}{f}+\frac{b\tan^3(e+fx)}{3f}-\frac{a\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -ax+\frac{a\tan(e+fx)}{f}+\frac{b\tan^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \left(a+b\sec^2(e+fx)\right)\tan^2(e+fx)\,dx = -\frac{a\arctan(\tan(e+fx))}{f} + \frac{a\tan(e+fx)}{f}+\frac{b\tan^3(e+fx)}{3f}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^2,x]
[Out] -((a*ArcTan[Tan[e + f*x]])/f) + (a*Tan[e + f*x])/f + (b*Tan[e + f*x]^3)/(3*
f)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

method	result	size
parts	$\frac{a(\tan(fx+e)-\arctan(\tan(fx+e)))}{f}+\frac{b\tan(fx+e)^3}{3f}$	37
derivativedivides	$\frac{a(\tan(fx+e)-fx-e)+\frac{b\sin(fx+e)^3}{3\cos(fx+e)^3}}{f}$	41
default	$\frac{a(\tan(fx+e)-fx-e)+\frac{b\sin(fx+e)^3}{3\cos(fx+e)^3}}{f}$	41
risch	$-ax-\frac{2i\left(-3a\,\mathrm{e}^{4i(fx+e)}+3b\,\mathrm{e}^{4i(fx+e)}-6a\,\mathrm{e}^{2i(fx+e)}-3a+b\right)}{3f\left(\mathrm{e}^{2i(fx+e)}+1\right)^3}$	66

```
[In] int((a+b*sec(f*x+e)^2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
[Out] a/f*(tan(f*x+e)-arctan(tan(f*x+e)))+1/3*b*tan(f*x+e)^3/f
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.66

$$\int \left(a+b\sec^2(e+fx)\right)\tan^2(e+fx)\,dx = -\frac{3\,afx\cos\left(fx+e\right)^3-\left(\left(3\,a-b\right)\cos\left(fx+e\right)^2+b\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^2,x, algorithm="fricas")
[Out] -1/3*(3*a*f*x*cos(f*x + e)^3 - ((3*a - b)*cos(f*x + e)^2 + b)*sin(f*x + e))
/(f*cos(f*x + e)^3)
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \left(a+b\sec^2(e+fx)\right)\tan^2(e+fx)\,dx = a\left(\begin{cases}-x+\frac{\tan\left(e+fx\right)}{f} & \text{for}\ f\neq 0\\ x\tan^2\left(e\right) & \text{otherwise}\end{cases}\right) + b\left(\begin{cases}x\tan^2\left(e\right)\sec^2\left(e\right) & \text{for}\ f=0\\ \frac{\tan^3\left(e+fx\right)}{3f} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**2,x)
```

```
[Out] a*Piecewise((-x + tan(e + f*x)/f, Ne(f, 0)), (x*tan(e)**2, True)) + b*Piece
wise((x*tan(e)**2*sec(e)**2, Eq(f, 0)), (tan(e + f*x)**3/(3*f), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int (a+b\sec^2(e+fx))\tan^2(e+fx)\,dx=\frac{b\tan(fx+e)^3-3(fx+e)a+3a\tan(fx+e)}{3f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b*tan(f*x + e)^3 - 3*(f*x + e)*a + 3*a*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int (a+b\sec^2(e+fx))\tan^2(e+fx)\,dx=\frac{b\tan(fx+e)^3-3(fx+e)a+3a\tan(fx+e)}{3f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b*tan(f*x + e)^3 - 3*(f*x + e)*a + 3*a*tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 19.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int (a+b\sec^2(e+fx))\tan^2(e+fx)\,dx=\frac{\frac{b\tan(e+fx)^3}{3}+a\tan(e+fx)-afx}{f}$$

```
[In] int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2),x)
```

```
[Out] (a*tan(e + f*x) + (b*tan(e + f*x)^3)/3 - a*f*x)/f
```

3.320 $\int (a+b\sec^2(e+fx))\,dx$

Optimal result

Integrand size = 12, antiderivative size = 15

$$\int (a+b\sec^2(e+fx))\,dx = ax+\frac{b\tan(e+fx)}{f}$$

```
[Out] a*x+b*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3852, 8}

$$\int (a+b\sec^2(e+fx))\,dx = ax+\frac{b\tan(e+fx)}{f}$$

```
[In] Int[a + b*Sec[e + f*x]^2,x]
[Out] a*x + (b*Tan[e + f*x])/f
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\text{integral} = ax + b\int \sec^2(e+fx)\,dx$$
$$= ax - \frac{b\text{Subst}(\int 1\,dx, x, -\tan(e+fx))}{f}$$
$$= ax + \frac{b\tan(e+fx)}{f}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \left(a+b\sec^2(e+fx)\right)\,dx = ax + \frac{b\tan(e+fx)}{f}$$

```
[In] Integrate[a + b*Sec[e + f*x]^2,x]
[Out] a*x + (b*Tan[e + f*x])/f
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$ax+\frac{b\tan(fx+e)}{f}$	16
parts	$ax+\frac{b\tan(fx+e)}{f}$	16
derivativedivides	$\frac{(fx+e)a+b\tan(fx+e)}{f}$	21
risch	$ax+\frac{2ib}{f\left(e^{2i(fx+e)}+1\right)}$	25
parallelrisch	$-\frac{2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}+ax$	35
norman	$\frac{ax\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-ax-\frac{2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}$	51

```
[In] int(a+b*sec(f*x+e)^2,x,method=_RETURNVERBOSE)
[Out] a*x+b*tan(f*x+e)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(15) = 30.

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \left(a+b\sec^2(e+fx)\right)\,dx=\frac{afx\cos\left(fx+e\right)+b\sin\left(fx+e\right)}{f\cos\left(fx+e\right)}$$

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="fricas")
[Out] (a*f*x*cos(f*x + e) + b*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)\,dx$$

```
[In] integrate(a+b*sec(f*x+e)**2,x)
[Out] Integral(a + b*sec(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \left(a+b\sec^2(e+fx)\right)\,dx=ax+\frac{b\tan\left(fx+e\right)}{f}$$

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="maxima")
[Out] a*x + b*tan(f*x + e)/f
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \left(a+b\sec^2(e+fx)\right)\,dx=ax+\frac{b\tan\left(fx+e\right)}{f}$$

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="giac")
[Out] a*x + b*tan(f*x + e)/f
```

Mupad [B] (verification not implemented)

Time = 19.59 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \left(a + b\sec^2(e + fx)\right) \, dx = \frac{b\tan(e + f\,x) + a\,f\,x}{f}$$

```
[In] int(a + b/cos(e + f*x)^2,x)
[Out] (b*tan(e + f*x) + a*f*x)/f
```

3.321 $\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 19

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-ax-\frac{(a+b)\cot(e+fx)}{f}$$

```
[Out] -a*x-(a+b)*cot(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4226, 1816, 209}

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{(a+b)\cot(e+fx)}{f}-ax$$

```
[In] Int[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(a*x) - ((a + b)*Cot[e + f*x])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{a+b(1+x^2)}{x^2(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a+b}{x^2}-\frac{a}{1+x^2}\right)\,dx, x, \tan(e+fx)\right)}{f}\\ &= -\frac{(a+b)\cot(e+fx)}{f}-\frac{a\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}\\ &= -ax-\frac{(a+b)\cot(e+fx)}{f}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.26

$$\begin{aligned}&\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx\\ &= -\frac{b\cot(e+fx)}{f}-\frac{a\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(e+fx)\right)}{f}\end{aligned}$$

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((b*Cot[e + f*x])/f) - (a*Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Ta
n[e + f*x]^2])/f
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

method	result	size
derivativedivides	$\frac{a(-\cot(fx+e)-fx-e)-b\cot(fx+e)}{f}$	33
default	$\frac{a(-\cot(fx+e)-fx-e)-b\cot(fx+e)}{f}$	33
risch	$-ax-\frac{2ia}{f\left(e^{2i(fx+e)}-1\right)}-\frac{2ib}{f\left(e^{2i(fx+e)}-1\right)}$	46

```
[In] int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(a*(-cot(f*x+e)-f*x-e)-b*cot(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.79

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{afx\sin\left(fx+e\right)+(a+b)\cos\left(fx+e\right)}{f\sin\left(fx+e\right)}$$

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
[Out] -(a*f*x*sin(f*x + e) + (a + b)*cos(f*x + e))/(f*sin(f*x + e))
```

Sympy [F]

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\int\left(a+b\sec^2\left(e+fx\right)\right)\cot^2\left(e+fx\right)dx$$

```
[In] integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2),x)
[Out] Integral((a + b*sec(e + f*x)**2)*cot(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{(fx+e)a+\frac{a+b}{\tan(fx+e)}}{f}$$

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] -((f*x + e)*a + (a + b)/tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(19) = 38.

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.79

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{2(fx+e)a-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+\frac{a+b}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}}{2f}$$

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] -1/2*(2*(f*x + e)*a - a*tan(1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*e) + (a
+ b)/tan(1/2*f*x + 1/2*e))/f
```

Mupad [B] (verification not implemented)

Time = 19.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-a\,x-\frac{\cot(e+f\,x)\,(a+b)}{f}$$

```
[In] int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2),x)
[Out] - a*x - (cot(e + f*x)*(a + b))/f
```

3.322 $\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 33

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = ax+\frac{a\cot(e+fx)}{f}-\frac{(a+b)\cot^3(e+fx)}{3f}$$

```
[Out] a*x+a*cot(f*x+e)/f-1/3*(a+b)*cot(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4226, 1816, 209}

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{(a+b)\cot^3(e+fx)}{3f}+\frac{a\cot(e+fx)}{f}+ax$$

```
[In] Int[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] a*x + (a*Cot[e + f*x])/f - ((a + b)*Cot[e + f*x]^3)/(3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{a+b(1+x^2)}{x^4(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \left(\frac{a+b}{x^4} - \frac{a}{x^2} + \frac{a}{1+x^2}\right)\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{a\cot(e+fx)}{f} - \frac{(a+b)\cot^3(e+fx)}{3f} + \frac{a\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= ax + \frac{a\cot(e+fx)}{f} - \frac{(a+b)\cot^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{b\cot^3(e+fx)}{3f} - \frac{a\cot^3(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(e+fx)\right)}{3f}$$

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
[Out] -1/3*(b*Cot[e + f*x]^3)/f - (a*Cot[e + f*x]^3*Hypergeometric2F1[-3/2, 1, -1
/2, -Tan[e + f*x]^2])/(3*f)
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

method	result	size
derivativedivides	$\frac{a\left(-\frac{\cot(fx+e)^3}{3}+\cot(fx+e)+fx+e\right)-\frac{b\cos(fx+e)^3}{3\sin(fx+e)^3}}{f}$	48
default	$\frac{a\left(-\frac{\cot(fx+e)^3}{3}+\cot(fx+e)+fx+e\right)-\frac{b\cos(fx+e)^3}{3\sin(fx+e)^3}}{f}$	48
risch	$ax+\frac{2i\left(6a\,e^{4i(fx+e)}+3b\,e^{4i(fx+e)}-6a\,e^{2i(fx+e)}+4a+b\right)}{3f\left(e^{2i(fx+e)}-1\right)^3}$	65

```
[In] int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*(-1/3*cot(f*x+e)^3+cot(f*x+e)+f*x+e)-1/3*b/sin(f*x+e)^3*cos(f*x+e)^3
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(31) = 62.

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.30

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(4\,a+b)\cos\left(fx+e\right)^3-3\,a\cos\left(fx+e\right)+3\left(afx\cos\left(fx+e\right)^2-afx\right)\sin\left(fx+e\right)}{3\left(f\cos\left(fx+e\right)^2-f\right)\sin\left(fx+e\right)}$$

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/3*((4*a + b)*cos(f*x + e)^3 - 3*a*cos(f*x + e) + 3*(a*f*x*cos(f*x + e)^2
- a*f*x)*sin(f*x + e))/((f*cos(f*x + e)^2 - f)*sin(f*x + e))
```

Sympy [F]

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)\cot^4\left(e+fx\right)dx$$

```
[In] integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*cot(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{3\,(fx+e)a+\frac{3\,a\tan(fx+e)^2-a-b}{\tan(fx+e)^3}}{3\,f}$$

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(f*x + e)*a + (3*a*tan(f*x + e)^2 - a - b)/tan(f*x + e)^3)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(31) = 62.

Time = 0.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.36

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$= \frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+24\,(fx+e)a-15\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-3\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+}{24\,f}$$

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*f*x + 1/2*e)^3 + b*tan(1/2*f*x + 1/2*e)^3 + 24*(f*x + e)*a
- 15*a*tan(1/2*f*x + 1/2*e) - 3*b*tan(1/2*f*x + 1/2*e) + (15*a*tan(1/2*f*x
+ 1/2*e)^2 + 3*b*tan(1/2*f*x + 1/2*e)^2 - a - b)/tan(1/2*f*x + 1/2*e)^3)/f
```

Mupad [B] (verification not implemented)

Time = 19.44 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = a\,x-\frac{-a\tan\left(e+f\,x\right)^2+\frac{a}{3}+\frac{b}{3}}{f\tan\left(e+f\,x\right)^3}$$

```
[In] int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2),x)
```

```
[Out] a*x - (a/3 + b/3 - a*tan(e + f*x)^2)/(f*tan(e + f*x)^3)
```

3.323 $\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-ax-\frac{a\cot(e+fx)}{f}+\frac{a\cot^3(e+fx)}{3f}-\frac{(a+b)\cot^5(e+fx)}{5f}$$

```
[Out] -a*x-a*cot(f*x+e)/f+1/3*a*cot(f*x+e)^3/f-1/5*(a+b)*cot(f*x+e)^5/f
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4226, 1816, 209}

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{(a+b)\cot^5(e+fx)}{5f}+\frac{a\cot^3(e+fx)}{3f}-\frac{a\cot(e+fx)}{f}-ax$$

```
[In] Int[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(a*x) - (a*Cot[e + f*x])/f + (a*Cot[e + f*x]^3)/(3*f) - ((a + b)*Cot[e + f
*x]^5)/(5*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{a+b(1+x^2)}{x^6(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a+b}{x^6} - \frac{a}{x^4} + \frac{a}{x^2} - \frac{a}{1+x^2}\right)\,dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{a\cot(e+fx)}{f} + \frac{a\cot^3(e+fx)}{3f} - \frac{(a+b)\cot^5(e+fx)}{5f} - \frac{a\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -ax - \frac{a\cot(e+fx)}{f} + \frac{a\cot^3(e+fx)}{3f} - \frac{(a+b)\cot^5(e+fx)}{5f}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{b\cot^5(e+fx)}{5f} - \frac{a\cot^5(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(e+fx)\right)}{5f}$$

```
[In] Integrate[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
[Out] -1/5*(b*Cot[e + f*x]^5)/f - (a*Cot[e + f*x]^5*Hypergeometric2F1[-5/2, 1, -3
/2, -Tan[e + f*x]^2])/(5*f)
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

method	result	siz
derivativedivides	$\frac{a\left(-\frac{\cot(fx+e)^5}{5}+\frac{\cot(fx+e)^3}{3}-\cot(fx+e)-fx-e\right)-\frac{b\cos(fx+e)^5}{5\sin(fx+e)^5}}{f}$	63
default	$\frac{a\left(-\frac{\cot(fx+e)^5}{5}+\frac{\cot(fx+e)^3}{3}-\cot(fx+e)-fx-e\right)-\frac{b\cos(fx+e)^5}{5\sin(fx+e)^5}}{f}$	63
risch	$-ax-\frac{2i\left(45a\,e^{8i(fx+e)}+15b\,e^{8i(fx+e)}-90a\,e^{6i(fx+e)}+140a\,e^{4i(fx+e)}+30b\,e^{4i(fx+e)}-70a\,e^{2i(fx+e)}+23a+3b\right)}{15f\left(e^{2i(fx+e)}-1\right)^5}$	10

```
[In] int(cot(f*x+e)^6*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*(-1/5*cot(f*x+e)^5+1/3*cot(f*x+e)^3-cot(f*x+e)-f*x-e)-1/5*b/sin(f*x+
e)^5*cos(f*x+e)^5)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(47) = 94.

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.16

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{(23\,a+3\,b)\cos\left(fx+e\right)^5-35\,a\cos\left(fx+e\right)^3+15\,a\cos\left(fx+e\right)+15\left(afx\cos\left(fx+e\right)^4-2\,afx\cos\left(f\right.\right.}{15\left(f\cos\left(fx+e\right)^4-2\,f\cos\left(fx+e\right)^2+f\right)\sin\left(fx+e\right)}$$

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/15*((23*a + 3*b)*cos(f*x + e)^5 - 35*a*cos(f*x + e)^3 + 15*a*cos(f*x + e
) + 15*(a*f*x*cos(f*x + e)^4 - 2*a*f*x*cos(f*x + e)^2 + a*f*x)*sin(f*x + e)
)/((f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 + f)*sin(f*x + e))
```

Sympy [F]

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)\cot^6\left(e+fx\right)dx$$

```
[In] integrate(cot(f*x+e)**6*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*cot(e + f*x)**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{15\,(fx+e)a+\frac{15\,a\tan(fx+e)^4-5\,a\tan(fx+e)^2+3\,a+3\,b}{\tan(fx+e)^5}}{15\,f}$$

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*(f*x + e)*a + (15*a*tan(f*x + e)^4 - 5*a*tan(f*x + e)^2 + 3*a + 3
*b)/tan(f*x + e)^5)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(47) = 94.

Time = 0.34 (sec) , antiderivative size = 170, normalized size of antiderivative = 3.33

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$=\frac{3\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+3\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-35\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-15\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-480\,(fx+$$

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/480*(3*a*tan(1/2*f*x + 1/2*e)^5 + 3*b*tan(1/2*f*x + 1/2*e)^5 - 35*a*tan(1
/2*f*x + 1/2*e)^3 - 15*b*tan(1/2*f*x + 1/2*e)^3 - 480*(f*x + e)*a + 330*a*t
an(1/2*f*x + 1/2*e) + 30*b*tan(1/2*f*x + 1/2*e) - (330*a*tan(1/2*f*x + 1/2*
e)^4 + 30*b*tan(1/2*f*x + 1/2*e)^4 - 35*a*tan(1/2*f*x + 1/2*e)^2 - 15*b*tan
(1/2*f*x + 1/2*e)^2 + 3*a + 3*b)/tan(1/2*f*x + 1/2*e)^5)/f
```

Mupad [B] (verification not implemented)

Time = 19.80 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-a\,x-\frac{a\tan(e+f\,x)^4-\frac{a\tan(e+f\,x)^2}{3}+\frac{a}{5}+\frac{b}{5}}{f\tan\left(e+f\,x\right)^5}$$

```
[In] int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2),x)
```

```
[Out] - a*x - (a/5 + b/5 - (a*tan(e + f*x)^2)/3 + a*tan(e + f*x)^4)/(f*tan(e + f*
x)^5)
```

3.324 $\int (a+b\sec^2(e+fx))^2\tan^5(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 100

$$\int (a+b\sec^2(e+fx))^2\tan^5(e+fx)\,dx = -\frac{a^2\log(\cos(e+fx))}{f}-\frac{a(a-b)\sec^2(e+fx)}{f}+\frac{(a^2-4ab+b^2)\sec^4(e+fx)}{4f}+\frac{(a-b)b\sec^6(e+fx)}{3f}+\frac{b^2\sec^8(e+fx)}{8f}$$

```
[Out] -a^2*ln(cos(f*x+e))/f-a*(a-b)*sec(f*x+e)^2/f+1/4*(a^2-4*a*b+b^2)*sec(f*x+e)
^4/f+1/3*(a-b)*b*sec(f*x+e)^6/f+1/8*b^2*sec(f*x+e)^8/f
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4223, 457, 90}

$$\int (a+b\sec^2(e+fx))^2\tan^5(e+fx)\,dx = \frac{(a^2-4ab+b^2)\sec^4(e+fx)}{4f}-\frac{a^2\log(\cos(e+fx))}{f}+\frac{b(a-b)\sec^6(e+fx)}{3f}-\frac{a(a-b)\sec^2(e+fx)}{f}+\frac{b^2\sec^8(e+fx)}{8f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^5,x]
```

```
[Out] -((a^2*Log[Cos[e + f*x]])/f) - (a*(a - b)*Sec[e + f*x]^2)/f + ((a^2 - 4*a*b
 + b^2)*Sec[e + f*x]^4)/(4*f) + ((a - b)*b*Sec[e + f*x]^6)/(3*f) + (b^2*Sec
[e + f*x]^8)/(8*f)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2(b+ax^2)^2}{x^9}\,dx, x, \cos(e+fx)\right)}{f} \\
&= -\frac{\text{Subst}\left(\int \frac{(1-x)^2(b+ax)^2}{x^5}\,dx, x, \cos^2(e+fx)\right)}{2f} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{x^5}+\frac{2(a-b)b}{x^4}+\frac{a^2-4ab+b^2}{x^3}-\frac{2a(a-b)}{x^2}+\frac{a^2}{x}\right)dx, x, \cos^2(e+fx)\right)}{2f} \\
&= -\frac{a^2\log(\cos(e+fx))}{f}-\frac{a(a-b)\sec^2(e+fx)}{f} \\
&\quad +\frac{(a^2-4ab+b^2)\sec^4(e+fx)}{4f}+\frac{(a-b)b\sec^6(e+fx)}{3f}+\frac{b^2\sec^8(e+fx)}{8f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.26

$$\int (a+b\sec^2(e+fx))^2\tan^5(e+fx)\,dx = -\frac{\cos^4(e+fx)\,(a+b\sec^2(e+fx))^2\,(24a^2\log(\cos(e+fx))+24a(a-b)\sec^2(e+fx)-6(a^2-4ab+b^2)\,\mathrm{s}}{6f(a+2b+a\cos(2e+2fx))^2}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^5,x]
```

```
[Out] -1/6*(Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*(24*a^2*Log[Cos[e + f*x]] + 2
4*a*(a - b)*Sec[e + f*x]^2 - 6*(a^2 - 4*a*b + b^2)*Sec[e + f*x]^4 - 8*(a -
b)*b*Sec[e + f*x]^6 - 3*b^2*Sec[e + f*x]^8))/(f*(a + 2*b + a*Cos[2*e + 2*f*
x])^2)
```

Maple [A] (verified)

Time = 9.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.86

method	result
parts	$\frac{a^2\left(\frac{\tan(fx+e)^4}{4}-\frac{\tan(fx+e)^2}{2}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}+\frac{b^2\left(\frac{\tan(fx+e)^8}{8}+\frac{\tan(fx+e)^6}{6}\right)}{f}+\frac{ab\tan(fx+e)^6}{3f}$
derivativedivides	$\frac{\frac{b^2\sec(fx+e)^8}{8}+\frac{b\sec(fx+e)^6a}{3}-\frac{b^2\sec(fx+e)^6}{3}+\frac{a^2\sec(fx+e)^4}{4}-ab\sec(fx+e)^4+\frac{\sec(fx+e)^4b^2}{4}-a^2\sec(fx+e)^2+\sec(fx+e)^2a}{f}$
default	$\frac{\frac{b^2\sec(fx+e)^8}{8}+\frac{b\sec(fx+e)^6a}{3}-\frac{b^2\sec(fx+e)^6}{3}+\frac{a^2\sec(fx+e)^4}{4}-ab\sec(fx+e)^4+\frac{\sec(fx+e)^4b^2}{4}-a^2\sec(fx+e)^2+\sec(fx+e)^2a}{f}$
risch	$ia^2x+\frac{2ia^2e}{f}+\frac{-4a^2\mathrm{e}^{14i(fx+e)}+4ab\,\mathrm{e}^{14i(fx+e)}-20a^2\mathrm{e}^{12i(fx+e)}+8ab\,\mathrm{e}^{12i(fx+e)}+4b^2\mathrm{e}^{12i(fx+e)}-44a^2\mathrm{e}^{10i(fx+e)}+5}{}$

```
[In] int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/f*(1/4*tan(f*x+e)^4-1/2*tan(f*x+e)^2+1/2*ln(1+tan(f*x+e)^2))+b^2/f*(1/8
*tan(f*x+e)^8+1/6*tan(f*x+e)^6)+1/3*a*b/f*tan(f*x+e)^6
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^5(e+fx)\,dx = -\frac{24\,a^2\cos\left(fx+e\right)^8\log\left(-\cos\left(fx+e\right)\right)+24\left(a^2-ab\right)\cos\left(fx+e\right)^6-6\left(a^2-4\,ab+b^2\right)\cos\left(fx+e\right)^4-}{24\,f\cos\left(fx+e\right)^8}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] -1/24*(24*a^2*cos(f*x + e)^8*log(-cos(f*x + e)) + 24*(a^2 - a*b)*cos(f*x +
e)^6 - 6*(a^2 - 4*a*b + b^2)*cos(f*x + e)^4 - 8*(a*b - b^2)*cos(f*x + e)^2
- 3*b^2)/(f*cos(f*x + e)^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(85) = 170.

Time = 1.13 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.90

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^5(e+fx)\,dx = \begin{cases} \frac{a^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a^2\tan^4\left(e+fx\right)}{4f}-\frac{a^2\tan^2\left(e+fx\right)}{2f}+\frac{ab\tan^4\left(e+fx\right)\sec^2\left(e+fx\right)}{3f}-\frac{ab\tan^2\left(e+fx\right)\sec^2\left(e+fx\right)}{3f}+\frac{ab\sec^2\left(e\right.}{3f} \\ x\left(a+b\sec^2\left(e\right)\right)^2\tan^5\left(e\right) \end{cases}$$

```
[In] integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**5,x)
```

```
[Out] Piecewise((a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a**2*tan(e + f*x)**4/(4*f)
 - a**2*tan(e + f*x)**2/(2*f) + a*b*tan(e + f*x)**4*sec(e + f*x)**2/(3*f) -
 a*b*tan(e + f*x)**2*sec(e + f*x)**2/(3*f) + a*b*sec(e + f*x)**2/(3*f) + b*
*2*tan(e + f*x)**4*sec(e + f*x)**4/(8*f) - b**2*tan(e + f*x)**2*sec(e + f*x
)**4/(12*f) + b**2*sec(e + f*x)**4/(24*f), Ne(f, 0)), (x*(a + b*sec(e)**2)*
*2*tan(e)**5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.47

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^5(e+fx)\,dx = -\frac{12\,a^2\log\left(\sin\left(fx+e\right)^2-1\right)-\frac{24\,(a^2-ab)\sin(fx+e)^6-6\,(11\,a^2-8\,ab-b^2)\sin(fx+e)^4+4\,(15\,a^2-8\,ab-b^2)\sin(fx+e)^2-18\,a^2+8\,ab}{\sin(fx+e)^8-4\,\sin(fx+e)^6+6\,\sin(fx+e)^4-4\,\sin(fx+e)^2+1}}{24\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -1/24*(12*a^2*log(sin(f*x + e)^2 - 1) - (24*(a^2 - a*b)*sin(f*x + e)^6 - 6*
(11*a^2 - 8*a*b - b^2)*sin(f*x + e)^4 + 4*(15*a^2 - 8*a*b - b^2)*sin(f*x +
e)^2 - 18*a^2 + 8*a*b + b^2)/(sin(f*x + e)^8 - 4*sin(f*x + e)^6 + 6*sin(f*x
 + e)^4 - 4*sin(f*x + e)^2 + 1))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. 2(94) = 188.

Time = 1.89 (sec) , antiderivative size = 431, normalized size of antiderivative = 4.31

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^5(e+fx)\,dx$$

$$= \frac{12\,a^2\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right)-12\,a^2\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-2\right|\right)+\frac{25\,a^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\right.}{}}{}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] 1/24*(12*a^2*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) + 2)) - 12*a^2*log(abs(-(cos(f*x + e) + 1)/(cos(f*
x + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2)) + (25*a^2*((cos(f
*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1))^4
+ 248*a^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(
f*x + e) + 1))^3 + 984*a^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*
x + e) - 1)/(cos(f*x + e) + 1))^2 + 1760*a^2*((cos(f*x + e) + 1)/(cos(f*x +
 e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 512*a*b*((cos(f*x + e)
+ 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 256*b^2*
((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) +
 1)) + 1168*a^2 - 1024*a*b + 256*b^2)/((cos(f*x + e) + 1)/(cos(f*x + e) - 1
) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2)^4)/f
```

Mupad [B] (verification not implemented)

Time = 19.52 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.24

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^5(e+fx)\,dx = \frac{\tan(e+f\,x)^4\,\left(\frac{(a+b)^2}{4}+\frac{b^2}{4}-\frac{b\,(a+b)}{2}\right)}{f} - \frac{\tan(e+f\,x)^2\,\left(\frac{(a+b)^2}{2}+\frac{b^2}{2}-b\,(a+b)\right)}{f} - \frac{\tan(e+f\,x)^6\,\left(\frac{b^2}{6}-\frac{b\,(a+b)}{3}\right)}{f} + \frac{a^2\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f} + \frac{b^2\tan(e+f\,x)^8}{8\,f}$$

```
[In] int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (tan(e + f*x)^4*((a + b)^2/4 + b^2/4 - (b*(a + b))/2))/f - (tan(e + f*x)^2*
((a + b)^2/2 + b^2/2 - b*(a + b)))/f - (tan(e + f*x)^6*(b^2/6 - (b*(a + b))
/3))/f + (a^2*log(tan(e + f*x)^2 + 1))/(2*f) + (b^2*tan(e + f*x)^8)/(8*f)
```

3.325 $\int (a+b\sec^2(e+fx))^2\tan^3(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int (a+b\sec^2(e+fx))^2\tan^3(e+fx)\,dx = \frac{a^2\log(\cos(e+fx))}{f}+\frac{a(a-2b)\sec^2(e+fx)}{2f}+\frac{(2a-b)b\sec^4(e+fx)}{4f}+\frac{b^2\sec^6(e+fx)}{6f}$$

```
[Out] a^2*ln(cos(f*x+e))/f+1/2*a*(a-2*b)*sec(f*x+e)^2/f+1/4*(2*a-b)*b*sec(f*x+e)^
4/f+1/6*b^2*sec(f*x+e)^6/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4223, 457, 77}

$$\int (a+b\sec^2(e+fx))^2\tan^3(e+fx)\,dx = \frac{a^2\log(\cos(e+fx))}{f}+\frac{b(2a-b)\sec^4(e+fx)}{4f}+\frac{a(a-2b)\sec^2(e+fx)}{2f}+\frac{b^2\sec^6(e+fx)}{6f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^3,x]
```

```
[Out] (a^2*Log[Cos[e + f*x]])/f + (a*(a - 2*b)*Sec[e + f*x]^2)/(2*f) + ((2*a - b)
*b*Sec[e + f*x]^4)/(4*f) + (b^2*Sec[e + f*x]^6)/(6*f)
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
```

```
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)(b+ax^2)^2}{x^7}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \frac{(1-x)(b+ax)^2}{x^4}\,dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{x^4}+\frac{(2a-b)b}{x^3}+\frac{a(a-2b)}{x^2}-\frac{a^2}{x}\right)\,dx, x, \cos^2(e+fx)\right)}{2f}\\
&= \frac{a^2\log(\cos(e+fx))}{f}+\frac{a(a-2b)\sec^2(e+fx)}{2f}+\frac{(2a-b)b\sec^4(e+fx)}{4f}+\frac{b^2\sec^6(e+fx)}{6f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.39

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^3(e+fx)\,dx = \frac{\cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\left(12a^2\log(\cos(e+fx))+6a(a-2b)\sec^2(e+fx)+3(2a-b)b\sec^4(e+\right.}{3f(a+2b+a\cos(2e+2fx))^2}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^3,x]
```

```
[Out] (Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*(12*a^2*Log[Cos[e + f*x]] + 6*a*(a
 - 2*b)*Sec[e + f*x]^2 + 3*(2*a - b)*b*Sec[e + f*x]^4 + 2*b^2*Sec[e + f*x]^
6))/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2)
```

Maple [A] (verified)

Time = 4.57 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

method	result
parts	$\frac{a^2\left(\frac{\tan(fx+e)^2}{2}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}+\frac{b^2\left(\frac{\sec(fx+e)^6}{6}-\frac{\sec(fx+e)^4}{4}\right)}{f}+\frac{ab\tan(fx+e)^4}{2f}$
derivativedivides	$\frac{\frac{b^2\sec(fx+e)^6}{6}+\frac{ab\sec(fx+e)^4}{2}-\frac{\sec(fx+e)^4b^2}{4}+\frac{a^2\sec(fx+e)^2}{2}-\sec(fx+e)^2ab-a^2\ln(\sec(fx+e))}{f}$
default	$\frac{\frac{b^2\sec(fx+e)^6}{6}+\frac{ab\sec(fx+e)^4}{2}-\frac{\sec(fx+e)^4b^2}{4}+\frac{a^2\sec(fx+e)^2}{2}-\sec(fx+e)^2ab-a^2\ln(\sec(fx+e))}{f}$
risch	$-ia^2x-\frac{2ia^2e}{f}-\frac{2\left(-3a^2e^{10i(fx+e)}+6ab\,e^{10i(fx+e)}-12a^2e^{8i(fx+e)}+12ab\,e^{8i(fx+e)}+6b^2e^{8i(fx+e)}-18a^2e^{6i(fx+e)}+\right.}{3f(e}$

```
[In] int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/f*(1/2*tan(f*x+e)^2-1/2*ln(1+tan(f*x+e)^2))+b^2/f*(1/6*sec(f*x+e)^6-1/4
*sec(f*x+e)^4)+1/2*a*b/f*tan(f*x+e)^4
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^3(e+fx)\,dx$$
$$=\frac{12\,a^2\cos\left(fx+e\right)^6\log\left(-\cos\left(fx+e\right)\right)+6\left(a^2-2\,ab\right)\cos\left(fx+e\right)^4+3\left(2\,ab-b^2\right)\cos\left(fx+e\right)^2+2\,b^2}{12\,f\cos\left(fx+e\right)^6}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(12*a^2*cos(f*x + e)^6*log(-cos(f*x + e)) + 6*(a^2 - 2*a*b)*cos(f*x +
e)^4 + 3*(2*a*b - b^2)*cos(f*x + e)^2 + 2*b^2)/(f*cos(f*x + e)^6)
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.66

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^3(e+fx)\,dx$$
$$=\begin{cases} -\frac{a^2\log\left(\tan^2(e+fx)+1\right)}{2f}+\frac{a^2\tan^2(e+fx)}{2f}+\frac{ab\tan^2(e+fx)\sec^2(e+fx)}{2f}-\frac{ab\sec^2(e+fx)}{2f}+\frac{b^2\tan^2(e+fx)\sec^4(e+fx)}{6f}-\frac{b^2\sec^4}{12} \\ x\left(a+b\sec^2(e)\right)^2\tan^3(e) \end{cases}$$

```
[In] integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**3,x)
```

```
[Out] Piecewise((-a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a**2*tan(e + f*x)**2/(2*f
) + a*b*tan(e + f*x)**2*sec(e + f*x)**2/(2*f) - a*b*sec(e + f*x)**2/(2*f) +
 b**2*tan(e + f*x)**2*sec(e + f*x)**4/(6*f) - b**2*sec(e + f*x)**4/(12*f),
Ne(f, 0)), (x*(a + b*sec(e)**2)**2*tan(e)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.48

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^3(e+fx)\,dx$$
$$=\frac{6\,a^2\log\left(\sin(fx+e)^2-1\right)-\frac{6\,(a^2-2\,ab)\sin(fx+e)^4-3\,(4\,a^2-6\,ab-b^2)\sin(fx+e)^2+6\,a^2-6\,ab-b^2}{\sin(fx+e)^6-3\,\sin(fx+e)^4+3\,\sin(fx+e)^2-1}}{12\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(6*a^2*log(sin(f*x + e)^2 - 1) - (6*(a^2 - 2*a*b)*sin(f*x + e)^4 - 3*(
4*a^2 - 6*a*b - b^2)*sin(f*x + e)^2 + 6*a^2 - 6*a*b - b^2)/(sin(f*x + e)^6
- 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(71) = 142.

Time = 0.88 (sec) , antiderivative size = 385, normalized size of antiderivative = 5.00

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^3(e+fx)\,dx =$$
$$-\frac{6\,a^2\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right)-6\,a^2\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-2\right|\right)+\frac{11\,a^2\left(\frac{\cos(fx+e)+}{\cos(fx+e)-}\right.}{}}{}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] -1/12*(6*a^2*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) + 2)) - 6*a^2*log(abs(-(cos(f*x + e) + 1)/(cos(f*x
 + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2)) + (11*a^2*((cos(f*
x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1))^3 +
 90*a^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*
x + e) + 1))^2 + 228*a^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x
+ e) - 1)/(cos(f*x + e) + 1)) - 96*a*b*((cos(f*x + e) + 1)/(cos(f*x + e) -
1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 48*b^2*((cos(f*x + e) + 1)/(c
os(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 184*a^2 - 192*a
*b + 32*b^2)/((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(c
os(f*x + e) + 1) + 2)^3)/f
```

Mupad [B] (verification not implemented)

Time = 20.18 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.19

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^3(e+fx)\,dx=\frac{\tan(e+f\,x)^2\,\left(\frac{(a+b)^2}{2}+\frac{b^2}{2}-b\,(a+b)\right)}{f}-\frac{\tan(e+f\,x)^4\,\left(\frac{b^2}{4}-\frac{b\,(a+b)}{2}\right)}{f}-\frac{a^2\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f}+\frac{b^2\tan(e+f\,x)^6}{6\,f}$$

```
[In] int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (tan(e + f*x)^2*((a + b)^2/2 + b^2/2 - b*(a + b)))/f - (tan(e + f*x)^4*(b^2
/4 - (b*(a + b))/2))/f - (a^2*log(tan(e + f*x)^2 + 1))/(2*f) + (b^2*tan(e +
 f*x)^6)/(6*f)
```

3.326 $\int (a+b\sec^2(e+fx))^2\tan(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 48

$$\int (a+b\sec^2(e+fx))^2\tan(e+fx)\,dx=-\frac{a^2\log(\cos(e+fx))}{f}+\frac{ab\sec^2(e+fx)}{f}+\frac{b^2\sec^4(e+fx)}{4f}$$

```
[Out] -a^2*ln(cos(f*x+e))/f+a*b*sec(f*x+e)^2/f+1/4*b^2*sec(f*x+e)^4/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4223, 272, 45}

$$\int (a+b\sec^2(e+fx))^2\tan(e+fx)\,dx=-\frac{a^2\log(\cos(e+fx))}{f}+\frac{ab\sec^2(e+fx)}{f}+\frac{b^2\sec^4(e+fx)}{4f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x],x]
```

```
[Out] -((a^2*Log[Cos[e + f*x]])/f) + (a*b*Sec[e + f*x]^2)/f + (b^2*Sec[e + f*x]^4
)/(4*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(b+ax^2)^2}{x^5}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \frac{(b+ax)^2}{x^3}\,dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{x^3}+\frac{2ab}{x^2}+\frac{a^2}{x}\right)\,dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{a^2\log(\cos(e+fx))}{f}+\frac{ab\sec^2(e+fx)}{f}+\frac{b^2\sec^4(e+fx)}{4f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.73

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan(e+fx)\,dx = -\frac{\cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\left(4a^2\log(\cos(e+fx))-4ab\sec^2(e+fx)-b^2\sec^4(e+fx)\right)}{f(a+2b+a\cos(2e+2fx))^2}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x],x]
```

```
[Out] -((Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*(4*a^2*Log[Cos[e + f*x]] - 4*a*b
*Sec[e + f*x]^2 - b^2*Sec[e + f*x]^4))/(f*(a + 2*b + a*Cos[2*e + 2*f*x])^2)
)
```

Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\frac{\sec(fx+e)^4 b^2}{4}+\sec(fx+e)^2 ab+a^2\ln(\sec(fx+e))}{f}$	41
default	$\frac{\frac{\sec(fx+e)^4 b^2}{4}+\sec(fx+e)^2 ab+a^2\ln(\sec(fx+e))}{f}$	41
parts	$\frac{a^2\ln\left(1+\tan(fx+e)^2\right)}{2f}+\frac{b^2\sec(fx+e)^4}{4f}+\frac{ab\sec(fx+e)^2}{f}$	51
risch	$ia^2x+\frac{2ia^2e}{f}+\frac{4b\left(a\,e^{6i(fx+e)}+2a\,e^{4i(fx+e)}+b\,e^{4i(fx+e)}+a\,e^{2i(fx+e)}\right)}{f\left(e^{2i(fx+e)}+1\right)^4}-\frac{a^2\ln\left(e^{2i(fx+e)}+1\right)}{f}$	104

```
[In] int((a+b*sec(f*x+e)^2)^2*tan(f*x+e),x,method=_RETURNVERBOSE)
[Out] 1/f*(1/4*sec(f*x+e)^4*b^2+sec(f*x+e)^2*a*b+a^2*ln(sec(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan(e+fx)\,dx = -\frac{4\,a^2\cos\left(fx+e\right)^4\log\left(-\cos\left(fx+e\right)\right)-4\,ab\cos\left(fx+e\right)^2-b^2}{4\,f\cos\left(fx+e\right)^4}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e),x, algorithm="fricas")
[Out] -1/4*(4*a^2*cos(f*x + e)^4*log(-cos(f*x + e)) - 4*a*b*cos(f*x + e)^2 - b^2)
/(f*cos(f*x + e)^4)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan(e+fx)\,dx = \begin{cases} \frac{a^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{ab\sec^2\left(e+fx\right)}{f}+\frac{b^2\sec^4\left(e+fx\right)}{4f} & \text{for}\ f\neq 0 \\ x\left(a+b\sec^2\left(e\right)\right)^2\tan\left(e\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e),x)
[Out] Piecewise((a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a*b*sec(e + f*x)**2/f + b*
*2*sec(e + f*x)**4/(4*f), Ne(f, 0)), (x*(a + b*sec(e)**2)**2*tan(e), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.40

$$\int (a+b\sec^2(e+fx))^2\tan(e+fx)\,dx = -\frac{2\,a^2\log\left(\sin\left(fx+e\right)^2-1\right)+\frac{4\,ab\sin(fx+e)^2-4\,ab-b^2}{\sin(fx+e)^4-2\,\sin(fx+e)^2+1}}{4\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] -1/4*(2*a^2*log(sin(f*x + e)^2 - 1) + (4*a*b*sin(f*x + e)^2 - 4*a*b - b^2)/
(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(46) = 92.

Time = 0.42 (sec) , antiderivative size = 334, normalized size of antiderivative = 6.96

$$\int (a+b\sec^2(e+fx))^2\tan(e+fx)\,dx$$

$$= \frac{2\,a^2\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right)-2\,a^2\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-2\right|\right)+\frac{3\,a^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(}{\cos(}\right.}{}}{4\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e),x, algorithm="giac")
```

```
[Out] 1/4*(2*a^2*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) -
 1)/(cos(f*x + e) + 1) + 2)) - 2*a^2*log(abs(-(cos(f*x + e) + 1)/(cos(f*x +
 e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2)) + (3*a^2*((cos(f*x +
 e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1))^2 + 12
*a^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x +
 e) + 1)) - 16*a*b*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) -
 1)/(cos(f*x + e) + 1)) - 8*b^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (c
os(f*x + e) - 1)/(cos(f*x + e) + 1)) + 12*a^2 - 32*a*b)/((cos(f*x + e) + 1)
/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2)^2)/f
```

Mupad [B] (verification not implemented)

Time = 19.69 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan(e+fx)\,dx=\frac{a^2\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f}-\frac{\tan(e+f\,x)^2\,\left(\frac{b^2}{2}-b\,(a+b)\right)}{f}+\frac{b^2\tan(e+f\,x)^4}{4\,f}$$

```
[In] int(tan(e + f*x)*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (a^2*log(tan(e + f*x)^2 + 1))/(2*f) - (tan(e + f*x)^2*(b^2/2 - b*(a + b)))/
f + (b^2*tan(e + f*x)^4)/(4*f)
```

3.327 $\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=-\frac{b(2a+b)\log(\cos(e+fx))}{f}+\frac{(a+b)^2\log(\sin(e+fx))}{f}+\frac{b^2\sec^2(e+fx)}{2f}$$

```
[Out] -b*(2*a+b)*ln(cos(f*x+e))/f+(a+b)^2*ln(sin(f*x+e))/f+1/2*b^2*sec(f*x+e)^2/f
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4223, 457, 90}

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{(a+b)^2\log(\sin(e+fx))}{f}-\frac{b(2a+b)\log(\cos(e+fx))}{f}+\frac{b^2\sec^2(e+fx)}{2f}$$

```
[In] Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -((b*(2*a + b)*Log[Cos[e + f*x]])/f) + ((a + b)^2*Log[Sin[e + f*x]])/f + (b
^2*Sec[e + f*x]^2)/(2*f)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(b+ax^2)^2}{x^3(1-x^2)}\,dx, x, \cos(e+fx)\right)}{f} \\
&= -\frac{\text{Subst}\left(\int \frac{(b+ax)^2}{(1-x)x^2}\,dx, x, \cos^2(e+fx)\right)}{2f} \\
&= -\frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{-1+x}+\frac{b^2}{x^2}+\frac{b(2a+b)}{x}\right)\,dx, x, \cos^2(e+fx)\right)}{2f} \\
&= -\frac{b(2a+b)\log(\cos(e+fx))}{f}+\frac{(a+b)^2\log(\sin(e+fx))}{f}+\frac{b^2\sec^2(e+fx)}{2f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.58

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{2(b^2+2\cos^2(e+fx)\left(-b(2a+b)\log(\cos(e+fx))+(a+b)^2\log(\sin(e+fx))\right))\left(a\cos(e+fx)+b\sec(e\right.}{f(a+2b+a\cos(2(e+fx)))^2}$$

```
[In] Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (2*(b^2 + 2*Cos[e + f*x]^2*(-(b*(2*a + b)*Log[Cos[e + f*x]]) + (a + b)^2*Lo
g[Sin[e + f*x]]))*(a*Cos[e + f*x] + b*Sec[e + f*x])^2)/(f*(a + 2*b + a*Cos[
2*(e + f*x)])^2)
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{a^{2} \ln \left(\sin \left(f x+e\right)\right)+2 a b \ln \left(\tan \left(f x+e\right)\right)+b^{2} \left(\frac{1}{2 \cos \left(f x+e\right)^{2}}+\ln \left(\tan \left(f x+e\right)\right)\right)}{f}$
default	$\frac{a^{2} \ln \left(\sin \left(f x+e\right)\right)+2 a b \ln \left(\tan \left(f x+e\right)\right)+b^{2} \left(\frac{1}{2 \cos \left(f x+e\right)^{2}}+\ln \left(\tan \left(f x+e\right)\right)\right)}{f}$
risch	$-i a^{2} x-\frac{2 i a^{2} e}{f}+\frac{2 b^{2} {\mathrm e}^{2 i \left(f x+e\right)}}{f \left({\mathrm e}^{2 i \left(f x+e\right)}+1\right)^{2}}-\frac{2 b \ln \left({\mathrm e}^{2 i \left(f x+e\right)}+1\right) a}{f}-\frac{b^{2} \ln \left({\mathrm e}^{2 i \left(f x+e\right)}+1\right)}{f}+\frac{\ln \left({\mathrm e}^{2 i \left(f x+e\right)}-1\right) a^{2}}{f}+\frac{2 \ln}{}$

[In] int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/f*(a^2*ln(sin(f*x+e))+2*a*b*ln(tan(f*x+e))+b^2*(1/2/cos(f*x+e)^2+ln(tan(f
*x+e))))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.49

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{\left(2\,ab+b^{2}\right)\cos\left(fx+e\right)^{2}\log\left(\cos\left(fx+e\right)^{2}\right)-\left(a^{2}+2\,ab+b^{2}\right)\cos\left(fx+e\right)^{2}\log\left(-\frac{1}{4}\,\cos\left(fx+e\right)^{2}+\frac{1}{4}\right)-}{2\,f\cos\left(fx+e\right)^{2}}$$

[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")

[Out] -1/2*((2*a*b + b^2)*cos(f*x + e)^2*log(cos(f*x + e)^2) - (a^2 + 2*a*b + b^2
)*cos(f*x + e)^2*log(-1/4*cos(f*x + e)^2 + 1/4) - b^2)/(f*cos(f*x + e)^2)

Sympy [F]

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^{2}\left(e+fx\right)\right)^{2}\cot\left(e+fx\right)dx$$

[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2)**2,x)

[Out] Integral((a + b*sec(e + f*x)**2)**2*cot(e + f*x), x)

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.21

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=-\frac{(2\,ab+b^2)\log\left(\sin\left(fx+e\right)^2-1\right)-(a^2+2\,ab+b^2)\log\left(\sin\left(fx+e\right)^2\right)+\frac{b^2}{\sin(fx+e)^2-1}}{2\,f}$$

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((2*a*b + b^2)*log(sin(f*x + e)^2 - 1) - (a^2 + 2*a*b + b^2)*log(sin(f
*x + e)^2) + b^2/(sin(f*x + e)^2 - 1))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(51) = 102.

Time = 0.32 (sec) , antiderivative size = 247, normalized size of antiderivative = 4.66

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx =$$
$$-\frac{a^2\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right)+(2\,ab+b^2)\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-2\right|\right)-\frac{2\,ab\left(\frac{\cos(f}{\cos(f}\right.}{}}{2\,f}$$

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(a^2*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) -
1)/(cos(f*x + e) + 1) + 2)) + (2*a*b + b^2)*log(abs(-(cos(f*x + e) + 1)/(co
s(f*x + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2)) - (2*a*b*((co
s(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1))
 + b^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x
 + e) + 1)) + 4*a*b - 2*b^2)/((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(
f*x + e) - 1)/(cos(f*x + e) + 1) + 2))/f
```

Mupad [B] (verification not implemented)

Time = 20.39 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \cot(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \,dx = \frac{\ln\left(\tan(e+f\,x)\right)\,\left(a^2+2\,a\,b+b^2\right)}{f} - \frac{a^2\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f} + \frac{b^2\tan(e+f\,x)^2}{2\,f}$$

```
[In] int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^2,x)
[Out] (log(tan(e + f*x))*(2*a*b + a^2 + b^2))/f - (a^2*log(tan(e + f*x)^2 + 1))/(
2*f) + (b^2*tan(e + f*x)^2)/(2*f)
```

3.328 $\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 57

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=-\frac{(a+b)^2\csc^2(e+fx)}{2f}-\frac{b^2\log(\cos(e+fx))}{f}-\frac{(a^2-b^2)\log(\sin(e+fx))}{f}$$

```
[Out] -1/2*(a+b)^2*csc(f*x+e)^2/f-b^2*ln(cos(f*x+e))/f-(a^2-b^2)*ln(sin(f*x+e))/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4223, 457, 90}

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=-\frac{(a^2-b^2)\log(\sin(e+fx))}{f}-\frac{(a+b)^2\csc^2(e+fx)}{2f}-\frac{b^2\log(\cos(e+fx))}{f}$$

```
[In] Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/2*((a + b)^2*Csc[e + f*x]^2)/f - (b^2*Log[Cos[e + f*x]])/f - ((a^2 - b^2
)*Log[Sin[e + f*x]])/f
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(b+ax^2)^2}{x(1-x^2)^2}\,dx, x, \cos(e+fx)\right)}{f} \\
&= -\frac{\text{Subst}\left(\int \frac{(b+ax)^2}{(1-x)^2x}\,dx, x, \cos^2(e+fx)\right)}{2f} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{(a+b)^2}{(-1+x)^2}+\frac{a^2-b^2}{-1+x}+\frac{b^2}{x}\right)\,dx, x, \cos^2(e+fx)\right)}{2f} \\
&= -\frac{(a+b)^2\csc^2(e+fx)}{2f}-\frac{b^2\log(\cos(e+fx))}{f}-\frac{(a^2-b^2)\log(\sin(e+fx))}{f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.42

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{2(b+a\cos^2(e+fx))^2\left((a+b)^2\csc^2(e+fx)+2b^2\log(\cos(e+fx))+2(a^2-b^2)\log(\sin(e+fx))\right)}{f(a+2b+a\cos(2(e+fx)))^2}$$

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (-2*(b + a*Cos[e + f*x]^2)^2*((a + b)^2*Csc[e + f*x]^2 + 2*b^2*Log[Cos[e +
f*x]] + 2*(a^2 - b^2)*Log[Sin[e + f*x]]))/(f*(a + 2*b + a*Cos[2*(e + f*x)])
^2)
```

Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cot(fx+e)^2}{2}-\ln(\sin(fx+e))\right)-\frac{ab}{\sin(fx+e)^2}+b^2\left(-\frac{1}{2\sin(fx+e)^2}+\ln(\tan(fx+e))\right)}{f}$
default	$\frac{a^2\left(-\frac{\cot(fx+e)^2}{2}-\ln(\sin(fx+e))\right)-\frac{ab}{\sin(fx+e)^2}+b^2\left(-\frac{1}{2\sin(fx+e)^2}+\ln(\tan(fx+e))\right)}{f}$
risch	$ia^2x+\frac{2ia^2e}{f}+\frac{2(a^2+2ab+b^2)e^{2i(fx+e)}}{f\left(e^{2i(fx+e)}-1\right)^2}-\frac{\ln\left(e^{2i(fx+e)}-1\right)a^2}{f}+\frac{\ln\left(e^{2i(fx+e)}-1\right)b^2}{f}-\frac{b^2\ln\left(e^{2i(fx+e)}+1\right)}{f}$

```
[In] int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/f*(a^2*(-1/2*cot(f*x+e)^2-ln(sin(f*x+e)))-a*b/sin(f*x+e)^2+b^2*(-1/2/sin(
f*x+e)^2+ln(tan(f*x+e))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.75

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{a^2+2\,ab+b^2-\left(b^2\cos\left(fx+e\right)^2-b^2\right)\log\left(\cos\left(fx+e\right)^2\right)-\left(\left(a^2-b^2\right)\cos\left(fx+e\right)^2-a^2+b^2\right)\log\left(-\frac{1}{4}\right.}{2\left(f\cos\left(fx+e\right)^2-f\right)}$$

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
[Out] 1/2*(a^2 + 2*a*b + b^2 - (b^2*cos(f*x + e)^2 - b^2)*log(cos(f*x + e)^2) - (
(a^2 - b^2)*cos(f*x + e)^2 - a^2 + b^2)*log(-1/4*cos(f*x + e)^2 + 1/4))/(f*
cos(f*x + e)^2 - f)
```

Sympy [F]

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^2\cot^3\left(e+fx\right)dx$$

```
[In] integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2)**2,x)
[Out] Integral((a + b*sec(e + f*x)**2)**2*cot(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= -\frac{b^2\log\left(\sin\left(fx+e\right)^2-1\right)+\left(a^2-b^2\right)\log\left(\sin\left(fx+e\right)^2\right)+\frac{a^2+2\,ab+b^2}{\sin(fx+e)^2}}{2\,f}$$

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*log(sin(f*x + e)^2 - 1) + (a^2 - b^2)*log(sin(f*x + e)^2) + (a^2
+ 2*a*b + b^2)/sin(f*x + e)^2)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(55) = 110.

Time = 0.33 (sec) , antiderivative size = 233, normalized size of antiderivative = 4.09

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{a^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+2\,ab\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+b^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+4\,a^2\log\left(\left|\cdot\right.\right.}{8\,f}$$

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(a^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f
*x + e) + 1)) + 2*a*b*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e
) - 1)/(cos(f*x + e) + 1)) + b^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (
cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 4*a^2*log(abs(-(cos(f*x + e) + 1)/(
cos(f*x + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2)) - 4*b^2*log
(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x +
 e) + 1) - 2)))/f
```

Mupad [B] (verification not implemented)

Time = 20.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{a^2\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f}-\frac{\ln\left(\tan(e+f\,x)\right)\,\left(a^2-b^2\right)}{f}-\frac{\cot(e+f\,x)^2\,\left(\frac{a^2}{2}+a\,b+\frac{b^2}{2}\right)}{f}$$

```
[In] int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (a^2*log(tan(e + f*x)^2 + 1))/(2*f) - (log(tan(e + f*x))*(a^2 - b^2))/f - (
cot(e + f*x)^2*(a*b + a^2/2 + b^2/2))/f
```

3.329 $\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 51

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{a(a+b)\csc^2(e+fx)}{f}-\frac{(a+b)^2\csc^4(e+fx)}{4f}+\frac{a^2\log(\sin(e+fx))}{f}$$

```
[Out] a*(a+b)*csc(f*x+e)^2/f-1/4*(a+b)^2*csc(f*x+e)^4/f+a^2*ln(sin(f*x+e))/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4223, 455, 45}

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{a^2\log(\sin(e+fx))}{f}-\frac{(a+b)^2\csc^4(e+fx)}{4f}+\frac{a(a+b)\csc^2(e+fx)}{f}$$

```
[In] Int[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (a*(a + b)*Csc[e + f*x]^2)/f - ((a + b)^2*Csc[e + f*x]^4)/(4*f) + (a^2*Log[
Sin[e + f*x]])/f
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x(b+ax^2)^2}{(1-x^2)^3}\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \frac{(b+ax)^2}{(1-x)^3}\,dx, x, \cos^2(e+fx)\right)}{2f}\\ &= -\frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{(-1+x)^3}-\frac{2a(a+b)}{(-1+x)^2}-\frac{a^2}{-1+x}\right)dx, x, \cos^2(e+fx)\right)}{2f}\\ &= \frac{a(a+b)\csc^2(e+fx)}{f}-\frac{(a+b)^2\csc^4(e+fx)}{4f}+\frac{a^2\log(\sin(e+fx))}{f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.51

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{\left(b+a\cos^2(e+fx)\right)^2\left(-4a(a+b)\csc^2(e+fx)+(a+b)^2\csc^4(e+fx)-4a^2\log(\sin(e+fx))\right)}{f(a+2b+a\cos(2(e+fx)))^2}$$

```
[In] Integrate[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
[Out] -(((b + a*Cos[e + f*x]^2)^2*(-4*a*(a + b)*Csc[e + f*x]^2 + (a + b)^2*Csc[e
+ f*x]^4 - 4*a^2*Log[Sin[e + f*x]]))/(f*(a + 2*b + a*Cos[2*(e + f*x)])^2))
```

Maple [A] (verified)

Time = 5.40 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cot(fx+e)^4}{4}+\frac{\cot(fx+e)^2}{2}+\ln(\sin(fx+e))\right)-\frac{ab\cos(fx+e)^4}{2\sin(fx+e)^4}-\frac{b^2}{4\sin(fx+e)^4}}{f}$
default	$\frac{a^2\left(-\frac{\cot(fx+e)^4}{4}+\frac{\cot(fx+e)^2}{2}+\ln(\sin(fx+e))\right)-\frac{ab\cos(fx+e)^4}{2\sin(fx+e)^4}-\frac{b^2}{4\sin(fx+e)^4}}{f}$
risch	$-ia^2x-\frac{2ia^2e}{f}-\frac{4\left(a^2e^{6i(fx+e)}+ab\,e^{6i(fx+e)}-a^2e^{4i(fx+e)}+b^2e^{4i(fx+e)}+a^2e^{2i(fx+e)}+ab\,e^{2i(fx+e)}\right)}{f\left(e^{2i(fx+e)}-1\right)^4}+\frac{\ln(e^{2i(fx-}}{f}$

```
[In] int(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(-1/4*cot(f*x+e)^4+1/2*cot(f*x+e)^2+ln(sin(f*x+e)))-1/2*a*b/sin(f*
x+e)^4*cos(f*x+e)^4-1/4*b^2/sin(f*x+e)^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.90

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{4\left(a^2+ab\right)\cos\left(fx+e\right)^2-3\,a^2-2\,ab+b^2-4\left(a^2\cos\left(fx+e\right)^4-2\,a^2\cos\left(fx+e\right)^2+a^2\right)\log\left(\frac{1}{2}\,\sin\left(fx\right.\right.}{4\left(f\cos\left(fx+e\right)^4-2\,f\cos\left(fx+e\right)^2+f\right)}$$

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*(a^2 + a*b)*cos(f*x + e)^2 - 3*a^2 - 2*a*b + b^2 - 4*(a^2*cos(f*x +
 e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*log(1/2*sin(f*x + e)))/(f*cos(f*x + e)^
4 - 2*f*cos(f*x + e)^2 + f)
```

Sympy [F]

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^2\cot^5\left(e+fx\right)dx$$

```
[In] integrate(cot(f*x+e)**5*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*cot(e + f*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20

$$\int \cot^5(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = \frac{2\,a^2 \log\left(\sin\left(fx+e\right)^2\right) + \frac{4\,\left(a^2+ab\right)\sin(fx+e)^2-a^2-2\,ab-b^2}{\sin(fx+e)^4}}{4\,f}$$

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*a^2*log(sin(f*x + e)^2) + (4*(a^2 + a*b)*sin(f*x + e)^2 - a^2 - 2*a*
b - b^2)/sin(f*x + e)^4)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(49) = 98.

Time = 0.36 (sec) , antiderivative size = 332, normalized size of antiderivative = 6.51

$$\int \cot^5(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx$$

$$= \frac{32\,a^2 \log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right) - 64\,a^2 \log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right) - \frac{12\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{8\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{4\,b^2(\cos(fx-}{\cos(fx+e}}{}$$

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/64*(32*a^2*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - 64*a^2*log
(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)) - 12*a^2*(cos(f*x + e) -
1)/(cos(f*x + e) + 1) - 8*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 4*b^2
*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - a^2*(cos(f*x + e) - 1)^2/(cos(f*x
+ e) + 1)^2 - 2*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - b^2*(cos(f*
x + e) - 1)^2/(cos(f*x + e) + 1)^2 - (a^2 + 2*a*b + b^2 + 12*a^2*(cos(f*x +
 e) - 1)/(cos(f*x + e) + 1) + 8*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) -
 4*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 48*a^2*(cos(f*x + e) - 1)^2/
(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(cos(f*x + e) - 1)^2)/f
```

Mupad [B] (verification not implemented)

Time = 20.71 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.63

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{a^2\,\ln\left(\tan(e+f\,x)\right)}{f}-\frac{\frac{a\,b}{2}+\frac{a^2}{4}+\frac{b^2}{4}-\tan(e+f\,x)^2\,\left(\frac{a^2}{2}-\frac{b^2}{2}\right)}{f\tan\left(e+f\,x\right)^4}-\frac{a^2\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f}$$

```
[In] int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (a^2*log(tan(e + f*x)))/f - ((a*b)/2 + a^2/4 + b^2/4 - tan(e + f*x)^2*(a^2/
2 - b^2/2))/(f*tan(e + f*x)^4) - (a^2*log(tan(e + f*x)^2 + 1))/(2*f)
```

3.330 $\int (a+b\sec^2(e+fx))^2\tan^6(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 95

$$\int (a+b\sec^2(e+fx))^2\tan^6(e+fx)\,dx = -a^2x+\frac{a^2\tan(e+fx)}{f}-\frac{a^2\tan^3(e+fx)}{3f}+\frac{a^2\tan^5(e+fx)}{5f}+\frac{b(2a+b)\tan^7(e+fx)}{7f}+\frac{b^2\tan^9(e+fx)}{9f}$$

```
[Out] -a^2*x+a^2*tan(f*x+e)/f-1/3*a^2*tan(f*x+e)^3/f+1/5*a^2*tan(f*x+e)^5/f+1/7*b
*(2*a+b)*tan(f*x+e)^7/f+1/9*b^2*tan(f*x+e)^9/f
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4226, 1816, 209}

$$\int (a+b\sec^2(e+fx))^2\tan^6(e+fx)\,dx = \frac{a^2\tan^5(e+fx)}{5f}-\frac{a^2\tan^3(e+fx)}{3f}+\frac{a^2\tan(e+fx)}{f}-a^2x+\frac{b(2a+b)\tan^7(e+fx)}{7f}+\frac{b^2\tan^9(e+fx)}{9f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^6,x]
```

```
[Out] -(a^2*x) + (a^2*Tan[e + f*x])/f - (a^2*Tan[e + f*x]^3)/(3*f) + (a^2*Tan[e +
 f*x]^5)/(5*f) + (b*(2*a + b)*Tan[e + f*x]^7)/(7*f) + (b^2*Tan[e + f*x]^9)/
(9*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^6\left(a+b\left(1+x^2\right)\right)^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(a^2-a^2x^2+a^2x^4+b(2a+b)x^6+b^2x^8-\frac{a^2}{1+x^2}\right)\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{a^2\tan(e+fx)}{f}-\frac{a^2\tan^3(e+fx)}{3f}+\frac{a^2\tan^5(e+fx)}{5f}+\frac{b(2a+b)\tan^7(e+fx)}{7f}\\
&\quad+\frac{b^2\tan^9(e+fx)}{9f}-\frac{a^2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -a^2x+\frac{a^2\tan(e+fx)}{f}-\frac{a^2\tan^3(e+fx)}{3f}+\frac{a^2\tan^5(e+fx)}{5f}\\
&\quad+\frac{b(2a+b)\tan^7(e+fx)}{7f}+\frac{b^2\tan^9(e+fx)}{9f}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 275 vs. $2(95) = 190$.

Time = 3.71 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.89

$$\int (a + b\sec^2(e + fx))^2 \tan^6(e + fx)\,dx =$$
$$-\frac{4(b + a\cos^2(e + fx))^2 \sec^9(e + fx)\,(315a^2 fx\cos^9(e + fx) - 35b^2 \sec(e)\sin(fx) - 5(18a - 19b)b\cos^2(e}{}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^6,x]
```

```
[Out] (-4*(b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^9*(315*a^2*f*x*Cos[e + f*x]^9 - 3
5*b^2*Sec[e]*Sin[f*x] - 5*(18*a - 19*b)*b*Cos[e + f*x]^2*Sec[e]*Sin[f*x] -
3*(21*a^2 - 90*a*b + 25*b^2)*Cos[e + f*x]^4*Sec[e]*Sin[f*x] + (231*a^2 - 27
0*a*b + 5*b^2)*Cos[e + f*x]^6*Sec[e]*Sin[f*x] - (483*a^2 - 90*a*b - 10*b^2)
*Cos[e + f*x]^8*Sec[e]*Sin[f*x] - 35*b^2*Cos[e + f*x]*Tan[e] - 5*(18*a - 19
*b)*b*Cos[e + f*x]^3*Tan[e] - 3*(21*a^2 - 90*a*b + 25*b^2)*Cos[e + f*x]^5*T
an[e] + (231*a^2 - 270*a*b + 5*b^2)*Cos[e + f*x]^7*Tan[e]))/(315*f*(a + 2*b
 + a*Cos[2*(e + f*x)])^2)
```

Maple [A] (verified)

Time = 12.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

method	result
parts	$\frac{a^2\left(\frac{\tan(fx+e)^5}{5}-\frac{\tan(fx+e)^3}{3}+\tan(fx+e)-\arctan(\tan(fx+e))\right)}{f}+\frac{b^2\left(\frac{\tan(fx+e)^9}{9}+\frac{\tan(fx+e)^7}{7}\right)}{f}+\frac{2ab\tan(fx+e)^7}{7f}$
derivativedivides	$\frac{a^2\left(\frac{\tan(fx+e)^5}{5}-\frac{\tan(fx+e)^3}{3}+\tan(fx+e)-fx-e\right)+\frac{2ab\sin(fx+e)^7}{7\cos(fx+e)^7}+b^2\left(\frac{\sin(fx+e)^7}{9\cos(fx+e)^9}+\frac{2\sin(fx+e)^7}{63\cos(fx+e)^7}\right)}{f}$
default	$\frac{a^2\left(\frac{\tan(fx+e)^5}{5}-\frac{\tan(fx+e)^3}{3}+\tan(fx+e)-fx-e\right)+\frac{2ab\sin(fx+e)^7}{7\cos(fx+e)^7}+b^2\left(\frac{\sin(fx+e)^7}{9\cos(fx+e)^9}+\frac{2\sin(fx+e)^7}{63\cos(fx+e)^7}\right)}{f}$
risch	$-a^2x-\frac{2i(90ab-483a^2+10b^2-28350a^2e^{10i(fx+e)}+5040ab\,e^{8i(fx+e)}+3780ab\,e^{6i(fx+e)}+1980ab\,e^{4i(fx+e)}+180ab\,e}{}$

```
[In] int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^6,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/f*(1/5*tan(f*x+e)^5-1/3*tan(f*x+e)^3+tan(f*x+e)-arctan(tan(f*x+e)))+b^2
/f*(1/9*tan(f*x+e)^9+1/7*tan(f*x+e)^7)+2/7*a*b/f*tan(f*x+e)^7
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.44

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^6(e+fx)\,dx = -\frac{315\,a^2fx\cos\left(fx+e\right)^9-\left((483\,a^2-90\,ab-10\,b^2)\cos\left(fx+e\right)^8-(231\,a^2-270\,ab+5\,b^2)\cos\left(fx+e\right)^6 \ldots}{315\,f\cos\left(fx+\ldots\right.}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] -1/315*(315*a^2*f*x*cos(f*x + e)^9 - ((483*a^2 - 90*a*b - 10*b^2)*cos(f*x +
 e)^8 - (231*a^2 - 270*a*b + 5*b^2)*cos(f*x + e)^6 + 3*(21*a^2 - 90*a*b + 2
5*b^2)*cos(f*x + e)^4 + 5*(18*a*b - 19*b^2)*cos(f*x + e)^2 + 35*b^2)*sin(f*
x + e))/(f*cos(f*x + e)^9)
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^6(e+fx)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^2\tan^6\left(e+fx\right)dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**6,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*tan(e + f*x)**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^6(e+fx)\,dx = \frac{35\,b^2\tan\left(fx+e\right)^9+45\left(2\,ab+b^2\right)\tan\left(fx+e\right)^7+63\,a^2\tan\left(fx+e\right)^5-105\,a^2\tan\left(fx+e\right)^3-315\left(fx+\ldots\right.}{315\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] 1/315*(35*b^2*tan(f*x + e)^9 + 45*(2*a*b + b^2)*tan(f*x + e)^7 + 63*a^2*tan
(f*x + e)^5 - 105*a^2*tan(f*x + e)^3 - 315*(f*x + e)*a^2 + 315*a^2*tan(f*x
+ e))/f
```

Giac [A] (verification not implemented)

none

Time = 2.52 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^6(e+fx)\,dx$$
$$=\frac{35\,b^2\tan\left(fx+e\right)^9+90\,ab\tan\left(fx+e\right)^7+45\,b^2\tan\left(fx+e\right)^7+63\,a^2\tan\left(fx+e\right)^5-105\,a^2\tan\left(fx+e\right)}{315\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^6,x, algorithm="giac")
```

```
[Out] 1/315*(35*b^2*tan(f*x + e)^9 + 90*a*b*tan(f*x + e)^7 + 45*b^2*tan(f*x + e)^
7 + 63*a^2*tan(f*x + e)^5 - 105*a^2*tan(f*x + e)^3 - 315*(f*x + e)*a^2 + 31
5*a^2*tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 19.69 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.33

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^6(e+fx)\,dx$$
$$=\frac{\tan(e+f\,x)\,\left((a+b)^2+b^2-2\,b\,(a+b)\right)-\tan(e+f\,x)^3\left(\frac{(a+b)^2}{3}+\frac{b^2}{3}-\frac{2\,b\,(a+b)}{3}\right)+\tan(e+f\,x)^5\left(\frac{(a+b)^2}{5}\right.}{f}$$

```
[In] int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (tan(e + f*x)*((a + b)^2 + b^2 - 2*b*(a + b)) - tan(e + f*x)^3*((a + b)^2/3
 + b^2/3 - (2*b*(a + b))/3) + tan(e + f*x)^5*((a + b)^2/5 + b^2/5 - (2*b*(a
 + b))/5) - tan(e + f*x)^7*(b^2/7 - (2*b*(a + b))/7) + (b^2*tan(e + f*x)^9)
/9 - a^2*f*x)/f
```

3.331 $\int (a+b\sec^2(e+fx))^2 \tan^4(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int (a+b\sec^2(e+fx))^2 \tan^4(e+fx)\,dx = a^2x - \frac{a^2\tan(e+fx)}{f} + \frac{a^2\tan^3(e+fx)}{3f} + \frac{b(2a+b)\tan^5(e+fx)}{5f} + \frac{b^2\tan^7(e+fx)}{7f}$$

```
[Out] a^2*x-a^2*tan(f*x+e)/f+1/3*a^2*tan(f*x+e)^3/f+1/5*b*(2*a+b)*tan(f*x+e)^5/f+
1/7*b^2*tan(f*x+e)^7/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4226, 1816, 209}

$$\int (a+b\sec^2(e+fx))^2 \tan^4(e+fx)\,dx = \frac{a^2\tan^3(e+fx)}{3f} - \frac{a^2\tan(e+fx)}{f} + a^2x + \frac{b(2a+b)\tan^5(e+fx)}{5f} + \frac{b^2\tan^7(e+fx)}{7f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^4,x]
```

```
[Out] a^2*x - (a^2*Tan[e + f*x])/f + (a^2*Tan[e + f*x]^3)/(3*f) + (b*(2*a + b)*Ta
n[e + f*x]^5)/(5*f) + (b^2*Tan[e + f*x]^7)/(7*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4(a+b(1+x^2))^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(-a^2+a^2x^2+b(2a+b)x^4+b^2x^6+\frac{a^2}{1+x^2}\right)dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{a^2\tan(e+fx)}{f}+\frac{a^2\tan^3(e+fx)}{3f}+\frac{b(2a+b)\tan^5(e+fx)}{5f}\\
&\quad+\frac{b^2\tan^7(e+fx)}{7f}+\frac{a^2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= a^2x-\frac{a^2\tan(e+fx)}{f}+\frac{a^2\tan^3(e+fx)}{3f}+\frac{b(2a+b)\tan^5(e+fx)}{5f}+\frac{b^2\tan^7(e+fx)}{7f}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 395 vs. $2(77) = 154$.

Time = 2.30 (sec) , antiderivative size = 395, normalized size of antiderivative = 5.13

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^4(e+fx)\,dx$$
$$= \frac{\sec(e)\sec^7(e+fx)\left(3675a^2fx\cos(fx)+3675a^2fx\cos(2e+fx)+2205a^2fx\cos(2e+3fx)+2205a^2fx\,\mathrm{c}\right.}{}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^4,x]
```

```
[Out] (Sec[e]*Sec[e + f*x]^7*(3675*a^2*f*x*Cos[f*x] + 3675*a^2*f*x*Cos[2*e + f*x]
 + 2205*a^2*f*x*Cos[2*e + 3*f*x] + 2205*a^2*f*x*Cos[4*e + 3*f*x] + 735*a^2*
f*x*Cos[4*e + 5*f*x] + 735*a^2*f*x*Cos[6*e + 5*f*x] + 105*a^2*f*x*Cos[6*e +
 7*f*x] + 105*a^2*f*x*Cos[8*e + 7*f*x] - 5320*a^2*Sin[f*x] + 1680*a*b*Sin[f
*x] + 840*b^2*Sin[f*x] + 4480*a^2*Sin[2*e + f*x] - 1260*a*b*Sin[2*e + f*x]
+ 420*b^2*Sin[2*e + f*x] - 3780*a^2*Sin[2*e + 3*f*x] + 924*a*b*Sin[2*e + 3*
f*x] - 168*b^2*Sin[2*e + 3*f*x] + 2100*a^2*Sin[4*e + 3*f*x] - 840*a*b*Sin[4
*e + 3*f*x] - 420*b^2*Sin[4*e + 3*f*x] - 1540*a^2*Sin[4*e + 5*f*x] + 168*a*
b*Sin[4*e + 5*f*x] + 84*b^2*Sin[4*e + 5*f*x] + 420*a^2*Sin[6*e + 5*f*x] - 4
20*a*b*Sin[6*e + 5*f*x] - 280*a^2*Sin[6*e + 7*f*x] + 84*a*b*Sin[6*e + 7*f*x
] + 12*b^2*Sin[6*e + 7*f*x]))/(13440*f)
```

Maple [A] (verified)

Time = 6.53 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

method	result
parts	$\frac{a^2\left(\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+\arctan(\tan(fx+e))\right)}{f}+\frac{b^2\left(\frac{\tan(fx+e)^7}{7}+\frac{\tan(fx+e)^5}{5}\right)}{f}+\frac{2ab\tan(fx+e)^5}{5f}$
derivativedivides	$\frac{a^2\left(\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+fx+e\right)+\frac{2ab\sin(fx+e)^5}{5\cos(fx+e)^5}+b^2\left(\frac{\sin(fx+e)^5}{7\cos(fx+e)^7}+\frac{2\sin(fx+e)^5}{35\cos(fx+e)^5}\right)}{f}$
default	$\frac{a^2\left(\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+fx+e\right)+\frac{2ab\sin(fx+e)^5}{5\cos(fx+e)^5}+b^2\left(\frac{\sin(fx+e)^5}{7\cos(fx+e)^7}+\frac{2\sin(fx+e)^5}{35\cos(fx+e)^5}\right)}{f}$
risch	$a^2x+\frac{4i\left(-105a^2e^{12i(fx+e)}+105ab\,e^{12i(fx+e)}-525a^2e^{10i(fx+e)}+210ab\,e^{10i(fx+e)}+105b^2e^{10i(fx+e)}-1120a^2e^{8i(fx+e)}\right.}{}$

```
[In] int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/f*(1/3*tan(f*x+e)^3-tan(f*x+e)+arctan(tan(f*x+e)))+b^2/f*(1/7*tan(f*x+e
)^7+1/5*tan(f*x+e)^5)+2/5*a*b/f*tan(f*x+e)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.47

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^4(e+fx)\,dx$$
$$=\frac{105\,a^2fx\cos\left(fx+e\right)^7-\left(2\left(70\,a^2-21\,ab-3\,b^2\right)\cos\left(fx+e\right)^6-\left(35\,a^2-84\,ab+3\,b^2\right)\cos\left(fx+e\right)^4-6\,(7}{105\,f\cos\left(fx+e\right)^7}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(105*a^2*f*x*cos(f*x + e)^7 - (2*(70*a^2 - 21*a*b - 3*b^2)*cos(f*x +
e)^6 - (35*a^2 - 84*a*b + 3*b^2)*cos(f*x + e)^4 - 6*(7*a*b - 4*b^2)*cos(f*x
 + e)^2 - 15*b^2)*sin(f*x + e))/(f*cos(f*x + e)^7)
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^4(e+fx)\,dx=\int \left(a+b\sec^2{(e+fx)}\right)^2\tan^4{(e+fx)}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**4,x)
[Out] Integral((a + b*sec(e + f*x)**2)**2*tan(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^4(e+fx)\,dx$$
$$=\frac{15\,b^2\tan{(fx+e)}^7+21\,(2\,ab+b^2)\tan{(fx+e)}^5+35\,a^2\tan{(fx+e)}^3+105\,(fx+e)a^2-105\,a^2\tan{(fx}{105\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^4,x, algorithm="maxima")
[Out] 1/105*(15*b^2*tan(f*x + e)^7 + 21*(2*a*b + b^2)*tan(f*x + e)^5 + 35*a^2*tan
(f*x + e)^3 + 105*(f*x + e)*a^2 - 105*a^2*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 1.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^4(e+fx)\,dx$$
$$=\frac{15\,b^2\tan{(fx+e)}^7+42\,ab\tan{(fx+e)}^5+21\,b^2\tan{(fx+e)}^5+35\,a^2\tan{(fx+e)}^3+105\,(fx+e)a^2-1}{105\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^4,x, algorithm="giac")
[Out] 1/105*(15*b^2*tan(f*x + e)^7 + 42*a*b*tan(f*x + e)^5 + 21*b^2*tan(f*x + e)^
5 + 35*a^2*tan(f*x + e)^3 + 105*(f*x + e)*a^2 - 105*a^2*tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 19.43 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.26

$$\int \left(a + b\sec^2(e + fx)\right)^2 \tan^4(e + fx)\, dx$$
$$= \frac{\tan(e + f\,x)^3\,\left(\frac{(a+b)^2}{3} + \frac{b^2}{3} - \frac{2\,b\,(a+b)}{3}\right) - \tan(e + f\,x)\,\left((a + b)^2 + b^2 - 2\,b\,(a + b)\right) - \tan(e + f\,x)^5\,\left(\frac{b^2}{5} - \frac{2\,b}{}\right.}{f}$$

```
[In] int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (tan(e + f*x)^3*((a + b)^2/3 + b^2/3 - (2*b*(a + b))/3) - tan(e + f*x)*((a
+ b)^2 + b^2 - 2*b*(a + b)) - tan(e + f*x)^5*(b^2/5 - (2*b*(a + b))/5) + (b
^2*tan(e + f*x)^7)/7 + a^2*f*x)/f
```

3.332 $\int (a+b\sec^2(e+fx))^2\tan^2(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 59

$$\int (a+b\sec^2(e+fx))^2\tan^2(e+fx)\,dx=-a^2x+\frac{a^2\tan(e+fx)}{f}+\frac{b(2a+b)\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+fx)}{5f}$$

```
[Out] -a^2*x+a^2*tan(f*x+e)/f+1/3*b*(2*a+b)*tan(f*x+e)^3/f+1/5*b^2*tan(f*x+e)^5/f
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4226, 1816, 209}

$$\int (a+b\sec^2(e+fx))^2\tan^2(e+fx)\,dx=\frac{a^2\tan(e+fx)}{f}-a^2x+\frac{b(2a+b)\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+fx)}{5f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^2,x]
```

```
[Out] -(a^2*x) + (a^2*Tan[e + f*x])/f + (b*(2*a + b)*Tan[e + f*x]^3)/(3*f) + (b^2
*Tan[e + f*x]^5)/(5*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2\left(a+b\left(1+x^2\right)\right)^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(a^2+b(2a+b)x^2+b^2x^4-\frac{a^2}{1+x^2}\right)\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{a^2\tan(e+fx)}{f}+\frac{b(2a+b)\tan^3(e+fx)}{3f}\\
&\quad+\frac{b^2\tan^5(e+fx)}{5f}-\frac{a^2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -a^2x+\frac{a^2\tan(e+fx)}{f}+\frac{b(2a+b)\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+fx)}{5f}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 281 vs. $2(59) = 118$.

Time = 1.71 (sec) , antiderivative size = 281, normalized size of antiderivative = 4.76

$$
\int \left(a+b\sec^2(e+fx)\right)^2\tan^2(e+fx)\,dx = -\frac{\sec(e)\sec^5(e+fx)\left(150a^2fx\cos(fx)+150a^2fx\cos(2e+fx)+75a^2fx\cos(2e+3fx)+75a^2fx\cos(4e\right.}{}
$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^2,x]
```

```
[Out] -1/480*(Sec[e]*Sec[e + f*x]^5*(150*a^2*f*x*Cos[f*x] + 150*a^2*f*x*Cos[2*e +
 f*x] + 75*a^2*f*x*Cos[2*e + 3*f*x] + 75*a^2*f*x*Cos[4*e + 3*f*x] + 15*a^2*
```

```
f*x*Cos[4*e + 5*f*x] + 15*a^2*f*x*Cos[6*e + 5*f*x] - 180*a^2*Sin[f*x] + 80*
a*b*Sin[f*x] - 20*b^2*Sin[f*x] + 120*a^2*Sin[2*e + f*x] - 120*a*b*Sin[2*e +
 f*x] - 60*b^2*Sin[2*e + f*x] - 120*a^2*Sin[2*e + 3*f*x] + 40*a*b*Sin[2*e +
 3*f*x] + 20*b^2*Sin[2*e + 3*f*x] + 30*a^2*Sin[4*e + 3*f*x] - 60*a*b*Sin[4*
e + 3*f*x] - 30*a^2*Sin[4*e + 5*f*x] + 20*a*b*Sin[4*e + 5*f*x] + 4*b^2*Sin[
4*e + 5*f*x]))/f
```

Maple [A] (verified)

Time = 3.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.15

method	result
parts	$\frac{a^2(\tan(fx+e)-\arctan(\tan(fx+e)))}{f}+\frac{b^2\left(\frac{\tan(fx+e)^5}{5}+\frac{\tan(fx+e)^3}{3}\right)}{f}+\frac{2ab\tan(fx+e)^3}{3f}$
derivativedivides	$\frac{a^2(\tan(fx+e)-fx-e)+\frac{2ab\sin(fx+e)^3}{3\cos(fx+e)^3}+b^2\left(\frac{\sin(fx+e)^3}{5\cos(fx+e)^5}+\frac{2\sin(fx+e)^3}{15\cos(fx+e)^3}\right)}{f}$
default	$\frac{a^2(\tan(fx+e)-fx-e)+\frac{2ab\sin(fx+e)^3}{3\cos(fx+e)^3}+b^2\left(\frac{\sin(fx+e)^3}{5\cos(fx+e)^5}+\frac{2\sin(fx+e)^3}{15\cos(fx+e)^3}\right)}{f}$
risch	$-a^2x-\frac{2i(-15a^2e^{8i(fx+e)}+30ab\,e^{8i(fx+e)}-60a^2e^{6i(fx+e)}+60ab\,e^{6i(fx+e)}+30b^2e^{6i(fx+e)}-90a^2e^{4i(fx+e)}+40ab\,e}{15f(e^{2i(fx+e)}+1)}$

```
[In] int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/f*(tan(f*x+e)-arctan(tan(f*x+e)))+b^2/f*(1/5*tan(f*x+e)^5+1/3*tan(f*x+e
)^3)+2/3*a*b/f*tan(f*x+e)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.46

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^2(e+fx)\,dx = -\frac{15\,a^2fx\cos\left(fx+e\right)^5-\left(\left(15\,a^2-10\,ab-2\,b^2\right)\cos\left(fx+e\right)^4+\left(10\,ab-b^2\right)\cos\left(fx+e\right)^2+3\,b^2\right)\sin\left(fx\right.}{15\,f\cos\left(fx+e\right)^5}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(15*a^2*f*x*cos(f*x + e)^5 - ((15*a^2 - 10*a*b - 2*b^2)*cos(f*x + e)^
4 + (10*a*b - b^2)*cos(f*x + e)^2 + 3*b^2)*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^2(e+fx)\,dx=\int \left(a+b\sec^2{(e+fx)}\right)^2\tan^2{(e+fx)}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**2,x)
[Out] Integral((a + b*sec(e + f*x)**2)**2*tan(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^2(e+fx)\,dx = \frac{3\,b^2\tan\left(fx+e\right)^5+5\,(2\,ab+b^2)\tan\left(fx+e\right)^3-15\,(fx+e)a^2+15\,a^2\tan\left(fx+e\right)}{15\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^2,x, algorithm="maxima")
[Out] 1/15*(3*b^2*tan(f*x + e)^5 + 5*(2*a*b + b^2)*tan(f*x + e)^3 - 15*(f*x + e)*
a^2 + 15*a^2*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.65 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^2(e+fx)\,dx = \frac{3\,b^2\tan\left(fx+e\right)^5+10\,ab\tan\left(fx+e\right)^3+5\,b^2\tan\left(fx+e\right)^3-15\,(fx+e)a^2+15\,a^2\tan\left(fx+e\right)}{15\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^2,x, algorithm="giac")
[Out] 1/15*(3*b^2*tan(f*x + e)^5 + 10*a*b*tan(f*x + e)^3 + 5*b^2*tan(f*x + e)^3 -
 15*(f*x + e)*a^2 + 15*a^2*tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 19.41 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.17

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^2(e+fx)\,dx$$
$$=\frac{\tan(e+f\,x)\,\left((a+b)^2+b^2-2\,b\,(a+b)\right)-\tan(e+f\,x)^3\,\left(\frac{b^2}{3}-\frac{2\,b\,(a+b)}{3}\right)+\frac{b^2\tan(e+f\,x)^5}{5}-a^2\,f\,x}{f}$$

```
[In] int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (tan(e + f*x)*((a + b)^2 + b^2 - 2*b*(a + b)) - tan(e + f*x)^3*(b^2/3 - (2*
b*(a + b))/3) + (b^2*tan(e + f*x)^5)/5 - a^2*f*x)/f
```

3.333 $\int (a+b\sec^2(e+fx))^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 40

$$\int (a+b\sec^2(e+fx))^2\,dx = a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] a^2*x+b*(2*a+b)*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4213, 398, 209}

$$\int (a+b\sec^2(e+fx))^2\,dx = a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] a^2*x + (b*(2*a + b)*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(b(2a+b)+b^2x^2+\frac{a^2}{1+x^2}\right)\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}+\frac{a^2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{3a^2fx+3b(2a+b)\tan(e+fx)+b^2\tan^3(e+fx)}{3f}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2,x]
[Out] (3*a^2*f*x + 3*b*(2*a + b)*Tan[e + f*x] + b^2*Tan[e + f*x]^3)/(3*f)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

method	result
parts	$a^{2} x -\frac{b^{2} \left(-\frac{2}{3}-\frac{\sec\left(f x +e \right)^{2}}{3}\right) \tan\left(f x +e \right)}{f}+\frac{2 a b \tan\left(f x +e \right)}{f}$
derivativedivides	$\frac{a^{2} \left(f x +e \right)+2 a b \tan\left(f x +e \right)-b^{2} \left(-\frac{2}{3}-\frac{\sec\left(f x +e \right)^{2}}{3}\right) \tan\left(f x +e \right)}{f}$
default	$\frac{a^{2} \left(f x +e \right)+2 a b \tan\left(f x +e \right)-b^{2} \left(-\frac{2}{3}-\frac{\sec\left(f x +e \right)^{2}}{3}\right) \tan\left(f x +e \right)}{f}$
risch	$a^{2} x +\frac{4 i b \left(3 a \,{\mathrm e}^{4 i \left(f x +e \right)}+6 a \,{\mathrm e}^{2 i \left(f x +e \right)}+3 b \,{\mathrm e}^{2 i \left(f x +e \right)}+3 a +b \right)}{3 f \left({\mathrm e}^{2 i \left(f x +e \right)}+1\right)^{3}}$
norman	$\frac{a^{2} x \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{6}-a^{2} x +3 a^{2} x \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-3 a^{2} x \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{4}-\frac{2 b \left(2 a +b \right) \tan\left(\frac{f x}{2}+\frac{e}{2}\right)}{f}-\frac{2 b \left(2 a +b \right) \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{f}+\frac{4 b \left(6 a +b\right.}{}}{\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right)^{3}}$
parallelrisch	$\frac{3 x \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{6} a^{2} f +\left(-12 a b -6 b^{2}\right) \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}-9 x \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{4} a^{2} f +\left(24 a b +4 b^{2}\right) \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}+9 x \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2} a}{3 f \left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)^{3} \left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{3}}$

```
[In] int((a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
[Out] a^2*x-b^2/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+2*a*b/f*tan(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{3\,a^{2} f x \cos\left(f x + e\right)^{3} + \left(2\,{\left(3\,a b + b^{2}\right)} \cos\left(f x + e\right)^{2} + b^{2}\right)\sin\left(f x + e\right)}{3\,f \cos\left(f x + e\right)^{3}}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
[Out] 1/3*(3*a^2*f*x*cos(f*x + e)^3 + (2*(3*a*b + b^2)*cos(f*x + e)^2 + b^2)*sin(
f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx=\int \left(a+b\sec^2{(e+fx)}\right)^2\,dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**2,x)
[Out] Integral((a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx=a^2x+\frac{\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)b^2}{3\,f}+\frac{2\,ab\tan\left(fx+e\right)}{f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
[Out] a^2*x + 1/3*(tan(f*x + e)^3 + 3*tan(f*x + e))*b^2/f + 2*a*b*tan(f*x + e)/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{b^2\tan\left(fx+e\right)^3+3\left(fx+e\right)a^2+6\,ab\tan\left(fx+e\right)+3\,b^2\tan\left(fx+e\right)}{3\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
[Out] 1/3*(b^2*tan(f*x + e)^3 + 3*(f*x + e)*a^2 + 6*a*b*tan(f*x + e) + 3*b^2*tan(
f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 19.57 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{\frac{b^2\tan(e+f\,x)^3}{3}-\tan(e+f\,x)\,\left(b^2-2\,b\,(a+b)\right)+a^2\,f\,x}{f}$$

```
[In] int((a + b/cos(e + f*x)^2)^2,x)
[Out] ((b^2*tan(e + f*x)^3)/3 - tan(e + f*x)*(b^2 - 2*b*(a + b)) + a^2*f*x)/f
```

3.334 $\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 36

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=-a^2x-\frac{(a+b)^2\cot(e+fx)}{f}+\frac{b^2\tan(e+fx)}{f}$$

```
[Out] -a^2*x-(a+b)^2*cot(f*x+e)/f+b^2*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4226, 1816, 209}

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=a^2(-x)-\frac{(a+b)^2\cot(e+fx)}{f}+\frac{b^2\tan(e+fx)}{f}$$

```
[In] Int[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(a^2*x) - ((a + b)^2*Cot[e + f*x])/f + (b^2*Tan[e + f*x])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b(1+x^2))^2}{x^2(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \left(b^2+\frac{(a+b)^2}{x^2}-\frac{a^2}{1+x^2}\right)\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{(a+b)^2\cot(e+fx)}{f}+\frac{b^2\tan(e+fx)}{f}-\frac{a^2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -a^2x-\frac{(a+b)^2\cot(e+fx)}{f}+\frac{b^2\tan(e+fx)}{f}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 82 vs. $2(36) = 72$.

Time = 3.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.28

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{4(b+a\cos^2(e+fx))^2\sec(e+fx)\left(a^2fx\cos(e+fx)-((a+b)^2\cot(e+fx)\csc(e)+b^2\sec(e))\sin(fx)\right)}{f(a+2b+a\cos(2(e+fx)))^2}$$

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (-4*(b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]*(a^2*f*x*Cos[e + f*x] - ((a + b)^
2*Cot[e + f*x]*Csc[e] + b^2*Sec[e])*Sin[f*x]))/(f*(a + 2*b + a*Cos[2*(e + f
*x)])^2)
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.83

method	result	size
derivativedivides	$\frac{a^{2}(-\cot(fx+e)-fx-e)-2ab\cot(fx+e)+b^{2}\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)}{f}$	66
default	$\frac{a^{2}(-\cot(fx+e)-fx-e)-2ab\cot(fx+e)+b^{2}\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)}{f}$	66
risch	$-a^{2}x-\frac{2i\left(a^{2}\mathrm{e}^{2i(fx+e)}+2ab\,\mathrm{e}^{2i(fx+e)}+a^{2}+2ab+2b^{2}\right)}{f\left(\mathrm{e}^{2i(fx+e)}-1\right)\left(\mathrm{e}^{2i(fx+e)}+1\right)}$	79

```
[In] int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(-cot(f*x+e)-f*x-e)-2*a*b*cot(f*x+e)+b^2*(1/sin(f*x+e)/cos(f*x+e)-
2*cot(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.86

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{a^2fx\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(a^2+2\,ab+2\,b^2\right)\cos\left(fx+e\right)^2-b^2}{f\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -(a^2*f*x*cos(f*x + e)*sin(f*x + e) + (a^2 + 2*a*b + 2*b^2)*cos(f*x + e)^2
- b^2)/(f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F]

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^2\cot^2\left(e+fx\right)dx$$

```
[In] integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*cot(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{(fx+e)a^2 - b^2\tan\left(fx+e\right) + \frac{a^2+2\,ab+b^2}{\tan(fx+e)}}{f}$$

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
[Out] -((f*x + e)*a^2 - b^2*tan(f*x + e) + (a^2 + 2*a*b + b^2)/tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{(fx+e)a^2 - b^2\tan\left(fx+e\right) + \frac{a^2+2\,ab+b^2}{\tan(fx+e)}}{f}$$

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
[Out] -((f*x + e)*a^2 - b^2*tan(f*x + e) + (a^2 + 2*a*b + b^2)/tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 19.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{b^2\tan(e+f\,x)}{f} - a^2\,x - \frac{a^2+2\,a\,b+b^2}{f\tan\left(e+f\,x\right)}$$

```
[In] int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^2,x)
[Out] (b^2*tan(e + f*x))/f - a^2*x - (2*a*b + a^2 + b^2)/(f*tan(e + f*x))
```

3.335 $\int \cot^4(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 45

$$\int \cot^4(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = a^2x+\frac{(a^2-b^2)\cot(e+fx)}{f}-\frac{(a+b)^2\cot^3(e+fx)}{3f}$$

```
[Out] a^2*x+(a^2-b^2)*cot(f*x+e)/f-1/3*(a+b)^2*cot(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4226, 1816, 209}

$$\int \cot^4(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = \frac{(a^2-b^2)\cot(e+fx)}{f}+a^2x-\frac{(a+b)^2\cot^3(e+fx)}{3f}$$

```
[In] Int[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] a^2*x + ((a^2 - b^2)*Cot[e + f*x])/f - ((a + b)^2*Cot[e + f*x]^3)/(3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b(1+x^2))^2}{x^4(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \left(\frac{(a+b)^2}{x^4} + \frac{-a^2+b^2}{x^2} + \frac{a^2}{1+x^2}\right)dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{(a^2-b^2)\cot(e+fx)}{f} - \frac{(a+b)^2\cot^3(e+fx)}{3f} + \frac{a^2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= a^2x + \frac{(a^2-b^2)\cot(e+fx)}{f} - \frac{(a+b)^2\cot^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. $2(45) = 90$.

Time = 1.57 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.56

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\csc(e)\csc^3(e+fx)\left(9a^2fx\cos(fx) - 9a^2fx\cos(2e+fx) - 3a^2fx\cos(2e+3fx) + 3a^2fx\cos(4e+3fx) - \right.}{}$$

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (Csc[e]*Csc[e + f*x]^3*(9*a^2*f*x*Cos[f*x] - 9*a^2*f*x*Cos[2*e + f*x] - 3*a
^2*f*x*Cos[2*e + 3*f*x] + 3*a^2*f*x*Cos[4*e + 3*f*x] - 12*a^2*Sin[f*x] + 12
*b^2*Sin[f*x] - 12*a^2*Sin[2*e + f*x] - 12*a*b*Sin[2*e + f*x] + 8*a^2*Sin[2
*e + 3*f*x] + 4*a*b*Sin[2*e + 3*f*x] - 4*b^2*Sin[2*e + 3*f*x]))/(24*f)
```

Maple [A] (verified)

Time = 3.91 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.62

method	result	size
derivativedivides	$\frac{a^{2}\left(-\frac{\cot(fx+e)^{3}}{3}+\cot(fx+e)+fx+e\right)-\frac{2ab\cos(fx+e)^{3}}{3\sin(fx+e)^{3}}+b^{2}\left(-\frac{2}{3}-\frac{\csc(fx+e)^{2}}{3}\right)\cot(fx+e)}{f}$	73
default	$\frac{a^{2}\left(-\frac{\cot(fx+e)^{3}}{3}+\cot(fx+e)+fx+e\right)-\frac{2ab\cos(fx+e)^{3}}{3\sin(fx+e)^{3}}+b^{2}\left(-\frac{2}{3}-\frac{\csc(fx+e)^{2}}{3}\right)\cot(fx+e)}{f}$	73
risch	$a^{2}x+\frac{4i\left(3a^{2}e^{4i(fx+e)}+3ab\,e^{4i(fx+e)}-3a^{2}e^{2i(fx+e)}+3b^{2}e^{2i(fx+e)}+2a^{2}+ab-b^{2}\right)}{3f\left(e^{2i(fx+e)}-1\right)^{3}}$	95

```
[In] int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/f*(a^2*(-1/3*cot(f*x+e)^3+cot(f*x+e)+f*x+e)-2/3*a*b/sin(f*x+e)^3*cos(f*x+
e)^3+b^2*(-2/3-1/3*csc(f*x+e)^2)*cot(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(43) = 86.

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.18

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{2\left(2a^2+ab-b^2\right)\cos\left(fx+e\right)^3-3\left(a^2-b^2\right)\cos\left(fx+e\right)+3\left(a^2fx\cos\left(fx+e\right)^2-a^2fx\right)\sin\left(fx+e\right)}{3\left(f\cos\left(fx+e\right)^2-f\right)\sin\left(fx+e\right)}$$

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
[Out] 1/3*(2*(2*a^2 + a*b - b^2)*cos(f*x + e)^3 - 3*(a^2 - b^2)*cos(f*x + e) + 3*
(a^2*f*x*cos(f*x + e)^2 - a^2*f*x)*sin(f*x + e))/((f*cos(f*x + e)^2 - f)*si
n(f*x + e))
```

Sympy [F]

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^2\cot^4\left(e+fx\right)dx$$

```
[In] integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2)**2,x)
[Out] Integral((a + b*sec(e + f*x)**2)**2*cot(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.31

$$\int \cot^4(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = \frac{3\,(fx+e)a^2 + \frac{3\,(a^2-b^2)\tan(fx+e)^2-a^2-2\,ab-b^2}{\tan(fx+e)^3}}{3\,f}$$

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(f*x + e)*a^2 + (3*(a^2 - b^2)*tan(f*x + e)^2 - a^2 - 2*a*b - b^2)/t
an(f*x + e)^3)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(43) = 86.

Time = 0.32 (sec) , antiderivative size = 176, normalized size of antiderivative = 3.91

$$\int \cot^4(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx$$

$$= \frac{a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 2\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + b^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 24\,(fx+e)a^2 - 15\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right.}{2}$$

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*f*x + 1/2*e)^3 + 2*a*b*tan(1/2*f*x + 1/2*e)^3 + b^2*tan(1
/2*f*x + 1/2*e)^3 + 24*(f*x + e)*a^2 - 15*a^2*tan(1/2*f*x + 1/2*e) - 6*a*b*
tan(1/2*f*x + 1/2*e) + 9*b^2*tan(1/2*f*x + 1/2*e) + (15*a^2*tan(1/2*f*x + 1
/2*e)^2 + 6*a*b*tan(1/2*f*x + 1/2*e)^2 - 9*b^2*tan(1/2*f*x + 1/2*e)^2 - a^2
 - 2*a*b - b^2)/tan(1/2*f*x + 1/2*e)^3)/f
```

Mupad [B] (verification not implemented)

Time = 19.97 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \cot^4(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = a^2\,x - \frac{\frac{2\,a\,b}{3} - \tan(e+f\,x)^2\,(a^2-b^2) + \frac{a^2}{3} + \frac{b^2}{3}}{f\tan\left(e+f\,x\right)^3}$$

```
[In] int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] a^2*x - ((2*a*b)/3 - tan(e + f*x)^2*(a^2 - b^2) + a^2/3 + b^2/3)/(f*tan(e +
 f*x)^3)
```

3.336 $\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 65

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=-a^2x-\frac{a^2\cot(e+fx)}{f}+\frac{(a^2-b^2)\cot^3(e+fx)}{3f}-\frac{(a+b)^2\cot^5(e+fx)}{5f}$$

```
[Out] -a^2*x-a^2*cot(f*x+e)/f+1/3*(a^2-b^2)*cot(f*x+e)^3/f-1/5*(a+b)^2*cot(f*x+e)
^5/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4226, 1816, 209}

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{(a^2-b^2)\cot^3(e+fx)}{3f}-\frac{a^2\cot(e+fx)}{f}-a^2x-\frac{(a+b)^2\cot^5(e+fx)}{5f}$$

```
[In] Int[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(a^2*x) - (a^2*Cot[e + f*x])/f + ((a^2 - b^2)*Cot[e + f*x]^3)/(3*f) - ((a
+ b)^2*Cot[e + f*x]^5)/(5*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b(1+x^2))^2}{x^6(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(\frac{(a+b)^2}{x^6}+\frac{-a^2+b^2}{x^4}+\frac{a^2}{x^2}-\frac{a^2}{1+x^2}\right)dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{a^2\cot(e+fx)}{f}+\frac{(a^2-b^2)\cot^3(e+fx)}{3f}\\
&\quad -\frac{(a+b)^2\cot^5(e+fx)}{5f}-\frac{a^2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -a^2x-\frac{a^2\cot(e+fx)}{f}+\frac{(a^2-b^2)\cot^3(e+fx)}{3f}-\frac{(a+b)^2\cot^5(e+fx)}{5f}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 256 vs. $2(65) = 130$.

Time = 3.89 (sec) , antiderivative size = 256, normalized size of antiderivative = 3.94

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{\csc(e)\csc^5(e+fx)\left(-150a^2fx\cos(fx)+150a^2fx\cos(2e+fx)+75a^2fx\cos(2e+3fx)-75a^2fx\cos(4e\right.}{}$$

```
[In] Integrate[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

[Out] (Csc[e]*Csc[e + f*x]^5*(-150*a^2*f*x*Cos[f*x] + 150*a^2*f*x*Cos[2*e + f*x] + 75*a^2*f*x*Cos[2*e + 3*f*x] - 75*a^2*f*x*Cos[4*e + 3*f*x] - 15*a^2*f*x*Cos[4*e + 5*f*x] + 15*a^2*f*x*Cos[6*e + 5*f*x] + 280*a^2*Sin[f*x] + 120*a*b*Sin[f*x] + 20*b^2*Sin[f*x] + 180*a^2*Sin[2*e + f*x] - 60*b^2*Sin[2*e + f*x] - 140*a^2*Sin[2*e + 3*f*x] + 20*b^2*Sin[2*e + 3*f*x] - 90*a^2*Sin[4*e + 3*f*x] - 60*a*b*Sin[4*e + 3*f*x] + 46*a^2*Sin[4*e + 5*f*x] + 12*a*b*Sin[4*e + 5*f*x] - 4*b^2*Sin[4*e + 5*f*x]))/(480*f)

Maple [A] (verified)

Time = 7.66 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.65

method	result
derivativedivides	$\frac{a^{2}\left(-\frac{\cot(fx+e)^{5}}{5}+\frac{\cot(fx+e)^{3}}{3}-\cot(fx+e)-fx-e\right)-\frac{2ab\cos(fx+e)^{5}}{5\sin(fx+e)^{5}}+b^{2}\left(-\frac{\cos(fx+e)^{3}}{5\sin(fx+e)^{5}}-\frac{2\cos(fx+e)^{3}}{15\sin(fx+e)^{3}}\right)}{f}$
default	$\frac{a^{2}\left(-\frac{\cot(fx+e)^{5}}{5}+\frac{\cot(fx+e)^{3}}{3}-\cot(fx+e)-fx-e\right)-\frac{2ab\cos(fx+e)^{5}}{5\sin(fx+e)^{5}}+b^{2}\left(-\frac{\cos(fx+e)^{3}}{5\sin(fx+e)^{5}}-\frac{2\cos(fx+e)^{3}}{15\sin(fx+e)^{3}}\right)}{f}$
risch	$-a^{2}x-\frac{2i\left(45a^{2}e^{8i(fx+e)}+30ab\,e^{8i(fx+e)}-90a^{2}e^{6i(fx+e)}+30b^{2}e^{6i(fx+e)}+140a^{2}e^{4i(fx+e)}+60ab\,e^{4i(fx+e)}+10b^{2}e^{4\cdots}{15f\left(e^{2i(fx+e)}-1\right)^{5}}$

[In] int(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/f*(a^2*(-1/5*cot(f*x+e)^5+1/3*cot(f*x+e)^3-cot(f*x+e)-f*x-e)-2/5*a*b/sin(f*x+e)^5*cos(f*x+e)^5+b^2*(-1/5/sin(f*x+e)^5*cos(f*x+e)^3-2/15*cos(f*x+e)^3/sin(f*x+e)^3))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(61) = 122.

Time = 0.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.09

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{(23\,a^2+6\,ab-2\,b^2)\cos(fx+e)^5-5\,(7\,a^2-b^2)\cos(fx+e)^3+15\,a^2\cos(fx+e)+15\,\left(a^2fx\cos(fx\cdots\right.}{15\,\left(f\cos(fx+e)^4-2\,f\cos(fx+e)^2+f\right)\sin(fx+e\cdots}$$

[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")

[Out] -1/15*((23*a^2 + 6*a*b - 2*b^2)*cos(f*x + e)^5 - 5*(7*a^2 - b^2)*cos(f*x + e)^3 + 15*a^2*cos(f*x + e) + 15*(a^2*f*x*cos(f*x + e)^4 - 2*a^2*f*x*cos(f*x + e)^2 + a^2*f*x)*sin(f*x + e))/((f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 + f)*sin(f*x + e))

Sympy [F]

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^2\cot^6\left(e+fx\right)dx$$

```
[In] integrate(cot(f*x+e)**6*(a+b*sec(f*x+e)**2)**2,x)
[Out] Integral((a + b*sec(e + f*x)**2)**2*cot(e + f*x)**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=-\frac{15\,(fx+e)a^2+\frac{15\,a^2\tan(fx+e)^4-5\,(a^2-b^2)\tan(fx+e)^2+3\,a^2+6\,ab+3\,b^2}{\tan(fx+e)^5}}{15\,f}$$

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
[Out] -1/15*(15*(f*x + e)*a^2 + (15*a^2*tan(f*x + e)^4 - 5*(a^2 - b^2)*tan(f*x +
e)^2 + 3*a^2 + 6*a*b + 3*b^2)/tan(f*x + e)^5)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(61) = 122$.

Time = 0.35 (sec) , antiderivative size = 273, normalized size of antiderivative = 4.20

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{3\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+6\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+3\,b^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-35\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-30\,ab\tan$$

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
[Out] 1/480*(3*a^2*tan(1/2*f*x + 1/2*e)^5 + 6*a*b*tan(1/2*f*x + 1/2*e)^5 + 3*b^2*
tan(1/2*f*x + 1/2*e)^5 - 35*a^2*tan(1/2*f*x + 1/2*e)^3 - 30*a*b*tan(1/2*f*x
 + 1/2*e)^3 + 5*b^2*tan(1/2*f*x + 1/2*e)^3 - 480*(f*x + e)*a^2 + 330*a^2*ta
n(1/2*f*x + 1/2*e) + 60*a*b*tan(1/2*f*x + 1/2*e) - 30*b^2*tan(1/2*f*x + 1/2
*e) - (330*a^2*tan(1/2*f*x + 1/2*e)^4 + 60*a*b*tan(1/2*f*x + 1/2*e)^4 - 30*
b^2*tan(1/2*f*x + 1/2*e)^4 - 35*a^2*tan(1/2*f*x + 1/2*e)^2 - 30*a*b*tan(1/2
*f*x + 1/2*e)^2 + 5*b^2*tan(1/2*f*x + 1/2*e)^2 + 3*a^2 + 6*a*b + 3*b^2)/tan
(1/2*f*x + 1/2*e)^5)/f
```

Mupad [B] (verification not implemented)

Time = 19.66 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=-a^2\,x-\frac{\frac{2\,a\,b}{5}+\frac{a^2}{5}+\frac{b^2}{5}-\tan(e+f\,x)^2\,\left(\frac{a^2}{3}-\frac{b^2}{3}\right)+a^2\tan(e+f\,x)^4}{f\tan\left(e+f\,x\right)^5}$$

```
[In] int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2)^2,x)
[Out] - a^2*x - ((2*a*b)/5 + a^2/5 + b^2/5 - tan(e + f*x)^2*(a^2/3 - b^2/3) + a^2
*tan(e + f*x)^4)/(f*tan(e + f*x)^5)
```

3.337 $\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 69

$$\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a+2b)\log(\cos(e+fx))}{b^2f} - \frac{(a+b)^2\log\left(b+a\cos^2(e+fx)\right)}{2ab^2f} + \frac{\sec^2(e+fx)}{2bf}$$

```
[Out] (a+2*b)*ln(cos(f*x+e))/b^2/f-1/2*(a+b)^2*ln(b+a*cos(f*x+e)^2)/a/b^2/f+1/2*s
ec(f*x+e)^2/b/f
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4223, 457, 90}

$$\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{(a+b)^2\log\left(a\cos^2(e+fx)+b\right)}{2ab^2f} + \frac{(a+2b)\log(\cos(e+fx))}{b^2f} + \frac{\sec^2(e+fx)}{2bf}$$

```
[In] Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b)*Log[Cos[e + f*x]])/(b^2*f) - ((a + b)^2*Log[b + a*Cos[e + f*x]^2
])/(2*a*b^2*f) + Sec[e + f*x]^2/(2*b*f)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^3(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^2(b+ax)}\,dx, x, \cos^2(e+fx)\right)}{2f}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{bx^2}+\frac{-a-2b}{b^2x}+\frac{(a+b)^2}{b^2(b+ax)}\right)\,dx, x, \cos^2(e+fx)\right)}{2f}\\ &= \frac{(a+2b)\log(\cos(e+fx))}{b^2f}-\frac{(a+b)^2\log\left(b+a\cos^2(e+fx)\right)}{2ab^2f}+\frac{\sec^2(e+fx)}{2bf}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.43

$$\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(2a(a+2b)\log(\cos(e+fx))-(a+b)^2\log\left(a+b-a\sin^2(e+f\right.\right.}{4ab^2f\left(a+b\sec^2(e+fx)\right)}$$

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(2*a*(a + 2*b)*Log[Cos[e + f
*x]] - (a + b)^2*Log[a + b - a*Sin[e + f*x]^2] + a*b*Sec[e + f*x]^2))/(4*a*
b^2*f*(a + b*Sec[e + f*x]^2))
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{\frac{(a+2b)\ln(\cos(fx+e))}{b^2}+\frac{1}{2b\cos(fx+e)^2}-\frac{\left(a^2+2ab+b^2\right)\ln\left(b+a\cos(fx+e)^2\right)}{2b^2a}}{f}$
default	$\frac{\frac{(a+2b)\ln(\cos(fx+e))}{b^2}+\frac{1}{2b\cos(fx+e)^2}-\frac{\left(a^2+2ab+b^2\right)\ln\left(b+a\cos(fx+e)^2\right)}{2b^2a}}{f}$
risch	$\frac{ix}{a}+\frac{2ie}{af}+\frac{2\,\mathrm{e}^{2i(fx+e)}}{fb\left(\mathrm{e}^{2i(fx+e)}+1\right)^2}+\frac{\ln\left(\mathrm{e}^{2i(fx+e)}+1\right)a}{b^2f}+\frac{2\ln\left(\mathrm{e}^{2i(fx+e)}+1\right)}{bf}-\frac{a\ln\left(\mathrm{e}^{4i(fx+e)}+\frac{2(a+2b)\mathrm{e}^{2i(fx+e)}}{a}+1\right)}{2b^2f}$

```
[In] int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*((a+2*b)/b^2*ln(cos(f*x+e))+1/2/b/cos(f*x+e)^2-1/2*(a^2+2*a*b+b^2)/b^2/
a*ln(b+a*cos(f*x+e)^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.22

$$\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\left(a^2+2\,ab+b^2\right)\cos\left(fx+e\right)^2\log\left(a\cos\left(fx+e\right)^2+b\right)-2\left(a^2+2\,ab\right)\cos\left(fx+e\right)^2\log\left(-\cos\left(fx+e\right)\right)}{2\,ab^2f\cos\left(fx+e\right)^2}$$

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/2*((a^2 + 2*a*b + b^2)*cos(f*x + e)^2*log(a*cos(f*x + e)^2 + b) - 2*(a^2
 + 2*a*b)*cos(f*x + e)^2*log(-cos(f*x + e)) - a*b)/(a*b^2*f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\tan^5\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

```
[In] integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(tan(e + f*x)**5/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.17

$$\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{(a+2b)\log\left(\sin(fx+e)^2-1\right)}{b^2} - \frac{1}{b\sin(fx+e)^2-b} - \frac{(a^2+2ab+b^2)\log\left(a\sin(fx+e)^2-a-b\right)}{ab^2}}{2f}$$

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*((a + 2*b)*log(sin(f*x + e)^2 - 1)/b^2 - 1/(b*sin(f*x + e)^2 - b) - (a^
2 + 2*a*b + b^2)*log(a*sin(f*x + e)^2 - a - b)/(a*b^2))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(65) = 130$.

Time = 1.52 (sec) , antiderivative size = 373, normalized size of antiderivative = 5.41

$$\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{(a^3+3a^2b+3ab^2+b^3)\log\left(\left|-a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-b\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-2a+2b\right|\right)}{a^2b^2+ab^3} - \frac{\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-}{\cos(fx+e)+}\right.\right.}{a}}{2f}$$

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(abs(-a*((cos(f*x + e) + 1)/(cos(f
*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - b*((cos(f*x + e) +
1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 2*a + 2*b)
)/(a^2*b^2 + a*b^3) - log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos
(f*x + e) - 1)/(cos(f*x + e) + 1) + 2))/a - (a + 2*b)*log(abs(-(cos(f*x + e
) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2))/b^2
 + (a*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x
+ e) + 1)) + 2*b*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1
)/(cos(f*x + e) + 1)) + 2*a + 8*b)/(b^2*((cos(f*x + e) + 1)/(cos(f*x + e) -
 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2)))/f
```

Mupad [B] (verification not implemented)

Time = 19.46 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.49

$$\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a\,f} - \frac{\ln\left(b\tan(e+f\,x)^2+a+b\right)}{b\,f} - \frac{\ln\left(b\tan(e+f\,x)^2+a+b\right)}{2\,a\,f} + \frac{\tan(e+f\,x)^2}{2\,b\,f} - \frac{a\,\ln\left(b\tan(e+f\,x)^2+a+b\right)}{2\,b^2\,f}$$

```
[In] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2),x)
```

```
[Out] log(tan(e + f*x)^2 + 1)/(2*a*f) - log(a + b + b*tan(e + f*x)^2)/(b*f) - log
(a + b + b*tan(e + f*x)^2)/(2*a*f) + tan(e + f*x)^2/(2*b*f) - (a*log(a + b
+ b*tan(e + f*x)^2))/(2*b^2*f)
```

3.338 $\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 45

$$\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx=-\frac{\log(\cos(e+fx))}{bf}+\frac{(a+b)\log\left(b+a\cos^2(e+fx)\right)}{2abf}$$

```
[Out] -ln(cos(f*x+e))/b/f+1/2*(a+b)*ln(b+a*cos(f*x+e)^2)/a/b/f
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4223, 457, 78}

$$\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx=\frac{(a+b)\log\left(a\cos^2(e+fx)+b\right)}{2abf}-\frac{\log(\cos(e+fx))}{bf}$$

```
[In] Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(Log[Cos[e + f*x]]/(b*f)) + ((a + b)*Log[b + a*Cos[e + f*x]^2])/(2*a*b*f)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1-x^2}{x(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{f} \\
&= -\frac{\text{Subst}\left(\int \frac{1-x}{x(b+ax)}\,dx, x, \cos^2(e+fx)\right)}{2f} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{bx}+\frac{-a-b}{b(b+ax)}\right)\,dx, x, \cos^2(e+fx)\right)}{2f} \\
&= -\frac{\log(\cos(e+fx))}{bf}+\frac{(a+b)\log\left(b+a\cos^2(e+fx)\right)}{2abf}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$
\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{-2a\log(\cos(e+fx))+(a+b)\log\left(b+a\cos^2(e+fx)\right)}{2abf}
$$

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
[Out] (-2*a*Log[Cos[e + f*x]] + (a + b)*Log[b + a*Cos[e + f*x]^2])/(2*a*b*f)
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{\frac{(a+b)\ln\left(b+a\cos(fx+e)^2\right)}{2ba}-\frac{\ln(\cos(fx+e))}{b}}{f}$
default	$\frac{\frac{(a+b)\ln\left(b+a\cos(fx+e)^2\right)}{2ba}-\frac{\ln(\cos(fx+e))}{b}}{f}$
risch	$-\frac{ix}{a}-\frac{2ie}{af}-\frac{\ln(e^{2i(fx+e)}+1)}{bf}+\frac{\ln\left(e^{4i(fx+e)}+\frac{2(a+2b)e^{2i(fx+e)}}{a}+1\right)}{2bf}+\frac{\ln\left(e^{4i(fx+e)}+\frac{2(a+2b)e^{2i(fx+e)}}{a}+1\right)}{2af}$

```
[In] int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(1/2*(a+b)/b/a*ln(b+a*cos(f*x+e)^2)-1/b*ln(cos(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a+b)\log\left(a\cos\left(fx+e\right)^2+b\right)-2\,a\log\left(-\cos\left(fx+e\right)\right)}{2\,abf}$$

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
[Out] 1/2*((a + b)*log(a*cos(f*x + e)^2 + b) - 2*a*log(-cos(f*x + e)))/(a*b*f)
```

Sympy [F]

$$\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\tan^3\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

```
[In] integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
[Out] Integral(tan(e + f*x)**3/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{(a+b)\log\left(a\sin(fx+e)^2-a-b\right)}{ab} - \frac{\log\left(\sin(fx+e)^2-1\right)}{b}}{2f}$$

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*((a + b)*log(a*sin(f*x + e)^2 - a - b)/(a*b) - log(sin(f*x + e)^2 - 1)/
b)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(43) = 86.

Time = 0.61 (sec) , antiderivative size = 222, normalized size of antiderivative = 4.93

$$\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{(a^2+2ab+b^2)\log\left(\left|-a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-b\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-2a+2b\right|\right)}{a^2b+ab^2} - \frac{\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right)}{a} - \text{lo}}{2f}$$

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*((a^2 + 2*a*b + b^2)*log(abs(-a*((cos(f*x + e) + 1)/(cos(f*x + e) - 1)
+ (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - b*((cos(f*x + e) + 1)/(cos(f*x +
 e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 2*a + 2*b))/(a^2*b + a*
b^2) - log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/
(cos(f*x + e) + 1) + 2))/a - log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1)
 - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2))/b)/f
```

Mupad [B] (verification not implemented)

Time = 20.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.42

$$\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\ln\left(b\tan(e+f\,x)^2+a+b\right)}{2\,a\,f} + \frac{\ln\left(b\tan(e+f\,x)^2+a+b\right)}{2\,b\,f} - \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a\,f}$$

```
[In] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2),x)
```

```
[Out] log(a + b + b*tan(e + f*x)^2)/(2*a*f) + log(a + b + b*tan(e + f*x)^2)/(2*b*
f) - log(tan(e + f*x)^2 + 1)/(2*a*f)
```

3.339 $\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 23

$$\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\log\left(b+a\cos^2(e+fx)\right)}{2af}$$

```
[Out] -1/2*ln(b+a*cos(f*x+e)^2)/a/f
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4223, 266}

$$\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\log\left(a\cos^2(e+fx)+b\right)}{2af}$$

```
[In] Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/2*Log[b + a*Cos[e + f*x]^2]/(a*f)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
```

```
x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\log\left(b+a\cos^2(e+fx)\right)}{2af}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\log(a+2b+a\cos(2(e+fx)))}{2af}$$

```
[In] Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2),x]
[Out] -1/2*Log[a + 2*b + a*Cos[2*(e + f*x)]]/(a*f)
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

method	result	size
derivativedivides	$\frac{-\frac{\ln\left(a+b\sec(fx+e)^2\right)}{2a}+\frac{\ln(\sec(fx+e))}{a}}{f}$	35
default	$\frac{-\frac{\ln\left(a+b\sec(fx+e)^2\right)}{2a}+\frac{\ln(\sec(fx+e))}{a}}{f}$	35
risch	$\frac{ix}{a}+\frac{2ie}{af}-\frac{\ln\left(e^{4i(fx+e)}+\frac{2(a+2b)e^{2i(fx+e)}}{a}+1\right)}{2af}$	58

```
[In] int(tan(f*x+e)/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(-1/2/a*ln(a+b*sec(f*x+e)^2)+1/a*ln(sec(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\log\left(a\cos\left(fx+e\right)^2+b\right)}{2\,af}$$

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
[Out] -1/2*log(a*cos(f*x + e)^2 + b)/(a*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(19) = 38.

Time = 5.56 (sec) , antiderivative size = 117, normalized size of antiderivative = 5.09

$$\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx = \begin{cases} \frac{\tilde{\infty}x\tan(e)}{\sec^2(e)} & \text{for } a=0 \wedge b=0 \wedge f=0 \\ -\frac{1}{2bf\sec^2(e+fx)} & \text{for } a=0 \\ \frac{\log(\tan^2(e+fx)+1)}{2af} & \text{for } b=0 \\ \frac{x\tan(e)}{a+b\sec^2(e)} & \text{for } f=0 \\ -\frac{\log\left(-\sqrt{-\frac{a}{b}}+\sec(e+fx)\right)}{2af} - \frac{\log\left(\sqrt{-\frac{a}{b}}+\sec(e+fx)\right)}{2af} + \frac{\log(\tan^2(e+fx)+1)}{2af} & \text{otherwise} \end{cases}$$

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2),x)
[Out] Piecewise((zoo*x*tan(e)/sec(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (-1/(2*
b*f*sec(e + f*x)**2), Eq(a, 0)), (log(tan(e + f*x)**2 + 1)/(2*a*f), Eq(b, 0
)), (x*tan(e)/(a + b*sec(e)**2), Eq(f, 0)), (-log(-sqrt(-a/b) + sec(e + f*x
))/(2*a*f) - log(sqrt(-a/b) + sec(e + f*x))/(2*a*f) + log(tan(e + f*x)**2 +
 1)/(2*a*f), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\log\left(a\sin\left(fx+e\right)^2-a-b\right)}{2\,af}$$

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*log(a*sin(f*x + e)^2 - a - b)/(a*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(21) = 42.

Time = 0.33 (sec) , antiderivative size = 129, normalized size of antiderivative = 5.61

$$\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{\log\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a}-\frac{2\,\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a}}{2\,f}$$

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/2*(log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
+ b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/a - 2*log(abs(-(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) + 1))/a)/f
```

Mupad [B] (verification not implemented)

Time = 19.83 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.74

$$\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\mathrm{atanh}\left(\frac{a}{2\left(\frac{3\,a}{2}+2\,b+\frac{a\,\cos(2\,e+2\,f\,x)}{2}\right)}-\frac{a\,\cos(2\,e+2\,f\,x)}{2\left(\frac{3\,a}{2}+2\,b+\frac{a\,\cos(2\,e+2\,f\,x)}{2}\right)}\right)}{a\,f}$$

```
[In] int(tan(e + f*x)/(a + b/cos(e + f*x)^2),x)
```

```
[Out] atanh(a/(2*((3*a)/2 + 2*b + (a*cos(2*e + 2*f*x))/2)) - (a*cos(2*e + 2*f*x))
/(2*((3*a)/2 + 2*b + (a*cos(2*e + 2*f*x))/2)))/(a*f)
```

3.340 $\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 46

$$\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{b\log\left(b+a\cos^2(e+fx)\right)}{2a(a+b)f} + \frac{\log(\sin(e+fx))}{(a+b)f}$$

```
[Out] 1/2*b*ln(b+a*cos(f*x+e)^2)/a/(a+b)/f+ln(sin(f*x+e))/(a+b)/f
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4223, 457, 78}

$$\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\log(\sin(e+fx))}{f(a+b)} + \frac{b\log\left(a\cos^2(e+fx)+b\right)}{2af(a+b)}$$

```
[In] Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (b*Log[b + a*Cos[e + f*x]^2])/(2*a*(a + b)*f) + Log[Sin[e + f*x]]/((a + b)*
f)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^3}{(1-x^2)(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{f} \\
&= -\frac{\text{Subst}\left(\int \frac{x}{(1-x)(b+ax)}\,dx, x, \cos^2(e+fx)\right)}{2f} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{(-a-b)(-1+x)} - \frac{b}{(a+b)(b+ax)}\right)\,dx, x, \cos^2(e+fx)\right)}{2f} \\
&= \frac{b\log\left(b+a\cos^2(e+fx)\right)}{2a(a+b)f} + \frac{\log(\sin(e+fx))}{(a+b)f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{2a\log(\sin(e+fx)) + b\log\left(a+b-a\sin^2(e+fx)\right)}{2a^2f + 2abf}$$

```
[In] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2),x]
[Out] (2*a*Log[Sin[e + f*x]] + b*Log[a + b - a*Sin[e + f*x]^2])/(2*a^2*f + 2*a*b*
f)
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

method	result	siz
derivativedivides	$\frac{\frac{b\ln\left(b+a\cos(fx+e)^2\right)}{2(a+b)a}+\frac{\ln(1+\cos(fx+e))}{2a+2b}+\frac{\ln(-1+\cos(fx+e))}{2a+2b}}{f}$	68
default	$\frac{\frac{b\ln\left(b+a\cos(fx+e)^2\right)}{2(a+b)a}+\frac{\ln(1+\cos(fx+e))}{2a+2b}+\frac{\ln(-1+\cos(fx+e))}{2a+2b}}{f}$	68
risch	$\frac{ix}{a}-\frac{2ix}{a+b}-\frac{2ie}{f(a+b)}-\frac{2ibx}{a(a+b)}-\frac{2ibe}{af(a+b)}+\frac{\ln\left(e^{2i(fx+e)}-1\right)}{f(a+b)}+\frac{b\ln\left(e^{4i(fx+e)}+\frac{2(a+2b)e^{2i(fx+e)}}{a}+1\right)}{2af(a+b)}$	12

```
[In] int(cot(f*x+e)/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(1/2/(a+b)*b/a*ln(b+a*cos(f*x+e)^2)+1/(2*a+2*b)*ln(1+cos(f*x+e))+1/(2*a
+2*b)*ln(-1+cos(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx=\frac{b\log\left(a\cos\left(fx+e\right)^2+b\right)+2\,a\log\left(\frac{1}{2}\,\sin\left(fx+e\right)\right)}{2\left(a^2+ab\right)f}$$

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
[Out] 1/2*(b*log(a*cos(f*x + e)^2 + b) + 2*a*log(1/2*sin(f*x + e)))/((a^2 + a*b)*
f)
```

Sympy [F]

$$\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx=\int \frac{\cot\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2),x)
[Out] Integral(cot(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{b\log\left(a\sin(fx+e)^2-a-b\right)}{a^2+ab}+\frac{\log\left(\sin(fx+e)^2\right)}{a+b}}{2\,f}$$

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] 1/2*(b*log(a*sin(f*x + e)^2 - a - b)/(a^2 + a*b) + log(sin(f*x + e)^2)/(a +
 b))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(44) = 88.

Time = 0.30 (sec) , antiderivative size = 166, normalized size of antiderivative = 3.61

$$\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{b\log\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^2+ab}+\frac{\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a+b}-\frac{2\,\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a}}{2\,f}$$

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] 1/2*(b*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x
 + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^2 + a*b) + log(abs(-cos(
f*x + e) + 1)/abs(cos(f*x + e) + 1))/(a + b) - 2*log(abs(-(cos(f*x + e) - 1
)/(cos(f*x + e) + 1) + 1))/a)/f
```

Mupad [B] (verification not implemented)

Time = 20.64 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.41

$$\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\ln\left(\tan(e+f\,x)\right)}{f\,\left(a+b\right)}-\frac{\ln\left({\tan(e+f\,x)}^2+1\right)}{2\,a\,f}+\frac{b\,\ln\left(b\,{\tan(e+f\,x)}^2+a+b\right)}{2\,f\,\left(a^2+b\,a\right)}$$

```
[In] int(cot(e + f*x)/(a + b/cos(e + f*x)^2),x)
[Out] log(tan(e + f*x))/(f*(a + b)) - log(tan(e + f*x)^2 + 1)/(2*a*f) + (b*log(a
+ b + b*tan(e + f*x)^2))/(2*f*(a*b + a^2))
```

3.341 $\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\csc^2(e+fx)}{2(a+b)f} - \frac{b^2\log\left(b+a\cos^2(e+fx)\right)}{2a(a+b)^2f} - \frac{(a+2b)\log(\sin(e+fx))}{(a+b)^2f}$$

```
[Out] -1/2*csc(f*x+e)^2/(a+b)/f-1/2*b^2*ln(b+a*cos(f*x+e)^2)/a/(a+b)^2/f-(a+2*b)*
ln(sin(f*x+e))/(a+b)^2/f
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4223, 457, 90}

$$\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{b^2\log\left(a\cos^2(e+fx)+b\right)}{2af(a+b)^2} - \frac{\csc^2(e+fx)}{2f(a+b)} - \frac{(a+2b)\log(\sin(e+fx))}{f(a+b)^2}$$

```
[In] Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/2*Csc[e + f*x]^2/((a + b)*f) - (b^2*Log[b + a*Cos[e + f*x]^2])/(2*a*(a +
 b)^2*f) - ((a + 2*b)*Log[Sin[e + f*x]])/((a + b)^2*f)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^5}{(1-x^2)^2(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \frac{x^2}{(1-x)^2(b+ax)}\,dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)(-1+x)^2}+\frac{a+2b}{(a+b)^2(-1+x)}+\frac{b^2}{(a+b)^2(b+ax)}\right)dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{\csc^2(e+fx)}{2(a+b)f}-\frac{b^2\log\left(b+a\cos^2(e+fx)\right)}{2a(a+b)^2f}-\frac{(a+2b)\log(\sin(e+fx))}{(a+b)^2f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35

$$\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left(a(a+b)\csc^2(e+fx)+2a(a+2b)\log(\sin(e+fx))+b^2\log\left(a+b-a\sin^2(\right.\right.}{4a(a+b)^2f\left(a+b\sec^2(e+fx)\right)}$$

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/4*((a + 2*b + a*Cos[2*(e + f*x)])*(a*(a + b)*Csc[e + f*x]^2 + 2*a*(a + 2
*b)*Log[Sin[e + f*x]] + b^2*Log[a + b - a*Sin[e + f*x]^2])*Sec[e + f*x]^2)/
(a*(a + b)^2*f*(a + b*Sec[e + f*x]^2))
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.61

method	result
derivativedivides	$\frac{-\frac{1}{(4a+4b)(1+\cos(fx+e))}+\frac{(-a-2b)\ln(1+\cos(fx+e))}{2(a+b)^2}-\frac{b^2\ln\left(b+a\cos(fx+e)^2\right)}{2(a+b)^2a}+\frac{1}{(4a+4b)(-1+\cos(fx+e))}+\frac{(-a-2b)\ln(-1+\cos(f}{2(a+b)^2}}{f}$
default	$\frac{-\frac{1}{(4a+4b)(1+\cos(fx+e))}+\frac{(-a-2b)\ln(1+\cos(fx+e))}{2(a+b)^2}-\frac{b^2\ln\left(b+a\cos(fx+e)^2\right)}{2(a+b)^2a}+\frac{1}{(4a+4b)(-1+\cos(fx+e))}+\frac{(-a-2b)\ln(-1+\cos(f}{2(a+b)^2}}{f}$
risch	$-\frac{ix}{a}+\frac{2iax}{a^2+2ab+b^2}+\frac{2iae}{f(a^2+2ab+b^2)}+\frac{4ibx}{a^2+2ab+b^2}+\frac{4ibe}{f(a^2+2ab+b^2)}+\frac{2ib^2x}{a(a^2+2ab+b^2)}+\frac{2ib^2e}{af(a^2+2ab+b^2)}$ -

```
[In] int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/(4*a+4*b)/(1+cos(f*x+e))+1/2*(-a-2*b)/(a+b)^2*ln(1+cos(f*x+e))-1/2*
b^2/(a+b)^2/a*ln(b+a*cos(f*x+e)^2)+1/(4*a+4*b)/(-1+cos(f*x+e))+1/2*(-a-2*b)
/(a+b)^2*ln(-1+cos(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.70

$$\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{a^2+ab-\left(b^2\cos\left(fx+e\right)^2-b^2\right)\log\left(a\cos\left(fx+e\right)^2+b\right)-2\left(\left(a^2+2\,ab\right)\cos\left(fx+e\right)^2-a^2-2\,ab\right)\log\left(\right.}{2\left(\left(a^3+2\,a^2b+ab^2\right)f\cos\left(fx+e\right)^2-\left(a^3+2\,a^2b+ab^2\right)f\right)}$$

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2 + a*b - (b^2*cos(f*x + e)^2 - b^2)*log(a*cos(f*x + e)^2 + b) - 2*(
(a^2 + 2*a*b)*cos(f*x + e)^2 - a^2 - 2*a*b)*log(1/2*sin(f*x + e)))/((a^3 +
2*a^2*b + a*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f)
```

Sympy [F]

$$\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\cot^3\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

```
[In] integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cot(e + f*x)**3/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.18

$$\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{b^2\log\left(a\sin(fx+e)^2-a-b\right)}{a^3+2\,a^2b+ab^2}+\frac{(a+2\,b)\log\left(\sin(fx+e)^2\right)}{a^2+2\,ab+b^2}+\frac{1}{(a+b)\sin(fx+e)^2}}{2\,f}$$

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*log(a*sin(f*x + e)^2 - a - b)/(a^3 + 2*a^2*b + a*b^2) + (a + 2*b)
*log(sin(f*x + e)^2)/(a^2 + 2*a*b + b^2) + 1/((a + b)*sin(f*x + e)^2))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(70) = 140.

Time = 0.34 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.97

$$\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{4\,b^2\log\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^3+2\,a^2b+ab^2}+\frac{4\,(a+2\,b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^2+2\,ab+b^2}-\left(a+b+\frac{4\,a(\cos(\cdots}{\cos(\cdots}\right.\cdots}{8\,f}$$

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/8*(4*b^2*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^3 + 2*a^2*b + a*b^2
) + 4*(a + 2*b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(a^2 + 2*
a*b + b^2) - (a + b + 4*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 8*b*(cos(
f*x + e) - 1)/(cos(f*x + e) + 1))*(cos(f*x + e) + 1)/((a^2 + 2*a*b + b^2)*(
cos(f*x + e) - 1)) - 8*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1))
/a - (cos(f*x + e) - 1)/((a + b)*(cos(f*x + e) + 1)))/f
```

Mupad [B] (verification not implemented)

Time = 20.76 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.32

$$\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a\,f} - \frac{\cot(e+f\,x)^2}{2\,f\,\left(a+b\right)} - \frac{\ln\left(\tan(e+f\,x)\right)\,\left(a+2\,b\right)}{f\,\left(a^2+2\,a\,b+b^2\right)} - \frac{b^2\,\ln\left(b\tan(e+f\,x)^2+a+b\right)}{2\,a\,f\,\left(a+b\right)^2}$$

```
[In] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2),x)
```

```
[Out] log(tan(e + f*x)^2 + 1)/(2*a*f) - cot(e + f*x)^2/(2*f*(a + b)) - (log(tan(e
 + f*x))*(a + 2*b))/(f*(2*a*b + a^2 + b^2)) - (b^2*log(a + b + b*tan(e + f*
x)^2))/(2*a*f*(a + b)^2)
```

3.342 $\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)} dx$

Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(2a+3b)\csc^2(e+fx)}{2(a+b)^2f} - \frac{\csc^4(e+fx)}{4(a+b)f} + \frac{b^3\log\left(b+a\cos^2(e+fx)\right)}{2a(a+b)^3f} + \frac{(a^2+3ab+3b^2)\log(\sin(e+fx))}{(a+b)^3f}$$

```
[Out] 1/2*(2*a+3*b)*csc(f*x+e)^2/(a+b)^2/f-1/4*csc(f*x+e)^4/(a+b)/f+1/2*b^3*ln(b+
a*cos(f*x+e)^2)/a/(a+b)^3/f+(a^2+3*a*b+3*b^2)*ln(sin(f*x+e))/(a+b)^3/f
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4223, 457, 90}

$$\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a^2+3ab+3b^2)\log(\sin(e+fx))}{f(a+b)^3} + \frac{b^3\log\left(a\cos^2(e+fx)+b\right)}{2af(a+b)^3} - \frac{\csc^4(e+fx)}{4f(a+b)} + \frac{(2a+3b)\csc^2(e+fx)}{2f(a+b)^2}$$

```
[In] Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((2*a + 3*b)*Csc[e + f*x]^2)/(2*(a + b)^2*f) - Csc[e + f*x]^4/(4*(a + b)*f)
 + (b^3*Log[b + a*Cos[e + f*x]^2])/(2*a*(a + b)^3*f) + ((a^2 + 3*a*b + 3*b^
2)*Log[Sin[e + f*x]])/((a + b)^3*f)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^7}{(1-x^2)^3(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^3}{(1-x)^3(b+ax)}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)(-1+x)^3} + \frac{-2a-3b}{(a+b)^2(-1+x)^2} + \frac{-a^2-3ab-3b^2}{(a+b)^3(-1+x)} - \frac{b^3}{(a+b)^3(b+ax)}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= \frac{(2a+3b)\csc^2(e+fx)}{2(a+b)^2 f} - \frac{\csc^4(e+fx)}{4(a+b)f} + \frac{b^3\log\left(b+a\cos^2(e+fx)\right)}{2a(a+b)^3 f} + \frac{(a^2+3ab+3b^2)\log(\sin(e+fx))}{(a+b)^3 f}$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.28

$$\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a+2b+a\cos(2e+2fx))\left(\frac{2(2a+3b)\csc^2(e+fx)}{(a+b)^2}-\frac{\csc^4(e+fx)}{a+b}+\frac{4(a^2+3ab+3b^2)\log(\sin(e+fx))}{(a+b)^3}+\frac{2b^3\log(a+b-a\sin^2(e+fx))}{a(a+b)^3}\right.}{8f\,(a+b\sec^2(e+fx))}$$

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*e + 2*f*x])*((2*(2*a + 3*b)*Csc[e + f*x]^2)/(a + b)^2 -
 Csc[e + f*x]^4/(a + b) + (4*(a^2 + 3*a*b + 3*b^2)*Log[Sin[e + f*x]])/(a +
b)^3 + (2*b^3*Log[a + b - a*Sin[e + f*x]^2])/(a*(a + b)^3))*Sec[e + f*x]^2)
/(8*f*(a + b*Sec[e + f*x]^2))
```

Maple [A] (verified)

Time = 2.61 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.67

method	result
derivativedivides	$\frac{\frac{b^3\ln\left(b+a\cos(fx+e)^2\right)}{2(a+b)^3a}-\frac{1}{2(8a+8b)(-1+\cos(fx+e))^2}-\frac{7a+11b}{16(a+b)^2(-1+\cos(fx+e))}+\frac{\left(a^2+3ab+3b^2\right)\ln(-1+\cos(fx+e))}{2(a+b)^3}-\frac{}{2(8a+8b)(1+}}{f}$
default	$\frac{\frac{b^3\ln\left(b+a\cos(fx+e)^2\right)}{2(a+b)^3a}-\frac{1}{2(8a+8b)(-1+\cos(fx+e))^2}-\frac{7a+11b}{16(a+b)^2(-1+\cos(fx+e))}+\frac{\left(a^2+3ab+3b^2\right)\ln(-1+\cos(fx+e))}{2(a+b)^3}-\frac{}{2(8a+8b)(1+}}{f}$
risch	$\frac{ix}{a}-\frac{2ia^2x}{a^3+3a^2b+3a\,b^2+b^3}-\frac{2ia^2e}{f(a^3+3a^2b+3a\,b^2+b^3)}-\frac{6iabx}{a^3+3a^2b+3a\,b^2+b^3}-\frac{6iabe}{f(a^3+3a^2b+3a\,b^2+b^3)}-\frac{6ib}{a^3+3a^2b+}$

```
[In] int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*b^3/(a+b)^3/a*ln(b+a*cos(f*x+e)^2)-1/2/(8*a+8*b)/(-1+cos(f*x+e))^2
-1/16*(7*a+11*b)/(a+b)^2/(-1+cos(f*x+e))+1/2*(a^2+3*a*b+3*b^2)/(a+b)^3*ln(-
1+cos(f*x+e))-1/2/(8*a+8*b)/(1+cos(f*x+e))^2-1/16*(-7*a-11*b)/(a+b)^2/(1+co
s(f*x+e))+1/2*(a^2+3*a*b+3*b^2)/(a+b)^3*ln(1+cos(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(102) = 204$.

Time = 0.46 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.45

$$\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{3a^3+8a^2b+5ab^2-2(2a^3+5a^2b+3ab^2)\cos(fx+e)^2+2\left(b^3\cos(fx+e)^4-2b^3\cos(fx+e)^2+b^3\right)}{4\left((a^4+3a^3b+3a^2b^2+ab^3)f\cos(fx+e\right.}$$

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^3 + 8*a^2*b + 5*a*b^2 - 2*(2*a^3 + 5*a^2*b + 3*a*b^2)*cos(f*x + e)
^2 + 2*(b^3*cos(f*x + e)^4 - 2*b^3*cos(f*x + e)^2 + b^3)*log(a*cos(f*x + e)
^2 + b) + 4*((a^3 + 3*a^2*b + 3*a*b^2)*cos(f*x + e)^4 + a^3 + 3*a^2*b + 3*a
*b^2 - 2*(a^3 + 3*a^2*b + 3*a*b^2)*cos(f*x + e)^2)*log(1/2*sin(f*x + e)))/(
(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3
*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)
```

Sympy [F]

$$\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\cot^5\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

```
[In] integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cot(e + f*x)**5/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.34

$$\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{2b^3\log\left(a\sin(fx+e)^2-a-b\right)}{a^4+3a^3b+3a^2b^2+ab^3}+\frac{2(a^2+3ab+3b^2)\log\left(\sin(fx+e)^2\right)}{a^3+3a^2b+3ab^2+b^3}+\frac{2(2a+3b)\sin(fx+e)^2-a-b}{(a^2+2ab+b^2)\sin(fx+e)^4}}{4f}$$

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*b^3*log(a*sin(f*x + e)^2 - a - b)/(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3
) + 2*(a^2 + 3*a*b + 3*b^2)*log(sin(f*x + e)^2)/(a^3 + 3*a^2*b + 3*a*b^2 +
b^3) + (2*(2*a + 3*b)*sin(f*x + e)^2 - a - b)/((a^2 + 2*a*b + b^2)*sin(f*x
+ e)^4))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. 2(102) = 204.

Time = 0.40 (sec) , antiderivative size = 513, normalized size of antiderivative = 4.75

$$\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{32\,b^3\log\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^4+3\,a^3b+3\,a^2b^2+ab^3}+\frac{32\,(a^2+3\,ab+3\,b^2)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^3+3\,a^2b+3\,ab^2+b^3}-\cdots}{\cdots}$$

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/64*(32*b^3*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(c
os(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e)
+ 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^4 + 3*a^3*b + 3*a^
2*b^2 + a*b^3) + 32*(a^2 + 3*a*b + 3*b^2)*log(abs(-cos(f*x + e) + 1)/abs(co
s(f*x + e) + 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (12*a*(cos(f*x + e) - 1)
/(cos(f*x + e) + 1) + 20*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f
*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e)
+ 1)^2)/(a^2 + 2*a*b + b^2) - (a^2 + 2*a*b + b^2 + 12*a^2*(cos(f*x + e) - 1
)/(cos(f*x + e) + 1) + 32*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 20*b^
2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 48*a^2*(cos(f*x + e) - 1)^2/(cos(
f*x + e) + 1)^2 + 144*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 144*b
^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/((a^3 +
3*a^2*b + 3*a*b^2 + b^3)*(cos(f*x + e) - 1)^2) - 64*log(abs(-(cos(f*x + e)
- 1)/(cos(f*x + e) + 1) + 1))/a)/f
```

Mupad [B] (verification not implemented)

Time = 20.66 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.48

$$\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\ln\left(\tan(e+f\,x)\right)\,\left(a^2+3\,a\,b+3\,b^2\right)}{f\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}-\frac{\ln\left({\tan(e+f\,x)}^2+1\right)}{2\,a\,f}-\frac{\ln\left(b\,{\tan(e+f\,x)}^2+a+b\right)\,\left(\frac{b}{2\,{(a+b)}^2}+\frac{1}{2\,(a+b)}+\frac{b^2}{2\,{(a+b)}^3}-\frac{1}{2\,a}\right)}{f}-\frac{{\cot(e+f\,x)}^4\,\left(\frac{1}{4\,(a+b)}-\frac{{\tan(e+f\,x)}^2\,(a+2\,b)}{2\,{(a+b)}^2}\right)}{f}$$

```
[In] int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2),x)
```

```
[Out] (log(tan(e + f*x))*(3*a*b + a^2 + 3*b^2))/(f*(3*a*b^2 + 3*a^2*b + a^3 + b^3
)) - log(tan(e + f*x)^2 + 1)/(2*a*f) - (log(a + b + b*tan(e + f*x)^2)*(b/(2
*(a + b)^2) + 1/(2*(a + b)) + b^2/(2*(a + b)^3) - 1/(2*a)))/f - (cot(e + f*
x)^4*(1/(4*(a + b)) - (tan(e + f*x)^2*(a + 2*b))/(2*(a + b)^2)))/f
```

3.343 $\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{x}{a}+\frac{(a+b)^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{ab^{5/2}f}-\frac{(a+2b)\tan(e+fx)}{b^2f}+\frac{\tan^3(e+fx)}{3bf}$$

```
[Out] -x/a+(a+b)^(5/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a/b^(5/2)/f-(a+2*b)
*tan(f*x+e)/b^2/f+1/3*tan(f*x+e)^3/b/f
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4226, 2000, 490, 596, 536, 209, 211}

$$\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a+b)^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{ab^{5/2}f}-\frac{(a+2b)\tan(e+fx)}{b^2f}-\frac{x}{a}+\frac{\tan^3(e+fx)}{3bf}$$

```
[In] Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(x/a) + ((a + b)^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*b^(5
/2)*f) - ((a + 2*b)*Tan[e + f*x])/(b^2*f) + Tan[e + f*x]^3/(3*b*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+b(1+x^2))}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\tan^3(e+fx)}{3bf} - \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)+3(a+2b)x^2)}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{3bf} \\
&= -\frac{(a+2b)\tan(e+fx)}{b^2 f} + \frac{\tan^3(e+fx)}{3bf} \\
&\quad + \frac{\text{Subst}\left(\int \frac{3(a+b)(a+2b)+3(a^2+3ab+3b^2)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{3b^2 f} \\
&= -\frac{(a+2b)\tan(e+fx)}{b^2 f} + \frac{\tan^3(e+fx)}{3bf} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{af} \\
&\quad + \frac{(a+b)^3\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{ab^2 f} \\
&= -\frac{x}{a} + \frac{(a+b)^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{ab^{5/2} f} - \frac{(a+2b)\tan(e+fx)}{b^2 f} + \frac{\tan^3(e+fx)}{3bf}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.95 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.76

$$\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(-\frac{3x}{a} - \frac{3(a+b)^{5/2}\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)}{ab^2 f\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{6\left(a+b\sec^2(e+fx)\right)}$$

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*((-3*x)/a - (3*(a + b)^(5/2)
*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*
e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin
[2*e]))/(a*b^2*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) - ((3*a + 7*b)*Sec[e]*Sec[e
 + f*x]*Sin[f*x])/(b^2*f) + (Sec[e]*Sec[e + f*x]^3*Sin[f*x])/(b*f) + (Sec[e
 + f*x]^2*Tan[e])/(b*f)))/(6*(a + b*Sec[e + f*x]^2))
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{-\frac{-\frac{b\tan(fx+e)^3}{3}+a\tan(fx+e)+2b\tan(fx+e)}{b^2}+\frac{\left(a^3+3a^2b+3a\,b^2+b^3\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^2a\sqrt{(a+b)b}}-\frac{\arctan(\tan(fx+e))}{a}}{f}$
default	$\frac{-\frac{-\frac{b\tan(fx+e)^3}{3}+a\tan(fx+e)+2b\tan(fx+e)}{b^2}+\frac{\left(a^3+3a^2b+3a\,b^2+b^3\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^2a\sqrt{(a+b)b}}-\frac{\arctan(\tan(fx+e))}{a}}{f}$
risch	$-\frac{x}{a}-\frac{2i\left(3a\,e^{4i(fx+e)}+9b\,e^{4i(fx+e)}+6a\,e^{2i(fx+e)}+12b\,e^{2i(fx+e)}+3a+7b\right)}{3f\,b^2\left(e^{2i(fx+e)}+1\right)^3}-\frac{\sqrt{-(a+b)b}\,a\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}}{a}\right.}{2b^3f}$

```
[In] int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/b^2*(-1/3*b*tan(f*x+e)^3+a*tan(f*x+e)+2*b*tan(f*x+e))+1/b^2*(a^3+3*
a^2*b+3*a*b^2+b^3)/a/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))-1
/a*arctan(tan(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(73) = 146$.

Time = 0.31 (sec) , antiderivative size = 373, normalized size of antiderivative = 4.49

$$\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \left[-\frac{12\,b^2fx\cos(fx+e)^3-3\left(a^2+2\,ab+b^2\right)\sqrt{-\frac{a+b}{b}}\cos(fx+e)^3\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos(fx+e)^4-2\left(3\,ab+4\,b^2\right)\cos(\right.}{a^2\,\mathrm{co}}\right.}{12\,ab^2f\,\mathrm{c}}\right.$$

$$-\frac{6\,b^2fx\cos(fx+e)^3+3\left(a^2+2\,ab+b^2\right)\sqrt{\frac{a+b}{b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{a+b}{b}}}{2\,(a+b)\cos(fx+e)\sin(fx+e)}\right)\cos(fx+e)^3+2\left((3\,a\right.}{6\,ab^2f\cos(fx+e)^3}$$

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/12*(12*b^2*f*x*cos(f*x + e)^3 - 3*(a^2 + 2*a*b + b^2)*sqrt(-(a + b)/b)*
cos(f*x + e)^3*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2
)*cos(f*x + e)^2 - 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt
(-(a + b)/b)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2
 + b^2)) + 4*((3*a^2 + 7*a*b)*cos(f*x + e)^2 - a*b)*sin(f*x + e))/(a*b^2*f*
cos(f*x + e)^3), -1/6*(6*b^2*f*x*cos(f*x + e)^3 + 3*(a^2 + 2*a*b + b^2)*sqr
t((a + b)/b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a + b)/b)/((a
+ b)*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e)^3 + 2*((3*a^2 + 7*a*b)*cos(f*
x + e)^2 - a*b)*sin(f*x + e))/(a*b^2*f*cos(f*x + e)^3)]
```

Sympy [F]

$$\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\tan^6{(e+fx)}}{a+b\sec^2{(e+fx)}}\,dx$$

```
[In] integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(tan(e + f*x)**6/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.14

$$\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{3\,(fx+e)}{a} - \frac{b\tan(fx+e)^3 - 3\,(a+2\,b)\tan(fx+e)}{b^2} - \frac{3\,(a^3+3\,a^2b+3\,ab^2+b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}ab^2}}{3\,f}$$

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*(f*x + e)/a - (b*tan(f*x + e)^3 - 3*(a + 2*b)*tan(f*x + e))/b^2 - 3
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sq
rt((a + b)*b)*a*b^2))/f
```

Giac [A] (verification not implemented)

none

Time = 2.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.52

$$\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{3\,(fx+e)}{a}-\frac{3\,(a^3+3\,a^2b+3\,ab^2+b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}ab^2}-\frac{b^2\tan(fx+e)^3-3\,ab\tan(fx+e)-6\,b^2\tan(fx+e)}{b^3}}{3\,f}$$

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*(f*x + e)/a - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(pi*floor((f*x + e)
/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/(sqrt(a*b + b^2
)*a*b^2) - (b^2*tan(f*x + e)^3 - 3*a*b*tan(f*x + e) - 6*b^2*tan(f*x + e))/b
^3)/f
```

Mupad [B] (verification not implemented)

Time = 19.88 (sec) , antiderivative size = 1109, normalized size of antiderivative = 13.36

$$\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\tan(e+f\,x)^3}{3\,b\,f} - \frac{\mathrm{atan}\left(\frac{40\,a^2\tan(e+f\,x)}{30\,a\,b+40\,a^2+10\,b^2+\frac{30\,a^3}{b}+\frac{12\,a^4}{b^2}+\frac{2\,a^5}{b^3}}+\frac{30\,a^3\tan(e+f\,x)}{30\,a\,b^2+40\,a^2\,b+30\,a^3+10\,b^3+\frac{12\,a^4}{b}+\frac{2\,a^5}{b^2}}+\frac{12\,a^4\tan(e+f\,x)}{30\,a\,b^3+30\,a^3\,b+12\,a^4+10\,b^4+40\,a^2\,b^2+\frac{2\,a^5}{b}}\cdots\right.}{a\cdots} - \frac{\tan(e+f\,x)\,(a+2\,b)}{b^2\,f} - \frac{\mathrm{atan}\left(\frac{\frac{\sqrt{-b^5\,(a+b)^5}\left(\frac{2\tan(e+f\,x)\left(a^6+6\,a^5\,b+15\,a^4\,b^2+20\,a^3\,b^3+15\,a^2\,b^4+6\,a\,b^5+2\,b^6\right)}{b^3}+\frac{\sqrt{-b^5\,(a+b)^5}\left(\frac{4\,a^4\,b^3+12\,a^3\,b^4+8\,a^2\,b^5\cdots}{b^3}\cdots\right.}{\cdots}\right.}{2\,a\,b^5}}{\frac{2\left(a^5+6\,a^4\,b+15\,a^3\,b^2+19\,a^2\,b^3+12\,a\,b^4+3\,b^5\right)}{b^3}-\frac{\sqrt{-b^5\,(a+b)^5}\left(\frac{2\tan(e+f\,x)\left(a^6+6\,a^5\,b+15\,a^4\,b^2+20\,a^3\,b^3+15\,a^2\,b^4+6\,a\,b^5+2\,b^6\right)}{b^3}+\frac{\sqrt{-b^5\,(a+b)^5}\cdots}{\cdots}\right.}{2\,a\,b^5}}\cdots\right.}{\cdots}$$

```
[In] int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2),x)
```

```
[Out] tan(e + f*x)^3/(3*b*f) - atan((40*a^2*tan(e + f*x))/(30*a*b + 40*a^2 + 10*b
^2 + (30*a^3)/b + (12*a^4)/b^2 + (2*a^5)/b^3) + (30*a^3*tan(e + f*x))/(30*a
*b^2 + 40*a^2*b + 30*a^3 + 10*b^3 + (12*a^4)/b + (2*a^5)/b^2) + (12*a^4*tan
```

```
(e + f*x))/(30*a*b^3 + 30*a^3*b + 12*a^4 + 10*b^4 + 40*a^2*b^2 + (2*a^5)/b)
 + (2*a^5*tan(e + f*x))/(30*a*b^4 + 12*a^4*b + 2*a^5 + 10*b^5 + 40*a^2*b^3
+ 30*a^3*b^2) + (10*b^2*tan(e + f*x))/(30*a*b + 40*a^2 + 10*b^2 + (30*a^3)/
b + (12*a^4)/b^2 + (2*a^5)/b^3) + (30*a*b*tan(e + f*x))/(30*a*b + 40*a^2 +
10*b^2 + (30*a^3)/b + (12*a^4)/b^2 + (2*a^5)/b^3))/(a*f) - (tan(e + f*x)*(a
 + 2*b))/(b^2*f) - (atan((((-b^5*(a + b)^5)^(1/2)*((2*tan(e + f*x)*(6*a*b^5
 + 6*a^5*b + a^6 + 2*b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2))/b^3 + ((-
b^5*(a + b)^5)^(1/2)*((8*a^2*b^5 + 12*a^3*b^4 + 4*a^4*b^3)/b^3 + (tan(e + f
*x)*(8*a^2*b^6 + 4*a^3*b^5)*(-b^5*(a + b)^5)^(1/2))/(a*b^8)))/(2*a*b^5))*1i
)/(2*a*b^5) + ((-b^5*(a + b)^5)^(1/2)*((2*tan(e + f*x)*(6*a*b^5 + 6*a^5*b +
 a^6 + 2*b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2))/b^3 - ((-b^5*(a + b)^
5)^(1/2)*((8*a^2*b^5 + 12*a^3*b^4 + 4*a^4*b^3)/b^3 - (tan(e + f*x)*(8*a^2*b
^6 + 4*a^3*b^5)*(-b^5*(a + b)^5)^(1/2))/(a*b^8)))/(2*a*b^5))*1i)/(2*a*b^5))
/((2*(12*a*b^4 + 6*a^4*b + a^5 + 3*b^5 + 19*a^2*b^3 + 15*a^3*b^2))/b^3 - ((
-b^5*(a + b)^5)^(1/2)*((2*tan(e + f*x)*(6*a*b^5 + 6*a^5*b + a^6 + 2*b^6 + 1
5*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2))/b^3 + ((-b^5*(a + b)^5)^(1/2)*((8*a^2
*b^5 + 12*a^3*b^4 + 4*a^4*b^3)/b^3 + (tan(e + f*x)*(8*a^2*b^6 + 4*a^3*b^5)*
(-b^5*(a + b)^5)^(1/2))/(a*b^8)))/(2*a*b^5)))/(2*a*b^5) + ((-b^5*(a + b)^5)
^(1/2)*((2*tan(e + f*x)*(6*a*b^5 + 6*a^5*b + a^6 + 2*b^6 + 15*a^2*b^4 + 20*
a^3*b^3 + 15*a^4*b^2))/b^3 - ((-b^5*(a + b)^5)^(1/2)*((8*a^2*b^5 + 12*a^3*b
^4 + 4*a^4*b^3)/b^3 - (tan(e + f*x)*(8*a^2*b^6 + 4*a^3*b^5)*(-b^5*(a + b)^5
)^(1/2))/(a*b^8)))/(2*a*b^5)))/(2*a*b^5)))*(-b^5*(a + b)^5)^(1/2)*1i)/(a*b^
5*f)
```

3.344 $\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 59

$$\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{x}{a} - \frac{(a+b)^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{ab^{3/2}f} + \frac{\tan(e+fx)}{bf}$$

```
[Out] x/a-(a+b)^(3/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a/b^(3/2)/f+tan(f*x+
e)/b/f
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4226, 2000, 490, 536, 209, 211}

$$\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{(a+b)^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{ab^{3/2}f} + \frac{x}{a} + \frac{\tan(e+fx)}{bf}$$

```
[In] Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] x/a - ((a + b)^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*b^(3/2)
*f) + Tan[e + f*x]/(b*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{\text{Subst}\left(\int\frac{x^4}{(1+x^2)(a+b(1+x^2))}\,dx,x,\tan(e+fx)\right)}{f}\\&=\frac{\text{Subst}\left(\int\frac{x^4}{(1+x^2)(a+b+bx^2)}\,dx,x,\tan(e+fx)\right)}{f}\\&=\frac{\tan(e+fx)}{bf}-\frac{\text{Subst}\left(\int\frac{a+b+(a+2b)x^2}{(1+x^2)(a+b+bx^2)}\,dx,x,\tan(e+fx)\right)}{bf}\end{aligned}$$

$$= \frac{\tan(e+fx)}{bf} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \tan(e+fx)\right)}{af} - \frac{(a+b)^2 \text{Subst}\left(\int \frac{1}{a+b+bx^2} dx, x, \tan(e+fx)\right)}{abf}$$

$$= \frac{x}{a} - \frac{(a+b)^{3/2} \arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{ab^{3/2}f} + \frac{\tan(e+fx)}{bf}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.12 (sec) , antiderivative size = 206, normalized size of antiderivative = 3.49

$$\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)} dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left((a+b)^2\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)\right.}{2ab\sqrt{a+b}f\left(a+b\sec^2(e+fx)\right)\sqrt{b}}$$

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*((a + b)^2*ArcTan[(Sec[f*x]*
(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt
[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a +
b]*Sqrt[b*(I*Cos[e] + Sin[e])^4]*(b*f*x + a*Sec[e]*Sec[e + f*x]*Sin[f*x])))
/(2*a*b*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)}{b}+\frac{\arctan(\tan(fx+e))}{a}+\frac{\left(-a^2-2ab-b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{ab\sqrt{(a+b)b}}}{f}$
default	$\frac{\frac{\tan(fx+e)}{b}+\frac{\arctan(\tan(fx+e))}{a}+\frac{\left(-a^2-2ab-b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{ab\sqrt{(a+b)b}}}{f}$
risch	$\frac{x}{a}+\frac{2i}{fb\left(e^{2i(fx+e)}+1\right)}+\frac{\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{2b^2f}+\frac{\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a-}{a}\right.}{2bfa}$

```
[In] int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(tan(f*x+e)/b+1/a*arctan(tan(f*x+e))+(-a^2-2*a*b-b^2)/a/b/((a+b)*b)^(1/
2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(51) = 102$.

Time = 0.28 (sec) , antiderivative size = 297, normalized size of antiderivative = 5.03

$$\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \left[\frac{4bfx\cos(fx+e) + (a+b)\sqrt{-\frac{a+b}{b}}\cos(fx+e)\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4 - 2(3ab+4b^2)\cos(fx+e)^2 + 4\left((ab+2b^2 \cdots}{a^2\cos(fx+e)^4 + 2ab\cos(\cdots}\right. \cdots}{4abf\cos(fx+e)} \cdots\right.$$

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*b*f*x*cos(f*x + e) + (a + b)*sqrt(-(a + b)/b)*cos(f*x + e)*log(((a^
2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((
a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(-(a + b)/b)*sin(f*x +
e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 4*a*sin(f*x
+ e))/(a*b*f*cos(f*x + e)), 1/2*(2*b*f*x*cos(f*x + e) + (a + b)*sqrt((a + b
)/b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a + b)/b)/((a + b)*cos
(f*x + e)*sin(f*x + e)))*cos(f*x + e) + 2*a*sin(f*x + e))/(a*b*f*cos(f*x +
e))]
```

Sympy [F]

$$\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

```
[In] integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(tan(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

$$\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{fx+e}{a}+\frac{\tan(fx+e)}{b}-\frac{(a^2+2\,ab+b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}ab}}{f}$$

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] ((f*x + e)/a + tan(f*x + e)/b - (a^2 + 2*a*b + b^2)*arctan(b*tan(f*x + e)/s
qrt((a + b)*b))/(sqrt((a + b)*b)*a*b))/f
```

Giac [A] (verification not implemented)

none

Time = 0.87 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.47

$$\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{fx+e}{a}+\frac{\tan(fx+e)}{b}-\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(a^2+2\,ab+b^2)}{\sqrt{ab+b^2}ab}}{f}$$

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] ((f*x + e)/a + tan(f*x + e)/b - (pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arct
an(b*tan(f*x + e)/sqrt(a*b + b^2)))*(a^2 + 2*a*b + b^2)/(sqrt(a*b + b^2)*a*
b))/f
```

Mupad [B] (verification not implemented)

Time = 19.16 (sec) , antiderivative size = 410, normalized size of antiderivative = 6.95

$$\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\mathrm{atan}\left(\frac{8\,a^2\tan(e+f\,x)}{12\,a\,b+8\,a^2+6\,b^2+\frac{2\,a^3}{b}}+\frac{2\,a^3\tan(e+f\,x)}{2\,a^3+8\,a^2\,b+12\,a\,b^2+6\,b^3}+\frac{6\,b^2\tan(e+f\,x)}{12\,a\,b+8\,a^2+6\,b^2+\frac{2\,a^3}{b}}+\frac{12\,a\,b\tan(e+f\,x)}{12\,a\,b+8\,a^2+6\,b^2+\frac{2\,a^3}{b}}\right)}{a\,f}$$
$$+\frac{\tan(e+f\,x)}{b\,f}$$
$$+\frac{\mathrm{atanh}\left(\frac{6\tan(e+f\,x)\sqrt{-a^3\,b^3-3\,a^2\,b^4-3\,a\,b^5-b^6}}{18\,a\,b^2+20\,a^2\,b+10\,a^3+6\,b^3+\frac{2\,a^4}{b}}+\frac{6\,a\tan(e+f\,x)\sqrt{-a^3\,b^3-3\,a^2\,b^4-3\,a\,b^5-b^6}}{2\,a^4+10\,a^3\,b+20\,a^2\,b^2+18\,a\,b^3+6\,b^4}+\frac{2\,a^2\tan(e+f\,x)\sqrt{-a^3\,b^3-3\,a^2\,b^4-3\,a\,b}}{2\,a^4\,b+10\,a^3\,b^2+20\,a^2\,b^3+18\,a\,b^4+6}\right.}{a\,b^3\,f}$$

```
[In] int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2),x)
```

```
[Out] atan((8*a^2*tan(e + f*x))/(12*a*b + 8*a^2 + 6*b^2 + (2*a^3)/b) + (2*a^3*tan
(e + f*x))/(12*a*b^2 + 8*a^2*b + 2*a^3 + 6*b^3) + (6*b^2*tan(e + f*x))/(12*
a*b + 8*a^2 + 6*b^2 + (2*a^3)/b) + (12*a*b*tan(e + f*x))/(12*a*b + 8*a^2 +
6*b^2 + (2*a^3)/b))/(a*f) + tan(e + f*x)/(b*f) + (atanh((6*tan(e + f*x)*(-
3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2))/(18*a*b^2 + 20*a^2*b + 10*a^3 +
 6*b^3 + (2*a^4)/b) + (6*a*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*
b^3)^(1/2))/(18*a*b^3 + 10*a^3*b + 2*a^4 + 6*b^4 + 20*a^2*b^2) + (2*a^2*tan
(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2))/(18*a*b^4 + 2*a^4*
b + 6*b^5 + 20*a^2*b^3 + 10*a^3*b^2))*(-b^3*(a + b)^3)^(1/2))/(a*b^3*f)
```

3.345 $\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 46

$$\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{x}{a}+\frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}f}$$

```
[Out] -x/a+arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*(a+b)^(1/2)/a/f/b^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4226, 2000, 492, 209, 211}

$$\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}f}-\frac{x}{a}$$

```
[In] Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(x/a) + (Sqrt[a + b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[b
]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+b(1+x^2))}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{af} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{af} \\
&= -\frac{x}{a} + \frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}f}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 184, normalized size of antiderivative = 4.00

$$\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(\sqrt{a+b}fx\sqrt{b(\cos(e)-i\sin(e))^4}+(a+b)\arctan\left(\frac{\sec(fx)(\cos(2e}{}\right.\right.}{2a\sqrt{a+b}f\left(a+b\sec^2(e+fx)\right)\sqrt{b(\cos(e)-i\sin(}}$$

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/2*((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[a + b]*f*x*Sqrt[b
*(Cos[e] - I*Sin[e])^4] + (a + b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*
(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] -
 I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e])))/(a*Sqrt[a + b]*f*(a + b*Sec[e + f
*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{a}+\frac{(a+b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}}{f}$	48
default	$\frac{-\frac{\arctan(\tan(fx+e))}{a}+\frac{(a+b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}}{f}$	48
risch	$-\frac{x}{a}-\frac{\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{2bfa}+\frac{\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-(a+b)b}-a-2b}{a}\right)}{2bfa}$	111

```
[In] int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a*arctan(tan(f*x+e))+(a+b)/a/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((
a+b)*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 4.91

$$\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[-\frac{4\,fx - \sqrt{-\frac{a+b}{b}}\log\Bigg(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2-4\,\big((ab+2\,b^2)\cos(fx+e)^3-b^2\cos(fx+e)\big)\sqrt{-\frac{a+b}{b}}\sin(}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}}{4\,af}, -\frac{2\,fx+\sqrt{\frac{a+b}{b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{a+b}{b}}}{2\,(a+b)\cos(fx+e)\sin(fx+e)}\right)}{2\,af} \right]$$

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*f*x - sqrt(-(a + b)/b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*c
os(f*x + e))*sqrt(-(a + b)/b)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a
*b*cos(f*x + e)^2 + b^2)))/(a*f), -1/2*(2*f*x + sqrt((a + b)/b)*arctan(1/2*
((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a + b)/b)/((a + b)*cos(f*x + e)*sin(f*
x + e))))/(a*f)]
```

Sympy [F]

$$\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\tan^2{(e+fx)}}{a+b\sec^2{(e+fx)}}\,dx$$

```
[In] integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{(a+b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a} - \frac{fx+e}{a}}{f}$$

[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")

[Out] ((a + b)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a) - (f*x + e)/a)/f

Giac [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.43

$$\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(a+b)}{\sqrt{ab+b^2}a} - \frac{fx+e}{a}}{f}$$

[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")

[Out] ((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))*(a + b)/(sqrt(a*b + b^2)*a) - (f*x + e)/a)/f

Mupad [B] (verification not implemented)

Time = 19.66 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.74

$$\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\mathrm{atan}\left(\frac{2\,a\,b^2\tan(e+f\,x)}{2\,a^2\,b+2\,a\,b^2}+\frac{2\,a^2\,b\tan(e+f\,x)}{2\,a^2\,b+2\,a\,b^2}\right)}{a\,f} - \frac{\mathrm{atanh}\left(\frac{2\,a\,b^2\tan(e+f\,x)\,\sqrt{-b^2-a\,b}}{2\,a^2\,b^2+2\,a\,b^3}\right)\,\sqrt{-b\,(a+b)}}{a\,b\,f}$$

[In] int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2),x)

[Out] - atan((2*a*b^2*tan(e + f*x))/(2*a*b^2 + 2*a^2*b) + (2*a^2*b*tan(e + f*x))/(2*a*b^2 + 2*a^2*b))/(a*f) - (atanh((2*a*b^2*tan(e + f*x)*(- a*b - b^2)^(1/2))/(2*a*b^3 + 2*a^2*b^2))*(-b*(a + b))^(1/2))/(a*b*f)

3.346 $\int \frac{1}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 45

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx=\frac{x}{a}+\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{a\sqrt{a+b}f}$$

```
[Out] x/a+arctan(cot(f*x+e)*(a+b)^(1/2)/b^(1/2))*b^(1/2)/a/f/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4212, 3260, 211}

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx=\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{af\sqrt{a+b}}+\frac{x}{a}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-1),x]
```

```
[Out] x/a + (Sqrt[b]*ArcTan[(Sqrt[a + b]*Cot[e + f*x])/Sqrt[b]])/(a*Sqrt[a + b]*f
)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
```

```
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 4212

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/a, x
] - Dist[b/a, Int[1/(b + a*Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f}, x
] && NeQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x}{a} - \frac{b \int \frac{1}{b+a\cos^2(e+fx)}\,dx}{a} \\
&= \frac{x}{a} + \frac{b\text{Subst}\left(\int \frac{1}{b+(a+b)x^2}\,dx, x, \cot(e+fx)\right)}{af} \\
&= \frac{x}{a} + \frac{\sqrt{b}\arctan\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{a\sqrt{a+b}f}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.04

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(\sqrt{a+b}fx\sqrt{b(\cos(e)-i\sin(e))^4}+b\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))}{2\sqrt{a+b}\sqrt{}}\right.\right.}{2a\sqrt{a+b}f\,(a+b\sec^2(e+fx))\,\sqrt{b(\cos(e)-i\sin(e))^4}}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-1),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[a + b]*f*x*Sqrt[b*(Cos
[e] - I*Sin[e])^4] + b*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b
)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^
4])]*(Cos[2*e] - I*Sin[2*e])))/(2*a*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)*Sq
rt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$\frac{-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}+\frac{\arctan(\tan(fx+e))}{a}}{f}$	46
default	$\frac{-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}+\frac{\arctan(\tan(fx+e))}{a}}{f}$	46
risch	$\frac{x}{a}+\frac{\sqrt{-(a+b)b}\,\ln\left(\mathrm{e}^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{2(a+b)fa}-\frac{\sqrt{-(a+b)b}\,\ln\left(\mathrm{e}^{2i(fx+e)}-\frac{2i\sqrt{-(a+b)b}-a-2b}{a}\right)}{2(a+b)fa}$	114

```
[In] int(1/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-b/a/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))+1/a*arctan(t
an(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 231, normalized size of antiderivative = 5.13

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \left[\frac{4\,fx+\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-\cdots}}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right.}{4\,af}\right.$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*f*x + sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3
- (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f
*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(a*f), 1/2*(2*f*x + sqrt(b/(a + b
))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e
)*sin(f*x + e))))/(a*f)]
```

Sympy [F]

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \int \frac{1}{a+b\sec^2\left(e+fx\right)}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e)**2),x)
[Out] Integral(1/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a} - \frac{fx+e}{a}}{f}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] -(b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a) - (f*x + e)/
a)/f
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{\sqrt{ab+b^2}a} - \frac{fx+e}{a}}{f}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] -((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b
^2)))*b/(sqrt(a*b + b^2)*a) - (f*x + e)/a)/f
```

Mupad [B] (verification not implemented)

Time = 19.37 (sec) , antiderivative size = 460, normalized size of antiderivative = 10.22

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx=\frac{x}{a}-\frac{\operatorname{atan}\left(\frac{\frac{\left(2\,b^3\,\tan(e+f\,x)-\frac{\left(2\,a^2\,b^2-\frac{\tan(e+f\,x)\,\left(8\,a^3\,b^2+16\,a^2\,b^3\right)\,\sqrt{-b\,(a+b)}}{4\,\left(a^2+b\,a\right)}\right)\,\sqrt{-b\,(a+b)}}{2\,\left(a^2+b\,a\right)}\right)\,\sqrt{-b\,(a+b)}\,1\mathrm{i}}{a^2+b\,a}+\frac{\left(2\,b^3\,\tan(e+f\,x)+\frac{\left(2\,a^2\,b^2+\frac{\tan(e+f\,x)\,\left(8\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}}{\frac{\left(2\,b^3\,\tan(e+f\,x)-\frac{\left(2\,a^2\,b^2-\frac{\tan(e+f\,x)\,\left(8\,a^3\,b^2+16\,a^2\,b^3\right)\,\sqrt{-b\,(a+b)}}{4\,\left(a^2+b\,a\right)}\right)\,\sqrt{-b\,(a+b)}}{2\,\left(a^2+b\,a\right)}\right)\,\sqrt{-b\,(a+b)}}{a^2+b\,a}-\frac{\left(2\,b^3\,\tan(e+f\,x)+\frac{\left(2\,a^2\,b^2+\frac{\tan(e+f\,x)\,\left(8\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}}\right)\cdots}{f\,\left(a^2+b\,a\right)}$$

```
[In] int(1/(a + b/cos(e + f*x)^2),x)
```

```
[Out] x/a - (atan((((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^2*b^3
 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(
a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(a*b + a^2) + ((2*b^3*tan(e + f*x) + ((
2*a^2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(
a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(a
*b + a^2))/(((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^2*b^3
+ 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a
*b + a^2)))*(-b*(a + b))^(1/2))/(a*b + a^2) - ((2*b^3*tan(e + f*x) + ((2*a^
2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b
+ a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2))/(a*b + a^
2)))*(-b*(a + b))^(1/2)*1i)/(f*(a*b + a^2))
```

3.347 $\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 62

$$\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{x}{a}+\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a(a+b)^{3/2}f}-\frac{\cot(e+fx)}{(a+b)f}$$

```
[Out] -x/a+b^(3/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a/(a+b)^(3/2)/f-cot(f*x
+e)/(a+b)/f
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4226, 2000, 491, 536, 209, 211}

$$\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{af(a+b)^{3/2}}-\frac{\cot(e+fx)}{f(a+b)}-\frac{x}{a}$$

```
[In] Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(x/a) + (b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*(a + b)^(3
/2)*f) - Cot[e + f*x]/((a + b)*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+b(1+x^2))}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{\cot(e+fx)}{(a+b)f} + \frac{\text{Subst}\left(\int \frac{-a-2b-bx^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{(a+b)f}
\end{aligned}
$$

$$=-\frac{\cot(e+fx)}{(a+b)f}-\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e+fx)\right)}{af}+\frac{b^2\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx,x,\tan(e+fx)\right)}{a(a+b)f}$$

$$=-\frac{x}{a}+\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a(a+b)^{3/2}f}-\frac{\cot(e+fx)}{(a+b)f}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.25 (sec) , antiderivative size = 204, normalized size of antiderivative = 3.29

$$\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx =$$
$$-\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(b^2\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos\right.}{2a(a+b)^{3/2}f\left(a+b\sec^2(e+fx)\right)}$$

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/2*((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(b^2*ArcTan[(Sec[f*x]*(
Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[
a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a + b
]*Sqrt[b*(Cos[e] - I*Sin[e])^4]*((a + b)*f*x - a*Csc[e]*Csc[e + f*x]*Sin[f*
x])))/(a*(a + b)^(3/2)*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^
4])
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{\frac{b^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)a\sqrt{(a+b)b}}-\frac{1}{(a+b)\tan(fx+e)}-\frac{\arctan(\tan(fx+e))}{a}}{f}$
default	$\frac{\frac{b^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)a\sqrt{(a+b)b}}-\frac{1}{(a+b)\tan(fx+e)}-\frac{\arctan(\tan(fx+e))}{a}}{f}$
risch	$-\frac{x}{a}-\frac{2i}{f(a+b)\left(e^{2i(fx+e)}-1\right)}-\frac{\sqrt{-(a+b)b}\,b\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{2(a+b)^2fa}+\frac{\sqrt{-(a+b)b}\,b\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-}\right.}{2(a+b)^2fa}$

```
[In] int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a+b)*b^2/a/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))-1/(
a+b)/tan(f*x+e)-1/a*arctan(tan(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(54) = 108$.

Time = 0.28 (sec) , antiderivative size = 310, normalized size of antiderivative = 5.00

$$\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \left[-\frac{4\,(a+b)fx\sin\left(fx+e\right) - b\sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos(fx+e)^4-2\,\left(3\,ab+4\,b^2\right)\cos(fx+e)^2-4\,\left(\left(a^2+3\,ab+2\,b^2\right)\cos(fx\cdots}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+\cdots}\right.\cdots}{4\,(a^2+ab)f\sin\left(fx+e\right)}, -\frac{2\,(a+b)fx\sin\left(fx+e\right)+b\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)\sin\left(fx+e\right)+2\,a\cos\left(fx+e\right)}{2\,(a^2+ab)f\sin\left(fx+e\right)} \right]$$

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a + b)*f*x*sin(f*x + e) - b*sqrt(-b/(a + b))*log(((a^2 + 8*a*b +
8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b
+ 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*
x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x +
e) + 4*a*cos(f*x + e))/((a^2 + a*b)*f*sin(f*x + e)), -1/2*(2*(a + b)*f*x*si
n(f*x + e) + b*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sq
rt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 2*a*cos(f*x + e
))/((a^2 + a*b)*f*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\cot^2\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

```
[In] integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cot(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06

$$\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{b^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^2+ab)\sqrt{(a+b)b}} - \frac{fx+e}{a} - \frac{1}{(a+b)\tan(fx+e)}}{f}$$

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] (b^2*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^2 + a*b)*sqrt((a + b)*b)) -
 (f*x + e)/a - 1/((a + b)*tan(f*x + e)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.40

$$\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^2}{(a^2+ab)\sqrt{ab+b^2}} - \frac{fx+e}{a} - \frac{1}{(a+b)\tan(fx+e)}}{f}$$

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] ((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^
2)))*b^2/((a^2 + a*b)*sqrt(a*b + b^2)) - (f*x + e)/a - 1/((a + b)*tan(f*x +
 e)))/f
```

Mupad [B] (verification not implemented)

Time = 21.80 (sec) , antiderivative size = 637, normalized size of antiderivative = 10.27

$$\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx =$$

$$-\frac{a\,b^2+2\,a^2\,b+a^3+a^3\tan(e+f\,x)\,\mathrm{atan}(\tan(e+f\,x))+b^3\tan(e+f\,x)\,\mathrm{atan}(\tan(e+f\,x))+3\,a\,b^2\tan(}{}$$

```
[In] int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2),x)
```

```
[Out] -(a*b^2 + 2*a^2*b + a^3 - atan((a*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4
 - a^3*b^3)^(3/2)*1i + b*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^
3)^(3/2)*2i + b^7*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2
```

```
)*2i + a*b^6*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*10i
 + a^6*b*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*1i + a^
2*b^5*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*21i + a^3*
b^4*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*24i + a^4*b^
3*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*16i + a^5*b^2*
tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*6i)/(3*a*b^9 + 1
8*a^2*b^8 + 46*a^3*b^7 + 65*a^4*b^6 + 55*a^5*b^5 + 28*a^6*b^4 + 8*a^7*b^3 +
 a^8*b^2))*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*1i +
a^3*tan(e + f*x)*atan(tan(e + f*x)) + b^3*tan(e + f*x)*atan(tan(e + f*x)) +
 3*a*b^2*tan(e + f*x)*atan(tan(e + f*x)) + 3*a^2*b*tan(e + f*x)*atan(tan(e
+ f*x)))/(a^4*f*tan(e + f*x) + a*b^3*f*tan(e + f*x) + 3*a^3*b*f*tan(e + f*x
) + 3*a^2*b^2*f*tan(e + f*x))
```

3.348 $\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{x}{a}-\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a(a+b)^{5/2}f}+\frac{(a+2b)\cot(e+fx)}{(a+b)^2f}-\frac{\cot^3(e+fx)}{3(a+b)f}$$

```
[Out] x/a-b^(5/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a/(a+b)^(5/2)/f+(a+2*b)*
cot(f*x+e)/(a+b)^2/f-1/3*cot(f*x+e)^3/(a+b)/f
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4226, 2000, 491, 597, 536, 209, 211}

$$\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{af(a+b)^{5/2}}-\frac{\cot^3(e+fx)}{3f(a+b)}+\frac{(a+2b)\cot(e+fx)}{f(a+b)^2}+\frac{x}{a}$$

```
[In] Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] x/a - (b^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*(a + b)^(5/2)
*f) + ((a + 2*b)*Cot[e + f*x])/((a + b)^2*f) - Cot[e + f*x]^3/(3*(a + b)*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+b(1+x^2))}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{\cot^3(e+fx)}{3(a+b)f} + \frac{\text{Subst}\left(\int \frac{-3(a+2b)-3bx^2}{x^2(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{3(a+b)f} \\
&= \frac{(a+2b)\cot(e+fx)}{(a+b)^2 f} - \frac{\cot^3(e+fx)}{3(a+b)f} - \frac{\text{Subst}\left(\int \frac{-3(a^2+3ab+3b^2)-3b(a+2b)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{3(a+b)^2 f} \\
&= \frac{(a+2b)\cot(e+fx)}{(a+b)^2 f} - \frac{\cot^3(e+fx)}{3(a+b)f} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{af} \\
&\quad - \frac{b^3\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{a(a+b)^2 f} \\
&= \frac{x}{a} - \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a(a+b)^{5/2} f} + \frac{(a+2b)\cot(e+fx)}{(a+b)^2 f} - \frac{\cot^3(e+fx)}{3(a+b)f}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.69 (sec) , antiderivative size = 390, normalized size of antiderivative = 4.53

$$\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(3b^3\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(\right.}{}$$

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(3*b^3*ArcTan[(Sec[f*x]*(Cos
[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a +
 b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + (Sqrt[a + b]*
```

```
Csc[e]*Csc[e + f*x]^3*Sqrt[b*(Cos[e] - I*Sin[e])^4]*(9*(a + b)^2*f*x*Cos[f*
x] - 9*(a + b)^2*f*x*Cos[2*e + f*x] - 3*a^2*f*x*Cos[2*e + 3*f*x] - 6*a*b*f*
x*Cos[2*e + 3*f*x] - 3*b^2*f*x*Cos[2*e + 3*f*x] + 3*a^2*f*x*Cos[4*e + 3*f*x
] + 6*a*b*f*x*Cos[4*e + 3*f*x] + 3*b^2*f*x*Cos[4*e + 3*f*x] - 12*a^2*Sin[f*
x] - 24*a*b*Sin[f*x] - 12*a^2*Sin[2*e + f*x] - 18*a*b*Sin[2*e + f*x] + 8*a^
2*Sin[2*e + 3*f*x] + 14*a*b*Sin[2*e + 3*f*x]))/8))/(6*a*(a + b)^(5/2)*f*(a
+ b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a}-\frac{1}{3(a+b)\tan(fx+e)^3}-\frac{-a-2b}{(a+b)^2\tan(fx+e)}-\frac{b^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)^2a\sqrt{(a+b)b}}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{a}-\frac{1}{3(a+b)\tan(fx+e)^3}-\frac{-a-2b}{(a+b)^2\tan(fx+e)}-\frac{b^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)^2a\sqrt{(a+b)b}}}{f}$
risch	$\frac{x}{a}+\frac{2i(6a\,e^{4i(fx+e)}+9b\,e^{4i(fx+e)}-6a\,e^{2i(fx+e)}-12b\,e^{2i(fx+e)}+4a+7b)}{3f(a+b)^2(e^{2i(fx+e)}-1)^3}+\frac{\sqrt{-(a+b)b}\,b^2\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}}{a}\right.}{2(a+b)^3fa}$

```
[In] int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a*arctan(tan(f*x+e))-1/3/(a+b)/tan(f*x+e)^3-(-a-2*b)/(a+b)^2/tan(f*x
+e)-1/(a+b)^2*b^3/a/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(76) = 152.

Time = 0.31 (sec) , antiderivative size = 533, normalized size of antiderivative = 6.20

$$\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{4\left(4a^2+7ab\right)\cos\left(fx+e\right)^3+3\left(b^2\cos\left(fx+e\right)^2-b^2\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^2+8ab+8b^2\right)\cos(fx+e)^4-2\left(3ab+4b^2\right)\cos(f}{}\right.}{}\right.$$

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(4*a^2 + 7*a*b)*cos(f*x + e)^3 + 3*(b^2*cos(f*x + e)^2 - b^2)*sqrt
(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*
cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(
```

```
f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*
cos(f*x + e)^2 + b^2))*sin(f*x + e) - 12*(a^2 + 2*a*b)*cos(f*x + e) + 12*((
a^2 + 2*a*b + b^2)*f*x*cos(f*x + e)^2 - (a^2 + 2*a*b + b^2)*f*x)*sin(f*x +
e))/(((a^3 + 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f)
*sin(f*x + e)), 1/6*(2*(4*a^2 + 7*a*b)*cos(f*x + e)^3 + 3*(b^2*cos(f*x + e)
^2 - b^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/
(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) - 6*(a^2 + 2*a*b)*cos(
f*x + e) + 6*((a^2 + 2*a*b + b^2)*f*x*cos(f*x + e)^2 - (a^2 + 2*a*b + b^2)*
f*x)*sin(f*x + e))/(((a^3 + 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^
2*b + a*b^2)*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx=\int \frac{\cot^4{(e+fx)}}{a+b\sec^2{(e+fx)}}\,dx$$

```
[In] integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cot(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.23

$$\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx=-\frac{\frac{3\,b^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^3+2\,a^2b+ab^2)\sqrt{(a+b)b}}-\frac{3\,(fx+e)}{a}-\frac{3\,(a+2\,b)\tan(fx+e)^2-a-b}{(a^2+2\,ab+b^2)\tan(fx+e)^3}}{3\,f}$$

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^3*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + 2*a^2*b + a*b^2)
*sqrt((a + b)*b)) - 3*(f*x + e)/a - (3*(a + 2*b)*tan(f*x + e)^2 - a - b)/((
a^2 + 2*a*b + b^2)*tan(f*x + e)^3))/f
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.56

$$\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^3}{(a^3+2a^2b+ab^2)\sqrt{ab+b^2}}-\frac{3(fx+e)}{a}-\frac{3a\tan(fx+e)^2+6b\tan(fx+e)^2-a-b}{(a^2+2ab+b^2)\tan(fx+e)^3}}{3f}$$

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b + b^2)))*b^3/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b + b^2)) - 3*(f*x + e)/a -
 (3*a*tan(f*x + e)^2 + 6*b*tan(f*x + e)^2 - a - b)/((a^2 + 2*a*b + b^2)*tan
(f*x + e)^3))/f
```

Mupad [B] (verification not implemented)

Time = 24.73 (sec) , antiderivative size = 2644, normalized size of antiderivative = 30.74

$$\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

```
[In] int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2),x)
```

```
[Out] atan((10*b^12*tan(e + f*x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 630*a^3*b
^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110*a^8*b^4 +
22*a^9*b^3 + 2*a^10*b^2) + (80*a*b^11*tan(e + f*x))/(80*a*b^11 + 10*b^12 +
290*a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6 + 330*
a^7*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^10*b^2) + (290*a^2*b^10*tan(e + f*
x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 + 922*a
^5*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^10*b^2)
 + (630*a^3*b^9*tan(e + f*x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 630*a^3
*b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110*a^8*b^4
+ 22*a^9*b^3 + 2*a^10*b^2) + (912*a^4*b^8*tan(e + f*x))/(80*a*b^11 + 10*b^1
2 + 290*a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6 +
330*a^7*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^10*b^2) + (922*a^5*b^7*tan(e +
 f*x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 + 92
2*a^5*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^10*b
^2) + (660*a^6*b^6*tan(e + f*x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 630*
a^3*b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110*a^8*b
^4 + 22*a^9*b^3 + 2*a^10*b^2) + (330*a^7*b^5*tan(e + f*x))/(80*a*b^11 + 10*
```

b^12 + 290*a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6
 + 330*a^7*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^10*b^2) + (110*a^8*b^4*tan(
e + f*x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 +
 922*a^5*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^1
0*b^2) + (22*a^9*b^3*tan(e + f*x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 63
0*a^3*b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110*a^8
*b^4 + 22*a^9*b^3 + 2*a^10*b^2) + (2*a^10*b^2*tan(e + f*x))/(80*a*b^11 + 10
*b^12 + 290*a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^
6 + 330*a^7*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^10*b^2))/(a*f) - (1/(3*(a
+ b)) - (tan(e + f*x)^2*(a + 2*b))/(a + b)^2)/(f*tan(e + f*x)^3) - (atan(((
(-b^5*(a + b)^5)^(1/2)*((tan(e + f*x)*(32*a*b^12 + 4*b^13 + 120*a^2*b^11 +
280*a^3*b^10 + 450*a^4*b^9 + 516*a^5*b^8 + 422*a^6*b^7 + 240*a^7*b^6 + 90*a
^8*b^5 + 20*a^9*b^4 + 2*a^10*b^3))/2 - ((-b^5*(a + b)^5)^(1/2)*(6*a^2*b^12
+ 54*a^3*b^11 + 218*a^4*b^10 + 520*a^5*b^9 + 812*a^6*b^8 + 868*a^7*b^7 + 64
4*a^8*b^6 + 328*a^9*b^5 + 110*a^10*b^4 + 22*a^11*b^3 + 2*a^12*b^2 - (tan(e
+ f*x)*(-b^5*(a + b)^5)^(1/2)*(16*a^2*b^13 + 168*a^3*b^12 + 800*a^4*b^11 +
2280*a^5*b^10 + 4320*a^6*b^9 + 5712*a^7*b^8 + 5376*a^8*b^7 + 3600*a^9*b^6 +
 1680*a^10*b^5 + 520*a^11*b^4 + 96*a^12*b^3 + 8*a^13*b^2))/(4*a*(a + b)^5))
)/(2*a*(a + b)^5))*1i)/(a*(a + b)^5) + ((-b^5*(a + b)^5)^(1/2)*((tan(e + f*
x)*(32*a*b^12 + 4*b^13 + 120*a^2*b^11 + 280*a^3*b^10 + 450*a^4*b^9 + 516*a^
5*b^8 + 422*a^6*b^7 + 240*a^7*b^6 + 90*a^8*b^5 + 20*a^9*b^4 + 2*a^10*b^3))/
2 + ((-b^5*(a + b)^5)^(1/2)*(6*a^2*b^12 + 54*a^3*b^11 + 218*a^4*b^10 + 520*
a^5*b^9 + 812*a^6*b^8 + 868*a^7*b^7 + 644*a^8*b^6 + 328*a^9*b^5 + 110*a^10*
b^4 + 22*a^11*b^3 + 2*a^12*b^2 + (tan(e + f*x)*(-b^5*(a + b)^5)^(1/2)*(16*a
^2*b^13 + 168*a^3*b^12 + 800*a^4*b^11 + 2280*a^5*b^10 + 4320*a^6*b^9 + 5712
*a^7*b^8 + 5376*a^8*b^7 + 3600*a^9*b^6 + 1680*a^10*b^5 + 520*a^11*b^4 + 96*
a^12*b^3 + 8*a^13*b^2))/(4*a*(a + b)^5)))/(2*a*(a + b)^5))*1i)/(a*(a + b)^5
))/(26*a*b^11 + 4*b^12 + 72*a^2*b^10 + 110*a^3*b^9 + 100*a^4*b^8 + 54*a^5*b
^7 + 16*a^6*b^6 + 2*a^7*b^5 + ((-b^5*(a + b)^5)^(1/2)*((tan(e + f*x)*(32*a*
b^12 + 4*b^13 + 120*a^2*b^11 + 280*a^3*b^10 + 450*a^4*b^9 + 516*a^5*b^8 + 4
22*a^6*b^7 + 240*a^7*b^6 + 90*a^8*b^5 + 20*a^9*b^4 + 2*a^10*b^3))/2 - ((-b^
5*(a + b)^5)^(1/2)*(6*a^2*b^12 + 54*a^3*b^11 + 218*a^4*b^10 + 520*a^5*b^9 +
 812*a^6*b^8 + 868*a^7*b^7 + 644*a^8*b^6 + 328*a^9*b^5 + 110*a^10*b^4 + 22*
a^11*b^3 + 2*a^12*b^2 - (tan(e + f*x)*(-b^5*(a + b)^5)^(1/2)*(16*a^2*b^13 +
 168*a^3*b^12 + 800*a^4*b^11 + 2280*a^5*b^10 + 4320*a^6*b^9 + 5712*a^7*b^8
+ 5376*a^8*b^7 + 3600*a^9*b^6 + 1680*a^10*b^5 + 520*a^11*b^4 + 96*a^12*b^3
+ 8*a^13*b^2))/(4*a*(a + b)^5)))/(2*a*(a + b)^5)))/(a*(a + b)^5) - ((-b^5*(
a + b)^5)^(1/2)*((tan(e + f*x)*(32*a*b^12 + 4*b^13 + 120*a^2*b^11 + 280*a^3
*b^10 + 450*a^4*b^9 + 516*a^5*b^8 + 422*a^6*b^7 + 240*a^7*b^6 + 90*a^8*b^5
+ 20*a^9*b^4 + 2*a^10*b^3))/2 + ((-b^5*(a + b)^5)^(1/2)*(6*a^2*b^12 + 54*a^
3*b^11 + 218*a^4*b^10 + 520*a^5*b^9 + 812*a^6*b^8 + 868*a^7*b^7 + 644*a^8*b
^6 + 328*a^9*b^5 + 110*a^10*b^4 + 22*a^11*b^3 + 2*a^12*b^2 + (tan(e + f*x)*
(-b^5*(a + b)^5)^(1/2)*(16*a^2*b^13 + 168*a^3*b^12 + 800*a^4*b^11 + 2280*a^
5*b^10 + 4320*a^6*b^9 + 5712*a^7*b^8 + 5376*a^8*b^7 + 3600*a^9*b^6 + 1680*a
^10*b^5 + 520*a^11*b^4 + 96*a^12*b^3 + 8*a^13*b^2))/(4*a*(a + b)^5)))/(2*a*

```
(a + b)^5)))/(a*(a + b)^5)))*(-b^5*(a + b)^5)^(1/2)*1i)/(a*f*(a + b)^5)
```

3.349 $\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 120

$$\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{x}{a}+\frac{b^{7/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a(a+b)^{7/2}f}-\frac{(a^2+3ab+3b^2)\cot(e+fx)}{(a+b)^3f}+\frac{(a+2b)\cot^3(e+fx)}{3(a+b)^2f}-\frac{\cot^5(e+fx)}{5(a+b)f}$$

```
[Out] -x/a+b^(7/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a/(a+b)^(7/2)/f-(a^2+3*
a*b+3*b^2)*cot(f*x+e)/(a+b)^3/f+1/3*(a+2*b)*cot(f*x+e)^3/(a+b)^2/f-1/5*cot(
f*x+e)^5/(a+b)/f
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4226, 2000, 491, 597, 536, 209, 211}

$$\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{(a^2+3ab+3b^2)\cot(e+fx)}{f(a+b)^3}+\frac{b^{7/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{af(a+b)^{7/2}}-\frac{\cot^5(e+fx)}{5f(a+b)}+\frac{(a+2b)\cot^3(e+fx)}{3f(a+b)^2}-\frac{x}{a}$$

```
[In] Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(x/a) + (b^(7/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*(a + b)^(7
/2)*f) - ((a^2 + 3*a*b + 3*b^2)*Cot[e + f*x])/((a + b)^3*f) + ((a + 2*b)*Co
t[e + f*x]^3)/(3*(a + b)^2*f) - Cot[e + f*x]^5/(5*(a + b)*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+b(1+x^2))}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{\cot^5(e+fx)}{5(a+b)f} + \frac{\text{Subst}\left(\int \frac{-5(a+2b)-5bx^2}{x^4(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{5(a+b)f} \\
&= \frac{(a+2b)\cot^3(e+fx)}{3(a+b)^2f} - \frac{\cot^5(e+fx)}{5(a+b)f} \\
&\quad - \frac{\text{Subst}\left(\int \frac{-15(a^2+3ab+3b^2)-15b(a+2b)x^2}{x^2(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{15(a+b)^2f} \\
&= -\frac{(a^2+3ab+3b^2)\cot(e+fx)}{(a+b)^3f} + \frac{(a+2b)\cot^3(e+fx)}{3(a+b)^2f} - \frac{\cot^5(e+fx)}{5(a+b)f} \\
&\quad + \frac{\text{Subst}\left(\int \frac{-15(a+2b)(a^2+2ab+2b^2)-15b(a^2+3ab+3b^2)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{15(a+b)^3f} \\
&= -\frac{(a^2+3ab+3b^2)\cot(e+fx)}{(a+b)^3f} + \frac{(a+2b)\cot^3(e+fx)}{3(a+b)^2f} - \frac{\cot^5(e+fx)}{5(a+b)f} \\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{af} + \frac{b^4\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{a(a+b)^3f} \\
&= -\frac{x}{a} + \frac{b^{7/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a(a+b)^{7/2}f} - \frac{(a^2+3ab+3b^2)\cot(e+fx)}{(a+b)^3f} \\
&\quad + \frac{(a+2b)\cot^3(e+fx)}{3(a+b)^2f} - \frac{\cot^5(e+fx)}{5(a+b)f}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.36 (sec) , antiderivative size = 671, normalized size of antiderivative = 5.59

$$\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(-\frac{480b^4\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-i}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{}$$

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*((-480*b^4*ArcTan[(Sec[f*x]*
(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt
[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a +
b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + Csc[e]*Csc[e + f*x]^5*(-150*(a + b)^3*f
*x*Cos[f*x] + 150*(a + b)^3*f*x*Cos[2*e + f*x] + 75*a^3*f*x*Cos[2*e + 3*f*x
] + 225*a^2*b*f*x*Cos[2*e + 3*f*x] + 225*a*b^2*f*x*Cos[2*e + 3*f*x] + 75*b^
3*f*x*Cos[2*e + 3*f*x] - 75*a^3*f*x*Cos[4*e + 3*f*x] - 225*a^2*b*f*x*Cos[4*
e + 3*f*x] - 225*a*b^2*f*x*Cos[4*e + 3*f*x] - 75*b^3*f*x*Cos[4*e + 3*f*x] -
 15*a^3*f*x*Cos[4*e + 5*f*x] - 45*a^2*b*f*x*Cos[4*e + 5*f*x] - 45*a*b^2*f*x
*Cos[4*e + 5*f*x] - 15*b^3*f*x*Cos[4*e + 5*f*x] + 15*a^3*f*x*Cos[6*e + 5*f*
x] + 45*a^2*b*f*x*Cos[6*e + 5*f*x] + 45*a*b^2*f*x*Cos[6*e + 5*f*x] + 15*b^3
*f*x*Cos[6*e + 5*f*x] + 280*a^3*Sin[f*x] + 780*a^2*b*Sin[f*x] + 680*a*b^2*S
in[f*x] + 180*a^3*Sin[2*e + f*x] + 540*a^2*b*Sin[2*e + f*x] + 480*a*b^2*Sin
[2*e + f*x] - 140*a^3*Sin[2*e + 3*f*x] - 420*a^2*b*Sin[2*e + 3*f*x] - 400*a
*b^2*Sin[2*e + 3*f*x] - 90*a^3*Sin[4*e + 3*f*x] - 240*a^2*b*Sin[4*e + 3*f*x
] - 180*a*b^2*Sin[4*e + 3*f*x] + 46*a^3*Sin[4*e + 5*f*x] + 132*a^2*b*Sin[4*
e + 5*f*x] + 116*a*b^2*Sin[4*e + 5*f*x])))/(960*a*(a + b)^3*f*(a + b*Sec[e
+ f*x]^2))
```

Maple [A] (verified)

Time = 3.66 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{a}+\frac{b^4\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)^3 a\sqrt{(a+b)b}}-\frac{1}{5(a+b)\tan(fx+e)^5}-\frac{-a-2b}{3(a+b)^2\tan(fx+e)^3}-\frac{a^2+3ab+3b^2}{(a+b)^3\tan(fx+e)}}{f}$
default	$\frac{-\frac{\arctan(\tan(fx+e))}{a}+\frac{b^4\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)^3 a\sqrt{(a+b)b}}-\frac{1}{5(a+b)\tan(fx+e)^5}-\frac{-a-2b}{3(a+b)^2\tan(fx+e)^3}-\frac{a^2+3ab+3b^2}{(a+b)^3\tan(fx+e)}}{f}$
risch	$-\frac{x}{a}-\frac{2i\left(45a^2\mathrm{e}^{8i(fx+e)}+120ab\,\mathrm{e}^{8i(fx+e)}+90b^2\mathrm{e}^{8i(fx+e)}-90a^2\mathrm{e}^{6i(fx+e)}-270ab\,\mathrm{e}^{6i(fx+e)}-240b^2\mathrm{e}^{6i(fx+e)}+140a^2\mathrm{e}^{4i(}\right.}{15f(a+b)^3(\mathrm{e}^{2}}$

```
[In] int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a*arctan(tan(f*x+e))+1/(a+b)^3*b^4/a/((a+b)*b)^(1/2)*arctan(b*tan(f
*x+e)/((a+b)*b)^(1/2))-1/5/(a+b)/tan(f*x+e)^5-1/3*(-a-2*b)/(a+b)^2/tan(f*x+
e)^3-(a^2+3*a*b+3*b^2)/(a+b)^3/tan(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(108) = 216.

Time = 0.32 (sec) , antiderivative size = 833, normalized size of antiderivative = 6.94

$$\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \left[-\frac{4\,(23\,a^3+66\,a^2b+58\,ab^2)\cos{(fx+e)}^5-20\,(7\,a^3+21\,a^2b+20\,ab^2)\cos{(fx+e)}^3-15\,(b^3\cos{(fx+e)}\right.}{}\right.$$

$$-\frac{2\,(23\,a^3+66\,a^2b+58\,ab^2)\cos{(fx+e)}^5-10\,(7\,a^3+21\,a^2b+20\,ab^2)\cos{(fx+e)}^3+15\,\left(b^3\cos{(fx+e)}\right.}{}$$

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(4*(23*a^3 + 66*a^2*b + 58*a*b^2)*cos(f*x + e)^5 - 20*(7*a^3 + 21*a^
2*b + 20*a*b^2)*cos(f*x + e)^3 - 15*(b^3*cos(f*x + e)^4 - 2*b^3*cos(f*x + e
)^2 + b^3)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(
3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (
a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x
+ e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*(a^3 + 3*a^2*b + 3*
```

```
a*b^2)*cos(f*x + e) + 60*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*x*cos(f*x + e)^
4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*x*cos(f*x + e)^2 + (a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*f*x)*sin(f*x + e))/(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*
cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (
a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)*sin(f*x + e)), -1/30*(2*(23*a^3 + 66*
a^2*b + 58*a*b^2)*cos(f*x + e)^5 - 10*(7*a^3 + 21*a^2*b + 20*a*b^2)*cos(f*x
 + e)^3 + 15*(b^3*cos(f*x + e)^4 - 2*b^3*cos(f*x + e)^2 + b^3)*sqrt(b/(a +
b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x +
e)*sin(f*x + e)))*sin(f*x + e) + 30*(a^3 + 3*a^2*b + 3*a*b^2)*cos(f*x + e)
+ 30*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*x*cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*f*x*cos(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*x)
*sin(f*x + e))/(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - 2*(
a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^4 + 3*a^3*b + 3*a^
2*b^2 + a*b^3)*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx=\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

```
[In] integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
[Out] Integral(cot(e + f*x)**6/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.34

$$\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{15\,b^4\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^4+3\,a^3b+3\,a^2b^2+ab^3)\sqrt{(a+b)b}}-\frac{15\,(fx+e)}{a}-\frac{15\,(a^2+3\,ab+3\,b^2)\tan(fx+e)^4-5\,(a^2+3\,ab+2\,b^2)\tan(fx+e)^2+3\,a^2+6\,ab+3\,b^2}{(a^3+3\,a^2b+3\,ab^2+b^3)\tan(fx+e)^5}}{15\,f}$$

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] 1/15*(15*b^4*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4 + 3*a^3*b + 3*a^2
*b^2 + a*b^3)*sqrt((a + b)*b)) - 15*(f*x + e)/a - (15*(a^2 + 3*a*b + 3*b^2)
*tan(f*x + e)^4 - 5*(a^2 + 3*a*b + 2*b^2)*tan(f*x + e)^2 + 3*a^2 + 6*a*b +
3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(f*x + e)^5))/f
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.77

$$\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^4}{(a^4+3a^3b+3a^2b^2+ab^3)\sqrt{ab+b^2}} - \frac{15(fx+e)}{a} - \frac{15a^2\tan(fx+e)^4+45ab\tan(fx+e)^4+45b^2\tan(fx+e)^4-5a^2\tan(fx+e)^2-}{(a^3+3a^2b+3ab^2+b^3)\tan(f}}{15f}$$

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(
a*b + b^2)))*b^4/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b + b^2)) - 15
*(f*x + e)/a - (15*a^2*tan(f*x + e)^4 + 45*a*b*tan(f*x + e)^4 + 45*b^2*tan(
f*x + e)^4 - 5*a^2*tan(f*x + e)^2 - 15*a*b*tan(f*x + e)^2 - 10*b^2*tan(f*x
+ e)^2 + 3*a^2 + 6*a*b + 3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(f*x +
e)^5))/f
```

Mupad [B] (verification not implemented)

Time = 25.76 (sec) , antiderivative size = 4324, normalized size of antiderivative = 36.03

$$\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

```
[In] int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2),x)
```

```
[Out] (atan(((((tan(e + f*x)*(48*a*b^17 + 4*b^18 + 282*a^2*b^16 + 1078*a^3*b^15 +
 2982*a^4*b^14 + 6258*a^5*b^13 + 10178*a^6*b^12 + 12942*a^7*b^11 + 12888*a^
8*b^10 + 10012*a^9*b^9 + 6006*a^10*b^8 + 2730*a^11*b^7 + 910*a^12*b^6 + 210
*a^13*b^5 + 30*a^14*b^4 + 2*a^15*b^3))/2 - ((-b^7*(a + b)^7)^(1/2)*(8*a^2*b
^17 + 108*a^3*b^16 + 680*a^4*b^15 + 2650*a^5*b^14 + 7152*a^6*b^13 + 14168*a
^7*b^12 + 21296*a^8*b^11 + 24750*a^9*b^10 + 22440*a^10*b^9 + 15884*a^11*b^8
 + 8712*a^12*b^7 + 3638*a^13*b^6 + 1120*a^14*b^5 + 240*a^15*b^4 + 32*a^16*b
^3 + 2*a^17*b^2 - (tan(e + f*x)*(-b^7*(a + b)^7)^(1/2)*(16*a^2*b^18 + 248*a
^3*b^17 + 1800*a^4*b^16 + 8120*a^5*b^15 + 25480*a^6*b^14 + 58968*a^7*b^13 +
 104104*a^8*b^12 + 143000*a^9*b^11 + 154440*a^10*b^10 + 131560*a^11*b^9 + 8
8088*a^12*b^8 + 45864*a^13*b^7 + 18200*a^14*b^6 + 5320*a^15*b^5 + 1080*a^16
*b^4 + 136*a^17*b^3 + 8*a^18*b^2))/(4*a*(a + b)^7)))/(2*a*(a + b)^7))*(-b^7
*(a + b)^7)^(1/2)*1i)/(a*(a + b)^7) + (((tan(e + f*x)*(48*a*b^17 + 4*b^18 +
 282*a^2*b^16 + 1078*a^3*b^15 + 2982*a^4*b^14 + 6258*a^5*b^13 + 10178*a^6*b
^12 + 12942*a^7*b^11 + 12888*a^8*b^10 + 10012*a^9*b^9 + 6006*a^10*b^8 + 273
```

0*a^11*b^7 + 910*a^12*b^6 + 210*a^13*b^5 + 30*a^14*b^4 + 2*a^15*b^3))/2 + (
(-b^7*(a + b)^7)^(1/2)*(8*a^2*b^17 + 108*a^3*b^16 + 680*a^4*b^15 + 2650*a^5
*b^14 + 7152*a^6*b^13 + 14168*a^7*b^12 + 21296*a^8*b^11 + 24750*a^9*b^10 +
22440*a^10*b^9 + 15884*a^11*b^8 + 8712*a^12*b^7 + 3638*a^13*b^6 + 1120*a^14
*b^5 + 240*a^15*b^4 + 32*a^16*b^3 + 2*a^17*b^2 + (tan(e + f*x)*(-b^7*(a + b
)^7)^(1/2)*(16*a^2*b^18 + 248*a^3*b^17 + 1800*a^4*b^16 + 8120*a^5*b^15 + 25
480*a^6*b^14 + 58968*a^7*b^13 + 104104*a^8*b^12 + 143000*a^9*b^11 + 154440*
a^10*b^10 + 131560*a^11*b^9 + 88088*a^12*b^8 + 45864*a^13*b^7 + 18200*a^14*
b^6 + 5320*a^15*b^5 + 1080*a^16*b^4 + 136*a^17*b^3 + 8*a^18*b^2))/(4*a*(a +
b)^7)))/(2*a*(a + b)^7))*(-b^7*(a + b)^7)^(1/2)*1i)/(a*(a + b)^7))/(60*a*b
^16 + 6*b^17 + 272*a^2*b^15 + 738*a^3*b^14 + 1332*a^4*b^13 + 1680*a^5*b^12
+ 1512*a^6*b^11 + 972*a^7*b^10 + 438*a^8*b^9 + 132*a^9*b^8 + 24*a^10*b^7 +
2*a^11*b^6 + (((tan(e + f*x)*(48*a*b^17 + 4*b^18 + 282*a^2*b^16 + 1078*a^3*
b^15 + 2982*a^4*b^14 + 6258*a^5*b^13 + 10178*a^6*b^12 + 12942*a^7*b^11 + 12
888*a^8*b^10 + 10012*a^9*b^9 + 6006*a^10*b^8 + 2730*a^11*b^7 + 910*a^12*b^6
+ 210*a^13*b^5 + 30*a^14*b^4 + 2*a^15*b^3))/2 - ((-b^7*(a + b)^7)^(1/2)*(8
*a^2*b^17 + 108*a^3*b^16 + 680*a^4*b^15 + 2650*a^5*b^14 + 7152*a^6*b^13 + 1
4168*a^7*b^12 + 21296*a^8*b^11 + 24750*a^9*b^10 + 22440*a^10*b^9 + 15884*a^
11*b^8 + 8712*a^12*b^7 + 3638*a^13*b^6 + 1120*a^14*b^5 + 240*a^15*b^4 + 32*
a^16*b^3 + 2*a^17*b^2 - (tan(e + f*x)*(-b^7*(a + b)^7)^(1/2)*(16*a^2*b^18 +
248*a^3*b^17 + 1800*a^4*b^16 + 8120*a^5*b^15 + 25480*a^6*b^14 + 58968*a^7*
b^13 + 104104*a^8*b^12 + 143000*a^9*b^11 + 154440*a^10*b^10 + 131560*a^11*b
^9 + 88088*a^12*b^8 + 45864*a^13*b^7 + 18200*a^14*b^6 + 5320*a^15*b^5 + 108
0*a^16*b^4 + 136*a^17*b^3 + 8*a^18*b^2))/(4*a*(a + b)^7)))/(2*a*(a + b)^7))
(-b^7(a + b)^7)^(1/2))/(a*(a + b)^7) - (((tan(e + f*x)*(48*a*b^17 + 4*b^1
8 + 282*a^2*b^16 + 1078*a^3*b^15 + 2982*a^4*b^14 + 6258*a^5*b^13 + 10178*a^
6*b^12 + 12942*a^7*b^11 + 12888*a^8*b^10 + 10012*a^9*b^9 + 6006*a^10*b^8 +
2730*a^11*b^7 + 910*a^12*b^6 + 210*a^13*b^5 + 30*a^14*b^4 + 2*a^15*b^3))/2
+ ((-b^7*(a + b)^7)^(1/2)*(8*a^2*b^17 + 108*a^3*b^16 + 680*a^4*b^15 + 2650*
a^5*b^14 + 7152*a^6*b^13 + 14168*a^7*b^12 + 21296*a^8*b^11 + 24750*a^9*b^10
+ 22440*a^10*b^9 + 15884*a^11*b^8 + 8712*a^12*b^7 + 3638*a^13*b^6 + 1120*a
^14*b^5 + 240*a^15*b^4 + 32*a^16*b^3 + 2*a^17*b^2 + (tan(e + f*x)*(-b^7*(a
+ b)^7)^(1/2)*(16*a^2*b^18 + 248*a^3*b^17 + 1800*a^4*b^16 + 8120*a^5*b^15 +
25480*a^6*b^14 + 58968*a^7*b^13 + 104104*a^8*b^12 + 143000*a^9*b^11 + 1544
40*a^10*b^10 + 131560*a^11*b^9 + 88088*a^12*b^8 + 45864*a^13*b^7 + 18200*a^
14*b^6 + 5320*a^15*b^5 + 1080*a^16*b^4 + 136*a^17*b^3 + 8*a^18*b^2))/(4*a*(
a + b)^7)))/(2*a*(a + b)^7))*(-b^7*(a + b)^7)^(1/2))/(a*(a + b)^7)))*(-b^7*
(a + b)^7)^(1/2)*1i)/(a*f*(a + b)^7) - (1/(5*(a + b)) + (tan(e + f*x)^4*(3*
a*b + a^2 + 3*b^2))/(a + b)^3 - (tan(e + f*x)^2*(a + 2*b))/(3*(a + b)^2))/(
f*tan(e + f*x)^5) - atan((14*b^17*tan(e + f*x))/(168*a*b^16 + 14*b^17 + 952
*a^2*b^15 + 3388*a^3*b^14 + 8484*a^4*b^13 + 15848*a^5*b^12 + 22808*a^6*b^11
+ 25722*a^7*b^10 + 22878*a^8*b^9 + 16016*a^9*b^8 + 8736*a^10*b^7 + 3640*a^
11*b^6 + 1120*a^12*b^5 + 240*a^13*b^4 + 32*a^14*b^3 + 2*a^15*b^2) + (952*a^
2*b^15*tan(e + f*x))/(168*a*b^16 + 14*b^17 + 952*a^2*b^15 + 3388*a^3*b^14 +
8484*a^4*b^13 + 15848*a^5*b^12 + 22808*a^6*b^11 + 25722*a^7*b^10 + 22878*a

^8*b^9 + 16016*a^9*b^8 + 8736*a^10*b^7 + 3640*a^11*b^6 + 1120*a^12*b^5 + 240*a^13*b^4 + 32*a^14*b^3 + 2*a^15*b^2) + (3388*a^3*b^14*tan(e + f*x))/(168*a*b^16 + 14*b^17 + 952*a^2*b^15 + 3388*a^3*b^14 + 8484*a^4*b^13 + 15848*a^5*b^12 + 22808*a^6*b^11 + 25722*a^7*b^10 + 22878*a^8*b^9 + 16016*a^9*b^8 + 8736*a^10*b^7 + 3640*a^11*b^6 + 1120*a^12*b^5 + 240*a^13*b^4 + 32*a^14*b^3 + 2*a^15*b^2) + (8484*a^4*b^13*tan(e + f*x))/(168*a*b^16 + 14*b^17 + 952*a^2*b^15 + 3388*a^3*b^14 + 8484*a^4*b^13 + 15848*a^5*b^12 + 22808*a^6*b^11 + 25722*a^7*b^10 + 22878*a^8*b^9 + 16016*a^9*b^8 + 8736*a^10*b^7 + 3640*a^11*b^6 + 1120*a^12*b^5 + 240*a^13*b^4 + 32*a^14*b^3 + 2*a^15*b^2) + (15848*a^5*b^12*tan(e + f*x))/(168*a*b^16 + 14*b^17 + 952*a^2*b^15 + 3388*a^3*b^14 + 8484*a^4*b^13 + 15848*a^5*b^12 + 22808*a^6*b^11 + 25722*a^7*b^10 + 22878*a^8*b^9 + 16016*a^9*b^8 + 8736*a^10*b^7 + 3640*a^11*b^6 + 1120*a^12*b^5 + 240*a^13*b^4 + 32*a^14*b^3 + 2*a^15*b^2) + (22808*a^6*b^11*tan(e + f*x))/(168*a*b^16 + 14*b^17 + 952*a^2*b^15 + 3388*a^3*b^14 + 8484*a^4*b^13 + 15848*a^5*b^12 + 22808*a^6*b^11 + 25722*a^7*b^10 + 22878*a^8*b^9 + 16016*a^9*b^8 + 8736*a^10*b^7 + 3640*a^11*b^6 + 1120*a^12*b^5 + 240*a^13*b^4 + 32*a^14*b^3 + 2*a^15*b^2) + (25722*a^7*b^10*tan(e + f*x))/(168*a*b^16 + 14*b^17 + 952*a^2*b^15 + 3388*a^3*b^14 + 8484*a^4*b^13 + 15848*a^5*b^12 + 22808*a^6*b^11 + 25722*a^7*b^10 + 22878*a^8*b^9 + 16016*a^9*b^8 + 8736*a^10*b^7 + 3640*a^11*b^6 + 1120*a^12*b^5 + 240*a^13*b^4 + 32*a^14*b^3 + 2*a^15*b^2) + (22878*a^8*b^9*tan(e + f*x))/(168*a*b^16 + 14*b^17 + 952*a^2*b^15 + 3388*a^3*b^14 + 8484*a^4*b^13 + 15848*a^5*b^12 + 22808*a^6*b^11 + 25722*a^7*b^10 + 22878*a^8*b^9 + 16016*a^9*b^8 + 8736*a^10*b^7 + 3640*a^11*b^6 + 1120*a^12*b^5 + 240*a^13*b^4 + 32*a^14*b^3 + 2*a^15*b^2) + (16016*a^9*b^8*tan(e + f*x))/(168*a*b^16 + 14*b^17 + 952*a^2*b^15 + 3388*a^3*b^14 + 8484*a^4*b^13 + 15848*a^5*b^12 + 22808*a^6*b^11 + 25722*a^7*b^10 + 22878*a^8*b^9 + 16016*a^9*b^8 + 8736*a^10*b^7 + 3640*a^11*b^6 + 1120*a^12*b^5 + 240*a^13*b^4 + 32*a^14*b^3 + 2*a^15*b^2) + (8736*a^10*b^7*tan(e + f*x))/(168*a*b^16 + 14*b^17 + 952*a^2*b^15 + 3388*a^3*b^14 + 8484*a^4*b^13 + 15848*a^5*b^12 + 22808*a^6*b^11 + 25722*a^7*b^10 + 22878*a^8*b^9 + 16016*a^9*b^8 + 8736*a^10*b^7 + 3640*a^11*b^6 + 1120*a^12*b^5 + 240*a^13*b^4 + 32*a^14*b^3 + 2*a^15*b^2) + (3640*a^11*b^6*tan(e + f*x))/(168*a*b^16 + 14*b^17 + 952*a^2*b^15 + 3388*a^3*b^14 + 8484*a^4*b^13 + 15848*a^5*b^12 + 22808*a^6*b^11 + 25722*a^7*b^10 + 22878*a^8*b^9 + 16016*a^9*b^8 + 8736*a^10*b^7 + 3640*a^11*b^6 + 1120*a^12*b^5 + 240*a^13*b^4 + 32*a^14*b^3 + 2*a^15*b^2) + (1120*a^12*b^5*tan(e + f*x))/(168*a*b^16 + 14*b^17 + 952*a^2*b^15 + 3388*a^3*b^14 + 8484*a^4*b^13 + 15848*a^5*b^12 + 22808*a^6*b^11 + 25722*a^7*b^10 + 22878*a^8*b^9 + 16016*a^9*b^8 + 8736*a^10*b^7 + 3640*a^11*b^6 + 1120*a^12*b^5 + 240*a^13*b^4 + 32*a^14*b^3 + 2*a^15*b^2) + (240*a^13*b^4*tan(e + f*x))/(168*a*b^16 + 14*b^17 + 952*a^2*b^15 + 3388*a^3*b^14 + 8484*a^4*b^13 + 15848*a^5*b^12 + 22808*a^6*b^11 + 25722*a^7*b^10 + 22878*a^8*b^9 + 16016*a^9*b^8 + 8736*a^10*b^7 + 3640*a^11*b^6 + 1120*a^12*b^5 + 240*a^13*b^4 + 32*a^14*b^3 + 2*a^15*b^2) + (32*a^14*b^3*tan(e + f*x))/(168*a*b^16 + 14*b^17 + 952*a^2*b^15 + 3388*a^3*b^14 + 8484*a^4*b^13 + 15848*a^5*b^12 + 22808*a^6*b^11 + 25722*a^7*b^10 + 22878*a^8*b^9 + 16016*a^9*b^8 + 8736*a^10*b^7 + 3640*a^11*b^6 + 1120*a^12*b^5 + 240*a^13*b^4 +

```
32*a^14*b^3 + 2*a^15*b^2) + (2*a^15*b^2*tan(e + f*x))/(168*a*b^16 + 14*b^1
7 + 952*a^2*b^15 + 3388*a^3*b^14 + 8484*a^4*b^13 + 15848*a^5*b^12 + 22808*a
^6*b^11 + 25722*a^7*b^10 + 22878*a^8*b^9 + 16016*a^9*b^8 + 8736*a^10*b^7 +
3640*a^11*b^6 + 1120*a^12*b^5 + 240*a^13*b^4 + 32*a^14*b^3 + 2*a^15*b^2) +
(168*a*b^16*tan(e + f*x))/(168*a*b^16 + 14*b^17 + 952*a^2*b^15 + 3388*a^3*b
^14 + 8484*a^4*b^13 + 15848*a^5*b^12 + 22808*a^6*b^11 + 25722*a^7*b^10 + 22
878*a^8*b^9 + 16016*a^9*b^8 + 8736*a^10*b^7 + 3640*a^11*b^6 + 1120*a^12*b^5
 + 240*a^13*b^4 + 32*a^14*b^3 + 2*a^15*b^2))/(a*f)
```

3.350 $\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{(a+b)^2}{2a^2bf\,(b+a\cos^2(e+fx))}-\frac{\log(\cos(e+fx))}{b^2f}-\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(b+a\cos^2(e+fx)\right)}{2f}$$

```
[Out] -1/2*(a+b)^2/a^2/b/f/(b+a*cos(f*x+e)^2)-ln(cos(f*x+e))/b^2/f-1/2*(1/a^2-1/b
^2)*ln(b+a*cos(f*x+e)^2)/f
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4223, 457, 90}

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(a\cos^2(e+fx)+b\right)}{2f}-\frac{(a+b)^2}{2a^2bf\,(a\cos^2(e+fx)+b)}-\frac{\log(\cos(e+fx))}{b^2f}$$

```
[In] Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/2*(a + b)^2/(a^2*b*f*(b + a*Cos[e + f*x]^2)) - Log[Cos[e + f*x]]/(b^2*f)
 - ((a^(-2) - b^(-2))*Log[b + a*Cos[e + f*x]^2])/(2*f)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \frac{(1-x)^2}{x(b+ax)^2}\,dx, x, \cos^2(e+fx)\right)}{2f}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{b^2x} - \frac{(a+b)^2}{ab(b+ax)^2} + \frac{-a^2+b^2}{ab^2(b+ax)}\right)dx, x, \cos^2(e+fx)\right)}{2f}\\ &= -\frac{(a+b)^2}{2a^2bf\left(b+a\cos^2(e+fx)\right)} - \frac{\log(\cos(e+fx))}{b^2f} - \frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(b+a\cos^2(e+fx)\right)}{2f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.42

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{(a+2b+a\cos(2e+2fx))^2\left(\frac{(a+b)^2}{a^2b(b+a\cos^2(e+fx))} + \frac{2\log(\cos(e+fx))}{b^2} + \left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(b+a\cos^2(e+fx)\right)\right)\sec$$
$$8f\left(a+b\sec^2(e+fx)\right)^2$$

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/8*((a + 2*b + a*Cos[2*e + 2*f*x])^2*((a + b)^2/(a^2*b*(b + a*Cos[e + f*x
]^2)) + (2*Log[Cos[e + f*x]])/b^2 + (a^(-2) - b^(-2))*Log[b + a*Cos[e + f*x
]^2])*Sec[e + f*x]^4)/(f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] (verified)

Time = 6.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\frac{(a+b)\left(\frac{(a-b)\ln\left(b+a\cos(fx+e)^2\right)}{a^2}-\frac{(a+b)b}{a^2\left(b+a\cos(fx+e)^2\right)}\right)}{2b^2}-\frac{\ln(\cos(fx+e))}{b^2}}{f}$
default	$\frac{\frac{(a+b)\left(\frac{(a-b)\ln\left(b+a\cos(fx+e)^2\right)}{a^2}-\frac{(a+b)b}{a^2\left(b+a\cos(fx+e)^2\right)}\right)}{2b^2}-\frac{\ln(\cos(fx+e))}{b^2}}{f}$
risch	$\frac{ix}{a^2}+\frac{2ie}{a^2f}-\frac{2(a^2+2ab+b^2)\mathrm{e}^{2i(fx+e)}}{a^2bf\left(a\,\mathrm{e}^{4i(fx+e)}+2a\,\mathrm{e}^{2i(fx+e)}+4b\,\mathrm{e}^{2i(fx+e)}+a\right)}-\frac{\ln(\mathrm{e}^{2i(fx+e)}+1)}{b^2f}+\frac{\ln\left(\mathrm{e}^{4i(fx+e)}+\frac{2(a+2b)\mathrm{e}^{2i(fx+e}}{a}\right.}{2b^2f}$

```
[In] int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2/b^2*(a+b)*((a-b)/a^2*ln(b+a*cos(f*x+e)^2)-(a+b)*b/a^2/(b+a*cos(f*x
+e)^2))-1/b^2*ln(cos(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.53

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{a^2b+2\,ab^2+b^3-\left(a^2b-b^3+(a^3-ab^2)\cos\left(fx+e\right)^2\right)\log\left(a\cos\left(fx+e\right)^2+b\right)+2\left(a^3\cos\left(fx+e\right)^2+\right.}{2\left(a^3b^2f\cos\left(fx+e\right)^2+a^2b^3f\right)}$$

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*b + 2*a*b^2 + b^3 - (a^2*b - b^3 + (a^3 - a*b^2)*cos(f*x + e)^2)*
log(a*cos(f*x + e)^2 + b) + 2*(a^3*cos(f*x + e)^2 + a^2*b)*log(-cos(f*x + e
)))/(a^3*b^2*f*cos(f*x + e)^2 + a^2*b^3*f)
```

Sympy [F]

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

```
[In] integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(tan(e + f*x)**5/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.27

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{a^2+2ab+b^2}{a^3b\sin(fx+e)^2-a^3b-a^2b^2} - \frac{\log\left(\sin(fx+e)^2-1\right)}{b^2} + \frac{(a^2-b^2)\log\left(a\sin(fx+e)^2-a-b\right)}{a^2b^2}}{2f}$$

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2 + 2*a*b + b^2)/(a^3*b*sin(f*x + e)^2 - a^3*b - a^2*b^2) - log(sin
(f*x + e)^2 - 1)/b^2 + (a^2 - b^2)*log(a*sin(f*x + e)^2 - a - b)/(a^2*b^2))
/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 532 vs. 2(73) = 146.

Time = 1.65 (sec) , antiderivative size = 532, normalized size of antiderivative = 6.91

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{(a^3+a^2b-ab^2-b^3)\log\left(\left|-a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-b\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-2a+2b\right|\right)}{a^3b^2+a^2b^3} + \frac{\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right)}{a^2}}{}$$

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((a^3 + a^2*b - a*b^2 - b^3)*log(abs(-a*((cos(f*x + e) + 1)/(cos(f*x +
e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - b*((cos(f*x + e) + 1)/(c
os(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 2*a + 2*b))/(a^
3*b^2 + a^2*b^3) + log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*
```

```
x + e) - 1)/(cos(f*x + e) + 1) + 2))/a^2 - log(abs(-(cos(f*x + e) + 1)/(cos
(f*x + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2))/b^2 - (a^3*((c
os(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)
) + a^2*b*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(
f*x + e) + 1)) - a*b^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x +
e) - 1)/(cos(f*x + e) + 1)) - b^3*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) +
(cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 2*a^3 - 6*a^2*b - 6*a*b^2 + 2*b^3)
/((a*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x +
 e) + 1)) + b*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(
cos(f*x + e) + 1)) + 2*a - 2*b)*a^2*b^2))/f
```

Mupad [B] (verification not implemented)

Time = 20.34 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.21

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\ln\left(b\tan(e+f\,x)^2+a+b\right)}{2\,b^2\,f} - \frac{\ln\left(b\tan(e+f\,x)^2+a+b\right)}{2\,a^2\,f} + \frac{a^2}{2\,f\,\left(a^2\,b^2+a\,b^3\tan\left(e+f\,x\right)^2+a\,b^3\right)} + \frac{b^2}{2\,f\,\left(a^2\,b^2+a\,b^3\tan\left(e+f\,x\right)^2+a\,b^3\right)} + \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a^2\,f} + \frac{a\,b}{f\,\left(a^2\,b^2+a\,b^3\tan\left(e+f\,x\right)^2+a\,b^3\right)}$$

```
[In] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] log(a + b + b*tan(e + f*x)^2)/(2*b^2*f) - log(a + b + b*tan(e + f*x)^2)/(2*
a^2*f) + a^2/(2*f*(a*b^3 + a^2*b^2 + a*b^3*tan(e + f*x)^2)) + b^2/(2*f*(a*b
^3 + a^2*b^2 + a*b^3*tan(e + f*x)^2)) + log(tan(e + f*x)^2 + 1)/(2*a^2*f) +
 (a*b)/(f*(a*b^3 + a^2*b^2 + a*b^3*tan(e + f*x)^2))
```

3.351 $\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 51

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{a+b}{2a^2f\,(b+a\cos^2(e+fx))}+\frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^2f}$$

```
[Out] 1/2*(a+b)/a^2/f/(b+a*cos(f*x+e)^2)+1/2*ln(b+a*cos(f*x+e)^2)/a^2/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4223, 455, 45}

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{a+b}{2a^2f\,(a\cos^2(e+fx)+b)}+\frac{\log\left(a\cos^2(e+fx)+b\right)}{2a^2f}$$

```
[In] Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (a + b)/(2*a^2*f*(b + a*Cos[e + f*x]^2)) + Log[b + a*Cos[e + f*x]^2]/(2*a^2
*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x(1-x^2)}{(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \frac{1-x}{(b+ax)^2}\,dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a+b}{a(b+ax)^2}-\frac{1}{a(b+ax)}\right)\,dx, x, \cos^2(e+fx)\right)}{2f}\\
&= \frac{a+b}{2a^2f\left(b+a\cos^2(e+fx)\right)}+\frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^2f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.59

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{2(a+b)+(a+2b)\log(a+2b+a\cos(2(e+fx)))+a\cos(2(e+fx))\log(a+2b+a\cos(2(e+fx)))}{2a^2f(a+2b+a\cos(2(e+fx)))}$$

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (2*(a + b) + (a + 2*b)*Log[a + 2*b + a*Cos[2*(e + f*x)]] + a*Cos[2*(e + f*x
)]*Log[a + 2*b + a*Cos[2*(e + f*x)]])/(2*a^2*f*(a + 2*b + a*Cos[2*(e + f*x)
]))
```

Maple [A] (verified)

Time = 3.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{\frac{\ln\left(b+a\cos(fx+e)^2\right)}{2a^2}-\frac{-a-b}{2a^2\left(b+a\cos(fx+e)^2\right)}}{f}$	50
default	$\frac{\frac{\ln\left(b+a\cos(fx+e)^2\right)}{2a^2}-\frac{-a-b}{2a^2\left(b+a\cos(fx+e)^2\right)}}{f}$	50
risch	$-\frac{ix}{a^2}-\frac{2ie}{a^2f}+\frac{2(a+b)e^{2i(fx+e)}}{a^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{\ln\left(e^{4i(fx+e)}+\frac{2(a+2b)e^{2i(fx+e)}}{a}+1\right)}{2a^2f}$	117

```
[In] int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/f*(1/2/a^2*ln(b+a*cos(f*x+e)^2)-1/2*(-a-b)/a^2/(b+a*cos(f*x+e)^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx=\frac{\left(a\cos\left(fx+e\right)^2+b\right)\log\left(a\cos\left(fx+e\right)^2+b\right)+a+b}{2\left(a^3f\cos\left(fx+e\right)^2+a^2bf\right)}$$

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
[Out] 1/2*((a*cos(f*x + e)^2 + b)*log(a*cos(f*x + e)^2 + b) + a + b)/(a^3*f*cos(f
*x + e)^2 + a^2*b*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{a+b}{a^3\sin(fx+e)^2-a^3-a^2b}-\frac{\log\left(a\sin(fx+e)^2-a-b\right)}{a^2}}{2f}$$

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((a + b)/(a^3*sin(f*x + e)^2 - a^3 - a^2*b) - log(a*sin(f*x + e)^2 - a
 - b)/a^2)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(47) = 94.

Time = 0.72 (sec) , antiderivative size = 314, normalized size of antiderivative = 6.16

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$= \frac{\frac{\log\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^2}-\frac{2\,\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a^2}-\frac{a+b+\frac{6\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\cdots}{\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\cdots\right.}}{2f}$$

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x +
 e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 +
 b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/a^2 - 2*log(abs(-(cos(f*x + e
) - 1)/(cos(f*x + e) + 1) + 1))/a^2 - (a + b + 6*a*(cos(f*x + e) - 1)/(cos(
f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e)
 - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)
/((a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) -
1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(co
s(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*a^2))/f
```

Mupad [B] (verification not implemented)

Time = 20.32 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.90

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\operatorname{atanh}\left(\frac{4\,b^2\,\tan(e+f\,x)^2}{8\,b^2+\frac{8\,b^3}{a}+4\,b^2\,\tan(e+f\,x)^2+\frac{8\,b^3\,\tan(e+f\,x)^2}{a}}\right)}{a^2\,f} - \frac{a+b}{2\,a\,b\,f\,\left(b\tan\left(e+f\,x\right)^2+a+b\right)}$$

```
[In] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] - atanh((4*b^2*tan(e + f*x)^2)/(8*b^2 + (8*b^3)/a + 4*b^2*tan(e + f*x)^2 +
(8*b^3*tan(e + f*x)^2)/a))/(a^2*f) - (a + b)/(2*a*b*f*(a + b + b*tan(e + f*
x)^2))
```

3.352 $\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^2} dx = -\frac{b}{2a^2f(b+a\cos^2(e+fx))} - \frac{\log(b+a\cos^2(e+fx))}{2a^2f}$$

```
[Out] -1/2*b/a^2/f/(b+a*cos(f*x+e)^2)-1/2*ln(b+a*cos(f*x+e)^2)/a^2/f
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4223, 272, 45}

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^2} dx = -\frac{b}{2a^2f(a\cos^2(e+fx)+b)} - \frac{\log(a\cos^2(e+fx)+b)}{2a^2f}$$

```
[In] Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/2*b/(a^2*f*(b + a*Cos[e + f*x]^2)) - Log[b + a*Cos[e + f*x]^2]/(2*a^2*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^3}{(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \frac{x}{(b+ax)^2}\,dx, x, \cos^2(e+fx)\right)}{2f}\\ &= -\frac{\text{Subst}\left(\int \left(-\frac{b}{a(b+ax)^2}+\frac{1}{a(b+ax)}\right)\,dx, x, \cos^2(e+fx)\right)}{2f}\\ &= -\frac{b}{2a^2f\left(b+a\cos^2(e+fx)\right)}-\frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^2f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.61

$$\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{2b+(a+2b)\log(a+2b+a\cos(2(e+fx)))+a\cos(2(e+fx))\log(a+2b+a\cos(2(e+fx)))}{2a^2f(a+2b+a\cos(2(e+fx)))}$$

```
[In] Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/2*(2*b + (a + 2*b)*Log[a + 2*b + a*Cos[2*(e + f*x)]] + a*Cos[2*(e + f*x)
]*Log[a + 2*b + a*Cos[2*(e + f*x)]])/(a^2*f*(a + 2*b + a*Cos[2*(e + f*x)]))
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$-\frac{\ln\left(a+b\sec(fx+e)^{2}\right)}{2f\,a^{2}}+\frac{1}{2fa\left(a+b\sec(fx+e)^{2}\right)}+\frac{\ln(\sec(fx+e))}{f\,a^{2}}$	59
default	$-\frac{\ln\left(a+b\sec(fx+e)^{2}\right)}{2f\,a^{2}}+\frac{1}{2fa\left(a+b\sec(fx+e)^{2}\right)}+\frac{\ln(\sec(fx+e))}{f\,a^{2}}$	59
risch	$\frac{ix}{a^{2}}+\frac{2ie}{a^{2}f}-\frac{2b\,e^{2i(fx+e)}}{a^{2}f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}-\frac{\ln\left(e^{4i(fx+e)}+\frac{2(a+2b)e^{2i(fx+e)}}{a}+1\right)}{2a^{2}f}$	115

```
[In] int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
[Out] -1/2/f/a^2*ln(a+b*sec(f*x+e)^2)+1/2/f/a/(a+b*sec(f*x+e)^2)+1/f/a^2*ln(sec(f
*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\left(a\cos\left(fx+e\right)^2+b\right)\log\left(a\cos\left(fx+e\right)^2+b\right)+b}{2\left(a^3f\cos\left(fx+e\right)^2+a^2bf\right)}$$

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
[Out] -1/2*((a*cos(f*x + e)^2 + b)*log(a*cos(f*x + e)^2 + b) + b)/(a^3*f*cos(f*x
+ e)^2 + a^2*b*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{b}{a^3\sin(fx+e)^2-a^3-a^2b} - \frac{\log\left(a\sin(fx+e)^2-a-b\right)}{a^2}}{2\,f}$$

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b/(a^3*sin(f*x + e)^2 - a^3 - a^2*b) - log(a*sin(f*x + e)^2 - a - b)/a
^2)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(45) = 90$.

Time = 0.43 (sec) , antiderivative size = 384, normalized size of antiderivative = 7.84

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \frac{\frac{a^2+2ab+b^2+\frac{2a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{4ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{2ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{(a^3+a^2b)\left(a+b+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)} - \frac{\log\left(a+b+\cdots\right.}{\cdots}}{2\,f}$$

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((a^2 + 2*a*b + b^2 + 2*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 4*a
*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b^2*(cos(f*x + e) - 1)/(cos(f*
x + e) + 1) + a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 2*a*b*(cos(f*
x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e)
 + 1)^2)/((a^3 + a^2*b)*(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1)
- 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f
*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)) - log(a + b
+ 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f
*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e
) - 1)^2/(cos(f*x + e) + 1)^2)/a^2 + 2*log(abs(-(cos(f*x + e) - 1)/(cos(f*x
 + e) + 1) + 1))/a^2)/f
```

Mupad [B] (verification not implemented)

Time = 19.57 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.84

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\operatorname{atanh}\left(\frac{4\,b^2\,\tan(e+f\,x)^2}{8\,b^2+\frac{8\,b^3}{a}+4\,b^2\,\tan(e+f\,x)^2+\frac{8\,b^3\,\tan(e+f\,x)^2}{a}}\right)}{a^2\,f} + \frac{1}{2\,a\,f\,\left(b\tan\left(e+f\,x\right)^2+a+b\right)}$$

[In] int(tan(e + f*x)/(a + b/cos(e + f*x)^2)^2,x)

[Out] atanh((4*b^2*tan(e + f*x)^2)/(8*b^2 + (8*b^3)/a + 4*b^2*tan(e + f*x)^2 + (8*b^3*tan(e + f*x)^2)/a))/(a^2*f) + 1/(2*a*f*(a + b + b*tan(e + f*x)^2))

3.353 $\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{b^2}{2a^2(a+b)f\,(b+a\cos^2(e+fx))} + \frac{b(2a+b)\log\left(b+a\cos^2(e+fx)\right)}{2a^2(a+b)^2f} + \frac{\log(\sin(e+fx))}{(a+b)^2f}$$

```
[Out] 1/2*b^2/a^2/(a+b)/f/(b+a*cos(f*x+e)^2)+1/2*b*(2*a+b)*ln(b+a*cos(f*x+e)^2)/a
^2/(a+b)^2/f+ln(sin(f*x+e))/(a+b)^2/f
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4223, 457, 90}

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{b^2}{2a^2f(a+b)\left(a\cos^2(e+fx)+b\right)} + \frac{b(2a+b)\log\left(a\cos^2(e+fx)+b\right)}{2a^2f(a+b)^2} + \frac{\log(\sin(e+fx))}{f(a+b)^2}$$

```
[In] Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] b^2/(2*a^2*(a + b)*f*(b + a*Cos[e + f*x]^2)) + (b*(2*a + b)*Log[b + a*Cos[e
 + f*x]^2])/(2*a^2*(a + b)^2*f) + Log[Sin[e + f*x]]/((a + b)^2*f)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^5}{(1-x^2)(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{f} \\
&= -\frac{\text{Subst}\left(\int \frac{x^2}{(1-x)(b+ax)^2}\,dx, x, \cos^2(e+fx)\right)}{2f} \\
&= -\frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)} + \frac{b^2}{a(a+b)(b+ax)^2} - \frac{b(2a+b)}{a(a+b)^2(b+ax)}\right)\,dx, x, \cos^2(e+fx)\right)}{2f} \\
&= \frac{b^2}{2a^2(a+b)f\left(b+a\cos^2(e+fx)\right)} + \frac{b(2a+b)\log\left(b+a\cos^2(e+fx)\right)}{2a^2(a+b)^2f} + \frac{\log(\sin(e+fx))}{(a+b)^2f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.35

$$\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{(a+2b+a\cos(2(e+fx)))^2\sec^4(e+fx)\left(2\log(\sin(e+fx)) + \frac{b(2a+b)\log\left(a+b-a\sin^2(e+fx)\right)}{a^2} + \frac{b^2(a+b)}{a^2(a+b-a\sin^2(e+f}\right.}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^2}$$

```
[In] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x]^4*(2*Log[Sin[e + f*x]] + (b*
(2*a + b)*Log[a + b - a*Sin[e + f*x]^2])/a^2 + (b^2*(a + b))/(a^2*(a + b -
a*Sin[e + f*x]^2))))/(8*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] (verified)

Time = 3.66 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{\frac{\ln(-1+\cos(fx+e))}{2(a+b)^2}+\frac{b\left(\frac{(a+b)b}{a^2\left(b+a\cos(fx+e)^2\right)}+\frac{(2a+b)\ln\left(b+a\cos(fx+e)^2\right)}{a^2}\right)}{2(a+b)^2}+\frac{\ln(1+\cos(fx+e))}{2(a+b)^2}}{f}$
default	$\frac{\frac{\ln(-1+\cos(fx+e))}{2(a+b)^2}+\frac{b\left(\frac{(a+b)b}{a^2\left(b+a\cos(fx+e)^2\right)}+\frac{(2a+b)\ln\left(b+a\cos(fx+e)^2\right)}{a^2}\right)}{2(a+b)^2}+\frac{\ln(1+\cos(fx+e))}{2(a+b)^2}}{f}$
risch	$\frac{ix}{a^2}-\frac{2ix}{a^2+2ab+b^2}-\frac{2ie}{f(a^2+2ab+b^2)}-\frac{4ibx}{a(a^2+2ab+b^2)}-\frac{4ibe}{af(a^2+2ab+b^2)}-\frac{2ib^2x}{a^2(a^2+2ab+b^2)}-\frac{2ib^2e}{a^2f(a^2+2ab+b}$

```
[In] int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2/(a+b)^2*ln(-1+cos(f*x+e))+1/2*b/(a+b)^2*((a+b)*b/a^2/(b+a*cos(f*x+
e)^2)+(2*a+b)/a^2*ln(b+a*cos(f*x+e)^2))+1/2/(a+b)^2*ln(1+cos(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.66

$$\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{ab^2+b^3+\left(2\,ab^2+b^3+\left(2\,a^2b+ab^2\right)\cos\left(fx+e\right)^2\right)\log\left(a\cos\left(fx+e\right)^2+b\right)+2\left(a^3\cos\left(fx+e\right)^2+a^2b\right)}{2\left(\left(a^5+2\,a^4b+a^3b^2\right)f\cos\left(fx+e\right)^2+\left(a^4b+2\,a^3b^2+a^2b^3\right)f\right)}$$

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*b^2 + b^3 + (2*a*b^2 + b^3 + (2*a^2*b + a*b^2)*cos(f*x + e)^2)*log(a
*cos(f*x + e)^2 + b) + 2*(a^3*cos(f*x + e)^2 + a^2*b)*log(1/2*sin(f*x + e))
)/((a^5 + 2*a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^
3)*f)
```

Sympy [F]

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(cot(e + f*x)/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.41

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{b^2}{a^4+2a^3b+a^2b^2-(a^4+a^3b)\sin(fx+e)^2}+\frac{(2ab+b^2)\log\left(a\sin(fx+e)^2-a-b\right)}{a^4+2a^3b+a^2b^2}+\frac{\log\left(\sin(fx+e)^2\right)}{a^2+2ab+b^2}}{2f}$$

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2/(a^4 + 2*a^3*b + a^2*b^2 - (a^4 + a^3*b)*sin(f*x + e)^2) + (2*a*b
+ b^2)*log(a*sin(f*x + e)^2 - a - b)/(a^4 + 2*a^3*b + a^2*b^2) + log(sin(f*
x + e)^2)/(a^2 + 2*a*b + b^2))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. 2(79) = 158.

Time = 0.34 (sec) , antiderivative size = 396, normalized size of antiderivative = 4.77

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{(2ab+b^2)\log\left(a+b+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^4+2a^3b+a^2b^2}+\frac{\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^2+2ab+b^2}-\frac{2ab+b^2+\frac{4ab(\cos(fx}{\cos(fx+}}{(a^3+a^2b)\left(a+b+\frac{2a(c}{\cos}\right.}}{2f}$$

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((2*a*b + b^2)*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) -
2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x
 + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^4 + 2*a^3*b
```

```
+ a^2*b^2) + log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(a^2 + 2*a*b
 + b^2) - (2*a*b + b^2 + 4*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b^
2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2*a*b*(cos(f*x + e) - 1)^2/(cos(f
*x + e) + 1)^2 + b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((a^3 + a^2
*b)*(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e)
- 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(
cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)) - 2*log(abs(-(cos(f*x + e) - 1)/
(cos(f*x + e) + 1) + 1))/a^2)/f
```

Mupad [B] (verification not implemented)

Time = 19.76 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.28

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\ln(\tan(e+f\,x))}{f\,(a^2+2\,a\,b+b^2)} - \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a^2\,f} - \frac{b}{2\,a\,f\,(a+b)\,\left(b\tan(e+f\,x)^2+a+b\right)} + \frac{b\,\ln\left(b\tan(e+f\,x)^2+a+b\right)\,(2\,a+b)}{2\,a^2\,f\,(a+b)^2}$$

```
[In] int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] log(tan(e + f*x))/(f*(2*a*b + a^2 + b^2)) - log(tan(e + f*x)^2 + 1)/(2*a^2*
f) - b/(2*a*f*(a + b)*(a + b + b*tan(e + f*x)^2)) + (b*log(a + b + b*tan(e
+ f*x)^2)*(2*a + b))/(2*a^2*f*(a + b)^2)
```

3.354 $\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 111

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^2} dx = -\frac{b^3}{2a^2(a+b)^2f(b+a\cos^2(e+fx))} - \frac{\csc^2(e+fx)}{2(a+b)^2f} - \frac{b^2(3a+b)\log(b+a\cos^2(e+fx))}{2a^2(a+b)^3f} - \frac{(a+3b)\log(\sin(e+fx))}{(a+b)^3f}$$

```
[Out] -1/2*b^3/a^2/(a+b)^2/f/(b+a*cos(f*x+e)^2)-1/2*csc(f*x+e)^2/(a+b)^2/f-1/2*b^
2*(3*a+b)*ln(b+a*cos(f*x+e)^2)/a^2/(a+b)^3/f-(a+3*b)*ln(sin(f*x+e))/(a+b)^3
/f
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4223, 457, 90}

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^2} dx = -\frac{b^3}{2a^2f(a+b)^2(a\cos^2(e+fx)+b)} - \frac{b^2(3a+b)\log(a\cos^2(e+fx)+b)}{2a^2f(a+b)^3} - \frac{\csc^2(e+fx)}{2f(a+b)^2} - \frac{(a+3b)\log(\sin(e+fx))}{f(a+b)^3}$$

```
[In] Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/2*b^3/(a^2*(a + b)^2*f*(b + a*Cos[e + f*x]^2)) - Csc[e + f*x]^2/(2*(a +
b)^2*f) - (b^2*(3*a + b)*Log[b + a*Cos[e + f*x]^2])/(2*a^2*(a + b)^3*f) - (
(a + 3*b)*Log[Sin[e + f*x]])/((a + b)^3*f)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^7}{(1-x^2)^2(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \frac{x^3}{(1-x)^2(b+ax)^2}\,dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)^2(-1+x)^2}+\frac{a+3b}{(a+b)^3(-1+x)}-\frac{b^3}{a(a+b)^2(b+ax)^2}+\frac{b^2(3a+b)}{a(a+b)^3(b+ax)}\right)dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{b^3}{2a^2(a+b)^2f\left(b+a\cos^2(e+fx)\right)}-\frac{\csc^2(e+fx)}{2(a+b)^2f}\\
&\quad -\frac{b^2(3a+b)\log\left(b+a\cos^2(e+fx)\right)}{2a^2(a+b)^3f}-\frac{(a+3b)\log(\sin(e+fx))}{(a+b)^3f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.17

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))^2\left((a+b)\csc^2(e+fx)+2(a+3b)\log(\sin(e+fx))+\frac{b^2\left(\frac{2b(a+b)}{a+2b+a\cos(2(e+fx))}+(3a\cdots\right)}{a\cdots}\right.}{8(a+b)^3f\left(a+b\sec^2(e+fx)\right)^2}$$

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/8*((a + 2*b + a*Cos[2*(e + f*x)])^2*((a + b)*Csc[e + f*x]^2 + 2*(a + 3*b
)*Log[Sin[e + f*x]] + (b^2*((2*b*(a + b))/(a + 2*b + a*Cos[2*(e + f*x)]) +
(3*a + b)*Log[a + b - a*Sin[e + f*x]^2]))/a^2)*Sec[e + f*x]^4)/((a + b)^3*f
*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] (verified)

Time = 7.28 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{-\frac{b^2\left(\frac{(a+b)b}{a^2\left(b+a\cos(fx+e)^2\right)}+\frac{(3a+b)\ln\left(b+a\cos(fx+e)^2\right)}{a^2}\right)}{2(a+b)^3}-\frac{1}{4(a+b)^2(1+\cos(fx+e))}+\frac{(-a-3b)\ln(1+\cos(fx+e))}{2(a+b)^3}+\frac{1}{4(a+b)^2(-1+\cos\cdots}}{f}$
default	$\frac{-\frac{b^2\left(\frac{(a+b)b}{a^2\left(b+a\cos(fx+e)^2\right)}+\frac{(3a+b)\ln\left(b+a\cos(fx+e)^2\right)}{a^2}\right)}{2(a+b)^3}-\frac{1}{4(a+b)^2(1+\cos(fx+e))}+\frac{(-a-3b)\ln(1+\cos(fx+e))}{2(a+b)^3}+\frac{1}{4(a+b)^2(-1+\cos\cdots}}{f}$
risch	$-\frac{ix}{a^2}+\frac{2iax}{a^3+3a^2b+3a\,b^2+b^3}+\frac{2iae}{f(a^3+3a^2b+3a\,b^2+b^3)}+\frac{6ibx}{a^3+3a^2b+3a\,b^2+b^3}+\frac{6ibe}{f(a^3+3a^2b+3a\,b^2+b^3)}+\frac{\cdots}{a(a^3+3\cdots}$

```
[In] int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/2*b^2/(a+b)^3*((a+b)*b/a^2/(b+a*cos(f*x+e)^2)+(3*a+b)/a^2*ln(b+a*co
s(f*x+e)^2))-1/4/(a+b)^2/(1+cos(f*x+e))+1/2*(-a-3*b)/(a+b)^3*ln(1+cos(f*x+e
))+1/4/(a+b)^2/(-1+cos(f*x+e))+1/2*(-a-3*b)/(a+b)^3*ln(-1+cos(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(105) = 210$.

Time = 0.50 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.81

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \frac{a^3b + a^2b^2 + ab^3 + b^4 + (a^4 + a^3b - ab^3 - b^4)\cos(fx+e)^2 - \left((3a^2b^2 + ab^3)\cos(fx+e)^4 - 3ab^3 - b^4 - \right.}{2\left((a^6 + 3a^5b + 3a^4b^2 + a^3b^3)f\,\mathrm{c}\right.}$$

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
[Out] 1/2*(a^3*b + a^2*b^2 + a*b^3 + b^4 + (a^4 + a^3*b - a*b^3 - b^4)*cos(f*x +
e)^2 - ((3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - 3*a*b^3 - b^4 - (3*a^2*b^2 - 2
*a*b^3 - b^4)*cos(f*x + e)^2)*log(a*cos(f*x + e)^2 + b) - 2*((a^4 + 3*a^3*b
)*cos(f*x + e)^4 - a^3*b - 3*a^2*b^2 - (a^4 + 2*a^3*b - 3*a^2*b^2)*cos(f*x
+ e)^2)*log(1/2*sin(f*x + e)))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos
(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^2 - (a^5
*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f)
```

Sympy [F]

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

```
[In] integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
[Out] Integral(cot(e + f*x)**3/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.73

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx =$$
$$-\frac{\frac{(3ab^2+b^3)\log\left(a\sin(fx+e)^2-a-b\right)}{a^5+3a^4b+3a^3b^2+a^2b^3} + \frac{(a+3b)\log\left(\sin(fx+e)^2\right)}{a^3+3a^2b+3ab^2+b^3} - \frac{a^3+a^2b-(a^3-b^3)\sin(fx+e)^2}{(a^5+2a^4b+a^3b^2)\sin(fx+e)^4-(a^5+3a^4b+3a^3b^2+a^2b^3)\sin(fx+e)^2}}{2f}$$

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
[Out] -1/2*((3*a*b^2 + b^3)*log(a*sin(f*x + e)^2 - a - b)/(a^5 + 3*a^4*b + 3*a^3*
b^2 + a^2*b^3) + (a + 3*b)*log(sin(f*x + e)^2)/(a^3 + 3*a^2*b + 3*a*b^2 + b
^3) - (a^3 + a^2*b - (a^3 - b^3)*sin(f*x + e)^2)/((a^5 + 2*a^4*b + a^3*b^2)
*sin(f*x + e)^4 - (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sin(f*x + e)^2))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 811 vs. 2(105) = 210.

Time = 0.42 (sec) , antiderivative size = 811, normalized size of antiderivative = 7.31

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/24*(12*(3*a*b^2 + b^3)*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e)
+ 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(
cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^5 + 3
*a^4*b + 3*a^3*b^2 + a^2*b^3) + 12*(a + 3*b)*log(abs(-cos(f*x + e) + 1)/abs
(cos(f*x + e) + 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a^4 + 6*a^3*b + 3*
a^2*b^2 + 10*a^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 16*a^3*b*(cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) + 30*a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e)
+ 1) + 32*a*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 8*b^4*(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) + 11*a^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
 + 22*a^3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 27*a^2*b^2*(cos(f*x
 + e) - 1)^2/(cos(f*x + e) + 1)^2 + 16*a*b^3*(cos(f*x + e) - 1)^2/(cos(f*x
+ e) + 1)^2 - 16*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 4*a^4*(cos
(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 16*a^3*b*(cos(f*x + e) - 1)^3/(cos(
f*x + e) + 1)^3 + 36*a^2*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 32
*a*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 8*b^4*(cos(f*x + e) - 1)
^3/(cos(f*x + e) + 1)^3)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*(a*(cos(f*x
 + e) - 1)/(cos(f*x + e) + 1) + b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2
*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 2*b*(cos(f*x + e) - 1)^2/(co
s(f*x + e) + 1)^2 + a*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + b*(cos(f*
x + e) - 1)^3/(cos(f*x + e) + 1)^3)) - 3*(cos(f*x + e) - 1)/((a^2 + 2*a*b +
 b^2)*(cos(f*x + e) + 1)) - 24*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) +
1) + 1))/a^2)/f
```

Mupad [B] (verification not implemented)

Time = 20.51 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.44

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a^2\,f} - \frac{\frac{1}{2\,(a+b)}+\frac{\tan(e+f\,x)^2\,(a\,b-b^2)}{2\,a\,(a+b)^2}}{f\,\left(b\tan\left(e+f\,x\right)^4+(a+b)\,\tan\left(e+f\,x\right)^2\right)} - \frac{\ln\left(\tan(e+f\,x)\right)\,(a+3\,b)}{f\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)} - \frac{b^2\,\ln\left(b\tan(e+f\,x)^2+a+b\right)\,(3\,a+b)}{2\,a^2\,f\,(a+b)^3}$$

```
[In] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] log(tan(e + f*x)^2 + 1)/(2*a^2*f) - (1/(2*(a + b)) + (tan(e + f*x)^2*(a*b -
 b^2))/(2*a*(a + b)^2))/(f*(tan(e + f*x)^2*(a + b) + b*tan(e + f*x)^4)) - (
log(tan(e + f*x))*(a + 3*b))/(f*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) - (b^2*log
(a + b + b*tan(e + f*x)^2)*(3*a + b))/(2*a^2*f*(a + b)^3)
```

3.355 $\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 140

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{b^4}{2a^2(a+b)^3f\,(b+a\cos^2(e+fx))}+\frac{(a+2b)\csc^2(e+fx)}{(a+b)^3f} - \frac{\csc^4(e+fx)}{4(a+b)^2f}+\frac{b^3(4a+b)\log\left(b+a\cos^2(e+fx)\right)}{2a^2(a+b)^4f} + \frac{(a^2+4ab+6b^2)\log(\sin(e+fx))}{(a+b)^4f}$$

```
[Out] 1/2*b^4/a^2/(a+b)^3/f/(b+a*cos(f*x+e)^2)+(a+2*b)*csc(f*x+e)^2/(a+b)^3/f-1/4
*csc(f*x+e)^4/(a+b)^2/f+1/2*b^3*(4*a+b)*ln(b+a*cos(f*x+e)^2)/a^2/(a+b)^4/f+
(a^2+4*a*b+6*b^2)*ln(sin(f*x+e))/(a+b)^4/f
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4223, 457, 90}

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{b^4}{2a^2f(a+b)^3\,(a\cos^2(e+fx)+b)} + \frac{b^3(4a+b)\log\left(a\cos^2(e+fx)+b\right)}{2a^2f(a+b)^4} + \frac{(a^2+4ab+6b^2)\log(\sin(e+fx))}{f(a+b)^4} - \frac{\csc^4(e+fx)}{4f(a+b)^2}+\frac{(a+2b)\csc^2(e+fx)}{f(a+b)^3}$$

```
[In] Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] b^4/(2*a^2*(a + b)^3*f*(b + a*Cos[e + f*x]^2)) + ((a + 2*b)*Csc[e + f*x]^2)
/((a + b)^3*f) - Csc[e + f*x]^4/(4*(a + b)^2*f) + (b^3*(4*a + b)*Log[b + a*
Cos[e + f*x]^2])/(2*a^2*(a + b)^4*f) + ((a^2 + 4*a*b + 6*b^2)*Log[Sin[e + f
*x]])/((a + b)^4*f)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^9}{(1-x^2)^3(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^4}{(1-x)^3(b+ax)^2}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)^3} - \frac{2(a+2b)}{(a+b)^3(-1+x)^2} + \frac{-a^2-4ab-6b^2}{(a+b)^4(-1+x)} + \frac{b^4}{a(a+b)^3(b+ax)^2} - \frac{b^3(4a+b)}{a(a+b)^4(b+ax)}\right)\,dx, x, \text{co}\right.}{2f}$$

$$= \frac{b^4}{2a^2(a+b)^3f\left(b+a\cos^2(e+fx)\right)} + \frac{(a+2b)\csc^2(e+fx)}{(a+b)^3f} - \frac{\csc^4(e+fx)}{4(a+b)^2f} + \frac{b^3(4a+b)\log\left(b+a\cos^2(e+fx)\right)}{2a^2(a+b)^4f} + \frac{(a^2+4ab+6b^2)\log(\sin(e+fx))}{(a+b)^4f}$$

Mathematica [A] (verified)

Time = 2.96 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.16

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{(a+2b+a\cos(2(e+fx)))^2\sec^4(e+fx)\Big(4(a+b)(a+2b)\csc^2(e+fx)-(a+b)^2\csc^4(e+fx)+4(a^2+}{16(a+b)^4f\,(a+b\sec^2(e+fx)}$$

[In] Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]

[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x]^4*(4*(a + b)*(a + 2*b)*Csc[e + f*x]^2 - (a + b)^2*Csc[e + f*x]^4 + 4*(a^2 + 4*a*b + 6*b^2)*Log[Sin[e + f*x]] + (2*b^3*(4*a + b)*Log[a + b - a*Sin[e + f*x]^2])/a^2 + (2*b^4*(a + b))/(a^2*(a + b - a*Sin[e + f*x]^2))))/(16*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^2)

Maple [A] (verified)

Time = 13.27 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.44

method	result
derivativedivides	$\frac{\frac{b^3\left(\frac{(a+b)b}{a^2\left(b+a\cos(fx+e)^2\right)}+\frac{(4a+b)\ln\left(b+a\cos(fx+e)^2\right)}{a^2}\right)}{2(a+b)^4}-\frac{1}{16(a+b)^2(1+\cos(fx+e))^2}-\frac{-7a-15b}{16(a+b)^3(1+\cos(fx+e))}+\frac{\left(a^2+4ab+6b^2\right)\ln(}{2(a+b}}{f}$
default	$\frac{\frac{b^3\left(\frac{(a+b)b}{a^2\left(b+a\cos(fx+e)^2\right)}+\frac{(4a+b)\ln\left(b+a\cos(fx+e)^2\right)}{a^2}\right)}{2(a+b)^4}-\frac{1}{16(a+b)^2(1+\cos(fx+e))^2}-\frac{-7a-15b}{16(a+b)^3(1+\cos(fx+e))}+\frac{\left(a^2+4ab+6b^2\right)\ln(}{2(a+b}}{f}$
risch	$-\frac{2ia^2x}{a^4+4a^3b+6a^2b^2+4a\,b^3+b^4}-\frac{8iabx}{a^4+4a^3b+6a^2b^2+4a\,b^3+b^4}-\frac{8iabe}{f(a^4+4a^3b+6a^2b^2+4a\,b^3+b^4)}-\frac{2ib^4e}{a^2f(a^4+4a^3b+6a^2b}$

[In] int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/f*(1/2*b^3/(a+b)^4*((a+b)*b/a^2/(b+a*cos(f*x+e)^2)+(4*a+b)/a^2*ln(b+a*cos(f*x+e)^2))-1/16/(a+b)^2/(1+cos(f*x+e))^2-1/16*(-7*a-15*b)/(a+b)^3/(1+cos(f*x+e))+1/2*(a^2+4*a*b+6*b^2)/(a+b)^4*ln(1+cos(f*x+e))-1/16/(a+b)^2/(-1+cos(f*x+e))^2-1/16*(7*a+15*b)/(a+b)^3/(-1+cos(f*x+e))+1/2*(a^2+4*a*b+6*b^2)/(a+b)^4*ln(-1+cos(f*x+e)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 557 vs. $2(134) = 268$.

Time = 0.80 (sec) , antiderivative size = 557, normalized size of antiderivative = 3.98

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \frac{3\,a^4b+10\,a^3b^2+7\,a^2b^3+2\,ab^4+2\,b^5-2\,(2\,a^5+6\,a^4b+4\,a^3b^2-ab^4-b^5)\cos{(fx+e)}^4+(3\,a^5+6\,a^4b-}{}$$

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^4*b + 10*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4 + 2*b^5 - 2*(2*a^5 + 6*a^4*
b + 4*a^3*b^2 - a*b^4 - b^5)*cos(f*x + e)^4 + (3*a^5 + 6*a^4*b - 5*a^3*b^2
- 8*a^2*b^3 - 4*a*b^4 - 4*b^5)*cos(f*x + e)^2 + 2*((4*a^2*b^3 + a*b^4)*cos(
f*x + e)^6 + 4*a*b^4 + b^5 - (8*a^2*b^3 - 2*a*b^4 - b^5)*cos(f*x + e)^4 + (
4*a^2*b^3 - 7*a*b^4 - 2*b^5)*cos(f*x + e)^2)*log(a*cos(f*x + e)^2 + b) + 4*
((a^5 + 4*a^4*b + 6*a^3*b^2)*cos(f*x + e)^6 + a^4*b + 4*a^3*b^2 + 6*a^2*b^3
 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 - 6*a^2*b^3)*cos(f*x + e)^4 + (a^5 + 2*a^4*
b - 2*a^3*b^2 - 12*a^2*b^3)*cos(f*x + e)^2)*log(1/2*sin(f*x + e)))/((a^7 +
4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a^7 + 7*a^
6*b + 8*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)^4 + (a^7
+ 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^2
 + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f)
```

Sympy [F]

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\cot^5{(e+fx)}}{(a+b\sec^2{(e+fx)})^2}\,dx$$

```
[In] integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(cot(e + f*x)**5/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(134) = 268$.

Time = 0.20 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.99

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \frac{\frac{2\,(4\,ab^3+b^4)\log\left(a\sin(fx+e)^2-a-b\right)}{a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4}+\frac{2\,(a^2+4\,ab+6\,b^2)\log\left(\sin(fx+e)^2\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{2\,(2\,a^4+4\,a^3b-b^4)\sin(fx+e)^4+a^4+2\,a^3b+a^2b^2-(5\,a^4+13\,a^3}{(a^6+3\,a^5b+3\,a^4b^2+a^3b^3)\sin(fx+e)^6-(a^6+4\,a^5b+6\,a^4b^2+4\,a}}{4\,f}$$

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(4*a*b^3 + b^4)*log(a*sin(f*x + e)^2 - a - b)/(a^6 + 4*a^5*b + 6*a^4
*b^2 + 4*a^3*b^3 + a^2*b^4) + 2*(a^2 + 4*a*b + 6*b^2)*log(sin(f*x + e)^2)/(
a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + (2*(2*a^4 + 4*a^3*b - b^4)*sin
(f*x + e)^4 + a^4 + 2*a^3*b + a^2*b^2 - (5*a^4 + 13*a^3*b + 8*a^2*b^2)*sin(
f*x + e)^2)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sin(f*x + e)^6 - (a^6 +
4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sin(f*x + e)^4))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 913 vs. $2(134) = 268$.

Time = 0.50 (sec) , antiderivative size = 913, normalized size of antiderivative = 6.52

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/64*(32*(4*a*b^3 + b^4)*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) +
 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(c
os(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^6 + 4*
a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4) + 32*(a^2 + 4*a*b + 6*b^2)*log(abs
(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a
*b^3 + b^4) - (12*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 40*a*b*(cos(f
*x + e) - 1)/(cos(f*x + e) + 1) + 28*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) +
 1) + a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 2*a*b*(cos(f*x + e) -
 1)^2/(cos(f*x + e) + 1)^2 + b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)
/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - (a^2 + 2*a*b + b^2 + 12*a^2*
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 40*a*b*(cos(f*x + e) - 1)/(cos(f*x
+ e) + 1) + 28*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 48*a^2*(cos(f*x
+ e) - 1)^2/(cos(f*x + e) + 1)^2 + 192*a*b*(cos(f*x + e) - 1)^2/(cos(f*x +
e) + 1)^2 + 288*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e
) + 1)^2/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(cos(f*x + e) - 1)^2)
 - 32*(4*a^2*b^3 + 5*a*b^4 + b^5 + 8*a^2*b^3*(cos(f*x + e) - 1)/(cos(f*x +
e) + 1) - 2*a*b^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b^5*(cos(f*x +
e) - 1)/(cos(f*x + e) + 1) + 4*a^2*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
 1)^2 + 5*a*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b^5*(cos(f*x +
e) - 1)^2/(cos(f*x + e) + 1)^2)/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a
^2*b^4)*(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x +
 e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 +
 b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)) - 64*log(abs(-(cos(f*x + e)
- 1)/(cos(f*x + e) + 1) + 1))/a^2)/f
```

Mupad [B] (verification not implemented)

Time = 21.17 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.47

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{\tan(e+f\,x)^2\,(2\,a+5\,b)}{4\,(a+b)^2}-\frac{1}{4\,(a+b)}+\frac{\tan(e+f\,x)^4\,(a^2\,b+3\,a\,b^2-b^3)}{2\,a\,(a+b)^3}}{f\,\left(b\tan\left(e+f\,x\right)^6+(a+b)\,\tan\left(e+f\,x\right)^4\right)} - \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a^2\,f} + \frac{\ln\left(\tan(e+f\,x)\right)\,(a^2+4\,a\,b+6\,b^2)}{f\,(a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4)} + \frac{b^3\,\ln\left(b\tan(e+f\,x)^2+a+b\right)\,(4\,a+b)}{2\,a^2\,f\,(a+b)^4}$$

```
[In] int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] ((tan(e + f*x)^2*(2*a + 5*b))/(4*(a + b)^2) - 1/(4*(a + b)) + (tan(e + f*x)
^4*(3*a*b^2 + a^2*b - b^3))/(2*a*(a + b)^3))/(f*(tan(e + f*x)^4*(a + b) + b
*tan(e + f*x)^6)) - log(tan(e + f*x)^2 + 1)/(2*a^2*f) + (log(tan(e + f*x))*
(4*a*b + a^2 + 6*b^2))/(f*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)) + (b
^3*log(a + b + b*tan(e + f*x)^2)*(4*a + b))/(2*a^2*f*(a + b)^4)
```

3.356 $\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 119

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^2} dx = -\frac{x}{a^2} - \frac{(3a-2b)(a+b)^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2b^{5/2}f} + \frac{(3a+b)\tan(e+fx)}{2ab^2f} - \frac{(a+b)\tan^3(e+fx)}{2abf(a+b+b\tan^2(e+fx))}$$

```
[Out] -x/a^2-1/2*(3*a-2*b)*(a+b)^(3/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^2
/b^(5/2)/f+1/2*(3*a+b)*tan(f*x+e)/a/b^2/f-1/2*(a+b)*tan(f*x+e)^3/a/b/f/(a+b
+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4226, 2000, 481, 596, 536, 209, 211}

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^2} dx = -\frac{(3a-2b)(a+b)^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2b^{5/2}f} - \frac{x}{a^2} + \frac{(3a+b)\tan(e+fx)}{2ab^2f} - \frac{(a+b)\tan^3(e+fx)}{2abf(a+b\tan^2(e+fx)+b)}$$

```
[In] Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(x/a^2) - ((3*a - 2*b)*(a + b)^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a
+ b]])/(2*a^2*b^(5/2)*f) + ((3*a + b)*Tan[e + f*x])/(2*a*b^2*f) - ((a + b)*
Tan[e + f*x]^3)/(2*a*b*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+b(1+x^2))^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{(a+b)\tan^3(e+fx)}{2abf\,(a+b+b\tan^2(e+fx))} + \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)+(3a+b)x^2)}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{2abf}\\
&= \frac{(3a+b)\tan(e+fx)}{2ab^2f} - \frac{(a+b)\tan^3(e+fx)}{2abf\,(a+b+b\tan^2(e+fx))}\\
&\quad - \frac{\text{Subst}\left(\int \frac{(a+b)(3a+b)+(3a^2+4ab-b^2)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{2ab^2f}\\
&= \frac{(3a+b)\tan(e+fx)}{2ab^2f} - \frac{(a+b)\tan^3(e+fx)}{2abf\,(a+b+b\tan^2(e+fx))}\\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^2f}\\
&\quad - \frac{((3a-2b)(a+b)^2)\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{2a^2b^2f}\\
&= -\frac{x}{a^2} - \frac{(3a-2b)(a+b)^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2b^{5/2}f}\\
&\quad + \frac{(3a+b)\tan(e+fx)}{2ab^2f} - \frac{(a+b)\tan^3(e+fx)}{2abf\,(a+b+b\tan^2(e+fx))}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.80 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.40

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(-\frac{2x(a+2b+a\cos(2(e+fx)))}{a^2}+\frac{(3a-2b)(a+b)^{3/2}\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-}{2\sqrt{a+b}\sqrt{b(}}\right.}{a^2}\right.}{8}$$

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((-2*x*(a + 2*b + a*Cos[2*(e
 + f*x)]))/a^2 + ((3*a - 2*b)*(a + b)^(3/2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*
Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b
*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[
2*e]))/(a^2*b^2*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (2*(a + 2*b + a*Cos[2*(e
 + f*x)])*Sec[e]*Sec[e + f*x]*Sin[f*x])/(b^2*f) - ((a + b)^2*((a + 2*b)*Sin
[2*e] - a*Sin[2*f*x]))/(a^2*b^2*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8
*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] (verified)

Time = 8.53 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)}{b^2}-\frac{\frac{\left(-\frac{1}{2}a^3-a^2b-\frac{1}{2}ab^2\right)\tan(fx+e)}{a+b+b\tan(fx+e)^2}+\frac{\left(3a^3+4a^2b-ab^2-2b^3\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}}{a^2b^2}-\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
default	$\frac{\frac{\tan(fx+e)}{b^2}-\frac{\frac{\left(-\frac{1}{2}a^3-a^2b-\frac{1}{2}ab^2\right)\tan(fx+e)}{a+b+b\tan(fx+e)^2}+\frac{\left(3a^3+4a^2b-ab^2-2b^3\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}}{a^2b^2}-\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
risch	$-\frac{x}{a^2}+\frac{i\left(3a^3e^{4i(fx+e)}+4a^2b\,e^{4i(fx+e)}+5a\,b^2e^{4i(fx+e)}+2b^3e^{4i(fx+e)}+6a^3e^{2i(fx+e)}+14a^2b\,e^{2i(fx+e)}+6a\,b^2e^{2i(fx+e}\right.}{a^2b^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)\left(e^{2i(fx+e)}+1\right)}$

```
[In] int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(tan(f*x+e)/b^2-1/a^2/b^2*((-1/2*a^3-a^2*b-1/2*a*b^2)*tan(f*x+e)/(a+b+b
*tan(f*x+e)^2)+1/2*(3*a^3+4*a^2*b-a*b^2-2*b^3)/((a+b)*b)^(1/2)*arctan(b*tan
(f*x+e)/((a+b)*b)^(1/2)))-1/a^2*arctan(tan(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(105) = 210.

Time = 0.30 (sec) , antiderivative size = 514, normalized size of antiderivative = 4.32

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \left[-\frac{8\,ab^2fx\cos\left(fx+e\right)^3 + 8\,b^3fx\cos\left(fx+e\right) + \left((3\,a^3+a^2b-2\,ab^2)\cos\left(fx+e\right)^3 + (3\,a^2b+ab^2-2\,b^3)\,c\right.}{} \right.$$

$$-\frac{4\,ab^2fx\cos\left(fx+e\right)^3 + 4\,b^3fx\cos\left(fx+e\right) - \left((3\,a^3+a^2b-2\,ab^2)\cos\left(fx+e\right)^3 + (3\,a^2b+ab^2-2\,b^3)\,c\right.}{4\left(a^3b^2f\cos\left(fx+\right.\right.}$$

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(8*a*b^2*f*x*cos(f*x + e)^3 + 8*b^3*f*x*cos(f*x + e) + ((3*a^3 + a^2*
b - 2*a*b^2)*cos(f*x + e)^3 + (3*a^2*b + a*b^2 - 2*b^3)*cos(f*x + e))*sqrt(
-(a + b)/b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*c
os(f*x + e)^2 - 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(-(
a + b)/b)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 +
b^2)) - 4*(2*a^2*b + (3*a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sin(f*x + e)
)/(a^3*b^2*f*cos(f*x + e)^3 + a^2*b^3*f*cos(f*x + e)), -1/4*(4*a*b^2*f*x*co
s(f*x + e)^3 + 4*b^3*f*x*cos(f*x + e) - ((3*a^3 + a^2*b - 2*a*b^2)*cos(f*x
+ e)^3 + (3*a^2*b + a*b^2 - 2*b^3)*cos(f*x + e))*sqrt((a + b)/b)*arctan(1/2
*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a + b)/b)/((a + b)*cos(f*x + e)*sin(f
*x + e))) - 2*(2*a^2*b + (3*a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sin(f*x
+ e))/(a^3*b^2*f*cos(f*x + e)^3 + a^2*b^3*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\tan^6\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

```
[In] integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(tan(e + f*x)**6/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.07

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{(a^2+2\,ab+b^2)\tan(fx+e)}{ab^3\tan(fx+e)^2+a^2b^2+ab^3} - \frac{2\,(fx+e)}{a^2} + \frac{2\,\tan(fx+e)}{b^2} - \frac{(3\,a^3+4\,a^2b-ab^2-2\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^2b^2}}{2\,f}$$

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2 + 2*a*b + b^2)*tan(f*x + e)/(a*b^3*tan(f*x + e)^2 + a^2*b^2 + a*b
^3) - 2*(f*x + e)/a^2 + 2*tan(f*x + e)/b^2 - (3*a^3 + 4*a^2*b - a*b^2 - 2*b
^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^2*b^2))/f
```

Giac [A] (verification not implemented)

none

Time = 2.24 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.30

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{2\,(fx+e)}{a^2} - \frac{2\,\tan(fx+e)}{b^2} + \frac{(3\,a^3+4\,a^2b-ab^2-2\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}a^2b^2} - \frac{a^2\tan(fx+e)+2\,ab\tan(fx+e)+b^2\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)ab^2}}{2\,f}$$

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(f*x + e)/a^2 - 2*tan(f*x + e)/b^2 + (3*a^3 + 4*a^2*b - a*b^2 - 2*b
^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b +
 b^2)))/(sqrt(a*b + b^2)*a^2*b^2) - (a^2*tan(f*x + e) + 2*a*b*tan(f*x + e)
+ b^2*tan(f*x + e))/((b*tan(f*x + e)^2 + a + b)*a*b^2))/f
```

Mupad [B] (verification not implemented)

Time = 20.05 (sec) , antiderivative size = 765, normalized size of antiderivative = 6.43

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\tan(e+f\,x)}{b^2\,f}$$

$$-\frac{\operatorname{atan}\left(\frac{5\tan(e+f\,x)}{\frac{12\,a}{b}-\frac{10\,b}{a}-\frac{15\,b^2}{2\,a^2}+\frac{9\,a^2}{2\,b^2}+5}-\frac{10\tan(e+f\,x)}{\frac{5\,a}{b}-\frac{15\,b}{2\,a}+\frac{12\,a^2}{b^2}+\frac{9\,a^3}{2\,b^3}-10}+\frac{12\,a\tan(e+f\,x)}{12\,a+5\,b-\frac{10\,b^2}{a}+\frac{9\,a^2}{2\,b}-\frac{15\,b^3}{2\,a^2}}-\frac{15\,b\tan(e+f\,x)}{2\left(\frac{5\,a^2}{b}-\frac{15\,b}{2}-10\,a+\frac{12\,a^3}{b^2}+\frac{9\,a^4}{2\,b^3}\right)}+\right.}{a^2\,f}$$

$$+\frac{\tan(e+f\,x)\,\left(a^2+2\,a\,b+b^2\right)}{2\,a\,f\,\left(b^3\tan\left(e+f\,x\right)^2+b^3+a\,b^2\right)}$$

$$-\frac{\operatorname{atan}\left(-\frac{\tan(e+f\,x)\sqrt{-a^3\,b^5-3\,a^2\,b^6-3\,a\,b^7-b^8}\,35\mathrm{i}}{4\left(9\,a^3\,b-\frac{85\,a\,b^3}{4}+\frac{81\,a^4}{4}+\frac{25\,b^4}{4}-\frac{49\,a^2\,b^2}{2}+\frac{15\,b^5}{2\,a}+\frac{27\,a^5}{4\,b}\right)}+\frac{\tan(e+f\,x)\sqrt{-a^3\,b^5-3\,a^2\,b^6-3\,a\,b^7-b^8}\,15\mathrm{i}}{2\left(\frac{25\,a\,b^3}{4}-\frac{49\,a^3\,b}{2}+9\,a^4+\frac{15\,b^4}{2}-\frac{85\,a^2\,b^2}{4}+\frac{81\,a^5}{4\,b}+\frac{27\,a^6}{4\,b^2}\right)}-\frac{\tan(}{4\left(\frac{81\,a^3\,b}{4}-\right.}\right.}{}$$

```
[In] int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] tan(e + f*x)/(b^2*f) - atan((5*tan(e + f*x))/((12*a)/b - (10*b)/a - (15*b^2
)/(2*a^2) + (9*a^2)/(2*b^2) + 5) - (10*tan(e + f*x))/((5*a)/b - (15*b)/(2*a
) + (12*a^2)/b^2 + (9*a^3)/(2*b^3) - 10) + (12*a*tan(e + f*x))/(12*a + 5*b
- (10*b^2)/a + (9*a^2)/(2*b) - (15*b^3)/(2*a^2)) - (15*b*tan(e + f*x))/(2*(
(5*a^2)/b - (15*b)/2 - 10*a + (12*a^3)/b^2 + (9*a^4)/(2*b^3))) + (9*a^2*tan
(e + f*x))/(2*(12*a*b + (9*a^2)/2 + 5*b^2 - (10*b^3)/a - (15*b^4)/(2*a^2)))
)/(a^2*f) + (tan(e + f*x)*(2*a*b + a^2 + b^2))/(2*a*f*(a*b^2 + b^3 + b^3*ta
n(e + f*x)^2)) - (atan((tan(e + f*x)*(- 3*a*b^7 - b^8 - 3*a^2*b^6 - a^3*b^5
)^(1/2)*15i)/(2*((25*a*b^3)/4 - (49*a^3*b)/2 + 9*a^4 + (15*b^4)/2 - (85*a^2
*b^2)/4 + (81*a^5)/(4*b) + (27*a^6)/(4*b^2))) - (tan(e + f*x)*(- 3*a*b^7 -
b^8 - 3*a^2*b^6 - a^3*b^5)^(1/2)*35i)/(4*(9*a^3*b - (85*a*b^3)/4 + (81*a^4)
/4 + (25*b^4)/4 - (49*a^2*b^2)/2 + (15*b^5)/(2*a) + (27*a^5)/(4*b))) - (tan
(e + f*x)*(- 3*a*b^7 - b^8 - 3*a^2*b^6 - a^3*b^5)^(1/2)*45i)/(4*((81*a^3*b)
/4 - (49*a*b^3)/2 + (27*a^4)/4 - (85*b^4)/4 + 9*a^2*b^2 + (25*b^5)/(4*a) +
(15*b^6)/(2*a^2))) + (a^2*tan(e + f*x)*(- 3*a*b^7 - b^8 - 3*a^2*b^6 - a^3*b
^5)^(1/2)*27i)/(4*(9*a^2*b^4 - (85*b^6)/4 - (49*a*b^5)/2 + (81*a^3*b^3)/4 +
 (27*a^4*b^2)/4 + (25*b^7)/(4*a) + (15*b^8)/(2*a^2))) + (a*tan(e + f*x)*(-
3*a*b^7 - b^8 - 3*a^2*b^6 - a^3*b^5)^(1/2)*27i)/(4*((27*a^4*b)/4 - (49*a*b^
4)/2 - (85*b^5)/4 + 9*a^2*b^3 + (81*a^3*b^2)/4 + (25*b^6)/(4*a) + (15*b^7)/
(2*a^2))))*(-b^5*(a + b)^3)^(1/2)*(3*a - 2*b)*1i)/(2*a^2*b^5*f)
```

3.357 $\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 90

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{x}{a^2}+\frac{(a-2b)\sqrt{a+b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2b^{3/2}f} - \frac{(a+b)\tan(e+fx)}{2abf\,(a+b+b\tan^2(e+fx))}$$

```
[Out] x/a^2+1/2*(a-2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*(a+b)^(1/2)/a^2/b^
(3/2)/f-1/2*(a+b)*tan(f*x+e)/a/b/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4226, 2000, 481, 536, 209, 211}

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{(a-2b)\sqrt{a+b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2b^{3/2}f} + \frac{x}{a^2} - \frac{(a+b)\tan(e+fx)}{2abf\,(a+b\tan^2(e+fx)+b)}$$

```
[In] Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] x/a^2 + ((a - 2*b)*Sqrt[a + b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/
(2*a^2*b^(3/2)*f) - ((a + b)*Tan[e + f*x])/(2*a*b*f*(a + b + b*Tan[e + f*x]
^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+b(1+x^2))^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\tan(e+fx)}{2abf\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{a+b+(a-b)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{2abf}$$

$$= -\frac{(a+b)\tan(e+fx)}{2abf\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^2f} + \frac{((a-2b)(a+b))\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{2a^2bf}$$

$$= \frac{x}{a^2} + \frac{(a-2b)\sqrt{a+b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2b^{3/2}f} - \frac{(a+b)\tan(e+fx)}{2abf\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.73 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.77

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(2x(a+2b+a\cos(2(e+fx))) + \frac{(-a^2+ab+2b^2)\arctan\left(\frac{\sec(fx)(\cos(2e)-\ldots}{\ldots}\right)}{\ldots}\right.}{8a^2\left(a+b\sec^2(e+\ldots\right.}$$

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(2*x*(a + 2*b + a*Cos[2*(e +
 f*x)]) + ((-a^2 + a*b + 2*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-
((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I
*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/(b*Sq
rt[a + b]*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + ((a + b)*((a + 2*b)*Sin[2*e] -
 a*Sin[2*f*x]))/(b*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*a^2*(a + b*S
ec[e + f*x]^2)^2)
```

Maple [A] (verified)

Time = 4.55 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a^2}+\frac{(a+b)\left(-\frac{a\tan(fx+e)}{2b\left(a+b+b\tan(fx+e)^2\right)}+\frac{(a-2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2b\sqrt{(a+b)b}}\right)}{a^2}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{a^2}+\frac{(a+b)\left(-\frac{a\tan(fx+e)}{2b\left(a+b+b\tan(fx+e)^2\right)}+\frac{(a-2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2b\sqrt{(a+b)b}}\right)}{a^2}}{f}$
risch	$\frac{x}{a^2}-\frac{i(a^2e^{2i(fx+e)}+3ab\,e^{2i(fx+e)}+2b^2e^{2i(fx+e)}+a^2+ab)}{a^2bf\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-(a+b)b}-a-2b}{a}\right)}{4b^2fa}-\frac{\sqrt{-(}}{}$

```
[In] int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^2*arctan(tan(f*x+e))+(a+b)/a^2*(-1/2*a/b*tan(f*x+e)/(a+b+b*tan(f*x
+e)^2)+1/2*(a-2*b)/b/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(78) = 156.

Time = 0.29 (sec) , antiderivative size = 393, normalized size of antiderivative = 4.37

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$=\left[\frac{8\,abfx\cos(fx+e)^2+8\,b^2fx-4\,(a^2+ab)\cos(fx+e)\sin(fx+e)-\left((a^2-2\,ab)\cos(fx+e)^2+ab-2\right.}{8\left(a^3bf\cos(f\right.}\right.$$

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*a*b*f*x*cos(f*x + e)^2 + 8*b^2*f*x - 4*(a^2 + a*b)*cos(f*x + e)*sin
(f*x + e) - ((a^2 - 2*a*b)*cos(f*x + e)^2 + a*b - 2*b^2)*sqrt(-(a + b)/b)*l
og(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2
 + 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(-(a + b)/b)*sin
(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(a^3*b
*f*cos(f*x + e)^2 + a^2*b^2*f), 1/4*(4*a*b*f*x*cos(f*x + e)^2 + 4*b^2*f*x -
 2*(a^2 + a*b)*cos(f*x + e)*sin(f*x + e) - ((a^2 - 2*a*b)*cos(f*x + e)^2 +
a*b - 2*b^2)*sqrt((a + b)/b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt
((a + b)/b)/((a + b)*cos(f*x + e)*sin(f*x + e))))/(a^3*b*f*cos(f*x + e)^2 +
 a^2*b^2*f)]
```

Sympy [F]

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

```
[In] integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
[Out] Integral(tan(e + f*x)**4/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.07

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{(a+b)\tan(fx+e)}{ab^2\tan(fx+e)^2+a^2b+ab^2} - \frac{2\,(fx+e)}{a^2} - \frac{(a^2-ab-2\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^2b}}{2\,f}$$

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
[Out] -1/2*((a + b)*tan(f*x + e)/(a*b^2*tan(f*x + e)^2 + a^2*b + a*b^2) - 2*(f*x
+ e)/a^2 - (a^2 - a*b - 2*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt
((a + b)*b)*a^2*b))/f
```

Giac [A] (verification not implemented)

none

Time = 1.02 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.33

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{2\,(fx+e)}{a^2} + \frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(a^2-ab-2\,b^2)}{\sqrt{ab+b^2}a^2b} - \frac{a\tan(fx+e)+b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)ab}}{2\,f}$$

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
[Out] 1/2*(2*(f*x + e)/a^2 + (pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(
f*x + e)/sqrt(a*b + b^2)))*(a^2 - a*b - 2*b^2)/(sqrt(a*b + b^2)*a^2*b) - (a
*tan(f*x + e) + b*tan(f*x + e))/((b*tan(f*x + e)^2 + a + b)*a*b))/f
```

Mupad [B] (verification not implemented)

Time = 19.79 (sec) , antiderivative size = 285, normalized size of antiderivative = 3.17

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{\tan(e+f\,x)}{\frac{3\,b}{2\,a}-\frac{a}{2\,b}+1}-\frac{\tan(e+f\,x)}{2\,\left(\frac{b}{a}+\frac{3\,b^2}{2\,a^2}-\frac{1}{2}\right)}+\frac{3\,b\tan(e+f\,x)}{2\,\left(a+\frac{3\,b}{2}-\frac{a^2}{2\,b}\right)}\right)}{a^2\,f}-\frac{\tan(e+f\,x)\,\left(a+b\right)}{2\,a\,b\,f\,\left(b\tan\left(e+f\,x\right)^2+a+b\right)}$$
$$-\frac{\operatorname{atanh}\left(\frac{3\tan(e+f\,x)\,\sqrt{-b^4-a\,b^3}}{2\,\left(\frac{a\,b}{4}-a^2+\frac{3\,b^2}{2}+\frac{a^3}{4\,b}\right)}-\frac{5\tan(e+f\,x)\,\sqrt{-b^4-a\,b^3}}{4\,\left(\frac{a^2}{4}-a\,b+\frac{b^2}{4}+\frac{3\,b^3}{2\,a}\right)}+\frac{\tan(e+f\,x)\,\sqrt{-b^4-a\,b^3}}{4\,\left(\frac{a\,b}{4}-b^2+\frac{b^3}{4\,a}+\frac{3\,b^4}{2\,a^2}\right)}\right)\,\sqrt{-b^3\,\left(a+b\right)}\,\left(a-2\,b\right)}{2\,a^2\,b^3\,f}$$

```
[In] int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] atan(tan(e + f*x)/((3*b)/(2*a) - a/(2*b) + 1) - tan(e + f*x)/(2*(b/a + (3*b
^2)/(2*a^2) - 1/2)) + (3*b*tan(e + f*x))/(2*(a + (3*b)/2 - a^2/(2*b))))/(a^
2*f) - (tan(e + f*x)*(a + b))/(2*a*b*f*(a + b + b*tan(e + f*x)^2)) - (atanh
((3*tan(e + f*x)*(- a*b^3 - b^4)^(1/2))/(2*((a*b)/4 - a^2 + (3*b^2)/2 + a^3
/(4*b))) - (5*tan(e + f*x)*(- a*b^3 - b^4)^(1/2))/(4*(a^2/4 - a*b + b^2/4 +
 (3*b^3)/(2*a))) + (tan(e + f*x)*(- a*b^3 - b^4)^(1/2))/(4*((a*b)/4 - b^2 +
 b^3/(4*a) + (3*b^4)/(2*a^2))))*(-b^3*(a + b))^(1/2)*(a - 2*b))/(2*a^2*b^3*
f)
```

3.358 $\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^2} dx = -\frac{x}{a^2} + \frac{(a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2\sqrt{b}\sqrt{a+b}f} + \frac{\tan(e+fx)}{2af\,(a+b+b\tan^2(e+fx))}$$

```
[Out] -x/a^2+1/2*(a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^2/f/b^(1/2)/(a+
b)^(1/2)+1/2*tan(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4226, 2000, 482, 536, 209, 211}

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^2} dx = \frac{(a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2\sqrt{b}f\sqrt{a+b}} - \frac{x}{a^2} + \frac{\tan(e+fx)}{2af\,(a+b\tan^2(e+fx)+b)}$$

```
[In] Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(x/a^2) + ((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*a^2*Sq
rt[b]*Sqrt[a + b]*f) + Tan[e + f*x]/(2*a*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+b(1+x^2))^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}
\end{aligned}$$

$$= \frac{\tan(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{1-x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= \frac{\tan(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^2f} + \frac{(a+2b)\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{2a^2f}$$

$$= -\frac{x}{a^2} + \frac{(a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2\sqrt{b}\sqrt{a+b}f} + \frac{\tan(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.55 (sec) , antiderivative size = 346, normalized size of antiderivative = 4.07

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))^2\sec^4(e+fx)\left(-\frac{16x+\frac{\left(-a^3+6a^2b+24ab^2+16b^3\right)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{b(a+b)^{3/2}f\sqrt{b(\cos(e)-i\sin(e))^4}}}{}\right.}{64\,(a+b}$$

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x]^4*(-((16*x + ((-a^3 + 6*a^2*
b + 24*a*b^2 + 16*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b
)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^
4])]*(Cos[2*e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])
^4]) + ((a^2 + 8*a*b + 8*b^2)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a +
b)*f*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])))/a
^2) + (((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2)
 - (a*Sqrt[b]*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])))/(
b^(3/2)*f)))/(64*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}}{a^2}-\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
default	$\frac{\frac{\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}}{a^2}-\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
risch	$-\frac{x}{a^2}+\frac{i\left(a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a\right)}{a^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{\ln\left(e^{2i(fx+e)}+\frac{-2iba-2ib^2+a\sqrt{-ab-b^2}+2b\sqrt{-ab-b^2}}{a\sqrt{-ab-b^2}}\right)}{4\sqrt{-ab-b^2}\,fa}+$

```
[In] int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^2*(1/2*a*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(a+2*b)/((a+b)*b)^(1/
2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))-1/a^2*arctan(tan(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(73) = 146.

Time = 0.30 (sec) , antiderivative size = 458, normalized size of antiderivative = 5.39

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$=\left[-\frac{8\left(a^2b+ab^2\right)fx\cos\left(fx+e\right)^2+8\left(ab^2+b^3\right)fx-4\left(a^2b+ab^2\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(\left(a^2+2\,ab\right)c\right.}{8\left(\left(a^4b+a\right.\right.}\right.$$

$$-\frac{4\left(a^2b+ab^2\right)fx\cos\left(fx+e\right)^2+4\left(ab^2+b^3\right)fx-2\left(a^2b+ab^2\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(\left(a^2+2\,ab\right)co\right.}{4\left(\left(a^4b+a^3b^2\right)f\cos\left(fx+e\right)^2+\left(a^3b^2+a^2\right.\right.}$$

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(8*(a^2*b + a*b^2)*f*x*cos(f*x + e)^2 + 8*(a*b^2 + b^3)*f*x - 4*(a^2*
b + a*b^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + 2*a*b)*cos(f*x + e)^2 + a*b
+ 2*b^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*
a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))
*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x +
 e)^2 + b^2)))/((a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^3*b^2 + a^2*b^3)*f)
, -1/4*(4*(a^2*b + a*b^2)*f*x*cos(f*x + e)^2 + 4*(a*b^2 + b^3)*f*x - 2*(a^2
```

```
*b + a*b^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + 2*a*b)*cos(f*x + e)^2 + a*b
 + 2*b^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a
*b + b^2)*cos(f*x + e)*sin(f*x + e))))/((a^4*b + a^3*b^2)*f*cos(f*x + e)^2
+ (a^3*b^2 + a^2*b^3)*f)]
```

Sympy [F]

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

```
[In] integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
[Out] Integral(tan(e + f*x)**2/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{\tan(fx+e)}{ab\tan(fx+e)^2+a^2+ab} + \frac{(a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^2} - \frac{2(fx+e)}{a^2}}{2f}$$

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
[Out] 1/2*(tan(f*x + e)/(a*b*tan(f*x + e)^2 + a^2 + a*b) + (a + 2*b)*arctan(b*tan
(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^2) - 2*(f*x + e)/a^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.55 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(a+2b)}{\sqrt{ab+b^2}a^2} - \frac{2(fx+e)}{a^2} + \frac{\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)a}}{2f}$$

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
[Out] 1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
+ b^2)))*(a + 2*b)/(sqrt(a*b + b^2)*a^2) - 2*(f*x + e)/a^2 + tan(f*x + e)/(
(b*tan(f*x + e)^2 + a + b)*a))/f
```

Mupad [B] (verification not implemented)

Time = 20.67 (sec) , antiderivative size = 711, normalized size of antiderivative = 8.36

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\tan(e+f\,x)}{2\,a\,f\,\left(b\tan\left(e+f\,x\right)^2+a+b\right)}-\frac{x}{a^2} - \frac{\operatorname{atan}\left(\frac{\frac{\left(\frac{\tan(e+f\,x)\,\left(a^2\,b+4\,a\,b^2+8\,b^3\right)}{2\,a^2}-\frac{\left(2\,a\,b^2-\frac{\tan(e+f\,x)\,\left(16\,a^5\,b^2+32\,a^4\,b^3\right)\,(a+2\,b)\,\sqrt{-b\,(a+b)}}{8\,a^2\,\left(a^3\,b+a^2\,b^2\right)}\right)\,(a+2\,b)\,\sqrt{-b\,(a+b)}}{4\,\left(a^3\,b+a^2\,b^2\right)}\right)\,(a+2\,b)\,\sqrt{-b\,(a+b)}\,1\mathrm{i}}{4\,\left(a^3\,b+a^2\,b^2\right)}+\cdots}{\frac{b^2+\frac{a\,b}{2}}{a^3}-\frac{\left(\frac{\tan(e+f\,x)\,\left(a^2\,b+4\,a\,b^2+8\,b^3\right)}{2\,a^2}-\frac{\left(2\,a\,b^2-\frac{\tan(e+f\,x)\,\left(16\,a^5\,b^2+32\,a^4\,b^3\right)\,(a+2\,b)\,\sqrt{-b\,(a+b)}}{8\,a^2\,\left(a^3\,b+a^2\,b^2\right)}\right)\,(a+2\,b)\,\sqrt{-b\,(a+b)}}{4\,\left(a^3\,b+a^2\,b^2\right)}\right)\,(a+2\,b)\,\sqrt{-b\,(a+b)}}{4\,\left(a^3\,b+a^2\,b^2\right)}\cdots}\right)\cdots}{2\cdots}$$

```
[In] int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] tan(e + f*x)/(2*a*f*(a + b + b*tan(e + f*x)^2)) - x/a^2 - (atan(((((tan(e +
 f*x)*(4*a*b^2 + a^2*b + 8*b^3))/(2*a^2) - ((2*a*b^2 - (tan(e + f*x)*(32*a^
4*b^3 + 16*a^5*b^2)*(a + 2*b)*(-b*(a + b))^(1/2))/(8*a^2*(a^3*b + a^2*b^2))
)*(a + 2*b)*(-b*(a + b))^(1/2))/(4*(a^3*b + a^2*b^2)))*(a + 2*b)*(-b*(a + b
))^(1/2)*1i)/(4*(a^3*b + a^2*b^2)) + (((tan(e + f*x)*(4*a*b^2 + a^2*b + 8*b
^3))/(2*a^2) + ((2*a*b^2 + (tan(e + f*x)*(32*a^4*b^3 + 16*a^5*b^2)*(a + 2*b
)*(-b*(a + b))^(1/2))/(8*a^2*(a^3*b + a^2*b^2)))*(a + 2*b)*(-b*(a + b))^(1/
2))/(4*(a^3*b + a^2*b^2)))*(a + 2*b)*(-b*(a + b))^(1/2)*1i)/(4*(a^3*b + a^2
*b^2)))/(((a*b)/2 + b^2)/a^3 - (((tan(e + f*x)*(4*a*b^2 + a^2*b + 8*b^3))/(
2*a^2) - ((2*a*b^2 - (tan(e + f*x)*(32*a^4*b^3 + 16*a^5*b^2)*(a + 2*b)*(-b*
(a + b))^(1/2))/(8*a^2*(a^3*b + a^2*b^2)))*(a + 2*b)*(-b*(a + b))^(1/2))/(4
*(a^3*b + a^2*b^2)))*(a + 2*b)*(-b*(a + b))^(1/2))/(4*(a^3*b + a^2*b^2)) +
(((tan(e + f*x)*(4*a*b^2 + a^2*b + 8*b^3))/(2*a^2) + ((2*a*b^2 + (tan(e + f
*x)*(32*a^4*b^3 + 16*a^5*b^2)*(a + 2*b)*(-b*(a + b))^(1/2))/(8*a^2*(a^3*b +
 a^2*b^2)))*(a + 2*b)*(-b*(a + b))^(1/2))/(4*(a^3*b + a^2*b^2)))*(a + 2*b)*
(-b*(a + b))^(1/2))/(4*(a^3*b + a^2*b^2))))*(a + 2*b)*(-b*(a + b))^(1/2)*1i
)/(2*f*(a^3*b + a^2*b^2))
```

3.359 $\int \frac{1}{(a+b\sec^2(e+fx))^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 92

$$\int \frac{1}{(a+b\sec^2(e+fx))^2} dx = \frac{x}{a^2} - \frac{\sqrt{b}(3a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}f} - \frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

```
[Out] x/a^2-1/2*(3*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/a^2/(a+b
)^(3/2)/f-1/2*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4213, 425, 536, 209, 211}

$$\int \frac{1}{(a+b\sec^2(e+fx))^2} dx = -\frac{\sqrt{b}(3a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{3/2}} + \frac{x}{a^2} - \frac{b\tan(e+fx)}{2af(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-2),x]
```

```
[Out] x/a^2 - (Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2
*a^2*(a + b)^(3/2)*f) - (b*Tan[e + f*x])/(2*a*(a + b)*f*(a + b + b*Tan[e +
f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{b\tan(e+fx)}{2a(a+b)f\,(a+b+b\tan^2(e+fx))} + \frac{\text{Subst}\left(\int \frac{2a+b-bx^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{2a(a+b)f} \\
&= -\frac{b\tan(e+fx)}{2a(a+b)f\,(a+b+b\tan^2(e+fx))} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^2f} \\
&\quad - \frac{(b(3a+2b))\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{2a^2(a+b)f}
\end{aligned}
$$

$$= \frac{x}{a^2} - \frac{\sqrt{b}(3a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}f} - \frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.20 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.61

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(2x(a+2b+a\cos(2(e+fx)))+\frac{b(3a+2b)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e}{2\sqrt{a+b}}\right.}{}\right.}{8a^2\left(a+b\sec^2(e+\right.}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(2*x*(a + 2*b + a*Cos[2*(e +
 f*x)]) + (b*(3*a + 2*b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2
*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e]
)^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/((a + b)^(3/
2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (b*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]
))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*a^2*(a + b*Sec[e +
f*x]^2)^2)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(3a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^2}+\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
default	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(3a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^2}+\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
risch	$\frac{x}{a^2}-\frac{ib\left(a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a\right)}{a^2(a+b)f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{3\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{4(a+b)^2fa}+$

```
[In] int(1/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-b/a^2*(1/2*a/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a+2*b)/(a+b
)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^2*arctan(tan(f*
x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(80) = 160$.

Time = 0.29 (sec) , antiderivative size = 435, normalized size of antiderivative = 4.73

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \left[\frac{8\,(a^2+ab)fx\cos{(fx+e)}^2 - 4\,ab\cos{(fx+e)}\sin{(fx+e)} + 8\,(ab+b^2)fx + \left((3\,a^2+2\,ab)\cos{(fx+e)}^2 \right.}{8\,\left((a^4+a^3b \right.} \right.$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 4*a*b*cos(f*x + e)*sin(f*x + e) +
8*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt(-
b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*co
s(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*
x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*co
s(f*x + e)^2 + b^2)))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*b^2)*f
), 1/4*(4*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 2*a*b*cos(f*x + e)*sin(f*x + e)
+ 4*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt
(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*co
s(f*x + e)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*b
^2)*f)]
```

Sympy [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{1}{(a+b\sec^2{(e+fx)})^2}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{b\tan(fx+e)}{a^3+2\,a^2b+ab^2+(a^2b+ab^2)\tan(fx+e)^2}+\frac{(3\,ab+2\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^3+a^2b)\sqrt{(a+b)b}}-\frac{2\,(fx+e)}{a^2}}{2\,f}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*tan(f*x + e)/(a^3 + 2*a^2*b + a*b^2 + (a^2*b + a*b^2)*tan(f*x + e)^
2) + (3*a*b + 2*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + a^2*b)*
sqrt((a + b)*b)) - 2*(f*x + e)/a^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.24

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3\,ab+2\,b^2)}{(a^3+a^2b)\sqrt{ab+b^2}}+\frac{b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)(a^2+ab)}-\frac{2\,(fx+e)}{a^2}}{2\,f}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
 + b^2)))*(3*a*b + 2*b^2)/((a^3 + a^2*b)*sqrt(a*b + b^2)) + b*tan(f*x + e)/
((b*tan(f*x + e)^2 + a + b)*(a^2 + a*b)) - 2*(f*x + e)/a^2)/f
```

Mupad [B] (verification not implemented)

Time = 21.73 (sec) , antiderivative size = 2056, normalized size of antiderivative = 22.35

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] atan((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^
2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8
*a^2*(2*a^3*b + a^4 + a^2*b^2)))/(2*a^2) + (tan(e + f*x)*(20*a*b^4 + 8*b^5
+ 13*a^2*b^3))/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 - ((((2*a^4*b^4 + 6*a^5*b
^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^2)) + (tan(e + f*x)*(32*a^4*b
^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2
)))/(2*a^2) - (tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(4*(2*a^3*b +
a^4 + a^2*b^2)))/a^2)/((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^
4*b + a^5 + a^3*b^2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3
 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) + (tan(e + f
*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2
+ (((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^2))
 + (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^
2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) - (tan(e + f*x)*(20*a*b^4 + 8*b^5
 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 + ((3*a*b^3)/2 + b^4)
/(2*a^4*b + a^5 + a^3*b^2)))/(a^2*f) + (atan(((((tan(e + f*x)*(20*a*b^4 + 8
*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((-b*(a + b)^3)^(1/2)*(
(2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a^3*b^2) - (tan(e + f*
x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 +
 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3
*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(-b*(a + b)
^3)^(1/2)*(3*a + 2*b)*1i)/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)) + (((ta
n(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) +
 ((-b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5
+ a^3*b^2) + (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 8
0*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b
 + a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 +
3*a^3*b^2)))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*1i)/(4*(3*a^4*b + a^5 + a^2*b
^3 + 3*a^3*b^2)))/((((3*a*b^3)/2 + b^4)/(2*a^4*b + a^5 + a^3*b^2) - (((tan(e
 + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((
-b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a
^3*b^2) - (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a
^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b +
a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a
^3*b^2)))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3
*a^3*b^2)) + (((tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b +
 a^4 + a^2*b^2)) + ((-b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^
2)/(2*a^4*b + a^5 + a^3*b^2) + (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*
b)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 +
 a^2*b^2)*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b
+ a^5 + a^2*b^3 + 3*a^3*b^2)))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b))/(4*(3*a^4*
b + a^5 + a^2*b^3 + 3*a^3*b^2))))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*1i)/(2*f
*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)) - (b*tan(e + f*x))/(2*a*f*(a + b)*(
a + b + b*tan(e + f*x)^2))
```

3.360 $\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{x}{a^2} + \frac{b^{3/2}(5a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{5/2}f} - \frac{(2a-b)\cot(e+fx)}{2a(a+b)^2f} - \frac{b\cot(e+fx)}{2a(a+b)f\,(a+b+b\tan^2(e+fx))}$$

```
[Out] -x/a^2+1/2*b^(3/2)*(5*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^2/(a+
b)^(5/2)/f-1/2*(2*a-b)*cot(f*x+e)/a/(a+b)^2/f-1/2*b*cot(f*x+e)/a/(a+b)/f/(a
+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4226, 2000, 483, 597, 536, 209, 211}

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{b^{3/2}(5a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{5/2}} - \frac{x}{a^2} - \frac{(2a-b)\cot(e+fx)}{2af(a+b)^2} - \frac{b\cot(e+fx)}{2af(a+b)\,(a+b\tan^2(e+fx)+b)}$$

```
[In] Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(x/a^2) + (b^(3/2)*(5*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])
/(2*a^2*(a + b)^(5/2)*f) - ((2*a - b)*Cot[e + f*x])/(2*a*(a + b)^2*f) - (b*
Cot[e + f*x])/(2*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+b(1+x^2))^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{b\cot(e+fx)}{2a(a+b)f\,(a+b+b\tan^2(e+fx))} + \frac{\text{Subst}\left(\int \frac{2a-b-3bx^2}{x^2(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{2a(a+b)f}\\
&= -\frac{(2a-b)\cot(e+fx)}{2a(a+b)^2f} - \frac{b\cot(e+fx)}{2a(a+b)f\,(a+b+b\tan^2(e+fx))}\\
&\quad - \frac{\text{Subst}\left(\int \frac{2a^2+6ab+b^2+(2a-b)bx^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{2a(a+b)^2f}\\
&= -\frac{(2a-b)\cot(e+fx)}{2a(a+b)^2f} - \frac{b\cot(e+fx)}{2a(a+b)f\,(a+b+b\tan^2(e+fx))}\\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^2f}\\
&\quad + \frac{(b^2(5a+2b))\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{2a^2(a+b)^2f}\\
&= -\frac{x}{a^2} + \frac{b^{3/2}(5a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{5/2}f}\\
&\quad - \frac{(2a-b)\cot(e+fx)}{2a(a+b)^2f} - \frac{b\cot(e+fx)}{2a(a+b)f\,(a+b+b\tan^2(e+fx))}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.34 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.38

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(-\frac{2x(a+2b+a\cos(2(e+fx)))}{a^2}-\frac{b^2(5a+2b)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)s}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin}}\right.}{a^2(a+b)^{5/}}\right.}{8(}$$

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((-2*x*(a + 2*b + a*Cos[2*(e
 + f*x)]))/a^2 - (b^2*(5*a + 2*b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*
(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] -
 I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/(a^
2*(a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (2*(a + 2*b + a*Cos[2*(e
 + f*x)])*Csc[e]*Csc[e + f*x]*Sin[f*x])/((a + b)^2*f) + (b^2*(-((a + 2*b)*S
in[2*e]) + a*Sin[2*f*x]))/(a^2*(a + b)^2*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[
e]))))/(8*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] (verified)

Time = 5.16 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{a^2}-\frac{1}{(a+b)^2\tan(fx+e)}+\frac{b^2\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(5a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^2(a+b)^2}}{f}$
default	$\frac{-\frac{\arctan(\tan(fx+e))}{a^2}-\frac{1}{(a+b)^2\tan(fx+e)}+\frac{b^2\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(5a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^2(a+b)^2}}{f}$
risch	$-\frac{x}{a^2}+\frac{i\left(-2a^3e^{4i(fx+e)}+a\,b^2e^{4i(fx+e)}+2b^3e^{4i(fx+e)}-4a^3e^{2i(fx+e)}-8a^2b\,e^{2i(fx+e)}-2b^3e^{2i(fx+e)}-2a^3-a\,b^2\right)}{a^2(a+b)^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)\left(e^{2i(fx+e)}-1\right)}-\frac{5\sqrt{}}{}$

```
[In] int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^2*arctan(tan(f*x+e))-1/(a+b)^2/tan(f*x+e)+b^2/a^2/(a+b)^2*(1/2*a*
tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(5*a+2*b)/((a+b)*b)^(1/2)*arctan(b*tan(
f*x+e)/((a+b)*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(107) = 214$.

Time = 0.30 (sec) , antiderivative size = 604, normalized size of antiderivative = 4.99

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \left[-\frac{4\,(2\,a^3+ab^2)\cos(fx+e)^3 - \left(5\,ab^2+2\,b^3+(5\,a^2b+2\,ab^2)\cos(fx+e)^2\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos \cdots}{\cdots}\right.}{\cdots}\right.$$
$$-\frac{2\,(2\,a^3+ab^2)\cos(fx+e)^3 + \left(5\,ab^2+2\,b^3+(5\,a^2b+2\,ab^2)\cos(fx+e)^2\right)\sqrt{\frac{b}{a+b}}\arctan\left(\frac{((a+2\,b)\cos(fx+ \cdots}{2\,b\cos(fx+e)\cdots}\right.}{4\left((a^5+2\,a^4b+a^3b^2)f \cdots\right.}$$

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(2*a^3 + a*b^2)*cos(f*x + e)^3 - (5*a*b^2 + 2*b^3 + (5*a^2*b + 2*a
*b^2)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x +
 e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x
 + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a
^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 4*(2*a^2*b
- a*b^2)*cos(f*x + e) + 8*((a^3 + 2*a^2*b + a*b^2)*f*x*cos(f*x + e)^2 + (a^
2*b + 2*a*b^2 + b^3)*f*x)*sin(f*x + e))/(((a^5 + 2*a^4*b + a^3*b^2)*f*cos(f
*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f)*sin(f*x + e)), -1/4*(2*(2*a^3
+ a*b^2)*cos(f*x + e)^3 + (5*a*b^2 + 2*b^3 + (5*a^2*b + 2*a*b^2)*cos(f*x +
e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a +
 b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 2*(2*a^2*b - a*b^2)*cos(
f*x + e) + 4*((a^3 + 2*a^2*b + a*b^2)*f*x*cos(f*x + e)^2 + (a^2*b + 2*a*b^2
 + b^3)*f*x)*sin(f*x + e))/(((a^5 + 2*a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (
a^4*b + 2*a^3*b^2 + a^2*b^3)*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\cot^2{(e+fx)}}{(a+b\sec^2{(e+fx)})^2}\,dx$$

```
[In] integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(cot(e + f*x)**2/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.35

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{(5\,ab^2+2\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^4+2\,a^3b+a^2b^2)\sqrt{(a+b)b}} - \frac{(2\,ab-b^2)\tan(fx+e)^2+2\,a^2+2\,ab}{(a^3b+2\,a^2b^2+ab^3)\tan(fx+e)^3+(a^4+3\,a^3b+3\,a^2b^2+ab^3)\tan(fx+e)} - \frac{2\,(fx+e)}{a^2}}{2\,f}$$

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((5*a*b^2 + 2*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4 + 2*a^3
*b + a^2*b^2)*sqrt((a + b)*b)) - ((2*a*b - b^2)*tan(f*x + e)^2 + 2*a^2 + 2*
a*b)/((a^3*b + 2*a^2*b^2 + a*b^3)*tan(f*x + e)^3 + (a^4 + 3*a^3*b + 3*a^2*b
^2 + a*b^3)*tan(f*x + e)) - 2*(f*x + e)/a^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.45

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{(5\,ab^2+2\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^4+2\,a^3b+a^2b^2)\sqrt{ab+b^2}} - \frac{2\,ab\tan(fx+e)^2-b^2\tan(fx+e)^2+2\,a^2+2\,ab}{\left(b\tan(fx+e)^3+a\tan(fx+e)+b\tan(fx+e)\right)(a^3+2\,a^2b+ab^2)} - \frac{2\,(fx+e)}{a^2}}{2\,f}$$

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((5*a*b^2 + 2*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(
f*x + e)/sqrt(a*b + b^2)))/((a^4 + 2*a^3*b + a^2*b^2)*sqrt(a*b + b^2)) - (2
*a*b*tan(f*x + e)^2 - b^2*tan(f*x + e)^2 + 2*a^2 + 2*a*b)/((b*tan(f*x + e)^
3 + a*tan(f*x + e) + b*tan(f*x + e))*(a^3 + 2*a^2*b + a*b^2)) - 2*(f*x + e)
/a^2)/f
```

Mupad [B] (verification not implemented)

Time = 24.79 (sec) , antiderivative size = 3146, normalized size of antiderivative = 26.00

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (atan((((tan(e + f*x)*(128*a^3*b^13 + 1344*a^4*b^12 + 6160*a^5*b^11 + 16160
*a^6*b^10 + 26800*a^7*b^9 + 29312*a^8*b^8 + 21424*a^9*b^7 + 10400*a^10*b^6
+ 3280*a^11*b^5 + 640*a^12*b^4 + 64*a^13*b^3) - ((-b^3*(a + b)^5)^(1/2)*(5*
a + 2*b)*(64*a^6*b^12 + 896*a^7*b^11 + 4992*a^8*b^10 + 15360*a^9*b^9 + 2956
8*a^10*b^8 + 37632*a^11*b^7 + 32256*a^12*b^6 + 18432*a^13*b^5 + 6720*a^14*b
^4 + 1408*a^15*b^3 + 128*a^16*b^2 - (tan(e + f*x)*(-b^3*(a + b)^5)^(1/2)*(5
*a + 2*b)*(512*a^7*b^13 + 5376*a^8*b^12 + 25600*a^9*b^11 + 72960*a^10*b^10
+ 138240*a^11*b^9 + 182784*a^12*b^8 + 172032*a^13*b^7 + 115200*a^14*b^6 + 5
3760*a^15*b^5 + 16640*a^16*b^4 + 3072*a^17*b^3 + 256*a^18*b^2))/(4*(5*a^6*b
 + a^7 + a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2))))/(4*(5*a^6*b + a^
7 + a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2)))*(-b^3*(a + b)^5)^(1/2)
*(5*a + 2*b)*1i)/(4*(5*a^6*b + a^7 + a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*
a^5*b^2)) + ((tan(e + f*x)*(128*a^3*b^13 + 1344*a^4*b^12 + 6160*a^5*b^11 +
16160*a^6*b^10 + 26800*a^7*b^9 + 29312*a^8*b^8 + 21424*a^9*b^7 + 10400*a^10
*b^6 + 3280*a^11*b^5 + 640*a^12*b^4 + 64*a^13*b^3) + ((-b^3*(a + b)^5)^(1/2
)*(5*a + 2*b)*(64*a^6*b^12 + 896*a^7*b^11 + 4992*a^8*b^10 + 15360*a^9*b^9 +
 29568*a^10*b^8 + 37632*a^11*b^7 + 32256*a^12*b^6 + 18432*a^13*b^5 + 6720*a
^14*b^4 + 1408*a^15*b^3 + 128*a^16*b^2 + (tan(e + f*x)*(-b^3*(a + b)^5)^(1/
2)*(5*a + 2*b)*(512*a^7*b^13 + 5376*a^8*b^12 + 25600*a^9*b^11 + 72960*a^10*
b^10 + 138240*a^11*b^9 + 182784*a^12*b^8 + 172032*a^13*b^7 + 115200*a^14*b^
6 + 53760*a^15*b^5 + 16640*a^16*b^4 + 3072*a^17*b^3 + 256*a^18*b^2))/(4*(5*
a^6*b + a^7 + a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2))))/(4*(5*a^6*b
 + a^7 + a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2)))*(-b^3*(a + b)^5)^
(1/2)*(5*a + 2*b)*1i)/(4*(5*a^6*b + a^7 + a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3
+ 10*a^5*b^2)))/(80*a^5*b^9 - 208*a^3*b^11 - 416*a^4*b^10 - 32*a^2*b^12 + 1
600*a^6*b^8 + 2768*a^7*b^7 + 2272*a^8*b^6 + 944*a^9*b^5 + 160*a^10*b^4 + ((
tan(e + f*x)*(128*a^3*b^13 + 1344*a^4*b^12 + 6160*a^5*b^11 + 16160*a^6*b^10
 + 26800*a^7*b^9 + 29312*a^8*b^8 + 21424*a^9*b^7 + 10400*a^10*b^6 + 3280*a^
11*b^5 + 640*a^12*b^4 + 64*a^13*b^3) - ((-b^3*(a + b)^5)^(1/2)*(5*a + 2*b)*
(64*a^6*b^12 + 896*a^7*b^11 + 4992*a^8*b^10 + 15360*a^9*b^9 + 29568*a^10*b^
8 + 37632*a^11*b^7 + 32256*a^12*b^6 + 18432*a^13*b^5 + 6720*a^14*b^4 + 1408
*a^15*b^3 + 128*a^16*b^2 - (tan(e + f*x)*(-b^3*(a + b)^5)^(1/2)*(5*a + 2*b)
*(512*a^7*b^13 + 5376*a^8*b^12 + 25600*a^9*b^11 + 72960*a^10*b^10 + 138240*
a^11*b^9 + 182784*a^12*b^8 + 172032*a^13*b^7 + 115200*a^14*b^6 + 53760*a^15
*b^5 + 16640*a^16*b^4 + 3072*a^17*b^3 + 256*a^18*b^2))/(4*(5*a^6*b + a^7 +
a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2))))/(4*(5*a^6*b + a^7 + a^2*b
```

```
^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2)))*(-b^3*(a + b)^5)^(1/2)*(5*a + 2
*b))/(4*(5*a^6*b + a^7 + a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2)) -
((tan(e + f*x)*(128*a^3*b^13 + 1344*a^4*b^12 + 6160*a^5*b^11 + 16160*a^6*b^
10 + 26800*a^7*b^9 + 29312*a^8*b^8 + 21424*a^9*b^7 + 10400*a^10*b^6 + 3280*
a^11*b^5 + 640*a^12*b^4 + 64*a^13*b^3) + ((-b^3*(a + b)^5)^(1/2)*(5*a + 2*b
)*(64*a^6*b^12 + 896*a^7*b^11 + 4992*a^8*b^10 + 15360*a^9*b^9 + 29568*a^10*
b^8 + 37632*a^11*b^7 + 32256*a^12*b^6 + 18432*a^13*b^5 + 6720*a^14*b^4 + 14
08*a^15*b^3 + 128*a^16*b^2 + (tan(e + f*x)*(-b^3*(a + b)^5)^(1/2)*(5*a + 2*
b)*(512*a^7*b^13 + 5376*a^8*b^12 + 25600*a^9*b^11 + 72960*a^10*b^10 + 13824
0*a^11*b^9 + 182784*a^12*b^8 + 172032*a^13*b^7 + 115200*a^14*b^6 + 53760*a^
15*b^5 + 16640*a^16*b^4 + 3072*a^17*b^3 + 256*a^18*b^2))/(4*(5*a^6*b + a^7
+ a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2))))/(4*(5*a^6*b + a^7 + a^2
*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2)))*(-b^3*(a + b)^5)^(1/2)*(5*a +
 2*b))/(4*(5*a^6*b + a^7 + a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2)))
)*(-b^3*(a + b)^5)^(1/2)*(5*a + 2*b)*1i)/(2*f*(5*a^6*b + a^7 + a^2*b^5 + 5*
a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2)) - atan((240*a^3*b^11*tan(e + f*x))/(240
*a^3*b^11 + 2080*a^4*b^10 + 7760*a^5*b^9 + 16384*a^6*b^8 + 21584*a^7*b^7 +
18400*a^8*b^6 + 10160*a^9*b^5 + 3520*a^10*b^4 + 704*a^11*b^3 + 64*a^12*b^2)
 + (2080*a^4*b^10*tan(e + f*x))/(240*a^3*b^11 + 2080*a^4*b^10 + 7760*a^5*b^
9 + 16384*a^6*b^8 + 21584*a^7*b^7 + 18400*a^8*b^6 + 10160*a^9*b^5 + 3520*a^
10*b^4 + 704*a^11*b^3 + 64*a^12*b^2) + (7760*a^5*b^9*tan(e + f*x))/(240*a^3
*b^11 + 2080*a^4*b^10 + 7760*a^5*b^9 + 16384*a^6*b^8 + 21584*a^7*b^7 + 1840
0*a^8*b^6 + 10160*a^9*b^5 + 3520*a^10*b^4 + 704*a^11*b^3 + 64*a^12*b^2) + (
16384*a^6*b^8*tan(e + f*x))/(240*a^3*b^11 + 2080*a^4*b^10 + 7760*a^5*b^9 +
16384*a^6*b^8 + 21584*a^7*b^7 + 18400*a^8*b^6 + 10160*a^9*b^5 + 3520*a^10*b
^4 + 704*a^11*b^3 + 64*a^12*b^2) + (21584*a^7*b^7*tan(e + f*x))/(240*a^3*b^
11 + 2080*a^4*b^10 + 7760*a^5*b^9 + 16384*a^6*b^8 + 21584*a^7*b^7 + 18400*a
^8*b^6 + 10160*a^9*b^5 + 3520*a^10*b^4 + 704*a^11*b^3 + 64*a^12*b^2) + (184
00*a^8*b^6*tan(e + f*x))/(240*a^3*b^11 + 2080*a^4*b^10 + 7760*a^5*b^9 + 163
84*a^6*b^8 + 21584*a^7*b^7 + 18400*a^8*b^6 + 10160*a^9*b^5 + 3520*a^10*b^4
+ 704*a^11*b^3 + 64*a^12*b^2) + (10160*a^9*b^5*tan(e + f*x))/(240*a^3*b^11
+ 2080*a^4*b^10 + 7760*a^5*b^9 + 16384*a^6*b^8 + 21584*a^7*b^7 + 18400*a^8*
b^6 + 10160*a^9*b^5 + 3520*a^10*b^4 + 704*a^11*b^3 + 64*a^12*b^2) + (3520*a
^10*b^4*tan(e + f*x))/(240*a^3*b^11 + 2080*a^4*b^10 + 7760*a^5*b^9 + 16384*
a^6*b^8 + 21584*a^7*b^7 + 18400*a^8*b^6 + 10160*a^9*b^5 + 3520*a^10*b^4 + 7
04*a^11*b^3 + 64*a^12*b^2) + (704*a^11*b^3*tan(e + f*x))/(240*a^3*b^11 + 20
80*a^4*b^10 + 7760*a^5*b^9 + 16384*a^6*b^8 + 21584*a^7*b^7 + 18400*a^8*b^6
+ 10160*a^9*b^5 + 3520*a^10*b^4 + 704*a^11*b^3 + 64*a^12*b^2) + (64*a^12*b^
2*tan(e + f*x))/(240*a^3*b^11 + 2080*a^4*b^10 + 7760*a^5*b^9 + 16384*a^6*b^
8 + 21584*a^7*b^7 + 18400*a^8*b^6 + 10160*a^9*b^5 + 3520*a^10*b^4 + 704*a^1
1*b^3 + 64*a^12*b^2))/(a^2*f) - (1/(a + b) + (tan(e + f*x)^2*(2*a*b - b^2))
/(2*a*(a + b)^2))/(f*(b*tan(e + f*x)^3 + tan(e + f*x)*(a + b)))
```

3.361 $\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 160

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{x}{a^2} - \frac{b^{5/2}(7a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{7/2}f} + \frac{(2a^2+6ab-b^2)\cot(e+fx)}{2a(a+b)^3f} - \frac{(2a-3b)\cot^3(e+fx)}{6a(a+b)^2f} - \frac{b\cot^3(e+fx)}{2a(a+b)f\,(a+b+b\tan^2(e+fx))}$$

```
[Out] x/a^2-1/2*b^(5/2)*(7*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^2/(a+b
)^(7/2)/f+1/2*(2*a^2+6*a*b-b^2)*cot(f*x+e)/a/(a+b)^3/f-1/6*(2*a-3*b)*cot(f*
x+e)^3/a/(a+b)^2/f-1/2*b*cot(f*x+e)^3/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4226, 2000, 483, 597, 536, 209, 211}

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{b^{5/2}(7a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{7/2}} + \frac{(2a^2+6ab-b^2)\cot(e+fx)}{2af(a+b)^3} + \frac{x}{a^2} - \frac{(2a-3b)\cot^3(e+fx)}{6af(a+b)^2} - \frac{b\cot^3(e+fx)}{2af(a+b)\,(a+b\tan^2(e+fx)+b)}$$

```
[In] Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] x/a^2 - (b^(5/2)*(7*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2
*a^2*(a + b)^(7/2)*f) + ((2*a^2 + 6*a*b - b^2)*Cot[e + f*x])/(2*a*(a + b)^3
*f) - ((2*a - 3*b)*Cot[e + f*x]^3)/(6*a*(a + b)^2*f) - (b*Cot[e + f*x]^3)/(
2*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+b(1+x^2))^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^3(e+fx)}{2a(a+b)f\,(a+b+b\tan^2(e+fx))} + \frac{\text{Subst}\left(\int \frac{2a-3b-5bx^2}{x^4(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{2a(a+b)f}$$

$$= -\frac{(2a-3b)\cot^3(e+fx)}{6a(a+b)^2f} - \frac{b\cot^3(e+fx)}{2a(a+b)f\,(a+b+b\tan^2(e+fx))} - \frac{\text{Subst}\left(\int \frac{3(2a^2+6ab-b^2)+3(2a-3b)bx^2}{x^2(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{6a(a+b)^2f}$$

$$= \frac{(2a^2+6ab-b^2)\cot(e+fx)}{2a(a+b)^3f} - \frac{(2a-3b)\cot^3(e+fx)}{6a(a+b)^2f} - \frac{b\cot^3(e+fx)}{2a(a+b)f\,(a+b+b\tan^2(e+fx))} + \frac{\text{Subst}\left(\int \frac{3(2a^3+8a^2b+12ab^2+b^3)+3b(2a^2+6ab-b^2)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{6a(a+b)^3f}$$

$$= \frac{(2a^2+6ab-b^2)\cot(e+fx)}{2a(a+b)^3f} - \frac{(2a-3b)\cot^3(e+fx)}{6a(a+b)^2f} - \frac{b\cot^3(e+fx)}{2a(a+b)f\,(a+b+b\tan^2(e+fx))} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^2f} - \frac{(b^3(7a+2b))\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{2a^2(a+b)^3f}$$

$$= \frac{x}{a^2} - \frac{b^{5/2}(7a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{7/2}f} + \frac{(2a^2+6ab-b^2)\cot(e+fx)}{2a(a+b)^3 f} - \frac{(2a-3b)\cot^3(e+fx)}{6a(a+b)^2 f} - \frac{b\cot^3(e+fx)}{2a(a+b)f\,(a+b+b\tan^2(e+fx))}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.33 (sec) , antiderivative size = 1896, normalized size of antiderivative = 11.85

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \frac{(7a+2b)(a+2b+a\cos(2e+2fx))^2\sec^4(e+fx)\left(\frac{b^3\arctan\left(\sec(fx)\left(\frac{\cos(2e)}{2\sqrt{a+b}\sqrt{b\cos(4e)-ib\sin(4e)}}-\frac{i\sin(2e)}{2\sqrt{a+b}\sqrt{b\cos(4e)-ib\sin(4e)}}\right.\right.}{8a^2\sqrt{a+b}f\sqrt{b\cos(4e)-ib\sin}}\right.}{(\ldots}$$

$$+ \frac{(a+2b+a\cos(2e+2fx))\csc(e)\csc^3(e+fx)\sec(2e)\sec^4(e+fx)\,(-6a^4fx\cos(fx)-54a^3bfx\cos(fx)}{\ldots}$$

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((7*a + 2*b)*(a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*((b^3*ArcTan[S
ec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*
Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2
*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(8*a^2*Sqrt[a + b]*f*Sqrt[b*Cos[
4*e] - I*b*Sin[4*e]]) - ((I/8)*b^3*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]
*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Co
s[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]
*Sin[2*e])/(a^2*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]])))/((a + b)^3
*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[2*e + 2*f*x])*Csc[e]*Csc[e +
 f*x]^3*Sec[2*e]*Sec[e + f*x]^4*(-6*a^4*f*x*Cos[f*x] - 54*a^3*b*f*x*Cos[f*x
] - 126*a^2*b^2*f*x*Cos[f*x] - 114*a*b^3*f*x*Cos[f*x] - 36*b^4*f*x*Cos[f*x]
 + 3*a^4*f*x*Cos[3*f*x] - 3*a^3*b*f*x*Cos[3*f*x] - 27*a^2*b^2*f*x*Cos[3*f*x
] - 33*a*b^3*f*x*Cos[3*f*x] - 12*b^4*f*x*Cos[3*f*x] + 6*a^4*f*x*Cos[2*e - f
*x] + 54*a^3*b*f*x*Cos[2*e - f*x] + 126*a^2*b^2*f*x*Cos[2*e - f*x] + 114*a*
b^3*f*x*Cos[2*e - f*x] + 36*b^4*f*x*Cos[2*e - f*x] + 6*a^4*f*x*Cos[2*e + f*
x] + 54*a^3*b*f*x*Cos[2*e + f*x] + 126*a^2*b^2*f*x*Cos[2*e + f*x] + 114*a*b
^3*f*x*Cos[2*e + f*x] + 36*b^4*f*x*Cos[2*e + f*x] - 6*a^4*f*x*Cos[4*e + f*x
] - 54*a^3*b*f*x*Cos[4*e + f*x] - 126*a^2*b^2*f*x*Cos[4*e + f*x] - 114*a*b^
3*f*x*Cos[4*e + f*x] - 36*b^4*f*x*Cos[4*e + f*x] - 3*a^4*f*x*Cos[2*e + 3*f*
x] + 3*a^3*b*f*x*Cos[2*e + 3*f*x] + 27*a^2*b^2*f*x*Cos[2*e + 3*f*x] + 33*a*
b^3*f*x*Cos[2*e + 3*f*x] + 12*b^4*f*x*Cos[2*e + 3*f*x] + 3*a^4*f*x*Cos[4*e
+ 3*f*x] - 3*a^3*b*f*x*Cos[4*e + 3*f*x] - 27*a^2*b^2*f*x*Cos[4*e + 3*f*x] -
```

```
33*a*b^3*f*x*Cos[4*e + 3*f*x] - 12*b^4*f*x*Cos[4*e + 3*f*x] - 3*a^4*f*x*Co
s[6*e + 3*f*x] + 3*a^3*b*f*x*Cos[6*e + 3*f*x] + 27*a^2*b^2*f*x*Cos[6*e + 3*
f*x] + 33*a*b^3*f*x*Cos[6*e + 3*f*x] + 12*b^4*f*x*Cos[6*e + 3*f*x] - 3*a^4*
f*x*Cos[2*e + 5*f*x] - 9*a^3*b*f*x*Cos[2*e + 5*f*x] - 9*a^2*b^2*f*x*Cos[2*e
 + 5*f*x] - 3*a*b^3*f*x*Cos[2*e + 5*f*x] + 3*a^4*f*x*Cos[4*e + 5*f*x] + 9*a
^3*b*f*x*Cos[4*e + 5*f*x] + 9*a^2*b^2*f*x*Cos[4*e + 5*f*x] + 3*a*b^3*f*x*Co
s[4*e + 5*f*x] - 3*a^4*f*x*Cos[6*e + 5*f*x] - 9*a^3*b*f*x*Cos[6*e + 5*f*x]
- 9*a^2*b^2*f*x*Cos[6*e + 5*f*x] - 3*a*b^3*f*x*Cos[6*e + 5*f*x] + 3*a^4*f*x
*Cos[8*e + 5*f*x] + 9*a^3*b*f*x*Cos[8*e + 5*f*x] + 9*a^2*b^2*f*x*Cos[8*e +
5*f*x] + 3*a*b^3*f*x*Cos[8*e + 5*f*x] - 12*a^4*Sin[f*x] - 60*a^3*b*Sin[f*x]
 - 96*a^2*b^2*Sin[f*x] + 18*b^4*Sin[f*x] + 4*a^4*Sin[3*f*x] + 36*a^3*b*Sin[
3*f*x] + 80*a^2*b^2*Sin[3*f*x] - 6*a*b^3*Sin[3*f*x] + 6*b^4*Sin[3*f*x] + 4*
a^4*Sin[2*e - f*x] + 76*a^3*b*Sin[2*e - f*x] + 144*a^2*b^2*Sin[2*e - f*x] +
 18*b^4*Sin[2*e - f*x] - 4*a^4*Sin[2*e + f*x] - 76*a^3*b*Sin[2*e + f*x] - 1
44*a^2*b^2*Sin[2*e + f*x] + 6*a*b^3*Sin[2*e + f*x] + 18*b^4*Sin[2*e + f*x]
- 12*a^4*Sin[4*e + f*x] - 60*a^3*b*Sin[4*e + f*x] - 96*a^2*b^2*Sin[4*e + f*
x] - 6*a*b^3*Sin[4*e + f*x] - 18*b^4*Sin[4*e + f*x] - 12*a^4*Sin[2*e + 3*f*
x] - 24*a^3*b*Sin[2*e + 3*f*x] + 6*a*b^3*Sin[2*e + 3*f*x] - 6*b^4*Sin[2*e +
 3*f*x] + 4*a^4*Sin[4*e + 3*f*x] + 36*a^3*b*Sin[4*e + 3*f*x] + 80*a^2*b^2*S
in[4*e + 3*f*x] - 3*a*b^3*Sin[4*e + 3*f*x] - 6*b^4*Sin[4*e + 3*f*x] - 12*a^
4*Sin[6*e + 3*f*x] - 24*a^3*b*Sin[6*e + 3*f*x] + 3*a*b^3*Sin[6*e + 3*f*x] +
 6*b^4*Sin[6*e + 3*f*x] + 8*a^4*Sin[2*e + 5*f*x] + 20*a^3*b*Sin[2*e + 5*f*x
] + 3*a*b^3*Sin[2*e + 5*f*x] - 3*a*b^3*Sin[4*e + 5*f*x] + 8*a^4*Sin[6*e + 5
*f*x] + 20*a^3*b*Sin[6*e + 5*f*x]))/(384*a^2*(a + b)^3*f*(a + b*Sec[e + f*x
]^2)^2)
```

Maple [A] (verified)

Time = 9.73 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a^2}-\frac{1}{3(a+b)^2\tan(fx+e)^3}-\frac{-a-3b}{(a+b)^3\tan(fx+e)}-\frac{b^3\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(7a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^2(a+b)^3}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{a^2}-\frac{1}{3(a+b)^2\tan(fx+e)^3}-\frac{-a-3b}{(a+b)^3\tan(fx+e)}-\frac{b^3\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(7a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^2(a+b)^3}}{f}$
risch	$\frac{x}{a^2}-\frac{i\left(-12a^4 e^{8i(fx+e)}-24a^3b\, e^{8i(fx+e)}+3a\, b^3 e^{8i(fx+e)}+6b^4 e^{8i(fx+e)}-12a^4 e^{6i(fx+e)}-60a^3b\, e^{6i(fx+e)}-96a^2b^2 e^{6i(\right.}{}$

```
[In] int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^2*arctan(tan(f*x+e))-1/3/(a+b)^2/tan(f*x+e)^3-(-a-3*b)/(a+b)^3/tan
```

```
(f*x+e)-b^3/a^2/(a+b)^3*(1/2*a*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(7*a+2*b
)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(144) = 288$.

Time = 0.32 (sec) , antiderivative size = 979, normalized size of antiderivative = 6.12

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/24*(4*(8*a^4 + 20*a^3*b + 3*a*b^3)*cos(f*x + e)^5 - 8*(3*a^4 + 5*a^3*b -
 10*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^3 + 3*((7*a^2*b^2 + 2*a*b^3)*cos(f*x +
e)^4 - 7*a*b^3 - 2*b^4 - (7*a^2*b^2 - 5*a*b^3 - 2*b^4)*cos(f*x + e)^2)*sqrt
(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*
cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(
f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*
cos(f*x + e)^2 + b^2))*sin(f*x + e) - 12*(2*a^3*b + 6*a^2*b^2 - a*b^3)*cos(
f*x + e) + 24*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)^4 - (a^
4 + 2*a^3*b - 2*a*b^3 - b^4)*f*x*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*
b^3 + b^4)*f*x)*sin(f*x + e))/(((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos
(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^2 - (a^5
*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f)*sin(f*x + e)), 1/12*(2*(8*a^4 + 20
*a^3*b + 3*a*b^3)*cos(f*x + e)^5 - 4*(3*a^4 + 5*a^3*b - 10*a^2*b^2 + 3*a*b^
3)*cos(f*x + e)^3 + 3*((7*a^2*b^2 + 2*a*b^3)*cos(f*x + e)^4 - 7*a*b^3 - 2*b
^4 - (7*a^2*b^2 - 5*a*b^3 - 2*b^4)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1
/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x +
 e)))*sin(f*x + e) - 6*(2*a^3*b + 6*a^2*b^2 - a*b^3)*cos(f*x + e) + 12*((a^
4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)^4 - (a^4 + 2*a^3*b - 2*a*
b^3 - b^4)*f*x*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f*x)*si
n(f*x + e))/(((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos(f*x + e)^4 - (a^6
 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^2 - (a^5*b + 3*a^4*b^2 + 3
*a^3*b^3 + a^2*b^4)*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx=\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

```
[In] integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(cot(e + f*x)**4/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.47

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx=$$
$$-\frac{\frac{3\,(7\,ab^3+2\,b^4)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^5+3\,a^4b+3\,a^3b^2+a^2b^3)\sqrt{(a+b)b}}-\frac{3\,(2\,a^2b+6\,ab^2-b^3)\tan(fx+e)^4-2\,a^3-4\,a^2b-2\,ab^2+2\,(3\,a^3+11\,a^2b+8\,ab^2)\tan(fx+e)^2}{(a^4b+3\,a^3b^2+3\,a^2b^3+ab^4)\tan(fx+e)^5+(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4)\tan(fx+e)^3}-\frac{6\,(fx+e}{a^2}}{6\,f}$$

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(7*a*b^3 + 2*b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^5 + 3*
a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt((a + b)*b)) - (3*(2*a^2*b + 6*a*b^2 - b^3
)*tan(f*x + e)^4 - 2*a^3 - 4*a^2*b - 2*a*b^2 + 2*(3*a^3 + 11*a^2*b + 8*a*b^
2)*tan(f*x + e)^2)/((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*tan(f*x + e)^5
+ (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*tan(f*x + e)^3) - 6*(f*x
+ e)/a^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.32

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx=$$
$$-\frac{\frac{3\,b^3\tan(fx+e)}{(a^4+3\,a^3b+3\,a^2b^2+ab^3)\left(b\tan(fx+e)^2+a+b\right)}+\frac{3\,(7\,ab^3+2\,b^4)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^5+3\,a^4b+3\,a^3b^2+a^2b^3)\sqrt{ab+b^2}}-\frac{6\,(fx+e)}{a^2}-\frac{2\left(3\,a\tan(f\right.}{(a^3+3\,a}}{6\,f}$$

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*b^3*tan(f*x + e)/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*(b*tan(f*x +
e)^2 + a + b)) + 3*(7*a*b^3 + 2*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
 arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*
b^3)*sqrt(a*b + b^2)) - 6*(f*x + e)/a^2 - 2*(3*a*tan(f*x + e)^2 + 9*b*tan(f
*x + e)^2 - a - b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(f*x + e)^3))/f
```

Mupad [B] (verification not implemented)

Time = 25.73 (sec) , antiderivative size = 4987, normalized size of antiderivative = 31.17

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

```
[In] int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] ((tan(e + f*x)^2*(3*a + 8*b))/(3*(a + b)^2) - 1/(3*(a + b)) + (tan(e + f*x)
^4*(6*a*b^2 + 2*a^2*b - b^3))/(2*a*(a + b)^3))/(f*(tan(e + f*x)^3*(a + b) +
 b*tan(e + f*x)^5)) + atan((560*a^3*b^16*tan(e + f*x))/(560*a^3*b^16 + 7280
*a^4*b^15 + 42560*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8
*b^11 + 741120*a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b
^7 + 116480*a^13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*
a^17*b^2) + (7280*a^4*b^15*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42
560*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 741120
*a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b^7 + 116480*a^
13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17*b^2) + (4
2560*a^5*b^14*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42560*a^5*b^14
+ 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 741120*a^9*b^10 + 6
99840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b^7 + 116480*a^13*b^6 + 3584
0*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17*b^2) + (149184*a^6*b^1
3*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42560*a^5*b^14 + 149184*a^6
*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 741120*a^9*b^10 + 699840*a^10*b
^9 + 505008*a^11*b^8 + 278768*a^12*b^7 + 116480*a^13*b^6 + 35840*a^14*b^5 +
 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17*b^2) + (351904*a^7*b^12*tan(e + f*
x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42560*a^5*b^14 + 149184*a^6*b^13 + 3519
04*a^7*b^12 + 593440*a^8*b^11 + 741120*a^9*b^10 + 699840*a^10*b^9 + 505008*
a^11*b^8 + 278768*a^12*b^7 + 116480*a^13*b^6 + 35840*a^14*b^5 + 7680*a^15*b
^4 + 1024*a^16*b^3 + 64*a^17*b^2) + (593440*a^8*b^11*tan(e + f*x))/(560*a^3
*b^16 + 7280*a^4*b^15 + 42560*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^12
+ 593440*a^8*b^11 + 741120*a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 + 2
78768*a^12*b^7 + 116480*a^13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^
16*b^3 + 64*a^17*b^2) + (741120*a^9*b^10*tan(e + f*x))/(560*a^3*b^16 + 7280
*a^4*b^15 + 42560*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8
*b^11 + 741120*a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b
^7 + 116480*a^13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*
a^17*b^2) + (699840*a^10*b^9*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 +
```

42560*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 7411
20*a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b^7 + 116480*
a^13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17*b^2) +
(505008*a^11*b^8*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42560*a^5*b^
14 + 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 741120*a^9*b^10
+ 699840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b^7 + 116480*a^13*b^6 + 3
5840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17*b^2) + (278768*a^12
*b^7*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42560*a^5*b^14 + 149184*
a^6*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 741120*a^9*b^10 + 699840*a^1
0*b^9 + 505008*a^11*b^8 + 278768*a^12*b^7 + 116480*a^13*b^6 + 35840*a^14*b^
5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17*b^2) + (116480*a^13*b^6*tan(e +
f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42560*a^5*b^14 + 149184*a^6*b^13 + 3
51904*a^7*b^12 + 593440*a^8*b^11 + 741120*a^9*b^10 + 699840*a^10*b^9 + 5050
08*a^11*b^8 + 278768*a^12*b^7 + 116480*a^13*b^6 + 35840*a^14*b^5 + 7680*a^1
5*b^4 + 1024*a^16*b^3 + 64*a^17*b^2) + (35840*a^14*b^5*tan(e + f*x))/(560*a
^3*b^16 + 7280*a^4*b^15 + 42560*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^1
2 + 593440*a^8*b^11 + 741120*a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 +
278768*a^12*b^7 + 116480*a^13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*
a^16*b^3 + 64*a^17*b^2) + (7680*a^15*b^4*tan(e + f*x))/(560*a^3*b^16 + 7280
*a^4*b^15 + 42560*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8
*b^11 + 741120*a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b
^7 + 116480*a^13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*
a^17*b^2) + (1024*a^16*b^3*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42
560*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 741120
*a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b^7 + 116480*a^
13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17*b^2) + (6
4*a^17*b^2*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42560*a^5*b^14 + 1
49184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 741120*a^9*b^10 + 6998
40*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b^7 + 116480*a^13*b^6 + 35840*a
^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17*b^2))/(a^2*f) - (atan((((
tan(e + f*x)*(128*a^3*b^18 + 1984*a^4*b^17 + 13840*a^5*b^16 + 57680*a^6*b^1
5 + 161280*a^7*b^14 + 322560*a^8*b^13 + 480928*a^9*b^12 + 550560*a^10*b^11
+ 494400*a^11*b^10 + 352640*a^12*b^9 + 199696*a^13*b^8 + 88144*a^14*b^7 + 2
9120*a^15*b^6 + 6720*a^16*b^5 + 960*a^17*b^4 + 64*a^18*b^3) - ((-b^5*(a + b
)^7)^(1/2)*(7*a + 2*b)*(64*a^6*b^17 + 1536*a^7*b^16 + 13952*a^8*b^15 + 7104
0*a^9*b^14 + 235968*a^10*b^13 + 551936*a^11*b^12 + 948992*a^12*b^11 + 12291
84*a^13*b^10 + 1214400*a^14*b^9 + 918016*a^15*b^8 + 528000*a^16*b^7 + 22745
6*a^17*b^6 + 71232*a^18*b^5 + 15360*a^19*b^4 + 2048*a^20*b^3 + 128*a^21*b^2
- (tan(e + f*x)*(-b^5*(a + b)^7)^(1/2)*(7*a + 2*b)*(512*a^7*b^18 + 7936*a^
8*b^17 + 57600*a^9*b^16 + 259840*a^10*b^15 + 815360*a^11*b^14 + 1886976*a^1
2*b^13 + 3331328*a^13*b^12 + 4576000*a^14*b^11 + 4942080*a^15*b^10 + 420992
0*a^16*b^9 + 2818816*a^17*b^8 + 1467648*a^18*b^7 + 582400*a^19*b^6 + 170240
*a^20*b^5 + 34560*a^21*b^4 + 4352*a^22*b^3 + 256*a^23*b^2))/(4*(7*a^8*b + a
^9 + a^2*b^7 + 7*a^3*b^6 + 21*a^4*b^5 + 35*a^5*b^4 + 35*a^6*b^3 + 21*a^7*b^
2))))/(4*(7*a^8*b + a^9 + a^2*b^7 + 7*a^3*b^6 + 21*a^4*b^5 + 35*a^5*b^4 + 3

5*a^6*b^3 + 21*a^7*b^2)))*(-b^5*(a + b)^7)^(1/2)*(7*a + 2*b)*1i)/(4*(7*a^8*
b + a^9 + a^2*b^7 + 7*a^3*b^6 + 21*a^4*b^5 + 35*a^5*b^4 + 35*a^6*b^3 + 21*a
^7*b^2)) + ((tan(e + f*x)*(128*a^3*b^18 + 1984*a^4*b^17 + 13840*a^5*b^16 +
57680*a^6*b^15 + 161280*a^7*b^14 + 322560*a^8*b^13 + 480928*a^9*b^12 + 5505
60*a^10*b^11 + 494400*a^11*b^10 + 352640*a^12*b^9 + 199696*a^13*b^8 + 88144
*a^14*b^7 + 29120*a^15*b^6 + 6720*a^16*b^5 + 960*a^17*b^4 + 64*a^18*b^3) +
((-b^5*(a + b)^7)^(1/2)*(7*a + 2*b)*(64*a^6*b^17 + 1536*a^7*b^16 + 13952*a^
8*b^15 + 71040*a^9*b^14 + 235968*a^10*b^13 + 551936*a^11*b^12 + 948992*a^12
*b^11 + 1229184*a^13*b^10 + 1214400*a^14*b^9 + 918016*a^15*b^8 + 528000*a^1
6*b^7 + 227456*a^17*b^6 + 71232*a^18*b^5 + 15360*a^19*b^4 + 2048*a^20*b^3 +
128*a^21*b^2 + (tan(e + f*x)*(-b^5*(a + b)^7)^(1/2)*(7*a + 2*b)*(512*a^7*b
^18 + 7936*a^8*b^17 + 57600*a^9*b^16 + 259840*a^10*b^15 + 815360*a^11*b^14
+ 1886976*a^12*b^13 + 3331328*a^13*b^12 + 4576000*a^14*b^11 + 4942080*a^15*
b^10 + 4209920*a^16*b^9 + 2818816*a^17*b^8 + 1467648*a^18*b^7 + 582400*a^19
*b^6 + 170240*a^20*b^5 + 34560*a^21*b^4 + 4352*a^22*b^3 + 256*a^23*b^2))/(4
*(7*a^8*b + a^9 + a^2*b^7 + 7*a^3*b^6 + 21*a^4*b^5 + 35*a^5*b^4 + 35*a^6*b^
3 + 21*a^7*b^2))))/(4*(7*a^8*b + a^9 + a^2*b^7 + 7*a^3*b^6 + 21*a^4*b^5 + 3
5*a^5*b^4 + 35*a^6*b^3 + 21*a^7*b^2)))*(-b^5*(a + b)^7)^(1/2)*(7*a + 2*b)*1
i)/(4*(7*a^8*b + a^9 + a^2*b^7 + 7*a^3*b^6 + 21*a^4*b^5 + 35*a^5*b^4 + 35*a
^6*b^3 + 21*a^7*b^2)))/(304*a^4*b^15 - 208*a^3*b^16 - 32*a^2*b^17 + 7040*a^
5*b^14 + 31200*a^6*b^13 + 75936*a^7*b^12 + 118944*a^8*b^11 + 126528*a^9*b^1
0 + 92640*a^10*b^9 + 46000*a^11*b^8 + 14768*a^12*b^7 + 2752*a^13*b^6 + 224*
a^14*b^5 + ((tan(e + f*x)*(128*a^3*b^18 + 1984*a^4*b^17 + 13840*a^5*b^16 +
57680*a^6*b^15 + 161280*a^7*b^14 + 322560*a^8*b^13 + 480928*a^9*b^12 + 5505
60*a^10*b^11 + 494400*a^11*b^10 + 352640*a^12*b^9 + 199696*a^13*b^8 + 88144
*a^14*b^7 + 29120*a^15*b^6 + 6720*a^16*b^5 + 960*a^17*b^4 + 64*a^18*b^3) -
((-b^5*(a + b)^7)^(1/2)*(7*a + 2*b)*(64*a^6*b^17 + 1536*a^7*b^16 + 13952*a^
8*b^15 + 71040*a^9*b^14 + 235968*a^10*b^13 + 551936*a^11*b^12 + 948992*a^12
*b^11 + 1229184*a^13*b^10 + 1214400*a^14*b^9 + 918016*a^15*b^8 + 528000*a^1
6*b^7 + 227456*a^17*b^6 + 71232*a^18*b^5 + 15360*a^19*b^4 + 2048*a^20*b^3 +
128*a^21*b^2 - (tan(e + f*x)*(-b^5*(a + b)^7)^(1/2)*(7*a + 2*b)*(512*a^7*b
^18 + 7936*a^8*b^17 + 57600*a^9*b^16 + 259840*a^10*b^15 + 815360*a^11*b^14
+ 1886976*a^12*b^13 + 3331328*a^13*b^12 + 4576000*a^14*b^11 + 4942080*a^15*
b^10 + 4209920*a^16*b^9 + 2818816*a^17*b^8 + 1467648*a^18*b^7 + 582400*a^19
*b^6 + 170240*a^20*b^5 + 34560*a^21*b^4 + 4352*a^22*b^3 + 256*a^23*b^2))/(4
*(7*a^8*b + a^9 + a^2*b^7 + 7*a^3*b^6 + 21*a^4*b^5 + 35*a^5*b^4 + 35*a^6*b^
3 + 21*a^7*b^2))))/(4*(7*a^8*b + a^9 + a^2*b^7 + 7*a^3*b^6 + 21*a^4*b^5 + 3
5*a^5*b^4 + 35*a^6*b^3 + 21*a^7*b^2)))*(-b^5*(a + b)^7)^(1/2)*(7*a + 2*b))/
(4*(7*a^8*b + a^9 + a^2*b^7 + 7*a^3*b^6 + 21*a^4*b^5 + 35*a^5*b^4 + 35*a^6*
b^3 + 21*a^7*b^2)) - ((tan(e + f*x)*(128*a^3*b^18 + 1984*a^4*b^17 + 13840*a
^5*b^16 + 57680*a^6*b^15 + 161280*a^7*b^14 + 322560*a^8*b^13 + 480928*a^9*b
^12 + 550560*a^10*b^11 + 494400*a^11*b^10 + 352640*a^12*b^9 + 199696*a^13*b
^8 + 88144*a^14*b^7 + 29120*a^15*b^6 + 6720*a^16*b^5 + 960*a^17*b^4 + 64*a^
18*b^3) + ((-b^5*(a + b)^7)^(1/2)*(7*a + 2*b)*(64*a^6*b^17 + 1536*a^7*b^16
+ 13952*a^8*b^15 + 71040*a^9*b^14 + 235968*a^10*b^13 + 551936*a^11*b^12 + 9

48992*a^12*b^11 + 1229184*a^13*b^10 + 1214400*a^14*b^9 + 918016*a^15*b^8 + 528000*a^16*b^7 + 227456*a^17*b^6 + 71232*a^18*b^5 + 15360*a^19*b^4 + 2048*a^20*b^3 + 128*a^21*b^2 + (tan(e + f*x)*(-b^5*(a + b)^7)^(1/2)*(7*a + 2*b)*(512*a^7*b^18 + 7936*a^8*b^17 + 57600*a^9*b^16 + 259840*a^10*b^15 + 815360*a^11*b^14 + 1886976*a^12*b^13 + 3331328*a^13*b^12 + 4576000*a^14*b^11 + 4942080*a^15*b^10 + 4209920*a^16*b^9 + 2818816*a^17*b^8 + 1467648*a^18*b^7 + 582400*a^19*b^6 + 170240*a^20*b^5 + 34560*a^21*b^4 + 4352*a^22*b^3 + 256*a^23*b^2))/(4*(7*a^8*b + a^9 + a^2*b^7 + 7*a^3*b^6 + 21*a^4*b^5 + 35*a^5*b^4 + 35*a^6*b^3 + 21*a^7*b^2))))/(4*(7*a^8*b + a^9 + a^2*b^7 + 7*a^3*b^6 + 21*a^4*b^5 + 35*a^5*b^4 + 35*a^6*b^3 + 21*a^7*b^2)))*(-b^5*(a + b)^7)^(1/2)*(7*a + 2*b))/(4*(7*a^8*b + a^9 + a^2*b^7 + 7*a^3*b^6 + 21*a^4*b^5 + 35*a^5*b^4 + 35*a^6*b^3 + 21*a^7*b^2))))*(-b^5*(a + b)^7)^(1/2)*(7*a + 2*b)*1i)/(2*f*(7*a^8*b + a^9 + a^2*b^7 + 7*a^3*b^6 + 21*a^4*b^5 + 35*a^5*b^4 + 35*a^6*b^3 + 21*a^7*b^2))

3.362 $\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 207

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^2} dx = -\frac{x}{a^2} + \frac{b^{7/2}(9a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{9/2}f} - \frac{(2a^3+8a^2b+12ab^2-b^3)\cot(e+fx)}{2a(a+b)^4f} + \frac{(2a^2+6ab-3b^2)\cot^3(e+fx)}{6a(a+b)^3f} - \frac{(2a-5b)\cot^5(e+fx)}{10a(a+b)^2f} - \frac{b\cot^5(e+fx)}{2a(a+b)f\,(a+b+b\tan^2(e+fx))}$$

```
[Out] -x/a^2+1/2*b^(7/2)*(9*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^2/(a+
b)^(9/2)/f-1/2*(2*a^3+8*a^2*b+12*a*b^2-b^3)*cot(f*x+e)/a/(a+b)^4/f+1/6*(2*a
^2+6*a*b-3*b^2)*cot(f*x+e)^3/a/(a+b)^3/f-1/10*(2*a-5*b)*cot(f*x+e)^5/a/(a+b
)^2/f-1/2*b*cot(f*x+e)^5/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used

$= \{4226, 2000, 483, 597, 536, 209, 211\}$

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{b^{7/2}(9a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{9/2}} + \frac{(2a^2+6ab-3b^2)\cot^3(e+fx)}{6af(a+b)^3} - \frac{x}{a^2} - \frac{(2a^3+8a^2b+12ab^2-b^3)\cot(e+fx)}{2af(a+b)^4} - \frac{(2a-5b)\cot^5(e+fx)}{10af(a+b)^2} - \frac{b\cot^5(e+fx)}{2af(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

```
[In] Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(x/a^2) + (b^(7/2)*(9*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])
/(2*a^2*(a + b)^(9/2)*f) - ((2*a^3 + 8*a^2*b + 12*a*b^2 - b^3)*Cot[e + f*x]
)/(2*a*(a + b)^4*f) + ((2*a^2 + 6*a*b - 3*b^2)*Cot[e + f*x]^3)/(6*a*(a + b)
^3*f) - ((2*a - 5*b)*Cot[e + f*x]^5)/(10*a*(a + b)^2*f) - (b*Cot[e + f*x]^5
)/(2*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+b(1+x^2))^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{b\cot^5(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}+\frac{\text{Subst}\left(\int \frac{2a-5b-7bx^2}{x^6(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{2a(a+b)f}\\
&= -\frac{(2a-5b)\cot^5(e+fx)}{10a(a+b)^2f}-\frac{b\cot^5(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}\\
&\quad-\frac{\text{Subst}\left(\int \frac{5(2a^2+6ab-3b^2)+5(2a-5b)bx^2}{x^4(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{10a(a+b)^2f}
\end{aligned}$$

$$= \frac{(2a^2+6ab-3b^2)\cot^3(e+fx)}{6a(a+b)^3f} - \frac{(2a-5b)\cot^5(e+fx)}{10a(a+b)^2f} - \frac{b\cot^5(e+fx)}{2a(a+b)f(a+b+b\tan^2(e+fx))} + \frac{\text{Subst}\left(\int \frac{15(2a^3+8a^2b+12ab^2-b^3)+15b(2a^2+6ab-3b^2)x^2}{x^2(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{30a(a+b)^3f}$$

$$= -\frac{(2a^3+8a^2b+12ab^2-b^3)\cot(e+fx)}{2a(a+b)^4f} + \frac{(2a^2+6ab-3b^2)\cot^3(e+fx)}{6a(a+b)^3f} - \frac{(2a-5b)\cot^5(e+fx)}{10a(a+b)^2f} - \frac{b\cot^5(e+fx)}{2a(a+b)f(a+b+b\tan^2(e+fx))} - \frac{\text{Subst}\left(\int \frac{15(2a^4+10a^3b+20a^2b^2+20ab^3+b^4)+15b(2a^3+8a^2b+12ab^2-b^3)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{30a(a+b)^4f}$$

$$= -\frac{(2a^3+8a^2b+12ab^2-b^3)\cot(e+fx)}{2a(a+b)^4f} + \frac{(2a^2+6ab-3b^2)\cot^3(e+fx)}{6a(a+b)^3f} - \frac{(2a-5b)\cot^5(e+fx)}{10a(a+b)^2f} - \frac{b\cot^5(e+fx)}{2a(a+b)f(a+b+b\tan^2(e+fx))} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^2f} + \frac{(b^4(9a+2b))\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{2a^2(a+b)^4f}$$

$$= -\frac{x}{a^2} + \frac{b^{7/2}(9a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{9/2}f} - \frac{(2a^3+8a^2b+12ab^2-b^3)\cot(e+fx)}{2a(a+b)^4f} + \frac{(2a^2+6ab-3b^2)\cot^3(e+fx)}{6a(a+b)^3f} - \frac{(2a-5b)\cot^5(e+fx)}{10a(a+b)^2f} - \frac{b\cot^5(e+fx)}{2a(a+b)f(a+b+b\tan^2(e+fx))}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.49 (sec) , antiderivative size = 3028, normalized size of antiderivative = 14.63

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((9*a + 2*b)*(a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*(-1/8*(b^4*Arc
Tan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((
```

I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]
) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(a^2*Sqrt[a + b]*f*Sqrt[b*C
os[4*e] - I*b*Sin[4*e]]) + ((I/8)*b^4*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a +
 b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b
*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x
])]*Sin[2*e])/(a^2*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]])))/((a + b
)^4*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[2*e + 2*f*x])*Csc[e]*Csc[
e + f*x]^5*Sec[2*e]*Sec[e + f*x]^4*(75*a^5*f*x*Cos[f*x] + 900*a^4*b*f*x*Cos
[f*x] + 2850*a^3*b^2*f*x*Cos[f*x] + 3900*a^2*b^3*f*x*Cos[f*x] + 2475*a*b^4*
f*x*Cos[f*x] + 600*b^5*f*x*Cos[f*x] - 15*a^5*f*x*Cos[3*f*x] + 240*a^4*b*f*x
*Cos[3*f*x] + 1110*a^3*b^2*f*x*Cos[3*f*x] + 1740*a^2*b^3*f*x*Cos[3*f*x] + 1
185*a*b^4*f*x*Cos[3*f*x] + 300*b^5*f*x*Cos[3*f*x] - 75*a^5*f*x*Cos[2*e - f*
x] - 900*a^4*b*f*x*Cos[2*e - f*x] - 2850*a^3*b^2*f*x*Cos[2*e - f*x] - 3900*
a^2*b^3*f*x*Cos[2*e - f*x] - 2475*a*b^4*f*x*Cos[2*e - f*x] - 600*b^5*f*x*Co
s[2*e - f*x] - 75*a^5*f*x*Cos[2*e + f*x] - 900*a^4*b*f*x*Cos[2*e + f*x] - 2
850*a^3*b^2*f*x*Cos[2*e + f*x] - 3900*a^2*b^3*f*x*Cos[2*e + f*x] - 2475*a*b
^4*f*x*Cos[2*e + f*x] - 600*b^5*f*x*Cos[2*e + f*x] + 75*a^5*f*x*Cos[4*e + f
*x] + 900*a^4*b*f*x*Cos[4*e + f*x] + 2850*a^3*b^2*f*x*Cos[4*e + f*x] + 3900
*a^2*b^3*f*x*Cos[4*e + f*x] + 2475*a*b^4*f*x*Cos[4*e + f*x] + 600*b^5*f*x*C
os[4*e + f*x] + 15*a^5*f*x*Cos[2*e + 3*f*x] - 240*a^4*b*f*x*Cos[2*e + 3*f*x
] - 1110*a^3*b^2*f*x*Cos[2*e + 3*f*x] - 1740*a^2*b^3*f*x*Cos[2*e + 3*f*x] -
 1185*a*b^4*f*x*Cos[2*e + 3*f*x] - 300*b^5*f*x*Cos[2*e + 3*f*x] - 15*a^5*f*
x*Cos[4*e + 3*f*x] + 240*a^4*b*f*x*Cos[4*e + 3*f*x] + 1110*a^3*b^2*f*x*Cos[
4*e + 3*f*x] + 1740*a^2*b^3*f*x*Cos[4*e + 3*f*x] + 1185*a*b^4*f*x*Cos[4*e +
 3*f*x] + 300*b^5*f*x*Cos[4*e + 3*f*x] + 15*a^5*f*x*Cos[6*e + 3*f*x] - 240*
a^4*b*f*x*Cos[6*e + 3*f*x] - 1110*a^3*b^2*f*x*Cos[6*e + 3*f*x] - 1740*a^2*b
^3*f*x*Cos[6*e + 3*f*x] - 1185*a*b^4*f*x*Cos[6*e + 3*f*x] - 300*b^5*f*x*Cos
[6*e + 3*f*x] + 45*a^5*f*x*Cos[2*e + 5*f*x] + 120*a^4*b*f*x*Cos[2*e + 5*f*x
] + 30*a^3*b^2*f*x*Cos[2*e + 5*f*x] - 180*a^2*b^3*f*x*Cos[2*e + 5*f*x] - 19
5*a*b^4*f*x*Cos[2*e + 5*f*x] - 60*b^5*f*x*Cos[2*e + 5*f*x] - 45*a^5*f*x*Cos
[4*e + 5*f*x] - 120*a^4*b*f*x*Cos[4*e + 5*f*x] - 30*a^3*b^2*f*x*Cos[4*e + 5
*f*x] + 180*a^2*b^3*f*x*Cos[4*e + 5*f*x] + 195*a*b^4*f*x*Cos[4*e + 5*f*x] +
 60*b^5*f*x*Cos[4*e + 5*f*x] + 45*a^5*f*x*Cos[6*e + 5*f*x] + 120*a^4*b*f*x*
Cos[6*e + 5*f*x] + 30*a^3*b^2*f*x*Cos[6*e + 5*f*x] - 180*a^2*b^3*f*x*Cos[6*
e + 5*f*x] - 195*a*b^4*f*x*Cos[6*e + 5*f*x] - 60*b^5*f*x*Cos[6*e + 5*f*x] -
 45*a^5*f*x*Cos[8*e + 5*f*x] - 120*a^4*b*f*x*Cos[8*e + 5*f*x] - 30*a^3*b^2*
f*x*Cos[8*e + 5*f*x] + 180*a^2*b^3*f*x*Cos[8*e + 5*f*x] + 195*a*b^4*f*x*Cos
[8*e + 5*f*x] + 60*b^5*f*x*Cos[8*e + 5*f*x] - 15*a^5*f*x*Cos[4*e + 7*f*x] -
 60*a^4*b*f*x*Cos[4*e + 7*f*x] - 90*a^3*b^2*f*x*Cos[4*e + 7*f*x] - 60*a^2*b
^3*f*x*Cos[4*e + 7*f*x] - 15*a*b^4*f*x*Cos[4*e + 7*f*x] + 15*a^5*f*x*Cos[6*
e + 7*f*x] + 60*a^4*b*f*x*Cos[6*e + 7*f*x] + 90*a^3*b^2*f*x*Cos[6*e + 7*f*x
] + 60*a^2*b^3*f*x*Cos[6*e + 7*f*x] + 15*a*b^4*f*x*Cos[6*e + 7*f*x] - 15*a^
5*f*x*Cos[8*e + 7*f*x] - 60*a^4*b*f*x*Cos[8*e + 7*f*x] - 90*a^3*b^2*f*x*Cos
[8*e + 7*f*x] - 60*a^2*b^3*f*x*Cos[8*e + 7*f*x] - 15*a*b^4*f*x*Cos[8*e + 7*
f*x] + 15*a^5*f*x*Cos[10*e + 7*f*x] + 60*a^4*b*f*x*Cos[10*e + 7*f*x] + 90*a

```
^3*b^2*f*x*Cos[10*e + 7*f*x] + 60*a^2*b^3*f*x*Cos[10*e + 7*f*x] + 15*a*b^4*
f*x*Cos[10*e + 7*f*x] - 10*a^5*Sin[f*x] + 860*a^4*b*Sin[f*x] + 3120*a^3*b^2
*Sin[f*x] + 3600*a^2*b^3*Sin[f*x] - 300*b^5*Sin[f*x] + 46*a^5*Sin[3*f*x] -
508*a^4*b*Sin[3*f*x] - 2324*a^3*b^2*Sin[3*f*x] - 3120*a^2*b^3*Sin[3*f*x] +
75*a*b^4*Sin[3*f*x] - 150*b^5*Sin[3*f*x] - 240*a^5*Sin[2*e - f*x] - 1840*a^
4*b*Sin[2*e - f*x] - 4840*a^3*b^2*Sin[2*e - f*x] - 5040*a^2*b^3*Sin[2*e - f
*x] - 300*b^5*Sin[2*e - f*x] + 240*a^5*Sin[2*e + f*x] + 1840*a^4*b*Sin[2*e
+ f*x] + 4840*a^3*b^2*Sin[2*e + f*x] + 5040*a^2*b^3*Sin[2*e + f*x] - 75*a*b
^4*Sin[2*e + f*x] - 300*b^5*Sin[2*e + f*x] - 10*a^5*Sin[4*e + f*x] + 860*a^
4*b*Sin[4*e + f*x] + 3120*a^3*b^2*Sin[4*e + f*x] + 3600*a^2*b^3*Sin[4*e + f
*x] + 75*a*b^4*Sin[4*e + f*x] + 300*b^5*Sin[4*e + f*x] - 240*a^4*b*Sin[2*e
+ 3*f*x] - 900*a^3*b^2*Sin[2*e + 3*f*x] - 1200*a^2*b^3*Sin[2*e + 3*f*x] - 7
5*a*b^4*Sin[2*e + 3*f*x] + 150*b^5*Sin[2*e + 3*f*x] + 46*a^5*Sin[4*e + 3*f*
x] - 508*a^4*b*Sin[4*e + 3*f*x] - 2324*a^3*b^2*Sin[4*e + 3*f*x] - 3120*a^2*
b^3*Sin[4*e + 3*f*x] + 60*a*b^4*Sin[4*e + 3*f*x] + 150*b^5*Sin[4*e + 3*f*x]
 - 240*a^4*b*Sin[6*e + 3*f*x] - 900*a^3*b^2*Sin[6*e + 3*f*x] - 1200*a^2*b^3
*Sin[6*e + 3*f*x] - 60*a*b^4*Sin[6*e + 3*f*x] - 150*b^5*Sin[6*e + 3*f*x] -
48*a^5*Sin[2*e + 5*f*x] - 32*a^4*b*Sin[2*e + 5*f*x] + 340*a^3*b^2*Sin[2*e +
 5*f*x] + 864*a^2*b^3*Sin[2*e + 5*f*x] - 60*a*b^4*Sin[2*e + 5*f*x] + 30*b^5
*Sin[2*e + 5*f*x] - 90*a^5*Sin[4*e + 5*f*x] - 300*a^4*b*Sin[4*e + 5*f*x] -
300*a^3*b^2*Sin[4*e + 5*f*x] + 60*a*b^4*Sin[4*e + 5*f*x] - 30*b^5*Sin[4*e +
 5*f*x] - 48*a^5*Sin[6*e + 5*f*x] - 32*a^4*b*Sin[6*e + 5*f*x] + 340*a^3*b^2
*Sin[6*e + 5*f*x] + 864*a^2*b^3*Sin[6*e + 5*f*x] - 15*a*b^4*Sin[6*e + 5*f*x
] - 30*b^5*Sin[6*e + 5*f*x] - 90*a^5*Sin[8*e + 5*f*x] - 300*a^4*b*Sin[8*e +
 5*f*x] - 300*a^3*b^2*Sin[8*e + 5*f*x] + 15*a*b^4*Sin[8*e + 5*f*x] + 30*b^5
*Sin[8*e + 5*f*x] + 46*a^5*Sin[4*e + 7*f*x] + 172*a^4*b*Sin[4*e + 7*f*x] +
216*a^3*b^2*Sin[4*e + 7*f*x] + 15*a*b^4*Sin[4*e + 7*f*x] - 15*a*b^4*Sin[6*e
 + 7*f*x] + 46*a^5*Sin[8*e + 7*f*x] + 172*a^4*b*Sin[8*e + 7*f*x] + 216*a^3*
b^2*Sin[8*e + 7*f*x]))/(7680*a^2*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] (verified)

Time = 18.38 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{a^2}+\frac{b^4\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(9a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^4a^2}-\frac{1}{5(a+b)^2\tan(fx+e)^5}-\frac{-a-3b}{3(a+b)^3\tan(fx+e)^3}-}{f}$
default	$\frac{-\frac{\arctan(\tan(fx+e))}{a^2}+\frac{b^4\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(9a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^4a^2}-\frac{1}{5(a+b)^2\tan(fx+e)^5}-\frac{-a-3b}{3(a+b)^3\tan(fx+e)^3}-}{f}$
risch	$-\frac{x}{a^2}-\frac{i(172a^4b+216a^3b^2+15ab^4+46a^5+300a^4b\,e^{12i(fx+e)}+900a^3b^2e^{10i(fx+e)}+1200a^2b^3e^{10i(fx+e)}+60ab^4e^{10i(fx+}}{}$

```
[In] int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^2*arctan(tan(f*x+e))+b^4/(a+b)^4/a^2*(1/2*a*tan(f*x+e)/(a+b+b*tan
(f*x+e)^2)+1/2*(9*a+2*b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2
)))-1/5/(a+b)^2/tan(f*x+e)^5-1/3*(-a-3*b)/(a+b)^3/tan(f*x+e)^3-(a^2+4*a*b+6
*b^2)/(a+b)^4/tan(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. $2(189) = 378$.

Time = 0.36 (sec) , antiderivative size = 1505, normalized size of antiderivative = 7.27

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/120*(4*(46*a^5 + 172*a^4*b + 216*a^3*b^2 + 15*a*b^4)*cos(f*x + e)^7 - 4
*(70*a^5 + 234*a^4*b + 218*a^3*b^2 - 216*a^2*b^3 + 45*a*b^4)*cos(f*x + e)^5
 + 20*(6*a^5 + 10*a^4*b - 20*a^3*b^2 - 78*a^2*b^3 + 9*a*b^4)*cos(f*x + e)^3
 - 15*((9*a^2*b^3 + 2*a*b^4)*cos(f*x + e)^6 + 9*a*b^4 + 2*b^5 - (18*a^2*b^3
 - 5*a*b^4 - 2*b^5)*cos(f*x + e)^4 + (9*a^2*b^3 - 16*a*b^4 - 4*b^5)*cos(f*x
 + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*
a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*
b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x +
e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*(2*a^4*b + 8*a^3*b^2
+ 12*a^2*b^3 - a*b^4)*cos(f*x + e) + 120*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^
2*b^3 + a*b^4)*f*x*cos(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^
3 - 2*a*b^4 - b^5)*f*x*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*
b^3 - 7*a*b^4 - 2*b^5)*f*x*cos(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3
+ 4*a*b^4 + b^5)*f*x)*sin(f*x + e))/(((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^
3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a^7 + 7*a^6*b + 8*a^5*b^2 + 2*a^4*b^3 -
```

```
2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)^4 + (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*
b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^2 + (a^6*b + 4*a^5*b^2 + 6*a^4*
b^3 + 4*a^3*b^4 + a^2*b^5)*f)*sin(f*x + e)), -1/60*(2*(46*a^5 + 172*a^4*b +
 216*a^3*b^2 + 15*a*b^4)*cos(f*x + e)^7 - 2*(70*a^5 + 234*a^4*b + 218*a^3*b
^2 - 216*a^2*b^3 + 45*a*b^4)*cos(f*x + e)^5 + 10*(6*a^5 + 10*a^4*b - 20*a^3
*b^2 - 78*a^2*b^3 + 9*a*b^4)*cos(f*x + e)^3 + 15*((9*a^2*b^3 + 2*a*b^4)*cos
(f*x + e)^6 + 9*a*b^4 + 2*b^5 - (18*a^2*b^3 - 5*a*b^4 - 2*b^5)*cos(f*x + e)
^4 + (9*a^2*b^3 - 16*a*b^4 - 4*b^5)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(
1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x
+ e)))*sin(f*x + e) + 30*(2*a^4*b + 8*a^3*b^2 + 12*a^2*b^3 - a*b^4)*cos(f*x
 + e) + 60*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*f*x*cos(f*x + e
)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*f*x*cos(f*x
 + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*f*x*cos
(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f*x)*sin(f*x
+ e))/(((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6
- (2*a^7 + 7*a^6*b + 8*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x
 + e)^4 + (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f
*cos(f*x + e)^2 + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f)*
sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.54

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{15\,(9\,ab^4+2\,b^5)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4)\sqrt{(a+b)b}} - \frac{15\,(2\,a^3b+8\,a^2b^2+12\,ab^3-b^4)\tan(fx+e)^6+10\,(3\,a^4+14\,a^3b+26\,a^2b^2+15\,ab^3)\tan(fx+e)^4+6\,a^4 \cdots}{(a^5b+4\,a^4b^2+6\,a^3b^3+4\,a^2b^4+ab^5)\tan(fx+e)^7+(a^6+5\,a^5b+10\,a^4 \cdots}}{30\,f}$$

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/30*(15*(9*a*b^4 + 2*b^5)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^6 + 4
*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt((a + b)*b)) - (15*(2*a^3*b +
 8*a^2*b^2 + 12*a*b^3 - b^4)*tan(f*x + e)^6 + 10*(3*a^4 + 14*a^3*b + 26*a^2
*b^2 + 15*a*b^3)*tan(f*x + e)^4 + 6*a^4 + 18*a^3*b + 18*a^2*b^2 + 6*a*b^3 -
 2*(5*a^4 + 22*a^3*b + 29*a^2*b^2 + 12*a*b^3)*tan(f*x + e)^2)/((a^5*b + 4*a
^4*b^2 + 6*a^3*b^3 + 4*a^2*b^4 + a*b^5)*tan(f*x + e)^7 + (a^6 + 5*a^5*b + 1
0*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*tan(f*x + e)^5) - 30*(f*x + e)/
a^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.44

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{15\,b^4\tan(fx+e)}{(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4)\left(b\tan(fx+e)^2+a+b\right)}+\frac{15\,(9\,ab^4+2\,b^5)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4)\sqrt{ab+b^2}}-\frac{30\,(fx+e)}{a^2}-\frac{2\,(15\,a}{}}{30\,f}$$

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/30*(15*b^4*tan(f*x + e)/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*
(b*tan(f*x + e)^2 + a + b)) + 15*(9*a*b^4 + 2*b^5)*(pi*floor((f*x + e)/pi +
 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^6 + 4*a^5*b + 6*
a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b + b^2)) - 30*(f*x + e)/a^2 - 2*(15*
a^2*tan(f*x + e)^4 + 60*a*b*tan(f*x + e)^4 + 90*b^2*tan(f*x + e)^4 - 5*a^2*
tan(f*x + e)^2 - 20*a*b*tan(f*x + e)^2 - 15*b^2*tan(f*x + e)^2 + 3*a^2 + 6*
a*b + 3*b^2)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*tan(f*x + e)^5))/
f
```

Mupad [B] (verification not implemented)

Time = 26.62 (sec) , antiderivative size = 6017, normalized size of antiderivative = 29.07

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] atan(((((64*a^6*b^22 + 2304*a^7*b^21 + 29440*a^8*b^20 + 210560*a^9*b^19 + 9
97248*a^10*b^18 + 3404800*a^11*b^17 + 8806912*a^12*b^16 + 17809920*a^13*b^1
5 + 28745600*a^14*b^14 + 37533184*a^15*b^13 + 39975936*a^16*b^12 + 34874112
```

*a^17*b^11 + 24926720*a^18*b^10 + 14545920*a^19*b^9 + 6874624*a^20*b^8 + 25
95328*a^21*b^7 + 765504*a^22*b^6 + 170240*a^23*b^5 + 26880*a^24*b^4 + 2688*
a^25*b^3 + 128*a^26*b^2 + (tan(e + f*x)*(512*a^7*b^23 + 10496*a^8*b^22 + 10
2400*a^9*b^21 + 632320*a^10*b^20 + 2772480*a^11*b^19 + 9178368*a^12*b^18 +
23814144*a^13*b^17 + 49612800*a^14*b^16 + 84341760*a^15*b^15 + 118243840*a^
16*b^14 + 137592832*a^17*b^13 + 133293056*a^18*b^12 + 107494400*a^19*b^11 +
71938560*a^20*b^10 + 39690240*a^21*b^9 + 17860608*a^22*b^8 + 6449664*a^23*
b^7 + 1824000*a^24*b^6 + 389120*a^25*b^5 + 58880*a^26*b^4 + 5632*a^27*b^3 +
256*a^28*b^2)*1i)/(2*a^2))*1i)/(2*a^2) + tan(e + f*x)*(128*a^3*b^23 + 2624
*a^4*b^22 + 24592*a^5*b^21 + 140608*a^6*b^20 + 554016*a^7*b^19 + 1613184*a^
8*b^18 + 3637488*a^9*b^17 + 6570624*a^10*b^16 + 9747456*a^11*b^15 + 1207507
2*a^12*b^14 + 12596848*a^13*b^13 + 11073344*a^14*b^12 + 8154592*a^15*b^11 +
4977408*a^16*b^10 + 2481936*a^17*b^9 + 992256*a^18*b^8 + 310080*a^19*b^7 +
72960*a^20*b^6 + 12160*a^21*b^5 + 1280*a^22*b^4 + 64*a^23*b^3))/(2*a^2) -
(((64*a^6*b^22 + 2304*a^7*b^21 + 29440*a^8*b^20 + 210560*a^9*b^19 + 997248*
a^10*b^18 + 3404800*a^11*b^17 + 8806912*a^12*b^16 + 17809920*a^13*b^15 + 28
745600*a^14*b^14 + 37533184*a^15*b^13 + 39975936*a^16*b^12 + 34874112*a^17*
b^11 + 24926720*a^18*b^10 + 14545920*a^19*b^9 + 6874624*a^20*b^8 + 2595328*
a^21*b^7 + 765504*a^22*b^6 + 170240*a^23*b^5 + 26880*a^24*b^4 + 2688*a^25*b
^3 + 128*a^26*b^2 - (tan(e + f*x)*(512*a^7*b^23 + 10496*a^8*b^22 + 102400*a
^9*b^21 + 632320*a^10*b^20 + 2772480*a^11*b^19 + 9178368*a^12*b^18 + 238141
44*a^13*b^17 + 49612800*a^14*b^16 + 84341760*a^15*b^15 + 118243840*a^16*b^1
4 + 137592832*a^17*b^13 + 133293056*a^18*b^12 + 107494400*a^19*b^11 + 71938
560*a^20*b^10 + 39690240*a^21*b^9 + 17860608*a^22*b^8 + 6449664*a^23*b^7 +
1824000*a^24*b^6 + 389120*a^25*b^5 + 58880*a^26*b^4 + 5632*a^27*b^3 + 256*a
^28*b^2)*1i)/(2*a^2))*1i)/(2*a^2) - tan(e + f*x)*(128*a^3*b^23 + 2624*a^4*b
^22 + 24592*a^5*b^21 + 140608*a^6*b^20 + 554016*a^7*b^19 + 1613184*a^8*b^18
+ 3637488*a^9*b^17 + 6570624*a^10*b^16 + 9747456*a^11*b^15 + 12075072*a^12
*b^14 + 12596848*a^13*b^13 + 11073344*a^14*b^12 + 8154592*a^15*b^11 + 49774
08*a^16*b^10 + 2481936*a^17*b^9 + 992256*a^18*b^8 + 310080*a^19*b^7 + 72960
*a^20*b^6 + 12160*a^21*b^5 + 1280*a^22*b^4 + 64*a^23*b^3))/(2*a^2))/(2752*a
^4*b^20 - ((((64*a^6*b^22 + 2304*a^7*b^21 + 29440*a^8*b^20 + 210560*a^9*b^1
9 + 997248*a^10*b^18 + 3404800*a^11*b^17 + 8806912*a^12*b^16 + 17809920*a^1
3*b^15 + 28745600*a^14*b^14 + 37533184*a^15*b^13 + 39975936*a^16*b^12 + 348
74112*a^17*b^11 + 24926720*a^18*b^10 + 14545920*a^19*b^9 + 6874624*a^20*b^8
+ 2595328*a^21*b^7 + 765504*a^22*b^6 + 170240*a^23*b^5 + 26880*a^24*b^4 +
2688*a^25*b^3 + 128*a^26*b^2 - (tan(e + f*x)*(512*a^7*b^23 + 10496*a^8*b^22
+ 102400*a^9*b^21 + 632320*a^10*b^20 + 2772480*a^11*b^19 + 9178368*a^12*b^
18 + 23814144*a^13*b^17 + 49612800*a^14*b^16 + 84341760*a^15*b^15 + 1182438
40*a^16*b^14 + 137592832*a^17*b^13 + 133293056*a^18*b^12 + 107494400*a^19*b
^11 + 71938560*a^20*b^10 + 39690240*a^21*b^9 + 17860608*a^22*b^8 + 6449664*
a^23*b^7 + 1824000*a^24*b^6 + 389120*a^25*b^5 + 58880*a^26*b^4 + 5632*a^27*
b^3 + 256*a^28*b^2)*1i)/(2*a^2))*1i)/(2*a^2) - tan(e + f*x)*(128*a^3*b^23 +
2624*a^4*b^22 + 24592*a^5*b^21 + 140608*a^6*b^20 + 554016*a^7*b^19 + 16131
84*a^8*b^18 + 3637488*a^9*b^17 + 6570624*a^10*b^16 + 9747456*a^11*b^15 + 12

075072*a^12*b^14 + 12596848*a^13*b^13 + 11073344*a^14*b^12 + 8154592*a^15*b
^11 + 4977408*a^16*b^10 + 2481936*a^17*b^9 + 992256*a^18*b^8 + 310080*a^19*
b^7 + 72960*a^20*b^6 + 12160*a^21*b^5 + 1280*a^22*b^4 + 64*a^23*b^3))*1i)/(
2*a^2) - 32*a^2*b^22 - 144*a^3*b^21 - ((((64*a^6*b^22 + 2304*a^7*b^21 + 294
40*a^8*b^20 + 210560*a^9*b^19 + 997248*a^10*b^18 + 3404800*a^11*b^17 + 8806
912*a^12*b^16 + 17809920*a^13*b^15 + 28745600*a^14*b^14 + 37533184*a^15*b^1
3 + 39975936*a^16*b^12 + 34874112*a^17*b^11 + 24926720*a^18*b^10 + 14545920
*a^19*b^9 + 6874624*a^20*b^8 + 2595328*a^21*b^7 + 765504*a^22*b^6 + 170240*
a^23*b^5 + 26880*a^24*b^4 + 2688*a^25*b^3 + 128*a^26*b^2 + (tan(e + f*x)*(5
12*a^7*b^23 + 10496*a^8*b^22 + 102400*a^9*b^21 + 632320*a^10*b^20 + 2772480
*a^11*b^19 + 9178368*a^12*b^18 + 23814144*a^13*b^17 + 49612800*a^14*b^16 +
84341760*a^15*b^15 + 118243840*a^16*b^14 + 137592832*a^17*b^13 + 133293056*
a^18*b^12 + 107494400*a^19*b^11 + 71938560*a^20*b^10 + 39690240*a^21*b^9 +
17860608*a^22*b^8 + 6449664*a^23*b^7 + 1824000*a^24*b^6 + 389120*a^25*b^5 +
58880*a^26*b^4 + 5632*a^27*b^3 + 256*a^28*b^2)*1i)/(2*a^2))*1i)/(2*a^2) +
tan(e + f*x)*(128*a^3*b^23 + 2624*a^4*b^22 + 24592*a^5*b^21 + 140608*a^6*b^
20 + 554016*a^7*b^19 + 1613184*a^8*b^18 + 3637488*a^9*b^17 + 6570624*a^10*b
^16 + 9747456*a^11*b^15 + 12075072*a^12*b^14 + 12596848*a^13*b^13 + 1107334
4*a^14*b^12 + 8154592*a^15*b^11 + 4977408*a^16*b^10 + 2481936*a^17*b^9 + 99
2256*a^18*b^8 + 310080*a^19*b^7 + 72960*a^20*b^6 + 12160*a^21*b^5 + 1280*a^
22*b^4 + 64*a^23*b^3))*1i)/(2*a^2) + 33824*a^5*b^19 + 182784*a^6*b^18 + 613
648*a^7*b^17 + 1429120*a^8*b^16 + 2433024*a^9*b^15 + 3113088*a^10*b^14 + 30
34768*a^11*b^13 + 2261952*a^12*b^12 + 1281952*a^13*b^11 + 543872*a^14*b^10
+ 167664*a^15*b^9 + 35584*a^16*b^8 + 4672*a^17*b^7 + 288*a^18*b^6))/(a^2*f)
- (1/(5*(a + b)) + (tan(e + f*x)^4*(11*a*b + 3*a^2 + 15*b^2))/(3*(a + b)^3
) - (tan(e + f*x)^2*(5*a + 12*b))/(15*(a + b)^2) + (tan(e + f*x)^6*(12*a*b^
3 + 2*a^3*b - b^4 + 8*a^2*b^2))/(2*a*(a + b)^4))/(f*(tan(e + f*x)^5*(a + b)
+ b*tan(e + f*x)^7)) + (atan((((-b^7*(a + b)^9)^(1/2)*(9*a + 2*b)*(tan(e +
f*x)*(128*a^3*b^23 + 2624*a^4*b^22 + 24592*a^5*b^21 + 140608*a^6*b^20 + 55
4016*a^7*b^19 + 1613184*a^8*b^18 + 3637488*a^9*b^17 + 6570624*a^10*b^16 + 9
747456*a^11*b^15 + 12075072*a^12*b^14 + 12596848*a^13*b^13 + 11073344*a^14*
b^12 + 8154592*a^15*b^11 + 4977408*a^16*b^10 + 2481936*a^17*b^9 + 992256*a^
18*b^8 + 310080*a^19*b^7 + 72960*a^20*b^6 + 12160*a^21*b^5 + 1280*a^22*b^4
+ 64*a^23*b^3) - ((-b^7*(a + b)^9)^(1/2)*(9*a + 2*b)*(64*a^6*b^22 + 2304*a^
7*b^21 + 29440*a^8*b^20 + 210560*a^9*b^19 + 997248*a^10*b^18 + 3404800*a^11
*b^17 + 8806912*a^12*b^16 + 17809920*a^13*b^15 + 28745600*a^14*b^14 + 37533
184*a^15*b^13 + 39975936*a^16*b^12 + 34874112*a^17*b^11 + 24926720*a^18*b^1
0 + 14545920*a^19*b^9 + 6874624*a^20*b^8 + 2595328*a^21*b^7 + 765504*a^22*b
^6 + 170240*a^23*b^5 + 26880*a^24*b^4 + 2688*a^25*b^3 + 128*a^26*b^2 - (tan
(e + f*x)*(-b^7*(a + b)^9)^(1/2)*(9*a + 2*b)*(512*a^7*b^23 + 10496*a^8*b^22
+ 102400*a^9*b^21 + 632320*a^10*b^20 + 2772480*a^11*b^19 + 9178368*a^12*b^
18 + 23814144*a^13*b^17 + 49612800*a^14*b^16 + 84341760*a^15*b^15 + 1182438
40*a^16*b^14 + 137592832*a^17*b^13 + 133293056*a^18*b^12 + 107494400*a^19*b
^11 + 71938560*a^20*b^10 + 39690240*a^21*b^9 + 17860608*a^22*b^8 + 6449664*
a^23*b^7 + 1824000*a^24*b^6 + 389120*a^25*b^5 + 58880*a^26*b^4 + 5632*a^27*

b^3 + 256*a^28*b^2))/(4*(9*a^10*b + a^11 + a^2*b^9 + 9*a^3*b^8 + 36*a^4*b^7 + 84*a^5*b^6 + 126*a^6*b^5 + 126*a^7*b^4 + 84*a^8*b^3 + 36*a^9*b^2))))/(4*(9*a^10*b + a^11 + a^2*b^9 + 9*a^3*b^8 + 36*a^4*b^7 + 84*a^5*b^6 + 126*a^6*b^5 + 126*a^7*b^4 + 84*a^8*b^3 + 36*a^9*b^2)))*1i)/(4*(9*a^10*b + a^11 + a^2*b^9 + 9*a^3*b^8 + 36*a^4*b^7 + 84*a^5*b^6 + 126*a^6*b^5 + 126*a^7*b^4 + 84*a^8*b^3 + 36*a^9*b^2)) + ((-b^7*(a + b)^9)^(1/2)*(9*a + 2*b)*(tan(e + f*x)*(128*a^3*b^23 + 2624*a^4*b^22 + 24592*a^5*b^21 + 140608*a^6*b^20 + 554016*a^7*b^19 + 1613184*a^8*b^18 + 3637488*a^9*b^17 + 6570624*a^10*b^16 + 9747456*a^11*b^15 + 12075072*a^12*b^14 + 12596848*a^13*b^13 + 11073344*a^14*b^12 + 8154592*a^15*b^11 + 4977408*a^16*b^10 + 2481936*a^17*b^9 + 992256*a^18*b^8 + 310080*a^19*b^7 + 72960*a^20*b^6 + 12160*a^21*b^5 + 1280*a^22*b^4 + 64*a^23*b^3) + ((-b^7*(a + b)^9)^(1/2)*(9*a + 2*b)*(64*a^6*b^22 + 2304*a^7*b^21 + 29440*a^8*b^20 + 210560*a^9*b^19 + 997248*a^10*b^18 + 3404800*a^11*b^17 + 8806912*a^12*b^16 + 17809920*a^13*b^15 + 28745600*a^14*b^14 + 37533184*a^15*b^13 + 39975936*a^16*b^12 + 34874112*a^17*b^11 + 24926720*a^18*b^10 + 14545920*a^19*b^9 + 6874624*a^20*b^8 + 2595328*a^21*b^7 + 765504*a^22*b^6 + 170240*a^23*b^5 + 26880*a^24*b^4 + 2688*a^25*b^3 + 128*a^26*b^2 + (tan(e + f*x)*(-b^7*(a + b)^9)^(1/2)*(9*a + 2*b)*(512*a^7*b^23 + 10496*a^8*b^22 + 102400*a^9*b^21 + 632320*a^10*b^20 + 2772480*a^11*b^19 + 9178368*a^12*b^18 + 23814144*a^13*b^17 + 49612800*a^14*b^16 + 84341760*a^15*b^15 + 118243840*a^16*b^14 + 137592832*a^17*b^13 + 133293056*a^18*b^12 + 107494400*a^19*b^11 + 71938560*a^20*b^10 + 39690240*a^21*b^9 + 17860608*a^22*b^8 + 6449664*a^23*b^7 + 1824000*a^24*b^6 + 389120*a^25*b^5 + 58880*a^26*b^4 + 5632*a^27*b^3 + 256*a^28*b^2))/(4*(9*a^10*b + a^11 + a^2*b^9 + 9*a^3*b^8 + 36*a^4*b^7 + 84*a^5*b^6 + 126*a^6*b^5 + 126*a^7*b^4 + 84*a^8*b^3 + 36*a^9*b^2))))/(4*(9*a^10*b + a^11 + a^2*b^9 + 9*a^3*b^8 + 36*a^4*b^7 + 84*a^5*b^6 + 126*a^6*b^5 + 126*a^7*b^4 + 84*a^8*b^3 + 36*a^9*b^2)))*1i)/(4*(9*a^10*b + a^11 + a^2*b^9 + 9*a^3*b^8 + 36*a^4*b^7 + 84*a^5*b^6 + 126*a^6*b^5 + 126*a^7*b^4 + 84*a^8*b^3 + 36*a^9*b^2)))/(2752*a^4*b^20 - 144*a^3*b^21 - 32*a^2*b^22 + 33824*a^5*b^19 + 182784*a^6*b^18 + 613648*a^7*b^17 + 1429120*a^8*b^16 + 2433024*a^9*b^15 + 3113088*a^10*b^14 + 3034768*a^11*b^13 + 2261952*a^12*b^12 + 1281952*a^13*b^11 + 543872*a^14*b^10 + 167664*a^15*b^9 + 35584*a^16*b^8 + 4672*a^17*b^7 + 288*a^18*b^6 + ((-b^7*(a + b)^9)^(1/2)*(9*a + 2*b)*(tan(e + f*x)*(128*a^3*b^23 + 2624*a^4*b^22 + 24592*a^5*b^21 + 140608*a^6*b^20 + 554016*a^7*b^19 + 1613184*a^8*b^18 + 3637488*a^9*b^17 + 6570624*a^10*b^16 + 9747456*a^11*b^15 + 12075072*a^12*b^14 + 12596848*a^13*b^13 + 11073344*a^14*b^12 + 8154592*a^15*b^11 + 4977408*a^16*b^10 + 2481936*a^17*b^9 + 992256*a^18*b^8 + 310080*a^19*b^7 + 72960*a^20*b^6 + 12160*a^21*b^5 + 1280*a^22*b^4 + 64*a^23*b^3) - ((-b^7*(a + b)^9)^(1/2)*(9*a + 2*b)*(64*a^6*b^22 + 2304*a^7*b^21 + 29440*a^8*b^20 + 210560*a^9*b^19 + 997248*a^10*b^18 + 3404800*a^11*b^17 + 8806912*a^12*b^16 + 17809920*a^13*b^15 + 28745600*a^14*b^14 + 37533184*a^15*b^13 + 39975936*a^16*b^12 + 34874112*a^17*b^11 + 24926720*a^18*b^10 + 14545920*a^19*b^9 + 6874624*a^20*b^8 + 2595328*a^21*b^7 + 765504*a^22*b^6 + 170240*a^23*b^5 + 26880*a^24*b^4 + 2688*a^25*b^3 + 128*a^26*b^2 - (tan(e + f*x)*(-b^7*(a + b)^9)^(1/2)*(9*a + 2*b)*(512*a^7*b^23 + 10496*a^8*b^22 + 1024

00*a^9*b^21 + 632320*a^10*b^20 + 2772480*a^11*b^19 + 9178368*a^12*b^18 + 23
814144*a^13*b^17 + 49612800*a^14*b^16 + 84341760*a^15*b^15 + 118243840*a^16
*b^14 + 137592832*a^17*b^13 + 133293056*a^18*b^12 + 107494400*a^19*b^11 + 7
1938560*a^20*b^10 + 39690240*a^21*b^9 + 17860608*a^22*b^8 + 6449664*a^23*b^
7 + 1824000*a^24*b^6 + 389120*a^25*b^5 + 58880*a^26*b^4 + 5632*a^27*b^3 + 2
56*a^28*b^2))/(4*(9*a^10*b + a^11 + a^2*b^9 + 9*a^3*b^8 + 36*a^4*b^7 + 84*a
^5*b^6 + 126*a^6*b^5 + 126*a^7*b^4 + 84*a^8*b^3 + 36*a^9*b^2))))/(4*(9*a^10
*b + a^11 + a^2*b^9 + 9*a^3*b^8 + 36*a^4*b^7 + 84*a^5*b^6 + 126*a^6*b^5 + 1
26*a^7*b^4 + 84*a^8*b^3 + 36*a^9*b^2))))/(4*(9*a^10*b + a^11 + a^2*b^9 + 9*
a^3*b^8 + 36*a^4*b^7 + 84*a^5*b^6 + 126*a^6*b^5 + 126*a^7*b^4 + 84*a^8*b^3
+ 36*a^9*b^2)) - ((-b^7*(a + b)^9)^(1/2)*(9*a + 2*b)*(tan(e + f*x)*(128*a^3
*b^23 + 2624*a^4*b^22 + 24592*a^5*b^21 + 140608*a^6*b^20 + 554016*a^7*b^19
+ 1613184*a^8*b^18 + 3637488*a^9*b^17 + 6570624*a^10*b^16 + 9747456*a^11*b^
15 + 12075072*a^12*b^14 + 12596848*a^13*b^13 + 11073344*a^14*b^12 + 8154592
*a^15*b^11 + 4977408*a^16*b^10 + 2481936*a^17*b^9 + 992256*a^18*b^8 + 31008
0*a^19*b^7 + 72960*a^20*b^6 + 12160*a^21*b^5 + 1280*a^22*b^4 + 64*a^23*b^3)
 + ((-b^7*(a + b)^9)^(1/2)*(9*a + 2*b)*(64*a^6*b^22 + 2304*a^7*b^21 + 29440
*a^8*b^20 + 210560*a^9*b^19 + 997248*a^10*b^18 + 3404800*a^11*b^17 + 880691
2*a^12*b^16 + 17809920*a^13*b^15 + 28745600*a^14*b^14 + 37533184*a^15*b^13
+ 39975936*a^16*b^12 + 34874112*a^17*b^11 + 24926720*a^18*b^10 + 14545920*a
^19*b^9 + 6874624*a^20*b^8 + 2595328*a^21*b^7 + 765504*a^22*b^6 + 170240*a^
23*b^5 + 26880*a^24*b^4 + 2688*a^25*b^3 + 128*a^26*b^2 + (tan(e + f*x)*(-b^
7*(a + b)^9)^(1/2)*(9*a + 2*b)*(512*a^7*b^23 + 10496*a^8*b^22 + 102400*a^9*
b^21 + 632320*a^10*b^20 + 2772480*a^11*b^19 + 9178368*a^12*b^18 + 23814144*
a^13*b^17 + 49612800*a^14*b^16 + 84341760*a^15*b^15 + 118243840*a^16*b^14 +
 137592832*a^17*b^13 + 133293056*a^18*b^12 + 107494400*a^19*b^11 + 71938560
*a^20*b^10 + 39690240*a^21*b^9 + 17860608*a^22*b^8 + 6449664*a^23*b^7 + 182
4000*a^24*b^6 + 389120*a^25*b^5 + 58880*a^26*b^4 + 5632*a^27*b^3 + 256*a^28
b^2))/(4(9*a^10*b + a^11 + a^2*b^9 + 9*a^3*b^8 + 36*a^4*b^7 + 84*a^5*b^6
+ 126*a^6*b^5 + 126*a^7*b^4 + 84*a^8*b^3 + 36*a^9*b^2))))/(4*(9*a^10*b + a^
11 + a^2*b^9 + 9*a^3*b^8 + 36*a^4*b^7 + 84*a^5*b^6 + 126*a^6*b^5 + 126*a^7*
b^4 + 84*a^8*b^3 + 36*a^9*b^2))))/(4*(9*a^10*b + a^11 + a^2*b^9 + 9*a^3*b^8
 + 36*a^4*b^7 + 84*a^5*b^6 + 126*a^6*b^5 + 126*a^7*b^4 + 84*a^8*b^3 + 36*a^
9*b^2))))*(-b^7*(a + b)^9)^(1/2)*(9*a + 2*b)*1i)/(2*f*(9*a^10*b + a^11 + a^
2*b^9 + 9*a^3*b^8 + 36*a^4*b^7 + 84*a^5*b^6 + 126*a^6*b^5 + 126*a^7*b^4 + 8
4*a^8*b^3 + 36*a^9*b^2))

3.363 $\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{(a+b)^2}{4a^3f\,(b+a\cos^2(e+fx))^2} - \frac{a+b}{a^3f\,(b+a\cos^2(e+fx))} - \frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^3f}$$

```
[Out] 1/4*(a+b)^2/a^3/f/(b+a*cos(f*x+e)^2)^2+(-a-b)/a^3/f/(b+a*cos(f*x+e)^2)-1/2*
ln(b+a*cos(f*x+e)^2)/a^3/f
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4223, 455, 45}

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{(a+b)^2}{4a^3f\,(a\cos^2(e+fx)+b)^2} - \frac{a+b}{a^3f\,(a\cos^2(e+fx)+b)} - \frac{\log\left(a\cos^2(e+fx)+b\right)}{2a^3f}$$

```
[In] Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (a + b)^2/(4*a^3*f*(b + a*Cos[e + f*x]^2)^2) - (a + b)/(a^3*f*(b + a*Cos[e
+ f*x]^2)) - Log[b + a*Cos[e + f*x]^2]/(2*a^3*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x(1-x^2)^2}{(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \frac{(1-x)^2}{(b+ax)^3}\,dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{(a+b)^2}{a^2(b+ax)^3}-\frac{2(a+b)}{a^2(b+ax)^2}+\frac{1}{a^2(b+ax)}\right)dx, x, \cos^2(e+fx)\right)}{2f}\\
&= \frac{(a+b)^2}{4a^3f\left(b+a\cos^2(e+fx)\right)^2}-\frac{a+b}{a^3f\left(b+a\cos^2(e+fx)\right)}-\frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^3f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 3.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.74

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{2(a^2+4ab+3b^2)+(a+2b)^2\log(a+2b+a\cos(2(e+fx)))+a^2\cos^2(2(e+fx))\log(a+2b+a\cos(2(e}{2a^3f(a+2b+a\cos(2(e+f}$$

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -1/2*(2*(a^2 + 4*a*b + 3*b^2) + (a + 2*b)^2*Log[a + 2*b + a*Cos[2*(e + f*x)
]] + a^2*Cos[2*(e + f*x)]^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] + 2*a*Cos[2*(
e + f*x)]*(2*(a + b) + (a + 2*b)*Log[a + 2*b + a*Cos[2*(e + f*x)]]))/(a^3*f
*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Maple [A] (verified)

Time = 20.49 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{\frac{-2a-2b}{2a^{3}\left(b+a\cos\left(fx+e\right)^{2}\right)}+\frac{a^{2}+2ab+b^{2}}{4a^{3}\left(b+a\cos\left(fx+e\right)^{2}\right)^{2}}-\frac{\ln\left(b+a\cos\left(fx+e\right)^{2}\right)}{2a^{3}}}{f}$
default	$\frac{\frac{-2a-2b}{2a^{3}\left(b+a\cos\left(fx+e\right)^{2}\right)}+\frac{a^{2}+2ab+b^{2}}{4a^{3}\left(b+a\cos\left(fx+e\right)^{2}\right)^{2}}-\frac{\ln\left(b+a\cos\left(fx+e\right)^{2}\right)}{2a^{3}}}{f}$
risch	$\frac{ix}{a^{3}}+\frac{2ie}{a^{3}f}-\frac{4\left(a^{2}{\rm e}^{6i(fx+e)}+ab\,{\rm e}^{6i(fx+e)}+a^{2}{\rm e}^{4i(fx+e)}+4ab\,{\rm e}^{4i(fx+e)}+3b^{2}{\rm e}^{4i(fx+e)}+a^{2}{\rm e}^{2i(fx+e)}+ab\,{\rm e}^{2i(fx+e)}\right)}{a^{3}f\left(a\,{\rm e}^{4i(fx+e)}+2a\,{\rm e}^{2i(fx+e)}+4b\,{\rm e}^{2i(fx+e)}+a\right)^{2}}-$

```
[In] int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*(-2*a-2*b)/a^3/(b+a*cos(f*x+e)^2)+1/4*(a^2+2*a*b+b^2)/a^3/(b+a*cos
(f*x+e)^2)^2-1/2/a^3*ln(b+a*cos(f*x+e)^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.49

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{4\,(a^2+ab)\cos{(fx+e)}^2-a^2+2\,ab+3\,b^2+2\,\left(a^2\cos{(fx+e)}^4+2\,ab\cos{(fx+e)}^2+b^2\right)\log{\left(a\cos{(f}\right.}}{4\,\left(a^5f\cos{(fx+e)}^4+2\,a^4bf\cos{(fx+e)}^2+a^3b^2f\right)}$$

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(4*(a^2 + a*b)*cos(f*x + e)^2 - a^2 + 2*a*b + 3*b^2 + 2*(a^2*cos(f*x +
 e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*log(a*cos(f*x + e)^2 + b))/(a^5*f*cos(f
*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.44

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{4\,(a^2+ab)\sin(fx+e)^2-3\,a^2-6\,ab-3\,b^2}{a^5\sin(fx+e)^4+a^5+2\,a^4b+a^3b^2-2\,(a^5+a^4b)\sin(fx+e)^2}-\frac{2\,\log\left(a\sin(fx+e)^2-a-b\right)}{a^3}}{4\,f}$$

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
[Out] 1/4*((4*(a^2 + a*b)*sin(f*x + e)^2 - 3*a^2 - 6*a*b - 3*b^2)/(a^5*sin(f*x +
e)^4 + a^5 + 2*a^4*b + a^3*b^2 - 2*(a^5 + a^4*b)*sin(f*x + e)^2) - 2*log(a*
sin(f*x + e)^2 - a - b)/a^3)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. 2(74) = 148.

Time = 1.93 (sec) , antiderivative size = 531, normalized size of antiderivative = 6.81

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{2\,\log\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^3}-\frac{4\,\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a^3}-\frac{3\,a^2+6\,ab+3\,b^2+\frac{20\,a^2}{c}\cdots}{\cdots}}{\cdots}$$

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
[Out] -1/4*(2*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*
x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^
2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/a^3 - 4*log(abs(-(cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) + 1))/a^3 - (3*a^2 + 6*a*b + 3*b^2 + 20*a^2*(c
os(f*x + e) - 1)/(cos(f*x + e) + 1) + 8*a*b*(cos(f*x + e) - 1)/(cos(f*x + e
) + 1) - 12*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 50*a^2*(cos(f*x + e
) - 1)^2/(cos(f*x + e) + 1)^2 - 28*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
 1)^2 + 18*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 20*a^2*(cos(f*x
+ e) - 1)^3/(cos(f*x + e) + 1)^3 + 8*a*b*(cos(f*x + e) - 1)^3/(cos(f*x + e)
 + 1)^3 - 12*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 3*a^2*(cos(f*x
 + e) - 1)^4/(cos(f*x + e) + 1)^4 + 6*a*b*(cos(f*x + e) - 1)^4/(cos(f*x + e
) + 1)^4 + 3*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/((a + b + 2*a*(
cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e)
 + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^
2/(cos(f*x + e) + 1)^2)^2*a^3))/f
```

Mupad [B] (verification not implemented)

Time = 19.93 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.13

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{\operatorname{atanh}\left(\frac{4\,b^2\tan(e+f\,x)^2}{8\,b^2+\frac{8\,b^3}{a}+4\,b^2\tan(e+f\,x)^2+\frac{8\,b^3\tan(e+f\,x)^2}{a}}\right)}{a^3\,f}$$
$$+ \frac{\frac{-a^3+3\,a\,b^2+2\,b^3}{4\,a^2\,b^2}-\frac{\tan(e+f\,x)^2\,(a^2-b^2)}{2\,a^2\,b}}{f\,\left(2\,a\,b+a^2+b^2+\tan\left(e+f\,x\right)^2\,\left(2\,b^2+2\,a\,b\right)+b^2\tan\left(e+f\,x\right)^4\right)}$$

```
[In] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] atanh((4*b^2*tan(e + f*x)^2)/(8*b^2 + (8*b^3)/a + 4*b^2*tan(e + f*x)^2 + (8
*b^3*tan(e + f*x)^2)/a))/(a^3*f) + ((3*a*b^2 - a^3 + 2*b^3)/(4*a^2*b^2) - (
tan(e + f*x)^2*(a^2 - b^2))/(2*a^2*b))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)
^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4))
```

3.364 $\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 81

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{b(a+b)}{4a^3f\,(b+a\cos^2(e+fx))^2} + \frac{a+2b}{2a^3f\,(b+a\cos^2(e+fx))} + \frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^3f}$$

```
[Out] -1/4*b*(a+b)/a^3/f/(b+a*cos(f*x+e)^2)^2+1/2*(a+2*b)/a^3/f/(b+a*cos(f*x+e)^2
)+1/2*ln(b+a*cos(f*x+e)^2)/a^3/f
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4223, 457, 78}

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{b(a+b)}{4a^3f\,(a\cos^2(e+fx)+b)^2} + \frac{a+2b}{2a^3f\,(a\cos^2(e+fx)+b)} + \frac{\log\left(a\cos^2(e+fx)+b\right)}{2a^3f}$$

```
[In] Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -1/4*(b*(a + b))/(a^3*f*(b + a*Cos[e + f*x]^2)^2) + (a + 2*b)/(2*a^3*f*(b +
 a*Cos[e + f*x]^2)) + Log[b + a*Cos[e + f*x]^2]/(2*a^3*f)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^3(1-x^2)}{(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \frac{(1-x)x}{(b+ax)^3}\,dx, x, \cos^2(e+fx)\right)}{2f}\\ &= -\frac{\text{Subst}\left(\int \left(-\frac{b(a+b)}{a^2(b+ax)^3}+\frac{a+2b}{a^2(b+ax)^2}-\frac{1}{a^2(b+ax)}\right)dx, x, \cos^2(e+fx)\right)}{2f}\\ &= -\frac{b(a+b)}{4a^3f\left(b+a\cos^2(e+fx)\right)^2}+\frac{a+2b}{2a^3f\left(b+a\cos^2(e+fx)\right)}+\frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^3f}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.62

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{2(a^2+3ab+3b^2)+(a+2b)^2\log(a+2b+a\cos(2(e+fx)))+a^2\cos^2(2(e+fx))\log(a+2b+a\cos(2(e ...}{2a^3f(a+2b+a\cos(2(e+fx)))^2}$$

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (2*(a^2 + 3*a*b + 3*b^2) + (a + 2*b)^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] +
a^2*Cos[2*(e + f*x)]^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] + 2*a*(a + 2*b)*Co
s[2*(e + f*x)]*(1 + Log[a + 2*b + a*Cos[2*(e + f*x)]]))/(2*a^3*f*(a + 2*b +
 a*Cos[2*(e + f*x)])^2)
```

Maple [A] (verified)

Time = 11.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-\frac{(a+b)b}{4a^3\left(b+a\cos(fx+e)^2\right)^2}-\frac{-a-2b}{2a^3\left(b+a\cos(fx+e)^2\right)}+\frac{\ln\left(b+a\cos(fx+e)^2\right)}{2a^3}}{f}$
default	$\frac{-\frac{(a+b)b}{4a^3\left(b+a\cos(fx+e)^2\right)^2}-\frac{-a-2b}{2a^3\left(b+a\cos(fx+e)^2\right)}+\frac{\ln\left(b+a\cos(fx+e)^2\right)}{2a^3}}{f}$
risch	$-\frac{ix}{a^3}-\frac{2ie}{a^3f}+\frac{2a^2e^{6i(fx+e)}+4ab\,e^{6i(fx+e)}+4a^2e^{4i(fx+e)}+12ab\,e^{4i(fx+e)}+12b^2e^{4i(fx+e)}+2a^2e^{2i(fx+e)}+4ab\,e^{2i(fx+}}{a^3f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2}$

```
[In] int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/4*(a+b)*b/a^3/(b+a*cos(f*x+e)^2)^2-1/2*(-a-2*b)/a^3/(b+a*cos(f*x+e)
^2)+1/2/a^3*ln(b+a*cos(f*x+e)^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.37

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{2\,(a^2+2\,ab)\cos\left(fx+e\right)^2+ab+3\,b^2+2\,\left(a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2\right)\log\left(a\cos\left(fx+e\right)^2+\right.}{4\,\left(a^5f\cos\left(fx+e\right)^4+2\,a^4bf\cos\left(fx+e\right)^2+a^3b^2f\right)}$$

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a^2 + 2*a*b)*cos(f*x + e)^2 + a*b + 3*b^2 + 2*(a^2*cos(f*x + e)^4 +
 2*a*b*cos(f*x + e)^2 + b^2)*log(a*cos(f*x + e)^2 + b))/(a^5*f*cos(f*x + e)
^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.40

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{2\,(a^2+2\,ab)\sin(fx+e)^2-2\,a^2-5\,ab-3\,b^2}{a^5\sin(fx+e)^4+a^5+2\,a^4b+a^3b^2-2\,(a^5+a^4b)\sin(fx+e)^2}-\frac{2\log\left(a\sin(fx+e)^2-a-b\right)}{a^3}}{4\,f}$$

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
[Out] -1/4*((2*(a^2 + 2*a*b)*sin(f*x + e)^2 - 2*a^2 - 5*a*b - 3*b^2)/(a^5*sin(f*x
 + e)^4 + a^5 + 2*a^4*b + a^3*b^2 - 2*(a^5 + a^4*b)*sin(f*x + e)^2) - 2*log
(a*sin(f*x + e)^2 - a - b)/a^3)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. 2(75) = 150.

Time = 0.96 (sec) , antiderivative size = 656, normalized size of antiderivative = 8.10

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{3\,a^3+9\,a^2b+9\,ab^2+3\,b^3+\frac{20\,a^3(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{28\,a^2b(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4\,ab^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{12\,b^3(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{34\,a^3(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{22\,a^2b(\cos(f}{(\cos(fx+}}{(a^4}$$

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
[Out] -1/4*((3*a^3 + 9*a^2*b + 9*a*b^2 + 3*b^3 + 20*a^3*(cos(f*x + e) - 1)/(cos(f
*x + e) + 1) + 28*a^2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*a*b^2*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) - 12*b^3*(cos(f*x + e) - 1)/(cos(f*x + e
```

```
) + 1) + 34*a^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 22*a^2*b*(cos(f
*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 10*a*b^2*(cos(f*x + e) - 1)^2/(cos(f*
x + e) + 1)^2 + 18*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 20*a^3*(
cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 28*a^2*b*(cos(f*x + e) - 1)^3/(c
os(f*x + e) + 1)^3 - 4*a*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 12
*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 3*a^3*(cos(f*x + e) - 1)^4
/(cos(f*x + e) + 1)^4 + 9*a^2*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 +
 9*a*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 3*b^3*(cos(f*x + e) -
1)^4/(cos(f*x + e) + 1)^4)/((a^4 + a^3*b)*(a + b + 2*a*(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x
+ e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1
)^2)^2) - 2*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/a^3 + 4*log(abs(-(cos(
f*x + e) - 1)/(cos(f*x + e) + 1) + 1))/a^3)/f
```

Mupad [B] (verification not implemented)

Time = 19.47 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.89

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{a^2+3\,a\,b+2\,b^2}{4\,a^2\,b}+\frac{b\tan(e+f\,x)^2}{2\,a^2}}{f\,\left(2\,a\,b+a^2+b^2+\tan\left(e+f\,x\right)^2\,\left(2\,b^2+2\,a\,b\right)+b^2\tan\left(e+f\,x\right)^4\right)} - \frac{\operatorname{atanh}\left(\frac{4\,b^2\tan(e+f\,x)^2}{8\,b^2+\frac{8\,b^3}{a}+4\,b^2\tan(e+f\,x)^2+\frac{8\,b^3\tan(e+f\,x)^2}{a}}\right)}{a^3\,f}$$

```
[In] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] - ((3*a*b + a^2 + 2*b^2)/(4*a^2*b) + (b*tan(e + f*x)^2)/(2*a^2))/(f*(2*a*b
+ a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4)) - atanh
((4*b^2*tan(e + f*x)^2)/(8*b^2 + (8*b^3)/a + 4*b^2*tan(e + f*x)^2 + (8*b^3*
tan(e + f*x)^2)/a))/(a^3*f)
```

3.365 $\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 74

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^3} dx = \frac{b^2}{4a^3f(b+a\cos^2(e+fx))^2} - \frac{b}{a^3f(b+a\cos^2(e+fx))} - \frac{\log(b+a\cos^2(e+fx))}{2a^3f}$$

```
[Out] 1/4*b^2/a^3/f/(b+a*cos(f*x+e)^2)^2-b/a^3/f/(b+a*cos(f*x+e)^2)-1/2*ln(b+a*co
s(f*x+e)^2)/a^3/f
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4223, 272, 45}

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^3} dx = \frac{b^2}{4a^3f(a\cos^2(e+fx)+b)^2} - \frac{b}{a^3f(a\cos^2(e+fx)+b)} - \frac{\log(a\cos^2(e+fx)+b)}{2a^3f}$$

```
[In] Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] b^2/(4*a^3*f*(b + a*Cos[e + f*x]^2)^2) - b/(a^3*f*(b + a*Cos[e + f*x]^2)) -
 Log[b + a*Cos[e + f*x]^2]/(2*a^3*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^5}{(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f} \\
&= -\frac{\text{Subst}\left(\int \frac{x^2}{(b+ax)^3}\,dx, x, \cos^2(e+fx)\right)}{2f} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{a^2(b+ax)^3} - \frac{2b}{a^2(b+ax)^2} + \frac{1}{a^2(b+ax)}\right)\,dx, x, \cos^2(e+fx)\right)}{2f} \\
&= \frac{b^2}{4a^3f\left(b+a\cos^2(e+fx)\right)^2} - \frac{b}{a^3f\left(b+a\cos^2(e+fx)\right)} - \frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^3f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.91 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.74

$$\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{2b(2a+3b) + (a+2b)^2\log(a+2b+a\cos(2(e+fx))) + a^2\cos^2(2(e+fx))\log(a+2b+a\cos(2(e+fx}{2a^3f(a+2b+a\cos(2(e+fx)))^2}$$

```
[In] Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -1/2*(2*b*(2*a + 3*b) + (a + 2*b)^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] + a^2
*Cos[2*(e + f*x)]^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] + 2*a*Cos[2*(e + f*x)
]*(2*b + (a + 2*b)*Log[a + 2*b + a*Cos[2*(e + f*x)]]))/(a^3*f*(a + 2*b + a*
Cos[2*(e + f*x)])^2)
```

Maple [A] (verified)

Time = 11.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.09

method	result	si
derivativedivides	$\frac{1}{4fa\left(a+b\sec(fx+e)^2\right)^2}+\frac{1}{2f\,a^2\left(a+b\sec(fx+e)^2\right)}-\frac{\ln\left(a+b\sec(fx+e)^2\right)}{2f\,a^3}+\frac{\ln(\sec(fx+e))}{f\,a^3}$	8
default	$\frac{1}{4fa\left(a+b\sec(fx+e)^2\right)^2}+\frac{1}{2f\,a^2\left(a+b\sec(fx+e)^2\right)}-\frac{\ln\left(a+b\sec(fx+e)^2\right)}{2f\,a^3}+\frac{\ln(\sec(fx+e))}{f\,a^3}$	8
risch	$\frac{ix}{a^3}+\frac{2ie}{a^3 f}-\frac{4b\left(a\,\mathrm{e}^{6i(fx+e)}+2a\,\mathrm{e}^{4i(fx+e)}+3b\,\mathrm{e}^{4i(fx+e)}+a\,\mathrm{e}^{2i(fx+e)}\right)}{a^3\left(a\,\mathrm{e}^{4i(fx+e)}+2a\,\mathrm{e}^{2i(fx+e)}+4b\,\mathrm{e}^{2i(fx+e)}+a\right)^2 f}-\frac{\ln\left(\mathrm{e}^{4i(fx+e)}+\frac{2(a+2b)\mathrm{e}^{2i(fx+e)}}{a}+1\right)}{2a^3 f}$	1

```
[In] int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f/a/(a+b*sec(f*x+e)^2)^2+1/2/f/a^2/(a+b*sec(f*x+e)^2)-1/2/f/a^3*ln(a+b*
sec(f*x+e)^2)+1/f/a^3*ln(sec(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.38

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{4\,ab\cos\left(fx+e\right)^2+3\,b^2+2\,\left(a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2\right)\log\left(a\cos\left(fx+e\right)^2+b\right)}{4\,\left(a^5 f\cos\left(fx+e\right)^4+2\,a^4 bf\cos\left(fx+e\right)^2+a^3b^2 f\right)}$$

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*b*cos(f*x + e)^2 + 3*b^2 + 2*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x
+ e)^2 + b^2)*log(a*cos(f*x + e)^2 + b))/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f*
cos(f*x + e)^2 + a^3*b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.38

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{4\,ab\sin(fx+e)^2-4\,ab-3\,b^2}{a^5\sin(fx+e)^4+a^5+2\,a^4b+a^3b^2-2\,(a^5+a^4b)\sin(fx+e)^2}-\frac{2\,\log\left(a\sin(fx+e)^2-a-b\right)}{a^3}}{4\,f}$$

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
[Out] 1/4*((4*a*b*sin(f*x + e)^2 - 4*a*b - 3*b^2)/(a^5*sin(f*x + e)^4 + a^5 + 2*a
^4*b + a^3*b^2 - 2*(a^5 + a^4*b)*sin(f*x + e)^2) - 2*log(a*sin(f*x + e)^2 -
 a - b)/a^3)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 782 vs. 2(70) = 140.

Time = 0.58 (sec) , antiderivative size = 782, normalized size of antiderivative = 10.57

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
[Out] 1/4*((3*a^4 + 12*a^3*b + 18*a^2*b^2 + 12*a*b^3 + 3*b^4 + 12*a^4*(cos(f*x +
e) - 1)/(cos(f*x + e) + 1) + 40*a^3*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1)
 + 24*a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 16*a*b^3*(cos(f*x + e
) - 1)/(cos(f*x + e) + 1) - 12*b^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) +
18*a^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 56*a^3*b*(cos(f*x + e) -
 1)^2/(cos(f*x + e) + 1)^2 + 12*a^2*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e)
+ 1)^2 + 8*a*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 18*b^4*(cos(f*
x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 12*a^4*(cos(f*x + e) - 1)^3/(cos(f*x +
 e) + 1)^3 + 40*a^3*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 24*a^2*b^
2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 16*a*b^3*(cos(f*x + e) - 1)^3
/(cos(f*x + e) + 1)^3 - 12*b^4*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 +
3*a^4*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 12*a^3*b*(cos(f*x + e) -
1)^4/(cos(f*x + e) + 1)^4 + 18*a^2*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) +
 1)^4 + 12*a*b^3*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 3*b^4*(cos(f*x
 + e) - 1)^4/(cos(f*x + e) + 1)^4)/((a^5 + 2*a^4*b + a^3*b^2)*(a + b + 2*a*
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e
) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)
```

```
^2/(cos(f*x + e) + 1)^2)^2) - 2*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x
 + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) -
1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/a^
3 + 4*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1))/a^3)/f
```

Mupad [B] (verification not implemented)

Time = 19.33 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.92

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{3\,a+2\,b}{4\,a^2}+\frac{b\tan(e+f\,x)^2}{2\,a^2}}{f\,\left(2\,a\,b+a^2+b^2+\tan\left(e+f\,x\right)^2\,\left(2\,b^2+2\,a\,b\right)+b^2\tan\left(e+f\,x\right)^4\right)}+\frac{\mathrm{atanh}\left(\frac{4\,b^2\tan(e+f\,x)^2}{8\,b^2+\frac{8\,b^3}{a}+4\,b^2\tan(e+f\,x)^2+\frac{8\,b^3\tan(e+f\,x)^2}{a}}\right)}{a^3\,f}$$

```
[In] int(tan(e + f*x)/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] ((3*a + 2*b)/(4*a^2) + (b*tan(e + f*x)^2)/(2*a^2))/(f*(2*a*b + a^2 + b^2 +
tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4)) + atanh((4*b^2*tan(e
+ f*x)^2)/(8*b^2 + (8*b^3)/a + 4*b^2*tan(e + f*x)^2 + (8*b^3*tan(e + f*x)^2
)/a))/(a^3*f)
```

3.366 $\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 130

$$\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{b^3}{4a^3(a+b)f\left(b+a\cos^2(e+fx)\right)^2} + \frac{b^2(3a+2b)}{2a^3(a+b)^2f\left(b+a\cos^2(e+fx)\right)} + \frac{b(3a^2+3ab+b^2)\log\left(b+a\cos^2(e+fx)\right)}{2a^3(a+b)^3f} + \frac{\log(\sin(e+fx))}{(a+b)^3f}$$

```
[Out] -1/4*b^3/a^3/(a+b)/f/(b+a*cos(f*x+e)^2)^2+1/2*b^2*(3*a+2*b)/a^3/(a+b)^2/f/(
b+a*cos(f*x+e)^2)+1/2*b*(3*a^2+3*a*b+b^2)*ln(b+a*cos(f*x+e)^2)/a^3/(a+b)^3/
f+ln(sin(f*x+e))/(a+b)^3/f
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used

$= \{4223, 457, 90\}$

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{b^3}{4a^3f(a+b)\left(a\cos^2(e+fx)+b\right)^2} + \frac{b^2(3a+2b)}{2a^3f(a+b)^2\left(a\cos^2(e+fx)+b\right)} + \frac{b(3a^2+3ab+b^2)\log\left(a\cos^2(e+fx)+b\right)}{2a^3f(a+b)^3} + \frac{\log(\sin(e+fx))}{f(a+b)^3}$$

```
[In] Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -1/4*b^3/(a^3*(a + b)*f*(b + a*Cos[e + f*x]^2)^2) + (b^2*(3*a + 2*b))/(2*a^
3*(a + b)^2*f*(b + a*Cos[e + f*x]^2)) + (b*(3*a^2 + 3*a*b + b^2)*Log[b + a*
Cos[e + f*x]^2])/(2*a^3*(a + b)^3*f) + Log[Sin[e + f*x]]/((a + b)^3*f)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^7}{(1-x^2)(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^3}{(1-x)(b+ax)^3}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)} - \frac{b^3}{a^2(a+b)(b+ax)^3} + \frac{b^2(3a+2b)}{a^2(a+b)^2(b+ax)^2} - \frac{b(3a^2+3ab+b^2)}{a^2(a+b)^3(b+ax)}\right) dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{b^3}{4a^3(a+b)f\,(b+a\cos^2(e+fx))^2} + \frac{b^2(3a+2b)}{2a^3(a+b)^2f\,(b+a\cos^2(e+fx))} + \frac{b(3a^2+3ab+b^2)\log\left(b+a\cos^2(e+fx)\right)}{2a^3(a+b)^3f} + \frac{\log(\sin(e+fx))}{(a+b)^3f}$$

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.22

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{(a+2b+a\cos(2(e+fx)))^3\sec^6(e+fx)\left(4\log(\sin(e+fx)) + \frac{2b(3a^2+3ab+b^2)\log(a+b-a\sin^2(e+fx))}{a^3} - \frac{b^3(a}{a^3(a+b-a\sin}\right.}{32(a+b)^3f\,(a+b\sec^2(e+fx))^3}$$

```
[In] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*(4*Log[Sin[e + f*x]] + (2*
b*(3*a^2 + 3*a*b + b^2)*Log[a + b - a*Sin[e + f*x]^2])/a^3 - (b^3*(a + b)^2
)/(a^3*(a + b - a*Sin[e + f*x]^2)^2) + (2*b^2*(a + b)*(3*a + 2*b))/(a^3*(a
+ b - a*Sin[e + f*x]^2))))/(32*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A] (verified)

Time = 17.52 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{\frac{b\left(\frac{b(3a^2+5ab+2b^2)}{a^3(b+a\cos(fx+e)^2)}+\frac{(3a^2+3ab+b^2)\ln(b+a\cos(fx+e)^2)}{a^3}-\frac{b^2(a^2+2ab+b^2)}{2a^3(b+a\cos(fx+e)^2)^2}\right)}{2(a+b)^3}+\frac{\ln(1+\cos(fx+e))}{2(a+b)^3}+\frac{\ln(-1+\cos(fx+e))}{2(a+b)^3}}{f}$
default	$\frac{\frac{b\left(\frac{b(3a^2+5ab+2b^2)}{a^3(b+a\cos(fx+e)^2)}+\frac{(3a^2+3ab+b^2)\ln(b+a\cos(fx+e)^2)}{a^3}-\frac{b^2(a^2+2ab+b^2)}{2a^3(b+a\cos(fx+e)^2)^2}\right)}{2(a+b)^3}+\frac{\ln(1+\cos(fx+e))}{2(a+b)^3}+\frac{\ln(-1+\cos(fx+e))}{2(a+b)^3}}{f}$
risch	$\frac{ix}{a^3} - \frac{2ix}{a^3+3a^2b+3a\,b^2+b^3} - \frac{2ie}{f(a^3+3a^2b+3a\,b^2+b^3)} - \frac{6ibx}{a(a^3+3a^2b+3a\,b^2+b^3)} - \frac{6ibe}{af(a^3+3a^2b+3a\,b^2+b^3)} - \frac{}{a^2(a^3}$

```
[In] int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*b/(a+b)^3*(1/a^3*b*(3*a^2+5*a*b+2*b^2)/(b+a*cos(f*x+e)^2)+(3*a^2+3
*a*b+b^2)/a^3*ln(b+a*cos(f*x+e)^2)-1/2*b^2*(a^2+2*a*b+b^2)/a^3/(b+a*cos(f*x
+e)^2)^2)+1/2/(a+b)^3*ln(1+cos(f*x+e))+1/2/(a+b)^3*ln(-1+cos(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(124) = 248$.

Time = 0.61 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.36

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{5\,a^2b^3+8\,ab^4+3\,b^5+2\,(3\,a^3b^2+5\,a^2b^3+2\,ab^4)\cos{(fx+e)}^2+2\,\left(3\,a^2b^3+3\,ab^4+b^5+(3\,a^4b+3\,a^3b^2+\right.}{4\,\left((a^8+3\,a^7b+3\,a^6b^2+a^5b^3)f\cos{(fx\right.}}$$

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(5*a^2*b^3 + 8*a*b^4 + 3*b^5 + 2*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*cos(
f*x + e)^2 + 2*(3*a^2*b^3 + 3*a*b^4 + b^5 + (3*a^4*b + 3*a^3*b^2 + a^2*b^3)
*cos(f*x + e)^4 + 2*(3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(f*x + e)^2)*log(a*c
os(f*x + e)^2 + b) + 4*(a^5*cos(f*x + e)^4 + 2*a^4*b*cos(f*x + e)^2 + a^3*b
^2)*log(1/2*sin(f*x + e)))/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x
 + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a
^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.87

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{2\,(3\,a^2b+3\,ab^2+b^3)\log\left(a\sin(fx+e)^2-a-b\right)}{a^6+3\,a^5b+3\,a^4b^2+a^3b^3}+\frac{6\,a^2b^2+9\,ab^3+3\,b^4-2\,(3\,a^2b^2+2\,ab^3)\sin(fx+e)^2}{a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4+(a^7+2\,a^6b+a^5b^2)\sin(fx+e)^4-2\,(a^7+3\,a^6b+3\,a^5b^2+a^4b^3)\sin}}{4\,f}$$

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(3*a^2*b + 3*a*b^2 + b^3)*log(a*sin(f*x + e)^2 - a - b)/(a^6 + 3*a^5
*b + 3*a^4*b^2 + a^3*b^3) + (6*a^2*b^2 + 9*a*b^3 + 3*b^4 - 2*(3*a^2*b^2 + 2
*a*b^3)*sin(f*x + e)^2)/(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4 +
(a^7 + 2*a^6*b + a^5*b^2)*sin(f*x + e)^4 - 2*(a^7 + 3*a^6*b + 3*a^5*b^2 + a
^4*b^3)*sin(f*x + e)^2) + 2*log(sin(f*x + e)^2)/(a^3 + 3*a^2*b + 3*a*b^2 +
b^3))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 767 vs. $2(124) = 248$.

Time = 0.42 (sec) , antiderivative size = 767, normalized size of antiderivative = 5.90

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*(3*a^2*b + 3*a*b^2 + b^3)*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*
x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) -
 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(
a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3) + 2*log(abs(-cos(f*x + e) + 1)/abs(cos
(f*x + e) + 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (9*a^3*b + 18*a^2*b^2 + 1
2*a*b^3 + 3*b^4 + 36*a^3*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 24*a^2*b
^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 16*a*b^3*(cos(f*x + e) - 1)/(cos
(f*x + e) + 1) - 12*b^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 54*a^3*b*(c
os(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 12*a^2*b^2*(cos(f*x + e) - 1)^2/(
cos(f*x + e) + 1)^2 + 8*a*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 1
8*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 36*a^3*b*(cos(f*x + e) -
1)^3/(cos(f*x + e) + 1)^3 + 24*a^2*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) +
 1)^3 - 16*a*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 12*b^4*(cos(f*
x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 9*a^3*b*(cos(f*x + e) - 1)^4/(cos(f*x
+ e) + 1)^4 + 18*a^2*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 12*a*b
^3*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 3*b^4*(cos(f*x + e) - 1)^4/(
cos(f*x + e) + 1)^4)/((a^5 + 2*a^4*b + a^3*b^2)*(a + b + 2*a*(cos(f*x + e)
- 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(co
s(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x +
e) + 1)^2)^2) - 4*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1))/a^3)
/f
```

Mupad [B] (verification not implemented)

Time = 20.77 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.46

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{\ln(\tan(e+f\,x))}{f\,(a^3+3\,a^2\,b+3\,a\,b^2+b^3)} - \frac{\frac{2\,b^2+5\,a\,b}{4\,a^2\,(a+b)}+\frac{b\tan(e+f\,x)^2\,(b^2+2\,a\,b)}{2\,a^2\,(a+b)^2}}{f\,\left(2\,a\,b+a^2+b^2+\tan(e+f\,x)^2\,(2\,b^2+2\,a\,b)+b^2\tan(e+f\,x)^4\right)} - \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a^3\,f} + \frac{b\,\ln\left(b\tan(e+f\,x)^2+a+b\right)\,(3\,a^2+3\,a\,b+b^2)}{2\,a^3\,f\,(a+b)^3}$$

```
[In] int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] log(tan(e + f*x))/(f*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) - ((5*a*b + 2*b^2)/(4
*a^2*(a + b)) + (b*tan(e + f*x)^2*(2*a*b + b^2))/(2*a^2*(a + b)^2))/(f*(2*a
*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4)) - lo
g(tan(e + f*x)^2 + 1)/(2*a^3*f) + (b*log(a + b + b*tan(e + f*x)^2)*(3*a*b +
 3*a^2 + b^2))/(2*a^3*f*(a + b)^3)
```

3.367 $\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 154

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{b^4}{4a^3(a+b)^2f\,(b+a\cos^2(e+fx))^2} - \frac{b^3(2a+b)}{a^3(a+b)^3f\,(b+a\cos^2(e+fx))} - \frac{\csc^2(e+fx)}{2(a+b)^3f} - \frac{b^2(6a^2+4ab+b^2)\log\left(b+a\cos^2(e+fx)\right)}{2a^3(a+b)^4f} - \frac{(a+4b)\log(\sin(e+fx))}{(a+b)^4f}$$

```
[Out] 1/4*b^4/a^3/(a+b)^2/f/(b+a*cos(f*x+e)^2)^2-b^3*(2*a+b)/a^3/(a+b)^3/f/(b+a*c
os(f*x+e)^2)-1/2*csc(f*x+e)^2/(a+b)^3/f-1/2*b^2*(6*a^2+4*a*b+b^2)*ln(b+a*co
s(f*x+e)^2)/a^3/(a+b)^4/f-(a+4*b)*ln(sin(f*x+e))/(a+b)^4/f
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used

= {4223, 457, 90}

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{b^4}{4a^3f(a+b)^2\left(a\cos^2(e+fx)+b\right)^2} - \frac{b^3(2a+b)}{a^3f(a+b)^3\left(a\cos^2(e+fx)+b\right)} - \frac{b^2(6a^2+4ab+b^2)\log\left(a\cos^2(e+fx)+b\right)}{2a^3f(a+b)^4} - \frac{\csc^2(e+fx)}{2f(a+b)^3} - \frac{(a+4b)\log(\sin(e+fx))}{f(a+b)^4}$$

```
[In] Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] b^4/(4*a^3*(a + b)^2*f*(b + a*Cos[e + f*x]^2)^2) - (b^3*(2*a + b))/(a^3*(a
+ b)^3*f*(b + a*Cos[e + f*x]^2)) - Csc[e + f*x]^2/(2*(a + b)^3*f) - (b^2*(6
*a^2 + 4*a*b + b^2)*Log[b + a*Cos[e + f*x]^2])/(2*a^3*(a + b)^4*f) - ((a +
4*b)*Log[Sin[e + f*x]])/((a + b)^4*f)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^9}{(1-x^2)^2(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^4}{(1-x)^2(b+ax)^3}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)^3(-1+x)^2} + \frac{a+4b}{(a+b)^4(-1+x)} + \frac{b^4}{a^2(a+b)^2(b+ax)^3} - \frac{2b^3(2a+b)}{a^2(a+b)^3(b+ax)^2} + \frac{b^2(6a^2+4ab+b^2)}{a^2(a+b)^4(b+ax)}\right) dx, x, \text{co}}{2f}$$

$$= \frac{b^4}{4a^3(a+b)^2 f\,(b+a\cos^2(e+fx))^2} - \frac{b^3(2a+b)}{a^3(a+b)^3 f\,(b+a\cos^2(e+fx))} - \frac{\csc^2(e+fx)}{2(a+b)^3 f} - \frac{b^2(6a^2+4ab+b^2)\log\left(b+a\cos^2(e+fx)\right)}{2a^3(a+b)^4 f} - \frac{(a+4b)\log(\sin(e+fx))}{(a+b)^4 f}$$

Mathematica [A] (verified)

Time = 1.98 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.14

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))^3\sec^6(e+fx)\left(2(a+b)\csc^2(e+fx)+4(a+4b)\log(\sin(e+fx))+\frac{2b^2(6a^2+}{}\right.}{32(a+b)^4 f\,(a+b\sec^2(e+fx))^3}$$

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -1/32*((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*(2*(a + b)*Csc[e + f
*x]^2 + 4*(a + 4*b)*Log[Sin[e + f*x]] + (2*b^2*(6*a^2 + 4*a*b + b^2)*Log[a
+ b - a*Sin[e + f*x]^2])/a^3 - (b^4*(a + b)^2)/(a^3*(a + b - a*Sin[e + f*x]
^2)^2) + (4*b^3*(a + b)*(2*a + b))/(a^3*(a + b - a*Sin[e + f*x]^2))))/((a +
 b)^4*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A] (verified)

Time = 30.71 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{-\frac{1}{4(a+b)^3(1+\cos(fx+e))}+\frac{(-a-4b)\ln(1+\cos(fx+e))}{2(a+b)^4}-\frac{b^2\left(\frac{2b\left(2a^2+3ab+b^2\right)}{a^3\left(b+a\cos(fx+e)^2\right)}+\frac{\left(6a^2+4ab+b^2\right)\ln\left(b+a\cos(fx+e)^2\right)}{a^3}-\frac{b^2\left(a^2\right.}{2a^3\left(b+a\right.}\right.}{2(a+b)^4}}{f}$
default	$\frac{-\frac{1}{4(a+b)^3(1+\cos(fx+e))}+\frac{(-a-4b)\ln(1+\cos(fx+e))}{2(a+b)^4}-\frac{b^2\left(\frac{2b\left(2a^2+3ab+b^2\right)}{a^3\left(b+a\cos(fx+e)^2\right)}+\frac{\left(6a^2+4ab+b^2\right)\ln\left(b+a\cos(fx+e)^2\right)}{a^3}-\frac{b^2\left(a^2\right.}{2a^3\left(b+a\right.}\right.}{2(a+b)^4}}{f}$
risch	Expression too large to display

```
[In] int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/4/(a+b)^3/(1+cos(f*x+e))+1/2*(-a-4*b)/(a+b)^4*ln(1+cos(f*x+e))-1/2*
b^2/(a+b)^4*(2/a^3*b*(2*a^2+3*a*b+b^2)/(b+a*cos(f*x+e)^2)+(6*a^2+4*a*b+b^2)
/a^3*ln(b+a*cos(f*x+e)^2)-1/2*b^2*(a^2+2*a*b+b^2)/a^3/(b+a*cos(f*x+e)^2)^2)
+1/4/(a+b)^3/(-1+cos(f*x+e))+1/2*(-a-4*b)/(a+b)^4*ln(-1+cos(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. $2(148) = 296$.

Time = 0.93 (sec) , antiderivative size = 584, normalized size of antiderivative = 3.79

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$=\frac{2\,a^4b^2+2\,a^3b^3+7\,a^2b^4+10\,ab^5+3\,b^6+2\,(a^6+a^5b-4\,a^3b^3-6\,a^2b^4-2\,ab^5)\cos{(fx+e)}^4+(4\,a^5b+4\,a}{}$$

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^4*b^2 + 2*a^3*b^3 + 7*a^2*b^4 + 10*a*b^5 + 3*b^6 + 2*(a^6 + a^5*b
- 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^4 + (4*a^5*b + 4*a^4*b^2 +
8*a^3*b^3 + 5*a^2*b^4 - 6*a*b^5 - 3*b^6)*cos(f*x + e)^2 - 2*((6*a^4*b^2 + 4
*a^3*b^3 + a^2*b^4)*cos(f*x + e)^6 - 6*a^2*b^4 - 4*a*b^5 - b^6 - (6*a^4*b^2
 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^4 - (12*a^3*b^3 + 2*a^2*b^
4 - 2*a*b^5 - b^6)*cos(f*x + e)^2)*log(a*cos(f*x + e)^2 + b) - 4*((a^6 + 4*
a^5*b)*cos(f*x + e)^6 - a^4*b^2 - 4*a^3*b^3 - (a^6 + 2*a^5*b - 8*a^4*b^2)*c
os(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 - 4*a^3*b^3)*cos(f*x + e)^2)*log(1/2*s
in(f*x + e)))/((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*f*cos(f*x
+ e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2 - 8*a^6*b^3 - 7*a^5*b^4 - 2*a^4*b^5)*f*
cos(f*x + e)^4 - (2*a^8*b + 7*a^7*b^2 + 8*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 -
 a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 +
 a^3*b^6)*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(148) = 296$.

Time = 0.21 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.23

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{2\,(6\,a^2b^2+4\,ab^3+b^4)\log\left(a\sin(fx+e)^2-a-b\right)}{a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4}+\frac{2\,(a+4\,b)\log\left(\sin(fx+e)^2\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{2\,a^5+4\,a^4b+2\,a^3b^2+2\,(a^5-4\,a^2b^3-2\,ab\cdots}{(a^8+3\,a^7b+3\,a^6b^2+a^5b^3)\sin(fx+e)^6-2\,(a^8+4\,a^7b+6\,a^6b^2+\cdots}}{4\,f}$$

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(6*a^2*b^2 + 4*a*b^3 + b^4)*log(a*sin(f*x + e)^2 - a - b)/(a^7 + 4*
a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4) + 2*(a + 4*b)*log(sin(f*x + e)^2)/
(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + (2*a^5 + 4*a^4*b + 2*a^3*b^2
+ 2*(a^5 - 4*a^2*b^3 - 2*a*b^4)*sin(f*x + e)^4 - (4*a^5 + 4*a^4*b - 8*a^2*b
^3 - 11*a*b^4 - 3*b^5)*sin(f*x + e)^2)/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^
3)*sin(f*x + e)^6 - 2*(a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a^4*b^4)*sin
(f*x + e)^4 + (a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^
5)*sin(f*x + e)^2))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1069 vs. $2(148) = 296$.

Time = 0.56 (sec) , antiderivative size = 1069, normalized size of antiderivative = 6.94

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(4*(6*a^2*b^2 + 4*a*b^3 + b^4)*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos
(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e
```

```
) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
)/(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4) + 4*(a + 4*b)*log(abs(-
cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b
^3 + b^4) - (a + b + 4*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 16*b*(cos(
f*x + e) - 1)/(cos(f*x + e) + 1))*(cos(f*x + e) + 1)/((a^4 + 4*a^3*b + 6*a^
2*b^2 + 4*a*b^3 + b^4)*(cos(f*x + e) - 1)) - (cos(f*x + e) - 1)/((a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*(cos(f*x + e) + 1)) - 2*(18*a^4*b^2 + 48*a^3*b^3 + 45
*a^2*b^4 + 18*a*b^5 + 3*b^6 + 72*a^4*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) +
 1) + 80*a^3*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 20*a^2*b^4*(cos(f*
x + e) - 1)/(cos(f*x + e) + 1) - 40*a*b^5*(cos(f*x + e) - 1)/(cos(f*x + e)
+ 1) - 12*b^6*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 108*a^4*b^2*(cos(f*x
+ e) - 1)^2/(cos(f*x + e) + 1)^2 + 64*a^3*b^3*(cos(f*x + e) - 1)^2/(cos(f*x
 + e) + 1)^2 + 46*a^2*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 44*a*
b^5*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 18*b^6*(cos(f*x + e) - 1)^2
/(cos(f*x + e) + 1)^2 + 72*a^4*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^
3 + 80*a^3*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 20*a^2*b^4*(cos(
f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 40*a*b^5*(cos(f*x + e) - 1)^3/(cos(f
*x + e) + 1)^3 - 12*b^6*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 18*a^4*
b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 48*a^3*b^3*(cos(f*x + e) -
1)^4/(cos(f*x + e) + 1)^4 + 45*a^2*b^4*(cos(f*x + e) - 1)^4/(cos(f*x + e) +
 1)^4 + 18*a*b^5*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 3*b^6*(cos(f*x
 + e) - 1)^4/(cos(f*x + e) + 1)^4)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3
+ a^3*b^4)*(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*
x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^
2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)^2) - 8*log(abs(-(cos(f*x +
 e) - 1)/(cos(f*x + e) + 1) + 1))/a^3)/f
```

Mupad [B] (verification not implemented)

Time = 21.19 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.77

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{\tan(e+f\,x)^2\,(-4\,a^2\,b+7\,a\,b^2+2\,b^3)}{4\,a^2\,(a^2+2\,a\,b+b^2)}-\frac{1}{2\,(a+b)}+\frac{\tan(e+f\,x)^4\,(-a^2\,b^2+3\,a\,b^3+b^4)}{2\,a^2\,(a+b)\,(a^2+2\,a\,b+b^2)}}{f\,\left(\tan\left(e+f\,x\right)^2\,(a^2+2\,a\,b+b^2)+\tan\left(e+f\,x\right)^4\,(2\,b^2+2\,a\,b)+b^2\tan\left(e+f\,x\right)^6\right)} + \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a^3\,f} - \frac{\ln\left(\tan(e+f\,x)\right)\,(a+4\,b)}{f\,(a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4)} - \frac{b^2\,\ln\left(b\tan(e+f\,x)^2+a+b\right)\,(6\,a^2+4\,a\,b+b^2)}{2\,a^3\,f\,(a+b)^4}$$

```
[In] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] ((tan(e + f*x)^2*(7*a*b^2 - 4*a^2*b + 2*b^3))/(4*a^2*(2*a*b + a^2 + b^2)) -
 1/(2*(a + b)) + (tan(e + f*x)^4*(3*a*b^3 + b^4 - a^2*b^2))/(2*a^2*(a + b)*
```

```
(2*a*b + a^2 + b^2)))/(f*(tan(e + f*x)^2*(2*a*b + a^2 + b^2) + tan(e + f*x)
^4*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^6)) + log(tan(e + f*x)^2 + 1)/(2*a^3*
f) - (log(tan(e + f*x))*(a + 4*b))/(f*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^
2*b^2)) - (b^2*log(a + b + b*tan(e + f*x)^2)*(4*a*b + 6*a^2 + b^2))/(2*a^3*
f*(a + b)^4)
```

3.368 $\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 192

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{b^5}{4a^3(a+b)^3f\left(b+a\cos^2(e+fx)\right)^2} + \frac{b^4(5a+2b)}{2a^3(a+b)^4f\left(b+a\cos^2(e+fx)\right)} + \frac{(2a+5b)\csc^2(e+fx)}{2(a+b)^4f} - \frac{\csc^4(e+fx)}{4(a+b)^3f} + \frac{b^3(10a^2+5ab+b^2)\log\left(b+a\cos^2(e+fx)\right)}{2a^3(a+b)^5f} + \frac{(a^2+5ab+10b^2)\log(\sin(e+fx))}{(a+b)^5f}$$

```
[Out] -1/4*b^5/a^3/(a+b)^3/f/(b+a*cos(f*x+e)^2)^2+1/2*b^4*(5*a+2*b)/a^3/(a+b)^4/f
/(b+a*cos(f*x+e)^2)+1/2*(2*a+5*b)*csc(f*x+e)^2/(a+b)^4/f-1/4*csc(f*x+e)^4/(
a+b)^3/f+1/2*b^3*(10*a^2+5*a*b+b^2)*ln(b+a*cos(f*x+e)^2)/a^3/(a+b)^5/f+(a^2
+5*a*b+10*b^2)*ln(sin(f*x+e))/(a+b)^5/f
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used

$= \{4223, 457, 90\}$

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{b^5}{4a^3f(a+b)^3\left(a\cos^2(e+fx)+b\right)^2} + \frac{b^4(5a+2b)}{2a^3f(a+b)^4\left(a\cos^2(e+fx)+b\right)} + \frac{(a^2+5ab+10b^2)\log(\sin(e+fx))}{f(a+b)^5} + \frac{b^3(10a^2+5ab+b^2)\log\left(a\cos^2(e+fx)+b\right)}{2a^3f(a+b)^5} - \frac{\csc^4(e+fx)}{4f(a+b)^3} + \frac{(2a+5b)\csc^2(e+fx)}{2f(a+b)^4}$$

```
[In] Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -1/4*b^5/(a^3*(a + b)^3*f*(b + a*Cos[e + f*x]^2)^2) + (b^4*(5*a + 2*b))/(2*
a^3*(a + b)^4*f*(b + a*Cos[e + f*x]^2)) + ((2*a + 5*b)*Csc[e + f*x]^2)/(2*(
a + b)^4*f) - Csc[e + f*x]^4/(4*(a + b)^3*f) + (b^3*(10*a^2 + 5*a*b + b^2)*
Log[b + a*Cos[e + f*x]^2])/(2*a^3*(a + b)^5*f) + ((a^2 + 5*a*b + 10*b^2)*Lo
g[Sin[e + f*x]])/((a + b)^5*f)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^{11}}{(1-x^2)^3(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^5}{(1-x)^3(b+ax)^3}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)^3} + \frac{-2a-5b}{(a+b)^4(-1+x)^2} + \frac{-a^2-5ab-10b^2}{(a+b)^5(-1+x)} - \frac{b^5}{a^2(a+b)^3(b+ax)^3} + \frac{b^4(5a+2b)}{a^2(a+b)^4(b+ax)^2} - \frac{b^3(10}{a^2(a}\right.\right.}{2f}$$

$$= -\frac{b^5}{4a^3(a+b)^3f\left(b+a\cos^2(e+fx)\right)^2} + \frac{b^4(5a+2b)}{2a^3(a+b)^4f\left(b+a\cos^2(e+fx)\right)} + \frac{(2a+5b)\csc^2(e+fx)}{2(a+b)^4f} - \frac{\csc^4(e+fx)}{4(a+b)^3f} + \frac{b^3(10a^2+5ab+b^2)\log\left(b+a\cos^2(e+fx)\right)}{2a^3(a+b)^5f} + \frac{(a^2+5ab+10b^2)\log(\sin(e+fx))}{(a+b)^5f}$$

Mathematica [A] (verified)

Time = 5.53 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.08

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{(a+2b+a\cos(2(e+fx)))^3\sec^6(e+fx)\left(2(a+b)(2a+5b)\csc^2(e+fx)-(a+b)^2\csc^4(e+fx)+4(a^2\right.}{32(a+b)^5f\,(a}$$

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*(2*(a + b)*(2*a + 5*b)*Csc
[e + f*x]^2 - (a + b)^2*Csc[e + f*x]^4 + 4*(a^2 + 5*a*b + 10*b^2)*Log[Sin[e
 + f*x]] + (2*b^3*(10*a^2 + 5*a*b + b^2)*Log[a + b - a*Sin[e + f*x]^2])/a^3
 - (b^5*(a + b)^2)/(a^3*(a + b - a*Sin[e + f*x]^2)^2) + (2*b^4*(a + b)*(5*a
 + 2*b))/(a^3*(a + b - a*Sin[e + f*x]^2))))/(32*(a + b)^5*f*(a + b*Sec[e +
f*x]^2)^3)
```

Maple [A] (verified)

Time = 53.50 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{-\frac{1}{16(a+b)^3(-1+\cos(fx+e))^2}-\frac{7a+19b}{16(a+b)^4(-1+\cos(fx+e))}+\frac{\left(a^2+5ab+10b^2\right)\ln(-1+\cos(fx+e))}{2(a+b)^5}-\frac{1}{16(a+b)^3(1+\cos(fx+e))^2}-\frac{}{16(a+b}}{}$
default	$\frac{-\frac{1}{16(a+b)^3(-1+\cos(fx+e))^2}-\frac{7a+19b}{16(a+b)^4(-1+\cos(fx+e))}+\frac{\left(a^2+5ab+10b^2\right)\ln(-1+\cos(fx+e))}{2(a+b)^5}-\frac{1}{16(a+b)^3(1+\cos(fx+e))^2}-\frac{}{16(a+b}}{}$
risch	Expression too large to display

```
[In] int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/16/(a+b)^3/(-1+cos(f*x+e))^2-1/16*(7*a+19*b)/(a+b)^4/(-1+cos(f*x+e)
)+1/2*(a^2+5*a*b+10*b^2)/(a+b)^5*ln(-1+cos(f*x+e))-1/16/(a+b)^3/(1+cos(f*x+
e))^2-1/16*(-7*a-19*b)/(a+b)^4/(1+cos(f*x+e))+1/2*(a^2+5*a*b+10*b^2)/(a+b)^
5*ln(1+cos(f*x+e))+1/2*b^3/(a+b)^5*((10*a^2+5*a*b+b^2)/a^3*ln(b+a*cos(f*x+e
)^2)-1/2*b^2*(a^2+2*a*b+b^2)/a^3/(b+a*cos(f*x+e)^2)^2+1/a^3*b*(5*a^2+7*a*b+
2*b^2)/(b+a*cos(f*x+e)^2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 859 vs. 2(182) = 364.

Time = 1.68 (sec) , antiderivative size = 859, normalized size of antiderivative = 4.47

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{3\,a^5b^2+12\,a^4b^3+9\,a^3b^4+9\,a^2b^5+12\,ab^6+3\,b^7-2\left(2\,a^7+7\,a^6b+5\,a^5b^2-5\,a^3b^4-7\,a^2b^5-2\,ab^6\right)\cos\left(f\right.}{}$$

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^5*b^2 + 12*a^4*b^3 + 9*a^3*b^4 + 9*a^2*b^5 + 12*a*b^6 + 3*b^7 - 2*
(2*a^7 + 7*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 7*a^2*b^5 - 2*a*b^6)*cos(f*x + e
)^6 + (3*a^7 + 4*a^6*b - 19*a^5*b^2 - 20*a^4*b^3 - 20*a^3*b^4 - 19*a^2*b^5
+ 4*a*b^6 + 3*b^7)*cos(f*x + e)^4 + 2*(3*a^6*b + 10*a^5*b^2 + 2*a^4*b^3 - 2
*a^2*b^5 - 10*a*b^6 - 3*b^7)*cos(f*x + e)^2 + 2*((10*a^4*b^3 + 5*a^3*b^4 +
a^2*b^5)*cos(f*x + e)^8 + 10*a^2*b^5 + 5*a*b^6 + b^7 - 2*(10*a^4*b^3 - 5*a^
3*b^4 - 4*a^2*b^5 - a*b^6)*cos(f*x + e)^6 + (10*a^4*b^3 - 35*a^3*b^4 - 9*a^
2*b^5 + a*b^6 + b^7)*cos(f*x + e)^4 + 2*(10*a^3*b^4 - 5*a^2*b^5 - 4*a*b^6 -
 b^7)*cos(f*x + e)^2)*log(a*cos(f*x + e)^2 + b) + 4*((a^7 + 5*a^6*b + 10*a^
5*b^2)*cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 - 2*(a^7 + 4*a^6*b
```

```
 + 5*a^5*b^2 - 10*a^4*b^3)*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 35*a
^4*b^3 + 10*a^3*b^4)*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 10
*a^3*b^4)*cos(f*x + e)^2)*log(1/2*sin(f*x + e)))/((a^10 + 5*a^9*b + 10*a^8*
b^2 + 10*a^7*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8 - 2*(a^10 + 4*a^9*
b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6)*f*cos(f*x + e)^6 + (a^10 +
 a^9*b - 9*a^8*b^2 - 25*a^7*b^3 - 25*a^6*b^4 - 9*a^5*b^5 + a^4*b^6 + a^3*b^
7)*f*cos(f*x + e)^4 + 2*(a^9*b + 4*a^8*b^2 + 5*a^7*b^3 - 5*a^5*b^5 - 4*a^4*
b^6 - a^3*b^7)*f*cos(f*x + e)^2 + (a^8*b^2 + 5*a^7*b^3 + 10*a^6*b^4 + 10*a^
5*b^5 + 5*a^4*b^6 + a^3*b^7)*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. 2(182) = 364.

Time = 0.23 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.36

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{2\,(10\,a^2b^3+5\,ab^4+b^5)\log\left(a\sin(fx+e)^2-a-b\right)}{a^8+5\,a^7b+10\,a^6b^2+10\,a^5b^3+5\,a^4b^4+a^3b^5} + \frac{2\,(a^2+5\,ab+10\,b^2)\log\left(\sin(fx+e)^2\right)}{a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5} + \frac{2\,(2\,a^6+5\,a^5b-5\,a^2b^4-2\,ab^5)\sin(fx+e)^6-a^6-3\,a}{(a^9+4\,a^8b+6\,a^7b^2+4\,a^6b^3+a^5b^4)\sin(fx+e)^8-2}}{4\,f}$$

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(10*a^2*b^3 + 5*a*b^4 + b^5)*log(a*sin(f*x + e)^2 - a - b)/(a^8 + 5*
a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5) + 2*(a^2 + 5*a*b + 1
0*b^2)*log(sin(f*x + e)^2)/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b
^4 + b^5) + (2*(2*a^6 + 5*a^5*b - 5*a^2*b^4 - 2*a*b^5)*sin(f*x + e)^6 - a^6
 - 3*a^5*b - 3*a^4*b^2 - a^3*b^3 - (9*a^6 + 29*a^5*b + 20*a^4*b^2 - 10*a^2*
b^4 - 13*a*b^5 - 3*b^6)*sin(f*x + e)^4 + 2*(3*a^6 + 11*a^5*b + 13*a^4*b^2 +
 5*a^3*b^3)*sin(f*x + e)^2)/((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b
^4)*sin(f*x + e)^8 - 2*(a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b^4
 + a^4*b^5)*sin(f*x + e)^6 + (a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*
a^5*b^4 + 6*a^4*b^5 + a^3*b^6)*sin(f*x + e)^4))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1912 vs. 2(182) = 364.

Time = 0.53 (sec) , antiderivative size = 1912, normalized size of antiderivative = 9.96

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/64*(32*(10*a^2*b^3 + 5*a*b^4 + b^5)*log(a + b + 2*a*(cos(f*x + e) - 1)/(c
os(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x +
 e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)
^2)/(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5) + 32*(a
^2 + 5*a*b + 10*b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(a^5
 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - (12*a^3*(cos(f*x +
e) - 1)/(cos(f*x + e) + 1) + 60*a^2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1)
 + 84*a*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 36*b^3*(cos(f*x + e) -
1)/(cos(f*x + e) + 1) + a^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 3*a
^2*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 3*a*b^2*(cos(f*x + e) - 1)
^2/(cos(f*x + e) + 1)^2 + b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a
^6 + 6*a^5*b + 15*a^4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6) - (a^7
 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4 + 16*a^7*(cos(f*x + e) - 1)/(c
os(f*x + e) + 1) + 80*a^6*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 144*a^5
*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 112*a^4*b^3*(cos(f*x + e) - 1)
/(cos(f*x + e) + 1) + 32*a^3*b^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 70
*a^7*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 312*a^6*b*(cos(f*x + e) -
1)^2/(cos(f*x + e) + 1)^2 + 436*a^5*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e)
+ 1)^2 + 536*a^4*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 822*a^3*b^
4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 672*a^2*b^5*(cos(f*x + e) - 1
)^2/(cos(f*x + e) + 1)^2 + 224*a*b^6*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1
)^2 + 32*b^7*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 140*a^7*(cos(f*x +
 e) - 1)^3/(cos(f*x + e) + 1)^3 + 568*a^6*b*(cos(f*x + e) - 1)^3/(cos(f*x +
 e) + 1)^3 + 672*a^5*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 1096*a
^4*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 852*a^3*b^4*(cos(f*x + e
) - 1)^3/(cos(f*x + e) + 1)^3 - 384*a^2*b^5*(cos(f*x + e) - 1)^3/(cos(f*x +
 e) + 1)^3 - 512*a*b^6*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 128*b^7*
(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 145*a^7*(cos(f*x + e) - 1)^4/(c
os(f*x + e) + 1)^4 + 612*a^6*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 +
838*a^5*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 1572*a^4*b^3*(cos(f
*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 1777*a^3*b^4*(cos(f*x + e) - 1)^4/(co
s(f*x + e) + 1)^4 + 704*a^2*b^5*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 +
 576*a*b^6*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 192*b^7*(cos(f*x + e
) - 1)^4/(cos(f*x + e) + 1)^4 + 76*a^7*(cos(f*x + e) - 1)^5/(cos(f*x + e) +
 1)^5 + 392*a^6*b*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 + 720*a^5*b^2*(
```

```
cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 + 1080*a^4*b^3*(cos(f*x + e) - 1)^
5/(cos(f*x + e) + 1)^5 + 676*a^3*b^4*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1
)^5 - 384*a^2*b^5*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 - 512*a*b^6*(co
s(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 - 128*b^7*(cos(f*x + e) - 1)^5/(cos(
f*x + e) + 1)^5 + 16*a^7*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6 + 112*a^
6*b*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6 + 336*a^5*b^2*(cos(f*x + e) -
 1)^6/(cos(f*x + e) + 1)^6 + 720*a^4*b^3*(cos(f*x + e) - 1)^6/(cos(f*x + e)
 + 1)^6 + 960*a^3*b^4*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6 + 672*a^2*b
^5*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6 + 224*a*b^6*(cos(f*x + e) - 1)
^6/(cos(f*x + e) + 1)^6 + 32*b^7*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6)
/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*(a*(cos(f
*x + e) - 1)/(cos(f*x + e) + 1) + b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) +
 2*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 2*b*(cos(f*x + e) - 1)^2/(
cos(f*x + e) + 1)^2 + a*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + b*(cos(
f*x + e) - 1)^3/(cos(f*x + e) + 1)^3)^2) - 64*log(abs(-(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) + 1))/a^3)/f
```

Mupad [B] (verification not implemented)

Time = 22.49 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.70

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\ln\left(\tan(e+f\,x)\right)\,\left(a^2+5\,a\,b+10\,b^2\right)}{f\,\left(a^5+5\,a^4\,b+10\,a^3\,b^2+10\,a^2\,b^3+5\,a\,b^4+b^5\right)} - \frac{\frac{1}{4\,(a+b)}-\frac{\tan(e+f\,x)^2\,(a+3\,b)}{2\,(a+b)^2}+\frac{\tan(e+f\,x)^4\,\left(-4\,a^3\,b-15\,a^2\,b^2+9\,a\,b^3+2\,b^4\right)}{4\,a^2\,(a+b)\,\left(a^2+2\,a\,b+b^2\right)}+\frac{\tan(e+f\,x)^6\,\left(-a^3\,b^2-4\,a^2\,b^3+4\,a\,b^4+b^5\right)}{2\,a^2\,(a+b)^2\,\left(a^2+2\,a\,b+b^2\right)}}{f\,\left(\tan\left(e+f\,x\right)^4\,\left(a^2+2\,a\,b+b^2\right)+\tan\left(e+f\,x\right)^6\,\left(2\,b^2+2\,a\,b\right)+b^2\tan\left(e+f\,x\right)^8\right)} - \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a^3\,f} + \frac{b^3\,\ln\left(b\tan(e+f\,x)^2+a+b\right)\,\left(10\,a^2+5\,a\,b+b^2\right)}{2\,a^3\,f\,(a+b)^5}$$

```
[In] int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] (log(tan(e + f*x))*(5*a*b + a^2 + 10*b^2))/(f*(5*a*b^4 + 5*a^4*b + a^5 + b^
5 + 10*a^2*b^3 + 10*a^3*b^2)) - (1/(4*(a + b)) - (tan(e + f*x)^2*(a + 3*b))
/(2*(a + b)^2) + (tan(e + f*x)^4*(9*a*b^3 - 4*a^3*b + 2*b^4 - 15*a^2*b^2))/
(4*a^2*(a + b)*(2*a*b + a^2 + b^2)) + (tan(e + f*x)^6*(4*a*b^4 + b^5 - 4*a^
2*b^3 - a^3*b^2))/(2*a^2*(a + b)^2*(2*a*b + a^2 + b^2)))/(f*(tan(e + f*x)^4
*(2*a*b + a^2 + b^2) + tan(e + f*x)^6*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^8)
) - log(tan(e + f*x)^2 + 1)/(2*a^3*f) + (b^3*log(a + b + b*tan(e + f*x)^2)*
(5*a*b + 10*a^2 + b^2))/(2*a^3*f*(a + b)^5)
```

3.369 $\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{x}{a^3}+\frac{\sqrt{a+b}(3a^2-4ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3b^{5/2}f} - \frac{(a+b)\tan^3(e+fx)}{4abf\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{(3a-4b)(a+b)\tan(e+fx)}{8a^2b^2f\left(a+b+b\tan^2(e+fx)\right)}$$

```
[Out] -x/a^3+1/8*(3*a^2-4*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*(a+b)
^(1/2)/a^3/b^(5/2)/f-1/4*(a+b)*tan(f*x+e)^3/a/b/f/(a+b+b*tan(f*x+e)^2)^2-1/
8*(3*a-4*b)*(a+b)*tan(f*x+e)/a^2/b^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4226, 2000, 481, 592, 536, 209, 211}

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{x}{a^3}-\frac{(3a-4b)(a+b)\tan(e+fx)}{8a^2b^2f\left(a+b\tan^2(e+fx)+b\right)} + \frac{\sqrt{a+b}(3a^2-4ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3b^{5/2}f} - \frac{(a+b)\tan^3(e+fx)}{4abf\left(a+b\tan^2(e+fx)+b\right)^2}$$

```
[In] Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -(x/a^3) + (Sqrt[a + b]*(3*a^2 - 4*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x
])/Sqrt[a + b]])/(8*a^3*b^(5/2)*f) - ((a + b)*Tan[e + f*x]^3)/(4*a*b*f*(a +
 b + b*Tan[e + f*x]^2)^2) - ((3*a - 4*b)*(a + b)*Tan[e + f*x])/(8*a^2*b^2*f
*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+b(1+x^2))^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{4abf\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)+(3a-b)x^2)}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4abf}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{4abf\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{(3a-4b)(a+b)\tan(e+fx)}{8a^2b^2f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-((3a-4b)(a+b))+(-3a^2+ab-4b^2)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{8a^2b^2f}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{4abf\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{(3a-4b)(a+b)\tan(e+fx)}{8a^2b^2f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^3f} + \frac{\left((a+b)\left(3a^2-4ab+8b^2\right)\right)\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{8a^3b^2f}$$

$$= -\frac{x}{a^3} + \frac{\sqrt{a+b}(3a^2-4ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3b^{5/2}f} - \frac{(a+b)\tan^3(e+fx)}{4abf\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{(3a-4b)(a+b)\tan(e+fx)}{8a^2b^2f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.61 (sec) , antiderivative size = 760, normalized size of antiderivative = 5.17

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{(-3a^3+a^2b-4ab^2-8b^3)(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(\frac{\arctan\left(\sec(fx)\left(\frac{\cos(2e)}{2\sqrt{a+b}\sqrt{b\cos(4e)-ib\sin(4e)}}-\frac{}{2\sqrt{}}\right.\right.}{64a^3}\right.}{}$$
$$+\frac{(a+2b+a\cos(2e+2fx))\sec(2e)\sec^6(e+fx)(-24a^2b^2fx\cos(2e)-64ab^3fx\cos(2e)-64b^4fx\cos(2e}{}$$

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((-3*a^3 + a^2*b - 4*a*b^2 - 8*b^3)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e
+ f*x]^6*((ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*S
in[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))
*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(64*a^3*b^2*S
qrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/64)*ArcTan[Sec[f*x]*(Co
s[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(
Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x]
 + a*Sin[2*e + f*x])]*Sin[2*e])/(a^3*b^2*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*
b*Sin[4*e]])))/(a + b*Sec[e + f*x]^2)^3 + ((a + 2*b + a*Cos[2*e + 2*f*x])*S
ec[2*e]*Sec[e + f*x]^6*(-24*a^2*b^2*f*x*Cos[2*e] - 64*a*b^3*f*x*Cos[2*e] -
64*b^4*f*x*Cos[2*e] - 16*a^2*b^2*f*x*Cos[2*f*x] - 32*a*b^3*f*x*Cos[2*f*x] -
 16*a^2*b^2*f*x*Cos[4*e + 2*f*x] - 32*a*b^3*f*x*Cos[4*e + 2*f*x] - 4*a^2*b^
2*f*x*Cos[2*e + 4*f*x] - 4*a^2*b^2*f*x*Cos[6*e + 4*f*x] + 9*a^4*Sin[2*e] +
15*a^3*b*Sin[2*e] - 18*a^2*b^2*Sin[2*e] - 72*a*b^3*Sin[2*e] - 48*b^4*Sin[2*
e] - 9*a^4*Sin[2*f*x] - 13*a^3*b*Sin[2*f*x] + 28*a^2*b^2*Sin[2*f*x] + 32*a*
b^3*Sin[2*f*x] + 3*a^4*Sin[4*e + 2*f*x] - a^3*b*Sin[4*e + 2*f*x] - 20*a^2*b
^2*Sin[4*e + 2*f*x] - 16*a*b^3*Sin[4*e + 2*f*x] - 3*a^4*Sin[2*e + 4*f*x] +
3*a^3*b*Sin[2*e + 4*f*x] + 6*a^2*b^2*Sin[2*e + 4*f*x]))/(128*a^3*b^2*f*(a +
 b*Sec[e + f*x]^2)^3)
```

Maple [A] (verified)

Time = 27.17 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{(a+b)\left(\frac{-\frac{a(5a-4b)\tan(fx+e)^3}{8b}-\frac{a\left(3a^2-ab-4b^2\right)\tan(fx+e)}{8b^2}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(3a^2-4ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8b^2\sqrt{(a+b)b}}\right)}{a^3}-\frac{\arctan(\tan(fx+e))}{a^3}}{f}$
default	$\frac{\frac{(a+b)\left(\frac{-\frac{a(5a-4b)\tan(fx+e)^3}{8b}-\frac{a\left(3a^2-ab-4b^2\right)\tan(fx+e)}{8b^2}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(3a^2-4ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8b^2\sqrt{(a+b)b}}\right)}{a^3}-\frac{\arctan(\tan(fx+e))}{a^3}}{f}$
risch	$-\frac{x}{a^3}+\frac{i(-3a^4e^{6i(fx+e)}+a^3be^{6i(fx+e)}+20a^2b^2e^{6i(fx+e)}+16ab^3e^{6i(fx+e)}-9a^4e^{4i(fx+e)}-15a^3be^{4i(fx+e)}+18a^2b^2e^{4i}}{4a^3b^2f(ae^{4i(fx+}}$

```
[In] int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*((a+b)/a^3*((-1/8*a*(5*a-4*b)/b*tan(f*x+e)^3-1/8*a*(3*a^2-a*b-4*b^2)/b^
2*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(3*a^2-4*a*b+8*b^2)/b^2/((a+b)*b)^
(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))-1/a^3*arctan(tan(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(133) = 266.

Time = 0.30 (sec) , antiderivative size = 664, normalized size of antiderivative = 4.52

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$=\left[-\frac{32\,a^2b^2fx\cos(fx+e)^4+64\,ab^3fx\cos(fx+e)^2+32\,b^4fx-\left((3\,a^4-4\,a^3b+8\,a^2b^2)\cos(fx+e)^4+3\,\right.}{}\right.$$

$$-\frac{16\,a^2b^2fx\cos(fx+e)^4+32\,ab^3fx\cos(fx+e)^2+16\,b^4fx+\left((3\,a^4-4\,a^3b+8\,a^2b^2)\cos(fx+e)^4+3\,a\right.}{}$$

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(32*a^2*b^2*f*x*cos(f*x + e)^4 + 64*a*b^3*f*x*cos(f*x + e)^2 + 32*b^
4*f*x - ((3*a^4 - 4*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 - 4*a*b^3
```

```
 + 8*b^4 + 2*(3*a^3*b - 4*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-(a + b)/
b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
e)^2 - 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(-(a + b)/b)
*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 4
*(3*(a^4 - a^3*b - 2*a^2*b^2)*cos(f*x + e)^3 + (5*a^3*b + a^2*b^2 - 4*a*b^3
)*cos(f*x + e))*sin(f*x + e))/(a^5*b^2*f*cos(f*x + e)^4 + 2*a^4*b^3*f*cos(f
*x + e)^2 + a^3*b^4*f), -1/16*(16*a^2*b^2*f*x*cos(f*x + e)^4 + 32*a*b^3*f*x
*cos(f*x + e)^2 + 16*b^4*f*x + ((3*a^4 - 4*a^3*b + 8*a^2*b^2)*cos(f*x + e)^
4 + 3*a^2*b^2 - 4*a*b^3 + 8*b^4 + 2*(3*a^3*b - 4*a^2*b^2 + 8*a*b^3)*cos(f*x
 + e)^2)*sqrt((a + b)/b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a
+ b)/b)/((a + b)*cos(f*x + e)*sin(f*x + e))) + 2*(3*(a^4 - a^3*b - 2*a^2*b^
2)*cos(f*x + e)^3 + (5*a^3*b + a^2*b^2 - 4*a*b^3)*cos(f*x + e))*sin(f*x + e
))/(a^5*b^2*f*cos(f*x + e)^4 + 2*a^4*b^3*f*cos(f*x + e)^2 + a^3*b^4*f)]
```

Sympy [F]

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

```
[In] integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral(tan(e + f*x)**6/(a + b*sec(e + f*x)**2)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.31

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{(5a^2b+ab^2-4b^3)\tan(fx+e)^3+(3a^3+2a^2b-5ab^2-4b^3)\tan(fx+e)}{a^2b^4\tan(fx+e)^4+a^4b^2+2a^3b^3+a^2b^4+2(a^3b^3+a^2b^4)\tan(fx+e)^2}+\frac{8(fx+e)}{a^3}-\frac{(3a^3-a^2b+4ab^2+8b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^3b^2}}{8f}$$

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(((5*a^2*b + a*b^2 - 4*b^3)*tan(f*x + e)^3 + (3*a^3 + 2*a^2*b - 5*a*b^
2 - 4*b^3)*tan(f*x + e))/(a^2*b^4*tan(f*x + e)^4 + a^4*b^2 + 2*a^3*b^3 + a^
2*b^4 + 2*(a^3*b^3 + a^2*b^4)*tan(f*x + e)^2) + 8*(f*x + e)/a^3 - (3*a^3 -
a^2*b + 4*a*b^2 + 8*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a +
b)*b)*a^3*b^2))/f
```

Giac [A] (verification not implemented)

none

Time = 2.54 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.36

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{8(fx+e)}{a^3} - \frac{(3a^3-a^2b+4ab^2+8b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}a^3b^2} + \frac{5a^2b\tan(fx+e)^3+ab^2\tan(fx+e)^3-4b^3\tan(fx+e)^3+\cdots}{(b\tan(\cdots}}{8f}$$

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(8*(f*x + e)/a^3 - (3*a^3 - a^2*b + 4*a*b^2 + 8*b^3)*(pi*floor((f*x +
e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/(sqrt(a*b + b
^2)*a^3*b^2) + (5*a^2*b*tan(f*x + e)^3 + a*b^2*tan(f*x + e)^3 - 4*b^3*tan(f
*x + e)^3 + 3*a^3*tan(f*x + e) + 2*a^2*b*tan(f*x + e) - 5*a*b^2*tan(f*x + e
) - 4*b^3*tan(f*x + e))/((b*tan(f*x + e)^2 + a + b)^2*a^2*b^2))/f
```

Mupad [B] (verification not implemented)

Time = 20.95 (sec) , antiderivative size = 615, normalized size of antiderivative = 4.18

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\operatorname{atan}\left(\frac{25\tan(e+fx)}{32\left(\frac{5b}{4a}-\frac{3a}{16b}+\frac{9a^2}{32b^2}+\frac{25}{32}\right)} - \frac{3\tan(e+fx)}{16\left(\frac{9a}{32b}+\frac{25b}{32a}+\frac{5b^2}{4a^2}-\frac{3}{16}\right)} + \frac{9\tan(e+fx)}{32\left(\frac{25b^2}{32a^2}-\frac{3b}{16a}+\frac{5b^3}{4a^3}+\frac{9}{32}\right)} + \frac{5\tan(e+fx)}{4\left(\frac{25a}{32b}-\frac{3a^2}{16b^2}+\frac{9a^3}{32b^3}+\frac{5}{4}\right)}\right)}{a^3 f} - \frac{\frac{\tan(e+fx)^3(5a^2+ab-4b^2)}{8a^2b} - \frac{\tan(e+fx)(a+b)(-3a^2+ab+4b^2)}{8a^2b^2}}{f\left(2ab+a^2+b^2+\tan(e+fx)^2(2b^2+2ab)+b^2\tan(e+fx)^4\right)} - \frac{\operatorname{atanh}\left(\frac{27\tan(e+fx)\sqrt{-b^6-ab^5}}{256\left(\frac{27ab^2}{256}-\frac{27b^3}{128}+\frac{171b^4}{256a}-\frac{7b^5}{64a^2}+\frac{5b^6}{32a^3}+\frac{5b^7}{4a^4}\right)} - \frac{81\tan(e+fx)\sqrt{-b^6-ab^5}}{256\left(\frac{27a^2b}{256}-\frac{27ab^2}{128}+\frac{171b^3}{256}-\frac{7b^4}{64a}+\frac{5b^5}{32a^2}+\frac{5b^6}{4a^3}\right)} - \frac{35\tan(e+fx)\cdots}{32\left(\frac{171a^2b}{256}-\frac{7ab^2}{64}-\frac{27a^3}{128}\cdots\right)}\cdots}{\cdots}$$

```
[In] int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] - atan((25*tan(e + f*x))/(32*((5*b)/(4*a) - (3*a)/(16*b) + (9*a^2)/(32*b^2)
 + 25/32)) - (3*tan(e + f*x))/(16*((9*a)/(32*b) + (25*b)/(32*a) + (5*b^2)/(
4*a^2) - 3/16)) + (9*tan(e + f*x))/(32*((25*b^2)/(32*a^2) - (3*b)/(16*a) +
(5*b^3)/(4*a^3) + 9/32)) + (5*tan(e + f*x))/(4*((25*a)/(32*b) - (3*a^2)/(16
*b^2) + (9*a^3)/(32*b^3) + 5/4)))/(a^3*f) - ((tan(e + f*x)^3*(a*b + 5*a^2 -
 4*b^2))/(8*a^2*b) - (tan(e + f*x)*(a + b)*(a*b - 3*a^2 + 4*b^2))/(8*a^2*b^
```

```
2))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*
x)^4)) - (atanh((27*tan(e + f*x)*(- a*b^5 - b^6)^(1/2))/(256*((27*a*b^2)/25
6 - (27*b^3)/128 + (171*b^4)/(256*a) - (7*b^5)/(64*a^2) + (5*b^6)/(32*a^3)
+ (5*b^7)/(4*a^4))) - (81*tan(e + f*x)*(- a*b^5 - b^6)^(1/2))/(256*((27*a^2
*b)/256 - (27*a*b^2)/128 + (171*b^3)/256 - (7*b^4)/(64*a) + (5*b^5)/(32*a^2
) + (5*b^6)/(4*a^3))) - (35*tan(e + f*x)*(- a*b^5 - b^6)^(1/2))/(32*((171*a
^2*b)/256 - (7*a*b^2)/64 - (27*a^3)/128 + (5*b^3)/32 + (5*b^4)/(4*a) + (27*
a^4)/(256*b))) + (5*tan(e + f*x)*(- a*b^5 - b^6)^(1/2))/(4*((5*a*b^2)/32 -
(7*a^2*b)/64 + (171*a^3)/256 + (5*b^3)/4 - (27*a^4)/(128*b) + (27*a^5)/(256
*b^2))) + (63*tan(e + f*x)*(- a*b^5 - b^6)^(1/2))/(64*((171*a*b^2)/256 - (2
7*a^2*b)/128 + (27*a^3)/256 - (7*b^3)/64 + (5*b^4)/(32*a) + (5*b^5)/(4*a^2)
)))*(-b^5*(a + b))^(1/2)*(3*a^2 - 4*a*b + 8*b^2))/(8*a^3*b^5*f)
```

3.370 $\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3} dx = \frac{x}{a^3} + \frac{(a^2-4ab-8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3b^{3/2}\sqrt{a+b}f} - \frac{(a+b)\tan(e+fx)}{4abf\,(a+b+b\tan^2(e+fx))^2} + \frac{(a-4b)\tan(e+fx)}{8a^2bf\,(a+b+b\tan^2(e+fx))}$$

```
[Out] x/a^3+1/8*(a^2-4*a*b-8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^3/b^(3
/2)/f/(a+b)^(1/2)-1/4*(a+b)*tan(f*x+e)/a/b/f/(a+b+b*tan(f*x+e)^2)^2+1/8*(a-
4*b)*tan(f*x+e)/a^2/b/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4226, 2000, 481, 541, 536, 209, 211}

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3} dx = \frac{x}{a^3} + \frac{(a-4b)\tan(e+fx)}{8a^2bf\,(a+b\tan^2(e+fx)+b)} + \frac{(a^2-4ab-8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3b^{3/2}f\sqrt{a+b}} - \frac{(a+b)\tan(e+fx)}{4abf\,(a+b\tan^2(e+fx)+b)^2}$$

```
[In] Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] x/a^3 + ((a^2 - 4*a*b - 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/
(8*a^3*b^(3/2)*Sqrt[a + b]*f) - ((a + b)*Tan[e + f*x])/(4*a*b*f*(a + b + b*
Tan[e + f*x]^2)^2) + ((a - 4*b)*Tan[e + f*x])/(8*a^2*b*f*(a + b + b*Tan[e +
 f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+b(1+x^2))^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\tan(e+fx)}{4abf\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{a+b+(a-3b)x^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4abf}$$

$$= -\frac{(a+b)\tan(e+fx)}{4abf\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(a-4b)\tan(e+fx)}{8a^2bf\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{(a+b)(a+4b)+(a-4b)(a+b)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{8a^2b(a+b)f}$$

$$= -\frac{(a+b)\tan(e+fx)}{4abf\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(a-4b)\tan(e+fx)}{8a^2bf\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^3f} + \frac{(a^2-4ab-8b^2)\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{8a^3bf}$$

$$= \frac{x}{a^3} + \frac{(a^2-4ab-8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3b^{3/2}\sqrt{a+b}f} - \frac{(a+b)\tan(e+fx)}{4abf\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(a-4b)\tan(e+fx)}{8a^2bf\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 16.89 (sec) , antiderivative size = 1473, normalized size of antiderivative = 10.75

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \frac{(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(\frac{(3a^2+8ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{a\sqrt{b}(3a^2+16ab+16b^2+3a(a+2b)\cos(2(e+}{(a+b)^2(a+2b+a\cos(2(e+f}\right.}{1024b^{5/2}f\,(a+b\sec^2(e+fx))^3}$$

$$- \frac{(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(-\frac{3a(a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + \frac{\sqrt{b}(3a^3+14a^2b+24ab^2+16b^3+a(3a^2+4ab}{(a+b)^2(a+2b+a\cos}\right.}{2048b^{5/2}f\,(a+b\sec^2(e+fx))^3}$$

$$+ \frac{(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(\frac{2(3a^5-10a^4b+80a^3b^2+480a^2b^3+640ab^4+256b^5)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))}{2\sqrt{a+b}\sqrt{}}\right.}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{}$$

$$- \frac{(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(-\frac{6a^2\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{}$$

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((3*a^2 + 8*a*b + 8*b^2)*
ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*(3*a
^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/((
a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(1024*b^(5/2)*f*(a + b*Sec[e +
 f*x]^2)^3) - ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-3*a*(a +
2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + (Sqrt[b]*(
3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*Cos[2*(e +
 f*x)])*Sin[2*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(2
048*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3
*Sec[e + f*x]^6*((2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*b^
4 + 256*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]
) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[
2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (Sec[2*e]
*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*f*x*Cos[2*e] + 512*a*b^2*(a + b
)^2*(a + 2*b)*f*x*Cos[2*f*x] + 128*a^4*b^2*f*x*Cos[2*(e + 2*f*x)] + 256*a^3
*b^3*f*x*Cos[2*(e + 2*f*x)] + 128*a^2*b^4*f*x*Cos[2*(e + 2*f*x)] + 512*a^4*
b^2*f*x*Cos[4*e + 2*f*x] + 2048*a^3*b^3*f*x*Cos[4*e + 2*f*x] + 2560*a^2*b^4
*f*x*Cos[4*e + 2*f*x] + 1024*a*b^5*f*x*Cos[4*e + 2*f*x] + 128*a^4*b^2*f*x*C
```

os[6*e + 4*f*x] + 256*a^3*b^3*f*x*Cos[6*e + 4*f*x] + 128*a^2*b^4*f*x*Cos[6*
e + 4*f*x] - 9*a^6*Sin[2*e] + 12*a^5*b*Sin[2*e] + 684*a^4*b^2*Sin[2*e] + 28
80*a^3*b^3*Sin[2*e] + 5280*a^2*b^4*Sin[2*e] + 4608*a*b^5*Sin[2*e] + 1536*b^
6*Sin[2*e] + 9*a^6*Sin[2*f*x] - 14*a^5*b*Sin[2*f*x] - 608*a^4*b^2*Sin[2*f*x
] - 2112*a^3*b^3*Sin[2*f*x] - 2560*a^2*b^4*Sin[2*f*x] - 1024*a*b^5*Sin[2*f*
x] + 3*a^6*Sin[2*(e + 2*f*x)] - 12*a^5*b*Sin[2*(e + 2*f*x)] - 204*a^4*b^2*S
in[2*(e + 2*f*x)] - 384*a^3*b^3*Sin[2*(e + 2*f*x)] - 192*a^2*b^4*Sin[2*(e +
 2*f*x)] - 3*a^6*Sin[4*e + 2*f*x] + 10*a^5*b*Sin[4*e + 2*f*x] + 304*a^4*b^2
*Sin[4*e + 2*f*x] + 1056*a^3*b^3*Sin[4*e + 2*f*x] + 1280*a^2*b^4*Sin[4*e +
2*f*x] + 512*a*b^5*Sin[4*e + 2*f*x]))/(a + 2*b + a*Cos[2*(e + f*x)])^2))/(4
096*a^3*b^2*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3) - ((a + 2*b + a*Cos[2*e +
 2*f*x])^3*Sec[e + f*x]^6*((-6*a^2*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])
*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e]
- I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*S
in[e])^4]) + (a*Sec[2*e]*((-9*a^4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^3 + 64*
b^4)*Sin[2*f*x] + a*(-3*a^3 + 2*a^2*b + 24*a*b^2 + 16*b^3)*Sin[2*(e + 2*f*x
)] + (3*a^4 - 64*a^2*b^2 - 128*a*b^3 - 64*b^4)*Sin[4*e + 2*f*x]) + (9*a^5 +
 18*a^4*b - 64*a^3*b^2 - 256*a^2*b^3 - 320*a*b^4 - 128*b^5)*Tan[2*e])/(a^2*
(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(2048*b^2*(a + b)^2*f*(a + b*Sec[e + f*
x]^2)^3)

Maple [A] (verified)

Time = 15.43 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{\frac{\left(\frac{1}{8}a^2-\frac{1}{2}ab\right)\tan(fx+e)^3-\frac{a\left(a^2+5ab+4b^2\right)\tan(fx+e)}{8b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(a^2-4ab-8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8b\sqrt{(a+b)b}}}{a^3}+\frac{\arctan(\tan(fx+e))}{a^3}}{f}$
default	$\frac{\frac{\frac{\left(\frac{1}{8}a^2-\frac{1}{2}ab\right)\tan(fx+e)^3-\frac{a\left(a^2+5ab+4b^2\right)\tan(fx+e)}{8b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(a^2-4ab-8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8b\sqrt{(a+b)b}}}{a^3}+\frac{\arctan(\tan(fx+e))}{a^3}}{f}$
risch	$\frac{x}{a^3}-\frac{i\left(a^3\mathrm{e}^{6i(fx+e)}+12a^2b\,\mathrm{e}^{6i(fx+e)}+16a\,b^2\mathrm{e}^{6i(fx+e)}+3a^3\mathrm{e}^{4i(fx+e)}+26a^2b\,\mathrm{e}^{4i(fx+e)}+56a\,b^2\mathrm{e}^{4i(fx+e)}+48b^3\mathrm{e}^{4i(fx+e)}\right.}{4a^3f\left(a\,\mathrm{e}^{4i(fx+e)}+2a\,\mathrm{e}^{2i(fx+e)}+4b\,\mathrm{e}^{2i(fx+e)}+a\right)^2b}$

[In] int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)

[Out] 1/f*(1/a^3*(((1/8*a^2-1/2*a*b)*tan(f*x+e)^3-1/8*a*(a^2+5*a*b+4*b^2)/b*tan(f
*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(a^2-4*a*b-8*b^2)/b/((a+b)*b)^(1/2)*arcta
n(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^3*arctan(tan(f*x+e)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(123) = 246$.

Time = 0.31 (sec) , antiderivative size = 763, normalized size of antiderivative = 5.57

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \left[\frac{32\,(a^3b^2+a^2b^3)fx\cos{(fx+e)}^4+64\,(a^2b^3+ab^4)fx\cos{(fx+e)}^2+32\,(ab^4+b^5)fx+\left((a^4-4\,a^3b-8\right.}{}\right.$$

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*(a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e)^4 + 64*(a^2*b^3 + a*b^4)*f*x
*cos(f*x + e)^2 + 32*(a*b^4 + b^5)*f*x + ((a^4 - 4*a^3*b - 8*a^2*b^2)*cos(f
*x + e)^4 + a^2*b^2 - 4*a*b^3 - 8*b^4 + 2*(a^3*b - 4*a^2*b^2 - 8*a*b^3)*cos
(f*x + e)^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2
*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x +
 e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f
*x + e)^2 + b^2)) - 4*((a^4*b + 7*a^3*b^2 + 6*a^2*b^3)*cos(f*x + e)^3 - (a^
3*b^2 - 3*a^2*b^3 - 4*a*b^4)*cos(f*x + e))*sin(f*x + e))/((a^6*b^2 + a^5*b^
3)*f*cos(f*x + e)^4 + 2*(a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^4*b^4 + a
^3*b^5)*f), 1/16*(16*(a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e)^4 + 32*(a^2*b^3 +
 a*b^4)*f*x*cos(f*x + e)^2 + 16*(a*b^4 + b^5)*f*x - ((a^4 - 4*a^3*b - 8*a^2
*b^2)*cos(f*x + e)^4 + a^2*b^2 - 4*a*b^3 - 8*b^4 + 2*(a^3*b - 4*a^2*b^2 - 8
*a*b^3)*cos(f*x + e)^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^
2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) - 2*((a^4*b + 7*a^3*b^2
 + 6*a^2*b^3)*cos(f*x + e)^3 - (a^3*b^2 - 3*a^2*b^3 - 4*a*b^4)*cos(f*x + e)
)*sin(f*x + e))/((a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^5*b^3 + a^4*b^
4)*f*cos(f*x + e)^2 + (a^4*b^4 + a^3*b^5)*f)]
```

Sympy [F]

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{\tan^4{(e+fx)}}{(a+b\sec^2{(e+fx)})^3}\,dx$$

```
[In] integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral(tan(e + f*x)**4/(a + b*sec(e + f*x)**2)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.19

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{(ab-4\,b^2)\tan(fx+e)^3-(a^2+5\,ab+4\,b^2)\tan(fx+e)}{a^2b^3\tan(fx+e)^4+a^4b+2\,a^3b^2+a^2b^3+2\,(a^3b^2+a^2b^3)\tan(fx+e)^2}+\frac{8\,(fx+e)}{a^3}+\frac{(a^2-4\,ab-8\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^3b}}{8\,f}$$

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(((a*b - 4*b^2)*tan(f*x + e)^3 - (a^2 + 5*a*b + 4*b^2)*tan(f*x + e))/(a
^2*b^3*tan(f*x + e)^4 + a^4*b + 2*a^3*b^2 + a^2*b^3 + 2*(a^3*b^2 + a^2*b^3)
*tan(f*x + e)^2) + 8*(f*x + e)/a^3 + (a^2 - 4*a*b - 8*b^2)*arctan(b*tan(f*x
 + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^3*b))/f
```

Giac [A] (verification not implemented)

none

Time = 1.25 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.16

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{8\,(fx+e)}{a^3}+\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(a^2-4\,ab-8\,b^2)}{\sqrt{ab+b^2}a^3b}+\frac{ab\tan(fx+e)^3-4\,b^2\tan(fx+e)^3-a^2\tan(fx+e)-5\,ab\tan(fx+e)-}{\left(b\tan(fx+e)^2+a+b\right)^2a^2b}}{8\,f}$$

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*(f*x + e)/a^3 + (pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(
f*x + e)/sqrt(a*b + b^2)))*(a^2 - 4*a*b - 8*b^2)/(sqrt(a*b + b^2)*a^3*b) +
(a*b*tan(f*x + e)^3 - 4*b^2*tan(f*x + e)^3 - a^2*tan(f*x + e) - 5*a*b*tan(f
*x + e) - 4*b^2*tan(f*x + e))/((b*tan(f*x + e)^2 + a + b)^2*a^2*b))/f
```

Mupad [B] (verification not implemented)

Time = 21.39 (sec) , antiderivative size = 1117, normalized size of antiderivative = 8.15

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] ((tan(e + f*x)^3*(a - 4*b))/(8*a^2) - (tan(e + f*x)*(a + b)*(a + 4*b))/(8*a
^2*b))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e +
 f*x)^4)) - atan(tan(e + f*x)/(32*(b/(4*a) - 1/32)) + tan(e + f*x)/(4*(a/(3
2*b) - 1/4)))/(a^3*f) + (atan(-(((((-b^3*(a + b))^(1/2)*((2*a^6*b^3 + (a^7*
b^2)/2)/(a^6*b) - (tan(e + f*x)*(512*a^6*b^4 + 256*a^7*b^3)*(-b^3*(a + b))^
(1/2)*(4*a*b - a^2 + 8*b^2))/(512*a^4*b*(a^3*b^4 + a^4*b^3)))*(4*a*b - a^2
+ 8*b^2))/(16*(a^3*b^4 + a^4*b^3)) - (tan(e + f*x)*(64*a*b^3 - 8*a^3*b + a^
4 + 128*b^4))/(32*a^4*b))*(-b^3*(a + b))^(1/2)*(4*a*b - a^2 + 8*b^2)*1i)/(1
6*(a^3*b^4 + a^4*b^3)) - ((((-b^3*(a + b))^(1/2)*((2*a^6*b^3 + (a^7*b^2)/2)
/(a^6*b) + (tan(e + f*x)*(512*a^6*b^4 + 256*a^7*b^3)*(-b^3*(a + b))^(1/2)*(
4*a*b - a^2 + 8*b^2))/(512*a^4*b*(a^3*b^4 + a^4*b^3)))*(4*a*b - a^2 + 8*b^2
))/(16*(a^3*b^4 + a^4*b^3)) + (tan(e + f*x)*(64*a*b^3 - 8*a^3*b + a^4 + 128
*b^4))/(32*a^4*b))*(-b^3*(a + b))^(1/2)*(4*a*b - a^2 + 8*b^2)*1i)/(16*(a^3*
b^4 + a^4*b^3)))/(((a*b^2)/4 - (a^2*b)/4 + a^3/32 + b^3)/(a^6*b) + ((((-b^3
*(a + b))^(1/2)*((2*a^6*b^3 + (a^7*b^2)/2)/(a^6*b) - (tan(e + f*x)*(512*a^6
*b^4 + 256*a^7*b^3)*(-b^3*(a + b))^(1/2)*(4*a*b - a^2 + 8*b^2))/(512*a^4*b*
(a^3*b^4 + a^4*b^3)))*(4*a*b - a^2 + 8*b^2))/(16*(a^3*b^4 + a^4*b^3)) - (ta
n(e + f*x)*(64*a*b^3 - 8*a^3*b + a^4 + 128*b^4))/(32*a^4*b))*(-b^3*(a + b))
^(1/2)*(4*a*b - a^2 + 8*b^2))/(16*(a^3*b^4 + a^4*b^3)) + ((((-b^3*(a + b))^
(1/2)*((2*a^6*b^3 + (a^7*b^2)/2)/(a^6*b) + (tan(e + f*x)*(512*a^6*b^4 + 256
*a^7*b^3)*(-b^3*(a + b))^(1/2)*(4*a*b - a^2 + 8*b^2))/(512*a^4*b*(a^3*b^4 +
 a^4*b^3)))*(4*a*b - a^2 + 8*b^2))/(16*(a^3*b^4 + a^4*b^3)) + (tan(e + f*x)
*(64*a*b^3 - 8*a^3*b + a^4 + 128*b^4))/(32*a^4*b))*(-b^3*(a + b))^(1/2)*(4*
a*b - a^2 + 8*b^2))/(16*(a^3*b^4 + a^4*b^3))))*(-b^3*(a + b))^(1/2)*(4*a*b
- a^2 + 8*b^2)*1i)/(8*f*(a^3*b^4 + a^4*b^3))
```

3.371 $\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 138

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx=-\frac{x}{a^3}+\frac{(3a^2+12ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3\sqrt{b}(a+b)^{3/2}f}+\frac{\tan(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2}+\frac{(3a+4b)\tan(e+fx)}{8a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

```
[Out] -x/a^3+1/8*(3*a^2+12*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^3/
(a+b)^(3/2)/f/b^(1/2)+1/4*tan(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^2+1/8*(3*a+4*
b)*tan(f*x+e)/a^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4226, 2000, 482, 541, 536, 209, 211}

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx=-\frac{x}{a^3}+\frac{(3a+4b)\tan(e+fx)}{8a^2f(a+b)\left(a+b\tan^2(e+fx)+b\right)}+\frac{(3a^2+12ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3\sqrt{b}f(a+b)^{3/2}}+\frac{\tan(e+fx)}{4af\left(a+b\tan^2(e+fx)+b\right)^2}$$

```
[In] Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -(x/a^3) + ((3*a^2 + 12*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a +
 b]])/(8*a^3*Sqrt[b]*(a + b)^(3/2)*f) + Tan[e + f*x]/(4*a*f*(a + b + b*Tan[
e + f*x]^2)^2) + ((3*a + 4*b)*Tan[e + f*x])/(8*a^2*(a + b)*f*(a + b + b*Tan
[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
```

```
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+b(1+x^2))^3}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\tan(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{1-3x^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4af} \\
&= \frac{\tan(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(3a+4b)\tan(e+fx)}{8a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)} \\
&\quad - \frac{\text{Subst}\left(\int \frac{5a+4b+(-3a-4b)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{8a^2(a+b)f} \\
&= \frac{\tan(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(3a+4b)\tan(e+fx)}{8a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)} \\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^3f} \\
&\quad + \frac{(3a^2+12ab+8b^2)\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{8a^3(a+b)f} \\
&= -\frac{x}{a^3} + \frac{(3a^2+12ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3\sqrt{b}(a+b)^{3/2}f} \\
&\quad + \frac{\tan(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(3a+4b)\tan(e+fx)}{8a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.27 (sec) , antiderivative size = 1473, normalized size of antiderivative = 10.67

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \frac{(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(\frac{(3a^2+8ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{a\sqrt{b}(3a^2+16ab+16b^2+3a(a+2b)\cos(2(e+}{(a+b)^2(a+2b+a\cos(2(e+f}\right.}{1024b^{5/2}f\,(a+b\sec^2(e+fx))^3}$$

$$+ \frac{(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(-\frac{3a(a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + \frac{\sqrt{b}(3a^3+14a^2b+24ab^2+16b^3+a(3a^2+4ab}{(a+b)^2(a+2b+a\cos}\right.}{2048b^{5/2}f\,(a+b\sec^2(e+fx))^3}$$

$$- \frac{(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(\frac{2(3a^5-10a^4b+80a^3b^2+480a^2b^3+640ab^4+256b^5)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e)}{2\sqrt{a+b}\sqrt{}}\right.}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{}$$

$$- \frac{(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(-\frac{6a^2\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{}$$

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((3*a^2 + 8*a*b + 8*b^2)*
ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*(3*a
^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/((
a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(1024*b^(5/2)*f*(a + b*Sec[e +
 f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-3*a*(a +
2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + (Sqrt[b]*(
3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*Cos[2*(e +
 f*x)])*Sin[2*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(2
048*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3) - ((a + 2*b + a*Cos[2*e + 2*f*x])^3
*Sec[e + f*x]^6*((2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*b^
4 + 256*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]
) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[
2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (Sec[2*e]
*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*f*x*Cos[2*e] + 512*a*b^2*(a + b
)^2*(a + 2*b)*f*x*Cos[2*f*x] + 128*a^4*b^2*f*x*Cos[2*(e + 2*f*x)] + 256*a^3
*b^3*f*x*Cos[2*(e + 2*f*x)] + 128*a^2*b^4*f*x*Cos[2*(e + 2*f*x)] + 512*a^4*
b^2*f*x*Cos[4*e + 2*f*x] + 2048*a^3*b^3*f*x*Cos[4*e + 2*f*x] + 2560*a^2*b^4
*f*x*Cos[4*e + 2*f*x] + 1024*a*b^5*f*x*Cos[4*e + 2*f*x] + 128*a^4*b^2*f*x*C
```

```
os[6*e + 4*f*x] + 256*a^3*b^3*f*x*Cos[6*e + 4*f*x] + 128*a^2*b^4*f*x*Cos[6*
e + 4*f*x] - 9*a^6*Sin[2*e] + 12*a^5*b*Sin[2*e] + 684*a^4*b^2*Sin[2*e] + 28
80*a^3*b^3*Sin[2*e] + 5280*a^2*b^4*Sin[2*e] + 4608*a*b^5*Sin[2*e] + 1536*b^
6*Sin[2*e] + 9*a^6*Sin[2*f*x] - 14*a^5*b*Sin[2*f*x] - 608*a^4*b^2*Sin[2*f*x
] - 2112*a^3*b^3*Sin[2*f*x] - 2560*a^2*b^4*Sin[2*f*x] - 1024*a*b^5*Sin[2*f*
x] + 3*a^6*Sin[2*(e + 2*f*x)] - 12*a^5*b*Sin[2*(e + 2*f*x)] - 204*a^4*b^2*S
in[2*(e + 2*f*x)] - 384*a^3*b^3*Sin[2*(e + 2*f*x)] - 192*a^2*b^4*Sin[2*(e +
 2*f*x)] - 3*a^6*Sin[4*e + 2*f*x] + 10*a^5*b*Sin[4*e + 2*f*x] + 304*a^4*b^2
*Sin[4*e + 2*f*x] + 1056*a^3*b^3*Sin[4*e + 2*f*x] + 1280*a^2*b^4*Sin[4*e +
2*f*x] + 512*a*b^5*Sin[4*e + 2*f*x]))/(a + 2*b + a*Cos[2*(e + f*x)])^2))/(4
096*a^3*b^2*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3) - ((a + 2*b + a*Cos[2*e +
 2*f*x])^3*Sec[e + f*x]^6*((-6*a^2*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])
*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e]
- I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*S
in[e])^4]) + (a*Sec[2*e]*((-9*a^4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^3 + 64*
b^4)*Sin[2*f*x] + a*(-3*a^3 + 2*a^2*b + 24*a*b^2 + 16*b^3)*Sin[2*(e + 2*f*x
)] + (3*a^4 - 64*a^2*b^2 - 128*a*b^3 - 64*b^4)*Sin[4*e + 2*f*x]) + (9*a^5 +
 18*a^4*b - 64*a^3*b^2 - 256*a^2*b^3 - 320*a*b^4 - 128*b^5)*Tan[2*e])/(a^2*
(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(2048*b^2*(a + b)^2*f*(a + b*Sec[e + f*
x]^2)^3)
```

Maple [A] (verified)

Time = 11.63 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{a^3}+\frac{\frac{\frac{ab(3a+4b)\tan(fx+e)^3}{8a+8b}+\frac{(5a+4b)a\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(3a^2+12ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8(a+b)\sqrt{(a+b)b}}}{a^3}}{f}$
default	$\frac{-\frac{\arctan(\tan(fx+e))}{a^3}+\frac{\frac{\frac{ab(3a+4b)\tan(fx+e)^3}{8a+8b}+\frac{(5a+4b)a\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(3a^2+12ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8(a+b)\sqrt{(a+b)b}}}{a^3}}{f}$
risch	$-\frac{x}{a^3}+\frac{i\left(5a^3e^{6i(fx+e)}+20a^2b\,e^{6i(fx+e)}+16a\,b^2e^{6i(fx+e)}+15a^3e^{4i(fx+e)}+58a^2b\,e^{4i(fx+e)}+88a\,b^2e^{4i(fx+e)}+48b^3e^{4i(f}\right.}{4a^3(a+b)f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e}\right.}$

```
[In] int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^3*arctan(tan(f*x+e))+1/a^3*((1/8*a*b*(3*a+4*b)/(a+b)*tan(f*x+e)^3
+1/8*(5*a+4*b)*a*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(3*a^2+12*a*b+8*b^2
)/(a+b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. 2(124) = 248.

Time = 0.33 (sec) , antiderivative size = 860, normalized size of antiderivative = 6.23

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \Bigg[-\frac{32\,(a^4b+2\,a^3b^2+a^2b^3)fx\cos{(fx+e)}^4+64\,(a^3b^2+2\,a^2b^3+ab^4)fx\cos{(fx+e)}^2+32\,(a^2b^3+2\,ab^4}{}$$

$$-\frac{16\,(a^4b+2\,a^3b^2+a^2b^3)fx\cos{(fx+e)}^4+32\,(a^3b^2+2\,a^2b^3+ab^4)fx\cos{(fx+e)}^2+16\,(a^2b^3+2\,ab^4\,-}{}$$

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(32*(a^4*b + 2*a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e)^4 + 64*(a^3*b^2 +
 2*a^2*b^3 + a*b^4)*f*x*cos(f*x + e)^2 + 32*(a^2*b^3 + 2*a*b^4 + b^5)*f*x +
 ((3*a^4 + 12*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 12*a*b^3 + 8*
b^4 + 2*(3*a^3*b + 12*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-a*b - b^2)*l
og(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2
 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x +
 e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 4*((5*a^4*b
 + 11*a^3*b^2 + 6*a^2*b^3)*cos(f*x + e)^3 + (3*a^3*b^2 + 7*a^2*b^3 + 4*a*b^
4)*cos(f*x + e))*sin(f*x + e))/((a^7*b + 2*a^6*b^2 + a^5*b^3)*f*cos(f*x + e
)^4 + 2*(a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^5*b^3 + 2*a^4
*b^4 + a^3*b^5)*f), -1/16*(16*(a^4*b + 2*a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e
)^4 + 32*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*f*x*cos(f*x + e)^2 + 16*(a^2*b^3 + 2
*a*b^4 + b^5)*f*x + ((3*a^4 + 12*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*
b^2 + 12*a*b^3 + 8*b^4 + 2*(3*a^3*b + 12*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)
*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)
*cos(f*x + e)*sin(f*x + e))) - 2*((5*a^4*b + 11*a^3*b^2 + 6*a^2*b^3)*cos(f*
x + e)^3 + (3*a^3*b^2 + 7*a^2*b^3 + 4*a*b^4)*cos(f*x + e))*sin(f*x + e))/((
a^7*b + 2*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^6*b^2 + 2*a^5*b^3 + a^
4*b^4)*f*cos(f*x + e)^2 + (a^5*b^3 + 2*a^4*b^4 + a^3*b^5)*f)]
```

Sympy [F]

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{\tan^2{(e+fx)}}{(a+b\sec^2{(e+fx)})^3}\,dx$$

```
[In] integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
[Out] Integral(tan(e + f*x)**2/(a + b*sec(e + f*x)**2)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.38

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{(3a^2+12ab+8b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^4+a^3b)\sqrt{(a+b)b}} + \frac{(3ab+4b^2)\tan(fx+e)^3+(5a^2+9ab+4b^2)\tan(fx+e)}{a^5+3a^4b+3a^3b^2+a^2b^3+(a^3b^2+a^2b^3)\tan(fx+e)^4+2(a^4b+2a^3b^2+a^2b^3)\tan(fx+e)^2} - \frac{8(fx+e)}{a^3}}{8f}$$

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
[Out] 1/8*((3*a^2 + 12*a*b + 8*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4
+ a^3*b)*sqrt((a + b)*b)) + ((3*a*b + 4*b^2)*tan(f*x + e)^3 + (5*a^2 + 9*a*
b + 4*b^2)*tan(f*x + e))/(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + (a^3*b^2 +
a^2*b^3)*tan(f*x + e)^4 + 2*(a^4*b + 2*a^3*b^2 + a^2*b^3)*tan(f*x + e)^2) -
 8*(f*x + e)/a^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.74 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.25

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3a^2+12ab+8b^2)}{(a^4+a^3b)\sqrt{ab+b^2}} + \frac{3ab\tan(fx+e)^3+4b^2\tan(fx+e)^3+5a^2\tan(fx+e)+9ab\tan(fx+e)+4b^2\tan}{(a^3+a^2b)\left(b\tan(fx+e)^2+a+b\right)^2}}{8f}$$

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
[Out] 1/8*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
+ b^2)))*(3*a^2 + 12*a*b + 8*b^2)/((a^4 + a^3*b)*sqrt(a*b + b^2)) + (3*a*b*
tan(f*x + e)^3 + 4*b^2*tan(f*x + e)^3 + 5*a^2*tan(f*x + e) + 9*a*b*tan(f*x
+ e) + 4*b^2*tan(f*x + e))/((a^3 + a^2*b)*(b*tan(f*x + e)^2 + a + b)^2) - 8
*(f*x + e)/a^3)/f
```

Mupad [B] (verification not implemented)

Time = 22.82 (sec) , antiderivative size = 2405, normalized size of antiderivative = 17.43

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] ((tan(e + f*x)*(5*a + 4*b))/(8*a^2) + (tan(e + f*x)^3*(3*a*b + 4*b^2))/(8*a
^2*(a + b)))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*t
an(e + f*x)^4)) - atan((((((2*a^6*b^4 + (9*a^7*b^3)/2 + (5*a^8*b^2)/2)*1i)/
(2*(2*a^7*b + a^8 + a^6*b^2)) - (tan(e + f*x)*(512*a^6*b^5 + 1280*a^7*b^4 +
 1024*a^8*b^3 + 256*a^9*b^2))/(128*a^3*(2*a^5*b + a^6 + a^4*b^2)))/(2*a^3)
+ (tan(e + f*x)*(320*a*b^4 + 9*a^4*b + 128*b^5 + 256*a^2*b^3 + 72*a^3*b^2))
/(64*(2*a^5*b + a^6 + a^4*b^2)))/a^3 - ((((2*a^6*b^4 + (9*a^7*b^3)/2 + (5*a
^8*b^2)/2)*1i)/(2*(2*a^7*b + a^8 + a^6*b^2)) + (tan(e + f*x)*(512*a^6*b^5 +
 1280*a^7*b^4 + 1024*a^8*b^3 + 256*a^9*b^2))/(128*a^3*(2*a^5*b + a^6 + a^4*
b^2)))/(2*a^3) - (tan(e + f*x)*(320*a*b^4 + 9*a^4*b + 128*b^5 + 256*a^2*b^3
 + 72*a^3*b^2))/(64*(2*a^5*b + a^6 + a^4*b^2)))/a^3)/(((9*a*b^3)/4 + (9*a^3
*b)/32 + b^4 + (3*a^2*b^2)/2)/(2*a^7*b + a^8 + a^6*b^2) + (((((2*a^6*b^4 +
(9*a^7*b^3)/2 + (5*a^8*b^2)/2)*1i)/(2*(2*a^7*b + a^8 + a^6*b^2)) - (tan(e +
 f*x)*(512*a^6*b^5 + 1280*a^7*b^4 + 1024*a^8*b^3 + 256*a^9*b^2))/(128*a^3*(
2*a^5*b + a^6 + a^4*b^2)))*1i)/(2*a^3) + (tan(e + f*x)*(320*a*b^4 + 9*a^4*b
 + 128*b^5 + 256*a^2*b^3 + 72*a^3*b^2)*1i)/(64*(2*a^5*b + a^6 + a^4*b^2)))/
a^3 + (((((2*a^6*b^4 + (9*a^7*b^3)/2 + (5*a^8*b^2)/2)*1i)/(2*(2*a^7*b + a^8
 + a^6*b^2)) + (tan(e + f*x)*(512*a^6*b^5 + 1280*a^7*b^4 + 1024*a^8*b^3 + 2
56*a^9*b^2))/(128*a^3*(2*a^5*b + a^6 + a^4*b^2)))*1i)/(2*a^3) - (tan(e + f*
x)*(320*a*b^4 + 9*a^4*b + 128*b^5 + 256*a^2*b^3 + 72*a^3*b^2)*1i)/(64*(2*a^
5*b + a^6 + a^4*b^2)))/a^3))/(a^3*f) - (atan((((-b*(a + b)^3)^(1/2)*((tan(e
 + f*x)*(320*a*b^4 + 9*a^4*b + 128*b^5 + 256*a^2*b^3 + 72*a^3*b^2))/(32*(2*
a^5*b + a^6 + a^4*b^2)) - ((-b*(a + b)^3)^(1/2)*((2*a^6*b^4 + (9*a^7*b^3)/2
 + (5*a^8*b^2)/2)/(2*a^7*b + a^8 + a^6*b^2) - (tan(e + f*x)*(-b*(a + b)^3)^
(1/2)*(12*a*b + 3*a^2 + 8*b^2)*(512*a^6*b^5 + 1280*a^7*b^4 + 1024*a^8*b^3 +
 256*a^9*b^2))/(512*(2*a^5*b + a^6 + a^4*b^2)*(a^6*b + a^3*b^4 + 3*a^4*b^3
+ 3*a^5*b^2)))*(12*a*b + 3*a^2 + 8*b^2))/(16*(a^6*b + a^3*b^4 + 3*a^4*b^3 +
 3*a^5*b^2)))*(12*a*b + 3*a^2 + 8*b^2)*1i)/(16*(a^6*b + a^3*b^4 + 3*a^4*b^3
 + 3*a^5*b^2)) + ((-b*(a + b)^3)^(1/2)*((tan(e + f*x)*(320*a*b^4 + 9*a^4*b
+ 128*b^5 + 256*a^2*b^3 + 72*a^3*b^2))/(32*(2*a^5*b + a^6 + a^4*b^2)) + ((-
b*(a + b)^3)^(1/2)*((2*a^6*b^4 + (9*a^7*b^3)/2 + (5*a^8*b^2)/2)/(2*a^7*b +
a^8 + a^6*b^2) + (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(12*a*b + 3*a^2 + 8*b^2
)*(512*a^6*b^5 + 1280*a^7*b^4 + 1024*a^8*b^3 + 256*a^9*b^2))/(512*(2*a^5*b
+ a^6 + a^4*b^2)*(a^6*b + a^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2)))*(12*a*b + 3*a^
2 + 8*b^2))/(16*(a^6*b + a^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2)))*(12*a*b + 3*a^2
 + 8*b^2)*1i)/(16*(a^6*b + a^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2)))/(((9*a*b^3)/4
```

```
 + (9*a^3*b)/32 + b^4 + (3*a^2*b^2)/2)/(2*a^7*b + a^8 + a^6*b^2) - ((-b*(a
+ b)^3)^(1/2)*((tan(e + f*x)*(320*a*b^4 + 9*a^4*b + 128*b^5 + 256*a^2*b^3 +
 72*a^3*b^2))/(32*(2*a^5*b + a^6 + a^4*b^2)) - ((-b*(a + b)^3)^(1/2)*((2*a^
6*b^4 + (9*a^7*b^3)/2 + (5*a^8*b^2)/2)/(2*a^7*b + a^8 + a^6*b^2) - (tan(e +
 f*x)*(-b*(a + b)^3)^(1/2)*(12*a*b + 3*a^2 + 8*b^2)*(512*a^6*b^5 + 1280*a^7
*b^4 + 1024*a^8*b^3 + 256*a^9*b^2))/(512*(2*a^5*b + a^6 + a^4*b^2)*(a^6*b +
 a^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2)))*(12*a*b + 3*a^2 + 8*b^2))/(16*(a^6*b +
a^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2)))*(12*a*b + 3*a^2 + 8*b^2))/(16*(a^6*b + a
^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2)) + ((-b*(a + b)^3)^(1/2)*((tan(e + f*x)*(32
0*a*b^4 + 9*a^4*b + 128*b^5 + 256*a^2*b^3 + 72*a^3*b^2))/(32*(2*a^5*b + a^6
 + a^4*b^2)) + ((-b*(a + b)^3)^(1/2)*((2*a^6*b^4 + (9*a^7*b^3)/2 + (5*a^8*b
^2)/2)/(2*a^7*b + a^8 + a^6*b^2) + (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(12*a
*b + 3*a^2 + 8*b^2)*(512*a^6*b^5 + 1280*a^7*b^4 + 1024*a^8*b^3 + 256*a^9*b^
2))/(512*(2*a^5*b + a^6 + a^4*b^2)*(a^6*b + a^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2
)))*(12*a*b + 3*a^2 + 8*b^2))/(16*(a^6*b + a^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2)
))*(12*a*b + 3*a^2 + 8*b^2))/(16*(a^6*b + a^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2))
))*(-b*(a + b)^3)^(1/2)*(12*a*b + 3*a^2 + 8*b^2)*1i)/(8*f*(a^6*b + a^3*b^4
+ 3*a^4*b^3 + 3*a^5*b^2))
```

3.372 $\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 144

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx = \frac{x}{a^3} - \frac{\sqrt{b}(15a^2+20ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}f} - \frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

```
[Out] x/a^3-1/8*(15*a^2+20*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1
/2)/a^3/(a+b)^(5/2)/f-1/4*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/8
*b*(7*a+4*b)*tan(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4213, 425, 541, 536, 209, 211}

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx = \frac{x}{a^3} - \frac{b(7a+4b)\tan(e+fx)}{8a^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)} - \frac{\sqrt{b}(15a^2+20ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{5/2}} - \frac{b\tan(e+fx)}{4af(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-3),x]
```

```
[Out] x/a^3 - (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sq
rt[a + b]])/(8*a^3*(a + b)^(5/2)*f) - (b*Tan[e + f*x])/(4*a*(a + b)*f*(a +
b + b*Tan[e + f*x]^2)^2) - (b*(7*a + 4*b)*Tan[e + f*x])/(8*a^2*(a + b)^2*f*
(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
```

```
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a+b-3bx^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4a(a+b)f}$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{8a^2+9ab+4b^2-b(7a+4b)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{8a^2(a+b)^2f}$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^3f} - \frac{\left(b(15a^2+20ab+8b^2)\right)\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{8a^3(a+b)^2f}$$

$$= \frac{x}{a^3} - \frac{\sqrt{b}(15a^2+20ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}f} - \frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.52 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.31

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(8x(a+2b+a\cos(2(e+fx)))^2 + \frac{b(15a^2+20ab+8b^2)\arctan\left(\frac{\sec(fx)(\cos\ldots}{\ldots}\right)}{\ldots}\right.}{\ldots}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-3),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(8*x*(a + 2*b + a*Cos[2*(e +
 f*x)])^2 + (b*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin
[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(C
os[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2*(Cos[2*e] - I*Sin[2
*e]))/((a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) - (4*b^2*((a + 2*b)*S
in[2*e] - a*Sin[2*f*x]))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])) +
(b*(a + 2*b + a*Cos[2*(e + f*x)])*((9*a^2 + 28*a*b + 16*b^2)*Sin[2*e] - 3*a
*(3*a + 2*b)*Sin[2*f*x]))/((a + b)^2*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))
))/(64*a^3*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\frac{ab(7a+4b)\tan(fx+e)^3}{8a^2+16ab+8b^2}+\frac{(9a+4b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}\right)}{a^3}+\frac{\arctan(\tan(fx+e))}{a^3}}{f}$
default	$\frac{-\frac{b\left(\frac{\frac{ab(7a+4b)\tan(fx+e)^3}{8a^2+16ab+8b^2}+\frac{(9a+4b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}\right)}{a^3}+\frac{\arctan(\tan(fx+e))}{a^3}}{f}$
risch	$\frac{x}{a^3}-\frac{ib\left(9a^3e^{6i(fx+e)}+28a^2b\,e^{6i(fx+e)}+16a\,b^2e^{6i(fx+e)}+27a^3e^{4i(fx+e)}+90a^2b\,e^{4i(fx+e)}+120a\,b^2e^{4i(fx+e)}+48b^3e^{4i(f}\right.}{4a^3(a+b)^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e}\right.}$

```
[In] int(1/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-b/a^3*((1/8*a*b*(7*a+4*b)/(a^2+2*a*b+b^2)*tan(f*x+e)^3+1/8*(9*a+4*b)*
a/(a+b)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(15*a^2+20*a*b+8*b^2)/(a^2+2
*a*b+b^2)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^3*arcta
n(tan(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(130) = 260.

Time = 0.32 (sec) , antiderivative size = 819, normalized size of antiderivative = 5.69

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \left[\frac{32\left(a^4+2a^3b+a^2b^2\right)fx\cos\left(fx+e\right)^4+64\left(a^3b+2a^2b^2+ab^3\right)fx\cos\left(fx+e\right)^2+32\left(a^2b^2+2ab^3+b^4\right)f}{}\right.$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 64*(a^3*b + 2*a^2*
b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 32*(a^2*b^2 + 2*a*b^3 + b^4)*f*x + ((15*a
^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 + 8*b^4 +
 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log((
(a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4
*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/
(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b
^2)) - 4*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 + 4*a*b^3)*co
s(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 + 2*(
a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^3 + a^3*
b^4)*f), 1/16*(16*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 32*(a^3*b
+ 2*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 16*(a^2*b^2 + 2*a*b^3 + b^4)*f*x
+ ((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 +
 8*b^4 + 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(b/(a + b)
)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)
*sin(f*x + e))) - 2*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 +
4*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x +
 e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4
*b^3 + a^3*b^4)*f)]
```

Sympy [F]

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \int \frac{1}{\left(a+b\sec^2\left(e+fx\right)\right)^3}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.60

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\frac{\left(15\,a^2b+20\,ab^2+8\,b^3\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\left(a^5+2\,a^4b+a^3b^2\right)\sqrt{(a+b)b}}+\frac{\left(7\,ab^2+4\,b^3\right)\tan(fx+e)^3+\left(9\,a^2b+13\,ab^2+4\,b^3\right)\tan(fx+e)}{a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4+\left(a^4b^2+2\,a^3b^3+a^2b^4\right)\tan(fx+e)^4+2\left(a^5b+3\,a^4b^2+3\,a^3b^3+a\right.}}{8\,f}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

[Out] -1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)) + ((7*a*b^2 + 4*b^3)*tan(f*x + e)^3 + (9*a^2*b + 13*a*b^2 + 4*b^3)*tan(f*x + e))/(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4 + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*tan(f*x + e)^4 + 2*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*tan(f*x + e)^2) - 8*(f*x + e)/a^3)/f

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.36

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\frac{(15\,a^2b+20\,ab^2+8\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^5+2\,a^4b+a^3b^2)\sqrt{ab+b^2}}+\frac{7\,ab^2\tan(fx+e)^3+4\,b^3\tan(fx+e)^3+9\,a^2b\tan(fx+e)+13\,ab^2\tan(fx+}{(a^4+2\,a^3b+a^2b^2)\left(b\tan(fx+e)^2+a+b\right)^2}}{8\,f}$$

[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")

[Out] -1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(a*b + b^2)) + (7*a*b^2*tan(f*x + e)^3 + 4*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x + e) + 13*a*b^2*tan(f*x + e) + 4*b^3*tan(f*x + e))/((a^4 + 2*a^3*b + a^2*b^2)*(b*tan(f*x + e)^2 + a + b)^2) - 8*(f*x + e)/a^3)/f

Mupad [B] (verification not implemented)

Time = 23.81 (sec) , antiderivative size = 3271, normalized size of antiderivative = 22.72

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

[In] int(1/(a + b/cos(e + f*x)^2)^3,x)

[Out] atan((((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/(2*a^3) + (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/a^3 - ((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) + (tan(e + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536* a^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/(2*a^3) - (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/a^3)/(((17*a*b^5)/4 + b^6 + (25*a^2*b^4)/4 + (105*a^3*b^3)/32)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(2*a^3) + (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3)*1i)/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/a^3 + (((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) + (tan(e + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(2*a^3) - (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3)*1i)/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/a^3))/(a^3*f) - ((tan(e + f*x)^3*(7*a*b^2 + 4*b^3))/(8*a^2*(a + b)^2) + (tan(e + f*x)*(9*a*b + 4*b^2))/(8*a^2*(a + b)))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4)) + (atan(((((tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) - (((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) - (tan(e + f*x)*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(512*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)*1i)/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)) + (((tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) + (((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (tan(e + f*x)*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(512*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)*1i)/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))/(((17*a*b^5)/4 + b^6 + (25*a^2*b^4)/4 + (105*a^3*b^3)/32)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) - (((tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) - (((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)/(4*a^9*b + a^10 + a^

6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) - (tan(e + f*x)*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(512*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)) + (((tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) + (((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (tan(e + f*x)*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(512*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2))))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)*1i)/(8*f*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2))

3.373 $\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 181

$$\begin{aligned}\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^3} dx = & -\frac{x}{a^3}+\frac{b^{3/2}(35a^2+28ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{7/2}f} \\ & -\frac{(8a^2-11ab-4b^2)\cot(e+fx)}{8a^2(a+b)^3f} \\ & -\frac{b\cot(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} \\ & -\frac{b(9a+4b)\cot(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}\end{aligned}$$

```
[Out] -x/a^3+1/8*b^(3/2)*(35*a^2+28*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1
/2))/a^3/(a+b)^(7/2)/f-1/8*(8*a^2-11*a*b-4*b^2)*cot(f*x+e)/a^2/(a+b)^3/f-1/
4*b*cot(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/8*b*(9*a+4*b)*cot(f*x+e)/
a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used

$= \{4226, 2000, 483, 593, 597, 536, 209, 211\}$

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{x}{a^3} - \frac{(8a^2-11ab-4b^2)\cot(e+fx)}{8a^2f(a+b)^3} - \frac{b(9a+4b)\cot(e+fx)}{8a^2f(a+b)^2\,(a+b\tan^2(e+fx)+b)} + \frac{b^{3/2}(35a^2+28ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{7/2}} - \frac{b\cot(e+fx)}{4af(a+b)\,(a+b\tan^2(e+fx)+b)^2}$$

```
[In] Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -(x/a^3) + (b^(3/2)*(35*a^2 + 28*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])
/Sqrt[a + b]])/(8*a^3*(a + b)^(7/2)*f) - ((8*a^2 - 11*a*b - 4*b^2)*Cot[e +
f*x])/(8*a^2*(a + b)^3*f) - (b*Cot[e + f*x])/(4*a*(a + b)*f*(a + b + b*Tan[
e + f*x]^2)^2) - (b*(9*a + 4*b)*Cot[e + f*x])/(8*a^2*(a + b)^2*f*(a + b + b
*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+b(1+x^2))^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$=-\frac{b\cot(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}+\frac{\text{Subst}\left(\int\frac{4a-b-5bx^2}{x^2(1+x^2)(a+b+bx^2)^2}\,dx,x,\tan(e+fx)\right)}{4a(a+b)f}$$

$$=-\frac{b\cot(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}-\frac{b(9a+4b)\cot(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}+\frac{\text{Subst}\left(\int\frac{8a^2-11ab-4b^2-3b(9a+4b)x^2}{x^2(1+x^2)(a+b+bx^2)}\,dx,x,\tan(e+fx)\right)}{8a^2(a+b)^2f}$$

$$=-\frac{(8a^2-11ab-4b^2)\cot(e+fx)}{8a^2(a+b)^3f}-\frac{b\cot(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}-\frac{b(9a+4b)\cot(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}-\frac{\text{Subst}\left(\int\frac{8a^3+32a^2b+13ab^2+4b^3+b(8a^2-11ab-4b^2)x^2}{(1+x^2)(a+b+bx^2)}\,dx,x,\tan(e+fx)\right)}{8a^2(a+b)^3f}$$

$$=-\frac{(8a^2-11ab-4b^2)\cot(e+fx)}{8a^2(a+b)^3f}-\frac{b\cot(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}-\frac{b(9a+4b)\cot(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}-\frac{\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\tan(e+fx)\right)}{a^3f}+\frac{\left(b^2(35a^2+28ab+8b^2)\right)\text{Subst}\left(\int\frac{1}{a+b+bx^2}\,dx,x,\tan(e+fx)\right)}{8a^3(a+b)^3f}$$

$$=-\frac{x}{a^3}+\frac{b^{3/2}(35a^2+28ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{7/2}f}-\frac{(8a^2-11ab-4b^2)\cot(e+fx)}{8a^2(a+b)^3f}-\frac{b\cot(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}-\frac{b(9a+4b)\cot(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.53 (sec) , antiderivative size = 2089, normalized size of antiderivative = 11.54

$$\int\frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx=\text{Result too large to show}$$

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((35*a^2 + 28*a*b + 8*b^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*
(-1/64*(b^2*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*
Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]])
)*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(a^3*Sqrt[a
+ b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) + ((I/64)*b^2*ArcTan[Sec[f*x]*(Cos[
```

2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sq
rt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] +
 a*Sin[2*e + f*x])]*Sin[2*e])/(a^3*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[
4*e]])))/((a + b)^3*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f
*x])*Csc[e]*Csc[e + f*x]*Sec[2*e]*Sec[e + f*x]^6*(8*a^5*f*x*Cos[f*x] + 56*a
^4*b*f*x*Cos[f*x] + 184*a^3*b^2*f*x*Cos[f*x] + 296*a^2*b^3*f*x*Cos[f*x] + 2
24*a*b^4*f*x*Cos[f*x] + 64*b^5*f*x*Cos[f*x] - 12*a^5*f*x*Cos[3*f*x] - 68*a^
4*b*f*x*Cos[3*f*x] - 132*a^3*b^2*f*x*Cos[3*f*x] - 108*a^2*b^3*f*x*Cos[3*f*x
] - 32*a*b^4*f*x*Cos[3*f*x] - 8*a^5*f*x*Cos[2*e - f*x] - 56*a^4*b*f*x*Cos[2
*e - f*x] - 184*a^3*b^2*f*x*Cos[2*e - f*x] - 296*a^2*b^3*f*x*Cos[2*e - f*x]
 - 224*a*b^4*f*x*Cos[2*e - f*x] - 64*b^5*f*x*Cos[2*e - f*x] - 8*a^5*f*x*Cos
[2*e + f*x] - 56*a^4*b*f*x*Cos[2*e + f*x] - 184*a^3*b^2*f*x*Cos[2*e + f*x]
- 296*a^2*b^3*f*x*Cos[2*e + f*x] - 224*a*b^4*f*x*Cos[2*e + f*x] - 64*b^5*f*
x*Cos[2*e + f*x] + 8*a^5*f*x*Cos[4*e + f*x] + 56*a^4*b*f*x*Cos[4*e + f*x] +
 184*a^3*b^2*f*x*Cos[4*e + f*x] + 296*a^2*b^3*f*x*Cos[4*e + f*x] + 224*a*b^
4*f*x*Cos[4*e + f*x] + 64*b^5*f*x*Cos[4*e + f*x] + 12*a^5*f*x*Cos[2*e + 3*f
*x] + 68*a^4*b*f*x*Cos[2*e + 3*f*x] + 132*a^3*b^2*f*x*Cos[2*e + 3*f*x] + 10
8*a^2*b^3*f*x*Cos[2*e + 3*f*x] + 32*a*b^4*f*x*Cos[2*e + 3*f*x] - 12*a^5*f*x
*Cos[4*e + 3*f*x] - 68*a^4*b*f*x*Cos[4*e + 3*f*x] - 132*a^3*b^2*f*x*Cos[4*e
 + 3*f*x] - 108*a^2*b^3*f*x*Cos[4*e + 3*f*x] - 32*a*b^4*f*x*Cos[4*e + 3*f*x
] + 12*a^5*f*x*Cos[6*e + 3*f*x] + 68*a^4*b*f*x*Cos[6*e + 3*f*x] + 132*a^3*b
^2*f*x*Cos[6*e + 3*f*x] + 108*a^2*b^3*f*x*Cos[6*e + 3*f*x] + 32*a*b^4*f*x*C
os[6*e + 3*f*x] - 4*a^5*f*x*Cos[2*e + 5*f*x] - 12*a^4*b*f*x*Cos[2*e + 5*f*x
] - 12*a^3*b^2*f*x*Cos[2*e + 5*f*x] - 4*a^2*b^3*f*x*Cos[2*e + 5*f*x] + 4*a^
5*f*x*Cos[4*e + 5*f*x] + 12*a^4*b*f*x*Cos[4*e + 5*f*x] + 12*a^3*b^2*f*x*Cos
[4*e + 5*f*x] + 4*a^2*b^3*f*x*Cos[4*e + 5*f*x] - 4*a^5*f*x*Cos[6*e + 5*f*x]
 - 12*a^4*b*f*x*Cos[6*e + 5*f*x] - 12*a^3*b^2*f*x*Cos[6*e + 5*f*x] - 4*a^2*
b^3*f*x*Cos[6*e + 5*f*x] + 4*a^5*f*x*Cos[8*e + 5*f*x] + 12*a^4*b*f*x*Cos[8*
e + 5*f*x] + 12*a^3*b^2*f*x*Cos[8*e + 5*f*x] + 4*a^2*b^3*f*x*Cos[8*e + 5*f*
x] - 32*a^5*Sin[f*x] - 64*a^4*b*Sin[f*x] - 30*a^2*b^3*Sin[f*x] - 120*a*b^4*
Sin[f*x] - 48*b^5*Sin[f*x] + 32*a^5*Sin[3*f*x] + 64*a^4*b*Sin[3*f*x] + 26*a
^3*b^2*Sin[3*f*x] + 86*a^2*b^3*Sin[3*f*x] + 32*a*b^4*Sin[3*f*x] - 48*a^5*Si
n[2*e - f*x] - 128*a^4*b*Sin[2*e - f*x] - 128*a^3*b^2*Sin[2*e - f*x] - 30*a
^2*b^3*Sin[2*e - f*x] - 120*a*b^4*Sin[2*e - f*x] - 48*b^5*Sin[2*e - f*x] +
48*a^5*Sin[2*e + f*x] + 128*a^4*b*Sin[2*e + f*x] + 102*a^3*b^2*Sin[2*e + f*
x] - 86*a^2*b^3*Sin[2*e + f*x] - 136*a*b^4*Sin[2*e + f*x] - 48*b^5*Sin[2*e
+ f*x] - 32*a^5*Sin[4*e + f*x] - 64*a^4*b*Sin[4*e + f*x] + 26*a^3*b^2*Sin[4
*e + f*x] + 86*a^2*b^3*Sin[4*e + f*x] + 136*a*b^4*Sin[4*e + f*x] + 48*b^5*S
in[4*e + f*x] - 8*a^5*Sin[2*e + 3*f*x] - 26*a^3*b^2*Sin[2*e + 3*f*x] - 86*a
^2*b^3*Sin[2*e + 3*f*x] - 32*a*b^4*Sin[2*e + 3*f*x] + 32*a^5*Sin[4*e + 3*f*
x] + 64*a^4*b*Sin[4*e + 3*f*x] - 13*a^3*b^2*Sin[4*e + 3*f*x] - 36*a^2*b^3*S
in[4*e + 3*f*x] - 16*a*b^4*Sin[4*e + 3*f*x] - 8*a^5*Sin[6*e + 3*f*x] + 13*a
^3*b^2*Sin[6*e + 3*f*x] + 36*a^2*b^3*Sin[6*e + 3*f*x] + 16*a*b^4*Sin[6*e +
3*f*x] + 8*a^5*Sin[2*e + 5*f*x] + 13*a^3*b^2*Sin[2*e + 5*f*x] + 6*a^2*b^3*S
in[2*e + 5*f*x] - 13*a^3*b^2*Sin[4*e + 5*f*x] - 6*a^2*b^3*Sin[4*e + 5*f*x]

```
+ 8*a^5*Sin[6*e + 5*f*x]))/(512*a^3*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A] (verified)

Time = 23.49 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{-\frac{1}{(a+b)^3\tan(fx+e)}+\frac{b^2\left(\frac{\left(\frac{11}{8}a^2b+\frac{1}{2}a\,b^2\right)\tan(fx+e)^3+\frac{a\left(13a^2+17ab+4b^2\right)\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(35a^2+28ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}}\right.}{a^3(a+b)^3}}{f}$
default	$\frac{-\frac{1}{(a+b)^3\tan(fx+e)}+\frac{b^2\left(\frac{\left(\frac{11}{8}a^2b+\frac{1}{2}a\,b^2\right)\tan(fx+e)^3+\frac{a\left(13a^2+17ab+4b^2\right)\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(35a^2+28ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}}\right.}{a^3(a+b)^3}}{f}$
risch	$-\frac{x}{a^3}+\frac{i\left(-8a^5e^{8i(fx+e)}+13a^3b^2e^{8i(fx+e)}+36a^2b^3e^{8i(fx+e)}+16a\,b^4e^{8i(fx+e)}-32a^5e^{6i(fx+e)}-64a^4b\,e^{6i(fx+e)}+26a^3b\right.}{}$

```
[In] int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/(a+b)^3/tan(f*x+e)+b^2/a^3/(a+b)^3*(((11/8*a^2*b+1/2*a*b^2)*tan(f*x
+e)^3+1/8*a*(13*a^2+17*a*b+4*b^2)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(3
5*a^2+28*a*b+8*b^2)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))-1
/a^3*arctan(tan(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(165) = 330$.

Time = 0.34 (sec) , antiderivative size = 1060, normalized size of antiderivative = 5.86

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(4*(8*a^5 + 13*a^3*b^2 + 6*a^2*b^3)*cos(f*x + e)^5 + 4*(16*a^4*b - 1
3*a^3*b^2 + 5*a^2*b^3 + 4*a*b^4)*cos(f*x + e)^3 - (35*a^2*b^3 + 28*a*b^4 +
8*b^5 + (35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cos(f*x + e)^4 + 2*(35*a^3*b^2
+ 28*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b
+ 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*
b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(
f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x
+ e) + 4*(8*a^3*b^2 - 11*a^2*b^3 - 4*a*b^4)*cos(f*x + e) + 32*((a^5 + 3*a^4
*b + 3*a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2
```

```
*b^3 + a*b^4)*f*x*cos(f*x + e)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*f*
x)*sin(f*x + e))/(((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 +
 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 +
3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*f)*sin(f*x + e)), -1/16*(2*(8*a^5 + 13*a^3
*b^2 + 6*a^2*b^3)*cos(f*x + e)^5 + 2*(16*a^4*b - 13*a^3*b^2 + 5*a^2*b^3 + 4
*a*b^4)*cos(f*x + e)^3 + (35*a^2*b^3 + 28*a*b^4 + 8*b^5 + (35*a^4*b + 28*a^
3*b^2 + 8*a^2*b^3)*cos(f*x + e)^4 + 2*(35*a^3*b^2 + 28*a^2*b^3 + 8*a*b^4)*c
os(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sq
rt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 2*(8*a^3*b^2 -
11*a^2*b^3 - 4*a*b^4)*cos(f*x + e) + 16*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b
^3)*f*x*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*f*x*cos(
f*x + e)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*f*x)*sin(f*x + e))/(((a^
8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2
+ 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4
+ a^3*b^5)*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx=\text{Timed out}$$

```
[In] integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.72

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{(35\,a^2b^2+28\,ab^3+8\,b^4)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^6+3\,a^5b+3\,a^4b^2+a^3b^3)\sqrt{(a+b)b}}-\frac{(8\,a^2b^2-11\,ab^3-4\,b^4)\tan(fx+e)^4+8\,a^4+16\,a^3b+8\,a^2b^2+(16\,a^3b+3\,a^2}{(a^5b^2+3\,a^4b^3+3\,a^3b^4+a^2b^5)\tan(fx+e)^5+2\,(a^6b+4\,a^5b^2+6\,a^4b^3+4\,a^3b^4+a^2b^5)\tan(fx+e)^3+(}}{8\,f}$$

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
[Out] 1/8*((35*a^2*b^2 + 28*a*b^3 + 8*b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))
/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sqrt((a + b)*b)) - ((8*a^2*b^2 - 11
*a*b^3 - 4*b^4)*tan(f*x + e)^4 + 8*a^4 + 16*a^3*b + 8*a^2*b^2 + (16*a^3*b +
 3*a^2*b^2 - 17*a*b^3 - 4*b^4)*tan(f*x + e)^2)/((a^5*b^2 + 3*a^4*b^3 + 3*a^
3*b^4 + a^2*b^5)*tan(f*x + e)^5 + 2*(a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*
b^4 + a^2*b^5)*tan(f*x + e)^3 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 +
5*a^3*b^4 + a^2*b^5)*tan(f*x + e)) - 8*(f*x + e)/a^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.36

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{(35\,a^2b^2+28\,ab^3+8\,b^4)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^6+3\,a^5b+3\,a^4b^2+a^3b^3)\sqrt{ab+b^2}}+\frac{11\,ab^3\tan(fx+e)^3+4\,b^4\tan(fx+e)^3+13\,a^2b^2\tan(fx+e)+17\,ab^3\tan(fx+e)}{(a^5+3\,a^4b+3\,a^3b^2+a^2b^3)\left(b\tan(fx+e)^2+a+b\right)^2}}{8\,f}$$

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((35*a^2*b^2 + 28*a*b^3 + 8*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
 arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*
b^3)*sqrt(a*b + b^2)) + (11*a*b^3*tan(f*x + e)^3 + 4*b^4*tan(f*x + e)^3 + 1
3*a^2*b^2*tan(f*x + e) + 17*a*b^3*tan(f*x + e) + 4*b^4*tan(f*x + e))/((a^5
+ 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*(b*tan(f*x + e)^2 + a + b)^2) - 8*(f*x + e
)/a^3 - 8/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(f*x + e)))/f
```

Mupad [B] (verification not implemented)

Time = 25.32 (sec) , antiderivative size = 4890, normalized size of antiderivative = 27.02

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] ((tan(e + f*x)^4*(11*a*b^3 + 4*b^4 - 8*a^2*b^2))/(8*a^2*(a + b)^3) - 1/(a +
 b) + (tan(e + f*x)^2*(13*a*b^2 - 16*a^2*b + 4*b^3))/(8*a^2*(a + b)^2))/(f*
(tan(e + f*x)^3*(2*a*b + 2*b^2) + tan(e + f*x)*(2*a*b + a^2 + b^2) + b^2*ta
n(e + f*x)^5)) - atan((286720*a^6*b^15*tan(e + f*x))/(286720*a^6*b^15 + 361
9840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b^11
 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 22166323
2*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 10
48576*a^18*b^3 + 65536*a^19*b^2) + (3619840*a^7*b^14*tan(e + f*x))/(286720*
a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 17717
2480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*
b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320
*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (21052416*a^8*b^13*tan(e +
 f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a
^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 +
334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^1
```

6*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (74346496*a^9*b^12*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (177172480*a^10*b^11*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (299796480*a^11*b^10*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (369346560*a^12*b^9*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (334344192*a^13*b^8*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (221663232*a^14*b^7*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (105978880*a^15*b^6*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (35445760*a^16*b^5*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (7864320*a^17*b^4*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (1048576*a^18*b^3*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (65536*a^19*b^2*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369

346560*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2))/(a^3*f) - (atan((((-b^3*(a + b)^7)^(1/2)*(tan(e + f*x)*(131072*a^6*b^18 + 2031616*a^7*b^17 + 14745600*a^8*b^16 + 66232320*a^9*b^15 + 205112320*a^10*b^14 + 462013440*a^11*b^13 + 778473472*a^12*b^12 + 994283520*a^13*b^11 + 965376000*a^14*b^10 + 708392960*a^15*b^9 + 387272704*a^16*b^8 + 154054656*a^17*b^7 + 43115520*a^18*b^6 + 8135680*a^19*b^5 + 983040*a^20*b^4 + 65536*a^21*b^3) - ((-b^3*(a + b)^7)^(1/2)*(28*a*b + 35*a^2 + 8*b^2)*(65536*a^10*b^17 + 999424*a^11*b^16 + 7405568*a^12*b^15 + 34897920*a^13*b^14 + 115474432*a^14*b^13 + 281329664*a^15*b^12 + 517603328*a^16*b^11 + 728825856*a^17*b^10 + 789381120*a^18*b^9 + 656195584*a^19*b^8 + 414515200*a^20*b^7 + 195067904*a^21*b^6 + 66060288*a^22*b^5 + 15155200*a^23*b^4 + 2097152*a^24*b^3 + 131072*a^25*b^2 - (tan(e + f*x)*(-b^3*(a + b)^7)^(1/2)*(28*a*b + 35*a^2 + 8*b^2)*(524288*a^12*b^18 + 8126464*a^13*b^17 + 58982400*a^14*b^16 + 266076160*a^15*b^15 + 834928640*a^16*b^14 + 1932263424*a^17*b^13 + 3411279872*a^18*b^12 + 4685824000*a^19*b^11 + 5060689920*a^20*b^10 + 4310958080*a^21*b^9 + 2886467584*a^22*b^8 + 1502871552*a^23*b^7 + 596377600*a^24*b^6 + 174325760*a^25*b^5 + 35389440*a^26*b^4 + 4456448*a^27*b^3 + 262144*a^28*b^2))/(16*(7*a^9*b + a^10 + a^3*b^7 + 7*a^4*b^6 + 21*a^5*b^5 + 35*a^6*b^4 + 35*a^7*b^3 + 21*a^8*b^2))))/(16*(7*a^9*b + a^10 + a^3*b^7 + 7*a^4*b^6 + 21*a^5*b^5 + 35*a^6*b^4 + 35*a^7*b^3 + 21*a^8*b^2)))*(28*a*b + 35*a^2 + 8*b^2)*1i)/(16*(7*a^9*b + a^10 + a^3*b^7 + 7*a^4*b^6 + 21*a^5*b^5 + 35*a^6*b^4 + 35*a^7*b^3 + 21*a^8*b^2)) + ((-b^3*(a + b)^7)^(1/2)*(tan(e + f*x)*(131072*a^6*b^18 + 2031616*a^7*b^17 + 14745600*a^8*b^16 + 66232320*a^9*b^15 + 205112320*a^10*b^14 + 462013440*a^11*b^13 + 778473472*a^12*b^12 + 994283520*a^13*b^11 + 965376000*a^14*b^10 + 708392960*a^15*b^9 + 387272704*a^16*b^8 + 154054656*a^17*b^7 + 43115520*a^18*b^6 + 8135680*a^19*b^5 + 983040*a^20*b^4 + 65536*a^21*b^3) + ((-b^3*(a + b)^7)^(1/2)*(28*a*b + 35*a^2 + 8*b^2)*(65536*a^10*b^17 + 999424*a^11*b^16 + 7405568*a^12*b^15 + 34897920*a^13*b^14 + 115474432*a^14*b^13 + 281329664*a^15*b^12 + 517603328*a^16*b^11 + 728825856*a^17*b^10 + 789381120*a^18*b^9 + 656195584*a^19*b^8 + 414515200*a^20*b^7 + 195067904*a^21*b^6 + 66060288*a^22*b^5 + 15155200*a^23*b^4 + 2097152*a^24*b^3 + 131072*a^25*b^2 + (tan(e + f*x)*(-b^3*(a + b)^7)^(1/2)*(28*a*b + 35*a^2 + 8*b^2)*(524288*a^12*b^18 + 8126464*a^13*b^17 + 58982400*a^14*b^16 + 266076160*a^15*b^15 + 834928640*a^16*b^14 + 1932263424*a^17*b^13 + 3411279872*a^18*b^12 + 4685824000*a^19*b^11 + 5060689920*a^20*b^10 + 4310958080*a^21*b^9 + 2886467584*a^22*b^8 + 1502871552*a^23*b^7 + 596377600*a^24*b^6 + 174325760*a^25*b^5 + 35389440*a^26*b^4 + 4456448*a^27*b^3 + 262144*a^28*b^2))/(16*(7*a^9*b + a^10 + a^3*b^7 + 7*a^4*b^6 + 21*a^5*b^5 + 35*a^6*b^4 + 35*a^7*b^3 + 21*a^8*b^2))))/(16*(7*a^9*b + a^10 + a^3*b^7 + 7*a^4*b^6 + 21*a^5*b^5 + 35*a^6*b^4 + 35*a^7*b^3 + 21*a^8*b^2)))*(28*a*b + 35*a^2 + 8*b^2)*1i)/(16*(7*a^9*b + a^10 + a^3*b^7 + 7*a^4*b^6 + 21*a^5*b^5 + 35*a^6*b^4 + 35*a^7*b^3 + 21*a^8*b^2)))/(32768*a^4*b^17 + 499712*a^5*b^16 + 3416064*a^6*b^15 + 13829120*a^7*b^14 + 36684800*a^8*b^13 + 66318336*a^9*b^12 + 81629184*a^10*b^11 + 64616448*a^11*b^10 + 25344000*a^12*b^9 - 6246400*a^13*b^8 - 14405632*a^14*b^7 - 8444928

*a^15*b^6 - 2415616*a^16*b^5 - 286720*a^17*b^4 - ((-b^3*(a + b)^7)^(1/2)*(t
an(e + f*x)*(131072*a^6*b^18 + 2031616*a^7*b^17 + 14745600*a^8*b^16 + 66232
320*a^9*b^15 + 205112320*a^10*b^14 + 462013440*a^11*b^13 + 778473472*a^12*b
^12 + 994283520*a^13*b^11 + 965376000*a^14*b^10 + 708392960*a^15*b^9 + 3872
72704*a^16*b^8 + 154054656*a^17*b^7 + 43115520*a^18*b^6 + 8135680*a^19*b^5
+ 983040*a^20*b^4 + 65536*a^21*b^3) - ((-b^3*(a + b)^7)^(1/2)*(28*a*b + 35*
a^2 + 8*b^2)*(65536*a^10*b^17 + 999424*a^11*b^16 + 7405568*a^12*b^15 + 3489
7920*a^13*b^14 + 115474432*a^14*b^13 + 281329664*a^15*b^12 + 517603328*a^16
*b^11 + 728825856*a^17*b^10 + 789381120*a^18*b^9 + 656195584*a^19*b^8 + 414
515200*a^20*b^7 + 195067904*a^21*b^6 + 66060288*a^22*b^5 + 15155200*a^23*b^
4 + 2097152*a^24*b^3 + 131072*a^25*b^2 - (tan(e + f*x)*(-b^3*(a + b)^7)^(1/
2)*(28*a*b + 35*a^2 + 8*b^2)*(524288*a^12*b^18 + 8126464*a^13*b^17 + 589824
00*a^14*b^16 + 266076160*a^15*b^15 + 834928640*a^16*b^14 + 1932263424*a^17*
b^13 + 3411279872*a^18*b^12 + 4685824000*a^19*b^11 + 5060689920*a^20*b^10 +
4310958080*a^21*b^9 + 2886467584*a^22*b^8 + 1502871552*a^23*b^7 + 59637760
0*a^24*b^6 + 174325760*a^25*b^5 + 35389440*a^26*b^4 + 4456448*a^27*b^3 + 26
2144*a^28*b^2))/(16*(7*a^9*b + a^10 + a^3*b^7 + 7*a^4*b^6 + 21*a^5*b^5 + 35
*a^6*b^4 + 35*a^7*b^3 + 21*a^8*b^2))))/(16*(7*a^9*b + a^10 + a^3*b^7 + 7*a^
4*b^6 + 21*a^5*b^5 + 35*a^6*b^4 + 35*a^7*b^3 + 21*a^8*b^2)))*(28*a*b + 35*a
^2 + 8*b^2))/(16*(7*a^9*b + a^10 + a^3*b^7 + 7*a^4*b^6 + 21*a^5*b^5 + 35*a^
6*b^4 + 35*a^7*b^3 + 21*a^8*b^2)) + ((-b^3*(a + b)^7)^(1/2)*(tan(e + f*x)*(
131072*a^6*b^18 + 2031616*a^7*b^17 + 14745600*a^8*b^16 + 66232320*a^9*b^15
+ 205112320*a^10*b^14 + 462013440*a^11*b^13 + 778473472*a^12*b^12 + 9942835
20*a^13*b^11 + 965376000*a^14*b^10 + 708392960*a^15*b^9 + 387272704*a^16*b^
8 + 154054656*a^17*b^7 + 43115520*a^18*b^6 + 8135680*a^19*b^5 + 983040*a^20
*b^4 + 65536*a^21*b^3) + ((-b^3*(a + b)^7)^(1/2)*(28*a*b + 35*a^2 + 8*b^2)*
(65536*a^10*b^17 + 999424*a^11*b^16 + 7405568*a^12*b^15 + 34897920*a^13*b^1
4 + 115474432*a^14*b^13 + 281329664*a^15*b^12 + 517603328*a^16*b^11 + 72882
5856*a^17*b^10 + 789381120*a^18*b^9 + 656195584*a^19*b^8 + 414515200*a^20*b
^7 + 195067904*a^21*b^6 + 66060288*a^22*b^5 + 15155200*a^23*b^4 + 2097152*a
^24*b^3 + 131072*a^25*b^2 + (tan(e + f*x)*(-b^3*(a + b)^7)^(1/2)*(28*a*b +
35*a^2 + 8*b^2)*(524288*a^12*b^18 + 8126464*a^13*b^17 + 58982400*a^14*b^16
+ 266076160*a^15*b^15 + 834928640*a^16*b^14 + 1932263424*a^17*b^13 + 341127
9872*a^18*b^12 + 4685824000*a^19*b^11 + 5060689920*a^20*b^10 + 4310958080*a
^21*b^9 + 2886467584*a^22*b^8 + 1502871552*a^23*b^7 + 596377600*a^24*b^6 +
174325760*a^25*b^5 + 35389440*a^26*b^4 + 4456448*a^27*b^3 + 262144*a^28*b^2
))/(16*(7*a^9*b + a^10 + a^3*b^7 + 7*a^4*b^6 + 21*a^5*b^5 + 35*a^6*b^4 + 35
*a^7*b^3 + 21*a^8*b^2))))/(16*(7*a^9*b + a^10 + a^3*b^7 + 7*a^4*b^6 + 21*a^
5*b^5 + 35*a^6*b^4 + 35*a^7*b^3 + 21*a^8*b^2)))*(28*a*b + 35*a^2 + 8*b^2))/
(16*(7*a^9*b + a^10 + a^3*b^7 + 7*a^4*b^6 + 21*a^5*b^5 + 35*a^6*b^4 + 35*a^
7*b^3 + 21*a^8*b^2))))*(-b^3*(a + b)^7)^(1/2)*(28*a*b + 35*a^2 + 8*b^2)*1i)
/(8*f*(7*a^9*b + a^10 + a^3*b^7 + 7*a^4*b^6 + 21*a^5*b^5 + 35*a^6*b^4 + 35*
a^7*b^3 + 21*a^8*b^2))

3.374 $\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 230

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{x}{a^3} - \frac{b^{5/2}(63a^2+36ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{9/2}f} + \frac{(8a^3+32a^2b-15ab^2-4b^3)\cot(e+fx)}{8a^2(a+b)^4f} - \frac{(8a^2-39ab-12b^2)\cot^3(e+fx)}{24a^2(a+b)^3f} - \frac{b\cot^3(e+fx)}{4a(a+b)f\,(a+b+b\tan^2(e+fx))^2} - \frac{b(11a+4b)\cot^3(e+fx)}{8a^2(a+b)^2f\,(a+b+b\tan^2(e+fx))}$$

```
[Out] x/a^3-1/8*b^(5/2)*(63*a^2+36*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/
2))/a^3/(a+b)^(9/2)/f+1/8*(8*a^3+32*a^2*b-15*a*b^2-4*b^3)*cot(f*x+e)/a^2/(a
+b)^4/f-1/24*(8*a^2-39*a*b-12*b^2)*cot(f*x+e)^3/a^2/(a+b)^3/f-1/4*b*cot(f*x
+e)^3/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/8*b*(11*a+4*b)*cot(f*x+e)^3/a^2/(a
+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4226, 2000, 483, 593, 597, 536, 209, 211}

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{x}{a^3} - \frac{(8a^2-39ab-12b^2)\cot^3(e+fx)}{24a^2f(a+b)^3} - \frac{b(11a+4b)\cot^3(e+fx)}{8a^2f(a+b)^2\,(a+b\tan^2(e+fx)+b)} - \frac{b^{5/2}(63a^2+36ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{9/2}} + \frac{(8a^3+32a^2b-15ab^2-4b^3)\cot(e+fx)}{8a^2f(a+b)^4} - \frac{b\cot^3(e+fx)}{4af(a+b)\,(a+b\tan^2(e+fx)+b)^2}$$

```
[In] Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] x/a^3 - (b^(5/2)*(63*a^2 + 36*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sq
rt[a + b]])/(8*a^3*(a + b)^(9/2)*f) + ((8*a^3 + 32*a^2*b - 15*a*b^2 - 4*b^3
)*Cot[e + f*x])/(8*a^2*(a + b)^4*f) - ((8*a^2 - 39*a*b - 12*b^2)*Cot[e + f*
x]^3)/(24*a^2*(a + b)^3*f) - (b*Cot[e + f*x]^3)/(4*a*(a + b)*f*(a + b + b*T
an[e + f*x]^2)^2) - (b*(11*a + 4*b)*Cot[e + f*x]^3)/(8*a^2*(a + b)^2*f*(a +
 b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
```

```
IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+b(1+x^2))^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^3(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a-3b-7bx^2}{x^4(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4a(a+b)f}$$

$$= -\frac{b\cot^3(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(11a+4b)\cot^3(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{8a^2-39ab-12b^2-5b(11a+4b)x^2}{x^4(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{8a^2(a+b)^2f}$$

$$= -\frac{(8a^2-39ab-12b^2)\cot^3(e+fx)}{24a^2(a+b)^3f} - \frac{b\cot^3(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(11a+4b)\cot^3(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{3(8a^3+32a^2b-15ab^2-4b^3)+3b(8a^2-39ab-12b^2)x^2}{x^2(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{24a^2(a+b)^3f}$$

$$= \frac{(8a^3+32a^2b-15ab^2-4b^3)\cot(e+fx)}{8a^2(a+b)^4f} - \frac{(8a^2-39ab-12b^2)\cot^3(e+fx)}{24a^2(a+b)^3f} - \frac{b\cot^3(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(11a+4b)\cot^3(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{3(8a^4+40a^3b+80a^2b^2+17ab^3+4b^4)+3b(8a^3+32a^2b-15ab^2-4b^3)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{24a^2(a+b)^4f}$$

$$= \frac{(8a^3+32a^2b-15ab^2-4b^3)\cot(e+fx)}{8a^2(a+b)^4f} - \frac{(8a^2-39ab-12b^2)\cot^3(e+fx)}{24a^2(a+b)^3f} - \frac{b\cot^3(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(11a+4b)\cot^3(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^3f} - \frac{\left(b^3(63a^2+36ab+8b^2)\right)\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{8a^3(a+b)^4f}$$

$$
= \frac{x}{a^3} - \frac{b^{5/2}(63a^2+36ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{9/2}f} + \frac{(8a^3+32a^2b-15ab^2-4b^3)\cot(e+fx)}{8a^2(a+b)^4f} - \frac{(8a^2-39ab-12b^2)\cot^3(e+fx)}{24a^2(a+b)^3f} - \frac{b\cot^3(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(11a+4b)\cot^3(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.84 (sec) , antiderivative size = 3340, normalized size of antiderivative = 14.52

$$
\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((63*a^2 + 36*a*b + 8*b^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*
((b^3*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*
e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a
*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(64*a^3*Sqrt[a + b
]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/64)*b^3*ArcTan[Sec[f*x]*(Cos[2*e
]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[
a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*
Sin[2*e + f*x])]*Sin[2*e])/(a^3*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e
]])))/((a + b)^4*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x]
)*Csc[e]*Csc[e + f*x]^3*Sec[2*e]*Sec[e + f*x]^6*(-36*a^6*f*x*Cos[f*x] - 336
*a^5*b*f*x*Cos[f*x] - 1560*a^4*b^2*f*x*Cos[f*x] - 3600*a^3*b^3*f*x*Cos[f*x]
 - 4260*a^2*b^4*f*x*Cos[f*x] - 2496*a*b^5*f*x*Cos[f*x] - 576*b^6*f*x*Cos[f*
x] + 36*a^6*f*x*Cos[3*f*x] + 240*a^5*b*f*x*Cos[3*f*x] + 408*a^4*b^2*f*x*Cos
[3*f*x] - 48*a^3*b^3*f*x*Cos[3*f*x] - 732*a^2*b^4*f*x*Cos[3*f*x] - 672*a*b^
5*f*x*Cos[3*f*x] - 192*b^6*f*x*Cos[3*f*x] + 36*a^6*f*x*Cos[2*e - f*x] + 336
*a^5*b*f*x*Cos[2*e - f*x] + 1560*a^4*b^2*f*x*Cos[2*e - f*x] + 3600*a^3*b^3*
f*x*Cos[2*e - f*x] + 4260*a^2*b^4*f*x*Cos[2*e - f*x] + 2496*a*b^5*f*x*Cos[2
*e - f*x] + 576*b^6*f*x*Cos[2*e - f*x] + 36*a^6*f*x*Cos[2*e + f*x] + 336*a^
5*b*f*x*Cos[2*e + f*x] + 1560*a^4*b^2*f*x*Cos[2*e + f*x] + 3600*a^3*b^3*f*x
*Cos[2*e + f*x] + 4260*a^2*b^4*f*x*Cos[2*e + f*x] + 2496*a*b^5*f*x*Cos[2*e
+ f*x] + 576*b^6*f*x*Cos[2*e + f*x] - 36*a^6*f*x*Cos[4*e + f*x] - 336*a^5*b
*f*x*Cos[4*e + f*x] - 1560*a^4*b^2*f*x*Cos[4*e + f*x] - 3600*a^3*b^3*f*x*Co
s[4*e + f*x] - 4260*a^2*b^4*f*x*Cos[4*e + f*x] - 2496*a*b^5*f*x*Cos[4*e + f
*x] - 576*b^6*f*x*Cos[4*e + f*x] - 36*a^6*f*x*Cos[2*e + 3*f*x] - 240*a^5*b*
f*x*Cos[2*e + 3*f*x] - 408*a^4*b^2*f*x*Cos[2*e + 3*f*x] + 48*a^3*b^3*f*x*Co
s[2*e + 3*f*x] + 732*a^2*b^4*f*x*Cos[2*e + 3*f*x] + 672*a*b^5*f*x*Cos[2*e +
```

3*f*x] + 192*b^6*f*x*Cos[2*e + 3*f*x] + 36*a^6*f*x*Cos[4*e + 3*f*x] + 240*a^5*b*f*x*Cos[4*e + 3*f*x] + 408*a^4*b^2*f*x*Cos[4*e + 3*f*x] - 48*a^3*b^3*f*x*Cos[4*e + 3*f*x] - 732*a^2*b^4*f*x*Cos[4*e + 3*f*x] - 672*a*b^5*f*x*Cos[4*e + 3*f*x] - 192*b^6*f*x*Cos[4*e + 3*f*x] - 36*a^6*f*x*Cos[6*e + 3*f*x] - 240*a^5*b*f*x*Cos[6*e + 3*f*x] - 408*a^4*b^2*f*x*Cos[6*e + 3*f*x] + 48*a^3*b^3*f*x*Cos[6*e + 3*f*x] + 732*a^2*b^4*f*x*Cos[6*e + 3*f*x] + 672*a*b^5*f*x*Cos[6*e + 3*f*x] + 192*b^6*f*x*Cos[6*e + 3*f*x] - 12*a^6*f*x*Cos[2*e + 5*f*x] - 144*a^5*b*f*x*Cos[2*e + 5*f*x] - 456*a^4*b^2*f*x*Cos[2*e + 5*f*x] - 624*a^3*b^3*f*x*Cos[2*e + 5*f*x] - 396*a^2*b^4*f*x*Cos[2*e + 5*f*x] - 96*a*b^5*f*x*Cos[2*e + 5*f*x] + 12*a^6*f*x*Cos[4*e + 5*f*x] + 144*a^5*b*f*x*Cos[4*e + 5*f*x] + 456*a^4*b^2*f*x*Cos[4*e + 5*f*x] + 624*a^3*b^3*f*x*Cos[4*e + 5*f*x] + 396*a^2*b^4*f*x*Cos[4*e + 5*f*x] + 96*a*b^5*f*x*Cos[4*e + 5*f*x] - 12*a^6*f*x*Cos[6*e + 5*f*x] - 144*a^5*b*f*x*Cos[6*e + 5*f*x] - 456*a^4*b^2*f*x*Cos[6*e + 5*f*x] - 624*a^3*b^3*f*x*Cos[6*e + 5*f*x] - 396*a^2*b^4*f*x*Cos[6*e + 5*f*x] - 96*a*b^5*f*x*Cos[6*e + 5*f*x] + 12*a^6*f*x*Cos[8*e + 5*f*x] + 144*a^5*b*f*x*Cos[8*e + 5*f*x] + 456*a^4*b^2*f*x*Cos[8*e + 5*f*x] + 624*a^3*b^3*f*x*Cos[8*e + 5*f*x] + 396*a^2*b^4*f*x*Cos[8*e + 5*f*x] + 96*a*b^5*f*x*Cos[8*e + 5*f*x] - 12*a^6*f*x*Cos[4*e + 7*f*x] - 48*a^5*b*f*x*Cos[4*e + 7*f*x] - 72*a^4*b^2*f*x*Cos[4*e + 7*f*x] - 48*a^3*b^3*f*x*Cos[4*e + 7*f*x] - 12*a^2*b^4*f*x*Cos[4*e + 7*f*x] + 12*a^6*f*x*Cos[6*e + 7*f*x] + 48*a^5*b*f*x*Cos[6*e + 7*f*x] + 72*a^4*b^2*f*x*Cos[6*e + 7*f*x] + 48*a^3*b^3*f*x*Cos[6*e + 7*f*x] + 12*a^2*b^4*f*x*Cos[6*e + 7*f*x] - 12*a^6*f*x*Cos[8*e + 7*f*x] - 48*a^5*b*f*x*Cos[8*e + 7*f*x] - 72*a^4*b^2*f*x*Cos[8*e + 7*f*x] - 48*a^3*b^3*f*x*Cos[8*e + 7*f*x] - 12*a^2*b^4*f*x*Cos[8*e + 7*f*x] + 12*a^6*f*x*Cos[10*e + 7*f*x] + 48*a^5*b*f*x*Cos[10*e + 7*f*x] + 72*a^4*b^2*f*x*Cos[10*e + 7*f*x] + 48*a^3*b^3*f*x*Cos[10*e + 7*f*x] + 12*a^2*b^4*f*x*Cos[10*e + 7*f*x] - 128*a^6*Sin[f*x] - 440*a^5*b*Sin[f*x] - 1152*a^4*b^2*Sin[f*x] - 1920*a^3*b^3*Sin[f*x] + 228*a^2*b^4*Sin[f*x] + 1320*a*b^5*Sin[f*x] + 432*b^6*Sin[f*x] + 48*a^6*Sin[3*f*x] + 104*a^5*b*Sin[3*f*x] + 640*a^4*b^2*Sin[3*f*x] + 1511*a^3*b^3*Sin[3*f*x] - 528*a^2*b^4*Sin[3*f*x] + 264*a*b^5*Sin[3*f*x] + 144*b^6*Sin[3*f*x] - 32*a^6*Sin[2*e - f*x] + 384*a^5*b*Sin[2*e - f*x] + 2048*a^4*b^2*Sin[2*e - f*x] + 3072*a^3*b^3*Sin[2*e - f*x] + 228*a^2*b^4*Sin[2*e - f*x] + 1320*a*b^5*Sin[2*e - f*x] + 432*b^6*Sin[2*e - f*x] + 32*a^6*Sin[2*e + f*x] - 384*a^5*b*Sin[2*e + f*x] - 2048*a^4*b^2*Sin[2*e + f*x] - 2919*a^3*b^3*Sin[2*e + f*x] + 642*a^2*b^4*Sin[2*e + f*x] + 1416*a*b^5*Sin[2*e + f*x] + 432*b^6*Sin[2*e + f*x] - 128*a^6*Sin[4*e + f*x] - 440*a^5*b*Sin[4*e + f*x] - 1152*a^4*b^2*Sin[4*e + f*x] - 2073*a^3*b^3*Sin[4*e + f*x] - 642*a^2*b^4*Sin[4*e + f*x] - 1416*a*b^5*Sin[4*e + f*x] - 432*b^6*Sin[4*e + f*x] - 144*a^6*Sin[2*e + 3*f*x] - 672*a^5*b*Sin[2*e + 3*f*x] - 960*a^4*b^2*Sin[2*e + 3*f*x] + 153*a^3*b^3*Sin[2*e + 3*f*x] + 528*a^2*b^4*Sin[2*e + 3*f*x] - 264*a*b^5*Sin[2*e + 3*f*x] - 144*b^6*Sin[2*e + 3*f*x] + 48*a^6*Sin[4*e + 3*f*x] + 104*a^5*b*Sin[4*e + 3*f*x] + 640*a^4*b^2*Sin[4*e + 3*f*x] + 1664*a^3*b^3*Sin[4*e + 3*f*x] - 66*a^2*b^4*Sin[4*e + 3*f*x] - 408*a*b^5*Sin[4*e + 3*f*x] - 144*b^6*Sin[4*e + 3*f*x] - 144*a^6*Sin[6*e + 3*f*x] - 672*a^5*b*Sin[6*e + 3*f*x] - 960*a^4*b^2*Sin[6*e + 3*f*x] + 66*a^2*b^4*Sin[6*e +

```
 3*f*x] + 408*a*b^5*Sin[6*e + 3*f*x] + 144*b^6*Sin[6*e + 3*f*x] + 80*a^6*Si
n[2*e + 5*f*x] + 480*a^5*b*Sin[2*e + 5*f*x] + 832*a^4*b^2*Sin[2*e + 5*f*x]
+ 294*a^2*b^4*Sin[2*e + 5*f*x] + 96*a*b^5*Sin[2*e + 5*f*x] - 48*a^6*Sin[4*e
 + 5*f*x] - 120*a^5*b*Sin[4*e + 5*f*x] - 294*a^2*b^4*Sin[4*e + 5*f*x] - 96*
a*b^5*Sin[4*e + 5*f*x] + 80*a^6*Sin[6*e + 5*f*x] + 480*a^5*b*Sin[6*e + 5*f*
x] + 832*a^4*b^2*Sin[6*e + 5*f*x] - 51*a^3*b^3*Sin[6*e + 5*f*x] - 132*a^2*b
^4*Sin[6*e + 5*f*x] - 48*a*b^5*Sin[6*e + 5*f*x] - 48*a^6*Sin[8*e + 5*f*x] -
 120*a^5*b*Sin[8*e + 5*f*x] + 51*a^3*b^3*Sin[8*e + 5*f*x] + 132*a^2*b^4*Sin
[8*e + 5*f*x] + 48*a*b^5*Sin[8*e + 5*f*x] + 32*a^6*Sin[4*e + 7*f*x] + 104*a
^5*b*Sin[4*e + 7*f*x] + 51*a^3*b^3*Sin[4*e + 7*f*x] + 18*a^2*b^4*Sin[4*e +
7*f*x] - 51*a^3*b^3*Sin[6*e + 7*f*x] - 18*a^2*b^4*Sin[6*e + 7*f*x] + 32*a^6
*Sin[8*e + 7*f*x] + 104*a^5*b*Sin[8*e + 7*f*x]))/(6144*a^3*(a + b)^4*f*(a +
 b*Sec[e + f*x]^2)^3)
```

Maple [A] (verified)

Time = 41.58 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a^3}-\frac{1}{3(a+b)^3\tan(fx+e)^3}-\frac{-a-4b}{(a+b)^4\tan(fx+e)}-\frac{b^3\left(\frac{\left(\frac{15}{8}a^2b+\frac{1}{2}ab^2\right)\tan(fx+e)^3+\frac{a\left(17a^2+21ab+4b^2\right)\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\cdots\right.}{a^3(a+b)^4}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{a^3}-\frac{1}{3(a+b)^3\tan(fx+e)^3}-\frac{-a-4b}{(a+b)^4\tan(fx+e)}-\frac{b^3\left(\frac{\left(\frac{15}{8}a^2b+\frac{1}{2}ab^2\right)\tan(fx+e)^3+\frac{a\left(17a^2+21ab+4b^2\right)\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\cdots\right.}{a^3(a+b)^4}}{f}$
risch	Expression too large to display

```
[In] int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^3*arctan(tan(f*x+e))-1/3/(a+b)^3/tan(f*x+e)^3-(-a-4*b)/(a+b)^4/tan
(f*x+e)-b^3/a^3/(a+b)^4*(((15/8*a^2*b+1/2*a*b^2)*tan(f*x+e)^3+1/8*a*(17*a^2
+21*a*b+4*b^2)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(63*a^2+36*a*b+8*b^2)
/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 781 vs. 2(212) = 424.

Time = 0.37 (sec) , antiderivative size = 1649, normalized size of antiderivative = 7.17

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx=\text{Too large to display}$$

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/96*(4*(32*a^6 + 104*a^5*b + 51*a^3*b^3 + 18*a^2*b^4)*cos(f*x + e)^7 - 4*
(24*a^6 + 32*a^5*b - 208*a^4*b^2 + 102*a^3*b^3 - 9*a^2*b^4 - 12*a*b^5)*cos(
f*x + e)^5 - 4*(48*a^5*b + 160*a^4*b^2 - 155*a^3*b^3 + 72*a^2*b^4 + 24*a*b^
5)*cos(f*x + e)^3 + 3*((63*a^4*b^2 + 36*a^3*b^3 + 8*a^2*b^4)*cos(f*x + e)^6
 - 63*a^2*b^4 - 36*a*b^5 - 8*b^6 - (63*a^4*b^2 - 90*a^3*b^3 - 64*a^2*b^4 -
16*a*b^5)*cos(f*x + e)^4 - (126*a^3*b^3 + 9*a^2*b^4 - 20*a*b^5 - 8*b^6)*cos
(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2
*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 -
 (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*
x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) - 12*(8*a^4*b^2 + 32*a
^3*b^3 - 15*a^2*b^4 - 4*a*b^5)*cos(f*x + e) + 96*((a^6 + 4*a^5*b + 6*a^4*b^
2 + 4*a^3*b^3 + a^2*b^4)*f*x*cos(f*x + e)^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 -
8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*f*x*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2
+ 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*f*x*cos(f*x + e)^2 - (a^4*b^2 + 4*
a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6)*f*x)*sin(f*x + e))/(((a^9 + 4*a^8*b +
6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b - 2*a^7*
b^2 - 8*a^6*b^3 - 7*a^5*b^4 - 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b + 7*a^
7*b^2 + 8*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2 - (a^
7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3*b^6)*f)*sin(f*x + e)), 1/48
*(2*(32*a^6 + 104*a^5*b + 51*a^3*b^3 + 18*a^2*b^4)*cos(f*x + e)^7 - 2*(24*a
^6 + 32*a^5*b - 208*a^4*b^2 + 102*a^3*b^3 - 9*a^2*b^4 - 12*a*b^5)*cos(f*x +
 e)^5 - 2*(48*a^5*b + 160*a^4*b^2 - 155*a^3*b^3 + 72*a^2*b^4 + 24*a*b^5)*co
s(f*x + e)^3 + 3*((63*a^4*b^2 + 36*a^3*b^3 + 8*a^2*b^4)*cos(f*x + e)^6 - 63
*a^2*b^4 - 36*a*b^5 - 8*b^6 - (63*a^4*b^2 - 90*a^3*b^3 - 64*a^2*b^4 - 16*a*
b^5)*cos(f*x + e)^4 - (126*a^3*b^3 + 9*a^2*b^4 - 20*a*b^5 - 8*b^6)*cos(f*x
+ e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a
 + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) - 6*(8*a^4*b^2 + 32*a^3*
b^3 - 15*a^2*b^4 - 4*a*b^5)*cos(f*x + e) + 48*((a^6 + 4*a^5*b + 6*a^4*b^2 +
 4*a^3*b^3 + a^2*b^4)*f*x*cos(f*x + e)^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a
^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*f*x*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 + 8
*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*f*x*cos(f*x + e)^2 - (a^4*b^2 + 4*a^3
*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6)*f*x)*sin(f*x + e))/(((a^9 + 4*a^8*b + 6*a
^7*b^2 + 4*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2
 - 8*a^6*b^3 - 7*a^5*b^4 - 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b + 7*a^7*b
^2 + 8*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b
^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3*b^6)*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.78

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\frac{3\,(63\,a^2b^3+36\,ab^4+8\,b^5)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4)\sqrt{(a+b)b}} - \frac{3\,(8\,a^3b^2+32\,a^2b^3-15\,ab^4-4\,b^5)\tan(fx+e)^6-8\,a^5-24\,a^4b-24\,a^3b^2-8\,a^2b^3+(48\,a^4b+232\,a}{(a^6b^2+4\,a^5b^3+6\,a^4b^4+4\,a^3b^5+a^2b^6)\tan(fx+e)^7+2\,(a^7b+5\,a^6b^2+10\,a^5b^3+10\,a^4b^4+5\,a^3b}}{24\,f}$$

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/24*(3*(63*a^2*b^3 + 36*a*b^4 + 8*b^5)*arctan(b*tan(f*x + e)/sqrt((a + b)
*b))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*sqrt((a + b)*b)) -
(3*(8*a^3*b^2 + 32*a^2*b^3 - 15*a*b^4 - 4*b^5)*tan(f*x + e)^6 - 8*a^5 - 24*
a^4*b - 24*a^3*b^2 - 8*a^2*b^3 + (48*a^4*b + 232*a^3*b^2 + 133*a^2*b^3 - 63
*a*b^4 - 12*b^5)*tan(f*x + e)^4 + 8*(3*a^5 + 16*a^4*b + 23*a^3*b^2 + 10*a^2
*b^3)*tan(f*x + e)^2)/((a^6*b^2 + 4*a^5*b^3 + 6*a^4*b^4 + 4*a^3*b^5 + a^2*b
^6)*tan(f*x + e)^7 + 2*(a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3
*b^5 + a^2*b^6)*tan(f*x + e)^5 + (a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 +
 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*tan(f*x + e)^3) - 24*(f*x + e)/a^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.31

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\frac{3\,(63\,a^2b^3+36\,ab^4+8\,b^5)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4)\sqrt{ab+b^2}} + \frac{3\left(15\,ab^4\tan(fx+e)^3+4\,b^5\tan(fx+e)^3+17\,a^2b^3\tan(fx+e)+21\,ab^4}{(a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4)\left(b\tan(fx+e)^2\right.}}{24\,f}$$

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/24*(3*(63*a^2*b^3 + 36*a*b^4 + 8*b^5)*(pi*floor((f*x + e)/pi + 1/2)*sgn(
b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^7 + 4*a^6*b + 6*a^5*b^2 +
4*a^4*b^3 + a^3*b^4)*sqrt(a*b + b^2)) + 3*(15*a*b^4*tan(f*x + e)^3 + 4*b^5*
tan(f*x + e)^3 + 17*a^2*b^3*tan(f*x + e) + 21*a*b^4*tan(f*x + e) + 4*b^5*ta
n(f*x + e))/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*(b*tan(f*x +
 e)^2 + a + b)^2) - 24*(f*x + e)/a^3 - 8*(3*a*tan(f*x + e)^2 + 12*b*tan(f*x
 + e)^2 - a - b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*tan(f*x + e)^
3))/f
```

Mupad [B] (verification not implemented)

Time = 25.94 (sec) , antiderivative size = 7057, normalized size of antiderivative = 30.68

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] atan((860160*a^6*b^20*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 +
115347456*a^8*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 4965811200
*a^11*b^15 + 9577308160*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a^1
4*b^12 + 16066462720*a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17*b^
9 + 3502829568*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 871628
80*a^21*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) + (145
15200*a^7*b^19*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 + 1153474
56*a^8*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 4965811200*a^11*b
^15 + 9577308160*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a^14*b^12
+ 16066462720*a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17*b^9 + 350
2829568*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21
*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) + (115347456*
a^8*b^18*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8
*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 4965811200*a^11*b^15 +
9577308160*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a^14*b^12 + 1606
6462720*a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17*b^9 + 350282956
8*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5 +
 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) + (570587136*a^9*b^
17*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8*b^18
+ 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 4965811200*a^11*b^15 + 957730
8160*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a^14*b^12 + 1606646272
0*a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17*b^9 + 3502829568*a^18
*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5 + 13762
560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) + (1961717760*a^10*b^16*t
an(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8*b^18 + 57
```

0587136*a^9*b^17 + 1961717760*a^10*b^16 + 4965811200*a^11*b^15 + 9577308160
*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a^14*b^12 + 16066462720*a^
15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17*b^9 + 3502829568*a^18*b^8
 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5 + 13762560*
a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) + (4965811200*a^11*b^15*tan(e
 + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8*b^18 + 570587
136*a^9*b^17 + 1961717760*a^10*b^16 + 4965811200*a^11*b^15 + 9577308160*a^1
2*b^14 + 14379552768*a^13*b^13 + 17038737408*a^14*b^12 + 16066462720*a^15*b
^11 + 12106321920*a^16*b^10 + 7294187520*a^17*b^9 + 3502829568*a^18*b^8 + 1
329527808*a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5 + 13762560*a^22
*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) + (9577308160*a^12*b^14*tan(e + f
*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8*b^18 + 570587136*
a^9*b^17 + 1961717760*a^10*b^16 + 4965811200*a^11*b^15 + 9577308160*a^12*b^
14 + 14379552768*a^13*b^13 + 17038737408*a^14*b^12 + 16066462720*a^15*b^11
+ 12106321920*a^16*b^10 + 7294187520*a^17*b^9 + 3502829568*a^18*b^8 + 13295
27808*a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5 + 13762560*a^22*b^4
 + 1376256*a^23*b^3 + 65536*a^24*b^2) + (14379552768*a^13*b^13*tan(e + f*x)
)/(860160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8*b^18 + 570587136*a^9
*b^17 + 1961717760*a^10*b^16 + 4965811200*a^11*b^15 + 9577308160*a^12*b^14
+ 14379552768*a^13*b^13 + 17038737408*a^14*b^12 + 16066462720*a^15*b^11 + 1
2106321920*a^16*b^10 + 7294187520*a^17*b^9 + 3502829568*a^18*b^8 + 13295278
08*a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5 + 13762560*a^22*b^4 +
1376256*a^23*b^3 + 65536*a^24*b^2) + (17038737408*a^14*b^12*tan(e + f*x))/(
860160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8*b^18 + 570587136*a^9*b^
17 + 1961717760*a^10*b^16 + 4965811200*a^11*b^15 + 9577308160*a^12*b^14 + 1
4379552768*a^13*b^13 + 17038737408*a^14*b^12 + 16066462720*a^15*b^11 + 1210
6321920*a^16*b^10 + 7294187520*a^17*b^9 + 3502829568*a^18*b^8 + 1329527808*
a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5 + 13762560*a^22*b^4 + 137
6256*a^23*b^3 + 65536*a^24*b^2) + (16066462720*a^15*b^11*tan(e + f*x))/(860
160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8*b^18 + 570587136*a^9*b^17
+ 1961717760*a^10*b^16 + 4965811200*a^11*b^15 + 9577308160*a^12*b^14 + 1437
9552768*a^13*b^13 + 17038737408*a^14*b^12 + 16066462720*a^15*b^11 + 1210632
1920*a^16*b^10 + 7294187520*a^17*b^9 + 3502829568*a^18*b^8 + 1329527808*a^1
9*b^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5 + 13762560*a^22*b^4 + 137625
6*a^23*b^3 + 65536*a^24*b^2) + (12106321920*a^16*b^10*tan(e + f*x))/(860160
*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8*b^18 + 570587136*a^9*b^17 + 1
961717760*a^10*b^16 + 4965811200*a^11*b^15 + 9577308160*a^12*b^14 + 1437955
2768*a^13*b^13 + 17038737408*a^14*b^12 + 16066462720*a^15*b^11 + 1210632192
0*a^16*b^10 + 7294187520*a^17*b^9 + 3502829568*a^18*b^8 + 1329527808*a^19*b
^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5 + 13762560*a^22*b^4 + 1376256*a
^23*b^3 + 65536*a^24*b^2) + (7294187520*a^17*b^9*tan(e + f*x))/(860160*a^6*
b^20 + 14515200*a^7*b^19 + 115347456*a^8*b^18 + 570587136*a^9*b^17 + 196171
7760*a^10*b^16 + 4965811200*a^11*b^15 + 9577308160*a^12*b^14 + 14379552768*
a^13*b^13 + 17038737408*a^14*b^12 + 16066462720*a^15*b^11 + 12106321920*a^1
6*b^10 + 7294187520*a^17*b^9 + 3502829568*a^18*b^8 + 1329527808*a^19*b^7 +

392232960*a^20*b^6 + 87162880*a^21*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b
^3 + 65536*a^24*b^2) + (3502829568*a^18*b^8*tan(e + f*x))/(860160*a^6*b^20
+ 14515200*a^7*b^19 + 115347456*a^8*b^18 + 570587136*a^9*b^17 + 1961717760*
a^10*b^16 + 4965811200*a^11*b^15 + 9577308160*a^12*b^14 + 14379552768*a^13*
b^13 + 17038737408*a^14*b^12 + 16066462720*a^15*b^11 + 12106321920*a^16*b^1
0 + 7294187520*a^17*b^9 + 3502829568*a^18*b^8 + 1329527808*a^19*b^7 + 39223
2960*a^20*b^6 + 87162880*a^21*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b^3 +
65536*a^24*b^2) + (1329527808*a^19*b^7*tan(e + f*x))/(860160*a^6*b^20 + 145
15200*a^7*b^19 + 115347456*a^8*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*
b^16 + 4965811200*a^11*b^15 + 9577308160*a^12*b^14 + 14379552768*a^13*b^13
+ 17038737408*a^14*b^12 + 16066462720*a^15*b^11 + 12106321920*a^16*b^10 + 7
294187520*a^17*b^9 + 3502829568*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*
a^20*b^6 + 87162880*a^21*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536
*a^24*b^2) + (392232960*a^20*b^6*tan(e + f*x))/(860160*a^6*b^20 + 14515200*
a^7*b^19 + 115347456*a^8*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 +
4965811200*a^11*b^15 + 9577308160*a^12*b^14 + 14379552768*a^13*b^13 + 1703
8737408*a^14*b^12 + 16066462720*a^15*b^11 + 12106321920*a^16*b^10 + 7294187
520*a^17*b^9 + 3502829568*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b
^6 + 87162880*a^21*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*
b^2) + (87162880*a^21*b^5*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^1
9 + 115347456*a^8*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 496581
1200*a^11*b^15 + 9577308160*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408
*a^14*b^12 + 16066462720*a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^1
7*b^9 + 3502829568*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 87
162880*a^21*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) +
(13762560*a^22*b^4*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 + 115
347456*a^8*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 4965811200*a^
11*b^15 + 9577308160*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a^14*b
^12 + 16066462720*a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17*b^9 +
3502829568*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 87162880*
a^21*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) + (137625
6*a^23*b^3*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a
^8*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 4965811200*a^11*b^15
+ 9577308160*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a^14*b^12 + 16
066462720*a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17*b^9 + 3502829
568*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5
+ 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) + (65536*a^24*b^2
*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8*b^18 +
570587136*a^9*b^17 + 1961717760*a^10*b^16 + 4965811200*a^11*b^15 + 95773081
60*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a^14*b^12 + 16066462720*
a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17*b^9 + 3502829568*a^18*b
^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5 + 1376256
0*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2))/(a^3*f) - (1/(3*(a + b)) -
(tan(e + f*x)^2*(3*a + 10*b))/(3*(a + b)^2) + (tan(e + f*x)^6*(15*a*b^4 +
4*b^5 - 32*a^2*b^3 - 8*a^3*b^2))/(8*a^2*(a + b)^4) + (tan(e + f*x)^4*(51*a*

b^3 - 48*a^3*b + 12*b^4 - 184*a^2*b^2))/(24*a^2*(a + b)^3))/(f*(tan(e + f*x
)^3*(2*a*b + a^2 + b^2) + tan(e + f*x)^5*(2*a*b + 2*b^2) + b^2*tan(e + f*x)
^7)) - (atan((((-b^5*(a + b)^9)^(1/2)*(tan(e + f*x)*(131072*a^6*b^23 + 2686
976*a^7*b^22 + 26214400*a^8*b^21 + 161013760*a^9*b^20 + 695239680*a^10*b^19
 + 2234314752*a^11*b^18 + 5525833728*a^12*b^17 + 10739159040*a^13*b^16 + 16
625679360*a^14*b^15 + 20693114880*a^15*b^14 + 20844212224*a^16*b^13 + 17084
284928*a^17*b^12 + 11452103680*a^18*b^11 + 6309949440*a^19*b^10 + 286651392
0*a^20*b^9 + 1069486080*a^21*b^8 + 321586176*a^22*b^7 + 74711040*a^23*b^6 +
 12451840*a^24*b^5 + 1310720*a^25*b^4 + 65536*a^26*b^3) - ((-b^5*(a + b)^9)
^(1/2)*(36*a*b + 63*a^2 + 8*b^2)*(65536*a^10*b^22 + 1327104*a^11*b^21 + 136
31488*a^12*b^20 + 91750400*a^13*b^19 + 443154432*a^14*b^18 + 1607925760*a^1
5*b^17 + 4509663232*a^16*b^16 + 9971564544*a^17*b^15 + 17627217920*a^18*b^1
4 + 25149669376*a^19*b^13 + 29127081984*a^20*b^12 + 27445297152*a^21*b^11 +
 21016346624*a^22*b^10 + 13016432640*a^23*b^9 + 6461587456*a^24*b^8 + 25337
52832*a^25*b^7 + 767361024*a^26*b^6 + 173293568*a^27*b^5 + 27525120*a^28*b^
4 + 2752512*a^29*b^3 + 131072*a^30*b^2 - (tan(e + f*x)*(-b^5*(a + b)^9)^(1/
2)*(36*a*b + 63*a^2 + 8*b^2)*(524288*a^12*b^23 + 10747904*a^13*b^22 + 10485
7600*a^14*b^21 + 647495680*a^15*b^20 + 2839019520*a^16*b^19 + 9398648832*a^
17*b^18 + 24385683456*a^18*b^17 + 50803507200*a^19*b^16 + 86365962240*a^20*
b^15 + 121081692160*a^21*b^14 + 140895059968*a^22*b^13 + 136492089344*a^23*
b^12 + 110074265600*a^24*b^11 + 73665085440*a^25*b^10 + 40642805760*a^26*b^
9 + 18289262592*a^27*b^8 + 6604455936*a^28*b^7 + 1867776000*a^29*b^6 + 3984
58880*a^30*b^5 + 60293120*a^31*b^4 + 5767168*a^32*b^3 + 262144*a^33*b^2))/(
16*(9*a^11*b + a^12 + a^3*b^9 + 9*a^4*b^8 + 36*a^5*b^7 + 84*a^6*b^6 + 126*a
^7*b^5 + 126*a^8*b^4 + 84*a^9*b^3 + 36*a^10*b^2))))/(16*(9*a^11*b + a^12 +
a^3*b^9 + 9*a^4*b^8 + 36*a^5*b^7 + 84*a^6*b^6 + 126*a^7*b^5 + 126*a^8*b^4 +
 84*a^9*b^3 + 36*a^10*b^2)))*(36*a*b + 63*a^2 + 8*b^2)*1i)/(16*(9*a^11*b +
a^12 + a^3*b^9 + 9*a^4*b^8 + 36*a^5*b^7 + 84*a^6*b^6 + 126*a^7*b^5 + 126*a^
8*b^4 + 84*a^9*b^3 + 36*a^10*b^2)) + ((-b^5*(a + b)^9)^(1/2)*(tan(e + f*x)*
(131072*a^6*b^23 + 2686976*a^7*b^22 + 26214400*a^8*b^21 + 161013760*a^9*b^2
0 + 695239680*a^10*b^19 + 2234314752*a^11*b^18 + 5525833728*a^12*b^17 + 107
39159040*a^13*b^16 + 16625679360*a^14*b^15 + 20693114880*a^15*b^14 + 208442
12224*a^16*b^13 + 17084284928*a^17*b^12 + 11452103680*a^18*b^11 + 630994944
0*a^19*b^10 + 2866513920*a^20*b^9 + 1069486080*a^21*b^8 + 321586176*a^22*b^
7 + 74711040*a^23*b^6 + 12451840*a^24*b^5 + 1310720*a^25*b^4 + 65536*a^26*b
^3) + ((-b^5*(a + b)^9)^(1/2)*(36*a*b + 63*a^2 + 8*b^2)*(65536*a^10*b^22 +
1327104*a^11*b^21 + 13631488*a^12*b^20 + 91750400*a^13*b^19 + 443154432*a^1
4*b^18 + 1607925760*a^15*b^17 + 4509663232*a^16*b^16 + 9971564544*a^17*b^15
 + 17627217920*a^18*b^14 + 25149669376*a^19*b^13 + 29127081984*a^20*b^12 +
27445297152*a^21*b^11 + 21016346624*a^22*b^10 + 13016432640*a^23*b^9 + 6461
587456*a^24*b^8 + 2533752832*a^25*b^7 + 767361024*a^26*b^6 + 173293568*a^27
*b^5 + 27525120*a^28*b^4 + 2752512*a^29*b^3 + 131072*a^30*b^2 + (tan(e + f*
x)*(-b^5*(a + b)^9)^(1/2)*(36*a*b + 63*a^2 + 8*b^2)*(524288*a^12*b^23 + 107
47904*a^13*b^22 + 104857600*a^14*b^21 + 647495680*a^15*b^20 + 2839019520*a^
16*b^19 + 9398648832*a^17*b^18 + 24385683456*a^18*b^17 + 50803507200*a^19*b

^16 + 86365962240*a^20*b^15 + 121081692160*a^21*b^14 + 140895059968*a^22*b^13 + 136492089344*a^23*b^12 + 110074265600*a^24*b^11 + 73665085440*a^25*b^10 + 40642805760*a^26*b^9 + 18289262592*a^27*b^8 + 6604455936*a^28*b^7 + 1867776000*a^29*b^6 + 398458880*a^30*b^5 + 60293120*a^31*b^4 + 5767168*a^32*b^3 + 262144*a^33*b^2))/(16*(9*a^11*b + a^12 + a^3*b^9 + 9*a^4*b^8 + 36*a^5*b^7 + 84*a^6*b^6 + 126*a^7*b^5 + 126*a^8*b^4 + 84*a^9*b^3 + 36*a^10*b^2))))/(16*(9*a^11*b + a^12 + a^3*b^9 + 9*a^4*b^8 + 36*a^5*b^7 + 84*a^6*b^6 + 126*a^7*b^5 + 126*a^8*b^4 + 84*a^9*b^3 + 36*a^10*b^2)))*(36*a*b + 63*a^2 + 8*b^2)*1i)/(16*(9*a^11*b + a^12 + a^3*b^9 + 9*a^4*b^8 + 36*a^5*b^7 + 84*a^6*b^6 + 126*a^7*b^5 + 126*a^8*b^4 + 84*a^9*b^3 + 36*a^10*b^2)))/(763699200*a^12*b^14 - 663552*a^5*b^21 - 5955584*a^6*b^20 - 31360000*a^7*b^19 - 106229760*a^8*b^18 - 233375744*a^9*b^17 - 293113856*a^10*b^16 - 19971072*a^11*b^15 - 32768*a^4*b^22 + 1804718080*a^13*b^13 + 2475196416*a^14*b^12 + 2343814144*a^15*b^11 + 1598148608*a^16*b^10 + 785971200*a^17*b^9 + 272035840*a^18*b^8 + 62651392*a^19*b^7 + 8552448*a^20*b^6 + 516096*a^21*b^5 + ((-b^5*(a + b)^9)^(1/2)*(tan(e + f*x)*(131072*a^6*b^23 + 2686976*a^7*b^22 + 26214400*a^8*b^21 + 161013760*a^9*b^20 + 695239680*a^10*b^19 + 2234314752*a^11*b^18 + 5525833728*a^12*b^17 + 10739159040*a^13*b^16 + 16625679360*a^14*b^15 + 20693114880*a^15*b^14 + 20844212224*a^16*b^13 + 17084284928*a^17*b^12 + 11452103680*a^18*b^11 + 6309949440*a^19*b^10 + 2866513920*a^20*b^9 + 1069486080*a^21*b^8 + 321586176*a^22*b^7 + 74711040*a^23*b^6 + 12451840*a^24*b^5 + 1310720*a^25*b^4 + 65536*a^26*b^3) - ((-b^5*(a + b)^9)^(1/2)*(36*a*b + 63*a^2 + 8*b^2)*(65536*a^10*b^22 + 1327104*a^11*b^21 + 13631488*a^12*b^20 + 91750400*a^13*b^19 + 443154432*a^14*b^18 + 1607925760*a^15*b^17 + 4509663232*a^16*b^16 + 9971564544*a^17*b^15 + 17627217920*a^18*b^14 + 25149669376*a^19*b^13 + 29127081984*a^20*b^12 + 27445297152*a^21*b^11 + 21016346624*a^22*b^10 + 13016432640*a^23*b^9 + 6461587456*a^24*b^8 + 2533752832*a^25*b^7 + 767361024*a^26*b^6 + 173293568*a^27*b^5 + 27525120*a^28*b^4 + 2752512*a^29*b^3 + 131072*a^30*b^2 - (tan(e + f*x)*(-b^5*(a + b)^9)^(1/2)*(36*a*b + 63*a^2 + 8*b^2)*(524288*a^12*b^23 + 10747904*a^13*b^22 + 104857600*a^14*b^21 + 647495680*a^15*b^20 + 2839019520*a^16*b^19 + 9398648832*a^17*b^18 + 24385683456*a^18*b^17 + 50803507200*a^19*b^16 + 86365962240*a^20*b^15 + 121081692160*a^21*b^14 + 140895059968*a^22*b^13 + 136492089344*a^23*b^12 + 110074265600*a^24*b^11 + 73665085440*a^25*b^10 + 40642805760*a^26*b^9 + 18289262592*a^27*b^8 + 6604455936*a^28*b^7 + 1867776000*a^29*b^6 + 398458880*a^30*b^5 + 60293120*a^31*b^4 + 5767168*a^32*b^3 + 262144*a^33*b^2))/(16*(9*a^11*b + a^12 + a^3*b^9 + 9*a^4*b^8 + 36*a^5*b^7 + 84*a^6*b^6 + 126*a^7*b^5 + 126*a^8*b^4 + 84*a^9*b^3 + 36*a^10*b^2))))/(16*(9*a^11*b + a^12 + a^3*b^9 + 9*a^4*b^8 + 36*a^5*b^7 + 84*a^6*b^6 + 126*a^7*b^5 + 126*a^8*b^4 + 84*a^9*b^3 + 36*a^10*b^2)))*(36*a*b + 63*a^2 + 8*b^2))/(16*(9*a^11*b + a^12 + a^3*b^9 + 9*a^4*b^8 + 36*a^5*b^7 + 84*a^6*b^6 + 126*a^7*b^5 + 126*a^8*b^4 + 84*a^9*b^3 + 36*a^10*b^2)) - ((-b^5*(a + b)^9)^(1/2)*(tan(e + f*x)*(131072*a^6*b^23 + 2686976*a^7*b^22 + 26214400*a^8*b^21 + 161013760*a^9*b^20 + 695239680*a^10*b^19 + 2234314752*a^11*b^18 + 5525833728*a^12*b^17 + 10739159040*a^13*b^16 + 16625679360*a^14*b^15 + 20693114880*a^15*b^14 + 20844212224*a^16*b^13 + 17084284928*a^17* b^12 + 11452103680*a^18*b^11 + 6309949440*a^19*b^10 + 2866513920*a^20*b^9 + 1069486080*a^21*b^8 + 321586176*a^22*b^7 + 74711040*a^23*b^6 + 12451840*a^24*b^5 + 1310720*a^25*b^4 + 65536*a^26*b^3) + ((-b^5*(a + b)^9)^(1/2)*(36*a*b + 63*a^2 + 8*b^2)*(65536*a^10*b^22 + 1327104*a^11*b^21 + 13631488*a^12*b^20 + 91750400*a^13*b^19 + 443154432*a^14*b^18 + 1607925760*a^15*b^17 + 4509663232*a^16*b^16 + 9971564544*a^17*b^15 + 17627217920*a^18*b^14 + 25149669376*a^19*b^13 + 29127081984*a^20*b^12 + 27445297152*a^21*b^11 + 21016346624*a^22*b^10 + 13016432640*a^23*b^9 + 6461587456*a^24*b^8 + 2533752832*a^25*b^7 + 767361024*a^26*b^6 + 173293568*a^27*b^5 + 27525120*a^28*b^4 + 2752512*a^29*b^3 + 131072*a^30*b^2 + (tan(e + f*x)*(-b^5*(a + b)^9)^(1/2)*(36*a*b + 63*a^2 + 8*b^2)*(524288*a^12*b^23 + 10747904*a^13*b^22 + 104857600*a^14*b^21 + 647495680*a^15*b^20 + 2839019520*a^16*b^19 + 9398648832*a^17*b^18 + 24385683456*a^18*b^17 + 50803507200*a^19*b^16 + 86365962240*a^20*b^15 + 121081692160*a^21*b^14 + 140895059968*a^22*b^13 + 136492089344*a^23*b^12 + 110074265600*a^24*b^11 + 73665085440*a^25*b^10 + 40642805760*a^26*b^9 + 18289262592*a^27*b^8 + 6604455936*a^28*b^7 + 1867776000*a^29*b^6 + 398458880*a^30*b^5 + 60293120*a^31*b^4 + 5767168*a^32*b^3 + 262144*a^33*b^2))/(16*(9*a^11*b + a^12 + a^3*b^9 + 9*a^4*b^8 + 36*a^5*b^7 + 84*a^6*b^6 + 126*a^7*b^5 + 126*a^8*b^4 + 84*a^9*b^3 + 36*a^10*b^2))))/(16*(9*a^11*b + a^12 + a^3*b^9 + 9*a^4*b^8 + 36*a^5*b^7 + 84*a^6*b^6 + 126*a^7*b^5 + 126*a^8*b^4 + 84*a^9*b^3 + 36*a^10*b^2)))*(36*a*b + 63*a^2 + 8*b^2))/(16*(9*a^11*b + a^12 + a^3*b^9 + 9*a^4*b^8 + 36*a^5*b^7 + 84*a^6*b^6 + 126*a^7*b^5 + 126*a^8*b^4 + 84*a^9*b^3 + 36*a^10*b^2))))*(-b^5*(a + b)^9)^(1/2)*(36*a*b + 63*a^2 + 8*b^2)*1i)/(8*f*(9*a^11*b + a^12 + a^3*b^9 + 9*a^4*b^8 + 36*a^5*b^7 + 84*a^6*b^6 + 126*a^7*b^5 + 126*a^8*b^4 + 84*a^9*b^3 + 36*a^10*b^2))

3.375 $\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 285

$$\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{x}{a^3}+\frac{b^{7/2}(99a^2+44ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{11/2}f} - \frac{(8a^4+40a^3b+80a^2b^2-19ab^3-4b^4)\cot(e+fx)}{8a^2(a+b)^5f} + \frac{(8a^3+32a^2b-51ab^2-12b^3)\cot^3(e+fx)}{24a^2(a+b)^4f} - \frac{(8a^2-75ab-20b^2)\cot^5(e+fx)}{40a^2(a+b)^3f} - \frac{b\cot^5(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(13a+4b)\cot^5(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

```
[Out] -x/a^3+1/8*b^(7/2)*(99*a^2+44*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1
/2))/a^3/(a+b)^(11/2)/f-1/8*(8*a^4+40*a^3*b+80*a^2*b^2-19*a*b^3-4*b^4)*cot(
f*x+e)/a^2/(a+b)^5/f+1/24*(8*a^3+32*a^2*b-51*a*b^2-12*b^3)*cot(f*x+e)^3/a^2
/(a+b)^4/f-1/40*(8*a^2-75*a*b-20*b^2)*cot(f*x+e)^5/a^2/(a+b)^3/f-1/4*b*cot(
f*x+e)^5/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/8*b*(13*a+4*b)*cot(f*x+e)^5/a^2
/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4226, 2000, 483, 593, 597, 536, 209, 211}

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx=-\frac{x}{a^3}-\frac{(8a^2-75ab-20b^2)\cot^5(e+fx)}{40a^2f(a+b)^3}-\frac{b(13a+4b)\cot^5(e+fx)}{8a^2f(a+b)^2\,(a+b\tan^2(e+fx)+b)}+\frac{b^{7/2}(99a^2+44ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{11/2}}+\frac{(8a^3+32a^2b-51ab^2-12b^3)\cot^3(e+fx)}{24a^2f(a+b)^4}-\frac{(8a^4+40a^3b+80a^2b^2-19ab^3-4b^4)\cot(e+fx)}{8a^2f(a+b)^5}-\frac{b\cot^5(e+fx)}{4af(a+b)\,(a+b\tan^2(e+fx)+b)^2}$$

```
[In] Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -(x/a^3) + (b^(7/2)*(99*a^2 + 44*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])
/Sqrt[a + b]])/(8*a^3*(a + b)^(11/2)*f) - ((8*a^4 + 40*a^3*b + 80*a^2*b^2 -
 19*a*b^3 - 4*b^4)*Cot[e + f*x])/(8*a^2*(a + b)^5*f) + ((8*a^3 + 32*a^2*b -
 51*a*b^2 - 12*b^3)*Cot[e + f*x]^3)/(24*a^2*(a + b)^4*f) - ((8*a^2 - 75*a*b
 - 20*b^2)*Cot[e + f*x]^5)/(40*a^2*(a + b)^3*f) - (b*Cot[e + f*x]^5)/(4*a*(
a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) - (b*(13*a + 4*b)*Cot[e + f*x]^5)/(8
*a^2*(a + b)^2*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
```

```
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
```

```
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+b(1+x^2))^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^5(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a-5b-9bx^2}{x^6(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4a(a+b)f}$$

$$= -\frac{b\cot^5(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(13a+4b)\cot^5(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{8a^2-75ab-20b^2-7b(13a+4b)x^2}{x^6(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{8a^2(a+b)^2f}$$

$$= -\frac{(8a^2-75ab-20b^2)\cot^5(e+fx)}{40a^2(a+b)^3f} - \frac{b\cot^5(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(13a+4b)\cot^5(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{5(8a^3+32a^2b-51ab^2-12b^3)+5b(8a^2-75ab-20b^2)x^2}{x^4(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{40a^2(a+b)^3f}$$

$$= \frac{(8a^3+32a^2b-51ab^2-12b^3)\cot^3(e+fx)}{24a^2(a+b)^4f} - \frac{(8a^2-75ab-20b^2)\cot^5(e+fx)}{40a^2(a+b)^3f} - \frac{b\cot^5(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(13a+4b)\cot^5(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{15(8a^4+40a^3b+80a^2b^2-19ab^3-4b^4)+15b(8a^3+32a^2b-51ab^2-12b^3)x^2}{x^2(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{120a^2(a+b)^4f}$$

$$= -\frac{(8a^4+40a^3b+80a^2b^2-19ab^3-4b^4)\cot(e+fx)}{8a^2(a+b)^5f} + \frac{(8a^3+32a^2b-51ab^2-12b^3)\cot^3(e+fx)}{24a^2(a+b)^4f} - \frac{(8a^2-75ab-20b^2)\cot^5(e+fx)}{40a^2(a+b)^3f} - \frac{b\cot^5(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(13a+4b)\cot^5(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{15(8a^5+48a^4b+120a^3b^2+160a^2b^3+21ab^4+4b^5)+15b(8a^4+40a^3b+80a^2b^2-19ab^3-4b^4)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{120a^2(a+b)^5f}$$

$$= -\frac{(8a^4+40a^3b+80a^2b^2-19ab^3-4b^4)\cot(e+fx)}{8a^2(a+b)^5f} + \frac{(8a^3+32a^2b-51ab^2-12b^3)\cot^3(e+fx)}{24a^2(a+b)^4f} - \frac{(8a^2-75ab-20b^2)\cot^5(e+fx)}{40a^2(a+b)^3f} - \frac{b\cot^5(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(13a+4b)\cot^5(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^3f} + \frac{(b^4(99a^2+44ab+8b^2))\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{8a^3(a+b)^5f}$$

$$= -\frac{x}{a^3} + \frac{b^{7/2}(99a^2+44ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{11/2}f} - \frac{(8a^4+40a^3b+80a^2b^2-19ab^3-4b^4)\cot(e+fx)}{8a^2(a+b)^5f} + \frac{(8a^3+32a^2b-51ab^2-12b^3)\cot^3(e+fx)}{24a^2(a+b)^4f} - \frac{(8a^2-75ab-20b^2)\cot^5(e+fx)}{40a^2(a+b)^3f} - \frac{b\cot^5(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(13a+4b)\cot^5(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 12.57 (sec) , antiderivative size = 976, normalized size of antiderivative = 3.42

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{x(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)}{8a^3\left(a+b\sec^2(e+fx)\right)^3}$$
$$+\frac{(11a\cos(e)+26b\cos(e))(a+2b+a\cos(2e+2fx))^3\csc(e)\csc^2(e+fx)\sec^6(e+fx)}{120(a+b)^4f\left(a+b\sec^2(e+fx)\right)^3}$$
$$-\frac{(a+2b+a\cos(2e+2fx))^3\cot(e)\csc^4(e+fx)\sec^6(e+fx)}{40(a+b)^3f\left(a+b\sec^2(e+fx)\right)^3}$$
$$+\frac{(99a^2+44ab+8b^2)(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(-\frac{b^4\arctan\left(\sec(fx)\left(\frac{\cos(2e)}{2\sqrt{a+b}\sqrt{b\cos(4e)-ib\sin(4e)}}-\frac{}{2\sqrt{a+}}\right.\right.}{64a^3\sqrt{a}}\right.}{}$$
$$+\frac{(a+2b+a\cos(2e+2fx))^3\csc(e)\csc^5(e+fx)\sec^6(e+fx)\sin(fx)}{40(a+b)^3f\left(a+b\sec^2(e+fx)\right)^3}$$
$$+\frac{(a+2b+a\cos(2e+2fx))^3\csc(e)\csc^3(e+fx)\sec^6(e+fx)(-11a\sin(fx)-26b\sin(fx))}{120(a+b)^4f\left(a+b\sec^2(e+fx)\right)^3}$$
$$+\frac{(a+2b+a\cos(2e+2fx))^3\csc(e)\csc(e+fx)\sec^6(e+fx)\left(23a^2\sin(fx)+106ab\sin(fx)+173b^2\sin(f\right.}{120(a+b)^5f\left(a+b\sec^2(e+fx)\right)^3}$$
$$+\frac{(a+2b+a\cos(2e+2fx))\sec(2e)\sec^6(e+fx)\left(ab^5\sin(2e)+2b^6\sin(2e)-ab^5\sin(2fx)\right)}{16a^3(a+b)^4f\left(a+b\sec^2(e+fx)\right)^3}$$
$$+\frac{(a+2b+a\cos(2e+2fx))^2\sec(2e)\sec^6(e+fx)\left(-21a^2b^4\sin(2e)-52ab^5\sin(2e)-16b^6\sin(2e)+21a^2b\right.}{64a^3(a+b)^5f\left(a+b\sec^2(e+fx)\right)^3}$$

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -1/8*(x*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6)/(a^3*(a + b*Sec[e
+ f*x]^2)^3) + ((11*a*Cos[e] + 26*b*Cos[e])*(a + 2*b + a*Cos[2*e + 2*f*x])^
3*Csc[e]*Csc[e + f*x]^2*Sec[e + f*x]^6)/(120*(a + b)^4*f*(a + b*Sec[e + f*x
]^2)^3) - ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Cot[e]*Csc[e + f*x]^4*Sec[e + f
*x]^6)/(40*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^3) + ((99*a^2 + 44*a*b + 8*b^
2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(-1/64*(b^4*ArcTan[Sec[f
*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[
2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*S
in[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(a^3*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] -
 I*b*Sin[4*e]]) + ((I/64)*b^4*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt
[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e
] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Sin[
2*e])/(a^3*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]])))/((a + b)^5*(a +
 b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Csc[e]*Csc[e + f*
x]^5*Sec[e + f*x]^6*Sin[f*x])/(40*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^3) + (
(a + 2*b + a*Cos[2*e + 2*f*x])^3*Csc[e]*Csc[e + f*x]^3*Sec[e + f*x]^6*(-11*
a*Sin[f*x] - 26*b*Sin[f*x]))/(120*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^3) + (
(a + 2*b + a*Cos[2*e + 2*f*x])^3*Csc[e]*Csc[e + f*x]*Sec[e + f*x]^6*(23*a^2
```

```
*Sin[f*x] + 106*a*b*Sin[f*x] + 173*b^2*Sin[f*x]))/(120*(a + b)^5*f*(a + b*S
ec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])*Sec[2*e]*Sec[e + f*x]^6
*(a*b^5*Sin[2*e] + 2*b^6*Sin[2*e] - a*b^5*Sin[2*f*x]))/(16*a^3*(a + b)^4*f*
(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[2*e]*Sec[
e + f*x]^6*(-21*a^2*b^4*Sin[2*e] - 52*a*b^5*Sin[2*e] - 16*b^6*Sin[2*e] + 21
*a^2*b^4*Sin[2*f*x] + 6*a*b^5*Sin[2*f*x]))/(64*a^3*(a + b)^5*f*(a + b*Sec[e
 + f*x]^2)^3)
```

Maple [A] (verified)

Time = 69.09 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.70

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{a^3}+\frac{b^4\left(\frac{\left(\frac{19}{8}a^2b+\frac{1}{2}a\,b^2\right)\tan(fx+e)^3+\frac{a\left(21a^2+25ab+4b^2\right)\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(99a^2+44ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right.}{8\sqrt{(a+b)b}}\right.}{a^3(a+b)^5}}{f}$
default	$\frac{-\frac{\arctan(\tan(fx+e))}{a^3}+\frac{b^4\left(\frac{\left(\frac{19}{8}a^2b+\frac{1}{2}a\,b^2\right)\tan(fx+e)^3+\frac{a\left(21a^2+25ab+4b^2\right)\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(99a^2+44ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right.}{8\sqrt{(a+b)b}}\right.}{a^3(a+b)^5}}{f}$
risch	Expression too large to display

```
[In] int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^3*arctan(tan(f*x+e))+b^4/a^3/(a+b)^5*(((19/8*a^2*b+1/2*a*b^2)*tan
(f*x+e)^3+1/8*a*(21*a^2+25*a*b+4*b^2)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/
8*(99*a^2+44*a*b+8*b^2)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)
))-1/5/(a+b)^3/tan(f*x+e)^5-1/3*(-a-4*b)/(a+b)^4/tan(f*x+e)^3-(a^2+5*a*b+10
*b^2)/(a+b)^5/tan(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. 2(265) = 530.

Time = 0.42 (sec) , antiderivative size = 2229, normalized size of antiderivative = 7.82

$$\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx=\text{Too large to display}$$

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/480*(4*(184*a^7 + 848*a^6*b + 1384*a^5*b^2 + 315*a^3*b^4 + 90*a^2*b^5)*
cos(f*x + e)^9 - 4*(280*a^7 + 1032*a^6*b + 864*a^5*b^2 - 2768*a^4*b^3 + 945
*a^3*b^4 - 15*a^2*b^5 - 60*a*b^6)*cos(f*x + e)^7 + 4*(120*a^7 + 40*a^6*b -
1416*a^5*b^2 - 4272*a^4*b^3 + 2329*a^3*b^4 - 585*a^2*b^5 - 180*a*b^6)*cos(f
```

x + e)^5 + 20(48*a^6*b + 184*a^5*b^2 + 200*a^4*b^3 - 575*a^3*b^4 + 153*a^
2*b^5 + 36*a*b^6)*cos(f*x + e)^3 - 15*((99*a^4*b^3 + 44*a^3*b^4 + 8*a^2*b^5
)*cos(f*x + e)^8 + 99*a^2*b^5 + 44*a*b^6 + 8*b^7 - 2*(99*a^4*b^3 - 55*a^3*b
^4 - 36*a^2*b^5 - 8*a*b^6)*cos(f*x + e)^6 + (99*a^4*b^3 - 352*a^3*b^4 - 69*
a^2*b^5 + 12*a*b^6 + 8*b^7)*cos(f*x + e)^4 + 2*(99*a^3*b^4 - 55*a^2*b^5 - 3
6*a*b^6 - 8*b^7)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2
)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b
^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e
) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) +
60*(8*a^5*b^2 + 40*a^4*b^3 + 80*a^3*b^4 - 19*a^2*b^5 - 4*a*b^6)*cos(f*x + e
) + 480*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*
x*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a
*b^6)*f*x*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b
^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*x*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5
*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*x*cos(f*x + e)^2 + (a^5*b^2 + 5*a^4
*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f*x)*sin(f*x + e))/(((a^10
+ 5*a^9*b + 10*a^8*b^2 + 10*a^7*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8
 - 2*(a^10 + 4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6)*f*cos(f
*x + e)^6 + (a^10 + a^9*b - 9*a^8*b^2 - 25*a^7*b^3 - 25*a^6*b^4 - 9*a^5*b^5
 + a^4*b^6 + a^3*b^7)*f*cos(f*x + e)^4 + 2*(a^9*b + 4*a^8*b^2 + 5*a^7*b^3 -
 5*a^5*b^5 - 4*a^4*b^6 - a^3*b^7)*f*cos(f*x + e)^2 + (a^8*b^2 + 5*a^7*b^3 +
 10*a^6*b^4 + 10*a^5*b^5 + 5*a^4*b^6 + a^3*b^7)*f)*sin(f*x + e)), -1/240*(2
*(184*a^7 + 848*a^6*b + 1384*a^5*b^2 + 315*a^3*b^4 + 90*a^2*b^5)*cos(f*x +
e)^9 - 2*(280*a^7 + 1032*a^6*b + 864*a^5*b^2 - 2768*a^4*b^3 + 945*a^3*b^4 -
 15*a^2*b^5 - 60*a*b^6)*cos(f*x + e)^7 + 2*(120*a^7 + 40*a^6*b - 1416*a^5*b
^2 - 4272*a^4*b^3 + 2329*a^3*b^4 - 585*a^2*b^5 - 180*a*b^6)*cos(f*x + e)^5
+ 10*(48*a^6*b + 184*a^5*b^2 + 200*a^4*b^3 - 575*a^3*b^4 + 153*a^2*b^5 + 36
*a*b^6)*cos(f*x + e)^3 + 15*((99*a^4*b^3 + 44*a^3*b^4 + 8*a^2*b^5)*cos(f*x
+ e)^8 + 99*a^2*b^5 + 44*a*b^6 + 8*b^7 - 2*(99*a^4*b^3 - 55*a^3*b^4 - 36*a^
2*b^5 - 8*a*b^6)*cos(f*x + e)^6 + (99*a^4*b^3 - 352*a^3*b^4 - 69*a^2*b^5 +
12*a*b^6 + 8*b^7)*cos(f*x + e)^4 + 2*(99*a^3*b^4 - 55*a^2*b^5 - 36*a*b^6 -
8*b^7)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2
 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 30*(8*a
^5*b^2 + 40*a^4*b^3 + 80*a^3*b^4 - 19*a^2*b^5 - 4*a*b^6)*cos(f*x + e) + 240
*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*x*cos(f
x + e)^8 - 2(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f
*x*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*
a^2*b^5 + a*b^6 + b^7)*f*x*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^
3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*x*cos(f*x + e)^2 + (a^5*b^2 + 5*a^4*b^3 +
10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f*x)*sin(f*x + e))/(((a^10 + 5*a^9
*b + 10*a^8*b^2 + 10*a^7*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8 - 2*(a
^10 + 4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6)*f*cos(f*x + e)
^6 + (a^10 + a^9*b - 9*a^8*b^2 - 25*a^7*b^3 - 25*a^6*b^4 - 9*a^5*b^5 + a^4*
b^6 + a^3*b^7)*f*cos(f*x + e)^4 + 2*(a^9*b + 4*a^8*b^2 + 5*a^7*b^3 - 5*a^5*
b^5 - 4*a^4*b^6 - a^3*b^7)*f*cos(f*x + e)^2 + (a^8*b^2 + 5*a^7*b^3 + 10*a^6

```
*b^4 + 10*a^5*b^5 + 5*a^4*b^6 + a^3*b^7)*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.82

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{15\,(99\,a^2b^4+44\,ab^5+8\,b^6)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^8+5\,a^7b+10\,a^6b^2+10\,a^5b^3+5\,a^4b^4+a^3b^5)\sqrt{(a+b)b}} - \frac{15\,(8\,a^4b^2+40\,a^3b^3+80\,a^2b^4-19\,ab^5-4\,b^6)\tan(fx+e)^8+5\,(48\,a^5b+280\,a^4b^2+680\,a^3b^3}{(a^7b^2+5\,a^6b^3+10\,a^5b^4+10\,a^4b^5+5\,a^3b^6+a^2b^7}}{}$$

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
[Out] 1/120*(15*(99*a^2*b^4 + 44*a*b^5 + 8*b^6)*arctan(b*tan(f*x + e)/sqrt((a + b
)*b))/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*sqrt
((a + b)*b)) - (15*(8*a^4*b^2 + 40*a^3*b^3 + 80*a^2*b^4 - 19*a*b^5 - 4*b^6)
*tan(f*x + e)^8 + 5*(48*a^5*b + 280*a^4*b^2 + 680*a^3*b^3 + 385*a^2*b^4 - 7
5*a*b^5 - 12*b^6)*tan(f*x + e)^6 + 24*a^6 + 96*a^5*b + 144*a^4*b^2 + 96*a^3
*b^3 + 24*a^2*b^4 + 8*(15*a^6 + 95*a^5*b + 258*a^4*b^2 + 291*a^3*b^3 + 113*
a^2*b^4)*tan(f*x + e)^4 - 8*(5*a^6 + 29*a^5*b + 57*a^4*b^2 + 47*a^3*b^3 + 1
4*a^2*b^4)*tan(f*x + e)^2)/((a^7*b^2 + 5*a^6*b^3 + 10*a^5*b^4 + 10*a^4*b^5
+ 5*a^3*b^6 + a^2*b^7)*tan(f*x + e)^9 + 2*(a^8*b + 6*a^7*b^2 + 15*a^6*b^3 +
 20*a^5*b^4 + 15*a^4*b^5 + 6*a^3*b^6 + a^2*b^7)*tan(f*x + e)^7 + (a^9 + 7*a
^8*b + 21*a^7*b^2 + 35*a^6*b^3 + 35*a^5*b^4 + 21*a^4*b^5 + 7*a^3*b^6 + a^2*
b^7)*tan(f*x + e)^5) - 120*(f*x + e)/a^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.36

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{\frac{15\,(99\,a^2b^4+44\,ab^5+8\,b^6)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^8+5\,a^7b+10\,a^6b^2+10\,a^5b^3+5\,a^4b^4+a^3b^5)\sqrt{ab+b^2}}+\frac{15\left(19\,ab^5\tan(fx+e)^3+4\,b^6\tan(fx+e)^3+21\,a^2b^4\tan(fx+e)+25\,ab^5\,\mathrm{t}\cdots\right.}{(a^7+5\,a^6b+10\,a^5b^2+10\,a^4b^3+5\,a^3b^4+a^2b^5)\left(b\tan(fx\cdots\right.}}{}$$

[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")

```
[Out] 1/120*(15*(99*a^2*b^4 + 44*a*b^5 + 8*b^6)*(pi*floor((f*x + e)/pi + 1/2)*sgn
(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^8 + 5*a^7*b + 10*a^6*b^2
+ 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*sqrt(a*b + b^2)) + 15*(19*a*b^5*tan(f*x
 + e)^3 + 4*b^6*tan(f*x + e)^3 + 21*a^2*b^4*tan(f*x + e) + 25*a*b^5*tan(f*x
 + e) + 4*b^6*tan(f*x + e))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a
^3*b^4 + a^2*b^5)*(b*tan(f*x + e)^2 + a + b)^2) - 120*(f*x + e)/a^3 - 8*(15
*a^2*tan(f*x + e)^4 + 75*a*b*tan(f*x + e)^4 + 150*b^2*tan(f*x + e)^4 - 5*a^
2*tan(f*x + e)^2 - 25*a*b*tan(f*x + e)^2 - 20*b^2*tan(f*x + e)^2 + 3*a^2 +
6*a*b + 3*b^2)/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*t
an(f*x + e)^5))/f
```

Mupad [B] (verification not implemented)

Time = 27.97 (sec) , antiderivative size = 7460, normalized size of antiderivative = 26.18

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

[In] int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^3,x)

```
[Out] atan(((((65536*a^10*b^27 + 1654784*a^11*b^26 + 21954560*a^12*b^25 + 1944780
80*a^13*b^24 + 1247936512*a^14*b^23 + 6060916736*a^15*b^22 + 22968795136*a^
16*b^21 + 69506170880*a^17*b^20 + 170976215040*a^18*b^19 + 346596343808*a^1
9*b^18 + 585044721664*a^20*b^17 + 828584034304*a^21*b^16 + 989821665280*a^2
2*b^15 + 1000564490240*a^23*b^14 + 856970493952*a^24*b^13 + 621538574336*a^
25*b^12 + 380751118336*a^26*b^11 + 196065116160*a^27*b^10 + 84230471680*a^2
8*b^9 + 29853974528*a^29*b^8 + 8588754944*a^30*b^7 + 1957904384*a^31*b^6 +
340787200*a^32*b^5 + 42598400*a^33*b^4 + 3407872*a^34*b^3 + 131072*a^35*b^2
 + (tan(e + f*x)*(524288*a^12*b^28 + 13369344*a^13*b^27 + 163840000*a^14*b^
26 + 1284505600*a^15*b^25 + 7235174400*a^16*b^24 + 31171543040*a^17*b^23 +
106779115520*a^18*b^22 + 298450944000*a^19*b^21 + 693069414400*a^20*b^20 +
```

1354635673600*a^21*b^19 + 2249325281280*a^22*b^18 + 3193847152640*a^23*b^17 + 3894935552000*a^24*b^16 + 4089682329600*a^25*b^15 + 3700188774400*a^26*b^14 + 2882252308480*a^27*b^13 + 1927993098240*a^28*b^12 + 1102610432000*a^29*b^11 + 535553638400*a^30*b^10 + 218864025600*a^31*b^9 + 74281123840*a^32*b^8 + 20559953920*a^33*b^7 + 4521984000*a^34*b^6 + 760217600*a^35*b^5 + 91750400*a^36*b^4 + 7077888*a^37*b^3 + 262144*a^38*b^2)*1i)/(2*a^3))*1i)/(2*a^3) + tan(e + f*x)*(131072*a^6*b^28 + 3342336*a^7*b^27 + 40960000*a^8*b^26 + 319234048*a^9*b^25 + 1768817664*a^10*b^24 + 7390051328*a^11*b^23 + 24132297728*a^12*b^22 + 63100984320*a^13*b^21 + 134472684544*a^14*b^20 + 236839037952*a^15*b^19 + 348859675648*a^16*b^18 + 434405198848*a^17*b^17 + 461878103040*a^18*b^16 + 423083193344*a^19*b^15 + 336152947712*a^20*b^14 + 232369345536*a^21*b^13 + 139446393856*a^22*b^12 + 72073323520*a^23*b^11 + 31662619648*a^24*b^10 + 11616461824*a^25*b^9 + 3481927680*a^26*b^8 + 829030400*a^27*b^7 + 150732800*a^28*b^6 + 19660800*a^29*b^5 + 1638400*a^30*b^4 + 65536*a^31*b^3))/(2*a^3) - (((65536*a^10*b^27 + 1654784*a^11*b^26 + 21954560*a^12*b^25 + 194478080*a^13*b^24 + 1247936512*a^14*b^23 + 6060916736*a^15*b^22 + 22968795136*a^16*b^21 + 69506170880*a^17*b^20 + 170976215040*a^18*b^19 + 346596343808*a^19*b^18 + 585044721664*a^20*b^17 + 828584034304*a^21*b^16 + 989821665280*a^22*b^15 + 1000564490240*a^23*b^14 + 856970493952*a^24*b^13 + 621538574336*a^25*b^12 + 380751118336*a^26*b^11 + 196065116160*a^27*b^10 + 84230471680*a^28*b^9 + 29853974528*a^29*b^8 + 8588754944*a^30*b^7 + 1957904384*a^31*b^6 + 340787200*a^32*b^5 + 42598400*a^33*b^4 + 3407872*a^34*b^3 + 131072*a^35*b^2 - (tan(e + f*x)*(524288*a^12*b^28 + 13369344*a^13*b^27 + 163840000*a^14*b^26 + 1284505600*a^15*b^25 + 7235174400*a^16*b^24 + 31171543040*a^17*b^23 + 106779115520*a^18*b^22 + 298450944000*a^19*b^21 + 693069414400*a^20*b^20 + 1354635673600*a^21*b^19 + 2249325281280*a^22*b^18 + 3193847152640*a^23*b^17 + 3894935552000*a^24*b^16 + 4089682329600*a^25*b^15 + 3700188774400*a^26*b^14 + 2882252308480*a^27*b^13 + 1927993098240*a^28*b^12 + 1102610432000*a^29*b^11 + 535553638400*a^30*b^10 + 218864025600*a^31*b^9 + 74281123840*a^32*b^8 + 20559953920*a^33*b^7 + 4521984000*a^34*b^6 + 760217600*a^35*b^5 + 91750400*a^36*b^4 + 7077888*a^37*b^3 + 262144*a^38*b^2)*1i)/(2*a^3))*1i)/(2*a^3) - tan(e + f*x)*(131072*a^6*b^28 + 3342336*a^7*b^27 + 40960000*a^8*b^26 + 319234048*a^9*b^25 + 1768817664*a^10*b^24 + 7390051328*a^11*b^23 + 24132297728*a^12*b^22 + 63100984320*a^13*b^21 + 134472684544*a^14*b^20 + 236839037952*a^15*b^19 + 348859675648*a^16*b^18 + 434405198848*a^17*b^17 + 461878103040*a^18*b^16 + 423083193344*a^19*b^15 + 336152947712*a^20*b^14 + 232369345536*a^21*b^13 + 139446393856*a^22*b^12 + 72073323520*a^23*b^11 + 31662619648*a^24*b^10 + 11616461824*a^25*b^9 + 3481927680*a^26*b^8 + 829030400*a^27*b^7 + 150732800*a^28*b^6 + 19660800*a^29*b^5 + 1638400*a^30*b^4 + 65536*a^31*b^3))/(2*a^3))/(27354112*a^10*b^21 - ((((65536*a^10*b^27 + 1654784*a^11*b^26 + 21954560*a^12*b^25 + 194478080*a^13*b^24 + 1247936512*a^14*b^23 + 6060916736*a^15*b^22 + 22968795136*a^16*b^21 + 69506170880*a^17*b^20 + 170976215040*a^18*b^19 + 346596343808*a^19*b^18 + 585044721664*a^20*b^17 + 828584034304*a^21*b^16 + 989821665280*a^22*b^15 + 1000564490240*a^23*b^14 + 856970493952*a^24*b^13 + 621538574336*a^25*b^12 + 380751118336*a^26*b^1

1 + 196065116160*a^27*b^10 + 84230471680*a^28*b^9 + 29853974528*a^29*b^8 + 8588754944*a^30*b^7 + 1957904384*a^31*b^6 + 340787200*a^32*b^5 + 42598400*a^33*b^4 + 3407872*a^34*b^3 + 131072*a^35*b^2 - (tan(e + f*x)*(524288*a^12*b^28 + 13369344*a^13*b^27 + 163840000*a^14*b^26 + 1284505600*a^15*b^25 + 7235174400*a^16*b^24 + 31171543040*a^17*b^23 + 106779115520*a^18*b^22 + 298450944000*a^19*b^21 + 693069414400*a^20*b^20 + 1354635673600*a^21*b^19 + 2249325281280*a^22*b^18 + 3193847152640*a^23*b^17 + 3894935552000*a^24*b^16 + 4089682329600*a^25*b^15 + 3700188774400*a^26*b^14 + 2882252308480*a^27*b^13 + 1927993098240*a^28*b^12 + 1102610432000*a^29*b^11 + 535553638400*a^30*b^10 + 218864025600*a^31*b^9 + 74281123840*a^32*b^8 + 20559953920*a^33*b^7 + 4521984000*a^34*b^6 + 760217600*a^35*b^5 + 91750400*a^36*b^4 + 7077888*a^37*b^3 + 262144*a^38*b^2)*1i)/(2*a^3))*1i)/(2*a^3) - tan(e + f*x)*(131072*a^6*b^28 + 3342336*a^7*b^27 + 40960000*a^8*b^26 + 319234048*a^9*b^25 + 1768817664*a^10*b^24 + 7390051328*a^11*b^23 + 24132297728*a^12*b^22 + 63100984320*a^13*b^21 + 134472684544*a^14*b^20 + 236839037952*a^15*b^19 + 348859675648*a^16*b^18 + 434405198848*a^17*b^17 + 461878103040*a^18*b^16 + 423083193344*a^19*b^15 + 336152947712*a^20*b^14 + 232369345536*a^21*b^13 + 139446393856*a^22*b^12 + 72073323520*a^23*b^11 + 31662619648*a^24*b^10 + 11616461824*a^25*b^9 + 3481927680*a^26*b^8 + 829030400*a^27*b^7 + 150732800*a^28*b^6 + 19660800*a^29*b^5 + 1638400*a^30*b^4 + 65536*a^31*b^3))*1i)/(2*a^3) - 32768*a^4*b^27 - 827392*a^5*b^26 - 9084928*a^6*b^25 - 57263104*a^7*b^24 - 221133824*a^8*b^23 - 467977216*a^9*b^22 - ((((65536*a^10*b^27 + 1654784*a^11*b^26 + 21954560*a^12*b^25 + 194478080*a^13*b^24 + 1247936512*a^14*b^23 + 6060916736*a^15*b^22 + 22968795136*a^16*b^21 + 69506170880*a^17*b^20 + 170976215040*a^18*b^19 + 346596343808*a^19*b^18 + 585044721664*a^20*b^17 + 828584034304*a^21*b^16 + 989821665280*a^22*b^15 + 1000564490240*a^23*b^14 + 856970493952*a^24*b^13 + 621538574336*a^25*b^12 + 380751118336*a^26*b^11 + 196065116160*a^27*b^10 + 84230471680*a^28*b^9 + 29853974528*a^29*b^8 + 8588754944*a^30*b^7 + 1957904384*a^31*b^6 + 340787200*a^32*b^5 + 42598400*a^33*b^4 + 3407872*a^34*b^3 + 131072*a^35*b^2 + (tan(e + f*x)*(524288*a^12*b^28 + 13369344*a^13*b^27 + 163840000*a^14*b^26 + 1284505600*a^15*b^25 + 7235174400*a^16*b^24 + 31171543040*a^17*b^23 + 106779115520*a^18*b^22 + 298450944000*a^19*b^21 + 693069414400*a^20*b^20 + 1354635673600*a^21*b^19 + 2249325281280*a^22*b^18 + 3193847152640*a^23*b^17 + 3894935552000*a^24*b^16 + 4089682329600*a^25*b^15 + 3700188774400*a^26*b^14 + 2882252308480*a^27*b^13 + 1927993098240*a^28*b^12 + 1102610432000*a^29*b^11 + 535553638400*a^30*b^10 + 218864025600*a^31*b^9 + 74281123840*a^32*b^8 + 20559953920*a^33*b^7 + 4521984000*a^34*b^6 + 760217600*a^35*b^5 + 91750400*a^36*b^4 + 7077888*a^37*b^3 + 262144*a^38*b^2)*1i)/(2*a^3))*1i)/(2*a^3) + tan(e + f*x)*(131072*a^6*b^28 + 3342336*a^7*b^27 + 40960000*a^8*b^26 + 319234048*a^9*b^25 + 1768817664*a^10*b^24 + 7390051328*a^11*b^23 + 24132297728*a^12*b^22 + 63100984320*a^13*b^21 + 134472684544*a^14*b^20 + 236839037952*a^15*b^19 + 348859675648*a^16*b^18 + 434405198848*a^17*b^17 + 461878103040*a^18*b^16 + 423083193344*a^19*b^15 + 336152947712*a^20*b^14 + 232369345536*a^21*b^13 + 139446393856*a^22*b^12 + 72073323520*a^23*b^11 + 31662619648*a^24*b^10 + 11616461824*a^25*b^9 + 3481927680*a

^26*b^8 + 829030400*a^27*b^7 + 150732800*a^28*b^6 + 19660800*a^29*b^5 + 163
8400*a^30*b^4 + 65536*a^31*b^3))*1i)/(2*a^3) + 4041583616*a^11*b^20 + 16331
772928*a^12*b^19 + 40173472768*a^13*b^18 + 71534228480*a^14*b^17 + 97563767
808*a^15*b^16 + 104426556416*a^16*b^15 + 88612000768*a^17*b^14 + 5970848460
8*a^18*b^13 + 31782593536*a^19*b^12 + 13203725312*a^20*b^11 + 4193231872*a^
21*b^10 + 984308736*a^22*b^9 + 161366016*a^23*b^8 + 16580608*a^24*b^7 + 811
008*a^25*b^6))/(a^3*f) - (1/(5*(a + b)) + (tan(e + f*x)^4*(65*a*b + 15*a^2
+ 113*b^2))/(15*(a + b)^3) - (tan(e + f*x)^2*(5*a + 14*b))/(15*(a + b)^2) +
 (tan(e + f*x)^8*(80*a^2*b^4 - 4*b^6 - 19*a*b^5 + 40*a^3*b^3 + 8*a^4*b^2))/
(8*a^2*(a + b)^5) + (tan(e + f*x)^6*(48*a^4*b - 63*a*b^4 - 12*b^5 + 448*a^2
*b^3 + 232*a^3*b^2))/(24*a^2*(a + b)^4))/(f*(tan(e + f*x)^5*(2*a*b + a^2 +
b^2) + tan(e + f*x)^7*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^9)) + (atan((((tan
(e + f*x)*(131072*a^6*b^28 + 3342336*a^7*b^27 + 40960000*a^8*b^26 + 3192340
48*a^9*b^25 + 1768817664*a^10*b^24 + 7390051328*a^11*b^23 + 24132297728*a^1
2*b^22 + 63100984320*a^13*b^21 + 134472684544*a^14*b^20 + 236839037952*a^15
*b^19 + 348859675648*a^16*b^18 + 434405198848*a^17*b^17 + 461878103040*a^18
*b^16 + 423083193344*a^19*b^15 + 336152947712*a^20*b^14 + 232369345536*a^21
*b^13 + 139446393856*a^22*b^12 + 72073323520*a^23*b^11 + 31662619648*a^24*b
^10 + 11616461824*a^25*b^9 + 3481927680*a^26*b^8 + 829030400*a^27*b^7 + 150
732800*a^28*b^6 + 19660800*a^29*b^5 + 1638400*a^30*b^4 + 65536*a^31*b^3) -
((-b^7*(a + b)^11)^(1/2)*(44*a*b + 99*a^2 + 8*b^2)*(65536*a^10*b^27 + 16547
84*a^11*b^26 + 21954560*a^12*b^25 + 194478080*a^13*b^24 + 1247936512*a^14*b
^23 + 6060916736*a^15*b^22 + 22968795136*a^16*b^21 + 69506170880*a^17*b^20
+ 170976215040*a^18*b^19 + 346596343808*a^19*b^18 + 585044721664*a^20*b^17
+ 828584034304*a^21*b^16 + 989821665280*a^22*b^15 + 1000564490240*a^23*b^14
 + 856970493952*a^24*b^13 + 621538574336*a^25*b^12 + 380751118336*a^26*b^11
 + 196065116160*a^27*b^10 + 84230471680*a^28*b^9 + 29853974528*a^29*b^8 + 8
588754944*a^30*b^7 + 1957904384*a^31*b^6 + 340787200*a^32*b^5 + 42598400*a^
33*b^4 + 3407872*a^34*b^3 + 131072*a^35*b^2 - (tan(e + f*x)*(-b^7*(a + b)^1
1)^(1/2)*(44*a*b + 99*a^2 + 8*b^2)*(524288*a^12*b^28 + 13369344*a^13*b^27 +
 163840000*a^14*b^26 + 1284505600*a^15*b^25 + 7235174400*a^16*b^24 + 311715
43040*a^17*b^23 + 106779115520*a^18*b^22 + 298450944000*a^19*b^21 + 6930694
14400*a^20*b^20 + 1354635673600*a^21*b^19 + 2249325281280*a^22*b^18 + 31938
47152640*a^23*b^17 + 3894935552000*a^24*b^16 + 4089682329600*a^25*b^15 + 37
00188774400*a^26*b^14 + 2882252308480*a^27*b^13 + 1927993098240*a^28*b^12 +
 1102610432000*a^29*b^11 + 535553638400*a^30*b^10 + 218864025600*a^31*b^9 +
 74281123840*a^32*b^8 + 20559953920*a^33*b^7 + 4521984000*a^34*b^6 + 760217
600*a^35*b^5 + 91750400*a^36*b^4 + 7077888*a^37*b^3 + 262144*a^38*b^2))/(16
*(11*a^13*b + a^14 + a^3*b^11 + 11*a^4*b^10 + 55*a^5*b^9 + 165*a^6*b^8 + 33
0*a^7*b^7 + 462*a^8*b^6 + 462*a^9*b^5 + 330*a^10*b^4 + 165*a^11*b^3 + 55*a^
12*b^2))))/(16*(11*a^13*b + a^14 + a^3*b^11 + 11*a^4*b^10 + 55*a^5*b^9 + 16
5*a^6*b^8 + 330*a^7*b^7 + 462*a^8*b^6 + 462*a^9*b^5 + 330*a^10*b^4 + 165*a^
11*b^3 + 55*a^12*b^2)))*(-b^7*(a + b)^11)^(1/2)*(44*a*b + 99*a^2 + 8*b^2)*1
i)/(16*(11*a^13*b + a^14 + a^3*b^11 + 11*a^4*b^10 + 55*a^5*b^9 + 165*a^6*b^
8 + 330*a^7*b^7 + 462*a^8*b^6 + 462*a^9*b^5 + 330*a^10*b^4 + 165*a^11*b^3 +

55*a^12*b^2)) + ((tan(e + f*x)*(131072*a^6*b^28 + 3342336*a^7*b^27 + 40960
000*a^8*b^26 + 319234048*a^9*b^25 + 1768817664*a^10*b^24 + 7390051328*a^11*
b^23 + 24132297728*a^12*b^22 + 63100984320*a^13*b^21 + 134472684544*a^14*b^
20 + 236839037952*a^15*b^19 + 348859675648*a^16*b^18 + 434405198848*a^17*b^
17 + 461878103040*a^18*b^16 + 423083193344*a^19*b^15 + 336152947712*a^20*b^
14 + 232369345536*a^21*b^13 + 139446393856*a^22*b^12 + 72073323520*a^23*b^1
1 + 31662619648*a^24*b^10 + 11616461824*a^25*b^9 + 3481927680*a^26*b^8 + 82
9030400*a^27*b^7 + 150732800*a^28*b^6 + 19660800*a^29*b^5 + 1638400*a^30*b^
4 + 65536*a^31*b^3) + ((-b^7*(a + b)^11)^(1/2)*(44*a*b + 99*a^2 + 8*b^2)*(6
5536*a^10*b^27 + 1654784*a^11*b^26 + 21954560*a^12*b^25 + 194478080*a^13*b^
24 + 1247936512*a^14*b^23 + 6060916736*a^15*b^22 + 22968795136*a^16*b^21 +
69506170880*a^17*b^20 + 170976215040*a^18*b^19 + 346596343808*a^19*b^18 + 5
85044721664*a^20*b^17 + 828584034304*a^21*b^16 + 989821665280*a^22*b^15 + 1
000564490240*a^23*b^14 + 856970493952*a^24*b^13 + 621538574336*a^25*b^12 +
380751118336*a^26*b^11 + 196065116160*a^27*b^10 + 84230471680*a^28*b^9 + 29
853974528*a^29*b^8 + 8588754944*a^30*b^7 + 1957904384*a^31*b^6 + 340787200*
a^32*b^5 + 42598400*a^33*b^4 + 3407872*a^34*b^3 + 131072*a^35*b^2 + (tan(e
+ f*x)*(-b^7*(a + b)^11)^(1/2)*(44*a*b + 99*a^2 + 8*b^2)*(524288*a^12*b^28
+ 13369344*a^13*b^27 + 163840000*a^14*b^26 + 1284505600*a^15*b^25 + 7235174
400*a^16*b^24 + 31171543040*a^17*b^23 + 106779115520*a^18*b^22 + 2984509440
00*a^19*b^21 + 693069414400*a^20*b^20 + 1354635673600*a^21*b^19 + 224932528
1280*a^22*b^18 + 3193847152640*a^23*b^17 + 3894935552000*a^24*b^16 + 408968
2329600*a^25*b^15 + 3700188774400*a^26*b^14 + 2882252308480*a^27*b^13 + 192
7993098240*a^28*b^12 + 1102610432000*a^29*b^11 + 535553638400*a^30*b^10 + 2
18864025600*a^31*b^9 + 74281123840*a^32*b^8 + 20559953920*a^33*b^7 + 452198
4000*a^34*b^6 + 760217600*a^35*b^5 + 91750400*a^36*b^4 + 7077888*a^37*b^3 +
262144*a^38*b^2))/(16*(11*a^13*b + a^14 + a^3*b^11 + 11*a^4*b^10 + 55*a^5*
b^9 + 165*a^6*b^8 + 330*a^7*b^7 + 462*a^8*b^6 + 462*a^9*b^5 + 330*a^10*b^4
+ 165*a^11*b^3 + 55*a^12*b^2))))/(16*(11*a^13*b + a^14 + a^3*b^11 + 11*a^4*
b^10 + 55*a^5*b^9 + 165*a^6*b^8 + 330*a^7*b^7 + 462*a^8*b^6 + 462*a^9*b^5 +
330*a^10*b^4 + 165*a^11*b^3 + 55*a^12*b^2)))*(-b^7*(a + b)^11)^(1/2)*(44*a
*b + 99*a^2 + 8*b^2)*1i)/(16*(11*a^13*b + a^14 + a^3*b^11 + 11*a^4*b^10 + 5
5*a^5*b^9 + 165*a^6*b^8 + 330*a^7*b^7 + 462*a^8*b^6 + 462*a^9*b^5 + 330*a^1
0*b^4 + 165*a^11*b^3 + 55*a^12*b^2)))/(27354112*a^10*b^21 - 827392*a^5*b^26
- 9084928*a^6*b^25 - 57263104*a^7*b^24 - 221133824*a^8*b^23 - 467977216*a^
9*b^22 - 32768*a^4*b^27 + 4041583616*a^11*b^20 + 16331772928*a^12*b^19 + 40
173472768*a^13*b^18 + 71534228480*a^14*b^17 + 97563767808*a^15*b^16 + 10442
6556416*a^16*b^15 + 88612000768*a^17*b^14 + 59708484608*a^18*b^13 + 3178259
3536*a^19*b^12 + 13203725312*a^20*b^11 + 4193231872*a^21*b^10 + 984308736*a
^22*b^9 + 161366016*a^23*b^8 + 16580608*a^24*b^7 + 811008*a^25*b^6 + ((tan(
e + f*x)*(131072*a^6*b^28 + 3342336*a^7*b^27 + 40960000*a^8*b^26 + 31923404
8*a^9*b^25 + 1768817664*a^10*b^24 + 7390051328*a^11*b^23 + 24132297728*a^12
*b^22 + 63100984320*a^13*b^21 + 134472684544*a^14*b^20 + 236839037952*a^15*
b^19 + 348859675648*a^16*b^18 + 434405198848*a^17*b^17 + 461878103040*a^18*
b^16 + 423083193344*a^19*b^15 + 336152947712*a^20*b^14 + 232369345536*a^21*

b^13 + 139446393856*a^22*b^12 + 72073323520*a^23*b^11 + 31662619648*a^24*b^
10 + 11616461824*a^25*b^9 + 3481927680*a^26*b^8 + 829030400*a^27*b^7 + 1507
32800*a^28*b^6 + 19660800*a^29*b^5 + 1638400*a^30*b^4 + 65536*a^31*b^3) - (
(-b^7*(a + b)^11)^(1/2)*(44*a*b + 99*a^2 + 8*b^2)*(65536*a^10*b^27 + 165478
4*a^11*b^26 + 21954560*a^12*b^25 + 194478080*a^13*b^24 + 1247936512*a^14*b^
23 + 6060916736*a^15*b^22 + 22968795136*a^16*b^21 + 69506170880*a^17*b^20 +
170976215040*a^18*b^19 + 346596343808*a^19*b^18 + 585044721664*a^20*b^17 +
828584034304*a^21*b^16 + 989821665280*a^22*b^15 + 1000564490240*a^23*b^14
+ 856970493952*a^24*b^13 + 621538574336*a^25*b^12 + 380751118336*a^26*b^11
+ 196065116160*a^27*b^10 + 84230471680*a^28*b^9 + 29853974528*a^29*b^8 + 85
88754944*a^30*b^7 + 1957904384*a^31*b^6 + 340787200*a^32*b^5 + 42598400*a^3
3*b^4 + 3407872*a^34*b^3 + 131072*a^35*b^2 - (tan(e + f*x)*(-b^7*(a + b)^11
)^(1/2)*(44*a*b + 99*a^2 + 8*b^2)*(524288*a^12*b^28 + 13369344*a^13*b^27 +
163840000*a^14*b^26 + 1284505600*a^15*b^25 + 7235174400*a^16*b^24 + 3117154
3040*a^17*b^23 + 106779115520*a^18*b^22 + 298450944000*a^19*b^21 + 69306941
4400*a^20*b^20 + 1354635673600*a^21*b^19 + 2249325281280*a^22*b^18 + 319384
7152640*a^23*b^17 + 3894935552000*a^24*b^16 + 4089682329600*a^25*b^15 + 370
0188774400*a^26*b^14 + 2882252308480*a^27*b^13 + 1927993098240*a^28*b^12 +
1102610432000*a^29*b^11 + 535553638400*a^30*b^10 + 218864025600*a^31*b^9 +
74281123840*a^32*b^8 + 20559953920*a^33*b^7 + 4521984000*a^34*b^6 + 7602176
00*a^35*b^5 + 91750400*a^36*b^4 + 7077888*a^37*b^3 + 262144*a^38*b^2))/(16*
(11*a^13*b + a^14 + a^3*b^11 + 11*a^4*b^10 + 55*a^5*b^9 + 165*a^6*b^8 + 330
*a^7*b^7 + 462*a^8*b^6 + 462*a^9*b^5 + 330*a^10*b^4 + 165*a^11*b^3 + 55*a^1
2*b^2))))/(16*(11*a^13*b + a^14 + a^3*b^11 + 11*a^4*b^10 + 55*a^5*b^9 + 165
*a^6*b^8 + 330*a^7*b^7 + 462*a^8*b^6 + 462*a^9*b^5 + 330*a^10*b^4 + 165*a^1
1*b^3 + 55*a^12*b^2)))*(-b^7*(a + b)^11)^(1/2)*(44*a*b + 99*a^2 + 8*b^2))/(
16*(11*a^13*b + a^14 + a^3*b^11 + 11*a^4*b^10 + 55*a^5*b^9 + 165*a^6*b^8 +
330*a^7*b^7 + 462*a^8*b^6 + 462*a^9*b^5 + 330*a^10*b^4 + 165*a^11*b^3 + 55*
a^12*b^2)) - ((tan(e + f*x)*(131072*a^6*b^28 + 3342336*a^7*b^27 + 40960000*
a^8*b^26 + 319234048*a^9*b^25 + 1768817664*a^10*b^24 + 7390051328*a^11*b^23
+ 24132297728*a^12*b^22 + 63100984320*a^13*b^21 + 134472684544*a^14*b^20 +
236839037952*a^15*b^19 + 348859675648*a^16*b^18 + 434405198848*a^17*b^17 +
461878103040*a^18*b^16 + 423083193344*a^19*b^15 + 336152947712*a^20*b^14 +
232369345536*a^21*b^13 + 139446393856*a^22*b^12 + 72073323520*a^23*b^11 +
31662619648*a^24*b^10 + 11616461824*a^25*b^9 + 3481927680*a^26*b^8 + 829030
400*a^27*b^7 + 150732800*a^28*b^6 + 19660800*a^29*b^5 + 1638400*a^30*b^4 +
65536*a^31*b^3) + ((-b^7*(a + b)^11)^(1/2)*(44*a*b + 99*a^2 + 8*b^2)*(65536
*a^10*b^27 + 1654784*a^11*b^26 + 21954560*a^12*b^25 + 194478080*a^13*b^24 +
1247936512*a^14*b^23 + 6060916736*a^15*b^22 + 22968795136*a^16*b^21 + 6950
6170880*a^17*b^20 + 170976215040*a^18*b^19 + 346596343808*a^19*b^18 + 58504
4721664*a^20*b^17 + 828584034304*a^21*b^16 + 989821665280*a^22*b^15 + 10005
64490240*a^23*b^14 + 856970493952*a^24*b^13 + 621538574336*a^25*b^12 + 3807
51118336*a^26*b^11 + 196065116160*a^27*b^10 + 84230471680*a^28*b^9 + 298539
74528*a^29*b^8 + 8588754944*a^30*b^7 + 1957904384*a^31*b^6 + 340787200*a^32
*b^5 + 42598400*a^33*b^4 + 3407872*a^34*b^3 + 131072*a^35*b^2 + (tan(e + f*

x)*(-b^7*(a + b)^11)^(1/2)*(44*a*b + 99*a^2 + 8*b^2)*(524288*a^12*b^28 + 13
369344*a^13*b^27 + 163840000*a^14*b^26 + 1284505600*a^15*b^25 + 7235174400*
a^16*b^24 + 31171543040*a^17*b^23 + 106779115520*a^18*b^22 + 298450944000*a
^19*b^21 + 693069414400*a^20*b^20 + 1354635673600*a^21*b^19 + 2249325281280
*a^22*b^18 + 3193847152640*a^23*b^17 + 3894935552000*a^24*b^16 + 4089682329
600*a^25*b^15 + 3700188774400*a^26*b^14 + 2882252308480*a^27*b^13 + 1927993
098240*a^28*b^12 + 1102610432000*a^29*b^11 + 535553638400*a^30*b^10 + 21886
4025600*a^31*b^9 + 74281123840*a^32*b^8 + 20559953920*a^33*b^7 + 4521984000
*a^34*b^6 + 760217600*a^35*b^5 + 91750400*a^36*b^4 + 7077888*a^37*b^3 + 262
144*a^38*b^2))/(16*(11*a^13*b + a^14 + a^3*b^11 + 11*a^4*b^10 + 55*a^5*b^9
+ 165*a^6*b^8 + 330*a^7*b^7 + 462*a^8*b^6 + 462*a^9*b^5 + 330*a^10*b^4 + 16
5*a^11*b^3 + 55*a^12*b^2))))/(16*(11*a^13*b + a^14 + a^3*b^11 + 11*a^4*b^10
 + 55*a^5*b^9 + 165*a^6*b^8 + 330*a^7*b^7 + 462*a^8*b^6 + 462*a^9*b^5 + 330
*a^10*b^4 + 165*a^11*b^3 + 55*a^12*b^2))*(-b^7*(a + b)^11)^(1/2)*(44*a*b +
 99*a^2 + 8*b^2))/(16*(11*a^13*b + a^14 + a^3*b^11 + 11*a^4*b^10 + 55*a^5*b
^9 + 165*a^6*b^8 + 330*a^7*b^7 + 462*a^8*b^6 + 462*a^9*b^5 + 330*a^10*b^4 +
 165*a^11*b^3 + 55*a^12*b^2))))*(-b^7*(a + b)^11)^(1/2)*(44*a*b + 99*a^2 +
8*b^2)*1i)/(8*f*(11*a^13*b + a^14 + a^3*b^11 + 11*a^4*b^10 + 55*a^5*b^9 + 1
65*a^6*b^8 + 330*a^7*b^7 + 462*a^8*b^6 + 462*a^9*b^5 + 330*a^10*b^4 + 165*a
^11*b^3 + 55*a^12*b^2))

3.376 $\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 111

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx = -\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f} + \frac{\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f}$$

```
[Out] -1/3*(a+2*b)*(a+b*sec(f*x+e)^2)^(3/2)/b^2/f+1/5*(a+b*sec(f*x+e)^2)^(5/2)/b^
2/f-arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f+(a+b*sec(f*x+e)^2)^
(1/2)/f
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used

$= \{4224, 457, 90, 52, 65, 214\}$

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx = -\frac{\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{(a+b\sec^2(e+fx))^{5/2}}{5b^2f} - \frac{(a+2b)\,(a+b\sec^2(e+fx))^{3/2}}{3b^2f} + \frac{\sqrt{a+b\sec^2(e+fx)}}{f}$$

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^5,x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f) + Sqrt[a + b*Sec
[e + f*x]^2]/f - ((a + 2*b)*(a + b*Sec[e + f*x]^2)^(3/2))/(3*b^2*f) + (a +
b*Sec[e + f*x]^2)^(5/2)/(5*b^2*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)^2\sqrt{a+bx^2}}{x}\,dx, x, \sec(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{(-1+x)^2\sqrt{a+bx}}{x}\,dx, x, \sec^2(e+fx)\right)}{2f} \\
&= \frac{\text{Subst}\left(\int \left(\frac{(-a-2b)\sqrt{a+bx}}{b}+\frac{\sqrt{a+bx}}{x}+\frac{(a+bx)^{3/2}}{b}\right)dx, x, \sec^2(e+fx)\right)}{2f} \\
&= -\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f}+\frac{\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f} \\
&\quad+\frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sec^2(e+fx)\right)}{2f} \\
&= \frac{\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f} \\
&\quad+\frac{\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f}+\frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f} \\
&= \frac{\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f} \\
&\quad+\frac{\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f}+\frac{a\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf}
\end{aligned}
$$

$$= -\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f} + \frac{\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f}$$

Mathematica [A] (verified)

Time = 2.83 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.25

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx$$

$$= \frac{15b^2(a+b\sec^2(e+fx)) - 5a(a+b\sec^2(e+fx))^2 - 10b(a+b\sec^2(e+fx))^2 + 3(a+b\sec^2(e+fx))^3 - 1}{15b^2f\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^5,x]
```

```
[Out] (15*b^2*(a + b*Sec[e + f*x]^2) - 5*a*(a + b*Sec[e + f*x]^2)^2 - 10*b*(a + b
*Sec[e + f*x]^2)^2 + 3*(a + b*Sec[e + f*x]^2)^3 - 15*a*b^2*ArcTanh[Sqrt[1 +
 (b*Sec[e + f*x]^2)/a]]*Sqrt[1 + (b*Sec[e + f*x]^2)/a])/(15*b^2*f*Sqrt[a +
b*Sec[e + f*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. 2(95) = 190.

Time = 12.02 (sec) , antiderivative size = 568, normalized size of antiderivative = 5.12

method	result
default	$-\frac{\sqrt{a+b\sec(fx+e)^2}\left(15\cos(fx+e)\sqrt{a}\,\ln\left(4\cos(fx+e)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\sqrt{a}+4\cos(fx+e)a+4\sqrt{a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\right)b^2+2\cos(fx+e\right.}{}$

```
[In] int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/f/b^2*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(15*cos(f*x+e)*a^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2))*b^2+2*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*a^2+10*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*a*b-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*cos(f*x+e)+2*a^
2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+10*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*a*b-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2-((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b*sec(f*x+e)+10*b^2*((b+a*cos(f
```

```
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)-((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*a*b*sec(f*x+e)^2+10*b^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*sec(f*x+e)^2-3*b^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*
x+e)^3-3*b^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(95) = 190$.

Time = 1.97 (sec) , antiderivative size = 456, normalized size of antiderivative = 4.11

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx$$

$$= \left[\frac{15\sqrt{a}b^2\cos(fx+e)^4\log\left(128\,a^4\cos(fx+e)^8+256\,a^3b\cos(fx+e)^6+160\,a^2b^2\cos(fx+e)^4+32\,ab^3\right.}{}\right.$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] [1/120*(15*sqrt(a)*b^2*cos(f*x + e)^4*log(128*a^4*cos(f*x + e)^8 + 256*a^3*
b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b
^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x
+ e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)) - 8*((2*a^2 + 10*a*b - 15*b^2)*cos(f*x + e)^4 - (a*b - 10*b^2)*cos(f
*x + e)^2 - 3*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(b^2*f*cos(
f*x + e)^4), 1/60*(15*sqrt(-a)*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b
*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2))*cos(f*x + e)^4 -
4*((2*a^2 + 10*a*b - 15*b^2)*cos(f*x + e)^4 - (a*b - 10*b^2)*cos(f*x + e)^2
 - 3*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(b^2*f*cos(f*x + e)^
4)]
```

Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx = \int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**5,x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**5, x)
```

Maxima [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx = \int \sqrt{b\sec\left(fx+e\right)^2+a}\tan\left(fx+e\right)^5\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^5, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1280 vs. 2(95) = 190.

Time = 1.82 (sec) , antiderivative size = 1280, normalized size of antiderivative = 11.53

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="giac")
[Out] 2/15*(15*a*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2
*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) - 2*(1
5*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*t
an(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*
e)^2 + a + b))^9*a - 165*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^8*sqrt(a + b)*a + 20*(sqrt(a + b)*tan
(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e
)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^7*(
27*a^2 - 5*a*b - 16*b^2) - 20*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*
tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e
)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^6*(33*a^2 - 83*a*b + 32*b^2)*sqr
t(a + b) - 2*(15*a^3 + 1230*a^2*b - 625*a*b^2 - 416*b^3)*(sqrt(a + b)*tan(1
/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^
4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^5 + 1
0*(81*a^3 + 90*a^2*b - 391*a*b^2 + 256*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*
e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1
/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^4*sqrt(a + b) - 20
*(33*a^4 - 45*a^3*b - 157*a^2*b^2 + 161*a*b^3 - 16*b^4)*(sqrt(a + b)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4
 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^3 + 20
*(3*a^4 - 75*a^3*b + 49*a^2*b^2 + 159*a*b^3 - 160*b^4)*(sqrt(a + b)*tan(1/2
*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4
- 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2*sqrt(
```

```
a + b) + 5*(27*a^5 + 140*a^4*b - 286*a^3*b^2 + 76*a^2*b^3 + 667*a*b^4 - 576
*b^5)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
 b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x +
1/2*e)^2 + a + b)) - (45*a^5 + 100*a^4*b - 450*a^3*b^2 + 1028*a^2*b^3 - 153
9*a*b^4 + 768*b^5)*sqrt(a + b))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt
(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/
2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*(sqrt(a + b)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2
*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a + b
) + a - 3*b)^5)*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx=\int \tan(e+f\,x)^5\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

```
[In] int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.377 $\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 80

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx=\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}-\frac{\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3bf}$$

```
[Out] 1/3*(a+b*sec(f*x+e)^2)^(3/2)/b/f+arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))*
a^(1/2)/f-(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4224, 457, 81, 52, 65, 214}

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx=\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3bf}-\frac{\sqrt{a+b\sec^2(e+fx)}}{f}$$

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^3,x]
```

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - Sqrt[a + b*Sec[e
+ f*x]^2]/f + (a + b*Sec[e + f*x]^2)^(3/2)/(3*b*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)\sqrt{a+bx^2}}{x}\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{(-1+x)\sqrt{a+bx}}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= \frac{(a+b\sec^2(e+fx))^{3/2}}{3bf} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= -\frac{\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{(a+b\sec^2(e+fx))^{3/2}}{3bf} - \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= -\frac{\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{(a+b\sec^2(e+fx))^{3/2}}{3bf}\\
&\quad - \frac{a\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf}\\
&= \frac{\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{(a+b\sec^2(e+fx))^{3/2}}{3bf}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx = \frac{3\sqrt{a}\,b\,\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right) + \sqrt{a+b\sec^2(e+fx)}(a-3b+b\sec^2(e+fx))}{3bf}$$

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^3,x]
[Out] (3*Sqrt[a]*b*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]] + Sqrt[a + b*Sec[e
 + f*x]^2]*(a - 3*b + b*Sec[e + f*x]^2))/(3*b*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(68) = 136.

Time = 7.44 (sec) , antiderivative size = 338, normalized size of antiderivative = 4.22

method	result
default	$\frac{\sqrt{a+b\sec(fx+e)^2}\left(3\ln\left(4\cos(fx+e)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{a}+4\cos(fx+e)a+4\sqrt{a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\right)\sqrt{a}\,b\cos(fx+e)+\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\right.}{3fb(1}$

```
[In] int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f/b*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*(3*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2))*a^(1/2)*b*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*co
s(f*x+e)*a-3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*b+a*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*b+b*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)+b*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(68) = 136.

Time = 0.59 (sec) , antiderivative size = 386, normalized size of antiderivative = 4.82

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx$$

$$= \left[\frac{3\sqrt{a}b\cos(fx+e)^2\log\left(128a^4\cos(fx+e)^8+256a^3b\cos(fx+e)^6+160a^2b^2\cos(fx+e)^4+32ab^3\mathrm{co}\right.}{}\right.$$

$$-\frac{3\sqrt{-a}b\arctan\left(\frac{\left(8a^2\cos(fx+e)^4+8ab\cos(fx+e)^2+b^2\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2a^3\cos(fx+e)^4+3a^2b\cos(fx+e)^2+ab^2\right)}\right)\cos(fx+e)^2-4\left((a-3b)\cos(fx\right.}{12bf\cos(fx+e)^2}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(a)*b*cos(f*x + e)^2*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*co
s(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 +
```

```
8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)
^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)) + 8*((a - 3*b)*cos(f*x + e)^2 + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2))/(b*f*cos(f*x + e)^2), -1/12*(3*sqrt(-a)*b*arctan(1/4*(8*a^2*cos(f*x
 + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2))*co
s(f*x + e)^2 - 4*((a - 3*b)*cos(f*x + e)^2 + b)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2))/(b*f*cos(f*x + e)^2)]
```

Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx=\int \sqrt{a+b\sec^2{(e+fx)}}\tan^3{(e+fx)}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**3,x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**3, x)
```

Maxima [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx=\int \sqrt{b\sec{(fx+e)}^2+a}\tan{(fx+e)}^3\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 800 vs. $2(68) = 136$.

Time = 0.76 (sec) , antiderivative size = 800, normalized size of antiderivative = 10.00

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx=\text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="giac")
[Out] -2/3*(3*a*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*
f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*
b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) - 2*(3*
(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan
(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)
^2 + a + b))^5*a - 3*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f
```

```
*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b
*tan(1/2*f*x + 1/2*e)^2 + a + b))^4*(3*a - 4*b)*sqrt(a + b) + 2*(sqrt(a + b
)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x +
1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)
)^3*(3*a^2 - 9*a*b + 8*b^2) + 6*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2
*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2*(a^2 + a*b - 4*b^2)*sqrt(a +
 b) - 3*(3*a^3 - 2*a^2*b - 5*a*b^2 + 16*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2
*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(
1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)) + (3*a^3 - 6*a^2*
b + 19*a*b^2 - 20*b^3)*sqrt(a + b))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 -
sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x
+ 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*(sqrt(a + b)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4
 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a
 + b) + a - 3*b)^3)*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx=\int \tan(e+f\,x)^3\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

```
[In] int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.378 $\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx=-\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\sqrt{a+b\sec^2(e+fx)}}{f}$$

```
[Out] -arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f+(a+b*sec(f*x+e)^2)^(1/
2)/f
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4224, 272, 52, 65, 214}

$$\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx=\frac{\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f) + Sqrt[a + b*Sec
[e + f*x]^2]/f
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x}\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= \frac{\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= \frac{\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{a\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf}
\end{aligned}
$$

$$= -\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{\sqrt{a+b\sec^2(e+fx)}}{f}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 119 vs. 2(54) = 108.

Time = 0.56 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.20

$$\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx$$
$$= \frac{\left(-2\sqrt{a}\operatorname{arcsinh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)\cos(e+fx)+\sqrt{2}\sqrt{b}\sqrt{\frac{a+2b+a\cos(2(e+fx))}{b}}\right)\sqrt{a+b\sec^2(e+fx)}}{\sqrt{2}\sqrt{b}f\sqrt{\frac{a+2b+a\cos(2(e+fx))}{b}}}$$

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x],x]
```

```
[Out] ((-2*Sqrt[a]*ArcSinh[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]]*Cos[e + f*x] + Sqrt[2]
*Sqrt[b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/b])*Sqrt[a + b*Sec[e + f*x]^2]
)/(Sqrt[2]*Sqrt[b]*f*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/b])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{\sqrt{a+b\sec(fx+e)^2}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\sec(fx+e)^2}}{\sec(fx+e)}\right)}{f}$	58
default	$\frac{\sqrt{a+b\sec(fx+e)^2}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\sec(fx+e)^2}}{\sec(fx+e)}\right)}{f}$	58

```
[In] int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*((a+b*sec(f*x+e)^2)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sec(f*x+e)^2)^
(1/2))/sec(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(46) = 92$.

Time = 0.37 (sec) , antiderivative size = 312, normalized size of antiderivative = 5.78

$$\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx$$

$$= \left[\frac{\sqrt{a}\log\left(128\,a^4\cos\left(fx+e\right)^8 + 256\,a^3b\cos\left(fx+e\right)^6 + 160\,a^2b^2\cos\left(fx+e\right)^4 + 32\,ab^3\cos\left(fx+e\right)^2 + b^4\right.}{} \right.$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a
^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x +
 e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e
)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 8*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2))/f, 1/4*(sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x
+ e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) + 4
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/f]
```

Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx = \int \sqrt{a+b\sec^2\left(e+fx\right)}\tan\left(e+fx\right)dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x), x)
```

Maxima [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx = \int \sqrt{b\sec\left(fx+e\right)^2+a}\tan\left(fx+e\right)\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(46) = 92$.

Time = 0.46 (sec) , antiderivative size = 377, normalized size of antiderivative = 6.98

$$\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx$$
$$= \frac{2\left(\frac{a\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}+\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{\cdots}{\left(\sqrt{a+b}\tan\cdots\right.}\right.}{\cdots}$$

[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="giac")

[Out] 2*(a*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) + 2*((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*b + sqrt(a + b)*b)/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a + b) + a - 3*b))*sgn(cos(f*x + e))/f

Mupad [B] (verification not implemented)

Time = 20.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx = \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{f}-\frac{\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\sqrt{a}}\right)}{f}$$

[In] int(tan(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2),x)

[Out] (a + b/cos(e + f*x)^2)^(1/2)/f - (a^(1/2)*atanh((a + b/cos(e + f*x)^2)^(1/2)/a^(1/2)))/f

3.379 $\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f}$$

```
[Out] arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f-arctanh((a+b*sec(f*x+e)
^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4224, 457, 85, 65, 214}

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f}$$

```
[In] Int[Cot[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - (Sqrt[a + b]*ArcT
anh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/f
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x(-1+x^2)}\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{(-1+x)x}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= -\frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}
\end{aligned}
$$

$$= -\frac{a\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf}$$

$$= \frac{\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f}$$

```
[In] Integrate[Cot[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - (Sqrt[a + b]*ArcT
anh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/f
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. 2(58) = 116.

Time = 1.35 (sec) , antiderivative size = 540, normalized size of antiderivative = 7.71

method	result
default	$\frac{\sqrt{a+b\sec(fx+e)^2}\left(2\sqrt{a}\,\ln\left(4\cos(fx+e)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\sqrt{a}+4\cos(fx+e)a+4\sqrt{a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\right)\sqrt{a+b}+\ln\left(\frac{2\sqrt{\frac{b+a\cos(fx+e}{(1+\cos(fx+e)}}}{}\right.\right.}{}$

```
[In] int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f/(a+b)^(1/2)*(a+b*sec(f*x+e)^2)^(1/2)*(2*a^(1/2)*ln(4*cos(f*x+e)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*(a+b)^(1/2)+ln(2/(a+b)^(1/2)*(((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e))
```

```
)*a+b*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(
1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos
(f*x+e)*a+b)/(1+cos(f*x+e)))-ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+
b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a-ln(-4*(((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*b)*cos(f*x+e)/(1+c
os(f*x+e))/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(58) = 116$.

Time = 0.42 (sec) , antiderivative size = 963, normalized size of antiderivative = 13.76

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a
^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x +
 e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e
)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 2*sqrt(a + b)*l
og(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)
^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2
 + 1)))/f, 1/8*(4*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sq
rt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x
 + e)^2 + a*b + b^2)) + sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(
f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8
*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4
 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))
)/f, -1/4*(sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2
 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x
 + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - sqrt(a + b)*log(2*((8*a^2 + 8*
a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*
a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/f, -1/4*(
sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqr
t(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3
*a^2*b*cos(f*x + e)^2 + a*b^2)) - 2*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(
f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((
a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)))/f]
```

Sympy [F]

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{a+b\sec^2{(e+fx)}}\cot{(e+fx)}\,dx$$

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 3317, normalized size of antiderivative = 47.39

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\text{Too large to display}$$

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] 1/4*(a^(3/2)*log(4*sqrt(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4
*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e
)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(
a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*
cos(2*f*x + 2*e))*a*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*
f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + 4*s
qrt(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^
2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b
)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*a*
sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*
f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + 16*(a^2*cos(4*f*x + 4*e
)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 +
 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2
)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos
(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*(a + b)*sqrt(a)*cos
(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x
 + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 16*a^2 + 32*a*b + 16*b^2) -
2*sqrt(a + b)*a*log(4*(4*sqrt(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)
^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x
 + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2
 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2
*a*b)*cos(2*f*x + 2*e))*(a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a +
 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) +
 a))^2 + 4*sqrt(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 +
```

4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*
f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*
(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*
x + 2*e))*(a + b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*
x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a^2
+ 2*a*b + b^2)*abs(2*e^(2*I*f*x + 2*I*e) - 2)^2 + 4*(a^2*cos(4*f*x + 4*e)^2
 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*
(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*s
in(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*
f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*((a + b)*abs(2*e^(2*I*
f*x + 2*I*e) - 2) + 4*a + 4*b)*sqrt(a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*
e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f
*x + 2*e) + a)) + 16*a^2 + 32*a*b + 16*b^2 + 8*(a^2 + 2*a*b + b^2)*abs(2*e^
(2*I*f*x + 2*I*e) - 2))/abs(2*e^(2*I*f*x + 2*I*e) - 2)^2) - (a^(3/2) + 2*sq
rt(a)*b)*log(4*a^2*cos(2*f*x + 2*e)^2 + 4*a^2*sin(2*f*x + 2*e)^2 + 4*a^2 +
16*a*b + 16*b^2 + 8*(a^2 + 2*a*b)*cos(2*f*x + 2*e) + 8*(a^2*cos(4*f*x + 4*e
)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 +
 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2
)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos
(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*(a*sin(2*f*x + 2*e)
*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4
*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + (a*cos(2*f*x + 2*e) + a
+ 2*b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a
*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)))*sqrt(a) + 4*sqrt(a^
2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos
(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^
2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(
2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*(a*cos(1
/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x +
 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + a*sin(1/2*arctan2(a*sin(4*f*
x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*c
os(2*f*x + 2*e) + a))^2)) + 2*(a^(3/2) + sqrt(a)*b)*log(a^2*cos(2*f*x + 2*e
)^2 + a^2*sin(2*f*x + 2*e)^2 + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x +
4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4
*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 +
 a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2
 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*a^(3/2)*sin(2*f*x + 2*e)*sin(1/2*arctan2(
a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(
a + 2*b)*cos(2*f*x + 2*e) + a)) + sqrt(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f
*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*
sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e
)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4
*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*a*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(
a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e
) + a))^2 + sqrt(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 +

```
 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2
*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2
*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f
*x + 2*e))*a*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2
*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + 2*(a^2*cos
(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*
x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4
*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x
 + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*(a^(3/2
)*cos(2*f*x + 2*e) + a^(3/2) + 2*sqrt(a)*b)*cos(1/2*arctan2(a*sin(4*f*x + 4
*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*
f*x + 2*e) + a)) + a^2 + 4*a*b + 4*b^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e)))
/(a*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(58) = 116$.

Time = 0.69 (sec) , antiderivative size = 402, normalized size of antiderivative = 5.74

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx =$$

$$-\frac{\left(\frac{4\,a\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-\sqrt{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-2\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a+b}+\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}}+\sqrt{a+\cdots}\right.}{\cdots}$$

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(4*a*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*
f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*
b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) + sqrt(
a + b)*log(abs(-sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a*tan(1/2*f*x + 1
/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1
/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))) - sqrt(a + b)*log(abs(-sqrt(a +
b)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x +
 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b
) - sqrt(a + b))) + sqrt(a + b)*log(abs((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2
 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f
*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a + b) - sqrt(a + b)*
(a - b))))*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \cot(e+f\,x)\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

```
[In] int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.380 $\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 109

$$\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{(2a+b)\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2\sqrt{a+b}f} - \frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}$$

```
[Out] -arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f+1/2*(2*a+b)*arctanh((a
+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/f/(a+b)^(1/2)-1/2*cot(f*x+e)^2*(a+b*sec
(f*x+e)^2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4224, 457, 101, 162, 65, 214}

$$\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{(2a+b)\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f\sqrt{a+b}} - \frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}$$

```
[In] Int[Cot[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f) + ((2*a + b)*Arc
Tanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(2*Sqrt[a + b]*f) - (Cot[e +
f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(2*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
```

```
x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x(-1+x^2)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{(-1+x)^2x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{-a-\frac{bx}{2}}{(-1+x)x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f} - \frac{(2a+b)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{4f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{a\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf} - \frac{(2a+b)\text{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{2bf}$$

$$= -\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{(2a+b)\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2\sqrt{a+b}f} - \frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.91 (sec) , antiderivative size = 527, normalized size of antiderivative = 4.83

$$\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos(e+fx)\left(\frac{1+e^{2i(e+fx)}}{\left(-1+e^{2i(e+fx)}\right)^2}-\frac{-2i\sqrt{a}\sqrt{a+b}fx+(2a+b)\log\left(1-e^{2i(e+fx)}\right)+\sqrt{a}\cdots}{\cdots}\right)}{\cdots}$$

```
[In] Integrate[Cot[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Cos[e + f*x]*((1 + E^((2*I)*(e + f*x)))/(-1 + E^((2*I)*(e + f*x)))^2
- ((-2*I)*Sqrt[a]*Sqrt[a + b]*f*x + (2*a + b)*Log[1 - E^((2*I)*(e + f*x))]
+ Sqrt[a]*Sqrt[a + b]*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*
b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + Sqrt[a]*Sqrt[a +
b]*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4
*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 2*a*Log[a + b +
a*E^((2*I)*(e + f*x)) + b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*
I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - b*Log[a + b + a*E^((2*I)*
(e + f*x)) + b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x
)) + a*(1 + E^((2*I)*(e + f*x)))^2]])/(Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f
*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]))*Sqrt[a + b*Sec[e + f*x]^2])/(Sqrt[2
]*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2755 vs. 2(91) = 182.

Time = 1.39 (sec) , antiderivative size = 2756, normalized size of antiderivative = 25.28

method	result	size
default	Expression too large to display	2756

```
[In] int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f/(a+b)^(5/2)*((a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(
f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^
2+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^2)^(1/2)*((1-cos(f*x+e))^2*csc(f*x
+e)^2-1)*(8*a^(3/2)*ln(4*(-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+a^(1/2)*(a*(1-co
s(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))
^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)+a)/((1-cos(f*x
```

```
+e))^2*csc(f*x+e)^2+1))*(1-cos(f*x+e))^2*(a+b)^(3/2)*csc(f*x+e)^2-(a+b)^(3/
2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-
cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*a*(
1-cos(f*x+e))^4*csc(f*x+e)^4-(a+b)^(3/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b
*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos
(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*b*(1-cos(f*x+e))^4*csc(f*x+e)^4+8*a^(1/2
)*ln(4*(-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+a^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*
x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*
b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)+a)/((1-cos(f*x+e))^2*csc(f*x+e)^
2+1))*(1-cos(f*x+e))^2*(a+b)^(3/2)*b*csc(f*x+e)^2+a*(a*(1-cos(f*x+e))^4*csc
(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2
+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(1-cos(f*x+e))^2*(a+b)^(3/2)*
csc(f*x+e)^2-3*b*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*
x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+
a+b)^(1/2)*(1-cos(f*x+e))^2*(a+b)^(3/2)*csc(f*x+e)^2+4*ln((a*(1-cos(f*x+e))
^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+(a*(1-cos(f*x+e))^4*csc(f*x
+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b
*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)-a+b)/(a+b)^(1/2))*a^3
*(1-cos(f*x+e))^2*csc(f*x+e)^2+10*ln((a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-
cos(f*x+e))^2*csc(f*x+e)^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e)
)^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc
(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)-a+b)/(a+b)^(1/2))*a^2*(1-cos(f*x+e))^2*b*c
sc(f*x+e)^2+8*a*ln((a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(
f*x+e)^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2
*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/
2)*(a+b)^(1/2)-a+b)/(a+b)^(1/2))*b^2*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*ln((a*
(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+(a*(1-cos(f*x
+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*cs
c(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)-a+b)/(a
+b)^(1/2))*b^3*(1-cos(f*x+e))^2*csc(f*x+e)^2-4*a^3*ln(2/(1-cos(f*x+e))^2*(-
a*(1-cos(f*x+e))^2+b*(1-cos(f*x+e))^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1
-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*
x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2+b*s
in(f*x+e)^2))*(1-cos(f*x+e))^2*csc(f*x+e)^2-10*a^2*ln(2/(1-cos(f*x+e))^2*(-
a*(1-cos(f*x+e))^2+b*(1-cos(f*x+e))^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1
-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*
x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2+b*s
in(f*x+e)^2))*(1-cos(f*x+e))^2*b*csc(f*x+e)^2-8*b^2*ln(2/(1-cos(f*x+e))^2*(
-a*(1-cos(f*x+e))^2+b*(1-cos(f*x+e))^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(
1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f
*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2+b*
sin(f*x+e)^2))*(1-cos(f*x+e))^2*a*csc(f*x+e)^2-2*b^3*ln(2/(1-cos(f*x+e))^2*
(-a*(1-cos(f*x+e))^2+b*(1-cos(f*x+e))^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*
(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(
f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2+b
```

```
*sin(f*x+e)^2))*(1-cos(f*x+e))^2*csc(f*x+e)^2+(a*(1-cos(f*x+e))^4*csc(f*x+e
)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(
1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(3/2)*(a+b)^(3/2))/(a*(1-cos(f*x+e))^4*cs
c(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^
2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)/(1-cos(f*x+e))^2*sin(f*x+e)^
2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(91) = 182$.

Time = 0.56 (sec) , antiderivative size = 1342, normalized size of antiderivative = 12.31

$$\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2
+ ((a + b)*cos(f*x + e)^2 - a - b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256
*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^
2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos
(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)) + ((2*a + b)*cos(f*x + e)^2 - 2*a - b)*sqrt(a + b)*log(2*((8*a^
2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 +
4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a
 + b)*f*cos(f*x + e)^2 - (a + b)*f), 1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*cos(f*x + e)^2 - 2*((2*a + b)*cos(f*x + e)^2 - 2*a - b
)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b +
b^2)) + ((a + b)*cos(f*x + e)^2 - a - b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8
 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x
 + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b
^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)))/((a + b)*f*cos(f*x + e)^2 - (a + b)*f), 1/8*(4*(a + b)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 + 2*((a + b)*cos(
f*x + e)^2 - a - b)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f
*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^
3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) + ((2*a + b)*cos(f*x +
e)^2 - 2*a - b)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2
*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos
(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(
f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a + b)*f*cos(f*x + e)^2 - (a + b)*f)
, 1/4*(2*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2
 + ((a + b)*cos(f*x + e)^2 - a - b)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)
```

```
^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - ((2*a
 + b)*cos(f*x + e)^2 - 2*a - b)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x
+ e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2
+ a*b)*cos(f*x + e)^2 + a*b + b^2)))/((a + b)*f*cos(f*x + e)^2 - (a + b)*f)
]
```

Sympy [F]

$$\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\cot^3(e+fx)\,dx$$

```
[In] integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x)**3, x)
```

Maxima [F]

$$\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cot(fx+e)^3\,dx$$

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 574 vs. $2(91) = 182$.

Time = 0.90 (sec) , antiderivative size = 574, normalized size of antiderivative = 5.27

$$\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\left(\frac{16\,a\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2\,a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\,b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}+\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{4(2a+b)}{}\right.}{}$$

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] 1/8*(16*a*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*
f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*
```

```
b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) - 4*(2*
a + b)*arctan(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1
/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1
/2*f*x + 1/2*e)^2 + a + b))/sqrt(-a - b))/sqrt(-a - b) + 2*(2*a + b)*log(ab
s(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*
tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2
*e)^2 + a + b))*(a + b) + sqrt(a + b)*(a - b)))/sqrt(a + b) + sqrt(a*tan(1/
2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - 2*((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^
2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*
f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a - b) - (a + b)^(3/
2))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1
/2*e)^2 + a + b))^2 - a - b))*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \cot(e+f\,x)^3\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

```
[In] int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.381 $\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 161

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{a}\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{(8a^2+12ab+3b^2)\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{3/2}f} + \frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f}$$

```
[Out] -1/8*(8*a^2+12*a*b+3*b^2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+
b)^(3/2)/f+arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f+1/8*(4*a+3*b
)*cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2)/(a+b)/f-1/4*cot(f*x+e)^4*(a+b*sec(f
*x+e)^2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used

$= \{4224, 457, 101, 156, 162, 65, 214\}$

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{(8a^2+12ab+3b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{3/2}} + \frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f} + \frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f(a+b)}$$

```
[In] Int[Cot[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - ((8*a^2 + 12*a*b
+ 3*b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(8*(a + b)^(3/2)*
f) + ((4*a + 3*b)*Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(8*(a + b)*f)
- (Cot[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2])/(4*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x(-1+x^2)^3}\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{(-1+x)^3x}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= -\frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f} + \frac{\text{Subst}\left(\int \frac{-2a-\frac{3bx}{2}}{(-1+x)^2x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{4f}\\
&= \frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f}\\
&\quad - \frac{\text{Subst}\left(\int \frac{-2a(a+b)-\frac{1}{4}b(4a+3b)x}{(-1+x)x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{4(a+b)f}
\end{aligned}
$$

$$= \frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f} - \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,\sec^2(e+fx)\right)}{2f} + \frac{(8a^2+12ab+3b^2)\,\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\,dx,x,\sec^2(e+fx)\right)}{16(a+b)f}$$

$$= \frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f} - \frac{a\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\sec^2(e+fx)}\right)}{bf} + \frac{(8a^2+12ab+3b^2)\,\text{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\sec^2(e+fx)}\right)}{8b(a+b)f}$$

$$= \frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{(8a^2+12ab+3b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{3/2}f} + \frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f}$$

Mathematica [F]

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

```
[In] Integrate[Cot[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Cot[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 4322 vs. 2(139) = 278.

Time = 1.36 (sec) , antiderivative size = 4323, normalized size of antiderivative = 26.85

method	result	size
default	Expression too large to display	4323

```
[In] int(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] -1/64/f/(a+b)^(9/2)*((a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^2)^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)*(-18*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(5/2)*a*b*(1-cos(f*x+e))^6*csc(f*x+e)^6-14*a*b*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(5/2)*(1-cos(f*x+e))^4*csc(f*x+e)^4+64*(a+b)^(5/2)*a^(1/2)*ln(4*(-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+a^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)+a)/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*b^2*(1-cos(f*x+e))^4*csc(f*x+e)^4+128*(a+b)^(5/2)*a^(3/2)*ln(4*(-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+a^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)+a)/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*b*(1-cos(f*x+e))^4*csc(f*x+e)^4+252*ln((a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)-a+b)/(a+b)^(1/2))*a^3*b^2*(1-cos(f*x+e))^4*csc(f*x+e)^4+212*ln((a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)-a+b)/(a+b)^(1/2))*a^2*b^3*(1-cos(f*x+e))^4*csc(f*x+e)^4+84*ln((a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)-a+b)/(a+b)^(1/2))*a*b^4*(1-cos(f*x+e))^4*csc(f*x+e)^4-144*ln(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))^2+b*(1-cos(f*x+e))^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2+b*sin(f*x+e)^2))*a^4*(1-cos(f*x+e))^4*b*csc(f*x+e)^4-252*ln(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))^2+b*(1-cos(f*x+e))^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2+b*sin(f*x+e)^2))*a^3*b^2*(1-cos(f*x+e))^4*csc(f*x+e)^4-212*ln(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))^2+b*(1-cos(f*x+e))^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2+b*sin(f*x+e)^2))*a^2*b^3*(1-cos(f*x+e))^4*csc(f*x+e)^4-84*ln(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))^2+b*(1-cos(f*x+e))^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2+b*sin(f*x+e)^2))*b^4*(1-cos(f*x+e))^4*a*csc(f*x+e)^4-8*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+

e))^2*csc(f*x+e)^2+a+b)^(1/2)*(1-cos(f*x+e))^6*a^2*(a+b)^(5/2)*csc(f*x+e)^6
-10*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1
-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(1
-cos(f*x+e))^6*b^2*(a+b)^(5/2)*csc(f*x+e)^6+8*a^2*(a*(1-cos(f*x+e))^4*csc(f
x+e)^4+b(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2
b(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(5/2)*(1-cos(f*x+e))^4*cs
c(f*x+e)^4-30*b^2*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f
*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2
+a+b)^(1/2)*(a+b)^(5/2)*(1-cos(f*x+e))^4*csc(f*x+e)^4+9*a*(a*(1-cos(f*x+e))
^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*
x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(3/2)*(a+b)^(5/2)*(1-cos(f*x+
e))^2*csc(f*x+e)^2+11*b*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4
*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*
x+e)^2+a+b)^(3/2)*(a+b)^(5/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+144*ln((a*(1-co
s(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+(a*(1-cos(f*x+e))^
4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x
+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)-a+b)/(a+b)^(
1/2))*a^4*b*(1-cos(f*x+e))^4*csc(f*x+e)^4-(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+
b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-co
s(f*x+e))^2*csc(f*x+e)^2+a+b)^(3/2)*(a+b)^(5/2)*a-(a*(1-cos(f*x+e))^4*csc(f
x+e)^4+b(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2
b(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(3/2)*(a+b)^(5/2)*b+32*ln((a*(1-cos(f
*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+(a*(1-cos(f*x+e))^4*c
sc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)
^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)-a+b)/(a+b)^(1/2
))*a^5*(1-cos(f*x+e))^4*csc(f*x+e)^4+12*ln((a*(1-cos(f*x+e))^2*csc(f*x+e)^2
+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(
f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))
^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)-a+b)/(a+b)^(1/2))*b^5*(1-cos(f*x+e))
^4*csc(f*x+e)^4-32*ln(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))^2+b*(1-cos(f*x+
e))^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*
(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*
(a+b)^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2+b*sin(f*x+e)^2))*a^5*(1-cos(f*x+e))
^4*csc(f*x+e)^4-12*ln(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))^2+b*(1-cos(f*x+
e))^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*
(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*
(a+b)^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2+b*sin(f*x+e)^2))*b^5*(1-cos(f*x+e))
^4*csc(f*x+e)^4+64*(a+b)^(5/2)*a^(5/2)*ln(4*(-a*(1-cos(f*x+e))^2*csc(f*x+e)
^2+a^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4
-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(
1/2)+a)/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(1-cos(f*x+e))^4*csc(f*x+e)^4)/(
a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(
f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)/(1-cos(
f*x+e))^4*sin(f*x+e)^4

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(139) = 278$.

Time = 1.39 (sec) , antiderivative size = 1953, normalized size of antiderivative = 12.13

$$\int \cot^5(e + fx)\sqrt{a + b\sec^2(e + fx)}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(4*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*
x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*
b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b
^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x
+ e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)) + ((8*a^2 + 12*a*b + 3*b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 12*a*b + 3*b
^2)*cos(f*x + e)^2 + 8*a^2 + 12*a*b + 3*b^2)*sqrt(a + b)*log(2*((8*a^2 + 8*
a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*
a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*((6*a^
2 + 11*a*b + 5*b^2)*cos(f*x + e)^4 - (4*a^2 + 7*a*b + 3*b^2)*cos(f*x + e)^2
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f
*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f)
, 1/16*(((8*a^2 + 12*a*b + 3*b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 12*a*b + 3*b^
2)*cos(f*x + e)^2 + 8*a^2 + 12*a*b + 3*b^2)*sqrt(-a - b)*arctan(1/2*((2*a +
 b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 2*((a^2 + 2*a*b + b^2)*co
s(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sq
rt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*c
os(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 +
24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqr
t(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 2*((6*a^2 + 11*a*b + 5*
b^2)*cos(f*x + e)^4 - (4*a^2 + 7*a*b + 3*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(
a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f), -1/32*(8*((a^
2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^
2 + 2*a*b + b^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x
+ e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*c
os(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - ((8*a^2 + 12*a*b + 3*b^2
)*cos(f*x + e)^4 - 2*(8*a^2 + 12*a*b + 3*b^2)*cos(f*x + e)^2 + 8*a^2 + 12*a
*b + 3*b^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*
a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x
 + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x
+ e)^4 - 2*cos(f*x + e)^2 + 1)) + 4*((6*a^2 + 11*a*b + 5*b^2)*cos(f*x + e)^
4 - (4*a^2 + 7*a*b + 3*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos
```

```
(f*x + e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)
*f*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f), -1/16*(4*((a^2 + 2*a*b + b^2)*c
os(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*s
qrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt
(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*
a^2*b*cos(f*x + e)^2 + a*b^2)) - ((8*a^2 + 12*a*b + 3*b^2)*cos(f*x + e)^4 -
 2*(8*a^2 + 12*a*b + 3*b^2)*cos(f*x + e)^2 + 8*a^2 + 12*a*b + 3*b^2)*sqrt(-
a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) +
2*((6*a^2 + 11*a*b + 5*b^2)*cos(f*x + e)^4 - (4*a^2 + 7*a*b + 3*b^2)*cos(f*
x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^2 + 2*a*b + b^2)
*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 + 2*a*b +
 b^2)*f)]
```

Sympy [F]

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{a+b\sec^2{(e+fx)}}\cot^5{(e+fx)}\,dx$$

```
[In] integrate(cot(f*x+e)**5*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x)**5, x)
```

Maxima [F]

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec{(fx+e)}^2+a}\cot{(fx+e)}^5\,dx$$

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^5, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(139) = 278$.

Time = 1.30 (sec) , antiderivative size = 866, normalized size of antiderivative = 5.38

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\text{Too large to display}$$

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/64*(sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/
2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)*(tan(1/2*f*x + 1/2*e
)^2 - (11*a + 9*b)/(a + b)) - 128*a*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x +
1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*t
an(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/
sqrt(-a))/sqrt(-a) + 8*(8*a^2 + 12*a*b + 3*b^2)*arctan(-(sqrt(a + b)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4
 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))/sqrt(-
a - b))/((a + b)*sqrt(-a - b)) - 4*(8*a^2 + 12*a*b + 3*b^2)*log(abs((sqrt(a
 + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*
x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a
+ b))*(a + b) - sqrt(a + b)*(a - b)))/(a + b)^(3/2) + 4*(2*(sqrt(a + b)*tan
(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e
)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^3*(
3*a^2 - 2*b^2) - (sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan
(1/2*f*x + 1/2*e)^2 + a + b))^2*(7*a^2 + 10*a*b + 3*b^2)*sqrt(a + b) - 2*(2
*a^3 + 2*a^2*b - 3*a*b^2 - 3*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1
/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)) + 5*(a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*sqrt(a + b))/(((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2
*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - a - b)^2*(a + b)))*sgn(cos(f*x + e)
)/f
```

Mupad [F(-1)]

Timed out.

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \cot(e+f\,x)^5\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

```
[In] int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.382 $\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 219

$$\begin{aligned}\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx &= -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{(a^3+5a^2b+15ab^2-5b^3)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16b^{5/2}f}\\&\quad-\frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16b^2f}\\&\quad+\frac{(a-5b)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24bf}\\&\quad+\frac{\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f}\end{aligned}$$

```
[Out] 1/16*(a^3+5*a^2*b+15*a*b^2-5*b^3)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x
+e)^2)^(1/2))/b^(5/2)/f-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2
))*a^(1/2)/f-1/16*(a-b)*(a+5*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b^2/f
+1/24*(a-5*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^3/b/f+1/6*(a+b+b*tan(f*
x+e)^2)^(1/2)*tan(f*x+e)^5/f
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4226, 2000, 489, 596, 537, 223, 212, 385, 209}

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx$$
$$=\frac{(a^3+5a^2b+15ab^2-5b^3)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16b^{5/2}f}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$
$$-\frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16b^2f}$$
$$+\frac{\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6f}$$
$$+\frac{(a-5b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{24bf}$$

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^6,x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) + ((a^3 + 5*a^2*b + 15*a*b^2 - 5*b^3)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt
[a + b + b*Tan[e + f*x]^2]])/(16*b^(5/2)*f) - ((a - b)*(a + 5*b)*Tan[e + f*
x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(16*b^2*f) + ((a - 5*b)*Tan[e + f*x]^3*S
qrt[a + b + b*Tan[e + f*x]^2])/(24*b*f) + (Tan[e + f*x]^5*Sqrt[a + b + b*Ta
n[e + f*x]^2])/(6*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^6\sqrt{a+b(1+x^2)}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6\sqrt{a+b+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f} - \frac{\text{Subst}\left(\int \frac{x^4(5(a+b)+(-a+5b)x^2)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{6f}$$

$$= \frac{(a-5b)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24bf} + \frac{\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f} + \frac{\text{Subst}\left(\int \frac{x^2(-3(a-5b)(a+b)-3(a-b)(a+5b)x^2)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{24bf}$$

$$= -\frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16b^2f} + \frac{(a-5b)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24bf} + \frac{\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f} - \frac{\text{Subst}\left(\int \frac{-3(a+5b)(a^2-b^2)-3(a^3+5a^2b+15ab^2-5b^3)x^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{48b^2f}$$

$$= -\frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16b^2f} + \frac{(a-5b)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24bf} + \frac{\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f} - \frac{a\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} + \frac{(a^3+5a^2b+15ab^2-5b^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{16b^2f}$$

$$= -\frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16b^2f} + \frac{(a-5b)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24bf} + \frac{\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f} - \frac{a\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(a^3+5a^2b+15ab^2-5b^3)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16b^2f}$$

$$= -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(a^3+5a^2b+15ab^2-5b^3)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16b^{5/2}f} - \frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16b^2f} + \frac{(a-5b)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24bf} + \frac{\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f}$$

Mathematica [A] (verified)

Time = 4.13 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.20

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx = -\frac{\left(16\sqrt{a}b^2\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right) - \frac{(a^3+5a^2b+15ab^2-5b^3)\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)\cos(e+fx)\sqrt{a+b\text{s}}}{8\sqrt{2}b^2f\sqrt{a+2b+a\cos(2e+2fx)}} - \frac{(9a^2+34ab-59b^2+4(3a^2+12ab-7b^2)\cos(2(e+fx))+(3a^2+14ab-33b^2)\cos(4(e+fx)))\sec^4(e+}{384b^2f}$$

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^6,x]
```

```
[Out] -1/8*((16*Sqrt[a]*b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e +
f*x]^2]] - ((a^3 + 5*a^2*b + 15*a*b^2 - 5*b^3)*ArcTanh[(Sqrt[b]*Sin[e + f*x
])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqrt[b])*Cos[e + f*x]*Sqrt[a + b*Sec[e
+ f*x]^2])/(Sqrt[2]*b^2*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]) - ((9*a^2 + 3
4*a*b - 59*b^2 + 4*(3*a^2 + 12*a*b - 7*b^2)*Cos[2*(e + f*x)] + (3*a^2 + 14*
a*b - 33*b^2)*Cos[4*(e + f*x)])*Sec[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2]*T
an[e + f*x])/(384*b^2*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1580 vs. 2(193) = 386.

Time = 21.77 (sec) , antiderivative size = 1581, normalized size of antiderivative = 7.22

method	result	size
default	Expression too large to display	1581

```
[In] int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/96/f/(-a)^(1/2)/b^(11/2)*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(66*(-a)^(1/2)*b^(11/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*sin(f*x+e)-96*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*cos(f*x+e)*b^(11/2)*a+66*(-a)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(11/2)*tan(f*x+e)-28*(-a)^(1/2)
*b^(9/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*sin(f*x+e)-52*(-a)^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(11/2)*tan(f*x+e)*sec(f*
x+e)-28*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(9/2)*a*ta
n(f*x+e)-6*(-a)^(1/2)*b^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a
^2*sin(f*x+e)-52*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(
11/2)*tan(f*x+e)*sec(f*x+e)^2+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*b^(9/2)*a*tan(f*x+e)*sec(f*x+e)-6*(-a)^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*b^(7/2)*a^2*tan(f*x+e)+16*(-a)^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(11/2)*tan(f*x+e)*sec(f*x+e)^3+4*(-a)^(1/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(9/2)*a*tan(f*x+e)*sec(f*x
+e)^2+3*ln(4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+
e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(s
in(f*x+e)-1))*(-a)^(1/2)*cos(f*x+e)*a^3*b^3+15*ln(4*(-((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*(-a)^(1/2)*cos(f*x+e)*
a^2*b^4+45*ln(4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f
*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)
/(sin(f*x+e)-1))*(-a)^(1/2)*cos(f*x+e)*a*b^5-15*ln(4*(-((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*(-a)^(1/2)*cos(f*x+e)
*b^6+3*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+
e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(s
in(f*x+e)+1))*(-a)^(1/2)*cos(f*x+e)*a^3*b^3+15*ln(-4*(-((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*(-a)^(1/2)*cos(f*x+e)
*a^2*b^4+45*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos
(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a+
b)/(sin(f*x+e)+1))*(-a)^(1/2)*cos(f*x+e)*a*b^5-15*ln(-4*(-((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*co
```

```
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*(-a)^(1/2)*cos(f*x
+e)*b^6+16*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(11/2)*
tan(f*x+e)*sec(f*x+e)^4)
```

Fricas [A] (verification not implemented)

none

Time = 4.22 (sec) , antiderivative size = 1775, normalized size of antiderivative = 8.11

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] [1/192*(24*sqrt(-a)*b^3*cos(f*x + e)^5*log(128*a^4*cos(f*x + e)^8 - 256*(a^
4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^
2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)
*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*
a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e)) - 3*(a^3 + 5*a^2*b + 15*a*b^2 - 5*b^3)*sqrt(b)
*cos(f*x + e)^5*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos
(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)
- 4*((3*a^2*b + 14*a*b^2 - 33*b^3)*cos(f*x + e)^4 - 8*b^3 - 2*(a*b^2 - 13*b
^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))/(b^3*f*cos(f*x + e)^5), 1/96*(12*sqrt(-a)*b^3*cos(f*x + e)^5*log(128*a^4
*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b +
 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 -
 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x
+ e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*c
os(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 3*(a^3 + 5*a^2*b +
15*a*b^2 - 5*b^3)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*
x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x
+ e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^5 - 2*((3*a^2*b + 14*a*b^2 - 33*b
^3)*cos(f*x + e)^4 - 8*b^3 - 2*(a*b^2 - 13*b^3)*cos(f*x + e)^2)*sqrt((a*cos
(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^5), 1/19
2*(48*sqrt(a)*b^3*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x
+ e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b
)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^5 - 3*(a^3 + 5*a^2*b + 15*a*b
^2 - 5*b^3)*sqrt(b)*cos(f*x + e)^5*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
+ 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^
```

```
2)/cos(f*x + e)^4) - 4*((3*a^2*b + 14*a*b^2 - 33*b^3)*cos(f*x + e)^4 - 8*b^
3 - 2*(a*b^2 - 13*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^5), 1/96*(24*sqrt(a)*b^3*arctan(1
/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^
2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^
3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x
+ e)))*cos(f*x + e)^5 + 3*(a^3 + 5*a^2*b + 15*a*b^2 - 5*b^3)*sqrt(-b)*arcta
n(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos
(f*x + e)^5 - 2*((3*a^2*b + 14*a*b^2 - 33*b^3)*cos(f*x + e)^4 - 8*b^3 - 2*(
a*b^2 - 13*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e))/(b^3*f*cos(f*x + e)^5)]
```

Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx=\int \sqrt{a+b\sec^2{(e+fx)}}\tan^6{(e+fx)}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**6,x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**6, x)
```

Maxima [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx=\int \sqrt{b\sec{(fx+e)}^2+a}\tan{(fx+e)}^6\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^6, x)
```

Giac [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx=\int \sqrt{b\sec{(fx+e)}^2+a}\tan{(fx+e)}^6\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx=\int \tan(e+f\,x)^6\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

```
[In] int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.383 $\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 165

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx = \frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{(a^2+6ab-3b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^{3/2}f} + \frac{(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8bf} + \frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f}$$

```
[Out] -1/8*(a^2+6*a*b-3*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2
))/b^(3/2)/f+arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/
f+1/8*(a-3*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f+1/4*(a+b+b*tan(f*x+
e)^2)^(1/2)*tan(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used

$= \{4226, 2000, 489, 596, 537, 223, 212, 385, 209\}$

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx = -\frac{(a^2+6ab-3b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8b^{3/2}f} + \frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f} + \frac{(a-3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8bf} + \frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4f}$$

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^4,x]
```

```
[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f -
 ((a^2 + 6*a*b - 3*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e
 + f*x]^2]])/(8*b^(3/2)*f) + ((a - 3*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e +
 f*x]^2])/(8*b*f) + (Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(4*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
```

```
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4\sqrt{a+b(1+x^2)}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4\sqrt{a+b+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f} - \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)+(-a+3b)x^2)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{4f}$$

$$= \frac{(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8bf} + \frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f} + \frac{\text{Subst}\left(\int \frac{-((a-3b)(a+b))+(-a^2-6ab+3b^2)x^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{8bf}$$

$$= \frac{(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8bf} + \frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f} + \frac{a\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} - \frac{(a^2+6ab-3b^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{8bf}$$

$$= \frac{(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8bf} + \frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f} + \frac{a\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{(a^2+6ab-3b^2)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8bf}$$

$$= \frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{(a^2+6ab-3b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^{3/2}f} + \frac{(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8bf} + \frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f}$$

Mathematica [A] (verified)

Time = 3.05 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.26

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx = \frac{\left(8\sqrt{a}b\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)-\frac{(a^2+6ab-3b^2)\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)\cos(e+fx)\sqrt{a+b\sec^2(e+f}{4\sqrt{2}bf\sqrt{a+2b+a\cos(2e+2fx)}} + \frac{(a-b+(a-5b)\cos(2(e+fx)))\sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\tan(e+fx)}{16bf}$$

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^4,x]
```

```
[Out] ((8*Sqrt[a]*b*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]
 - ((a^2 + 6*a*b - 3*b^2)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin
[e + f*x]^2]])/Sqrt[b])*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(4*Sqrt[2]
*b*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]) + ((a - b + (a - 5*b)*Cos[2*(e + f
*x)])*Sec[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x])/(16*b*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1064 vs. 2(143) = 286.

Time = 16.74 (sec) , antiderivative size = 1065, normalized size of antiderivative = 6.45

method	result	size
default	Expression too large to display	1065

```
[In] int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/f/(-a)^(1/2)/b^2*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(3*cos(f*x+e)*ln(4*(((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*(-a)^(1/2)*b^(5/2)+3*cos(
f*x+e)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e
)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(si
n(f*x+e)-1))*(-a)^(1/2)*b^(5/2)-6*cos(f*x+e)*ln(4*(((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*(-a)^(1/2)*b^(3/2)*a-6*co
s(f*x+e)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x
+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(
sin(f*x+e)-1))*(-a)^(1/2)*b^(3/2)*a-cos(f*x+e)*ln(4*(((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
```

```
(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*(-a)^(1/2)*b^(1/2)*a^2-
cos(f*x+e)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f
*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)
/(sin(f*x+e)-1))*(-a)^(1/2)*b^(1/2)*a^2+2*(-a)^(1/2)*sin(f*x+e)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b-10*(-a)^(1/2)*sin(f*x+e)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2+16*cos(f*x+e)*ln(4*(-a)^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^2+2*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*(-a)^(1/2)*a*b*tan(f*x+e)-10*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*(-a)^(1/2)*b^2*tan(f*x+e)+4*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*(-a)^(1/2)*b^2*tan(f*x+e)*sec(f*x+e)+4*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*b^2*tan(f*x+e)*sec(f*x+e)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(143) = 286$.

Time = 1.31 (sec) , antiderivative size = 1621, normalized size of antiderivative = 9.82

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] [1/32*(4*sqrt(-a)*b^2*cos(f*x + e)^3*log(128*a^4*cos(f*x + e)^8 - 256*(a^4
- a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4
+ a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*
b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*c
os(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^
2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*sin(f*x + e)) - (a^2 + 6*a*b - 3*b^2)*sqrt(b)*cos(f*x + e)^3*l
og(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*(
(a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*((a*b - 5*b^2
)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f
*x + e))/(b^2*f*cos(f*x + e)^3), 1/16*(2*sqrt(-a)*b^2*cos(f*x + e)^3*log(12
8*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos
(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b
^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - (a^2 + 6*a*b -
 3*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sq
rt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b
^2)*sin(f*x + e)))*cos(f*x + e)^3 + 2*((a*b - 5*b^2)*cos(f*x + e)^2 + 2*b^2
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f*x
```

```
+ e)^3), -1/32*(8*sqrt(a)*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a
*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a
^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^3 + (a^2 + 6*a*b
- 3*b^2)*sqrt(b)*cos(f*x + e)^3*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8
*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))
*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/
cos(f*x + e)^4) - 4*((a*b - 5*b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f*x + e)^3), -1/16*(4*
sqrt(a)*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3
 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(
f*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^3 + (a^2 + 6*a*b - 3*b^2)*sqrt(-b)*
arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))
)*cos(f*x + e)^3 - 2*((a*b - 5*b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f*x + e)^3)]
```

Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx=\int \sqrt{a+b\sec^2{(e+fx)}}\tan^4{(e+fx)}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**4,x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**4, x)
```

Maxima [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx=\int \sqrt{b\sec{(fx+e)}^2+a}\tan{(fx+e)}^4\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^4, x)
```

Giac [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx=\int \sqrt{b\sec{(fx+e)}^2+a}\tan{(fx+e)}^4\;dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx=\int \tan(e+f\,x)^4\,\sqrt{a+\frac{b}{\cos{(e+f\,x)}^2}}\,dx$$

```
[In] int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.384 $\int \sqrt{a+b\sec^2(e+fx)}\tan^2(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 118

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^2(e+fx)\,dx = -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(a-b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{b}f} + \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f+1/2*(a-b)*
arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/b^(1/2)+1/2*(a+b+b
*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4226, 2000, 489, 537, 223, 212, 385, 209}

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^2(e+fx)\,dx = -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f} + \frac{(a-b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{b}f} + \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}$$

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^2,x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) + ((a - b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
)/(2*Sqrt[b]*f) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+b(1+x^2)}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+b+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} - \frac{\text{Subst}\left(\int \frac{a+b+(-a+b)x^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} - \frac{a\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} + \frac{(a-b)\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} - \frac{a\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(a-b)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}$$

$$= -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(a-b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{b}f} + \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.61 (sec) , antiderivative size = 526, normalized size of antiderivative = 4.46

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^2(e+fx)\,dx$$

$$= \frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos(e+fx)\left(-\frac{i\left(-1+e^{2i(e+fx)}\right)}{\left(1+e^{2i(e+fx)}\right)^2}+\frac{-2\sqrt{a}\sqrt{b}fx+i\sqrt{a}\sqrt{b}\log\left(a+2b+ae^{2i(e+fx)}+\right.}{}\right.}{}$$

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^2,x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Cos[e + f*x]*(((-I)*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*
x)))^2 + (-2*Sqrt[a]*Sqrt[b]*f*x + I*Sqrt[a]*Sqrt[b]*Log[a + 2*b + a*E^((2*
I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e +
 f*x)))^2]] - I*Sqrt[a]*Sqrt[b]*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I
)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e +
f*x)))^2]] - a*Log[(2*(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*E^((
2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*f)/((a - b)*(1 + E^((2*I)
*(e + f*x))))] + b*Log[(2*(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*
E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*f)/((a - b)*(1 + E^((
2*I)*(e + f*x))))])/(Sqrt[b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)
*(e + f*x)))^2]))*Sqrt[a + b*Sec[e + f*x]^2])/(Sqrt[2]*f*Sqrt[a + 2*b + a*C
os[2*e + 2*f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 660 vs. 2(100) = 200.

Time = 12.30 (sec) , antiderivative size = 661, normalized size of antiderivative = 5.60

method	result
default	$-\frac{\sqrt{a+b\sec(fx+e)^2}\left(\ln\left(\frac{4\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{b}\cos(fx+e)+4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a-4a-4b}{\sin(fx+e)+1}\right)\sqrt{-a}\cos(fx+e)b^{\frac{3}{2}}+b^{\frac{3}{2}}\ln\left(-\right.\right.}{}$

```
[In] int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f/(-a)^(1/2)/b*(a+b*sec(f*x+e)^2)^(1/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)/(1+cos(f*x+e))*(ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*(-a)^(1/2)*cos(f*x+e)*b^(3/2)+b^(3/2)*ln
(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-
1))*(-a)^(1/2)*cos(f*x+e)-ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin
(f*x+e)*a-a-b)/(sin(f*x+e)+1))*(-a)^(1/2)*cos(f*x+e)*b^(1/2)*a-ln(-4*(((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(-a)^(
1/2)*cos(f*x+e)*b^(1/2)*a-2*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(-a)^(1/2)*b+4*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
-4*sin(f*x+e)*a)*cos(f*x+e)*a*b-2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*b*tan(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(100) = 200$.

Time = 0.62 (sec) , antiderivative size = 1471, normalized size of antiderivative = 12.47

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^2(e+fx)\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*b*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b
)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a
*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
+ e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7
*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e)) - (a - b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)
*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3
+ 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*sin(f*x + e))/(b*f*cos(f*x + e)), 1/8*(2*(a - b)*sqrt(-b)*arctan(
-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f
*x + e) + sqrt(-a)*b*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a
^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a
^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(
f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b
```

```
 + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*sin(f*x + e)) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e))/(b*f*cos(f*x + e)), 1/8*(2*sqrt(a)*b*arctan(1/4*(8*a^2*cos(f*x
 + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*
sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4
- a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x +
e) - (a - b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 +
 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e
))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2
)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
 + e))/(b*f*cos(f*x + e)), 1/4*(sqrt(a)*b*arctan(1/4*(8*a^2*cos(f*x + e)^5
- 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b
+ a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) + (a
- b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(
-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)
*sin(f*x + e)))*cos(f*x + e) + 2*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(b*f*cos(f*x + e))]
```

Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^2(e+fx)\,dx = \int \sqrt{a+b\sec^2{(e+fx)}}\tan^2{(e+fx)}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**2,x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**2, x)
```

Maxima [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^2(e+fx)\,dx = \int \sqrt{b\sec{(fx+e)}^2+a}\tan{(fx+e)}^2\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^2, x)
```

Giac [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^2(e+fx)\,dx=\int \sqrt{b\sec\left(fx+e\right)^2+a}\tan\left(fx+e\right)^2\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^2(e+fx)\,dx=\int \tan(e+f\,x)^2\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

```
[In] int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.385 $\int \sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 79

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx=\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f+arctanh(b^(
1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4213, 399, 223, 212, 385, 209}

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx=\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{a\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{a\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}\\
&= \frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 284, normalized size of antiderivative = 3.59

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{i\left(1+e^{2i(e+fx)}\right)\left(2\sqrt{b}\arctan\left(\frac{\sqrt{b}\left(-1+e^{2i(e+fx)}\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)+\sqrt{a}\text{arctanh}\left(\frac{a+2b+ae^{2i(e+fx)}}{\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)-\right.}{2\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}f}$$

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((-1/2*I)*(1 + E^((2*I)*(e + f*x)))*(2*Sqrt[b]*ArcTan[(Sqrt[b]*(-1 + E^((2*
I)*(e + f*x))))/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^
2]] + Sqrt[a]*ArcTanh[(a + 2*b + a*E^((2*I)*(e + f*x)))/(Sqrt[a]*Sqrt[4*b*E
^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])] - Sqrt[a]*ArcTanh[(a
+ a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)))/(Sqrt[a]*Sqrt[4*b*E^((2*
I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])])*Sqrt[a + b*Sec[e + f*x]^2
])/(Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(67) = 134.

Time = 4.09 (sec) , antiderivative size = 351, normalized size of antiderivative = 4.44

method	result
default	$\frac{\sqrt{a+b\sec(fx+e)^2}\left(\sqrt{b}\,\ln\left(-\frac{4\left(\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{b}\,\cos(fx+e)+\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-\sin(fx+e)a+a+b\right)}{\sin(fx+e)-1}\right)\sqrt{-a}+\sqrt{b}\,\ln\left(\frac{4\sqrt{\frac{b+a\cos(fx}{(1+\cos(fx+}}}{}\right.\right.}{2f\sqrt{}}$

```
[In] int((a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f/(-a)^(1/2)*(a+b*sec(f*x+e)^2)^(1/2)*(b^(1/2)*ln(-4*(((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(-a)^(1/2)+b^(1/2)
*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e
)+1))*(-a)^(1/2)+2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*s
in(f*x+e)*a)*a)*cos(f*x+e)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(67) = 134.

Time = 0.49 (sec) , antiderivative size = 1227, normalized size of antiderivative = 15.53

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*
a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
 e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*c
os(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e)) + 2*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*co
s(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)
)/f, 1/8*(4*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e)
)*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2
 + b^2)*sin(f*x + e))) + sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a
^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a
^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(
f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b
 + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*sin(f*x + e)))/f, -1/4*(sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5
- 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b
+ a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - sqrt(b)*log(((a^
2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))/f, -1/4*(sqrt(a)*arctan
(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b +
b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*
a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*
x + e))) - 2*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e
))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^
2 + b^2)*sin(f*x + e))))/f]
```

Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2{(e+fx)}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 3227, normalized size of antiderivative = 40.85

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] -1/2*(2*sqrt(a)*b^(3/2)*arctan2(a*sin(2*f*x + 2*e) + (a^2*cos(4*f*x + 4*e)^
2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4
*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*
sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4
*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arcta
n2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) +
2*(a + 2*b)*cos(2*f*x + 2*e) + a)), a*cos(2*f*x + 2*e) + (a^2*cos(4*f*x + 4
*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2
 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b
^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*c
os(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*a
rctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e
) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + a + 2*b) + a^(3/2)*sqrt(b)*arctan2
(2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^
2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b
)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1
/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*
e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f
*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x +
2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b
 + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2
*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos
(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x
 + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 4*a + 4*b) + a*b*log(((a + b
```

)*sqrt((16*b^2*cos(2*f*x + 2*e)^4 + 16*b^2*sin(2*f*x + 2*e)^4 + (a^2 + 2*a*
b + b^2)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^4 - 64*b^2*cos(2*f*x + 2*e)^3 + 96*
b^2*cos(2*f*x + 2*e)^2 - 8*((a*b + b^2)*cos(2*f*x + 2*e)^2 - (a*b + b^2)*si
n(2*f*x + 2*e)^2 + a*b + b^2 - 2*(a*b + b^2)*cos(2*f*x + 2*e))*abs(2*e^(2*I
*f*x + 2*I*e) + 2)^2 - 64*b^2*cos(2*f*x + 2*e) + 32*(b^2*cos(2*f*x + 2*e)^2
 - 2*b^2*cos(2*f*x + 2*e) + b^2)*sin(2*f*x + 2*e)^2 + 16*b^2)/(a^2 + 2*a*b
+ b^2))*cos(1/2*arctan2(8*(b*cos(2*f*x + 2*e) - b)*sin(2*f*x + 2*e)/((a + b
)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2), ((a + b)*abs(2*e^(2*I*f*x + 2*I*e) + 2
)^2 - 4*b*cos(2*f*x + 2*e)^2 + 4*b*sin(2*f*x + 2*e)^2 + 8*b*cos(2*f*x + 2*e
) - 4*b)/((a + b)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2)))^2 + (a + b)*sqrt((16*
b^2*cos(2*f*x + 2*e)^4 + 16*b^2*sin(2*f*x + 2*e)^4 + (a^2 + 2*a*b + b^2)*ab
s(2*e^(2*I*f*x + 2*I*e) + 2)^4 - 64*b^2*cos(2*f*x + 2*e)^3 + 96*b^2*cos(2*f
*x + 2*e)^2 - 8*((a*b + b^2)*cos(2*f*x + 2*e)^2 - (a*b + b^2)*sin(2*f*x + 2
*e)^2 + a*b + b^2 - 2*(a*b + b^2)*cos(2*f*x + 2*e))*abs(2*e^(2*I*f*x + 2*I*
e) + 2)^2 - 64*b^2*cos(2*f*x + 2*e) + 32*(b^2*cos(2*f*x + 2*e)^2 - 2*b^2*co
s(2*f*x + 2*e) + b^2)*sin(2*f*x + 2*e)^2 + 16*b^2)/(a^2 + 2*a*b + b^2))*sin
(1/2*arctan2(8*(b*cos(2*f*x + 2*e) - b)*sin(2*f*x + 2*e)/((a + b)*abs(2*e^(
2*I*f*x + 2*I*e) + 2)^2), ((a + b)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2 - 4*b*c
os(2*f*x + 2*e)^2 + 4*b*sin(2*f*x + 2*e)^2 + 8*b*cos(2*f*x + 2*e) - 4*b)/((
a + b)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2)))^2 + 4*b*cos(2*f*x + 2*e)^2 + 4*b
*sin(2*f*x + 2*e)^2 - 4*sqrt(a*b + b^2)*((16*b^2*cos(2*f*x + 2*e)^4 + 16*b^
2*sin(2*f*x + 2*e)^4 + (a^2 + 2*a*b + b^2)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^4
 - 64*b^2*cos(2*f*x + 2*e)^3 + 96*b^2*cos(2*f*x + 2*e)^2 - 8*((a*b + b^2)*c
os(2*f*x + 2*e)^2 - (a*b + b^2)*sin(2*f*x + 2*e)^2 + a*b + b^2 - 2*(a*b + b
^2)*cos(2*f*x + 2*e))*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2 - 64*b^2*cos(2*f*x +
 2*e) + 32*(b^2*cos(2*f*x + 2*e)^2 - 2*b^2*cos(2*f*x + 2*e) + b^2)*sin(2*f*
x + 2*e)^2 + 16*b^2)/(a^2 + 2*a*b + b^2))^(1/4)*cos(1/2*arctan2(8*(b*cos(2*
f*x + 2*e) - b)*sin(2*f*x + 2*e)/((a + b)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2)
, ((a + b)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2 - 4*b*cos(2*f*x + 2*e)^2 + 4*b*
sin(2*f*x + 2*e)^2 + 8*b*cos(2*f*x + 2*e) - 4*b)/((a + b)*abs(2*e^(2*I*f*x
+ 2*I*e) + 2)^2)))*sin(2*f*x + 2*e) - 4*(sqrt(a*b + b^2)*cos(2*f*x + 2*e) -
 sqrt(a*b + b^2))*((16*b^2*cos(2*f*x + 2*e)^4 + 16*b^2*sin(2*f*x + 2*e)^4 +
 (a^2 + 2*a*b + b^2)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^4 - 64*b^2*cos(2*f*x +
2*e)^3 + 96*b^2*cos(2*f*x + 2*e)^2 - 8*((a*b + b^2)*cos(2*f*x + 2*e)^2 - (a
*b + b^2)*sin(2*f*x + 2*e)^2 + a*b + b^2 - 2*(a*b + b^2)*cos(2*f*x + 2*e))*
abs(2*e^(2*I*f*x + 2*I*e) + 2)^2 - 64*b^2*cos(2*f*x + 2*e) + 32*(b^2*cos(2*
f*x + 2*e)^2 - 2*b^2*cos(2*f*x + 2*e) + b^2)*sin(2*f*x + 2*e)^2 + 16*b^2)/(
a^2 + 2*a*b + b^2))^(1/4)*sin(1/2*arctan2(8*(b*cos(2*f*x + 2*e) - b)*sin(2*
f*x + 2*e)/((a + b)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2), ((a + b)*abs(2*e^(2*
I*f*x + 2*I*e) + 2)^2 - 4*b*cos(2*f*x + 2*e)^2 + 4*b*sin(2*f*x + 2*e)^2 + 8
*b*cos(2*f*x + 2*e) - 4*b)/((a + b)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2))) - 8
*b*cos(2*f*x + 2*e) + 4*b)/((a + b)*abs(2*e^(2*I*f*x + 2*I*e) + 2)^2)) - (a
^(3/2) + 2*sqrt(a)*b)*sqrt(b)*arctan2(2*a*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f
*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x +
2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b

```
 + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2
*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin
(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x
 + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*a*cos(2*f*x + 2*e) + 2*(a^2
*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(
2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2
 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2
*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqr
t(a)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*c
os(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 2*a + 4*b))/(a*sqrt(
b)*f)
```

Giac [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec\left(fx+e\right)^2+a}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int((a + b/cos(e + f*x)^2)^(1/2), x)
```

3.386 $\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 69

$$\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f-cot(f*x+e)
*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4226, 2000, 486, 12, 385, 209}

$$\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

```
[In] Int[Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) - (Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b(1+x^2)}}{x^2(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{x^2(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{\text{Subst}\left(\int \frac{a}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{a\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{a\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

$$= -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.88

$$\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx =$$

$$-\frac{\cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\sqrt{a+b}\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}} + \sqrt{2}\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)\sin(e+fx)\right)}{\sqrt{a+b}f\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}}$$

```
[In] Integrate[Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] -((Cot[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(Sqrt[a + b]*Sqrt[(a + 2*b + a*C
os[2*(e + f*x)])/(a + b)] + Sqrt[2]*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/S
qrt[a + b]]*Sin[e + f*x]))/(Sqrt[a + b]*f*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)
])/(a + b)]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(61) = 122.

Time = 3.22 (sec) , antiderivative size = 224, normalized size of antiderivative = 3.25

method	result
default	$-\frac{\left(\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)a\sin(fx+}{f\sqrt{-a}\,(1+\cos(fx+e))\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}}$

```
[In] int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f/(-a)^(1/2)*((-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos
(f*x+e)+ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x
+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a
)*a*sin(f*x+e)+(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*(a+b
*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*cot(f*x+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(61) = 122$.

Time = 0.43 (sec) , antiderivative size = 499, normalized size of antiderivative = 7.23

$$\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8-256\,(a^4-a^3b)\cos\left(fx+e\right)^6+32\,(5\,a^4-14\,a^3b+5\,a^2b^2)\cos\left(fx+e\right)^4+\right.}{}\right.$$

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*
a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
 e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*c
os(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e))*sin(f*x + e) - 8*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
 e))/(f*sin(f*x + e)), 1/4*(sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a
^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^
2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e) - 4*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/(f*sin(f*x + e))]
```

Sympy [F]

$$\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2{(e+fx)}}\cot^2{(e+fx)}\,dx$$

```
[In] integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x)**2, x)
```

Maxima [F]

$$\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec{(fx+e)}^2+a}\cot{(fx+e)}^2\,dx$$

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^2, x)
```

Giac [F]

$$\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec{(fx+e)}^2+a}\cot{(fx+e)}^2\,dx$$

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \cot(e+f\,x)^2\,\sqrt{a+\frac{b}{\cos{(e+f\,x)}^2}}\,dx$$

```
[In] int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.387 $\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 114

$$\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a+2b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f+1/3*(3*a+2*
b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f-1/3*cot(f*x+e)^3*(a+b+b*ta
n(f*x+e)^2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4226, 2000, 486, 597, 12, 385, 209}

$$\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f} - \frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f} + \frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f(a+b)}$$

```
[In] Int[Cot[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 ((3*a + 2*b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*(a + b)*f) -
(Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b(1+x^2)}}{x^4(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{x^4(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f} + \frac{\text{Subst}\left(\int \frac{b-3(a+b)-2bx^2}{x^2(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{3f}$$

$$= \frac{(3a+2b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f} - \frac{\text{Subst}\left(\int -\frac{3a(a+b)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{3(a+b)f}$$

$$= \frac{(3a+2b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f} + \frac{a\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{(3a+2b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f} + \frac{a\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

$$= \frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a+2b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f}$$

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.54

$$\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{\sqrt{2}\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\frac{\csc^3(e+fx)(a+b-a\sin^2(e+fx))^{3/2}}{a+b}-3\csc(e+fx)\sqrt{a+b-a\sin^2(e+}\right.}{3f\sqrt{a+2b+a\cos(2e+2fx)}}$$

```
[In] Integrate[Cot[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/3*(Sqrt[2]*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*((Csc[e + f*x]^3*(a +
 b - a*Sin[e + f*x]^2)^(3/2))/(a + b) - 3*Csc[e + f*x]*Sqrt[a + b - a*Sin[e
 + f*x]^2]*(1 + (Sqrt[a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e +
 f*x])/(Sqrt[a + b]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]))))/(f*Sqrt[a + 2*
b + a*Cos[2*e + 2*f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 595 vs. 2(100) = 200.

Time = 4.71 (sec) , antiderivative size = 596, normalized size of antiderivative = 5.23

method	result
default	$-\frac{\left(3\sin(fx+e)a^2\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)\cos(fx+e)+3\sin(fx+e)ab\ln\left(\right.\right.}{}$

```
[In] int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f/(a+b)/(-a)^(1/2)*(3*sin(f*x+e)*a^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*cos(f*x+e)+3*sin(f*x+e)*a*b*ln(4*(-a)^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*cos(f*x+e)+4*cos(
f*x+e)^2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a+3*cos(f*x
+e)^2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b-3*sin(f*x+e)
*a^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)
+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)-3
*sin(f*x+e)*a*b*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(
f*x+e)*a)-3*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a-2*(-a)
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b)*(a+b*sec(f*x+e)^2)^(1
/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cot(f*x+e)*csc(f*x+e)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(100) = 200.

Time = 0.72 (sec) , antiderivative size = 629, normalized size of antiderivative = 5.52

$$\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{3\left((a+b)\cos\left(fx+e\right)^2-a-b\right)\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8-256\,(a^4-a^3b)\cos\left(fx+e\right)^6+32\,(5\,a^4-\right.}{} \right.$$

$$-\frac{3\left((a+b)\cos\left(fx+e\right)^2-a-b\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\,(a^2-ab)\cos(fx+e)^3+(a^2-6\,ab+b^2)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos}{\cos}}}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3\,a^2b)\cos(fx+e)^2\right)\sin(fx+e)}\right.}{12\left((a+b)f\cos\left(fx+e\right)^2-(a\right.}$$

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*((a + b)*cos(f*x + e)^2 - a - b)*sqrt(-a)*log(128*a^4*cos(f*x + e)
^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*c
os(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*
a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(
a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3
 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) + 8*((4*a + 3*b)*cos(
f*x + e)^3 - (3*a + 2*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2))/(((a + b)*f*cos(f*x + e)^2 - (a + b)*f)*sin(f*x + e)), -1/12*(3*((
a + b)*cos(f*x + e)^2 - a - b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8
*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a
*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e) - 4*((4*
a + 3*b)*cos(f*x + e)^3 - (3*a + 2*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2))/(((a + b)*f*cos(f*x + e)^2 - (a + b)*f)*sin(f*x + e))
]
```

Sympy [F]

$$\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2{(e+fx)}}\cot^4{(e+fx)}\,dx$$

```
[In] integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x)**4, x)
```

Maxima [F]

$$\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec{(fx+e)}^2+a}\cot{(fx+e)}^4\,dx$$

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^4, x)
```

Giac [F]

$$\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec{(fx+e)}^2+a}\cot{(fx+e)}^4\,dx$$

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \cot(e+f\,x)^4\,\sqrt{a+\frac{b}{\cos{(e+f\,x)}^2}}\,dx$$

```
[In] int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.388 $\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 167

$$\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{(15a^2+25ab+8b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} - \frac{(b-5(a+b))\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)f} - \frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5f}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f-1/15*(15*a
^2+25*a*b+8*b^2)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^2/f-1/15*(-4*b
-5*a)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f-1/5*cot(f*x+e)^5*(a+b
+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used

= {4226, 2000, 486, 597, 12, 385, 209}

$$\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= -\frac{(15a^2+25ab+8b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)^2} - \frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f} - \frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5f} - \frac{(b-5(a+b))\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)}$$

```
[In] Int[Cot[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) - ((15*a^2 + 25*a*b + 8*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])
/(15*(a + b)^2*f) - ((b - 5*(a + b))*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e +
f*x]^2])/(15*(a + b)*f) - (Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(
5*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b(1+x^2)}}{x^6(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{x^6(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5f} + \frac{\text{Subst}\left(\int \frac{b-5(a+b)-4bx^2}{x^4(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{5f}$$

$$= -\frac{(b-5(a+b))\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)f} - \frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5f} - \frac{\text{Subst}\left(\int \frac{-15a^2-25ab-8b^2+2b(b-5(a+b))x^2}{x^2(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{15(a+b)f}$$

$$= -\frac{(15a^2+25ab+8b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} - \frac{(b-5(a+b))\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)f} - \frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5f} + \frac{\text{Subst}\left(\int -\frac{15a(a+b)^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{15(a+b)^2f}$$

$$= -\frac{(15a^2+25ab+8b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} - \frac{(b-5(a+b))\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)f} - \frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5f} - \frac{a\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(15a^2+25ab+8b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} - \frac{(b-5(a+b))\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)f} - \frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5f} - \frac{a\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

$$= -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{(15a^2+25ab+8b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} - \frac{(b-5(a+b))\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)f} - \frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5f}$$

Mathematica [A] (verified)

Time = 2.88 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.07

$$\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= -\frac{\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f\sqrt{a+2b+a\cos(2e+2fx)}}$$
$$-\frac{\cot(e+fx)\left(23a^2+40ab+15b^2-(11a^2+21ab+10b^2)\csc^2(e+fx)+3(a+b)^2\csc^4(e+fx)\right)\sqrt{a+b\sec^2(e+fx)}}{15(a+b)^2f}$$

```
[In] Integrate[Cot[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Sqrt[2]*Sqrt[a]*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x
]^2]]*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(f*Sqrt[a + 2*b + a*Cos[2*e
+ 2*f*x]])) - (Cot[e + f*x]*(23*a^2 + 40*a*b + 15*b^2 - (11*a^2 + 21*a*b +
10*b^2)*Csc[e + f*x]^2 + 3*(a + b)^2*Csc[e + f*x]^4)*Sqrt[a + b*Sec[e + f*x
]^2])/(15*(a + b)^2*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1015 vs. 2(149) = 298.

Time = 5.90 (sec) , antiderivative size = 1016, normalized size of antiderivative = 6.08

method	result	size
default	Expression too large to display	1016

```
[In] int(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15/f/(a+b)^2/(-a)^(1/2)*(15*sin(f*x+e)^3*a^3*ln(4*(-a)^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*cos(f*x+e)+30*sin(f*x+e)^3*a^2*b*l
n(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a
)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*cos(f*x
+e)+15*sin(f*x+e)^3*a*b^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)-4*sin(f*x+e)*a)*cos(f*x+e)-23*cos(f*x+e)^4*(-a)^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*a^2-40*cos(f*x+e)^4*(-a)^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*a*b-15*cos(f*x+e)^4*(-a)^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*b^2-15*sin(f*x+e)^3*a^3*ln(4*(-a)^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)-30*sin(f*x+e)^3*a^2*b*ln(4*(-a
)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)-15*sin(f*x+e)^
```

```
3*a*b^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x
+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a
)+35*cos(f*x+e)^2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^
2+59*cos(f*x+e)^2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*
b+20*cos(f*x+e)^2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^
2-15*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2-25*(-a)^(1/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b-8*(-a)^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2)*(a+b*sec(f*x+e)^2)^(1/2)/((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cot(f*x+e)*csc(f*x+e)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(149) = 298$.

Time = 2.30 (sec) , antiderivative size = 849, normalized size of antiderivative = 5.08

$$\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{15\left((a^2+2\,ab+b^2)\cos\left(fx+e\right)^4-2\,(a^2+2\,ab+b^2)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2\right)\sqrt{-a}\log\left(128\,a^4\cos\right.}{}\right.$$

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/120*(15*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(
f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(
a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e
)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*
a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*
b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 -
7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) - 8*((23*a^2 + 40*a*b + 15*b^2)
*cos(f*x + e)^5 - (35*a^2 + 59*a*b + 20*b^2)*cos(f*x + e)^3 + (15*a^2 + 25*
a*b + 8*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((
a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^
2 + (a^2 + 2*a*b + b^2)*f)*sin(f*x + e)), 1/60*(15*((a^2 + 2*a*b + b^2)*cos
(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqr
t(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2
 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e
)^2)*sin(f*x + e)))*sin(f*x + e) - 4*((23*a^2 + 40*a*b + 15*b^2)*cos(f*x +
e)^5 - (35*a^2 + 59*a*b + 20*b^2)*cos(f*x + e)^3 + (15*a^2 + 25*a*b + 8*b^2
```

```
)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^2 + 2*a*b
 + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 +
2*a*b + b^2)*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\cot^6(e+fx)\,dx$$

```
[In] integrate(cot(f*x+e)**6*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x)**6, x)
```

Maxima [F]

$$\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cot(fx+e)^6\,dx$$

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^6, x)
```

Giac [F]

$$\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cot(fx+e)^6\,dx$$

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \cot(e+f\,x)^6\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,dx$$

```
[In] int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.389 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx=-\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f}+\frac{\left(a+b\sec^2(e+fx)\right)^{7/2}}{7b^2f}$$

```
[Out] -a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f+1/3*(a+b*sec(f*x+e)^2)
^(3/2)/f-1/5*(a+2*b)*(a+b*sec(f*x+e)^2)^(5/2)/b^2/f+1/7*(a+b*sec(f*x+e)^2)^
(7/2)/b^2/f+a*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4224, 457, 90, 52, 65, 214}

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx=-\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{7/2}}{7b^2f}-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}+\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^5,x]
```

```
[Out] -((a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f) + (a*Sqrt[a + b*
Sec[e + f*x]^2])/f + (a + b*Sec[e + f*x]^2)^(3/2)/(3*f) - ((a + 2*b)*(a + b
*Sec[e + f*x]^2)^(5/2))/(5*b^2*f) + (a + b*Sec[e + f*x]^2)^(7/2)/(7*b^2*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
```

```
[2*n, p])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(-1+x^2)^2(a+bx^2)^{3/2}}{x}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(-1+x)^2(a+bx)^{3/2}}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(-a-2b)(a+bx)^{3/2}}{b} + \frac{(a+bx)^{3/2}}{x} + \frac{(a+bx)^{5/2}}{b}\right)\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f} + \frac{\left(a+b\sec^2(e+fx)\right)^{7/2}}{7b^2f} + \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f} - \frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f} + \frac{\left(a+b\sec^2(e+fx)\right)^{7/2}}{7b^2f} + \frac{a\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{a\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f} - \frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f} + \frac{\left(a+b\sec^2(e+fx)\right)^{7/2}}{7b^2f} + \frac{a^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{a\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f} - \frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f} + \frac{\left(a+b\sec^2(e+fx)\right)^{7/2}}{7b^2f} + \frac{a^2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf}$$

$$= -\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{a\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f} - \frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f} + \frac{\left(a+b\sec^2(e+fx)\right)^{7/2}}{7b^2f}$$

Mathematica [A] (verified)

Time = 2.40 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.10

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx = \frac{\sqrt{a+b\sec^2(e+fx)}\left(35b^2(a+b\sec^2(e+fx))-42b(a+b\sec^2(e+fx))^2+15(a+b\sec^2(e+fx)\right.}{105b^2f}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^5,x]
```

```
[Out] (Sqrt[a + b*Sec[e + f*x]^2]*(35*b^2*(a + b*Sec[e + f*x]^2) - 42*b*(a + b*Se
c[e + f*x]^2)^2 + 15*(a + b*Sec[e + f*x]^2)^3 - (105*a*b^2*ArcTanh[Sqrt[(a
+ b*Sec[e + f*x]^2)/a]])/Sqrt[(a + b*Sec[e + f*x]^2)/a] + 21*a*(5*b^2 - (a
+ b*Sec[e + f*x]^2)^2)))/(105*b^2*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 825 vs. 2(115) = 230.

Time = 16.96 (sec) , antiderivative size = 826, normalized size of antiderivative = 6.12

method	result
default	$-\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}\left(105\ln\left(4\cos(fx+e)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{a}+4\cos(fx+e)a+4\sqrt{a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\right)\cos(fx+e)^3a^{\frac{3}{2}}b^2+6\cos(f\right.}{}$

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/105/f/b^2*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(105*ln(4*cos(f*x+e)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*cos(f*x+e)^3*a^(3/2)*b^2+6*cos(f*x+e)^3*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3+42*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*cos(f*x+e)^3*a^2*b-140*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*cos(f*x+e)^3*b^2*a+6*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*a^3+42*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*a^2*b-140*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^2*b^2*a
-3*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b+84*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a*b^2-35*cos(f*x+e)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3-3*a^2*b*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)+84*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^2-35*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3-24*a*b^2*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)+42*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
```

```
^(1/2)*b^3*sec(f*x+e)-24*a*b^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
sec(f*x+e)^2+42*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3*sec(f*x+e)^
2-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3*sec(f*x+e)^3-15*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3*sec(f*x+e)^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(115) = 230$.

Time = 7.52 (sec) , antiderivative size = 527, normalized size of antiderivative = 3.90

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^5(e+fx)\,dx = \left[\frac{105\,a^{\frac{3}{2}}b^2\cos(fx+e)^6\log\left(128\,a^4\cos(fx+e)^8+256\,a^3b\cos(fx+e)^6+160\,a^2b^2\cos(fx+e)^4\right.}{}\right.$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] [1/840*(105*a^(3/2)*b^2*cos(f*x + e)^6*log(128*a^4*cos(f*x + e)^8 + 256*a^3
*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 +
b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x
 + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)) - 8*(2*(3*a^3 + 21*a^2*b - 70*a*b^2)*cos(f*x + e)^6 - (3*a^2*b - 84
*a*b^2 + 35*b^3)*cos(f*x + e)^4 - 15*b^3 - 6*(4*a*b^2 - 7*b^3)*cos(f*x + e)
^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(b^2*f*cos(f*x + e)^6), 1/
420*(105*sqrt(-a)*a*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x +
e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos
(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2))*cos(f*x + e)^6 - 4*(2*(3*a^3
 + 21*a^2*b - 70*a*b^2)*cos(f*x + e)^6 - (3*a^2*b - 84*a*b^2 + 35*b^3)*cos(
f*x + e)^4 - 15*b^3 - 6*(4*a*b^2 - 7*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2))/(b^2*f*cos(f*x + e)^6)]
```

Sympy [F]

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^5(e+fx)\,dx = \int (a+b\sec^2(e+fx))^{\frac{3}{2}}\tan^5(e+fx)\,dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**5,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x)**5, x)
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^5\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^5, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2026 vs. $2(115) = 230$.

Time = 3.26 (sec) , antiderivative size = 2026, normalized size of antiderivative = 15.01

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx=\text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="giac")
[Out] 2/105*(105*a^2*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan
(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2
 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))*sgn(cos(f*x
 + e))/sqrt(-a) - 2*(105*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^13*a^2*sgn(cos(f*x + e)) - 1575*(sqrt
(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*
f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 +
a + b))^12*sqrt(a + b)*a^2*sgn(cos(f*x + e)) + 70*(129*a^3 + 21*a^2*b - 96*
a*b^2 - 32*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(
1/2*f*x + 1/2*e)^2 + a + b))^11*sgn(cos(f*x + e)) - 70*(387*a^3 - 417*a^2*b
 - 96*a*b^2 + 128*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2
*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^10*sqrt(a + b)*sgn(cos(f*x + e)) + 7*(6
525*a^4 - 11790*a^3*b - 2235*a^2*b^2 + 9088*a*b^3 - 1344*b^4)*(sqrt(a + b)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/
2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^
9*sgn(cos(f*x + e)) - 7*(5355*a^4 - 28290*a^3*b + 28995*a^2*b^2 - 3008*a*b^
3 - 896*b^4)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2
*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2
*f*x + 1/2*e)^2 + a + b))^8*sqrt(a + b)*sgn(cos(f*x + e)) - 4*(1575*a^5 + 5
0505*a^4*b - 80115*a^3*b^2 - 4949*a^2*b^3 + 45920*a*b^4 - 5024*b^5)*(sqrt(a
 + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*
x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a
```

```
+ b))^7*sgn(cos(f*x + e)) + 28*(1755*a^5 + 2325*a^4*b - 17655*a^3*b^2 + 202
07*a^2*b^3 - 3232*a*b^4 + 768*b^5)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sq
rt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^6*sqrt(a + b)*sgn(cos(f*x +
 e)) - 7*(7335*a^6 - 9300*a^5*b - 47430*a^4*b^2 + 98572*a^3*b^3 - 13353*a^2
*b^4 - 55296*a*b^5 - 2944*b^6)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a
*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*
e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^5*sgn(cos(f*x + e)) + 7*(3225*a
^6 - 20460*a^5*b + 9990*a^4*b^2 + 79028*a^3*b^3 - 136023*a^2*b^4 + 48000*a*
b^5 + 256*b^6)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1
/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1
/2*f*x + 1/2*e)^2 + a + b))^4*sqrt(a + b)*sgn(cos(f*x + e)) + 14*(45*a^7 +
7125*a^6*b - 13230*a^5*b^2 - 8006*a^4*b^3 + 52305*a^3*b^4 - 35199*a^2*b^5 -
 21472*a*b^6 - 2592*b^7)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^3*sgn(cos(f*x + e)) - 14*(375*a^7 + 2
895*a^6*b - 11370*a^5*b^2 + 19246*a^4*b^3 + 547*a^3*b^4 - 43053*a^2*b^5 + 3
9072*a*b^6 + 3456*b^7)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2
*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2*sqrt(a + b)*sgn(cos(f*x + e)) + 7*(31
5*a^8 + 1530*a^7*b - 5955*a^6*b^2 + 18508*a^5*b^3 - 21499*a^4*b^4 - 10598*a
^3*b^5 + 58275*a^2*b^6 - 43392*a*b^7 - 3648*b^8)*(sqrt(a + b)*tan(1/2*f*x +
 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*
tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sgn(cos(f*x +
 e)) - (315*a^8 + 1050*a^7*b - 4515*a^6*b^2 + 28364*a^5*b^3 - 81403*a^4*b^4
 + 133050*a^3*b^5 - 133053*a^2*b^6 + 53952*a*b^7 + 4992*b^8)*sqrt(a + b)*sg
n(cos(f*x + e)))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
+ 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*ta
n(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 -
sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x
+ 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a + b) + a - 3*b)^7)
/f
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx=\int \tan(e+f\,x)^5\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

```
[In] int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.390 $\int (a+b\sec^2(e+fx))^{3/2}\tan^3(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 104

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^3(e+fx)\,dx = \frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{a\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{(a+b\sec^2(e+fx))^{3/2}}{3f} + \frac{(a+b\sec^2(e+fx))^{5/2}}{5bf}$$

```
[Out] a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f-1/3*(a+b*sec(f*x+e)^2)^
(3/2)/f+1/5*(a+b*sec(f*x+e)^2)^(5/2)/b/f-a*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4224, 457, 81, 52, 65, 214}

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^3(e+fx)\,dx = \frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{(a+b\sec^2(e+fx))^{5/2}}{5bf} - \frac{(a+b\sec^2(e+fx))^{3/2}}{3f} - \frac{a\sqrt{a+b\sec^2(e+fx)}}{f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^3,x]
```

```
[Out] (a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - (a*Sqrt[a + b*Sec
[e + f*x]^2])/f - (a + b*Sec[e + f*x]^2)^(3/2)/(3*f) + (a + b*Sec[e + f*x]^
2)^(5/2)/(5*b*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a+bx^2)^{3/2}}{x}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(-1+x)(a+bx)^{3/2}}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{(a+b\sec^2(e+fx))^{5/2}}{5bf} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{(a+b\sec^2(e+fx))^{3/2}}{3f} + \frac{(a+b\sec^2(e+fx))^{5/2}}{5bf} - \frac{a\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{a\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{(a+b\sec^2(e+fx))^{3/2}}{3f} + \frac{(a+b\sec^2(e+fx))^{5/2}}{5bf} - \frac{a^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{a\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{(a+b\sec^2(e+fx))^{3/2}}{3f} + \frac{(a+b\sec^2(e+fx))^{5/2}}{5bf} - \frac{a^2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf}$$

$$= \frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{a\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{(a+b\sec^2(e+fx))^{3/2}}{3f} + \frac{(a+b\sec^2(e+fx))^{5/2}}{5bf}$$

Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.95

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^3(e+fx)\,dx = \frac{15a^{3/2}b\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right) - 15ab\sqrt{a+b\sec^2(e+fx)} - 5b(a+b\sec^2(e+fx))^{3/2} + 3(a+b}{15bf}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^3,x]
[Out] (15*a^(3/2)*b*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]] - 15*a*b*Sqrt[a +
 b*Sec[e + f*x]^2] - 5*b*(a + b*Sec[e + f*x]^2)^(3/2) + 3*(a + b*Sec[e + f*
x]^2)^(5/2))/(15*b*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. 2(88) = 176.

Time = 14.22 (sec) , antiderivative size = 518, normalized size of antiderivative = 4.98

method	result
default	$\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}\left(15\cos(fx+e)^3a^{\frac{3}{2}}\ln\left(4\cos(fx+e)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{a}+4\cos(fx+e)a+4\sqrt{a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\right)b+3\cos(fx+\right.}{}$

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15/f/b*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(15*cos(f*x+e)^3*a^(3/2)*ln(4*cos(f*x+
e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*b+3*cos(f*x+e)^3*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2-20*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*a*b+3*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*a^2-20*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*a*b+6*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b-5*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*cos(f*x+e)+6*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*a*b-5*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b
^2+3*b^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)+3*b^2*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(88) = 176.

Time = 1.97 (sec) , antiderivative size = 443, normalized size of antiderivative = 4.26

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx = \left[\frac{15\,a^{\frac{3}{2}}b\cos\left(fx+e\right)^4\log\left(128\,a^4\cos\left(fx+e\right)^8+256\,a^3b\cos\left(fx+e\right)^6+160\,a^2b^2\cos\left(fx+e\right)^4+\right.}{}\right.$$

$$-\frac{15\sqrt{-a}ab\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^4+8\,ab\cos(fx+e)^2+b^2\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4+3\,a^2b\cos(fx+e)^2+ab^2\right)}\right)\cos\left(fx+e\right)^4-4\left(\left(3\,a^2-20\,ab\right)\cos\left(\right.\right.}{60\,bf\cos\left(fx+e\right)^4}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] [1/120*(15*a^(3/2)*b*cos(f*x + e)^4*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*
cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4
 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x +
e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)) + 8*((3*a^2 - 20*a*b)*cos(f*x + e)^4 + (6*a*b - 5*b^2)*cos(f*x + e)^2
+ 3*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(b*f*cos(f*x + e)^4),
 -1/60*(15*sqrt(-a)*a*b*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x +
e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos
(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2))*cos(f*x + e)^4 - 4*((3*a^2 -
 20*a*b)*cos(f*x + e)^4 + (6*a*b - 5*b^2)*cos(f*x + e)^2 + 3*b^2)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2))/(b*f*cos(f*x + e)^4)]
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\tan^3\left(e+fx\right)dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x)**3, x)
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^3\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1431 vs. $2(88) = 176$.

Time = 1.67 (sec) , antiderivative size = 1431, normalized size of antiderivative = 13.76

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx=\text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*a^2*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2
```

+ 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))*sgn(cos(f*x + e))/sqrt(-a) - 2*(15*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^9*a^2*sgn(cos(f*x + e)) - 15*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^8*(7*a^2 - 8*a*b - 4*b^2)*sqrt(a + b)*sgn(cos(f*x + e)) + 20*(15*a^3 - 21*a^2*b - 12*a*b^2 + 8*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^7*sgn(cos(f*x + e)) - 20*(21*a^3 - 63*a^2*b + 60*a*b^2 - 4*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^6*sqrt(a + b)*sgn(cos(f*x + e)) + 2*(105*a^4 - 630*a^3*b + 1065*a^2*b^2 + 360*a*b^3 - 16*b^4)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^5*sgn(cos(f*x + e)) + 10*(21*a^4 + 42*a^3*b - 303*a^2*b^2 + 336*a*b^3 + 4*b^4)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^4*sqrt(a + b)*sgn(cos(f*x + e)) - 20*(21*a^5 - 21*a^4*b - 69*a^3*b^2 + 225*a^2*b^3 + 4*a*b^4 + 8*b^5)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^3*sgn(cos(f*x + e)) + 20*(15*a^5 - 39*a^4*b + 57*a^3*b^2 + 87*a^2*b^3 - 244*a*b^4 - 28*b^5)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2*sqrt(a + b)*sgn(cos(f*x + e)) - 5*(21*a^6 - 60*a^5*b + 222*a^4*b^2 - 172*a^3*b^3 - 491*a^2*b^4 + 848*a*b^5 + 96*b^6)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sgn(cos(f*x + e)) + (15*a^6 - 60*a^5*b + 390*a^4*b^2 - 980*a^3*b^3 + 1543*a^2*b^4 - 984*a*b^5 - 132*b^6)*sqrt(a + b)*sgn(cos(f*x + e)))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a + b) + a - 3*b)^5)/f

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx=\int \tan(e+f\,x)^3\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

```
[In] int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.391 $\int (a+b\sec^2(e+fx))^{3/2}\tan(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int (a+b\sec^2(e+fx))^{3/2}\tan(e+fx)\,dx=-\frac{a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{(a+b\sec^2(e+fx))^{3/2}}{3f}$$

```
[Out] -a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f+1/3*(a+b*sec(f*x+e)^2)
^(3/2)/f+a*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4224, 272, 52, 65, 214}

$$\int (a+b\sec^2(e+fx))^{3/2}\tan(e+fx)\,dx=-\frac{a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{(a+b\sec^2(e+fx))^{3/2}}{3f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x],x]
```

```
[Out] -((a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f) + (a*Sqrt[a + b*
Sec[e + f*x]^2])/f + (a + b*Sec[e + f*x]^2)^(3/2)/(3*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f} + \frac{a\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$=\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{(a+b\sec^2(e+fx))^{3/2}}{3f}+\frac{a^2\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,\sec^2(e+fx)\right)}{2f}$$

$$=\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{(a+b\sec^2(e+fx))^{3/2}}{3f}+\frac{a^2\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\sec^2(e+fx)}\right)}{bf}$$

$$=-\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{(a+b\sec^2(e+fx))^{3/2}}{3f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.33 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08

$$\int(a+b\sec^2(e+fx))^{3/2}\tan(e+fx)\,dx=\frac{2b\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},-\frac{a\cos^2(e+fx)}{b}\right)(a+b\sec^2(e+fx))^{3/2}}{3f\sqrt{1+\frac{a\cos^2(e+fx)}{b}}(a+2b+a\cos(2(e+fx)))}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x],x]
```

```
[Out] (2*b*Hypergeometric2F1[-3/2, -3/2, -1/2, -((a*Cos[e + f*x]^2)/b)]*(a + b*Se
c[e + f*x]^2)^(3/2))/(3*f*Sqrt[1 + (a*Cos[e + f*x]^2)/b]*(a + 2*b + a*Cos[2
*(e + f*x)]))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}{3f}-\frac{a^{\frac{3}{2}}\ln\left(\frac{2a+2\sqrt{a}\sqrt{a+b\sec(fx+e)^2}}{\sec(fx+e)}\right)}{f}+\frac{a\sqrt{a+b\sec(fx+e)^2}}{f}$	81
default	$\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}{3f}-\frac{a^{\frac{3}{2}}\ln\left(\frac{2a+2\sqrt{a}\sqrt{a+b\sec(fx+e)^2}}{\sec(fx+e)}\right)}{f}+\frac{a\sqrt{a+b\sec(fx+e)^2}}{f}$	81

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(a+b*sec(f*x+e)^2)^(3/2)/f-1/f*a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sec(f*x+e
)^2)^(1/2))/sec(f*x+e))+a*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(66) = 132.

Time = 0.60 (sec) , antiderivative size = 373, normalized size of antiderivative = 4.78

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan(e+fx)\,dx = \left[\frac{3\,a^{\frac{3}{2}}\cos(fx+e)^2\log\left(128\,a^4\cos(fx+e)^8+256\,a^3b\cos(fx+e)^6+160\,a^2b^2\cos(fx+e)^4+32\right.}{}\right.$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] [1/24*(3*a^(3/2)*cos(f*x + e)^2*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(
f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8
*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4
 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))
 + 8*(4*a*cos(f*x + e)^2 + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/
(f*cos(f*x + e)^2), 1/12*(3*sqrt(-a)*a*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8
*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2))*cos(f*x + e)^
2 + 4*(4*a*cos(f*x + e)^2 + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))
/(f*cos(f*x + e)^2)]
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan(e+fx)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\tan\left(e+fx\right)dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x), x)
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 867 vs. $2(66) = 132$.

Time = 1.00 (sec) , antiderivative size = 867, normalized size of antiderivative = 11.12

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan(e+fx)\,dx=\text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="giac")
[Out] 2/3*(3*a^2*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2
*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))*sgn(cos(f*x + e
))/sqrt(-a) + 2*(3*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*t
an(1/2*f*x + 1/2*e)^2 + a + b))^5*(2*a*b + b^2)*sgn(cos(f*x + e)) - 3*(sqrt
(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*
f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 +
a + b))^4*(6*a*b - b^2)*sqrt(a + b)*sgn(cos(f*x + e)) + 2*(6*a^2*b - 15*a*b
^2 - b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)
^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*
x + 1/2*e)^2 + a + b))^3*sgn(cos(f*x + e)) + 6*(2*a^2*b + 7*a*b^2 + b^3)*(s
qrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1
/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2
 + a + b))^2*sqrt(a + b)*sgn(cos(f*x + e)) - 3*(6*a^3*b - a^2*b^2 - 28*a*b^
3 - 5*b^4)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e
)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f
*x + 1/2*e)^2 + a + b))*sgn(cos(f*x + e)) + (6*a^3*b - 21*a^2*b^2 + 28*a*b^
3 + 7*b^4)*sqrt(a + b)*sgn(cos(f*x + e)))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e
)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/
2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*(sqrt(a + b)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/
2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*
sqrt(a + b) + a - 3*b)^3)/f
```

Mupad [B] (verification not implemented)

Time = 23.60 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan(e+fx)\,dx=\frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{3\,f}-\frac{a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\sqrt{a}}\right)}{f}+\frac{a\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{f}$$

```
[In] int(tan(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] (a + b/cos(e + f*x)^2)^(3/2)/(3*f) - (a^(3/2)*atanh((a + b/cos(e + f*x)^2)^
(1/2)/a^(1/2)))/f + (a*(a + b/cos(e + f*x)^2)^(1/2))/f
```

3.392 $\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}-\frac{(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f}+\frac{b\sqrt{a+b\sec^2(e+fx)}}{f}$$

```
[Out] a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f-(a+b)^(3/2)*arctanh((a+
b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/f+b*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4224, 457, 86, 162, 65, 214}

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}-\frac{(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f}+\frac{b\sqrt{a+b\sec^2(e+fx)}}{f}$$

```
[In] Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - ((a + b)^(3/2)*Ar
cTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/f + (b*Sqrt[a + b*Sec[e + f*
x]^2])/f
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Dist[1/(b*d), I
nt[(b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{x(-1+x^2)}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{(-1+x)x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{b\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{\text{Subst}\left(\int \frac{a^2+b(2a+b)x}{(-1+x)x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{b\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{a^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f} + \frac{(a+b)^2\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{b\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{a^2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf} + \frac{(a+b)^2\text{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf}$$

$$= \frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f} + \frac{b\sqrt{a+b\sec^2(e+fx)}}{f}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.08 (sec) , antiderivative size = 506, normalized size of antiderivative = 5.56

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\sqrt{2}e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(\frac{2b}{1+e^{2i(e+fx)}}+\frac{-2ia^{3/2}fx+2(a+b)^{3/2}\log(1}{}\right.}{}$$

```
[In] Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*((2*b)/(1 + E^((2*I)*(e + f*x))) + ((-2*I)*a^(
3/2)*f*x + 2*(a + b)^(3/2)*Log[1 - E^((2*I)*(e + f*x))] + a^(3/2)*Log[a + 2
*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 +
E^((2*I)*(e + f*x)))^2]] + a^(3/2)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((
2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e
 + f*x)))^2]] - 2*a*Sqrt[a + b]*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^((2
*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)
```

```
*(e + f*x)))^2]] - 2*b*Sqrt[a + b]*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^
((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2
*I)*(e + f*x)))^2]])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*
x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3
/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1329 vs. $2(77) = 154$.

Time = 5.57 (sec) , antiderivative size = 1330, normalized size of antiderivative = 14.62

method	result	size
default	Expression too large to display	1330

```
[In] int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f/(a+b)^(5/2)*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(2*(a+b)^(5/2)*cos(f*x+e)^3*l
n(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*
x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^(3/2)+2*(a+
b)^(5/2)*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b+2*(a+b)
^(5/2)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b+cos(f*x+e
)^3*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/
2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f
*x+e)*a+b)/(1+cos(f*x+e)))*a^4+4*cos(f*x+e)^3*ln(2/(a+b)^(1/2)*(((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^3*b+
6*cos(f*x+e)^3*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^
(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^2*b^2+4*cos(f*x+e)^3*ln(2/(a+b)^(1/
2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(
f*x+e)))*a*b^3+cos(f*x+e)^3*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*b^4-cos(f*x+e)^3*ln(-4*(
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x
+e)))*a^4-4*cos(f*x+e)^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(
1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^3*b-6*cos(f*x+e)^3*ln(-4*(((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^2
*b^2-4*cos(f*x+e)^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b
)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+
cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a*b^3-cos(f*x+e)^3*ln(-4*(((b+a*cos(f*x+e)
```

```
^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*b^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(77) = 154$.

Time = 0.58 (sec) , antiderivative size = 1075, normalized size of antiderivative = 11.81

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(a^(3/2)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a
^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x +
 e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e
)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 2*(a + b)^(3/2)
*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x +
e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)
^2 + 1)) + 8*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/f, 1/8*(4*(a +
b)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b +
 b^2)) + a^(3/2)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 16
0*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*
x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x
+ e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 8*b*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2))/f, -1/4*(sqrt(-a)*a*arctan(1/4*(8*a^2*
cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b
^2)) - (a + b)^(3/2)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b
 + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x +
e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e
)^4 - 2*cos(f*x + e)^2 + 1)) - 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2))/f, -1/4*(sqrt(-a)*a*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x +
 e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*co
s(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 2*(a + b)*sqrt(-a - b)*ar
ctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) - 4*b*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/f]
```

Sympy [F]

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\cot\left(e+fx\right)dx$$

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*cot(e + f*x), x)
```

Maxima [F]

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)\,dx$$

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 718 vs. $2(77) = 154$.

Time = 4.13 (sec) , antiderivative size = 718, normalized size of antiderivative = 7.89

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\text{Too large to display}$$

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] -1/2*(4*a^2*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/
2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))*sgn(cos(f*x +
e))/sqrt(-a) - (a + b)^(3/2)*log(abs(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b)))*sgn(cos(f*x
 + e)) + (a + b)^(3/2)*log(abs(-sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a
*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*
e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b)))*sgn(cos(f*x + e)
) + (a^2 + 2*a*b + b^2)*log(abs(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt
(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/
2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a + b) + sqrt(a + b)*(a - b)
))*sgn(cos(f*x + e))/sqrt(a + b) - 8*((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 -
 sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x
 + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*b^2*sgn(cos(f*x + e)) +
```

```
sqrt(a + b)*b^2*sgn(cos(f*x + e)))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*(sqrt(a + b)*tan(1/2
*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4
- 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a
+ b) + a - 3*b))/f
```

Mupad [F(-1)]

Timed out.

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \cot(e+f\,x)\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

```
[In] int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.393 $\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 114

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=-\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{(2a-b)\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f}-\frac{(a+b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}$$

```
[Out] -a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f+1/2*(2*a-b)*arctanh((a
+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/f-1/2*(a+b)*cot(f*x+e)^2*(a
+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4224, 457, 100, 162, 65, 214}

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=-\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{(2a-b)\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f}-\frac{(a+b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}$$

```
[In] Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f) + ((2*a - b)*Sqr
t[a + b]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(2*f) - ((a + b)*
Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(2*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{x(-1+x^2)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{(-1+x)^2 x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{(a+b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} - \frac{\text{Subst}\left(\int \frac{a^2+\frac{1}{2}(a-b)bx}{(-1+x)x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{(a+b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{a^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f} - \frac{((2a-b)(a+b))\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{4f}$$

$$= -\frac{(a+b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{a^2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf} - \frac{((2a-b)(a+b))\text{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{2bf}$$

$$= -\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{(2a-b)\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f} - \frac{(a+b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.31 (sec) , antiderivative size = 622, normalized size of antiderivative = 5.46

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{\sqrt{2}e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(\frac{(a+b)\left(1+e^{2i(e+fx)}\right)}{\left(-1+e^{2i(e+fx)}\right)^2}-\frac{-2ia^{3/2}\sqrt{a+b}fx+(}{}\right.}{}$$

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*(((a + b)*(1 + E^((2*I)*(e + f*x))))/(-1 + E^(
(2*I)*(e + f*x)))^2 - ((-2*I)*a^(3/2)*Sqrt[a + b]*f*x + (2*a^2 + a*b - b^2)
*Log[1 - E^((2*I)*(e + f*x))] + a^(3/2)*Sqrt[a + b]*Log[a + 2*b + a*E^((2*I
)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e +
f*x)))^2]] + a^(3/2)*Sqrt[a + b]*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*
I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e +
 f*x)))^2]] - 2*a^2*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^((2*I)*(e + f*x
)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))
^2]] - a*b*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^((2*I)*(e + f*x)) + Sqrt
[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + b^
2*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*S
qrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]])/(Sqrt[a + b]
*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]))*(a + b*Sec
[e + f*x]^2)^(3/2))/(f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1568 vs. 2(96) = 192.

Time = 1.26 (sec) , antiderivative size = 1569, normalized size of antiderivative = 13.76

method	result	size
default	Expression too large to display	1569

```
[In] int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f/(a+b)^(3/2)*(4*cos(f*x+e)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2))*(a+b)^(3/2)*a^(3/2)-4*a^(3/2)*ln(4*cos(f*x+e)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*(a+b)^(3/2)-2*cos(f*x+e)*((b+a*cos(f
```

```
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(3/2)*a-2*cos(f*x+e)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(3/2)*b-2*cos(f*x+e)*ln(-4*(((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^3-3*
cos(f*x+e)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*c
os(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e
)*a+b)/(-1+cos(f*x+e)))*a^2*b+cos(f*x+e)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*b^3+2*cos(f*x+e)*ln(2
/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f
*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+
b)/(1+cos(f*x+e)))*a^3+3*cos(f*x+e)*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^2*b-cos(f*x+e)
*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*
cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+
e)*a+b)/(1+cos(f*x+e)))*b^3+2*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^3+3*ln(-4*(((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^2*b-ln(-4*(
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x
+e)))*b^3-2*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(
a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/
2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^3-3*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^2*b+ln(2/
(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*
x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b
)/(1+cos(f*x+e)))*b^3)*(a+b*sec(f*x+e)^2)^(3/2)*cos(f*x+e)^3/(b+a*cos(f*x+e
)^2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/(cos(f*x+e)^2-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(96) = 192$.

Time = 0.60 (sec) , antiderivative size = 1300, normalized size of antiderivative = 11.40

$$\int \cot^3(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2
+ (a*cos(f*x + e)^2 - a)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos
```

```
(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 -
8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^
4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
) - ((2*a - b)*cos(f*x + e)^2 - 2*a + b)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b
+ b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a +
b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/(f*cos(f*x +
e)^2 - f), 1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f
*x + e)^2 - 2*((2*a - b)*cos(f*x + e)^2 - 2*a + b)*sqrt(-a - b)*arctan(1/2*
((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + (a*cos(f*x + e)^2
- a)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^
2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x +
e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)
^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)))/(f*cos(f*x + e)^2
 - f), 1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
 e)^2 + 2*(a*cos(f*x + e)^2 - a)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4
+ 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - ((2*a -
b)*cos(f*x + e)^2 - 2*a + b)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f
*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x +
 e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/(f*cos(f*x + e)^2 - f), 1
/4*(2*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 +
(a*cos(f*x + e)^2 - a)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*co
s(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2
*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - ((2*a - b)*cos(f*x
 + e)^2 - 2*a + b)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*s
qrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*
x + e)^2 + a*b + b^2)))/(f*cos(f*x + e)^2 - f)]
```

Sympy [F]

$$\int \cot^3(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \int \left(a+b\sec^2{(e+fx)}\right)^{\frac{3}{2}} \cot^3{(e+fx)}\, dx$$

```
[In] integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*cot(e + f*x)**3, x)
```

Maxima [F]

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^3\,dx$$

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \cot(e+f\,x)^3\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

```
[In] int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.394 $\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 159

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}-\frac{(8a^2+4ab-b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8\sqrt{a+b}f}+\frac{(4a-b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f}-\frac{(a+b)\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f}$$

```
[Out] a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f-1/8*(8*a^2+4*a*b-b^2)*a
rctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/f/(a+b)^(1/2)+1/8*(4*a-b)*cot(
f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2)/f-1/4*(a+b)*cot(f*x+e)^4*(a+b*sec(f*x+e)^
2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4224, 457, 100, 156, 162, 65, 214}

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}-\frac{(8a^2+4ab-b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8f\sqrt{a+b}}-\frac{(a+b)\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f}+\frac{(4a-b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f}$$

```
[In] Int[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - ((8*a^2 + 4*a*b -
 b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(8*Sqrt[a + b]*f) +
((4*a - b)*Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(8*f) - ((a + b)*Cot[
e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2])/(4*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
```

```
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{x(-1+x^2)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{(-1+x)^3 x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{(a+b)\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f} - \frac{\text{Subst}\left(\int \frac{2a^2+\frac{1}{2}(3a-b)bx}{(-1+x)^2 x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{4f}$$

$$
\begin{aligned}
&= \frac{(4a-b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} \\
&\quad - \frac{(a+b)\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f} \\
&\quad + \frac{\text{Subst}\left(\int \frac{2a^2(a+b)+\frac{1}{4}(4a-b)b(a+b)x}{(-1+x)x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{4(a+b)f} \\
&= \frac{(4a-b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} \\
&\quad - \frac{(a+b)\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f} \\
&\quad - \frac{a^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f} \\
&\quad + \frac{(8a^2+4ab-b^2)\,\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{16f} \\
&= \frac{(4a-b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} \\
&\quad - \frac{(a+b)\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f} \\
&\quad - \frac{a^2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf} \\
&\quad + \frac{(8a^2+4ab-b^2)\,\text{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{8bf} \\
&= \frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{(8a^2+4ab-b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8\sqrt{a+b}f} \\
&\quad + \frac{(4a-b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} \\
&\quad - \frac{(a+b)\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.91 (sec) , antiderivative size = 684, normalized size of antiderivative = 4.30

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{\left(1+e^{2i(e+fx)}\right)\left(b\left(1+6e^{2i(e+fx)}+e^{4i(e+fx)}\right)+a\right.}{\left(-1+e^{2i(e+fx)}\right)^4}\right.}{}$$

```
[In] Integrate[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Cos[e + f*x]^3*(-(((1 + E^((2*I)*(e + f*x)))*(b*(1 + 6*E^((2*I)*(e +
f*x)) + E^((4*I)*(e + f*x))) + a*(6 - 4*E^((2*I)*(e + f*x)) + 6*E^((4*I)*(e
 + f*x)))))/(-1 + E^((2*I)*(e + f*x)))^4) + ((-8*I)*a^(3/2)*Sqrt[a + b]*f*x
 + (8*a^2 + 4*a*b - b^2)*Log[1 - E^((2*I)*(e + f*x))] + 4*a^(3/2)*Sqrt[a +
b]*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x
)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + 4*a^(3/2)*Sqrt[a + b]*Log[a + a*E^((
2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e +
f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 8*a^2*Log[a + b + a*E^((2*I)*(e +
 f*x)) + b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) +
 a*(1 + E^((2*I)*(e + f*x)))^2]] - 4*a*b*Log[a + b + a*E^((2*I)*(e + f*x))
+ b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 +
 E^((2*I)*(e + f*x)))^2]] + b^2*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^((2
*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)
*(e + f*x)))^2]])/(Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*
I)*(e + f*x)))^2]))*(a + b*Sec[e + f*x]^2)^(3/2))/(2*Sqrt[2]*f*(a + 2*b + a
*Cos[2*e + 2*f*x])^(3/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 4838 vs. 2(137) = 274.

Time = 1.18 (sec) , antiderivative size = 4839, normalized size of antiderivative = 30.43

method	result	size
default	Expression too large to display	4839

```
[In] int(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/f/(a+b)^(5/2)*(-8*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
```

2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^4+ln(2/(a+b)^(1/2)*(((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*b
^4+8*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x
+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)
/(-1+cos(f*x+e)))*a^4-ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2
)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*b^4-8*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(a+b)^(5/2)*a+12*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*(a+b)^(5/2)*a+16*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2))*(a+b)^(5/2)*a^(3/2)*cos(f*x+e)^2+20*ln(2/(a+b)^(1/
2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(
f*x+e)))*a^3*b*cos(f*x+e)^2+15*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^2*b^2*cos(f*x+e)^2+
2*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)
*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x
+e)*a+b)/(1+cos(f*x+e)))*a*b^3*cos(f*x+e)^2-20*ln(-4*(((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^3*b*cos(f*x+e
)^2-15*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f
*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+
b)/(-1+cos(f*x+e)))*a^2*b^2*cos(f*x+e)^2-2*ln(-4*(((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a*b^3*cos(f*x+e)^2+
8*cos(f*x+e)*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1
/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^4-cos(f*x+e)*ln(2/(a+b)^(1/2)*(((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*b
^4+20*cos(f*x+e)*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b
)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^3*b+15*cos(f*x+e)*ln(2/(a+b)^(1/2
)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f
*x+e)))*a^2*b^2+2*cos(f*x+e)*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f
x+e))^2)^(1/2)(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a*b^3-16*(a+b)^(5/2)*co
s(f*x+e)^3*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1
/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a
^(3/2)+2*(a+b)^(5/2)*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*b-20*cos(f*x+e)^3*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*

(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^3*b-15*cos(f*x+e)^3*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^2*b^2-2*cos(f*x+e)^3*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a*b^3+20*cos(f*x+e)^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^3*b+15*cos(f*x+e)^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^2*b^2+2*cos(f*x+e)^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a*b^3+16*(a+b)^(5/2)*cos(f*x+e)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^(3/2)+2*(a+b)^(5/2)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b-20*cos(f*x+e)*a^3*b*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))-15*cos(f*x+e)*a^2*b^2*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))-2*cos(f*x+e)*a*b^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))-8*cos(f*x+e)*a^4*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))+cos(f*x+e)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*b^4-8*cos(f*x+e)^3*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^4+cos(f*x+e)^3*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*b^4+8*cos(f*x+e)^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^4-cos(f*x+e)^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*b^4+8*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^4*cos(f*x+e)^2-ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*b^4*cos(f*x+e)^2-8*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+co

```
s(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^4*cos(f*x
+e)^2+ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*
x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b
)/(-1+cos(f*x+e)))*b^4*cos(f*x+e)^2-2*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a*b^3+20*ln(-4
*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f
*x+e)))*a^3*b+15*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(
1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos
(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^2*b^2+2*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a*b^3-16*ln(4*cos(f*x+
e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*(a+b)^(5/2)*a^(3/2)-20*ln
(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos
(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*
a+b)/(1+cos(f*x+e)))*a^3*b-15*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^2*b^2)*cos(f*x+e)^3*
(a+b*sec(f*x+e)^2)^(3/2)/(b+a*cos(f*x+e)^2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)/(-1+cos(f*x+e))^2/(1+cos(f*x+e))^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 404 vs. 2(137) = 274.

Time = 1.61 (sec) , antiderivative size = 1801, normalized size of antiderivative = 11.33

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(4*((a^2 + a*b)*cos(f*x + e)^4 - 2*(a^2 + a*b)*cos(f*x + e)^2 + a^2 +
 a*b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a
^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x +
 e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e
)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - ((8*a^2 + 4*a*b
 - b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 4*a*b - b^2)*cos(f*x + e)^2 + 8*a^2 + 4
*a*b - b^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*
a*b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x
 + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x
+ e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*((6*a^2 + 7*a*b + b^2)*cos(f*x + e)^4 -
 (4*a^2 + 3*a*b - b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2))/((a + b)*f*cos(f*x + e)^4 - 2*(a + b)*f*cos(f*x + e)^2 + (a + b)*f
```

```
), 1/16*(((8*a^2 + 4*a*b - b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 4*a*b - b^2)*co
s(f*x + e)^2 + 8*a^2 + 4*a*b - b^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(
f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((
a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 2*((a^2 + a*b)*cos(f*x + e)^4 - 2
*(a^2 + a*b)*cos(f*x + e)^2 + a^2 + a*b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8
 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x
 + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b
^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)) - 2*((6*a^2 + 7*a*b + b^2)*cos(f*x + e)^4 - (4*a^2 + 3*a*
b - b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a +
 b)*f*cos(f*x + e)^4 - 2*(a + b)*f*cos(f*x + e)^2 + (a + b)*f), -1/32*(8*((
a^2 + a*b)*cos(f*x + e)^4 - 2*(a^2 + a*b)*cos(f*x + e)^2 + a^2 + a*b)*sqrt(
-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*
b*cos(f*x + e)^2 + a*b^2)) + ((8*a^2 + 4*a*b - b^2)*cos(f*x + e)^4 - 2*(8*a
^2 + 4*a*b - b^2)*cos(f*x + e)^2 + 8*a^2 + 4*a*b - b^2)*sqrt(a + b)*log(2*(
(8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b
^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1))
 + 4*((6*a^2 + 7*a*b + b^2)*cos(f*x + e)^4 - (4*a^2 + 3*a*b - b^2)*cos(f*x
+ e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a + b)*f*cos(f*x + e
)^4 - 2*(a + b)*f*cos(f*x + e)^2 + (a + b)*f), -1/16*(4*((a^2 + a*b)*cos(f*
x + e)^4 - 2*(a^2 + a*b)*cos(f*x + e)^2 + a^2 + a*b)*sqrt(-a)*arctan(1/4*(8
*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2
+ a*b^2)) - ((8*a^2 + 4*a*b - b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 4*a*b - b^2)
*cos(f*x + e)^2 + 8*a^2 + 4*a*b - b^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*c
os(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 2*((6*a^2 + 7*a*b + b^2)*cos(f
*x + e)^4 - (4*a^2 + 3*a*b - b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2))/((a + b)*f*cos(f*x + e)^4 - 2*(a + b)*f*cos(f*x + e)^2
+ (a + b)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \cot^5(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \text{Timed out}$$

```
[In] integrate(cot(f*x+e)**5*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^5\,dx$$

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^5, x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \cot(e+f\,x)^5\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

```
[In] int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.395 $\int (a+b\sec^2(e+fx))^{3/2}\tan^6(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 290

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^6(e+fx)\,dx = -\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a^4+20a^3b+90a^2b^2-60ab^3-5b^4)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{128b^{5/2}f} - \frac{(3a^3+17a^2b-55ab^2-5b^3)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{128b^2f} + \frac{(3a^2-50ab-5b^2)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{192bf} + \frac{(9a+b)\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48f} + \frac{b\tan^7(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f}$$

```
[Out] -a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/128*(3*a
^4+20*a^3*b+90*a^2*b^2-60*a*b^3-5*b^4)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*ta
n(f*x+e)^2)^(1/2))/b^(5/2)/f-1/128*(3*a^3+17*a^2*b-55*a*b^2-5*b^3)*(a+b+b*t
an(f*x+e)^2)^(1/2)*tan(f*x+e)/b^2/f+1/192*(3*a^2-50*a*b-5*b^2)*(a+b+b*tan(f
*x+e)^2)^(1/2)*tan(f*x+e)^3/b/f+1/48*(9*a+b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan
(f*x+e)^5/f+1/8*b*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^7/f
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4226, 2000, 488, 596, 537, 223, 212, 385, 209}

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^6(e+fx)\,dx = -\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f} + \frac{(3a^2-50ab-5b^2)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{192bf} - \frac{(3a^3+17a^2b-55ab^2-5b^3)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{128b^2f} + \frac{(3a^4+20a^3b+90a^2b^2-60ab^3-5b^4)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{128b^{5/2}f} + \frac{b\tan^7(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8f} + \frac{(9a+b)\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{48f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^6,x]
```

```
[Out] -((a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) + ((3*a^4 + 20*a^3*b + 90*a^2*b^2 - 60*a*b^3 - 5*b^4)*ArcTanh[(Sqrt[b]*Ta
n[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(128*b^(5/2)*f) - ((3*a^3 + 17
*a^2*b - 55*a*b^2 - 5*b^3)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(12
8*b^2*f) + ((3*a^2 - 50*a*b - 5*b^2)*Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e +
f*x]^2])/(192*b*f) + ((9*a + b)*Tan[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^
2])/(48*f) + (b*Tan[e + f*x]^7*Sqrt[a + b + b*Tan[e + f*x]^2])/(8*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
```

```
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^6\left(a+b\left(1+x^2\right)\right)^{3/2}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6\left(a+b+bx^2\right)^{3/2}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{b\tan^7(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{\text{Subst}\left(\int \frac{x^6\left((a+b)(8a+b)+b(9a+b)x^2\right)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx,x,\tan(e+fx)\right)}{8f}$$

$$= \frac{(9a+b)\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48f} + \frac{b\tan^7(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} - \frac{\text{Subst}\left(\int \frac{x^4\left(5b(a+b)(9a+b)-b(3a^2-50ab-5b^2)x^2\right)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx,x,\tan(e+fx)\right)}{48bf}$$

$$= \frac{(3a^2-50ab-5b^2)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{192bf} + \frac{(9a+b)\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48f} + \frac{b\tan^7(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{\text{Subst}\left(\int \frac{x^2\left(-3b(a+b)(3a^2-50ab-5b^2)-3b(3a^3+17a^2b-55ab^2-5b^3)x^2\right)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx,x,\tan(e+fx)\right)}{192b^2f}$$

$$= -\frac{(3a^3+17a^2b-55ab^2-5b^3)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{128b^2f} + \frac{(3a^2-50ab-5b^2)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{192bf} + \frac{(9a+b)\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48f} + \frac{b\tan^7(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} - \frac{\text{Subst}\left(\int \frac{-3b(a+b)(3a^3+17a^2b-55ab^2-5b^3)-3b(3a^4+20a^3b+90a^2b^2-60ab^3-5b^4)x^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx,x,\tan(e+fx)\right)}{384b^3f}$$

$$= -\frac{(3a^3+17a^2b-55ab^2-5b^3)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{128b^2f} + \frac{(3a^2-50ab-5b^2)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{192bf} + \frac{(9a+b)\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48f} + \frac{b\tan^7(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} - \frac{a^2\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} + \frac{(3a^4+20a^3b+90a^2b^2-60ab^3-5b^4)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{128b^2f}$$

$$= -\frac{(3a^3+17a^2b-55ab^2-5b^3)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{128b^2f} + \frac{(3a^2-50ab-5b^2)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{192bf} + \frac{(9a+b)\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48f} + \frac{b\tan^7(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} - \frac{a^2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a^4+20a^3b+90a^2b^2-60ab^3-5b^4)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{128b^2f}$$

$$= -\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a^4+20a^3b+90a^2b^2-60ab^3-5b^4)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{128b^{5/2}f} - \frac{(3a^3+17a^2b-55ab^2-5b^3)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{128b^2f} + \frac{(3a^2-50ab-5b^2)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{192bf} + \frac{(9a+b)\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48f} + \frac{b\tan^7(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f}$$

Mathematica [A] (verified)

Time = 7.88 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.22

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^6(e+fx)\,dx =$$

$$-\frac{\left(128a^{3/2}b^2\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)-\frac{(3a^4+20a^3b+90a^2b^2-60ab^3-5b^4)\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)\cos^3(e+fx)}{32\sqrt{2}b^2f(a+2b+a\cos(2e+2fx))^{3/2}}$$

$$-\frac{(90a^3+498a^2b-1594ab^2-626b^3+(135a^3+759a^2b-2303ab^2+513b^3)\cos(2(e+fx))+2(27a^3+159a^2$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^6,x]
```

```
[Out] -1/32*((128*a^(3/2)*b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e
+ f*x]^2]] - ((3*a^4 + 20*a^3*b + 90*a^2*b^2 - 60*a*b^3 - 5*b^4)*ArcTanh[(S
qrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqrt[b])*Cos[e + f*x]
^3*(a + b*Sec[e + f*x]^2)^(3/2))/(Sqrt[2]*b^2*f*(a + 2*b + a*Cos[2*e + 2*f*
x])^(3/2)) - ((90*a^3 + 498*a^2*b - 1594*a*b^2 - 626*b^3 + (135*a^3 + 759*a
^2*b - 2303*a*b^2 + 513*b^3)*Cos[2*(e + f*x)] + 2*(27*a^3 + 159*a^2*b - 523
*a*b^2 - 191*b^3)*Cos[4*(e + f*x)] + 9*a^3*Cos[6*(e + f*x)] + 57*a^2*b*Cos[
6*(e + f*x)] - 337*a*b^2*Cos[6*(e + f*x)] + 15*b^3*Cos[6*(e + f*x)])*Sec[e
+ f*x]^6*Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x])/(12288*b^2*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2228 vs. 2(260) = 520.

Time = 30.40 (sec) , antiderivative size = 2229, normalized size of antiderivative = 7.69

method	result	size
default	Expression too large to display	2229

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/768/f/(-a)^(1/2)/b^(13/2)*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(-674*cos(f*x+e)*s
in(f*x+e)*(-a)^(1/2)*b^(13/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a
-9*cos(f*x+e)^3*(-a)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-si
n(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a^4*b^4-60*cos(f*x+e)^3*(-a)^(1/2)*ln(4*(((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a^3
*b^5-270*cos(f*x+e)^3*(-a)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
```

)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a^2*b^6+180*cos(f*x+e)^3*(-a)^(1/2)*l
n(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+
1))*a*b^7-9*cos(f*x+e)^3*(-a)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a^4*b^4-60*cos(f*x+e)^3*(-a)^(1/2
)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x
+e)-1))*a^3*b^5-270*cos(f*x+e)^3*(-a)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a^2*b^6+180*cos(f*x+e)^3*
(-a)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f
x+e)+b^(1/2)((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)
/(sin(f*x+e)-1))*a*b^7+488*(-a)^(1/2)*b^(13/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*a*sin(f*x+e)+488*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*(-a)^(1/2)*b^(13/2)*a*tan(f*x+e)-96*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(-a)^(1/2)*b^(15/2)*tan(f*x+e)*sec(f*x+e)^4-96*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*b^(15/2)*tan(f*x+e)*sec(f*x+e)^3-12*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*b^(11/2)*a^2*tan(f*x+e)+3
0*cos(f*x+e)^2*sin(f*x+e)*(-a)^(1/2)*b^(15/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)+30*cos(f*x+e)*sin(f*x+e)*(-a)^(1/2)*b^(15/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)+272*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*(-a)^(1/2)*b^(15/2)*tan(f*x+e)*sec(f*x+e)+272*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*(-a)^(1/2)*b^(15/2)*tan(f*x+e)*sec(f*x+e)^2-12*(-a)^(1/2)*
b^(11/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*sin(f*x+e)+18*cos(
f*x+e)^2*sin(f*x+e)*(-a)^(1/2)*b^(9/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*a^3+18*cos(f*x+e)*sin(f*x+e)*(-a)^(1/2)*b^(9/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*a^3-674*cos(f*x+e)^2*sin(f*x+e)*(-a)^(1/2)*b^(13/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a-144*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*b^(13/2)*a*tan(f*x+e)*sec(f*x+e)^2+114*cos(
f*x+e)^2*sin(f*x+e)*(-a)^(1/2)*b^(11/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*a^2+114*cos(f*x+e)*sin(f*x+e)*(-a)^(1/2)*b^(11/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*a^2-144*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(-a)^(1/2)*b^(13/2)*a*tan(f*x+e)*sec(f*x+e)+768*cos(f*x+e)^3*b^(13/2)*
ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-
a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2-23
6*(-a)^(1/2)*b^(15/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sin(f*x+e
)+15*cos(f*x+e)^3*(-a)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-
sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^8+15*cos(f*x+e)^3*(-a)^(1/2)*ln(-4*(((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*b^8-
236*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*b^(15/2)*tan(f*x
+e))

Fricas [A] (verification not implemented)

none

Time = 14.98 (sec) , antiderivative size = 1973, normalized size of antiderivative = 6.80

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^6(e+fx)\,dx=\text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] [1/1536*(192*sqrt(-a)*a*b^3*cos(f*x + e)^7*log(128*a^4*cos(f*x + e)^8 - 256
*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x +
 e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b +
7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^
2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3
- 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*sin(f*x + e)) - 3*(3*a^4 + 20*a^3*b + 90*a^2*b^2 - 60*a*
b^3 - 5*b^4)*sqrt(b)*cos(f*x + e)^7*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
 e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b
^2)/cos(f*x + e)^4) - 4*((9*a^3*b + 57*a^2*b^2 - 337*a*b^3 + 15*b^4)*cos(f*
x + e)^6 - 2*(3*a^2*b^2 - 122*a*b^3 + 59*b^4)*cos(f*x + e)^4 - 48*b^4 - 8*(
9*a*b^3 - 17*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^7), 1/768*(96*sqrt(-a)*a*b^3*cos(f*x +
 e)^7*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5
*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 -
 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8
*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a
^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x +
e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 3*
(3*a^4 + 20*a^3*b + 90*a^2*b^2 - 60*a*b^3 - 5*b^4)*sqrt(-b)*arctan(-1/2*((a
 - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^
7 - 2*((9*a^3*b + 57*a^2*b^2 - 337*a*b^3 + 15*b^4)*cos(f*x + e)^6 - 2*(3*a^
2*b^2 - 122*a*b^3 + 59*b^4)*cos(f*x + e)^4 - 48*b^4 - 8*(9*a*b^3 - 17*b^4)*
cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(
b^3*f*cos(f*x + e)^7), 1/1536*(384*a^(3/2)*b^3*arctan(1/4*(8*a^2*cos(f*x +
e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqr
t(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a
^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^
7 - 3*(3*a^4 + 20*a^3*b + 90*a^2*b^2 - 60*a*b^3 - 5*b^4)*sqrt(b)*cos(f*x +
e)^7*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2
 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((9*a^3
*b + 57*a^2*b^2 - 337*a*b^3 + 15*b^4)*cos(f*x + e)^6 - 2*(3*a^2*b^2 - 122*a
```

```
*b^3 + 59*b^4)*cos(f*x + e)^4 - 48*b^4 - 8*(9*a*b^3 - 17*b^4)*cos(f*x + e)^
2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*cos(f*x
 + e)^7), 1/768*(192*a^(3/2)*b^3*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2
- a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 -
 (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^7 + 3*(3*a^4 +
 20*a^3*b + 90*a^2*b^2 - 60*a*b^3 - 5*b^4)*sqrt(-b)*arctan(-1/2*((a - b)*co
s(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^7 - 2*((
9*a^3*b + 57*a^2*b^2 - 337*a*b^3 + 15*b^4)*cos(f*x + e)^6 - 2*(3*a^2*b^2 -
122*a*b^3 + 59*b^4)*cos(f*x + e)^4 - 48*b^4 - 8*(9*a*b^3 - 17*b^4)*cos(f*x
+ e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*co
s(f*x + e)^7)]
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^6(e+fx)\,dx=\int \left(a+b\sec^2{(e+fx)}\right)^{\frac{3}{2}}\tan^6{(e+fx)}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**6,x)
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x)**6, x)
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^6(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^6\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^6,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^6, x)
```

Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^6(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^6\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^6,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^6(e+fx)\,dx=\int \tan(e+f\,x)^6\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

```
[In] int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.396 $\int (a+b\sec^2(e+fx))^{3/2}\tan^4(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 214

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^4(e+fx)\,dx = \frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{(a-b)\,(a^2+10ab+b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16b^{3/2}f} + \frac{(a^2-8ab-b^2)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16bf} + \frac{(7a+b)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24f} + \frac{b\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f}$$

```
[Out] a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f-1/16*(a-b)*
(a^2+10*a*b+b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(
3/2)/f+1/16*(a^2-8*a*b-b^2)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f+1/24*
(7*a+b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^3/f+1/6*b*(a+b+b*tan(f*x+e)^2
)^(1/2)*tan(f*x+e)^5/f
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4226, 2000, 488, 596, 537, 223, 212, 385, 209}

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx = \frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f} - \frac{(a-b)\left(a^2+10ab+b^2\right)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16b^{3/2}f} + \frac{\left(a^2-8ab-b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16bf} + \frac{b\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6f} + \frac{(7a+b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{24f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^4,x]
```

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f -
 ((a - b)*(a^2 + 10*a*b + b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b +
b*Tan[e + f*x]^2]])/(16*b^(3/2)*f) + ((a^2 - 8*a*b - b^2)*Tan[e + f*x]*Sqrt
[a + b + b*Tan[e + f*x]^2])/(16*b*f) + ((7*a + b)*Tan[e + f*x]^3*Sqrt[a + b
 + b*Tan[e + f*x]^2])/(24*f) + (b*Tan[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x
]^2])/(6*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
```

`erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])`

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4(a+b(1+x^2))^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4(a+b+bx^2)^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{b\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f} + \frac{\text{Subst}\left(\int \frac{x^4((a+b)(6a+b)+b(7a+b)x^2)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{6f}$$

$$= \frac{(7a+b)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24f} + \frac{b\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f} - \frac{\text{Subst}\left(\int \frac{x^2(3b(a+b)(7a+b)-3b(a^2-8ab-b^2)x^2)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{24bf}$$

$$= \frac{(a^2-8ab-b^2)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16bf} + \frac{(7a+b)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24f} + \frac{b\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f} + \frac{\text{Subst}\left(\int \frac{-3b(a+b)(a^2-8ab-b^2)-3(a-b)b(a^2+10ab+b^2)x^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{48b^2f}$$

$$= \frac{(a^2-8ab-b^2)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16bf} + \frac{(7a+b)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24f} + \frac{b\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f} + \frac{a^2\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} - \frac{((a-b)(a^2+10ab+b^2))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{16bf}$$

$$= \frac{(a^2-8ab-b^2)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16bf} + \frac{(7a+b)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24f} + \frac{b\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f} + \frac{a^2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{((a-b)(a^2+10ab+b^2))\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16bf}$$

$$= \frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{(a-b)(a^2+10ab+b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16b^{3/2}f} + \frac{(a^2-8ab-b^2)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16bf} + \frac{(7a+b)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24f} + \frac{b\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f}$$

Mathematica [A] (verified)

Time = 5.29 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.21

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^4(e+fx)\,dx = \frac{\left(16a^{3/2}b\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right) - \frac{(a-b)(a^2+10ab+b^2)\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)\cos^3(e+fx)\,(a}{4\sqrt{2}bf(a+2b+a\cos(2e+2fx))^{3/2}} + \frac{(9a^2-58ab+17b^2+4(3a^2-24ab-11b^2)\cos(2(e+fx))+(3a^2-38ab+3b^2)\cos(4(e+fx)))\sec^4(e+}{384bf}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^4,x]
```

```
[Out] ((16*a^(3/2)*b*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]
] - ((a - b)*(a^2 + 10*a*b + b^2)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b
 - a*Sin[e + f*x]^2]])/Sqrt[b])*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2)
```

```
)/(4*Sqrt[2]*b*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)) + ((9*a^2 - 58*a*b +
 17*b^2 + 4*(3*a^2 - 24*a*b - 11*b^2)*Cos[2*(e + f*x)] + (3*a^2 - 38*a*b +
3*b^2)*Cos[4*(e + f*x)])*Sec[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2]*Tan[e +
f*x])/(384*b*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1618 vs. 2(188) = 376.

Time = 21.72 (sec) , antiderivative size = 1619, normalized size of antiderivative = 7.57

method	result	size
default	Expression too large to display	1619

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/96/f/(-a)^(1/2)/b^(9/2)*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(-6*cos(f*x+e)^2*sin
(f*x+e)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(11/2)-96*
cos(f*x+e)^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*co
s(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x
+e)*a)*b^(9/2)*a^2-6*cos(f*x+e)*sin(f*x+e)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*b^(11/2)+76*cos(f*x+e)^2*sin(f*x+e)*(-a)^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(9/2)*a+28*(-a)^(1/2)*b^(11/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sin(f*x+e)+76*cos(f*x+e)*sin(f*x+e
)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(9/2)*a-6*cos(f*
x+e)^2*sin(f*x+e)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^
(7/2)*a^2+28*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(11/2
)*tan(f*x+e)-28*(-a)^(1/2)*b^(9/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*a*sin(f*x+e)-6*cos(f*x+e)*sin(f*x+e)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*b^(7/2)*a^2-16*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*b^(11/2)*tan(f*x+e)*sec(f*x+e)-28*(-a)^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(9/2)*a*tan(f*x+e)+3*cos(f*x+e)^3*(-a)^(1/2
)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+
e)+1))*a^3*b^3+27*cos(f*x+e)^3*(-a)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a^2*b^4-27*cos(f*x+e)^3*(-a)
^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)
+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin
(f*x+e)+1))*a*b^5-3*cos(f*x+e)^3*(-a)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^6+3*cos(f*x+e)^3*(-a)^(1
/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f
*x+e)-1))*a^3*b^3+27*cos(f*x+e)^3*(-a)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+
```

```
cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a^2*b^4-27*cos(f*x+e)^3*
(-a)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f
*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)
/(sin(f*x+e)-1))*a*b^5-3*cos(f*x+e)^3*(-a)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*b^6-16*(-a)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(11/2)*tan(f*x+e)*sec(f*x+e)^2)
```

Fricas [A] (verification not implemented)

none

Time = 4.15 (sec) , antiderivative size = 1777, normalized size of antiderivative = 8.30

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx=\text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] [1/192*(24*sqrt(-a)*a*b^2*cos(f*x + e)^5*log(128*a^4*cos(f*x + e)^8 - 256*(
a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e
)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*
a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*
b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 -
7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e)) - 3*(a^3 + 9*a^2*b - 9*a*b^2 - b^3)*sqrt(b)*
cos(f*x + e)^5*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(
f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) +
 4*((3*a^2*b - 38*a*b^2 + 3*b^3)*cos(f*x + e)^4 + 8*b^3 + 14*(a*b^2 - b^3)*
cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(
b^2*f*cos(f*x + e)^5), 1/96*(12*sqrt(-a)*a*b^2*cos(f*x + e)^5*log(128*a^4*c
os(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5
*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 3
2*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x +
e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos
(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 3*(a^3 + 9*a^2*b - 9*
a*b^2 - b^3)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e
))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^
2 + b^2)*sin(f*x + e)))*cos(f*x + e)^5 + 2*((3*a^2*b - 38*a*b^2 + 3*b^3)*co
s(f*x + e)^4 + 8*b^3 + 14*(a*b^2 - b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f*x + e)^5), -1/192*(48*a
^(3/2)*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3
+ (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos
```

```
(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f
*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^5 + 3*(a^3 + 9*a^2*b - 9*a*b^2 - b^3
)*sqrt(b)*cos(f*x + e)^5*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b -
 b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x
 + e)^4) - 4*((3*a^2*b - 38*a*b^2 + 3*b^3)*cos(f*x + e)^4 + 8*b^3 + 14*(a*b
^2 - b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f
*x + e))/(b^2*f*cos(f*x + e)^5), -1/96*(24*a^(3/2)*b^2*arctan(1/4*(8*a^2*co
s(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x +
 e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x +
e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f
*x + e)^5 + 3*(a^3 + 9*a^2*b - 9*a*b^2 - b^3)*sqrt(-b)*arctan(-1/2*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^5 - 2
*((3*a^2*b - 38*a*b^2 + 3*b^3)*cos(f*x + e)^4 + 8*b^3 + 14*(a*b^2 - b^3)*co
s(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^
2*f*cos(f*x + e)^5)]
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx=\int \left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}\tan^{4}\left(e+fx\right)dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**4,x)
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x)**4, x)
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^{4}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^4, x)
```

Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^4\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx = \int \tan(e+f\,x)^4\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

```
[In] int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.397 $\int (a+b\sec^2(e+fx))^{3/2}\tan^2(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 166

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^2(e+fx)\,dx = -\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a^2-6ab-b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{b}f} + \frac{(5a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{b\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f}$$

```
[Out] -a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/8*(3*a^2
-6*a*b-b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/b^(1/2
)+1/8*(5*a+b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f+1/4*b*(a+b+b*tan(f*x+
e)^2)^(1/2)*tan(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used

$= \{4226, 2000, 488, 596, 537, 223, 212, 385, 209\}$

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx = -\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f} + \frac{(3a^2-6ab-b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8\sqrt{b}f} + \frac{(5a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8f} + \frac{b\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^2,x]
```

```
[Out] -((a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) + ((3*a^2 - 6*a*b - b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Ta
n[e + f*x]^2]])/(8*Sqrt[b]*f) + ((5*a + b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[
e + f*x]^2])/(8*f) + (b*Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(4*f
)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
```

```
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2(a+b(1+x^2))^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2(a+b+bx^2)^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{b\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f} + \frac{\text{Subst}\left(\int \frac{x^2((a+b)(4a+b)+b(5a+b)x^2)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{4f}$$

$$= \frac{(5a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{b\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f} - \frac{\text{Subst}\left(\int \frac{b(a+b)(5a+b)-b(3a^2-6ab-b^2)x^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{8bf}$$

$$= \frac{(5a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{b\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f} - \frac{a^2\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} + \frac{(3a^2-6ab-b^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{8f}$$

$$= \frac{(5a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{b\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f} - \frac{a^2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a^2-6ab-b^2)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8f}$$

$$= -\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a^2-6ab-b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{b}f} + \frac{(5a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{b\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.21 (sec) , antiderivative size = 703, normalized size of antiderivative = 4.23

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx = \frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{i\left(-1+e^{2i(e+fx)}\right)\left(5a\left(1+e^{2i(e+fx)}\right)^2-b\left(1-6e^{2i(e+fx)}\right.\right.}{\left(1+e^{2i(e+fx)}\right)^4}\right.}{}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^2,x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Cos[e + f*x]^3*(((-I)*(-1 + E^((2*I)*(e + f*x)))*(5*a*(1 + E^((2*I)*(
e + f*x)))^2 - b*(1 - 6*E^((2*I)*(e + f*x)) + E^((4*I)*(e + f*x)))))/(1 + E
^((2*I)*(e + f*x)))^4 + (-8*a^(3/2)*Sqrt[b]*f*x + (4*I)*a^(3/2)*Sqrt[b]*Log
[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a
*(1 + E^((2*I)*(e + f*x)))^2]] - (4*I)*a^(3/2)*Sqrt[b]*Log[a + a*E^((2*I)*(
e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x))
+ a*(1 + E^((2*I)*(e + f*x)))^2]] - 3*a^2*Log[(4*(Sqrt[b]*(-1 + E^((2*I)*(e
 + f*x))) - I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]
)*f)/((3*a^2 - 6*a*b - b^2)*(1 + E^((2*I)*(e + f*x))))] + 6*a*b*Log[(4*(Sqr
t[b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E
^((2*I)*(e + f*x)))^2])*f)/((3*a^2 - 6*a*b - b^2)*(1 + E^((2*I)*(e + f*x)))
)] + b^2*Log[(4*(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*E^((2*I)*(
e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*f)/((3*a^2 - 6*a*b - b^2)*(1 +
E^((2*I)*(e + f*x))))])/(Sqrt[b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((
2*I)*(e + f*x)))^2]))*(a + b*Sec[e + f*x]^2)^(3/2))/(2*Sqrt[2]*f*(a + 2*b +
 a*Cos[2*e + 2*f*x])^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1118 vs. 2(144) = 288.

Time = 17.36 (sec) , antiderivative size = 1119, normalized size of antiderivative = 6.74

method	result	size
default	Expression too large to display	1119

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/f/(-a)^(1/2)/b^(5/2)*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(-2*sin(f*x+e)*cos(f*
x+e)^2*(-a)^(1/2)*b^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-16*co
s(f*x+e)^3*b^(5/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*s
in(f*x+e)*a)*a^2-2*sin(f*x+e)*cos(f*x+e)*(-a)^(1/2)*b^(7/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)+10*sin(f*x+e)*cos(f*x+e)^2*(-a)^(1/2)*b^(5/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a+4*(-a)^(1/2)*b^(7/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sin(f*x+e)+10*sin(f*x+e)*cos(f*x+e)*(-a
)^(1/2)*b^(5/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a+3*cos(f*x+e)^
3*(-a)^(1/2)*ln(4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos
(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-
b)/(sin(f*x+e)-1))*a^2*b^2-6*cos(f*x+e)^3*(-a)^(1/2)*ln(4*(-((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*a*b^3-cos(f*x+e)
^3*(-a)^(1/2)*ln(4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*co
s(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a
-b)/(sin(f*x+e)-1))*b^4+3*cos(f*x+e)^3*(-a)^(1/2)*ln(-4*(-((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*a^2*b^2-6*cos(f*x+
e)^3*(-a)^(1/2)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)
*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)+a+b)/(sin(f*x+e)+1))*a*b^3-cos(f*x+e)^3*(-a)^(1/2)*ln(-4*(-((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*b^4+4*(-a)^(1/2
)*b^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*tan(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(144) = 288$.

Time = 1.27 (sec) , antiderivative size = 1627, normalized size of antiderivative = 9.80

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] [1/32*(4*sqrt(-a)*a*b*cos(f*x + e)^3*log(128*a^4*cos(f*x + e)^8 - 256*(a^4
- a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4
+ a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*
b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*c
os(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^
2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*sin(f*x + e)) - (3*a^2 - 6*a*b - b^2)*sqrt(b)*cos(f*x + e)^3*l
og(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*(
```

```
(a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*((5*a*b - b^2
)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f
*x + e))/(b*f*cos(f*x + e)^3), 1/16*(2*sqrt(-a)*a*b*cos(f*x + e)^3*log(128*
a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*
b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^
4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f
*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2
)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + (3*a^2 - 6*a*b -
 b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt
(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2
)*sin(f*x + e)))*cos(f*x + e)^3 + 2*((5*a*b - b^2)*cos(f*x + e)^2 + 2*b^2)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b*f*cos(f*x + e)
^3), 1/32*(8*a^(3/2)*b*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos
(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*
a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^3 - (3*a^2 - 6*a*b - b^2
)*sqrt(b)*cos(f*x + e)^3*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b -
 b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x
 + e)^4) + 4*((5*a*b - b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*sin(f*x + e))/(b*f*cos(f*x + e)^3), 1/16*(4*a^(3/2)*b*
arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*
a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*
sin(f*x + e)))*cos(f*x + e)^3 + (3*a^2 - 6*a*b - b^2)*sqrt(-b)*arctan(-1/2*
((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x +
e)^3 + 2*((5*a*b - b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e))/(b*f*cos(f*x + e)^3)]
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx=\int \left(a+b\sec^2{\left(e+fx\right)}\right)^{\frac{3}{2}}\tan^2{\left(e+fx\right)}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x)**2, x)
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^2\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^2, x)
```

Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^2\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx=\int \tan(e+f\,x)^2\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

```
[In] int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.398 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 118

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}(3a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}+\frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

```
[Out] a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/2*(3*a+b)
*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f+1/2*b*(a+
b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4213, 427, 537, 223, 212, 385, 209}

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\sqrt{b}(3a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}+\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*(3*a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f
*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{(a+b)(2a+b)+b(3a+b)x^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{a^2\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} + \frac{(b(3a+b))\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{a^2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(b(3a+b))\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}$$

$$= \frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}(3a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.09 (sec) , antiderivative size = 527, normalized size of antiderivative = 4.47

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\sqrt{2}e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{ib\left(-1+e^{2i(e+fx)}\right)}{\left(1+e^{2i(e+fx)}\right)^2}+\frac{2a^{3/2}fx-ia^{3/2}\log\left(a\right.}{}\right.}{}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*(((-I)*b*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((
2*I)*(e + f*x)))^2 + (2*a^(3/2)*f*x - I*a^(3/2)*Log[a + 2*b + a*E^((2*I)*(e
 + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)
))^2]] + I*a^(3/2)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x))
+ Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] -
3*a*Sqrt[b]*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (2*I)*Sqrt[4*b*E
^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(b*(3*a + b)*(1 + E^
((2*I)*(e + f*x))))] - b^(3/2)*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f
 + (2*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(
b*(3*a + b)*(1 + E^((2*I)*(e + f*x))))])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(
1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(f*(a + 2*b + a*
Cos[2*e + 2*f*x])^(3/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 708 vs. $2(100) = 200$.

Time = 8.03 (sec) , antiderivative size = 709, normalized size of antiderivative = 6.01

method	result
default	$\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}\left(\cos(fx+e)^3 b^{\frac{5}{2}}\ln\left(\frac{-4\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{b}\cos(fx+e)+4\sin(fx+e)a-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4a-4b}{\sin(fx+e)-1}\right)\sqrt{-a}+\cos(f}{}$

```
[In] int((a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f/(-a)^(1/2)/b*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(cos(f*x+e)^3*b^(5/2)*ln(4*(
-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*
a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*
(-a)^(1/2)+cos(f*x+e)^3*b^(5/2)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*(-a)^(1/2)+3*cos(f*x+e)^3*b^(3/2)*ln
(4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x
+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-
1))*(-a)^(1/2)*a+3*cos(f*x+e)^3*b^(3/2)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*(-a)^(1/2)*a+2*sin(f*x+e)*co
s(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*b^2+4*ln(
4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^
(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*cos(f*x+e
)^3*a^2*b+2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*cos(
f*x+e)*sin(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(100) = 200.

Time = 0.62 (sec) , antiderivative size = 1457, normalized size of antiderivative = 12.35

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b
)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a
*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
+ e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7
*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e)) + (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^
2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^
3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e)), 1/8*(2*(3*a + b)*sqrt(-b)*arcta
n(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos
(f*x + e) + sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 -
 a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 +
 a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b
^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*co
s(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2
*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e)) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e))/(f*cos(f*x + e)), -1/8*(2*a^(3/2)*arctan(1/4*(8*a^2*cos(f*x
+ e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*s
qrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 -
 a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e
) - (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
+ 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^
2)/cos(f*x + e)^4) - 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e))/(f*cos(f*x + e)), -1/4*(a^(3/2)*arctan(1/4*(8*a^2*cos(f*x + e)^5 -
8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b +
a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) - (3*a
+ b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(
-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)
*sin(f*x + e)))*cos(f*x + e) - 2*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(f*cos(f*x + e))]
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int {\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}\,dx$$

```
[In] int((a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int((a + b/cos(e + f*x)^2)^(3/2), x)
```

3.399 $\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 111

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=-\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}-\frac{(a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}$$

```
[Out] -a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+b^(3/2)*ar
ctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f-(a+b)*cot(f*x+e)*(a+
b+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4226, 2000, 485, 537, 223, 212, 385, 209}

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=-\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{(a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

```
[In] Int[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) + (b^(3/2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
)/f - ((a + b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b(1+x^2))^{3/2}}{x^2(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^{3/2}}{x^2(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{(a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} + \frac{\text{Subst}\left(\int \frac{-a^2+b^2+b^2x^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{(a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} \\
&\quad - \frac{a^2\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&\quad + \frac{b^2\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{(a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} \\
&\quad - \frac{a^2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} \\
&\quad + \frac{b^2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} \\
&= -\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} \\
&\quad - \frac{(a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.50 (sec) , antiderivative size = 410, normalized size of antiderivative = 3.69

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\sqrt{2}e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{2i(a+b)}{-1+e^{2i(e+fx)}}+\frac{ia^{3/2}\log\left(a+2b+ae^{2i(e}\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*(((-2*I)*(a + b))/(-1 + E^((2*I)*(e + f*x))) +
 (I*a^(3/2)*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)
*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - I*a^(3/2)*Log[a + a*E^((2*I
)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x
)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 2*(a^(3/2)*f*x + b^(3/2)*Log[((Sqrt[
b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^(
(2*I)*(e + f*x)))^2])*f)/(b^2*(1 + E^((2*I)*(e + f*x))))]))/Sqrt[4*b*E^((2*
I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2
))/(f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 538 vs. $2(97) = 194$.

Time = 9.30 (sec) , antiderivative size = 539, normalized size of antiderivative = 4.86

method	result
default	$\frac{\left(\sqrt{-a}\,\ln\left(-\frac{4\left(-\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{b}\cos(fx+e)+\sin(fx+e)a-\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}+a+b\right)}{\sin(fx+e)+1}\right)b^{\frac{3}{2}}\sin(fx+e)+\sqrt{-a}\,\ln\left(\frac{-4\sqrt{\frac{b+a\cos(fx+}{(1+\cos(fx+}}}{\cdots}\right.\right.}{\cdots}$

```
[In] int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f/(-a)^(1/2)*((-a)^(1/2)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*b^(3/2)*sin(f*x+e)+(-a)^(1/2)*ln(4*(-((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b
```

```
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*b^(
3/2)*sin(f*x+e)-2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*
cos(f*x+e)-2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b*cos(f
*x+e)-2*sin(f*x+e)*a^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
-4*sin(f*x+e)*a)-2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a
-2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b)*(a+b*sec(f*x+e
)^2)^(3/2)/(b+a*cos(f*x+e)^2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/(
1+cos(f*x+e))*cos(f*x+e)^2*cot(f*x+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(97) = 194.

Time = 0.65 (sec) , antiderivative size = 1446, normalized size of antiderivative = 13.03

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*a*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e
)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 7
0*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x
 + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5
*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)
*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
 + e))*sin(f*x + e) + 2*b^(3/2)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8
*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))
*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/
cos(f*x + e)^4)*sin(f*x + e) - 8*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*cos(f*x + e))/(f*sin(f*x + e)), 1/8*(4*sqrt(-b)*b*arctan(-1/2*((a
 - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x + e)
+ sqrt(-a)*a*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6
+ 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^
2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e
)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3
 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos
(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))*sin(f*x + e) - 8*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos
(f*x + e))/(f*sin(f*x + e)), 1/4*(a^(3/2)*arctan(1/4*(8*a^2*cos(f*x + e)^5
- 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b
+ a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e) + b^(
3/2)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2
```

```
 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e
) - 4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/(f*
sin(f*x + e)), 1/4*(a^(3/2)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b
)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3
 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e) + 2*sqrt(-b)*b*arct
an(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*si
n(f*x + e) - 4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
+ e))/(f*sin(f*x + e))]
```

Sympy [F]

$$\int \cot^2(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}} \cot^2\left(e+fx\right) dx$$

```
[In] integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*cot(e + f*x)**2, x)
```

Maxima [F]

$$\int \cot^2(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \cot\left(fx+e\right)^2 dx$$

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^2, x)
```

Giac [F]

$$\int \cot^2(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \cot\left(fx+e\right)^2 dx$$

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \cot(e+f\,x)^2\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}dx$$

```
[In] int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.400 $\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 112

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{(3a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f}-\frac{(a+b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f}$$

```
[Out] a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/3*(3*a-b)
*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f-1/3*(a+b)*cot(f*x+e)^3*(a+b+b*tan(
f*x+e)^2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4226, 2000, 485, 597, 12, 385, 209}

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{(a+b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f}+\frac{(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f}$$

```
[In] Int[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 ((3*a - b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*f) - ((a + b)*C
ot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+b(1+x^2))^{3/2}}{x^4(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^{3/2}}{x^4(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f} + \frac{\text{Subst}\left(\int \frac{-((3a-b)(a+b))-(2a-b)bx^2}{x^2(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{3f}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f} - \frac{(a+b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f} - \frac{\text{Subst}\left(\int -\frac{3a^2(a+b)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{3(a+b)f}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f} - \frac{(a+b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f} + \frac{a^2\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f} - \frac{(a+b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f} + \frac{a^2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

$$= \frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f} - \frac{(a+b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.58 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.89

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = -\frac{2(a+b)\cot^3(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},\frac{a\sin^2(e+fx)}{a+b}\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f(a+2b+a\cos(2(e+fx)))\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}$$

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*(a + b)*Cot[e + f*x]^3*Hypergeometric2F1[-3/2, -3/2, -1/2, (a*Sin[e + f
*x]^2)/(a + b)]*(a + b*Sec[e + f*x]^2)^(3/2))/(3*f*(a + 2*b + a*Cos[2*(e +
f*x)])*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. 2(98) = 196.

Time = 3.74 (sec) , antiderivative size = 369, normalized size of antiderivative = 3.29

method	result
default	$-\frac{\left(4\cos(fx+e)^2\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,a+3\sin(fx+e)a^2\ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx\right.\right.}{}$

```
[In] int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f/(-a)^(1/2)*(4*cos(f*x+e)^2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*a+3*sin(f*x+e)*a^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)-4*sin(f*x+e)*a)*cos(f*x+e)-3*sin(f*x+e)*a^2*ln(4*(-a)^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)-3*(-a)^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a+(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*b)*(a+b*sec(f*x+e)^2)^(3/2)/(b+a*cos(f*x+e)^2)/((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*cot(f*x+e)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(98) = 196$.

Time = 0.84 (sec) , antiderivative size = 597, normalized size of antiderivative = 5.33

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left[\frac{3\left(a\cos\left(fx+e\right)^2-a\right)\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6+32\,(5\right.}{}\right.$$

$$-\frac{3\left(a\cos\left(fx+e\right)^2-a\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\left(a^2-ab\right)\cos(fx+e)^3+\left(a^2-6\,ab+b^2\right)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-\left(a^3-3\,a^2b\right)\cos(fx+e)^2\right)\sin(fx+e)}\right)\sin}{12\left(f\cos\left(fx+e\right)^2-f\right)\sin\left(fx+\right.}$$

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(a*cos(f*x + e)^2 - a)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(
a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e
)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*
a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*
b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 -
7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) + 8*(4*a*cos(f*x + e)^3 - (3*a
- b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((f*cos(f*x
 + e)^2 - f)*sin(f*x + e)), -1/12*(3*(a*cos(f*x + e)^2 - a)*sqrt(a)*arctan(
1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b
^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a
^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x
 + e)))*sin(f*x + e) - 4*(4*a*cos(f*x + e)^3 - (3*a - b)*cos(f*x + e))*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((f*cos(f*x + e)^2 - f)*sin(f*x +
e))]
```

Sympy [F(-1)]

Timed out.

$$\int \cot^4(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx = \text{Timed out}$$

```
[In] integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \cot^4(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \cot\left(fx+e\right)^4 \, dx$$

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^4, x)
```

Giac [F]

$$\int \cot^4(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \cot\left(fx+e\right)^4 \, dx$$

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^4(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx = \int \cot(e+f\,x)^4 \left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2} dx$$

```
[In] int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.401 $\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 165

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = -\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{(15a^2+10ab-2b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)f} + \frac{(5a-b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15f} - \frac{(a+b)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5f}$$

```
[Out] -a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f-1/15*(15*a
^2+10*a*b-2*b^2)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f+1/15*(5*a-b)
*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/f-1/5*(a+b)*cot(f*x+e)^5*(a+b+b*ta
n(f*x+e)^2)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used

$= \{4226, 2000, 485, 597, 12, 385, 209\}$

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = -\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f} - \frac{(15a^2+10ab-2b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)} - \frac{(a+b)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5f} + \frac{(5a-b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f}$$

```
[In] Int[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) - ((15*a^2 + 10*a*b - 2*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])
/(15*(a + b)*f) + ((5*a - b)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])
/(15*f) - ((a + b)*Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(5*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b(1+x^2))^{3/2}}{x^6(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^{3/2}}{x^6(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{(a+b)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5f} \\
&\quad + \frac{\text{Subst}\left(\int \frac{-((5a-b)(a+b))-(4a-b)bx^2}{x^4(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{5f} \\
&= \frac{(5a-b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15f} \\
&\quad - \frac{(a+b)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5f} \\
&\quad - \frac{\text{Subst}\left(\int \frac{-((a+b)(15a^2+10ab-2b^2))-2(5a-b)b(a+b)x^2}{x^2(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{15(a+b)f}
\end{aligned}
$$

$$=-\frac{(15a^2+10ab-2b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)f}+\frac{(5a-b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15f}-\frac{(a+b)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5f}+\frac{\text{Subst}\left(\int -\frac{15a^2(a+b)^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx,x,\tan(e+fx)\right)}{15(a+b)^2f}$$

$$=-\frac{(15a^2+10ab-2b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)f}+\frac{(5a-b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15f}-\frac{(a+b)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5f}-\frac{a^2\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=-\frac{(15a^2+10ab-2b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)f}+\frac{(5a-b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15f}-\frac{(a+b)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5f}-\frac{a^2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx,x,\frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

$$=-\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}-\frac{(15a^2+10ab-2b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)f}+\frac{(5a-b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15f}-\frac{(a+b)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.50 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.84

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{2\cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\left(-\frac{3}{4}(a+2b+a\cos(2(e+fx)))^2\csc^2(e+fx)+\frac{5(a+b)}{}\right.}{15(a+b)f(a+2b+a\cos(2(e+fx)))}$$

[In] Integrate[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]

[Out] (2*Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2)*((-3*(a + 2*b + a*Cos[2*(e + f*x)])^2*Csc[e + f*x]^2)/4 + (5*(a + b)^2*Hypergeometric2F1[-3/2, -3/2, -1/2, (a*Sin[e + f*x]^2)/(a + b)])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/(15*(a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)]))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 792 vs. 2(147) = 294.

Time = 5.02 (sec) , antiderivative size = 793, normalized size of antiderivative = 4.81

method	result
default	$-\frac{\left(23\cos(fx+e)^4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,a^2+20\cos(fx+e)^4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,ab-15\sin(fx+e)^3a^3\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)}{(1+\cos(fx+e))}}\right.\right.}{}$

[In] int(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/15/f/(a+b)/(-a)^(1/2)*(23*cos(f*x+e)^4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+20*cos(f*x+e)^4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b-15*sin(f*x+e)^3*a^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*cos(f*x+e)-15*sin(f*x+e)^3*a^2*b*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*cos(f*x+e)+15*sin(f*x+e)^3*a^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)+15*sin(f*x+e)^3*a^2*b*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)-35*cos(f*x+e)^2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2-24*cos(f*x+e)^2*(-a)^(1/2)*((b+a*cos(f*x+e

```
)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b+5*cos(f*x+e)^2*(-a)^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*b^2+15*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*a^2+10*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*a*b-2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2)*(a+b*se
c(f*x+e)^2)^(3/2)/(b+a*cos(f*x+e)^2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*cot(f*x+e)^3*csc(f*x+e)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(147) = 294$.

Time = 3.06 (sec) , antiderivative size = 767, normalized size of antiderivative = 4.65

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left[\frac{15\left(\left(a^2+ab\right)\cos\left(fx+e\right)^4-2\left(a^2+ab\right)\cos\left(fx+e\right)^2+a^2+ab\right)\sqrt{-a}\log\left(128\,a^4\cos\left(fx+\right.\right.}{}\right.$$

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/120*(15*((a^2 + a*b)*cos(f*x + e)^4 - 2*(a^2 + a*b)*cos(f*x + e)^2 + a^2
 + a*b)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e
)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 7
0*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x
 + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5
*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)
*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
 + e))*sin(f*x + e) - 8*((23*a^2 + 20*a*b)*cos(f*x + e)^5 - (35*a^2 + 24*a*
b - 5*b^2)*cos(f*x + e)^3 + (15*a^2 + 10*a*b - 2*b^2)*cos(f*x + e))*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a + b)*f*cos(f*x + e)^4 - 2*(a + b
)*f*cos(f*x + e)^2 + (a + b)*f)*sin(f*x + e)), 1/60*(15*((a^2 + a*b)*cos(f*
x + e)^4 - 2*(a^2 + a*b)*cos(f*x + e)^2 + a^2 + a*b)*sqrt(a)*arctan(1/4*(8*
a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos
(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(
f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))
*sin(f*x + e) - 4*((23*a^2 + 20*a*b)*cos(f*x + e)^5 - (35*a^2 + 24*a*b - 5*
b^2)*cos(f*x + e)^3 + (15*a^2 + 10*a*b - 2*b^2)*cos(f*x + e))*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2))/(((a + b)*f*cos(f*x + e)^4 - 2*(a + b)*f*co
s(f*x + e)^2 + (a + b)*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \cot^6(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx = \text{Timed out}$$

```
[In] integrate(cot(f*x+e)**6*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \cot^6(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \cot\left(fx+e\right)^6 \, dx$$

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^6, x)
```

Giac [F]

$$\int \cot^6(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \cot\left(fx+e\right)^6 \, dx$$

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^6(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx = \int \cot(e+f\,x)^6\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2} \,d x$$

```
[In] int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.402 $\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 89

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} - \frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}}{b^2f} + \frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f}$$

```
[Out] 1/3*(a+b*sec(f*x+e)^2)^(3/2)/b^2/f-arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2)
)/f/a^(1/2)-(a+2*b)*(a+b*sec(f*x+e)^2)^(1/2)/b^2/f
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4224, 457, 90, 65, 214}

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} + \frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f} - \frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}}{b^2f}$$

```
[In] Int[Tan[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f)) - ((a + 2*b)*Sqr
t[a + b*Sec[e + f*x]^2])/(b^2*f) + (a + b*Sec[e + f*x]^2)^(3/2)/(3*b^2*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)^2}{x\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{(-1+x)^2}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f} \\
&= \frac{\text{Subst}\left(\int \left(\frac{-a-2b}{b\sqrt{a+bx}} + \frac{1}{x\sqrt{a+bx}} + \frac{\sqrt{a+bx}}{b}\right)dx, x, \sec^2(e+fx)\right)}{2f}
\end{aligned}$$

$$= -\frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}}{b^2f} + \frac{(a+b\sec^2(e+fx))^{3/2}}{3b^2f} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}}{b^2f} + \frac{(a+b\sec^2(e+fx))^{3/2}}{3b^2f} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf}$$

$$= -\frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} - \frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}}{b^2f} + \frac{(a+b\sec^2(e+fx))^{3/2}}{3b^2f}$$

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.20

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{-2a(a+3b) - b(a+6b)\sec^2(e+fx) + b^2\sec^4(e+fx) - 3b^2\text{arctanh}\left(\sqrt{1+\frac{b\sec^2(e+fx)}{a}}\right)\sqrt{1+\frac{b\sec^2(e+fx)}{a}}}{3b^2f\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Integrate[Tan[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (-2*a*(a + 3*b) - b*(a + 6*b)*Sec[e + f*x]^2 + b^2*Sec[e + f*x]^4 - 3*b^2*A
rcTanh[Sqrt[1 + (b*Sec[e + f*x]^2)/a]]*Sqrt[1 + (b*Sec[e + f*x]^2)/a])/(3*b
^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(77) = 154.

Time = 8.11 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.40

method	result
default	$-\frac{2a^{\frac{5}{2}}+3\ln\left(4\cos(fx+e)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\sqrt{a}+4\cos(fx+e)a+4\sqrt{a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\right)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,b^2+6a^{\frac{3}{2}}b+3\ln\left(4\cos(fx+e\right.}{3f}$

```
[In] int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f/b^2/a^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)*(2*a^(5/2)+3*ln(4*cos(f*x+e)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*b^2+6*a^(3/2)*b+3*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*sec(f*x
+e)+a^(3/2)*b*sec(f*x+e)^2+6*a^(1/2)*b^2*sec(f*x+e)^2-a^(1/2)*b^2*sec(f*x+e
)^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(77) = 154$.

Time = 0.62 (sec) , antiderivative size = 410, normalized size of antiderivative = 4.61

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[\frac{3\sqrt{a}b^2\cos{(fx+e)}^2\log\left(128\,a^4\cos{(fx+e)}^8+256\,a^3b\cos{(fx+e)}^6+160\,a^2b^2\cos{(fx+e)}^4+32\,ab^3\,\mathrm{c}\right.}{} \right.$$

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(a)*b^2*cos(f*x + e)^2*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*
cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4
 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x +
e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)) - 8*(2*(a^2 + 3*a*b)*cos(f*x + e)^2 - a*b)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2))/(a*b^2*f*cos(f*x + e)^2), 1/12*(3*sqrt(-a)*b^2*arctan(1/4
*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)
^2 + a*b^2))*cos(f*x + e)^2 - 4*(2*(a^2 + 3*a*b)*cos(f*x + e)^2 - a*b)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a*b^2*f*cos(f*x + e)^2)]
```

Sympy [F]

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan^5{(e+fx)}}{\sqrt{a+b\sec^2{(e+fx)}}}\,dx$$

```
[In] integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(tan(e + f*x)**5/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan{(fx+e)}^5}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan{(fx+e)}^5}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(e+f\,x)^5}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

```
[In] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.403 $\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 56

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} + \frac{\sqrt{a+b\sec^2(e+fx)}}{bf}$$

```
[Out] arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)+(a+b*sec(f*x+e)^2)^(1/2
)/b/f
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4224, 457, 81, 65, 214}

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} + \frac{\sqrt{a+b\sec^2(e+fx)}}{bf}$$

```
[In] Int[Tan[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f) + Sqrt[a + b*Sec[e
+ f*x]^2]/(b*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{-1+x^2}{x\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{-1+x}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= \frac{\sqrt{a+b\sec^2(e+fx)}}{bf} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= \frac{\sqrt{a+b\sec^2(e+fx)}}{bf} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf}
\end{aligned}
$$

$$
= \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} + \frac{\sqrt{a+b\sec^2(e+fx)}}{bf}
$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$
\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} + \frac{\sqrt{a+b\sec^2(e+fx)}}{bf}
$$

```
[In] Integrate[Tan[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f) + Sqrt[a + b*Sec[e
+ f*x]^2]/(b*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(48) = 96.

Time = 4.97 (sec) , antiderivative size = 256, normalized size of antiderivative = 4.57

method	result
default	$\frac{\ln\left(4\cos(fx+e)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{a}+4\cos(fx+e)a+4\sqrt{a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\right)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,b+a^{\frac{3}{2}}+\ln\left(4\cos(fx+e)\sqrt{\frac{b+a\cos(fx}{(1+\cos(fx}}\right.}{fb\sqrt{a}\sqrt{a+b\sec(fx+e)^2}}$

```
[In] int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/b/a^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)*(ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b+a
^(3/2)+ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+
4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b*sec(f*x+e)+a^(1/2)*b*sec(f*x+e)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(48) = 96.

Time = 0.37 (sec) , antiderivative size = 328, normalized size of antiderivative = 5.86

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \left[\frac{\sqrt{a}b\log\left(128\,a^4\cos(fx+e)^8+256\,a^3b\cos(fx+e)^6+160\,a^2b^2\cos(fx+e)^4+32\,ab^3\cos(fx+e)^2+b^4\right.}{}\right.$$

$$\left. -\frac{\sqrt{-a}b\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^4+8\,ab\cos(fx+e)^2+b^2\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4+3\,a^2b\cos(fx+e)^2+ab^2\right)}\right)-4\,a\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\,abf}\right]$$

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160
*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x
 + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x +
 e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 8*a*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2))/(a*b*f), -1/4*(sqrt(-a)*b*arctan(1/4*(8
*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2
+ a*b^2)) - 4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a*b*f)]
```

Sympy [F]

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

```
[In] integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(fx+e)^3}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(fx+e)^3}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(e+f\,x)^3}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

```
[In] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.404 $\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 33

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

```
[Out] -arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4224, 272, 65, 214}

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

```
[In] Int[Tan[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf}\\
&= -\frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

```
[In] Integrate[Tan[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

method	result	size
derivativedivides	$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\sec(fx+e)^2}}{\sec(fx+e)}\right)}{f\sqrt{a}}$	42
default	$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\sec(fx+e)^2}}{\sec(fx+e)}\right)}{f\sqrt{a}}$	42

```
[In] int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/f/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sec(f*x+e)^2)^(1/2))/sec(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(27) = 54.

Time = 0.32 (sec) , antiderivative size = 261, normalized size of antiderivative = 7.91

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[\frac{\log\left(128\,a^4\cos\left(fx+e\right)^8 + 256\,a^3b\cos\left(fx+e\right)^6 + 160\,a^2b^2\cos\left(fx+e\right)^4 + 32\,ab^3\cos\left(fx+e\right)^2 + b^4 - 8\right.}{8\sqrt{a}} \right.$$

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] [1/8*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*co
s(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 2
4*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt
(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(sqrt(a)*f), 1/4*sqrt(-a)*
arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*co
s(f*x + e)^2 + a*b^2))/(a*f)]
```

Sympy [F]

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(tan(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(fx+e)}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(fx+e)}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 20.53 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

```
[In] int(tan(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] -atanh((a + b/cos(e + f*x)^2)^(1/2)/a^(1/2))/(a^(1/2)*f)
```

3.405 $\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}f}$$

```
[Out] arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)-arctanh((a+b*sec(f*x+e)
^2)^(1/2)/(a+b)^(1/2))/f/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4224, 457, 88, 65, 214}

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f\sqrt{a+b}}$$

```
[In] Int[Cot[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f) - ArcTanh[Sqrt[a +
b*Sec[e + f*x]^2]/Sqrt[a + b]]/(Sqrt[a + b]*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf}$$

$$= \frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} - \frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}f}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.80 (sec) , antiderivative size = 294, normalized size of antiderivative = 4.20

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\left(-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b}\left(1+e^{2i(e+fx)}\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)+\sqrt{a+b}\left(\text{arctanh}\left(\frac{a+}{\sqrt{a}\sqrt{4be^{2i(e}}}\right.\right.\right.}{2\sqrt{a}\sqrt{a+b}\left(1+e^{2i(e+fx)}\right)f\sqrt{a+b\sec^2(e+}}$$

```
[In] Integrate[Cot[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*(-2*Sqrt[a]*
ArcTanh[(Sqrt[a + b]*(1 + E^((2*I)*(e + f*x))))/Sqrt[4*b*E^((2*I)*(e + f*x)
) + a*(1 + E^((2*I)*(e + f*x)))^2]] + Sqrt[a + b]*(ArcTanh[(a + 2*b + a*E^(
(2*I)*(e + f*x)))/(Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(
e + f*x)))^2])] + ArcTanh[(a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*
x)))/(Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]
)])))/(2*Sqrt[a]*Sqrt[a + b]*(1 + E^((2*I)*(e + f*x)))*f*Sqrt[a + b*Sec[e +
 f*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. 2(58) = 116.

Time = 1.46 (sec) , antiderivative size = 343, normalized size of antiderivative = 4.90

method	result
default	$\frac{\left(\ln\left(\frac{2\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{a+b}\,\cos(fx+e)+2\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{a+b}-2\cos(fx+e)a+2b}{\sqrt{a+b}\,(1+\cos(fx+e))}\right)\sqrt{a}-\ln\left(-\frac{4\left(\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{a+b}\,\cos(fx+e)+\sqrt{}\right.}{-1+\cos}\right.\right.}{}$

```
[In] int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f/(a+b)^(1/2)/a^(1/2)*(ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^(1/2)-ln(-4*(((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^(1
/2)+2*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4
*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*(a+b)^
(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)
*(sec(f*x+e)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(58) = 116$.

Time = 0.44 (sec) , antiderivative size = 1015, normalized size of antiderivative = 14.50

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((a + b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6
 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*c
os(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos
(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 2*sqrt(
a + b)*a*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*co
s(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a
 + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(
f*x + e)^2 + 1)))/((a^2 + a*b)*f), 1/8*(4*a*sqrt(-a - b)*arctan(1/2*((2*a +
 b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + (a + b)*sqrt(a)*log(128*a
^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 +
 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*
x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)))/((a^2 + a*b)*f), -1/4*(sqrt(-a)*(a + b)
*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*c
os(f*x + e)^2 + a*b^2)) - sqrt(a + b)*a*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*
x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x +
e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a^2 + a*b)*f), -1/4*(sqr
t(-a)*(a + b)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2
)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^
4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 2*a*sqrt(-a - b)*arctan(1/2*((2*a +
```

```
b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)))/((a^2 + a*b)*f)]
```

Sympy [F]

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(cot(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(fx+e)}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(cot(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(fx+e)}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(e+f\,x)}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

```
[In] int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.406 $\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} + \frac{(2a+3b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{3/2}f} - \frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f}$$

```
[Out] 1/2*(2*a+3*b)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/f-a
rctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)-1/2*cot(f*x+e)^2*(a+b*se
c(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4224, 457, 105, 162, 65, 214}

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} + \frac{(2a+3b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f(a+b)^{3/2}} - \frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f(a+b)}$$

```
[In] Int[Cot[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f)) + ((2*a + 3*b)*A
rcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(2*(a + b)^(3/2)*f) - (Cot[
e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(2*(a + b)*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
```

```
x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(-1+x^2)^2\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)^2x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f} \\
&= -\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f} - \frac{\text{Subst}\left(\int \frac{a+b+\frac{bx}{2}}{(-1+x)x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2(a+b)f} \\
&= -\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f} \\
&\quad - \frac{(2a+3b)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{4(a+b)f} \\
&= -\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf} \\
&\quad - \frac{(2a+3b)\text{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{2b(a+b)f} \\
&= -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} + \frac{(2a+3b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{3/2}f} \\
&\quad - \frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f}
\end{aligned}$$

Mathematica [F]

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

```
[In] Integrate[Cot[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Cot[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1943 vs. 2(98) = 196.

Time = 1.38 (sec) , antiderivative size = 1944, normalized size of antiderivative = 16.76

method	result	size
default	Expression too large to display	1944

```
[In] int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f/(a+b)^(5/2)/a^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))
^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(
f*x+e)^2+a+b)^(1/2)/((a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*cs
c(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e
)^2+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^2)^(1/2)/((1-cos(f*x+e))^2*csc(f
*x+e)^2-1)/(1-cos(f*x+e))^2*(4*a^(5/2)*ln((a*(1-cos(f*x+e))^2*csc(f*x+e)^2+
b*(1-cos(f*x+e))^2*csc(f*x+e)^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f
*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^
2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)-a+b)/(a+b)^(1/2))*(1-cos(f*x+e))^2-4*
a^(5/2)*ln(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))^2+b*(1-cos(f*x+e))^2+(a*(1
-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+
e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)
*sin(f*x+e)^2+a*sin(f*x+e)^2+b*sin(f*x+e)^2))*(1-cos(f*x+e))^2+10*ln((a*(1-
cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+(a*(1-cos(f*x+e)
)^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f
*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)-a+b)/(a+b)
^(1/2))*(1-cos(f*x+e))^2*a^(3/2)*b-10*ln(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+
e))^2+b*(1-cos(f*x+e))^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^
4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f
*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2+b*sin(f*x+e)^2))
*(1-cos(f*x+e))^2*a^(3/2)*b-(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e
))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*cs
c(f*x+e)^2+a+b)^(1/2)*(a+b)^(3/2)*(1-cos(f*x+e))^2*a^(1/2)+8*ln(4*(-a*(1-co
s(f*x+e))^2*csc(f*x+e)^2+a^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(
f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))
^2*csc(f*x+e)^2+a+b)^(1/2)+a)/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a+b)^(3/2
)*(1-cos(f*x+e))^2*a+8*ln(4*(-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+a^(1/2)*(a*(1
-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+
e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)+a)/((1-cos(
f*x+e))^2*csc(f*x+e)^2+1))*(a+b)^(3/2)*(1-cos(f*x+e))^2*b+6*a^(1/2)*ln((a*(
1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+(a*(1-cos(f*x+
e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc
(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)-a+b)/(a+
b)^(1/2))*b^2*(1-cos(f*x+e))^2-6*a^(1/2)*ln(2/(1-cos(f*x+e))^2*(-a*(1-cos(f
*x+e))^2+b*(1-cos(f*x+e))^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e
```

```
))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*cs
c(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2+b*sin(f*x+e)^
2))*b^2*(1-cos(f*x+e))^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^
4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f
*x+e)^2+a+b)^(1/2)*(a+b)^(3/2)*a^(1/2)*sin(f*x+e)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(98) = 196$.

Time = 0.62 (sec) , antiderivative size = 1550, normalized size of antiderivative = 13.36

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a^2 + a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e
)^2 + ((a^2 + 2*a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(a)*log(
128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e
)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*c
os(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + ((2*a^2 + 3*a*b)*cos(f*x + e)^2 -
 2*a^2 - 3*a*b)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2
*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos
(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(
f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a^3 + 2*a^2*b + a*b^2)*f*cos(f*x + e
)^2 - (a^3 + 2*a^2*b + a*b^2)*f), 1/8*(4*(a^2 + a*b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 - 2*((2*a^2 + 3*a*b)*cos(f*x + e)^2 -
2*a^2 - 3*a*b)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(
-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x +
e)^2 + a*b + b^2)) + ((a^2 + 2*a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^
2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*
b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)
^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2
)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)))/((a^3 + 2*a^2*b + a
*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f), 1/8*(4*(a^2 + a*b)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 + 2*((a^2 + 2*a*b +
 b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*
x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) +
 ((2*a^2 + 3*a*b)*cos(f*x + e)^2 - 2*a^2 - 3*a*b)*sqrt(a + b)*log(2*((8*a^2
 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4
*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a^
```

```
3 + 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f), 1/4*(2*
(a^2 + a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 + ((
a^2 + 2*a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(-a)*arctan(1/4*
(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^
2 + a*b^2)) - ((2*a^2 + 3*a*b)*cos(f*x + e)^2 - 2*a^2 - 3*a*b)*sqrt(-a - b)
*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)))/((a^3 +
 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f)]
```

Sympy [F]

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

```
[In] integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(cot(e + f*x)**3/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(fx+e)^3}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(cot(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(fx+e)^3}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(e+f\,x)^3}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

```
[In] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.407 $\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 166

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} - \frac{(8a^2+20ab+15b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{5/2}f} + \frac{(4a+7b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)f}$$

```
[Out] -1/8*(8*a^2+20*a*b+15*b^2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a
+b)^(5/2)/f+arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)+1/8*(4*a+7*
b)*cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2)/(a+b)^2/f-1/4*cot(f*x+e)^4*(a+b*se
c(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used

= {4224, 457, 105, 156, 162, 65, 214}

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(8a^2+20ab+15b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{5/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f(a+b)} + \frac{(4a+7b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f(a+b)^2}$$

```
[In] Int[Cot[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f) - ((8*a^2 + 20*a*b
+ 15*b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(8*(a + b)^(5/2)
*f) + ((4*a + 7*b)*Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(8*(a + b)^2*
f) - (Cot[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2])/(4*(a + b)*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(-1+x^2)^3\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)^3x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= -\frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)f} - \frac{\text{Subst}\left(\int \frac{2(a+b)+\frac{3bx}{2}}{(-1+x)^2x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{4(a+b)f}\\
&= \frac{(4a+7b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)f}\\
&\quad + \frac{\text{Subst}\left(\int \frac{2(a+b)^2+\frac{1}{4}b(4a+7b)x}{(-1+x)x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{4(a+b)^2f}
\end{aligned}$$

$$= \frac{(4a+7b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)f} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f} + \frac{(8a^2+20ab+15b^2)\,\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{16(a+b)^2f}$$

$$= \frac{(4a+7b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)f} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf} + \frac{(8a^2+20ab+15b^2)\,\text{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{8b(a+b)^2f}$$

$$= \frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} - \frac{(8a^2+20ab+15b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{5/2}f} + \frac{(4a+7b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)f}$$

Mathematica [F]

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

```
[In] Integrate[Cot[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Cot[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3595 vs. $2(144) = 288$.

Time = 1.55 (sec) , antiderivative size = 3596, normalized size of antiderivative = 21.66

method	result	size
default	Expression too large to display	3596

```
[In] int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64/f/(a+b)^(9/2)/a^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e
))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*cs
c(f*x+e)^2+a+b)^(1/2)/((a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*
csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x
+e)^2+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^2)^(1/2)/((1-cos(f*x+e))^2*csc
(f*x+e)^2-1)/(1-cos(f*x+e))^4*(252*a^(5/2)*ln((a*(1-cos(f*x+e))^2*csc(f*x+e
)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-c
os(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+
e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)-a+b)/(a+b)^(1/2))*b^2*(1-cos(f*x+
e))^4-252*a^(5/2)*ln(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))^2+b*(1-cos(f*x+e
))^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(
1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(
a+b)^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2+b*sin(f*x+e)^2))*b^2*(1-cos(f*x+e))^
4+(1-cos(f*x+e))^6*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(
f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^
2+a+b)^(1/2)*(a+b)^(5/2)*a^(3/2)*csc(f*x+e)^2+(1-cos(f*x+e))^6*(a*(1-cos(f*
x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*c
sc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(5/2)*a^(1/2
)*b*csc(f*x+e)^2-11*a^(3/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e
))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*cs
c(f*x+e)^2+a+b)^(1/2)*(a+b)^(5/2)*(1-cos(f*x+e))^4-17*b*(a*(1-cos(f*x+e))^4
*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+
e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(5/2)*(1-cos(f*x+e)
)^4*a^(1/2)+128*(a+b)^(5/2)*ln(4*(-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+a^(1/2)*
(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos
(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)+a)/((1
-cos(f*x+e))^2*csc(f*x+e)^2+1))*a*b*(1-cos(f*x+e))^4+11*a^(3/2)*(a*(1-cos(f
*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*
csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(5/2)*(1-co
s(f*x+e))^2*sin(f*x+e)^2+17*b*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x
+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*
csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(5/2)*(1-cos(f*x+e))^2*a^(1/2)*sin(f*x+e)^2+6
4*(a+b)^(5/2)*ln(4*(-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+a^(1/2)*(a*(1-cos(f*x+
e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc
(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)+a)/((1-cos(f*x+e))^2
*csc(f*x+e)^2+1))*b^2*(1-cos(f*x+e))^4+32*a^(9/2)*ln((a*(1-cos(f*x+e))^2*cs
c(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4
+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-c
os(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)-a+b)/(a+b)^(1/2))*(1-cos(f
*x+e))^4+60*a^(1/2)*ln((a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*
csc(f*x+e)^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)
^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)
^(1/2)*(a+b)^(1/2)-a+b)/(a+b)^(1/2))*b^4*(1-cos(f*x+e))^4-32*a^(9/2)*ln(2/(
1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))^2+b*(1-cos(f*x+e))^2+(a*(1-cos(f*x+e))^4
*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+
```

```
e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^2+
a*sin(f*x+e)^2+b*sin(f*x+e)^2))*(1-cos(f*x+e))^4-60*a^(1/2)*ln(2/(1-cos(f*x
+e))^2*(-a*(1-cos(f*x+e))^2+b*(1-cos(f*x+e))^2+(a*(1-cos(f*x+e))^4*csc(f*x+
e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*
(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^2+a*sin(f*x
+e)^2+b*sin(f*x+e)^2))*b^4*(1-cos(f*x+e))^4-(a*(1-cos(f*x+e))^4*csc(f*x+e)^
4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-
cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(5/2)*a^(3/2)*sin(f*x+e)^4-(a*(
1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x
+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(5/2
)*a^(1/2)*b*sin(f*x+e)^4+64*(a+b)^(5/2)*ln(4*(-a*(1-cos(f*x+e))^2*csc(f*x+e
)^2+a^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^
4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^
(1/2)+a)/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*a^2*(1-cos(f*x+e))^4+144*a^(7/2
)*ln((a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+(a*(1
-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+
e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)
-a+b)/(a+b)^(1/2))*(1-cos(f*x+e))^4*b-144*a^(7/2)*ln(2/(1-cos(f*x+e))^2*(-a
*(1-cos(f*x+e))^2+b*(1-cos(f*x+e))^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-
cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x
+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2+b*si
n(f*x+e)^2))*(1-cos(f*x+e))^4*b+200*a^(3/2)*ln((a*(1-cos(f*x+e))^2*csc(f*x+
e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-
cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x
+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)-a+b)/(a+b)^(1/2))*b^3*(1-cos(f*x
+e))^4-200*a^(3/2)*ln(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))^2+b*(1-cos(f*x+
e))^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*
(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*
(a+b)^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2+b*sin(f*x+e)^2))*b^3*(1-cos(f*x+e))
^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(144) = 288$.

Time = 1.38 (sec) , antiderivative size = 2257, normalized size of antiderivative = 13.60

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 + a^3 + 3*a^2*b +
3*a*b^2 + b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^2)*sqrt(a)*l
og(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x
```

+ e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*
b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + ((8*a^3 + 20*a^2*b + 15*a*b^2)
*cos(f*x + e)^4 + 8*a^3 + 20*a^2*b + 15*a*b^2 - 2*(8*a^3 + 20*a^2*b + 15*a*
b^2)*cos(f*x + e)^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^
4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 +
b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/
(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*(3*(2*a^3 + 5*a^2*b + 3*a*b^2)
*cos(f*x + e)^4 - (4*a^3 + 11*a^2*b + 7*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos
(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^4
 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f), 1/16*(((8*a^3 + 20*a^2*b + 15*a*b^2)*co
s(f*x + e)^4 + 8*a^3 + 20*a^2*b + 15*a*b^2 - 2*(8*a^3 + 20*a^2*b + 15*a*b^2
)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sq
rt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x
 + e)^2 + a*b + b^2)) + 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 +
 a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x
+ e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160
*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x
 + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x +
 e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 2*(3*(2*a^3 +
 5*a^2*b + 3*a*b^2)*cos(f*x + e)^4 - (4*a^3 + 11*a^2*b + 7*a*b^2)*cos(f*x +
 e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 + 3*a^3*b + 3*a^2
*b^2 + a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*co
s(f*x + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f), -1/32*(8*((a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 2*(a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*co
s(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2
)) - ((8*a^3 + 20*a^2*b + 15*a*b^2)*cos(f*x + e)^4 + 8*a^3 + 20*a^2*b + 15*
a*b^2 - 2*(8*a^3 + 20*a^2*b + 15*a*b^2)*cos(f*x + e)^2)*sqrt(a + b)*log(2*(
(8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b
^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1))
 + 4*(3*(2*a^3 + 5*a^2*b + 3*a*b^2)*cos(f*x + e)^4 - (4*a^3 + 11*a^2*b + 7*
a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 +
 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b
^2 + a*b^3)*f*cos(f*x + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f), -1/1
6*(4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 + a^3 + 3*a^2*b + 3*a*
b^2 + b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^2)*sqrt(-a)*arct
an(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*
x + e)^2 + a*b^2)) - ((8*a^3 + 20*a^2*b + 15*a*b^2)*cos(f*x + e)^4 + 8*a^3
+ 20*a^2*b + 15*a*b^2 - 2*(8*a^3 + 20*a^2*b + 15*a*b^2)*cos(f*x + e)^2)*sqr
t(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*co

```
s(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2))
 + 2*(3*(2*a^3 + 5*a^2*b + 3*a*b^2)*cos(f*x + e)^4 - (4*a^3 + 11*a^2*b + 7*
a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 +
 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b
^2 + a*b^3)*f*cos(f*x + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)]
```

Sympy [F]

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cot^5{(e+fx)}}{\sqrt{a+b\sec^2{(e+fx)}}}\,dx$$

```
[In] integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(cot(e + f*x)**5/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cot{(fx+e)}^5}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(cot(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cot{(fx+e)}^5}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(e+f\,x)^5}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

```
[In] int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.408 $\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 173

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} + \frac{(3a^2+10ab+15b^2)\,\mathrm{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^{5/2}f} - \frac{(3a+7b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8b^2f} + \frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4bf}$$

```
[Out] 1/8*(3*a^2+10*a*b+15*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(
1/2))/b^(5/2)/f-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(
1/2)-1/8*(3*a+7*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b^2/f+1/4*(a+b+b*t
an(f*x+e)^2)^(1/2)*tan(f*x+e)^3/b/f
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used

$= \{4226, 2000, 490, 596, 537, 223, 212, 385, 209\}$

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{(3a^2+10ab+15b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8b^{5/2}f} - \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f} - \frac{(3a+7b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8b^2f} + \frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4bf}$$

```
[In] Int[Tan[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
) + ((3*a^2 + 10*a*b + 15*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b +
b*Tan[e + f*x]^2]])/(8*b^(5/2)*f) - ((3*a + 7*b)*Tan[e + f*x]*Sqrt[a + b +
b*Tan[e + f*x]^2])/(8*b^2*f) + (Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^
2])/(4*b*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
```

```
1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\sqrt{a+b(1+x^2)}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$=\frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4bf}-\frac{\text{Subst}\left(\int\frac{x^2(3(a+b)+(3a+7b)x^2)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx,x,\tan(e+fx)\right)}{4bf}$$

$$=-\frac{(3a+7b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8b^2f}+\frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4bf}+\frac{\text{Subst}\left(\int\frac{(a+b)(3a+7b)+(3a^2+10ab+15b^2)x^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx,x,\tan(e+fx)\right)}{8b^2f}$$

$$=-\frac{(3a+7b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8b^2f}+\frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4bf}-\frac{\text{Subst}\left(\int\frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx,x,\tan(e+fx)\right)}{f}+\frac{(3a^2+10ab+15b^2)\,\text{Subst}\left(\int\frac{1}{\sqrt{a+b+bx^2}}\,dx,x,\tan(e+fx)\right)}{8b^2f}$$

$$=-\frac{(3a+7b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8b^2f}+\frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4bf}-\frac{\text{Subst}\left(\int\frac{1}{1+ax^2}\,dx,x,\frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{(3a^2+10ab+15b^2)\,\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^2f}$$

$$=-\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f}+\frac{(3a^2+10ab+15b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^{5/2}f}-\frac{(3a+7b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8b^2f}+\frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4bf}$$

Mathematica [A] (verified)

Time = 3.64 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.33

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\left(\frac{8b^2\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{a}}-\frac{(3a^2+10ab+15b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(}{8\sqrt{2}b^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{(a+2b+a\cos(2(e+fx)))(3a+5b+3(a+3b)\cos(2(e+fx)))\sec^4(e+fx)\tan(e+fx)}{32b^2f\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Integrate[Tan[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/8*(((8*b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]
)/Sqrt[a] - ((3*a^2 + 10*a*b + 15*b^2)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[
a + b - a*Sin[e + f*x]^2]])/Sqrt[b])*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sec
[e + f*x])/(Sqrt[2]*b^2*f*Sqrt[a + b*Sec[e + f*x]^2]) - ((a + 2*b + a*Cos[2
*(e + f*x)])*(3*a + 5*b + 3*(a + 3*b)*Cos[2*(e + f*x)])*Sec[e + f*x]^4*Tan[
e + f*x])/(32*b^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1929 vs. 2(151) = 302.

Time = 17.68 (sec) , antiderivative size = 1930, normalized size of antiderivative = 11.16

method	result	size
default	Expression too large to display	1930

```
[In] int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/f/b^(9/2)/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)*(-16*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)-4*sin(f*x+e)*a)*b^(9/2)-16*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*l
n(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a
)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^(9/2)
*sec(f*x+e)-18*(-a)^(1/2)*b^(7/2)*a*tan(f*x+e)-6*(-a)^(1/2)*b^(5/2)*a^2*tan
(f*x+e)-18*(-a)^(1/2)*b^(9/2)*tan(f*x+e)*sec(f*x+e)^2+3*(-a)^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a^2*b^2+10*(-a)^(1/2)*((b+a*cos(f
```

```
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a*b^3+15*(-a)^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-
sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*b^4+3*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/
2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e
)*a-a-b)/(sin(f*x+e)+1))*a^2*b^2+10*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*c
os(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-
a-b)/(sin(f*x+e)+1))*a*b^3+15*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x
+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(
sin(f*x+e)+1))*b^4-2*(-a)^(1/2)*b^(7/2)*a*tan(f*x+e)*sec(f*x+e)^2+3*(-a)^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a^2*b^2*sec(f*x+e)+10
*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a*b^3*sec(f*x
+e)+15*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*b^4*sec
(f*x+e)+3*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a^2*b
^2*sec(f*x+e)+10*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(
4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1)
)*a*b^3*sec(f*x+e)+15*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+
e)+1))*b^4*sec(f*x+e)+4*(-a)^(1/2)*b^(9/2)*tan(f*x+e)*sec(f*x+e)^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(151) = 302$.

Time = 1.39 (sec) , antiderivative size = 1673, normalized size of antiderivative = 9.67

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(4*sqrt(-a)*b^3*cos(f*x + e)^3*log(128*a^4*cos(f*x + e)^8 - 256*(a^4
 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4
 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2
*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*
cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a
^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*sin(f*x + e)) - (3*a^3 + 10*a^2*b + 15*a*b^2)*sqrt(b)*cos(f*x
 + e)^3*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e
)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*(2*a
*b^2 - 3*(a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e))/(a*b^3*f*cos(f*x + e)^3), -1/16*(2*sqrt(-a)*b^3*c
os(f*x + e)^3*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6
 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a
^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^
3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*co
s(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
e)) - (3*a^3 + 10*a^2*b + 15*a*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x +
 e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^3 - 2*(2*a*b^2
- 3*(a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)*sin(f*x + e))/(a*b^3*f*cos(f*x + e)^3), 1/32*(8*sqrt(a)*b^3*arctan(1
/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^
2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^
3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x
+ e)))*cos(f*x + e)^3 + (3*a^3 + 10*a^2*b + 15*a*b^2)*sqrt(b)*cos(f*x + e)^
3*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 +
4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*(2*a*b^2 -
 3*(a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e))/(a*b^3*f*cos(f*x + e)^3), 1/16*(4*sqrt(a)*b^3*arctan(1/
4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2
)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3
*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x +
 e)))*cos(f*x + e)^3 + (3*a^3 + 10*a^2*b + 15*a*b^2)*sqrt(-b)*arctan(-1/2*(
(a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e
)^3 + 2*(2*a*b^2 - 3*(a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*b^3*f*cos(f*x + e)^3)]
```

Sympy [F]

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

```
[In] integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(tan(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(fx+e)^6}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(fx+e)^6}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(e+f\,x)^6}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

```
[In] int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.409 $\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 120

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} - \frac{(a+3b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{3/2}f} + \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf}$$

```
[Out] -1/2*(a+3*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(3/2)
/f+arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)+1/2*(a+b
+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4226, 2000, 490, 537, 223, 212, 385, 209}

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f} - \frac{(a+3b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{3/2}f} + \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2bf}$$

```
[In] Int[Tan[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f) -
 ((a + 3*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])
/(2*b^(3/2)*f) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*b*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\sqrt{a+b(1+x^2)}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf} - \frac{\text{Subst}\left(\int \frac{a+b+(a+3b)x^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2bf}$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} - \frac{(a+3b)\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2bf}$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf} + \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{(a+3b)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2bf}$$

$$= \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} - \frac{(a+3b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{3/2}f} + \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf}$$

Mathematica [A] (verified)

Time = 2.54 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.63

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$
$$= \frac{\left(\frac{2b\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{a}}-\frac{(a+3b)\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)}{2\sqrt{2}bf\sqrt{a+b\sec^2(e+fx)}}$$
$$+\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\tan(e+fx)}{4bf\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Integrate[Tan[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (((2*b*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqrt[
a] - ((a + 3*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^
2]])/Sqrt[b])*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(2*Sqrt[2]*b
*f*Sqrt[a + b*Sec[e + f*x]^2]) + ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*
x]^2*Tan[e + f*x])/(4*b*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1333 vs. 2(102) = 204.

Time = 12.66 (sec) , antiderivative size = 1334, normalized size of antiderivative = 11.12

method	result	size
default	Expression too large to display	1334

```
[In] int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f/(-a)^(1/2)/b^(5/2)/(a+b*sec(f*x+e)^2)^(1/2)*(4*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-
4*sin(f*x+e)*a)*b^(5/2)+4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*
(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^(5/2)*sec
(f*x+e)+2*(-a)^(1/2)*b^(3/2)*a*tan(f*x+e)-(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b
^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)+a+b)/(sin(f*x+e)+1))*a*b-3*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*
cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
```

```
+a+b)/(sin(f*x+e)+1))*b^2-(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*ln(4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)
+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin
(f*x+e)-1))*a*b-3*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln
(4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x
+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-
1))*b^2+2*(-a)^(1/2)*b^(5/2)*tan(f*x+e)*sec(f*x+e)^2-(-a)^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*a*b*sec(f*x+e)-3*(-a)^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*b^2*sec(f*x+e)-(-a)^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*a*b*sec(f*x+e)-3*(-a)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*b^2*sec(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(102) = 204$.

Time = 0.67 (sec) , antiderivative size = 1507, normalized size of antiderivative = 12.56

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a)*b^2*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^
3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^
4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
- a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f
*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b
+ 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*sin(f*x + e)) - (a^2 + 3*a*b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a
*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*
x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*a*b*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*sin(f*x + e))/(a*b^2*f*cos(f*x + e)), -1/8*(sqrt(-a)*b
^2*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)
^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70
*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x
```

```
+ e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*
a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*
cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
+ e)) + 2*(a^2 + 3*a*b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*
cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*co
s(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) - 4*a*b*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*b^2*f*cos(f*x + e)), -1/8*(2*sqrt
(a)*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (
a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x
+ e)^2)*sin(f*x + e)))*cos(f*x + e) - (a^2 + 3*a*b)*sqrt(b)*cos(f*x + e)*lo
g(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((
a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*a*b*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*b^2*f*cos(f*x + e)), -1/
4*(sqrt(a)*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e
)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*c
os(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) + (a^2 + 3*a*b)*sqrt(-b)*arctan(
-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f
*x + e) - 2*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(
a*b^2*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\tan^4{(e+fx)}}{\sqrt{a+b\sec^2{(e+fx)}}}\,dx$$

```
[In] integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(tan(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\tan{(fx+e)}^4}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\tan\left(fx+e\right)^4}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\tan(e+f\,x)^4}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

```
[In] int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.410 $\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 80

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=-\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{b}f}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)+arctanh(b^
(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/b^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4226, 2000, 494, 223, 212, 385, 209}

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}f}-\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}$$

```
[In] Int[Tan[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
) + ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[b]
*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\sqrt{a+b(1+x^2)}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

$$= -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} + \frac{\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{b}f}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.48 (sec) , antiderivative size = 296, normalized size of antiderivative = 3.70

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \frac{i\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\left(-2\sqrt{a}\arctan\left(\frac{\sqrt{b}\left(-1+e^{2i(e+fx)}\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)+\sqrt{b}\text{arctanh}\left(\frac{a+2b+ae^{2i}\cdots}{\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\cdots}}\right.\right.}{2\sqrt{a}\sqrt{b}\left(1+e^{2i(e+fx)}\right)f\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Integrate[Tan[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((I/2)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*(-2*Sq
rt[a]*ArcTan[(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))))/Sqrt[4*b*E^((2*I)*(e + f*
x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + Sqrt[b]*ArcTanh[(a + 2*b + a*E^((2*
I)*(e + f*x)))/(Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e +
 f*x)))^2])] - Sqrt[b]*ArcTanh[(a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e
 + f*x)))/(Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)
)^2])]))/(Sqrt[a]*Sqrt[b]*(1 + E^((2*I)*(e + f*x)))*f*Sqrt[a + b*Sec[e + f
*x]^2])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(68) = 136.

Time = 8.97 (sec) , antiderivative size = 344, normalized size of antiderivative = 4.30

method	result
default	$\left(\ln\left(\frac{-4\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\sqrt{b}\,\cos(fx+e)+4\sin(fx+e)a-4\sqrt{b}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4a-4b}{\sin(fx+e)-1}\right)\sqrt{-a}+\ln\left(-\frac{4\left(-\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\sqrt{b}\,\cos(fx+e)\cdots\right.}{\sin(\cdots}\right.\right.$

```
[In] int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f/(-a)^(1/2)/b^(1/2)*(ln(4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)-a-b)/(sin(f*x+e)-1))*(-a)^(1/2)+ln(-4*(-((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*(-a)^(1/2)-2*ln(4*(-a)^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^(1/2))*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)*(sec(f*x+e)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(68) = 136$.

Time = 0.50 (sec) , antiderivative size = 1259, normalized size of antiderivative = 15.74

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a)*b*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x +
e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b +
70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*
x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(
5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3
)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e)) - 2*a*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^
2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x +
e)^4))/(a*b*f), 1/8*(4*a*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b
*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*c
os(f*x + e)^2 + b^2)*sin(f*x + e))) - sqrt(-a)*b*log(128*a^4*cos(f*x + e)^8
 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos
(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^
3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^
3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 -
 (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e
```

```
)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/(a*b*f), 1/4*(sqrt(a)*b*arctan(1/4*
(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*
cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*c
os(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e
))) + a*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos
(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))
/(a*b*f), 1/4*(sqrt(a)*b*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*c
os(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 -
3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 2*a*sqrt(-b)*arctan(-1/2*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))))/(a*b*f)]
```

Sympy [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\tan^2{(e+fx)}}{\sqrt{a+b\sec^2{(e+fx)}}}\,dx$$

```
[In] integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(tan(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\tan\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\tan\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\tan(e+f\,x)^2}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

```
[In] int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.411 $\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4213, 385, 209}

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}$$

```
[In] Int[1/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}\\ &= \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(39) = 78.

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.23

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)}{\sqrt{2}\sqrt{a}f\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Integrate[1/Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] (ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2*b
 + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[e +
f*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(33) = 66$.

Time = 3.32 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.54

method	result	size
default	$\frac{\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)(\sec(fx+e)+1)}{f\sqrt{-a}\sqrt{a+b\sec(fx+e)^2}}$	138

```
[In] int(1/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)/(a+b*sec(f*x+e)^2)^(1/
2)*(sec(f*x+e)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(33) = 66$.

Time = 0.37 (sec) , antiderivative size = 408, normalized size of antiderivative = 10.46

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[-\frac{\sqrt{-a}\log\left(128\,a^4\cos(fx+e)^8-256\,(a^4-a^3b)\cos(fx+e)^6+32\,(5\,a^4-14\,a^3b+5\,a^2b^2)\cos(fx+e)^4\right.}{}\right.$$

$$\left. -\frac{\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\,(a^2-ab)\cos(fx+e)^3+(a^2-6\,ab+b^2)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3\,a^2b)\cos(fx+e)^2\right)\sin(fx+e)}\right)}{4\,\sqrt{a}f}\right]$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*
a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
 e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*c
```

```
os(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e))/(a*f), -1/4*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x +
 e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)
*cos(f*x + e)^2)*sin(f*x + e)))/(sqrt(a)*f)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. $2(33) = 66$.

Time = 0.42 (sec) , antiderivative size = 992, normalized size of antiderivative = 25.44

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(arctan2(2*a*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f
*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*
sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e
)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4
*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x
+ 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos
(2*f*x + 2*e) + a)), 2*a*cos(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2
*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 +
 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f
*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x +
4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*si
n(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a +
2*b)*cos(2*f*x + 2*e) + a)) + 2*a + 4*b) - arctan2(2*(a^2*cos(4*f*x + 4*e)^
2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4
*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*
sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4
*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arcta
n2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) +
```

```
2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f
*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*
sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e
)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4
*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x
+ 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos
(2*f*x + 2*e) + a)) + 4*a + 4*b))/(sqrt(a)*f)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

```
[In] int(1/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(1/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.412 $\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 74

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} - \frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{(a+b)f}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)-cot(f*x+e)
*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4226, 2000, 491, 12, 385, 209}

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f} - \frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f(a+b)}$$

```
[In] Int[Cot[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
) - (Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/((a + b)*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)\sqrt{a+b(1+x^2)}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{(a+b)f} + \frac{\text{Subst}\left(\int \frac{-a-b}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{(a+b)f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{(a+b)f} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{(a+b)f} - \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

$$= -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} - \frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{(a+b)f}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.72

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx =$$

$$-\frac{\sqrt{a+2b+a\cos(2(e+fx))}\sec(e+fx)\left((a+b)\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)+\sqrt{a}\csc(e+fx)\sqrt{a+b-}\right.}{\sqrt{2}\sqrt{a}(a+b)f\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Integrate[Cot[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sec[e + f*x]*((a + b)*ArcTan[(Sqrt[a]
*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]] + Sqrt[a]*Csc[e + f*x]*Sqrt[
a + b - a*Sin[e + f*x]^2]))/(Sqrt[2]*Sqrt[a]*(a + b)*f*Sqrt[a + b*Sec[e + f
*x]^2]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(66) = 132$.

Time = 4.12 (sec) , antiderivative size = 507, normalized size of antiderivative = 6.85

method	result
default	$-\frac{\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)a+\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\ln\left(4\sqrt{-}\right.}{}$

```
[In] int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f/(a+b)/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin
(f*x+e)*a)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b+(-a)^(1/2)*a*cot(f*x+e
)+sec(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a+((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-
4*sin(f*x+e)*a)*b*sec(f*x+e)+(-a)^(1/2)*b*sec(f*x+e)*csc(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(66) = 132$.

Time = 0.44 (sec) , antiderivative size = 525, normalized size of antiderivative = 7.09

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[-\frac{\sqrt{-a}(a+b)\log\left(128\,a^4\cos\left(fx+e\right)^8 - 256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6 + 32\left(5\,a^4-14\,a^3b+5\,a^2b^2\right)\cos\left(f\right.\right.}{}\right.$$

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a)*(a + b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(
f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^
3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*
cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5
 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2
 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
sin(f*x + e))*sin(f*x + e) + 8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*cos(f*x + e))/((a^2 + a*b)*f*sin(f*x + e)), 1/4*((a + b)*sqrt(a)*arctan(1
/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^
2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^
3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x
+ e)))*sin(f*x + e) - 4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f
*x + e))/((a^2 + a*b)*f*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

```
[In] integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(cot(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(fx+e)^2}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(cot(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(fx+e)^2}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(e+f\,x)^2}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

```
[In] int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.413 $\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 119

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f}+\frac{(3a+5b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^2f}-\frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)+1/3*(3*a+5*
b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^2/f-1/3*cot(f*x+e)^3*(a+b+b*
tan(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4226, 2000, 491, 597, 12, 385, 209}

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f(a+b)}+\frac{(3a+5b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f(a+b)^2}$$

```
[In] Int[Cot[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f) +
 ((3*a + 5*b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*(a + b)^2*f)
- (Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*(a + b)*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
```

```
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)\sqrt{a+b(1+x^2)}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} + \frac{\text{Subst}\left(\int \frac{-3a-5b-2bx^2}{x^2(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{3(a+b)f}$$

$$= \frac{(3a+5b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^2f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} - \frac{\text{Subst}\left(\int -\frac{3(a+b)^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{3(a+b)^2f}$$

$$= \frac{(3a+5b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^2f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{(3a+5b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^2f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} + \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

$$= \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} + \frac{(3a+5b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^2f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f}$$

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.41

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)}{\sqrt{2}\sqrt{a}f\sqrt{a+b\sec^2(e+fx)}} - \frac{(a+2b+a\cos(2(e+fx)))(-a-2b+(2a+3b)\cos(2(e+fx)))\csc^3(e+fx)\sec(e+fx)}{6(a+b)^2f\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Integrate[Cot[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2*b
 + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[e +
f*x]^2]) - ((a + 2*b + a*Cos[2*(e + f*x)])*(-a - 2*b + (2*a + 3*b)*Cos[2*(e
 + f*x)])*Csc[e + f*x]^3*Sec[e + f*x])/(6*(a + b)^2*f*Sqrt[a + b*Sec[e + f*
x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 890 vs. 2(105) = 210.

Time = 5.28 (sec) , antiderivative size = 891, normalized size of antiderivative = 7.49

method	result	size
default	Expression too large to display	891

```
[In] int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f/(a+b)^2/(-a)^(1/2)*(3*sin(f*x+e)^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*a^2*cos(f*x+e)+6*sin(f*x+e)^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*
b*cos(f*x+e)+3*sin(f*x+e)^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
```

```
(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*cos(f
*x+e)+3*sin(f*x+e)^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4
*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+6*sin(f*x+e)
^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4
*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b+3*sin(f*x+e)^3*ln(4*(-a)^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*b^2-4*cos(f*x+e)^4*(-a)^(1/2)*a^2-6*cos(f*x+e)^4*(-a
)^(1/2)*a*b+3*cos(f*x+e)^2*(-a)^(1/2)*a^2+cos(f*x+e)^2*(-a)^(1/2)*a*b-6*cos
(f*x+e)^2*(-a)^(1/2)*b^2+3*(-a)^(1/2)*a*b+5*(-a)^(1/2)*b^2)/(a+b*sec(f*x+e)
^2)^(1/2)/(cos(f*x+e)^2-1)*sec(f*x+e)*csc(f*x+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(105) = 210$.

Time = 0.68 (sec) , antiderivative size = 723, normalized size of antiderivative = 6.08

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[-\frac{3\left((a^2+2ab+b^2)\cos(fx+e)^2 - a^2 - 2ab - b^2\right)\sqrt{-a}\log\left(128a^4\cos(fx+e)^8 - 256(a^4-a^3b)\cos(f\right.}{} \right.$$

$$-\frac{3\left((a^2+2ab+b^2)\cos(fx+e)^2 - a^2 - 2ab - b^2\right)\sqrt{a}\arctan\left(\frac{\left(8a^2\cos(fx+e)^5 - 8(a^2-ab)\cos(fx+e)^3 + (a^2-6ab+b^2\right.}{4\left(2a^3\cos(fx+e)^4 - a^2b + ab^2 - (a^3-3a^2b)\cos\right.}\right.}{12\left((a^3+2a^2b+ab^2)f\cos(f\right.}$$

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^2 + 2*a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(-a)
*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4
- 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a
*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*
a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b
+ 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*s
qrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x +
 e) - 8*(2*(2*a^2 + 3*a*b)*cos(f*x + e)^3 - (3*a^2 + 5*a*b)*cos(f*x + e))*s
```

```
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^3 + 2*a^2*b + a*b^2)*f*cos
(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f)*sin(f*x + e)), -1/12*(3*((a^2 + 2*
a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(a)*arctan(1/4*(8*a^2*co
s(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x +
 e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x +
e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f
*x + e) - 4*(2*(2*a^2 + 3*a*b)*cos(f*x + e)^3 - (3*a^2 + 5*a*b)*cos(f*x + e
))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^3 + 2*a^2*b + a*b^2)*f
*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot^4\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

```
[In] integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(cot(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot\left(fx+e\right)^4}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(cot(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot\left(fx+e\right)^4}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{{\cot\left(e+f\,x\right)}^4}{\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}}\,dx$$

```
[In] int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.414 $\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 172

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} - \frac{(15a^2+40ab+33b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^3f} + \frac{(5a+9b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} - \frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5(a+b)f}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)-1/15*(15*a
^2+40*a*b+33*b^2)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^3/f+1/15*(5*a
+9*b)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^2/f-1/5*cot(f*x+e)^5*(a
+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used

$= \{4226, 2000, 491, 597, 12, 385, 209\}$

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(15a^2+40ab+33b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)^3} - \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f} - \frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5f(a+b)} + \frac{(5a+9b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)^2}$$

```
[In] Int[Cot[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
) - ((15*a^2 + 40*a*b + 33*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2]
)/(15*(a + b)^3*f) + ((5*a + 9*b)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x
]^2])/(15*(a + b)^2*f) - (Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(5
*(a + b)*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)\sqrt{a+b(1+x^2)}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5(a+b)f} + \frac{\text{Subst}\left(\int \frac{-5a-9b-4bx^2}{x^4(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{5(a+b)f} \\
&= \frac{(5a+9b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} \\
&\quad - \frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5(a+b)f} \\
&\quad - \frac{\text{Subst}\left(\int \frac{-15a^2-40ab-33b^2-2b(5a+9b)x^2}{x^2(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{15(a+b)^2f}
\end{aligned}$$

$$= -\frac{(15a^2+40ab+33b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^3f} + \frac{(5a+9b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} - \frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5(a+b)f} + \frac{\text{Subst}\left(\int -\frac{15(a+b)^3}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{15(a+b)^3f}$$

$$= -\frac{(15a^2+40ab+33b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^3f} + \frac{(5a+9b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} - \frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5(a+b)f} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(15a^2+40ab+33b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^3f} + \frac{(5a+9b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} - \frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5(a+b)f} - \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

$$= -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} - \frac{(15a^2+40ab+33b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^3f} + \frac{(5a+9b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} - \frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5(a+b)f}$$

Mathematica [A] (verified)

Time = 3.34 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.16

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= -\frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)}{\sqrt{2}\sqrt{a}f\sqrt{a+b\sec^2(e+fx)}} - \frac{(a+2b+a\cos(2(e+fx)))\csc(e+fx)\left(23a^2+60ab+45b^2-(11a^2+26ab+15b^2)\csc^2(e+fx)+3(a}{30(a+b)^3f\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Integrate[Cot[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2
*b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[e
+ f*x]^2])) - ((a + 2*b + a*Cos[2*(e + f*x)])*Csc[e + f*x]*(23*a^2 + 60*a*b
 + 45*b^2 - (11*a^2 + 26*a*b + 15*b^2)*Csc[e + f*x]^2 + 3*(a + b)^2*Csc[e +
 f*x]^4)*Sec[e + f*x])/(30*(a + b)^3*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1278 vs. $2(154) = 308$.

Time = 6.78 (sec) , antiderivative size = 1279, normalized size of antiderivative = 7.44

method	result	size
default	Expression too large to display	1279

```
[In] int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/f/(a+b)^3/(-a)^(1/2)*(15*sin(f*x+e)^5*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*a^3*cos(f*x+e)+45*sin(f*x+e)^5*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*a^2*b*cos(f*x+e)+45*sin(f*x+e)^5*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b
^2*cos(f*x+e)+15*sin(f*x+e)^5*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3*cos
(f*x+e)+23*cos(f*x+e)^6*(-a)^(1/2)*a^3+60*cos(f*x+e)^6*(-a)^(1/2)*a^2*b+45*
cos(f*x+e)^6*(-a)^(1/2)*a*b^2+15*sin(f*x+e)^5*ln(4*(-a)^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*a^3+45*sin(f*x+e)^5*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b+45
*sin(f*x+e)^5*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*c
os(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*
x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^2+15*sin(f*x+e)^5*l
n(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a
)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3-35*cos(f*x+e)^4*(-a)^(1/2)*a^3-71*
(-a)^(1/2)*a^2*b*cos(f*x+e)^4-15*cos(f*x+e)^4*(-a)^(1/2)*a*b^2+45*cos(f*x+e
)^4*(-a)^(1/2)*b^3+15*cos(f*x+e)^2*(-a)^(1/2)*a^3+5*(-a)^(1/2)*a^2*b*cos(f*
```

```
x+e)^2-61*(-a)^(1/2)*a*b^2*cos(f*x+e)^2-75*cos(f*x+e)^2*(-a)^(1/2)*b^3+15*(
-a)^(1/2)*a^2*b+40*(-a)^(1/2)*a*b^2+33*(-a)^(1/2)*b^3)/(a+b*sec(f*x+e)^2)^(
1/2)/(-1+cos(f*x+e))^2/(1+cos(f*x+e))^2*sec(f*x+e)*csc(f*x+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(154) = 308$.

Time = 1.91 (sec) , antiderivative size = 987, normalized size of antiderivative = 5.74

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/120*(15*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 + a^3 + 3*a^2*b
 + 3*a*b^2 + b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^2)*sqrt(-
a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^
4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28
*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(1
6*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*
b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))
*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x
 + e) + 8*((23*a^3 + 60*a^2*b + 45*a*b^2)*cos(f*x + e)^5 - (35*a^3 + 94*a^2
*b + 75*a*b^2)*cos(f*x + e)^3 + (15*a^3 + 40*a^2*b + 33*a*b^2)*cos(f*x + e)
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^4 + 3*a^3*b + 3*a^2*b^2
 + a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*
x + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)*sin(f*x + e)), 1/60*(15*(
(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 + a^3 + 3*a^2*b + 3*a*b^2 +
b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*
(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*
cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*c
os(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e
)))*sin(f*x + e) - 4*((23*a^3 + 60*a^2*b + 45*a*b^2)*cos(f*x + e)^5 - (35*a
^3 + 94*a^2*b + 75*a*b^2)*cos(f*x + e)^3 + (15*a^3 + 40*a^2*b + 33*a*b^2)*c
os(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^4 + 3*a^3*b
+ 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^
3)*f*cos(f*x + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cot^6{(e+fx)}}{\sqrt{a+b\sec^2{(e+fx)}}}\,dx$$

```
[In] integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(cot(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cot\left(fx+e\right)^6}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(cot(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cot\left(fx+e\right)^6}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{{\cot\left(e+f\,x\right)}^6}{\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}}\,dx$$

```
[In] int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.415 $\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{(a+b)^2}{ab^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\sqrt{a+b\sec^2(e+fx)}}{b^2f}$$

```
[Out] -arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+(a+b)^2/a/b^2/f/(a+b*s
ec(f*x+e)^2)^(1/2)+(a+b*sec(f*x+e)^2)^(1/2)/b^2/f
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4224, 457, 89, 65, 214}

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{(a+b)^2}{ab^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\sqrt{a+b\sec^2(e+fx)}}{b^2f}$$

```
[In] Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + (a + b)^2/(a*b
^2*f*Sqrt[a + b*Sec[e + f*x]^2]) + Sqrt[a + b*Sec[e + f*x]^2]/(b^2*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(-1+x^2)^2}{x(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(-1+x)^2}{x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{ab(a+bx)^{3/2}} + \frac{1}{b\sqrt{a+bx}} + \frac{1}{ax\sqrt{a+bx}}\right)\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{(a+b)^2}{ab^2 f\sqrt{a+b\sec^2(e+fx)}} + \frac{\sqrt{a+b\sec^2(e+fx)}}{b^2 f} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2af}$$

$$= \frac{(a+b)^2}{ab^2 f\sqrt{a+b\sec^2(e+fx)}} + \frac{\sqrt{a+b\sec^2(e+fx)}}{b^2 f} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{abf}$$

$$= -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2} f} + \frac{(a+b)^2}{ab^2 f\sqrt{a+b\sec^2(e+fx)}} + \frac{\sqrt{a+b\sec^2(e+fx)}}{b^2 f}$$

Mathematica [A] (verified)

Time = 5.46 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.38

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2} f} + \frac{(a+2b+a\cos(2(e+fx)))\left(2a^2+4ab+b^2+(2a^2+2ab+b^2)\cos(2(e+fx))\right)\sec^4(e+fx)}{4ab^2 f\,(a+b\sec^2(e+fx))^{3/2}}$$

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + ((a + 2*b + a*
Cos[2*(e + f*x)])*(2*a^2 + 4*a*b + b^2 + (2*a^2 + 2*a*b + b^2)*Cos[2*(e + f
*x)])*Sec[e + f*x]^4)/(4*a*b^2*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. 2(78) = 156.

Time = 5.38 (sec) , antiderivative size = 562, normalized size of antiderivative = 6.39

method	result
default	$-\frac{-2a^{\frac{9}{2}}-2a^{\frac{7}{2}}b-b^2a^{\frac{5}{2}}+\ln\left(4\cos(fx+e)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\sqrt{a}+4\cos(fx+e)a+4\sqrt{a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\right)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,a^2b^2-3a^{\frac{7}{2}}b\,\text{s}}{}$

```
[In] int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f/b^2/a^(5/2)/(a+b*sec(f*x+e)^2)^(3/2)*(-2*a^(9/2)-2*a^(7/2)*b-b^2*a^(5/
2)+ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*co
s(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((b+a*cos
```

```
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b^2-3*a^(7/2)*b*sec(f*x+e)^2+ln(4*co
s(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a
+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*a^2*b^2*sec(f*x+e)-2*b^2*a^(5/2)*sec(f*x+e)^2+ln(4
*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e
)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^3*sec(f*x+e)^2-a^(3/2)*b^3*sec(f*x+e)^2+ln(
4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+
e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^3*sec(f*x+e)^3-b^2*a^(5/2)*sec(f*x+e)^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(78) = 156$.

Time = 0.83 (sec) , antiderivative size = 458, normalized size of antiderivative = 5.20

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[\frac{(ab^2\cos(fx+e)^2+b^3)\sqrt{a}\log\left(128\,a^4\cos(fx+e)^8+256\,a^3b\cos(fx+e\right.}{}\right.$$

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*((a*b^2*cos(f*x + e)^2 + b^3)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256
*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^
2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos
(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)) + 8*(a^2*b + (2*a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^3*b^2*f*cos(f*x + e)^2 + a^2*b^3*f)
, 1/4*((a*b^2*cos(f*x + e)^2 + b^3)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)
^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) + 4*(a^
2*b + (2*a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2))/(a^3*b^2*f*cos(f*x + e)^2 + a^2*b^3*f)]
```

Sympy [F]

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(tan(e + f*x)**5/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(fx+e)^5}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(fx+e)^5}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(e+f\,x)^5}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.416 $\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 63

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} - \frac{a+b}{abf\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+(-a-b)/a/b/f/(a+b*sec(f
*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4224, 457, 79, 65, 214}

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} - \frac{a+b}{abf\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f) - (a + b)/(a*b*f*Sq
rt[a + b*Sec[e + f*x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{-1+x^2}{x(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{-1+x}{x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2f} \\
&= -\frac{a+b}{abf\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2af}
\end{aligned}$$

$$= -\frac{a+b}{abf\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{abf}$$

$$= \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} - \frac{a+b}{abf\sqrt{a+b\sec^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{a+b}{a\sqrt{a+b\sec^2(e+fx)}}}{bf}$$

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((b*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/a^(3/2) - (a + b)/(a*Sqrt[
a + b*Sec[e + f*x]^2]))/(b*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2292 vs. 2(58) = 116.

Time = 1.59 (sec) , antiderivative size = 2293, normalized size of antiderivative = 36.40

method	result	size
default	Expression too large to display	2293

```
[In] int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f/a^(3/2)/((-a*b)^(1/2)-a)/((-a*b)^(1/2)+a)/b/(a+b*sec(f*x+e)^2)^(3/2)/(
b+a*cos(f*x+e)^2)/(a+b)*(2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4
*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e
)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^4*b^2+4*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2))*a^3*b^3+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*
cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^2*b^4
-a^(9/2)*b^2*sec(f*x+e)^2-3*a^(7/2)*b^3*sec(f*x+e)^2-3*a^(5/2)*b^4*sec(f*x+
e)^2-a^(3/2)*b^5*sec(f*x+e)^2-3*cos(f*x+e)^2*a^(11/2)*b-3*cos(f*x+e)^2*a^(9
/2)*b^2-cos(f*x+e)^2*a^(7/2)*b^3-2*a^(11/2)*b-6*a^(9/2)*b^2-6*a^(7/2)*b^3-2
*a^(5/2)*b^4+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*
```

((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^2*b^4*sec(f*x+e)^3+((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2))*a*b^5*sec(f*x+e)^3+2*cos(f*x+e)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2))*a^4*b^2+cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*c
os(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^5*b+co
s(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^5*b+2*cos(f*x+e)^2*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2))*a^4*b^2+cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2
)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^3
*b^3+cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^3*b^3+2*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2))*a^4*b^2*sec(f*x+e)+4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*
cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^3*b^3
*sec(f*x+e)+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^2*b^4*sec(f*x+e)+((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2))*a^3*b^3*sec(f*x+e)^2+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a
^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
)*a^2*b^4*sec(f*x+e)^2+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos
(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+
4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a*b^5*sec(f*x+e)^2+(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^3*b^3*sec(f*x+e)^3-cos(f*x+e)^2*a^(13/2))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(55) = 110.

Time = 0.39 (sec) , antiderivative size = 417, normalized size of antiderivative = 6.62

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[-\frac{8\,(a^2+ab)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)^2-\left(ab\cos(fx+e)^2+b^2\right)\sqrt{a}\log \right.$$

$$-\frac{4\,(a^2+ab)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)^2+\left(ab\cos(fx+e)^2+b^2\right)\sqrt{-a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^4+8\,ab\cos(fx+e)^2\right.}{4\left(2\,a^3\cos(fx+e)^4+3\,a^2b\right.}\right.}{4\left(a^3bf\cos(fx+e)^2+a^2b^2f\right)}$$

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(8*(a^2 + a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e)^2 - (a*b*cos(f*x + e)^2 + b^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*
a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2
 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(
f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)))/(a^3*b*f*cos(f*x + e)^2 + a^2*b^2*f), -1/4*(4*(a^2 + a*b)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 + (a*b*cos(f*x + e)^2
 + b^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 +
b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x +
e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)))/(a^3*b*f*cos(f*x + e)^2 + a^2*b^2*
f)]
```

Sympy [F]

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2532 vs. 2(55) = 110.

Time = 0.63 (sec) , antiderivative size = 2532, normalized size of antiderivative = 40.19

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*a*b*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e
), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^3 + 4*a*b*cos(1/
2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x +
4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))*sin(1/2*arctan2(a*sin(4*f*x + 4*e
) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*
x + 2*e) + a))^2 - 4*(a^2 + a*b)*sin(2*f*x + 2*e)*sin(1/2*arctan2(a*sin(4*f
*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*
cos(2*f*x + 2*e) + a)) - 4*(a^2 + 2*a*b + (a^2 + a*b)*cos(2*f*x + 2*e))*cos
(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x
 + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + (a^2*cos(4*f*x + 4*e)^2 + a^
2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2
+ 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*
f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x +
 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*((b*cos(1/2*arctan2(a*sin(4
*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b
)*cos(2*f*x + 2*e) + a))^2 + b*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a +
2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) +
a))^2)*log(4*a^2*cos(2*f*x + 2*e)^2 + 4*a^2*sin(2*f*x + 2*e)^2 + 4*a^2 + 16
*a*b + 16*b^2 + 8*(a^2 + 2*a*b)*cos(2*f*x + 2*e) + 8*(a^2*cos(4*f*x + 4*e)^
2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4
*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*
sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4
*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*(a*sin(2*f*x + 2*e)*s
in(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f
*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + (a*cos(2*f*x + 2*e) + a +
2*b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*c
os(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)))*sqrt(a) + 4*sqrt(a^2*
cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2
*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2
+ 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*
f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*(a*cos(1/2
*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4
*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + a*sin(1/2*arctan2(a*sin(4*f*x
+ 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos
(2*f*x + 2*e) + a))^2)) + (b*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*
```

```
b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)
)^2 + b*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e),
a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*log(4*sqrt(a^2*c
os(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*
f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 +
 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f
*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*a*cos(1/2*a
rctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e
) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + 4*sqrt(a^2*cos(4*f*x + 4*e)^2 +
a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^
2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(
2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x
 + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*a*sin(1/2*arctan2(a*sin(4*f*x +
 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(
2*f*x + 2*e) + a))^2 + 16*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2
+ 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x +
4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 +
2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*
b)*cos(2*f*x + 2*e))^(1/4)*(a + b)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x + 4*
e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f
*x + 2*e) + a)) + 16*a^2 + 32*a*b + 16*b^2))*sqrt(a))/((a^2*b*cos(1/2*arcta
n2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) +
2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + a^2*b*sin(1/2*arctan2(a*sin(4*f*x +
4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2
*f*x + 2*e) + a))^2)*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(
a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*
sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^
2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*co
s(2*f*x + 2*e))^(1/4)*f)
```

Giac [F]

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\tan\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(e+f\,x)^3}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.417 $\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 57

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{1}{af\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+1/a/f/(a+b*sec(f*x+e)^
2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4224, 272, 53, 65, 214}

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{1}{af\sqrt{a+b\sec^2(e+fx)}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

```
[In] Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + 1/(a*f*Sqrt[a
+ b*Sec[e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
```

```
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2f} \\
&= \frac{1}{af\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2af} \\
&= \frac{1}{af\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{abf}
\end{aligned}
$$

$$= -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{1}{af\sqrt{a+b\sec^2(e+fx)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.66 (sec) , antiderivative size = 382, normalized size of antiderivative = 6.70

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))^{3/2}\sec^2(e+fx)\left(-\frac{2}{b\sqrt{a+2b+a\cos(2(e+fx))}} + \frac{\sqrt{2}e^{i(e+}}{}\right.}{}$$

```
[In] Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*Sec[e + f*x]^2*(-2/(b*Sqrt[a + 2*b +
a*Cos[2*(e + f*x)]]) + (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)
*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*((Sqrt[a]*(a + 4*b)*(1 + E^((2*I)*(e +
 f*x))))/(b*(4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2)) + ((
4*I)*f*x - 2*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I
)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 2*Log[a + a*E^((2*I)*(e +
f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*
(1 + E^((2*I)*(e + f*x)))^2]])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*
I)*(e + f*x)))^2])*Sec[e + f*x])/a^(3/2)))/(16*f*(a + b*Sec[e + f*x]^2)^(3/
2))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\frac{1}{a\sqrt{a+b\sec(fx+e)^2}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\sqrt{a+b\sec(fx+e)^2}}{\sec(fx+e)}\right)}{a^{\frac{3}{2}}}}{f}$	62
default	$\frac{\frac{1}{a\sqrt{a+b\sec(fx+e)^2}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\sqrt{a+b\sec(fx+e)^2}}{\sec(fx+e)}\right)}{a^{\frac{3}{2}}}}{f}$	62

```
[In] int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a/(a+b*sec(f*x+e)^2)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sec(f*x+
e)^2)^(1/2))/sec(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(49) = 98$.

Time = 0.37 (sec) , antiderivative size = 392, normalized size of antiderivative = 6.88

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[\frac{8\,a\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos\left(fx+e\right)^2+\left(a\cos\left(fx+e\right)^2+b\right)\sqrt{a}\log\left(128\,a^4\cos\left(f\right.\right.}{}\right.$$

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 + (a*c
os(f*x + e)^2 + b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x +
 e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*
a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^
3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)))/(a^
3*f*cos(f*x + e)^2 + a^2*b*f), 1/4*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*cos(f*x + e)^2 + (a*cos(f*x + e)^2 + b)*sqrt(-a)*arctan(1/4*(8*a^2
*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*
b^2)))/(a^3*f*cos(f*x + e)^2 + a^2*b*f)]
```

Sympy [A] (verification not implemented)

Time = 5.89 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.32

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{b}{2af\sqrt{a+b\sec^2(e+fx)}}+\frac{b\,\mathrm{atan}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{-a}}\right)}{2af\sqrt{-a}}\right)}{b} & \text{for } b \neq 0 \\ \frac{\log\left(\sec^2(e+fx)\right)}{2a^{\frac{3}{2}}f} & \text{otherwise} \end{cases}$$

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Piecewise((2*(b/(2*a*f*sqrt(a + b*sec(e + f*x)**2)) + b*atan(sqrt(a + b*sec
(e + f*x)**2)/sqrt(-a))/(2*a*f*sqrt(-a)))/b, Ne(b, 0)), (log(sec(e + f*x)**
2)/(2*a**(3/2)*f), True))
```

Maxima [F]

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 21.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{1}{a\,f\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}} - \frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\sqrt{a}}\right)}{a^{3/2}\,f}$$

```
[In] int(tan(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] 1/(a*f*(a + b/cos(e + f*x)^2)^(1/2)) - atanh((a + b/cos(e + f*x)^2)^(1/2)/a
^(1/2))/(a^(3/2)*f)
```

3.418 $\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}f} - \frac{b}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f-arctanh((a+b*sec(f*x+e)
^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/f-b/a/(a+b)/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4224, 457, 87, 162, 65, 214}

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f(a+b)^{3/2}} - \frac{b}{af(a+b)\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f) - ArcTanh[Sqrt[a +
b*Sec[e + f*x]^2]/Sqrt[a + b]]/((a + b)^(3/2)*f) - b/(a*(a + b)*f*Sqrt[a +
b*Sec[e + f*x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x(-1+x^2)(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{b}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a+b-bx}{(-1+x)x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2a(a+b)f}$$

$$= -\frac{b}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2af} + \frac{\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2(a+b)f}$$

$$= -\frac{b}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{abf} + \frac{\text{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{b(a+b)f}$$

$$= \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}f} - \frac{b}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}}$$

Mathematica [F]

$$\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$$

```
[In] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 9461 vs. $2(86) = 172$.

Time = 1.81 (sec) , antiderivative size = 9462, normalized size of antiderivative = 94.62

method	result	size
default	Expression too large to display	9462

```
[In] int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(86) = 172$.

Time = 0.61 (sec) , antiderivative size = 1569, normalized size of antiderivative = 15.69

$$\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] [-1/8*(8*(a^2*b + a*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*
x + e)^2 - (a^2*b + 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)
*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^
2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8
 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*
sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 2*(a^3*cos(f*x + e)^
2 + a^2*b)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a
*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x
+ e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x +
 e)^4 - 2*cos(f*x + e)^2 + 1)))/((a^5 + 2*a^4*b + a^3*b^2)*f*cos(f*x + e)^2
 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f), -1/8*(8*(a^2*b + a*b^2)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 - 4*(a^3*cos(f*x + e)^2 + a^2*
b)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b +
 b^2)) - (a^2*b + 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*s
qrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*
cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 +
 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sq
rt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)))/((a^5 + 2*a^4*b + a^3*b
^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f), -1/4*(4*(a^2*b + a
*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 + (a^2*b +
 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/
4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(
```

```
f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e
)^2 + a*b^2)) - (a^3*cos(f*x + e)^2 + a^2*b)*sqrt(a + b)*log(2*((8*a^2 + 8*
a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*
a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a^5 + 2
*a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f), -1/4
*(4*(a^2*b + a*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e
)^2 + (a^2*b + 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt
(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2
*b*cos(f*x + e)^2 + a*b^2)) - 2*(a^3*cos(f*x + e)^2 + a^2*b)*sqrt(-a - b)*a
rctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)))/((a^5 + 2
*a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f)]
```

Sympy [F]

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(cot(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(cot(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(e+f\,x)}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.419 $\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 153

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{(2a+5b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}f} - \frac{(a-2b)b}{2a(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^2(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+1/2*(2*a+5*b)*arctanh(
(a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/f-1/2*(a-2*b)*b/a/(a+b)^2
/f/(a+b*sec(f*x+e)^2)^(1/2)-1/2*cot(f*x+e)^2/(a+b)/f/(a+b*sec(f*x+e)^2)^(1/
2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4224, 457, 105, 157, 162, 65, 214}

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{(2a+5b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f(a+b)^{5/2}} - \frac{b(a-2b)}{2af(a+b)^2\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^2(e+fx)}{2f(a+b)\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + ((2*a + 5*b)*A
rcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(2*(a + b)^(5/2)*f) - ((a -
 2*b)*b)/(2*a*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^2]) - Cot[e + f*x]^2/(2*(
a + b)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(-1+x^2)^2(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)^2x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= -\frac{\cot^2(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{a+b+\frac{3bx}{2}}{(-1+x)x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2(a+b)f}\\
&= -\frac{(a-2b)b}{2a(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^2(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}}\\
&\quad + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(a+b)^2-\frac{1}{4}(a-2b)bx}{(-1+x)x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{a(a+b)^2f}\\
&= -\frac{(a-2b)b}{2a(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^2(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}}\\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2af}\\
&\quad - \frac{(2a+5b)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{4(a+b)^2f}
\end{aligned}$$

$$= -\frac{(a-2b)b}{2a(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^2(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{abf} - \frac{(2a+5b)\text{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{2b(a+b)^2f}$$

$$= -\frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{(2a+5b)\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}f} - \frac{(a-2b)b}{2a(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^2(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}}$$

Mathematica [F]

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 13609 vs. 2(131) = 262.

Time = 1.16 (sec) , antiderivative size = 13610, normalized size of antiderivative = 88.95

method	result	size
default	Expression too large to display	13610

```
[In] int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(131) = 262$.

Time = 1.49 (sec) , antiderivative size = 2347, normalized size of antiderivative = 15.34

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3*b - 3*a^2*b
^2 - 3*a*b^3 - b^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cos(f*x + e)^2)*sqrt(a
)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f
*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a
^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + ((2*a^4 + 5*a^3*b)*cos(f*x
+ e)^4 - 2*a^3*b - 5*a^2*b^2 - (2*a^4 + 3*a^3*b - 5*a^2*b^2)*cos(f*x + e)^2
)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^
2)*cos(f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*s
qrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2
*cos(f*x + e)^2 + 1)) + 4*((a^4 + a^3*b + 2*a^2*b^2 + 2*a*b^3)*cos(f*x + e)
^4 + (a^3*b - a^2*b^2 - 2*a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b
)/cos(f*x + e)^2))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos(f*x + e)^4
- (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^2 - (a^5*b + 3*a^4*b
^2 + 3*a^3*b^3 + a^2*b^4)*f), -1/8*(2*((2*a^4 + 5*a^3*b)*cos(f*x + e)^4 - 2
*a^3*b - 5*a^2*b^2 - (2*a^4 + 3*a^3*b - 5*a^2*b^2)*cos(f*x + e)^2)*sqrt(-a
- b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) - ((
a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3*b - 3*a^2*b^2 - 3*a
*b^3 - b^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cos(f*x + e)^2)*sqrt(a)*log(12
8*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^
4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos
(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 4*((a^4 + a^3*b + 2*a^2*b^2 + 2*a*b
^3)*cos(f*x + e)^4 + (a^3*b - a^2*b^2 - 2*a*b^3)*cos(f*x + e)^2)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f
*cos(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^2 -
(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f), 1/8*(2*((a^4 + 3*a^3*b + 3*a^
2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3*b - 3*a^2*b^2 - 3*a*b^3 - b^4 - (a^4 +
2*a^3*b - 2*a*b^3 - b^4)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x
 + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) +
((2*a^4 + 5*a^3*b)*cos(f*x + e)^4 - 2*a^3*b - 5*a^2*b^2 - (2*a^4 + 3*a^3*b
- 5*a^2*b^2)*cos(f*x + e)^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f
*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x +
```

```
e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) + 4*((a^4 + a^3*b + 2*a^2*
b^2 + 2*a*b^3)*cos(f*x + e)^4 + (a^3*b - a^2*b^2 - 2*a*b^3)*cos(f*x + e)^2)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^6 + 3*a^5*b + 3*a^4*b^2 +
a^3*b^3)*f*cos(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*
x + e)^2 - (a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f), 1/4*(((a^4 + 3*a^3
*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3*b - 3*a^2*b^2 - 3*a*b^3 - b^4
- (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^
2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a
*b^2)) - ((2*a^4 + 5*a^3*b)*cos(f*x + e)^4 - 2*a^3*b - 5*a^2*b^2 - (2*a^4 +
3*a^3*b - 5*a^2*b^2)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*co
s(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/
((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 2*((a^4 + a^3*b + 2*a^2*b^2 + 2
*a*b^3)*cos(f*x + e)^4 + (a^3*b - a^2*b^2 - 2*a*b^3)*cos(f*x + e)^2)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^
3)*f*cos(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^
2 - (a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f)]
```

Sympy [F]

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(cot(e + f*x)**3/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)^3}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(e+f\,x)^3}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.420 $\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 213

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} - \frac{(8a^2+28ab+35b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}f} + \frac{b(4a^2+11ab-8b^2)}{8a(a+b)^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{(4a+9b)\cot^2(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^4(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f-1/8*(8*a^2+28*a*b+35*b^
2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(7/2)/f+1/8*b*(4*a^2
+11*a*b-8*b^2)/a/(a+b)^3/f/(a+b*sec(f*x+e)^2)^(1/2)+1/8*(4*a+9*b)*cot(f*x+e
)^2/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(1/2)-1/4*cot(f*x+e)^4/(a+b)/f/(a+b*sec(f*
x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used

$= \{4224, 457, 105, 156, 157, 162, 65, 214\}$

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} - \frac{(8a^2+28ab+35b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{7/2}} + \frac{b(4a^2+11ab-8b^2)}{8af(a+b)^3\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^4(e+fx)}{4f(a+b)\sqrt{a+b\sec^2(e+fx)}} + \frac{(4a+9b)\cot^2(e+fx)}{8f(a+b)^2\sqrt{a+b\sec^2(e+fx)}}$$

```
[In] Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f) - ((8*a^2 + 28*a*b
+ 35*b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(8*(a + b)^(7/2)
*f) + (b*(4*a^2 + 11*a*b - 8*b^2))/(8*a*(a + b)^3*f*Sqrt[a + b*Sec[e + f*x]
^2]) + ((4*a + 9*b)*Cot[e + f*x]^2)/(8*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^
2]) - Cot[e + f*x]^4/(4*(a + b)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x(-1+x^2)^3(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)^3x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^4(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{2(a+b)+\frac{5bx}{2}}{(-1+x)^2x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{4(a+b)f}$$

$$= \frac{(4a+9b)\cot^2(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^4(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{2(a+b)^2+\frac{3}{4}b(4a+9b)x}{(-1+x)x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{4(a+b)^2f}$$

$$= \frac{b(4a^2+11ab-8b^2)}{8a(a+b)^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{(4a+9b)\cot^2(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^4(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{-(a+b)^3-\frac{1}{8}b(4a^2+11ab-8b^2)x}{(-1+x)x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2a(a+b)^3f}$$

$$= \frac{b(4a^2+11ab-8b^2)}{8a(a+b)^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{(4a+9b)\cot^2(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^4(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2af} + \frac{(8a^2+28ab+35b^2)\,\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{16(a+b)^3f}$$

$$= \frac{b(4a^2+11ab-8b^2)}{8a(a+b)^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{(4a+9b)\cot^2(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^4(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{abf} + \frac{(8a^2+28ab+35b^2)\,\text{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{8b(a+b)^3f}$$

$$= \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} - \frac{(8a^2+28ab+35b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}f} + \frac{b(4a^2+11ab-8b^2)}{8a(a+b)^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{(4a+9b)\cot^2(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^4(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}}$$

Mathematica [F]

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 17867 vs. $2(187) = 374$.

Time = 2.34 (sec) , antiderivative size = 17868, normalized size of antiderivative = 83.89

method	result	size
default	Expression too large to display	17868

```
[In] int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 829 vs. $2(187) = 374$.

Time = 4.80 (sec) , antiderivative size = 3501, normalized size of antiderivative = 16.44

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] [1/32*(4*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(f*x + e)^6 +
a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5 - (2*a^5 + 7*a^4*b + 8*a^3*b^
2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2
- 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*cos(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x
+ e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*c
os(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 +
10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)) + ((8*a^5 + 28*a^4*b + 35*a^3*b^2)*cos(f*x + e)^6 +
 8*a^4*b + 28*a^3*b^2 + 35*a^2*b^3 - (16*a^5 + 48*a^4*b + 42*a^3*b^2 - 35*a
^2*b^3)*cos(f*x + e)^4 + (8*a^5 + 12*a^4*b - 21*a^3*b^2 - 70*a^2*b^3)*cos(f
*x + e)^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a
*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x
```

```
+ e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x +
 e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*((6*a^5 + 19*a^4*b + 13*a^3*b^2 + 8*a^2*
b^3 + 8*a*b^4)*cos(f*x + e)^6 - (4*a^5 + 9*a^4*b - 8*a^3*b^2 + 3*a^2*b^3 +
16*a*b^4)*cos(f*x + e)^4 - (4*a^4*b + 15*a^3*b^2 + 3*a^2*b^3 - 8*a*b^4)*cos
(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 4*a^6*b +
 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a^7 + 7*a^6*b + 8*a
^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)^4 + (a^7 + 2*a^6*b
 - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^2 + (a^6*b
 + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f), 1/16*(((8*a^5 + 28*a^4*
b + 35*a^3*b^2)*cos(f*x + e)^6 + 8*a^4*b + 28*a^3*b^2 + 35*a^2*b^3 - (16*a^
5 + 48*a^4*b + 42*a^3*b^2 - 35*a^2*b^3)*cos(f*x + e)^4 + (8*a^5 + 12*a^4*b
- 21*a^3*b^2 - 70*a^2*b^3)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((2*a +
b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 2*((a^5 + 4*a^4*b + 6*a^3*
b^2 + 4*a^2*b^3 + a*b^4)*cos(f*x + e)^6 + a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4
*a*b^4 + b^5 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*co
s(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*co
s(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6
 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*c
os(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos
(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 2*((6*a
^5 + 19*a^4*b + 13*a^3*b^2 + 8*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^6 - (4*a^5 +
 9*a^4*b - 8*a^3*b^2 + 3*a^2*b^3 + 16*a*b^4)*cos(f*x + e)^4 - (4*a^4*b + 15
*a^3*b^2 + 3*a^2*b^3 - 8*a*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(
f*x + e)^6 - (2*a^7 + 7*a^6*b + 8*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5
)*f*cos(f*x + e)^4 + (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2
*a^2*b^5)*f*cos(f*x + e)^2 + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a
^2*b^5)*f), -1/32*(8*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(f
*x + e)^6 + a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5 - (2*a^5 + 7*a^4*
b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*cos(f*x + e)^4 + (a^5 + 2*a^4*b
- 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*cos(f*x + e)^2)*sqrt(-a)*arctan(
1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x +
 e)^2 + a*b^2)) - ((8*a^5 + 28*a^4*b + 35*a^3*b^2)*cos(f*x + e)^6 + 8*a^4*b
 + 28*a^3*b^2 + 35*a^2*b^3 - (16*a^5 + 48*a^4*b + 42*a^3*b^2 - 35*a^2*b^3)*
cos(f*x + e)^4 + (8*a^5 + 12*a^4*b - 21*a^3*b^2 - 70*a^2*b^3)*cos(f*x + e)^
2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b
^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*
sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 -
2*cos(f*x + e)^2 + 1)) + 4*((6*a^5 + 19*a^4*b + 13*a^3*b^2 + 8*a^2*b^3 + 8*
a*b^4)*cos(f*x + e)^6 - (4*a^5 + 9*a^4*b - 8*a^3*b^2 + 3*a^2*b^3 + 16*a*b^4
)*cos(f*x + e)^4 - (4*a^4*b + 15*a^3*b^2 + 3*a^2*b^3 - 8*a*b^4)*cos(f*x + e
)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 4*a^6*b + 6*a^5*b
^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a^7 + 7*a^6*b + 8*a^5*b^2 +
```

```
2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)^4 + (a^7 + 2*a^6*b - 2*a^5
*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^2 + (a^6*b + 4*a^5
*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f), -1/16*(4*((a^5 + 4*a^4*b + 6*a^
3*b^2 + 4*a^2*b^3 + a*b^4)*cos(f*x + e)^6 + a^4*b + 4*a^3*b^2 + 6*a^2*b^3 +
 4*a*b^4 + b^5 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*
cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*
cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x +
 e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*co
s(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - ((8*a^5 + 28*a^4*b + 35*a
^3*b^2)*cos(f*x + e)^6 + 8*a^4*b + 28*a^3*b^2 + 35*a^2*b^3 - (16*a^5 + 48*a
^4*b + 42*a^3*b^2 - 35*a^2*b^3)*cos(f*x + e)^4 + (8*a^5 + 12*a^4*b - 21*a^3
*b^2 - 70*a^2*b^3)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f
*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a
^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 2*((6*a^5 + 19*a^4*b + 13*a^3*b^2
+ 8*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^6 - (4*a^5 + 9*a^4*b - 8*a^3*b^2 + 3*a^
2*b^3 + 16*a*b^4)*cos(f*x + e)^4 - (4*a^4*b + 15*a^3*b^2 + 3*a^2*b^3 - 8*a*
b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 4
*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a^7 + 7*a^6
*b + 8*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)^4 + (a^7 +
 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^2
+ (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f)]
```

Sympy [F]

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(cot(e + f*x)**5/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{\cot\left(fx+e\right)^5}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] sage0*x

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\text{Hanged}$$

[In] int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2),x)

[Out] \text{Hanged}

3.421 $\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 172

$$\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{(3a+5b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{5/2}f} - \frac{(a+b)\tan^3(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a+2b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2ab^2f}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-1/2*(3*a+5
*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(5/2)/f+1/2*(3
*a+2*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/a/b^2/f-(a+b)*tan(f*x+e)^3/a/
b/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used

$= \{4226, 2000, 481, 596, 537, 223, 212, 385, 209\}$

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f} - \frac{(3a+5b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{5/2}f} + \frac{(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2ab^2f} - \frac{(a+b)\tan^3(e+fx)}{abf\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[In] Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f)
) - ((3*a + 5*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]
^2]])/(2*b^(5/2)*f) - ((a + b)*Tan[e + f*x]^3)/(a*b*f*Sqrt[a + b + b*Tan[e
+ f*x]^2]) + ((3*a + 2*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*a
*b^2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
```

```
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+b(1+x^2))^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)+(3a+2b)x^2)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{abf}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a+2b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2ab^2f} - \frac{\text{Subst}\left(\int \frac{(a+b)(3a+2b)+a(3a+5b)x^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2ab^2f}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a+2b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2ab^2f} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{af} - \frac{(3a+5b)\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2b^2f}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a+2b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2ab^2f} - \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{af} - \frac{(3a+5b)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^2f}$$

$$= -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{(3a+5b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{5/2}f} - \frac{(a+b)\tan^3(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a+2b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2ab^2f}$$

Mathematica [A] (verified)

Time = 9.40 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.44

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\left(\frac{2b^2\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{a}} + \frac{a(3a+5b)\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)(a+2b+a\cos(2e+2fx))^{3/2}\sec^3(e+fx)}{4\sqrt{2}ab^2f(a+b\sec^2(e+fx))^{3/2}} + \frac{(a+2b+a\cos(2(e+fx)))\left(3a^2+6ab+2b^2+(3a^2+4ab+2b^2)\cos(2(e+fx))\right)\sec^4(e+fx)\tan(e+fx)}{8ab^2f(a+b\sec^2(e+fx))^{3/2}}$$

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/4*(((2*b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]
)/Sqrt[a] + (a*(3*a + 5*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Si
n[e + f*x]^2]])/Sqrt[b])*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^
3)/(Sqrt[2]*a*b^2*f*(a + b*Sec[e + f*x]^2)^(3/2)) + ((a + 2*b + a*Cos[2*(e
+ f*x)])*(3*a^2 + 6*a*b + 2*b^2 + (3*a^2 + 4*a*b + 2*b^2)*Cos[2*(e + f*x)])
*Sec[e + f*x]^4*Tan[e + f*x])/(8*a*b^2*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2791 vs. $2(152) = 304$.

Time = 13.01 (sec) , antiderivative size = 2792, normalized size of antiderivative = 16.23

method	result	size
default	Expression too large to display	2792

```
[In] int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f/b^(7/2)/(-a)^(1/2)/a/(a+b*sec(f*x+e)^2)^(3/2)*(3*ln(-4*(((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(-a)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b^2*sec(f*x+e)^2+5*ln(-4*(((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(-a)
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^3*sec(f*x+e)^2+3*ln(
4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1)
)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b^2*sec(f*x+e)
^2+3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+
b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(
f*x+e)-1))*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b^2*s
ec(f*x+e)^3+5*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos
(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-
b)/(sin(f*x+e)+1))*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a
*b^3*sec(f*x+e)^2+5*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1
/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+
e)*a+a+b)/(sin(f*x+e)-1))*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*a*b^3*sec(f*x+e)^3+5*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-si
n(f*x+e)*a-a-b)/(sin(f*x+e)+1))*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*a*b^3*sec(f*x+e)^3+3*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*a^2*b^2*sec(f*x+e)^3+3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(
```

f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b*sec(f*x+e)-4*(-a)^(1/2)*b^(7/2)*a*tan(f*x+e)-8*(-a)^(1/2)*b^(5/2)*a^2*tan(f*x+e)-4*(-a)^(1/2)*b^(9/2)*tan(f*x+e)*sec(f*x+e)^2+5*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a^2*b^2*sec(f*x+e)+5*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a^2*b^2*sec(f*x+e)+3*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b*sec(f*x+e)-6*(-a)^(1/2)*b^(3/2)*a^3*tan(f*x+e)+4*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(9/2)*sec(f*x+e)^2+4*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(9/2)*sec(f*x+e)^3+4*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(7/2)*a+3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b+3*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b+5*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a^2*b^2+5*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a^2*b^2-8*(-a)^(1/2)*b^(7/2)*a*tan(f*x+e)*sec(f*x+e)^2-2*(-a)^(1/2)*b^(7/2)*a*tan(f*x+e)*sec(f*x+e)^4+4*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(7/2)*a*sec(f*x+e)-8*(-a)^(1/2)*b^(5/2)*a^2*tan(f*x+e)*sec(f*x+e)^2)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(152) = 304$.

Time = 1.76 (sec) , antiderivative size = 1895, normalized size of antiderivative = 11.02

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] [-1/8*((a*b^3*cos(f*x + e)^3 + b^4*cos(f*x + e))*sqrt(-a)*log(128*a^4*cos(f
*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2
*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a
^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7
 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x
 + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - ((3*a^4 + 5*a^3*b)*cos(f*
x + e)^3 + (3*a^3*b + 5*a^2*b^2)*cos(f*x + e))*sqrt(b)*log(((a^2 - 6*a*b +
b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e
)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*(a^2*b^2 + (3*a^3*b + 4*a^2*b^2
+ 2*a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(
f*x + e))/(a^3*b^3*f*cos(f*x + e)^3 + a^2*b^4*f*cos(f*x + e)), -1/8*(2*((3*
a^4 + 5*a^3*b)*cos(f*x + e)^3 + (3*a^3*b + 5*a^2*b^2)*cos(f*x + e))*sqrt(-b
)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e
))) + (a*b^3*cos(f*x + e)^3 + b^4*cos(f*x + e))*sqrt(-a)*log(128*a^4*cos(f*
x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*
b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^
4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7
- 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x
+ e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos
(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 4*(a^2*b^2 + (3*a^3*b + 4*
a^2*b^2 + 2*a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(a^3*b^3*f*cos(f*x + e)^3 + a^2*b^4*f*cos(f*x + e)), 1/8*
(2*(a*b^3*cos(f*x + e)^3 + b^4*cos(f*x + e))*sqrt(a)*arctan(1/4*(8*a^2*cos(
f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e
))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)
^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + ((3*a
^4 + 5*a^3*b)*cos(f*x + e)^3 + (3*a^3*b + 5*a^2*b^2)*cos(f*x + e))*sqrt(b)*
log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*
((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*(a^2*b^2 + (
3*a^3*b + 4*a^2*b^2 + 2*a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)*sin(f*x + e))/(a^3*b^3*f*cos(f*x + e)^3 + a^2*b^4*f*cos(f*x
```

```
 + e)), 1/4*((a*b^3*cos(f*x + e)^3 + b^4*cos(f*x + e))*sqrt(a)*arctan(1/4*(
8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*c
os(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*co
s(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)
)) - ((3*a^4 + 5*a^3*b)*cos(f*x + e)^3 + (3*a^3*b + 5*a^2*b^2)*cos(f*x + e)
)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*si
n(f*x + e))) + 2*(a^2*b^2 + (3*a^3*b + 4*a^2*b^2 + 2*a*b^3)*cos(f*x + e)^2)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*b^3*f*cos(f
*x + e)^3 + a^2*b^4*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{\tan^6{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(tan(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{\tan(fx+e)^6}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(e+f\,x)^6}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.422 $\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{3/2}f} - \frac{(a+b)\tan(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f+arctanh(b^(
1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(3/2)/f-(a+b)*tan(f*x+e)/a/b/
f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4226, 2000, 481, 537, 223, 212, 385, 209}

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{b^{3/2}f} - \frac{(a+b)\tan(e+fx)}{abf\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[In] Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f) +
 ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(b^(3/2)*f)
 - ((a + b)*Tan[e + f*x])/(a*b*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+b(1+x^2))^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\tan(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a+b+ax^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{abf}$$

$$= -\frac{(a+b)\tan(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{af} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{bf}$$

$$= -\frac{(a+b)\tan(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{af} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{bf}$$

$$= \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} + \frac{\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{3/2}f} - \frac{(a+b)\tan(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 4.61 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.73

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\left(\frac{b\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{a}}+\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)(a+2b+a\cos(2e+ \ldots}{2\sqrt{2}abf\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(a+b)(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\tan(e+fx)}{2abf\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (((b*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqrt[a]
 + (a*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqrt[
b])*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(2*Sqrt[2]*a*b*f*(
a + b*Sec[e + f*x]^2)^(3/2)) - ((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[
e + f*x]^2*Tan[e + f*x])/(2*a*b*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1185 vs. 2(102) = 204.

Time = 10.59 (sec) , antiderivative size = 1186, normalized size of antiderivative = 10.22

method	result	size
default	Expression too large to display	1186

```
[In] int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f/b^(5/2)/(-a)^(1/2)/a*(4*a*(-a)^(1/2)*b^(3/2)*(csc(f*x+e)-cot(f*x+e))+
4*(-a)^(1/2)*b^(5/2)*(csc(f*x+e)-cot(f*x+e))-2*ln(4*((-a)^(1/2)*(a*(1-cos(f
*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*
csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-c
ot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*csc(f*x+
e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*
(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*b^(5/2)-ln(4*(a*(1-cos(f*x+e))^2*c
sc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+b^(1/2)*(a*(1-cos(f*x+e))^4*csc
(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2
+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e))+a
+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-2*csc(f*x+e)+2*cot(f*x+e)+1))*a*(a*(1-co
s(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))
^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(-a)^(1/2)*b-l
n(4*(-a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2+b^(1/
2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-
```

```
cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a
*(csc(f*x+e)-cot(f*x+e))-a-b)/((1-cos(f*x+e))^2*csc(f*x+e)^2+2*csc(f*x+e)-2
*cot(f*x+e)+1))*a*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f
*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2
+a+b)^(1/2)*(-a)^(1/2)*b)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))
^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(
f*x+e)^2+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^3/((a*(1-cos(f*x+e))^4*csc(
f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+
2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^2)
^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(102) = 204$.

Time = 0.71 (sec) , antiderivative size = 1655, normalized size of antiderivative = 14.27

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(8*(a^2*b + a*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*
x + e)*sin(f*x + e) + (a*b^2*cos(f*x + e)^2 + b^3)*sqrt(-a)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 2*(a^3*cos(f*x + e)^2 +
 a^2*b)*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos
(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))
/(a^3*b^2*f*cos(f*x + e)^2 + a^2*b^3*f), -1/8*(8*(a^2*b + a*b^2)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) - 4*(a^3*cos(f*
x + e)^2 + a^2*b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*
x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x
+ e)^2 + b^2)*sin(f*x + e))) + (a*b^2*cos(f*x + e)^2 + b^3)*sqrt(-a)*log(12
8*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos
(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b
^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/(a^3*b^2*f*cos(
f*x + e)^2 + a^2*b^3*f), -1/4*(4*(a^2*b + a*b^2)*sqrt((a*cos(f*x + e)^2 + b
```

```
)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + (a*b^2*cos(f*x + e)^2 + b^3)*
sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (
a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x
+ e)^2)*sin(f*x + e))) - (a^3*cos(f*x + e)^2 + a^2*b)*sqrt(b)*log(((a^2 - 6
*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(
f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))/(a^3*b^2*f*cos(f*x + e)^2 +
a^2*b^3*f), -1/4*(4*(a^2*b + a*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*cos(f*x + e)*sin(f*x + e) + (a*b^2*cos(f*x + e)^2 + b^3)*sqrt(a)*arcta
n(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b +
 b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2
*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f
*x + e))) - 2*(a^3*cos(f*x + e)^2 + a^2*b)*sqrt(-b)*arctan(-1/2*((a - b)*co
s(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))))/(a^3*b^2*f*cos(f*x +
 e)^2 + a^2*b^3*f)]
```

Sympy [F]

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan^4{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(tan(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan{(fx+e)}^4}{(b\sec{(fx+e)}^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(fx+e)^4}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(e+f\,x)^4}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.423 $\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 71

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} + \frac{\tan(e+fx)}{af\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f+tan(f*x+e)
/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4226, 2000, 482, 385, 209}

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\tan(e+fx)}{af\sqrt{a+b\tan^2(e+fx)+b}} - \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}$$

```
[In] Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f)
) + Tan[e + f*x]/(a*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+b(1+x^2))^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\tan(e+fx)}{af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{af}\\
&= \frac{\tan(e+fx)}{af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{af}
\end{aligned}$$

$$= -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} + \frac{\tan(e+fx)}{af\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 169 vs. 2(71) = 142.

Time = 2.61 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.38

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx =$$

$$-\frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a+2b+a\cos(2(e+fx)))-\sqrt{2}\sqrt{a}\sqrt{a+}\right.}{4a^{3/2}\sqrt{a+b}f\,(a+b\sec^2(e+fx))^{3/2}\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}$$

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/4*((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(ArcSin[(Sqrt[a]*Sin[e
+ f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - Sqrt[2]*Sqrt[a]*Sqrt[
a + b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(a^(3/2)
*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)
/(a + b)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. 2(63) = 126.

Time = 4.09 (sec) , antiderivative size = 282, normalized size of antiderivative = 3.97

method	result
default	$-\frac{\left(b+a\cos(fx+e)^2\right)\left(\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+\right.}{fa\sqrt{-a}\left(a\right.}$

```
[In] int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f/a/(-a)^(1/2)*(b+a*cos(f*x+e)^2)*(ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*cos(f*x+e)+ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos
(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+
e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-(-a)^(1/2)*sin(f*x+e))/(a
+b*sec(f*x+e)^2)^(3/2)*sec(f*x+e)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(63) = 126$.

Time = 0.43 (sec) , antiderivative size = 548, normalized size of antiderivative = 7.72

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[\frac{8\,a\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e) - \left(a\cos(fx+e)^2+b\right)\sqrt{-a}\log\right.\right.$$

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x
+ e) - (a*cos(f*x + e)^2 + b)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^
4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^
2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)
*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*
a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e)))/(a^3*f*cos(f*x + e)^2 + a^2*b*f), 1/4*(4*a*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + (a*co
s(f*x + e)^2 + b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*
cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 -
 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))))/(a^3*f*cos(f*x + e)^2 + a^2*b*f)]
```

Sympy [F]

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2005 vs. 2(63) = 126.

Time = 0.50 (sec) , antiderivative size = 2005, normalized size of antiderivative = 28.24

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e),
 a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^3 + 2*a*cos(1/2*ar
ctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e)
 + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))*sin(2*f*x + 2*e) + 2*(a*cos(1/2*arcta
n2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) +
2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 - a*cos(2*f*x + 2*e))*sin(1/2*arctan2(
a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(
a + 2*b)*cos(2*f*x + 2*e) + a)) - (a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x +
 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(
4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2
+ a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^
2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*((cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2
*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2
*e) + a))^2 + sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x +
2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*
a*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4
*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e
)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(
a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*
cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a +
 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) +
 a)), 2*a*cos(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*
e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f
*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a
^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 +
 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x + 4*e)
+ 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x
+ 2*e) + a)) + 2*a + 4*b) - (cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*
b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)
)^2 + sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*
cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*(a^2*cos
(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*
x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4
*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x
 + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)
```

```
*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4
*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f*x + 4*e)^2
 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*
(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*s
in(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*
f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan
2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2
*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 4*a + 4*b))*sqrt(a))/((a^2*cos(4*f*x +
4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^
2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*
b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*
cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*(a^2*cos(1/2*arc
tan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e)
+ 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + a^2*sin(1/2*arctan2(a*sin(4*f*x +
4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2
*f*x + 2*e) + a))^2)*f)
```

Giac [F]

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{\tan\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{\tan(e+f\,x)^2}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.424 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 77

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f}-\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*tan(f*x+e
)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4213, 390, 385, 209}

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{b\tan(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f) -
 (b*Tan[e + f*x])/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{af}\\
&= -\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{af}\\
&= \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 168 vs. $2(77) = 154$.

Time = 1.45 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.18

$$\int \frac{1}{(a + b\sec^2(e + fx))^{3/2}}\,dx = \frac{(a + 2b + a\cos(2(e + fx)))\sec^3(e + fx)\left(\sqrt{a+b}\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a + \right.}{4a^{3/2}(a + b)f\,(a + b\sec^2(e + fx}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(Sqrt[a + b]*ArcSin[(Sqrt[a]
*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - Sqrt[2]*Sqrt[a
]*b*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(4*a^(3/2)*
(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a +
 b)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(69) = 138$.

Time = 4.43 (sec) , antiderivative size = 515, normalized size of antiderivative = 6.69

method	result
default	$-\frac{\left(b+a\cos(fx+e)^2\right)\left(-\cos(fx+e)\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)\sqrt{\frac{b+a\cos(fx-}{(1+\cos(fx+}}\right.}{}$

```
[In] int(1/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f/(a+b)/a/(-a)^(1/2)*(b+a*cos(f*x+e)^2)*(-cos(f*x+e)*ln(4*(-a)^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*a-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^
(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b*cos(f*x+e)+(-a
)^(1/2)*b*sin(f*x+e)-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^
(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a-((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)-4*sin(f*x+e)*a)*b)/(a+b*sec(f*x+e)^2)^(3/2)*sec(f*x+e)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(69) = 138$.

Time = 0.43 (sec) , antiderivative size = 601, normalized size of antiderivative = 7.81

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[-\frac{8\,ab\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e) + \left((a^2+ab)\cos(fx+e)^2 + a\right.}{}\right.$$

$$-\frac{4\,ab\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e) + \left((a^2+ab)\cos(fx+e)^2 + ab + b^2\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5\right.}{4\left(2\,a\right.}\right.}{4\left((a^4+a^3b)f\cos(fx+e)^2 + (a^3b+a^2b^2)f\right)}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(8*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f
*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*sqrt(-a)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/((a^4 + a^3*b)*f*cos(f*x
 + e)^2 + (a^3*b + a^2*b^2)*f), -1/4*(4*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b
+ b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e
)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*c
os(f*x + e)^2)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a
^2*b^2)*f)]
```

Sympy [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2055 vs. 2(69) = 138.

Time = 0.54 (sec) , antiderivative size = 2055, normalized size of antiderivative = 26.69

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*b*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*
e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))*sin(2*f*x + 2*e
) - 2*(a^2 + a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*
x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^3 - 2*(a*
b*cos(2*f*x + 2*e) + (a^2 + a*b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a
+ 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e)
+ a))^2 - a^2 - 2*a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin
(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) - (a
^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*co
s(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a
^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos
(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*(
((a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e),
 a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a + b)*sin(1/
2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x +
4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*a*sin(2*f*x + 2*e) +
 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^
2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b
)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1
/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*
e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*a*cos(2*f*x
+ 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*
b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x
+ 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2
 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x +
2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*
f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 2*a +
 4*b) - ((a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x
 + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a + b
)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(
4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*(a^2*cos(4*f
*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x +
2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b
 + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2
```

```
*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin
(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x
 + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f*x + 4*e)^2 + a
^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2
 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2
*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x
+ 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*
sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a
+ 2*b)*cos(2*f*x + 2*e) + a)) + 4*a + 4*b))*sqrt(a))/((a^2*cos(4*f*x + 4*e)
^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 +
4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)
*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(
4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*((a^3 + a^2*b)*cos(1
/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x +
 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a^3 + a^2*b)*sin(1/2*arctan
2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2
*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*f)
```

Giac [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{1}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(1/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(1/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.425 $\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 119

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{a(a+b)^2f}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*cot(f*x+
e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-(a-b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^
2)^(1/2)/a/(a+b)^2/f
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4226, 2000, 483, 597, 12, 385, 209}

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f} - \frac{(a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{af(a+b)^2} - \frac{b\cot(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[In] Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f)
) - (b*Cot[e + f*x])/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2]) - ((a - b
)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(a*(a + b)^2*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+b(1+x^2))^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-b-2bx^2}{x^2(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{a(a+b)f}$$

$$= -\frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{a(a+b)^2f} - \frac{\text{Subst}\left(\int \frac{(a+b)^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{a(a+b)^2f}$$

$$= -\frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{a(a+b)^2f} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{af}$$

$$= -\frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{a(a+b)^2f} - \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{af}$$

$$= -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{a(a+b)^2f}$$

Mathematica [A] (verified)

Time = 4.46 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.53

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)(a+2b+a\cos(2e+2fx))^{3/2}\sec^3(e+fx)}{2\sqrt{2}a^{3/2}f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{(a+2b+a\cos(2(e+fx)))\,(a^2+2ab-b^2+(a^2+b^2)\cos(2(e+fx)))\csc(e+fx)\sec^3(e+fx)}{4a(a+b)^2f\,(a+b\sec^2(e+fx))^{3/2}}$$

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2*
b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(Sqrt[2]*a^(3/2)*f*(a + b*Sec
[e + f*x]^2)^(3/2)) - ((a + 2*b + a*Cos[2*(e + f*x)])*(a^2 + 2*a*b - b^2 +
(a^2 + b^2)*Cos[2*(e + f*x)])*Csc[e + f*x]*Sec[e + f*x]^3)/(4*a*(a + b)^2*f
*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1227 vs. 2(109) = 218.

Time = 5.74 (sec) , antiderivative size = 1228, normalized size of antiderivative = 10.32

method	result	size
default	Expression too large to display	1228

```
[In] int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f/(a+b)^2/a/(-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e)
)^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc
(f*x+e)^2+a+b)*((-a)^(1/2)*a^2*(1-cos(f*x+e))^4*csc(f*x+e)^4+(-a)^(1/2)*a*b
*(1-cos(f*x+e))^4*csc(f*x+e)^4+2*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f
*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2
*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((
1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(
f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))
^2*csc(f*x+e)^2+a+b)^(1/2)*a^2*(csc(f*x+e)-cot(f*x+e))+4*ln(4*((-a)^(1/2)*(
a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(
f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(cs
c(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^
4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x
+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*a*b*(csc(f*x+e)-cot(f*x+
```

```
e))+2*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*
csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x
+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e)^
2+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*
(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*
b^2*(csc(f*x+e)-cot(f*x+e))-2*a^2*(1-cos(f*x+e))^2*(-a)^(1/2)*csc(f*x+e)^2+
2*(1-cos(f*x+e))^2*a*(-a)^(1/2)*b*csc(f*x+e)^2-4*(-a)^(1/2)*b^2*(1-cos(f*x+
e))^2*csc(f*x+e)^2+(-a)^(1/2)*a^2+(-a)^(1/2)*a*b)/((a*(1-cos(f*x+e))^4*csc(
f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+
2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^2)
^(3/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^3/(1-cos(f*x+e))*sin(f*x+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(109) = 218$.

Time = 0.73 (sec) , antiderivative size = 741, normalized size of antiderivative = 6.23

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[-\frac{(a^2b+2\,ab^2+b^3+(a^3+2\,a^2b+ab^2)\cos{(fx+e)}^2)\sqrt{-a}\log\Big(128\,a^4\cos\right.$$

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((a^2*b + 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqr
t(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5
*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 -
 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8
*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a
^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x +
e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(
f*x + e) + 8*((a^3 + a*b^2)*cos(f*x + e)^3 + (a^2*b - a*b^2)*cos(f*x + e))*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^5 + 2*a^4*b + a^3*b^2)*f*
cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f)*sin(f*x + e)), 1/4*((a^2*
b + 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt(a)*arctan(
1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b
^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a
^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x
 + e)))*sin(f*x + e) - 4*((a^3 + a*b^2)*cos(f*x + e)^3 + (a^2*b - a*b^2)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^5 + 2*a^4*b +
 a^3*b^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f)*sin(f*x + e))
]
```

Sympy [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(cot(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)^2}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(cot(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)^2}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(e+f\,x)^2}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.426 $\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 174

$$\begin{aligned}\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx &= \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} \\ &\quad - \frac{b\cot^3(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} \\ &\quad + \frac{(3a-b)(a+3b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a(a+b)^3f} \\ &\quad - \frac{(a-3b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a(a+b)^2f}\end{aligned}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*cot(f*x+e
)^3/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/3*(3*a-b)*(a+3*b)*cot(f*x+e)*(a+
b+b*tan(f*x+e)^2)^(1/2)/a/(a+b)^3/f-1/3*(a-3*b)*cot(f*x+e)^3*(a+b+b*tan(f*x
+e)^2)^(1/2)/a/(a+b)^2/f
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used

$= \{4226, 2000, 483, 597, 12, 385, 209\}$

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f} - \frac{(a-3b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3af(a+b)^2} - \frac{b\cot^3(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}} + \frac{(3a-b)(a+3b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3af(a+b)^3}$$

```
[In] Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f) -
 (b*Cot[e + f*x]^3)/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2]) + ((3*a -
b)*(a + 3*b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*a*(a + b)^3*f)
 - ((a - 3*b)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*a*(a + b)^2
*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+b(1+x^2))^{3/2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{b\cot^3(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-3b-4bx^2}{x^4(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{a(a+b)f} \\
&= -\frac{b\cot^3(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} \\
&\quad - \frac{(a-3b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a(a+b)^2f} \\
&\quad - \frac{\text{Subst}\left(\int \frac{(3a-b)(a+3b)+2(a-3b)bx^2}{x^2(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{3a(a+b)^2f}
\end{aligned}
$$

$$= -\frac{b\cot^3(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a-b)(a+3b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a(a+b)^3f} - \frac{(a-3b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a(a+b)^2f} + \frac{\text{Subst}\left(\int \frac{3(a+b)^3}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{3a(a+b)^3f}$$

$$= -\frac{b\cot^3(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a-b)(a+3b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a(a+b)^3f} - \frac{(a-3b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a(a+b)^2f} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{af}$$

$$= -\frac{b\cot^3(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a-b)(a+3b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a(a+b)^3f} - \frac{(a-3b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a(a+b)^2f} + \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{af}$$

$$= \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\cot^3(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a-b)(a+3b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a(a+b)^3f} - \frac{(a-3b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a(a+b)^2f}$$

Mathematica [A] (verified)

Time = 4.24 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.29

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)(a+2b+a\cos(2e+2fx))^{3/2}\sec^3(e+fx)}{2\sqrt{2}a^{3/2}f\,(a+b\sec^2(e+fx))^{3/2}} + \frac{(a+2b+a\cos(2e+2fx))^2\sec^3(e+fx)\left(\frac{(4a+9b)\csc(e+fx)}{12(a+b)^3f}-\frac{\csc^3(e+fx)}{12(a+b)^2f}-\frac{b^3\sin(e+fx)}{2a(a+b)^3f(a+2b+a\cos(2e+2fx))}\right)}{(a+b\sec^2(e+fx))^{3/2}}$$

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2*b + a
*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(2*Sqrt[2]*a^(3/2)*f*(a + b*Sec[e
+ f*x]^2)^(3/2)) + ((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^3*(((4*a
+ 9*b)*Csc[e + f*x])/(12*(a + b)^3*f) - Csc[e + f*x]^3/(12*(a + b)^2*f) - (
b^3*Sin[e + f*x])/(2*a*(a + b)^3*f*(a + 2*b + a*Cos[2*e + 2*f*x]))))/(a + b
*Sec[e + f*x]^2)^(3/2)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1778 vs. 2(158) = 316.

Time = 6.69 (sec) , antiderivative size = 1779, normalized size of antiderivative = 10.22

method	result	size
default	Expression too large to display	1779

```
[In] int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/f/(a+b)^3/a/(-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e
))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*cs
c(f*x+e)^2+a+b)*((-a)^(1/2)*a^3*(1-cos(f*x+e))^8*csc(f*x+e)^8+2*(-a)^(1/2)*
a^2*b*(1-cos(f*x+e))^8*csc(f*x+e)^8+(-a)^(1/2)*a*b^2*(1-cos(f*x+e))^8*csc(f
*x+e)^8-16*(-a)^(1/2)*a^3*(1-cos(f*x+e))^6*csc(f*x+e)^6-48*(-a)^(1/2)*a^2*b
*(1-cos(f*x+e))^6*csc(f*x+e)^6-32*(-a)^(1/2)*a*b^2*(1-cos(f*x+e))^6*csc(f*x
+e)^6+30*a^3*(1-cos(f*x+e))^4*(-a)^(1/2)*csc(f*x+e)^4+44*a^2*(1-cos(f*x+e))
^4*(-a)^(1/2)*b*csc(f*x+e)^4-66*(-a)^(1/2)*a*b^2*(1-cos(f*x+e))^4*csc(f*x+e
)^4+48*(-a)^(1/2)*b^3*(1-cos(f*x+e))^4*csc(f*x+e)^4-24*ln(4*((-a)^(1/2)*(a*
(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*
x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(
f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*
csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e
)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*a^3*(1-cos(f*x+e))^3*csc(f
*x+e)^3-72*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e
```

```
))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*cs
c(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*
x+e)^2+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4
-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(
1/2)*a^2*b*(1-cos(f*x+e))^3*csc(f*x+e)^3-72*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+
e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc
(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(
f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^
4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-
cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*a*b^2*(1-cos(f*x+e))^3*csc(f*x+e)^3-2
4*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(
f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^
2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1)
)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-c
os(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*b^3*
(1-cos(f*x+e))^3*csc(f*x+e)^3-16*(-a)^(1/2)*a^3*(1-cos(f*x+e))^2*csc(f*x+e)
^2-48*(-a)^(1/2)*a^2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2-32*(-a)^(1/2)*a*b^2*(1
-cos(f*x+e))^2*csc(f*x+e)^2+(-a)^(1/2)*a^3+2*(-a)^(1/2)*a^2*b+(-a)^(1/2)*a*
b^2)/((a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*
(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/((1-co
s(f*x+e))^2*csc(f*x+e)^2-1)^2)^(3/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^3/(1
-cos(f*x+e))^3*sin(f*x+e)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(158) = 316$.

Time = 2.11 (sec) , antiderivative size = 1061, normalized size of antiderivative = 6.10

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3*b - 3*a
^2*b^2 - 3*a*b^3 - b^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cos(f*x + e)^2)*sq
rt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(
5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2
- 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 +
8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*
a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x +
 e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin
(f*x + e) - 8*((4*a^4 + 9*a^3*b + 3*a*b^3)*cos(f*x + e)^5 - (3*a^4 + 4*a^3*
b - 9*a^2*b^2 + 6*a*b^3)*cos(f*x + e)^3 - (3*a^3*b + 8*a^2*b^2 - 3*a*b^3)*c
os(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^6 + 3*a^5*b
```

```
+ 3*a^4*b^2 + a^3*b^3)*f*cos(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*
b^4)*f*cos(f*x + e)^2 - (a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f)*sin(f*
x + e)), -1/12*(3*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3
*b - 3*a^2*b^2 - 3*a*b^3 - b^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cos(f*x +
e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)
^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*co
s(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e) - 4*((4*a^4 + 9*a^3*b + 3*a*b^3)*
cos(f*x + e)^5 - (3*a^4 + 4*a^3*b - 9*a^2*b^2 + 6*a*b^3)*cos(f*x + e)^3 - (
3*a^3*b + 8*a^2*b^2 - 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2))/(((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos(f*x + e)^4 - (
a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^2 - (a^5*b + 3*a^4*b^2
+ 3*a^3*b^3 + a^2*b^4)*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^4{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(cot(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)^4}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\cot(e+f\,x)^4}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.427 $\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 241

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(15a^3+55a^2b+73ab^2-15b^3)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a(a+b)^4f} + \frac{(5a^2+14ab-15b^2)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a(a+b)^3f} - \frac{(a-5b)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5a(a+b)^2f}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*cot(f*x+
e)^5/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/15*(15*a^3+55*a^2*b+73*a*b^2-15
*b^3)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/(a+b)^4/f+1/15*(5*a^2+14*a*b-
15*b^2)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/a/(a+b)^3/f-1/5*(a-5*b)*cot
(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(1/2)/a/(a+b)^2/f
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4226, 2000, 483, 597, 12, 385, 209}

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f} + \frac{(5a^2+14ab-15b^2)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15af(a+b)^3} - \frac{(15a^3+55a^2b+73ab^2-15b^3)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15af(a+b)^4} - \frac{(a-5b)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5af(a+b)^2} - \frac{b\cot^5(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[In] Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f)
) - (b*Cot[e + f*x]^5)/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2]) - ((15*
a^3 + 55*a^2*b + 73*a*b^2 - 15*b^3)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x
]^2])/(15*a*(a + b)^4*f) + ((5*a^2 + 14*a*b - 15*b^2)*Cot[e + f*x]^3*Sqrt[a
 + b + b*Tan[e + f*x]^2])/(15*a*(a + b)^3*f) - ((a - 5*b)*Cot[e + f*x]^5*Sq
rt[a + b + b*Tan[e + f*x]^2])/(5*a*(a + b)^2*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+b(1+x^2))^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-5b-6bx^2}{x^6(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{a(a+b)f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-5b)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5a(a+b)^2f} - \frac{\text{Subst}\left(\int \frac{5a^2+14ab-15b^2+4(a-5b)bx^2}{x^4(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{5a(a+b)^2f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5a^2+14ab-15b^2)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a(a+b)^3f} - \frac{(a-5b)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5a(a+b)^2f} + \frac{\text{Subst}\left(\int \frac{15a^3+55a^2b+73ab^2-15b^3+2b(5a^2+14ab-15b^2)x^2}{x^2(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{15a(a+b)^3f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(15a^3+55a^2b+73ab^2-15b^3)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a(a+b)^4f} + \frac{(5a^2+14ab-15b^2)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a(a+b)^3f} - \frac{(a-5b)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5a(a+b)^2f} - \frac{\text{Subst}\left(\int \frac{15(a+b)^4}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{15a(a+b)^4f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(15a^3+55a^2b+73ab^2-15b^3)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a(a+b)^4f} + \frac{(5a^2+14ab-15b^2)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a(a+b)^3f} - \frac{(a-5b)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5a(a+b)^2f} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{af}$$

$$= -\frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(15a^3+55a^2b+73ab^2-15b^3)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a(a+b)^4f} + \frac{(5a^2+14ab-15b^2)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a(a+b)^3f} - \frac{(a-5b)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5a(a+b)^2f} - \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{af}$$

$$= -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(15a^3+55a^2b+73ab^2-15b^3)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a(a+b)^4f} + \frac{(5a^2+14ab-15b^2)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a(a+b)^3f} - \frac{(a-5b)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5a(a+b)^2f}$$

Mathematica [A] (verified)

Time = 6.09 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.98

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)(a+2b+a\cos(2e+2fx))^{3/2}\sec^3(e+fx)}{2\sqrt{2}a^{3/2}f\,(a+b\sec^2(e+fx))^{3/2}} + \frac{(a+2b+a\cos(2(e+fx)))^2\left(\frac{30b^4}{a(a+2b+a\cos(2(e+fx)))} - (23a^2+80ab+90b^2)\csc^2(e+fx) + (a+b)(11a+20\right.}{60(a+b)^4f\,(a+b\sec^2(e+fx))^{3/2}}$$

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2*
b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(Sqrt[2]*a^(3/2)*f*(a + b*Sec
[e + f*x]^2)^(3/2)) + ((a + 2*b + a*Cos[2*(e + f*x)])^2*((30*b^4)/(a*(a + 2
*b + a*Cos[2*(e + f*x)])) - (23*a^2 + 80*a*b + 90*b^2)*Csc[e + f*x]^2 + (a
+ b)*(11*a + 20*b)*Csc[e + f*x]^4 - 3*(a + b)^2*Csc[e + f*x]^6)*Sec[e + f*x
]^2*Tan[e + f*x])/(60*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2448 vs. 2(221) = 442.

Time = 8.82 (sec) , antiderivative size = 2449, normalized size of antiderivative = 10.16

method	result	size
default	Expression too large to display	2449

```
[In] int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/480/f/(a+b)^4/a/(-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+
e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*c
sc(f*x+e)^2+a+b)*(3*(-a)^(1/2)*a*b^3+1920*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e)
)^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f
*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*
x+e)))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+
b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-co
s(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*a^3*b*(1-cos(f*x+e))^5*csc(f*x+e)^5+288
0*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(
f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^
2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1)
)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-c
os(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*a^2*
b^2*(1-cos(f*x+e))^5*csc(f*x+e)^5+1920*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4
*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+
e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e
)))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(
1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f
*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*a*b^3*(1-cos(f*x+e))^5*csc(f*x+e)^5-1804*a
^3*(1-cos(f*x+e))^6*(-a)^(1/2)*b*csc(f*x+e)^6-796*(-a)^(1/2)*a^2*b^2*(1-cos
(f*x+e))^6*csc(f*x+e)^6+2460*(-a)^(1/2)*a*b^3*(1-cos(f*x+e))^6*csc(f*x+e)^6
+480*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*c
sc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+
e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e)^2
+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(
1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*a
^4*(1-cos(f*x+e))^5*csc(f*x+e)^5+480*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*c
sc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)
^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e))
)/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-
cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x
+e))^2*csc(f*x+e)^2+a+b)^(1/2)*b^4*(1-cos(f*x+e))^5*csc(f*x+e)^5+1511*(-a)^
(1/2)*a^3*b*(1-cos(f*x+e))^4*csc(f*x+e)^4+2311*(-a)^(1/2)*a^2*b^2*(1-cos(f*
x+e))^4*csc(f*x+e)^4+1165*(-a)^(1/2)*a*b^3*(1-cos(f*x+e))^4*csc(f*x+e)^4-13
8*(-a)^(1/2)*a^3*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+2311*(-a)^(1/2)*a^2*b^2*(1
```

```
-cos(f*x+e))^8*csc(f*x+e)^8+1165*(-a)^(1/2)*a*b^3*(1-cos(f*x+e))^8*csc(f*x+
e)^8+9*(-a)^(1/2)*a^3*b*(1-cos(f*x+e))^12*csc(f*x+e)^12+9*(-a)^(1/2)*a^2*b^
2*(1-cos(f*x+e))^12*csc(f*x+e)^12+3*(-a)^(1/2)*a*b^3*(1-cos(f*x+e))^12*csc(
f*x+e)^12-138*(-a)^(1/2)*a^3*b*(1-cos(f*x+e))^10*csc(f*x+e)^10-162*(-a)^(1/
2)*a^2*b^2*(1-cos(f*x+e))^10*csc(f*x+e)^10-62*(-a)^(1/2)*a*b^3*(1-cos(f*x+e
))^10*csc(f*x+e)^10+1511*(-a)^(1/2)*a^3*b*(1-cos(f*x+e))^8*csc(f*x+e)^8-162
*(-a)^(1/2)*a^2*b^2*(1-cos(f*x+e))^2*csc(f*x+e)^2-62*(-a)^(1/2)*a*b^3*(1-co
s(f*x+e))^2*csc(f*x+e)^2+3*(-a)^(1/2)*a^4+9*(-a)^(1/2)*a^3*b+9*(-a)^(1/2)*a
^2*b^2-960*(-a)^(1/2)*b^4*(1-cos(f*x+e))^6*csc(f*x+e)^6+365*(-a)^(1/2)*a^4*
(1-cos(f*x+e))^4*csc(f*x+e)^4-38*(-a)^(1/2)*a^4*(1-cos(f*x+e))^2*csc(f*x+e)
^2+3*(-a)^(1/2)*a^4*(1-cos(f*x+e))^12*csc(f*x+e)^12-38*(-a)^(1/2)*a^4*(1-co
s(f*x+e))^10*csc(f*x+e)^10+365*(-a)^(1/2)*a^4*(1-cos(f*x+e))^8*csc(f*x+e)^8
-660*a^4*(1-cos(f*x+e))^6*(-a)^(1/2)*csc(f*x+e)^6)/((a*(1-cos(f*x+e))^4*csc
(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2
+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^2
)^(3/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^3/(1-cos(f*x+e))^5*sin(f*x+e)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 698 vs. $2(221) = 442$.

Time = 7.21 (sec) , antiderivative size = 1517, normalized size of antiderivative = 6.29

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/120*(15*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(f*x + e)^6
 + a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5 - (2*a^5 + 7*a^4*b + 8*a^3
*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b
^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(
f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^
2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(
a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^
7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*
x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) + 8*((23*a^5
+ 80*a^4*b + 90*a^3*b^2 + 15*a*b^4)*cos(f*x + e)^7 - (35*a^5 + 106*a^4*b +
80*a^3*b^2 - 90*a^2*b^3 + 45*a*b^4)*cos(f*x + e)^5 + (15*a^5 + 20*a^4*b - 5
6*a^3*b^2 - 160*a^2*b^3 + 45*a*b^4)*cos(f*x + e)^3 + (15*a^4*b + 55*a^3*b^2
 + 73*a^2*b^3 - 15*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2))/(((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e
)^6 - (2*a^7 + 7*a^6*b + 8*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos
(f*x + e)^4 + (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^
```

```
5)*f*cos(f*x + e)^2 + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)
*f)*sin(f*x + e)), 1/60*(15*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4
)*cos(f*x + e)^6 + a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5 - (2*a^5 +
 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*cos(f*x + e)^4 + (a^5 + 2
*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*cos(f*x + e)^2)*sqrt(a)*a
rctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a
*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*s
in(f*x + e)))*sin(f*x + e) - 4*((23*a^5 + 80*a^4*b + 90*a^3*b^2 + 15*a*b^4)
*cos(f*x + e)^7 - (35*a^5 + 106*a^4*b + 80*a^3*b^2 - 90*a^2*b^3 + 45*a*b^4)
*cos(f*x + e)^5 + (15*a^5 + 20*a^4*b - 56*a^3*b^2 - 160*a^2*b^3 + 45*a*b^4)
*cos(f*x + e)^3 + (15*a^4*b + 55*a^3*b^2 + 73*a^2*b^3 - 15*a*b^4)*cos(f*x +
 e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^7 + 4*a^6*b + 6*a^5*
b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a^7 + 7*a^6*b + 8*a^5*b^2
+ 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)^4 + (a^7 + 2*a^6*b - 2*a^
5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^2 + (a^6*b + 4*a^
5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(cot(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)^6}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Hanged}$$

```
[In] int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] \text{Hanged}
```

3.428 $\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{(a+b)^2}{3ab^2f\,(a+b\sec^2(e+fx))^{3/2}} + \frac{\frac{1}{a^2}-\frac{1}{b^2}}{f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/3*(a+b)^2/a/b^2/f/(a
+b*sec(f*x+e)^2)^(3/2)+(1/a^2-1/b^2)/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4224, 457, 89, 65, 214}

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{\frac{1}{a^2}-\frac{1}{b^2}}{f\sqrt{a+b\sec^2(e+fx)}} + \frac{(a+b)^2}{3ab^2f\,(a+b\sec^2(e+fx))^{3/2}}$$

```
[In] Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f)) + (a + b)^2/(3*a
*b^2*f*(a + b*Sec[e + f*x]^2)^(3/2)) + (a^(-2) - b^(-2))/(f*Sqrt[a + b*Sec[
e + f*x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)^2}{x(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{(-1+x)^2}{x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{ab(a+bx)^{5/2}}+\frac{a^2-b^2}{a^2b(a+bx)^{3/2}}+\frac{1}{a^2x\sqrt{a+bx}}\right)\,dx, x, \sec^2(e+fx)\right)}{2f}
\end{aligned}$$

$$= \frac{(a+b)^2}{3ab^2 f \left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\frac{1}{a^2}-\frac{1}{b^2}}{f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2a^2 f}$$

$$= \frac{(a+b)^2}{3ab^2 f \left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\frac{1}{a^2}-\frac{1}{b^2}}{f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{a^2 b f}$$

$$= -\frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2} f} + \frac{(a+b)^2}{3ab^2 f \left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\frac{1}{a^2}-\frac{1}{b^2}}{f\sqrt{a+b\sec^2(e+fx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 8.91 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.93

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{4(a+b)}{3f\left(a+b\sec^2(e+fx)\right)^{5/2}\left(8(a+b)\,\text{AppellF1}\left(3,\frac{1}{2},\frac{5}{2},4,\sin^2(e+fx),\frac{a\sin^2(e+}{a+b}\right.\right.}$$

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (4*(a + b)*AppellF1[3, 1/2, 5/2, 4, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)]*Tan[e + f*x]^6)/(3*f*(a + b*Sec[e + f*x]^2)^(5/2)*(8*(a + b)*AppellF1[
3, 1/2, 5/2, 4, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1
[4, 1/2, 7/2, 5, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a + b)*Appe
llF1[4, 3/2, 5/2, 5, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f
*x]^2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 6372 vs. 2(85) = 170.

Time = 2.94 (sec) , antiderivative size = 6373, normalized size of antiderivative = 65.70

method	result	size
default	Expression too large to display	6373

```
[In] int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(85) = 170$.

Time = 0.97 (sec) , antiderivative size = 564, normalized size of antiderivative = 5.81

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \left[\frac{3\left(a^2b^2\cos(fx+e)^4+2\,ab^3\cos(fx+e)^2+b^4\right)\sqrt{a}\log\left(128\,a^4\cos(fx+e\right.}{}\right.$$

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(a^2*b^2*cos(f*x + e)^4 + 2*a*b^3*cos(f*x + e)^2 + b^4)*sqrt(a)*lo
g(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x +
 e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b
*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 8*(2*(a^4 - a^3*b - 2*a^2*b^2)*
cos(f*x + e)^4 + 3*(a^3*b - a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2))/(a^5*b^2*f*cos(f*x + e)^4 + 2*a^4*b^3*f*cos(f*x + e)^2
 + a^3*b^4*f), 1/12*(3*(a^2*b^2*cos(f*x + e)^4 + 2*a*b^3*cos(f*x + e)^2 + b
^4)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)
*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4
 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 4*(2*(a^4 - a^3*b - 2*a^2*b^2)*cos(f*
x + e)^4 + 3*(a^3*b - a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2))/(a^5*b^2*f*cos(f*x + e)^4 + 2*a^4*b^3*f*cos(f*x + e)^2 + a^3
*b^4*f)]
```

Sympy [F]

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**5/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan\left(fx+e\right)^5}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(e+f\,x)^5}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

```
[In] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.429 $\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 89

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{a+b}{3abf\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/3*(-a-b)/a/b/f/(a+b*s
ec(f*x+e)^2)^(3/2)-1/a^2/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.240$, Rules used = {4224, 457, 79, 53, 65, 214}

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}}-\frac{a+b}{3abf\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

```
[In] Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f) - (a + b)/(3*a*b*f*
(a + b*Sec[e + f*x]^2)^(3/2)) - 1/(a^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

<u>Rule 65</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

<u>Rule 79</u>

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

<u>Rule 214</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

<u>Rule 457</u>

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

<u>Rule 4224</u>

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{-1+x}{x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{a+b}{3abf\,(a+b\sec^2(e+fx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2af}$$

$$= -\frac{a+b}{3abf\,(a+b\sec^2(e+fx))^{3/2}} - \frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2a^2f}$$

$$= -\frac{a+b}{3abf\,(a+b\sec^2(e+fx))^{3/2}} - \frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{a^2bf}$$

$$= \frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} - \frac{a+b}{3abf\,(a+b\sec^2(e+fx))^{3/2}} - \frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.45 (sec) , antiderivative size = 613, normalized size of antiderivative = 6.89

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$

$$-\frac{(a+3b+a\cos(2(e+fx)))(a+2b+a\cos(2e+2fx))^{5/2}\sec^4(e+fx)}{48b^2f(a+2b+a\cos(2(e+fx)))^{3/2}\,(a+b\sec^2(e+fx))^{5/2}}$$
$$+\frac{(a+b+(a-2b)\cos(2(e+fx)))(a+2b+a\cos(2e+2fx))^{5/2}\sec^4(e+fx)}{96b^2f(a+2b+a\cos(2(e+fx)))^{3/2}\,(a+b\sec^2(e+fx))^{5/2}}$$
$$-\frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\,(1+e^{2i(e+fx)})^2}(a+2b+a\cos(2e+2fx))^{5/2}\left(-\frac{\sqrt{a}(1+e^{2i(e+fx)})\left(-96b^3e^{2i(e+fx)}+a^3(1-\cdots\right.}{b^2\left(4\cdots\right.}\right.}{\cdots}$$

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/48*((a + 3*b + a*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*
Sec[e + f*x]^4)/(b^2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(a + b*Sec[e +
f*x]^2)^(5/2)) + ((a + b + (a - 2*b)*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e
 + 2*f*x])^(5/2)*Sec[e + f*x]^4)/(96*b^2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(
3/2)*(a + b*Sec[e + f*x]^2)^(5/2)) - (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^
((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*(a + 2*b + a*Cos[2*e + 2*f*x])^(
5/2)*(-((Sqrt[a]*(1 + E^((2*I)*(e + f*x)))*(-96*b^3*E^((2*I)*(e + f*x)) + a
^3*(1 + E^((2*I)*(e + f*x)))^2 - 32*a*b^2*(1 + E^((2*I)*(e + f*x)))^2 - 6*a
^2*b*(1 + E^((2*I)*(e + f*x)) + E^((4*I)*(e + f*x)))))/(b^2*(4*b*E^((2*I)*(
e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2)^2)) + ((24*I)*f*x - 12*Log[a + 2
*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 +
E^((2*I)*(e + f*x)))^2]] - 12*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*
(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*
x)))^2]])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*Se
c[e + f*x]^5)/(96*Sqrt[2]*a^(5/2)*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 6312 vs. $2(81) = 162$.

Time = 2.75 (sec) , antiderivative size = 6313, normalized size of antiderivative = 70.93

method	result	size
default	Expression too large to display	6313

```
[In] int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(77) = 154$.

Time = 0.84 (sec) , antiderivative size = 522, normalized size of antiderivative = 5.87

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \left[\frac{3\left(a^2b\cos(fx+e)^4+2\,ab^2\cos(fx+e)^2+b^3\right)\sqrt{a}\log\left(128\,a^4\cos(fx+e)^8\right.}{}\right.$$

$$-\frac{3\left(a^2b\cos(fx+e)^4+2\,ab^2\cos(fx+e)^2+b^3\right)\sqrt{-a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^4+8\,ab\cos(fx+e)^2+b^2\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4+3\,a^2b\cos(fx+e)^2+ab^2\right)}\right.}{12\left(a^5bf\cos(fx+e)^4+2\,a^4b^2f\cos(fx+e)^2\right.}$$

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(a^2*b*cos(f*x + e)^4 + 2*a*b^2*cos(f*x + e)^2 + b^3)*sqrt(a)*log(
128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e
)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*c
os(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 8*(3*a*b^2*cos(f*x + e)^2 + (a^3
+ 4*a^2*b)*cos(f*x + e)^4)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^
5*b*f*cos(f*x + e)^4 + 2*a^4*b^2*f*cos(f*x + e)^2 + a^3*b^3*f), -1/12*(3*(a
^2*b*cos(f*x + e)^4 + 2*a*b^2*cos(f*x + e)^2 + b^3)*sqrt(-a)*arctan(1/4*(8*
a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 +
 a*b^2)) + 4*(3*a*b^2*cos(f*x + e)^2 + (a^3 + 4*a^2*b)*cos(f*x + e)^4)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^5*b*f*cos(f*x + e)^4 + 2*a^4*b^
2*f*cos(f*x + e)^2 + a^3*b^3*f)]
```

Sympy [F]

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Giac [F]

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\tan\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\tan(e+f\,x)^3}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

```
[In] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.430 $\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{1}{3af\,(a+b\sec^2(e+fx))^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/3/a/f/(a+b*sec(f*x+e
)^2)^(3/2)+1/a^2/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4224, 272, 53, 65, 214}

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{1}{3af\,(a+b\sec^2(e+fx))^{3/2}}$$

```
[In] Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f)) + 1/(3*a*f*(a +
b*Sec[e + f*x]^2)^(3/2)) + 1/(a^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{2f} \\
&= \frac{1}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2af}
\end{aligned}
$$

$$= \frac{1}{3af\,(a+b\sec^2(e+fx))^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2a^2f}$$

$$= \frac{1}{3af\,(a+b\sec^2(e+fx))^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{a^2bf}$$

$$= -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{1}{3af\,(a+b\sec^2(e+fx))^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.12 (sec) , antiderivative size = 613, normalized size of antiderivative = 7.39

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{(a+3b+a\cos(2(e+fx)))(a+2b+a\cos(2e+2fx))^{5/2}\sec^4(e+fx)}{48b^2f(a+2b+a\cos(2(e+fx)))^{3/2}\,(a+b\sec^2(e+fx))^{5/2}} + \frac{(a+b+(a-2b)\cos(2(e+fx)))(a+2b+a\cos(2e+2fx))^{5/2}\sec^4(e+fx)}{32b^2f(a+2b+a\cos(2(e+fx)))^{3/2}\,(a+b\sec^2(e+fx))^{5/2}} + \frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}(a+2b+a\cos(2e+2fx))^{5/2}\left(-\frac{\sqrt{a}\left(1+e^{2i(e+fx)}\right)\left(-96b^3e^{2i(e+fx)}+a^3(1+\cdots\right.}{b^2\left(4b\cdots\right.}\right.}{\cdots}$$

```
[In] Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/48*((a + 3*b + a*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*
Sec[e + f*x]^4)/(b^2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(a + b*Sec[e +
f*x]^2)^(5/2)) + ((a + b + (a - 2*b)*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e
 + 2*f*x])^(5/2)*Sec[e + f*x]^4)/(32*b^2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(
3/2)*(a + b*Sec[e + f*x]^2)^(5/2)) + (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^
((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*(a + 2*b + a*Cos[2*e + 2*f*x])^(
5/2)*(-((Sqrt[a]*(1 + E^((2*I)*(e + f*x)))*(-96*b^3*E^((2*I)*(e + f*x)) + a
^3*(1 + E^((2*I)*(e + f*x)))^2 - 32*a*b^2*(1 + E^((2*I)*(e + f*x)))^2 - 6*a
^2*b*(1 + E^((2*I)*(e + f*x)) + E^((4*I)*(e + f*x)))))/(b^2*(4*b*E^((2*I)*(
e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2)^2)) + ((24*I)*f*x - 12*Log[a + 2
*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 +
E^((2*I)*(e + f*x)))^2]] - 12*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*
(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*
```

```
x)))^2]])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*Se
c[e + f*x]^5)/(96*Sqrt[2]*a^(5/2)*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{1}{3af\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}+\frac{1}{a^2f\sqrt{a+b\sec(fx+e)^2}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\sec(fx+e)^2}}{\sec(fx+e)}\right)}{f\,a^{\frac{5}{2}}}$	86
default	$\frac{1}{3af\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}+\frac{1}{a^2f\sqrt{a+b\sec(fx+e)^2}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\sec(fx+e)^2}}{\sec(fx+e)}\right)}{f\,a^{\frac{5}{2}}}$	86

```
[In] int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a/f/(a+b*sec(f*x+e)^2)^(3/2)+1/a^2/f/(a+b*sec(f*x+e)^2)^(1/2)-1/f/a^(5/
2)*ln((2*a+2*a^(1/2)*(a+b*sec(f*x+e)^2)^(1/2))/sec(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(71) = 142.

Time = 0.71 (sec) , antiderivative size = 494, normalized size of antiderivative = 5.95

$$\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \left[\frac{3\left(a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2\right)\sqrt{a}\log\left(128\,a^4\cos\left(fx+e\right)^8+\right.}{}\right.$$

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(a)*log(128*
a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4
+ 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f
*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)) + 8*(4*a^2*cos(f*x + e)^4 + 3*a*b*cos(f
*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^5*f*cos(f*x + e)
^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f), 1/12*(3*(a^2*cos(f*x + e)^4 + 2
*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*
b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) + 4*(4*a^2*cos(f
```

```
*x + e)^4 + 3*a*b*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2))/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f)]
```

Sympy [A] (verification not implemented)

Time = 9.79 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.20

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \begin{cases} \frac{2\left(\frac{b}{6af\left(a+b\sec^2(e+fx)\right)^{\frac{3}{2}}}+\frac{b}{2a^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{b\,\mathrm{atan}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{-a}}\right)}{2a^2f\sqrt{-a}}\right)}{b} & \text{for } b\neq 0 \\ \frac{\log(\sec^2(e+fx))}{2a^{\frac{5}{2}}f} & \text{otherwise} \end{cases}$$

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Piecewise((2*(b/(6*a*f*(a + b*sec(e + f*x)**2)**(3/2)) + b/(2*a**2*f*sqrt(a
 + b*sec(e + f*x)**2)) + b*atan(sqrt(a + b*sec(e + f*x)**2)/sqrt(-a))/(2*a*
*2*f*sqrt(-a)))/b, Ne(b, 0)), (log(sec(e + f*x)**2)/(2*a**(5/2)*f), True))
```

Maxima [F]

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(fx+e)}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(fx+e)}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 24.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.82

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\frac{a+\frac{b}{\cos(e+f\,x)^2}}{a^2}+\frac{1}{3\,a}}{f\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}} - \frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\sqrt{a}}\right)}{a^{5/2}\,f}$$

```
[In] int(tan(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] ((a + b/cos(e + f*x)^2)/a^2 + 1/(3*a))/(f*(a + b/cos(e + f*x)^2)^(3/2)) - a
tanh((a + b/cos(e + f*x)^2)^(1/2)/a^(1/2))/(a^(5/2)*f)
```

3.431 $\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}f} - \frac{b}{3a(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{b(2a+b)}{a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f-arctanh((a+b*sec(f*x+e)
^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/f-1/3*b/a/(a+b)/f/(a+b*sec(f*x+e)^2)^(3/
2)-b*(2*a+b)/a^2/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4224, 457, 87, 157, 162, 65, 214}

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} - \frac{b(2a+b)}{a^2f(a+b)^2\sqrt{a+b\sec^2(e+fx)}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f(a+b)^{5/2}} - \frac{b}{3af(a+b)\,(a+b\sec^2(e+fx))^{3/2}}$$

```
[In] Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f) - ArcTanh[Sqrt[a +
b*Sec[e + f*x]^2]/Sqrt[a + b]]/((a + b)^(5/2)*f) - b/(3*a*(a + b)*f*(a + b*
Sec[e + f*x]^2)^(3/2)) - (b*(2*a + b))/(a^2*(a + b)^2*f*Sqrt[a + b*Sec[e +
f*x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(-1+x^2)(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= -\frac{b}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}}+\frac{\text{Subst}\left(\int \frac{a+b-bx}{(-1+x)x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2a(a+b)f}\\
&= -\frac{b}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{b(2a+b)}{a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}}\\
&\quad-\frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(a+b)^2+\frac{1}{2}b(2a+b)x}{(-1+x)x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{a^2(a+b)^2f}\\
&= -\frac{b}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{b(2a+b)}{a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}}\\
&\quad-\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2a^2f}+\frac{\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2(a+b)^2f}\\
&= -\frac{b}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{b(2a+b)}{a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}}\\
&\quad-\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{a^2bf}\\
&\quad+\frac{\text{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{b(a+b)^2f}
\end{aligned}$$

$$= \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}f} - \frac{b}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b(2a+b)}{a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}}$$

Mathematica [F]

$$\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$$

```
[In] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
[Out] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 43854 vs. $2(119) = 238$.

Time = 4.92 (sec) , antiderivative size = 43855, normalized size of antiderivative = 320.11

method	result	size
default	Expression too large to display	43855

```
[In] int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 523 vs. $2(119) = 238$.

Time = 1.48 (sec) , antiderivative size = 2279, normalized size of antiderivative = 16.64

$$\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
[Out] [1/24*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 3*a^3*b^2
+ a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(f
*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 +
160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(
f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*
x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 6*(a^5*cos
```

```
(f*x + e)^4 + 2*a^4*b*cos(f*x + e)^2 + a^3*b^2)*sqrt(a + b)*log(2*((8*a^2 +
 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*(
(2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 8*((7
*a^4*b + 11*a^3*b^2 + 4*a^2*b^3)*cos(f*x + e)^4 + 3*(2*a^3*b^2 + 3*a^2*b^3
+ a*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^8
 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 +
 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 +
 a^3*b^5)*f), 1/24*(12*(a^5*cos(f*x + e)^4 + 2*a^4*b*cos(f*x + e)^2 + a^3*b
^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b
+ b^2)) + 3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 3*a^3*b
^2 + a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*co
s(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6
 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*c
os(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos
(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 8*((7*a
^4*b + 11*a^3*b^2 + 4*a^2*b^3)*cos(f*x + e)^4 + 3*(2*a^3*b^2 + 3*a^2*b^3 +
a*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^8 +
 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3
*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a
^3*b^5)*f), -1/12*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 + (a^5 + 3*a^4*b
+ 3*a^3*b^2 + a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 +
a*b^4)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*co
s(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2
*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 3*(a^5*cos(f*x + e
)^4 + 2*a^4*b*cos(f*x + e)^2 + a^3*b^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b +
 b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b
)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b
)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) + 4*((7*a^4*b +
 11*a^3*b^2 + 4*a^2*b^3)*cos(f*x + e)^4 + 3*(2*a^3*b^2 + 3*a^2*b^3 + a*b^4)
*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^8 + 3*a^7
*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b
^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5
)*f), -1/12*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 3*a^
3*b^2 + a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)
*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x
+ e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*c
os(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 6*(a^5*cos(f*x + e)^4 +
2*a^4*b*cos(f*x + e)^2 + a^3*b^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*
x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^
2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 4*((7*a^4*b + 11*a^3*b^2 + 4*a^2*b^
3)*cos(f*x + e)^4 + 3*(2*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(f*x + e)^2)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b
^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*f*cos(f*
```

```
x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*f)]
```

Sympy [F]

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(cot(e + f*x)/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(fx+e)}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(cot(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(fx+e)}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(e+f\,x)}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

```
[In] int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.432 $\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 200

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{(2a+7b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{7/2}f} - \frac{(3a-2b)b}{6a(a+b)^2f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{\cot^2(e+fx)}{2(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{b(a^2-6ab-2b^2)}{2a^2(a+b)^3f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/2*(2*a+7*b)*arctanh(
(a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(7/2)/f-1/6*(3*a-2*b)*b/a/(a+b)
^2/f/(a+b*sec(f*x+e)^2)^(3/2)-1/2*cot(f*x+e)^2/(a+b)/f/(a+b*sec(f*x+e)^2)^(
3/2)-1/2*b*(a^2-6*a*b-2*b^2)/a^2/(a+b)^3/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used

$= \{4224, 457, 105, 157, 162, 65, 214\}$

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} - \frac{b(a^2-6ab-2b^2)}{2a^2f(a+b)^3\sqrt{a+b\sec^2(e+fx)}} + \frac{(2a+7b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f(a+b)^{7/2}} - \frac{b(3a-2b)}{6af(a+b)^2\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^2(e+fx)}{2f(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

```
[In] Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f)) + ((2*a + 7*b)*A
rcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(2*(a + b)^(7/2)*f) - ((3*a
 - 2*b)*b)/(6*a*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^(3/2)) - Cot[e + f*x]^2/
(2*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (b*(a^2 - 6*a*b - 2*b^2))/(2*a
^2*(a + b)^3*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

```
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(-1+x^2)^2(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)^2x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{2f} \\
&= -\frac{\cot^2(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{a+b+\frac{5bx}{2}}{(-1+x)x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{2(a+b)f} \\
&= -\frac{(3a-2b)b}{6a(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^2(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} \\
&\quad + \frac{\text{Subst}\left(\int \frac{-\frac{3}{2}(a+b)^2-\frac{3}{4}(3a-2b)bx}{(-1+x)x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{3a(a+b)^2f}
\end{aligned}$$

$$= -\frac{(3a-2b)b}{6a(a+b)^2 f \left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^2(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b(a^2-6ab-2b^2)}{2a^2(a+b)^3 f\sqrt{a+b\sec^2(e+fx)}} - \frac{2\text{Subst}\left(\int \frac{\frac{3}{4}(a+b)^3+\frac{3}{8}b(a^2-6ab-2b^2)x}{(-1+x)x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{3a^2(a+b)^3 f}$$

$$= -\frac{(3a-2b)b}{6a(a+b)^2 f \left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^2(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b(a^2-6ab-2b^2)}{2a^2(a+b)^3 f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2a^2 f} - \frac{(2a+7b)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{4(a+b)^3 f}$$

$$= -\frac{(3a-2b)b}{6a(a+b)^2 f \left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^2(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b(a^2-6ab-2b^2)}{2a^2(a+b)^3 f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{a^2 b f} - \frac{(2a+7b)\text{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{2b(a+b)^3 f}$$

$$= -\frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2} f} + \frac{(2a+7b)\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{7/2} f} - \frac{(3a-2b)b}{6a(a+b)^2 f \left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^2(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b(a^2-6ab-2b^2)}{2a^2(a+b)^3 f\sqrt{a+b\sec^2(e+fx)}}$$

Mathematica [F]

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
[Out] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 73510 vs. $2(174) = 348$.

Time = 8.71 (sec) , antiderivative size = 73511, normalized size of antiderivative = 367.56

method	result	size
default	Expression too large to display	73511

```
[In] int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 830 vs. $2(174) = 348$.

Time = 4.92 (sec) , antiderivative size = 3507, normalized size of antiderivative = 17.54

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
[Out] [1/24*(3*((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^6
- a^4*b^2 - 4*a^3*b^3 - 6*a^2*b^4 - 4*a*b^5 - b^6 - (a^6 + 2*a^5*b - 2*a^4*
b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^
2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*cos(f*x + e)^2)*sqrt(a)*log(128*
a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4
+ 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f
*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)) + 3*((2*a^6 + 7*a^5*b)*cos(f*x + e)^6 -
 2*a^4*b^2 - 7*a^3*b^3 - (2*a^6 + 3*a^5*b - 14*a^4*b^2)*cos(f*x + e)^4 - (4
*a^5*b + 12*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^2)*sqrt(a + b)*log(2*((8*a^2
+ 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4*
((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x +
```

e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) + 4*((
3*a^6 + 3*a^5*b + 20*a^4*b^2 + 28*a^3*b^3 + 8*a^2*b^4)*cos(f*x + e)^6 + 2*(
3*a^5*b - 7*a^4*b^2 - 5*a^3*b^3 + 8*a^2*b^4 + 3*a*b^5)*cos(f*x + e)^4 + 3*(
a^4*b^2 - 5*a^3*b^3 - 8*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^2)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2))/((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*
b^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2 - 8*a^6*b^3 - 7*a^5*b^4
- 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b + 7*a^7*b^2 + 8*a^6*b^3 + 2*a^5*b^
4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^
4 + 4*a^4*b^5 + a^3*b^6)*f), -1/24*(6*((2*a^6 + 7*a^5*b)*cos(f*x + e)^6 - 2
*a^4*b^2 - 7*a^3*b^3 - (2*a^6 + 3*a^5*b - 14*a^4*b^2)*cos(f*x + e)^4 - (4*a
^5*b + 12*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((2*
a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) - 3*((a^6 + 4*a^5*b + 6*
a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^6 - a^4*b^2 - 4*a^3*b^3 - 6*a^2
*b^4 - 4*a*b^5 - b^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^4 -
2*a*b^5)*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2
*a*b^5 - b^6)*cos(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*
b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b
^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x
+ e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)) - 4*((3*a^6 + 3*a^5*b + 20*a^4*b^2 + 28*a^3*b^3 + 8*a^2*b^4)*cos(f*x
+ e)^6 + 2*(3*a^5*b - 7*a^4*b^2 - 5*a^3*b^3 + 8*a^2*b^4 + 3*a*b^5)*cos(f*x
+ e)^4 + 3*(a^4*b^2 - 5*a^3*b^3 - 8*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^2)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a
^6*b^3 + a^5*b^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2 - 8*a^6*b^3
- 7*a^5*b^4 - 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b + 7*a^7*b^2 + 8*a^6*b
^3 + 2*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b^2 + 4*a^6*b
^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3*b^6)*f), 1/24*(6*((a^6 + 4*a^5*b + 6*a^4*b
^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^6 - a^4*b^2 - 4*a^3*b^3 - 6*a^2*b^4
- 4*a*b^5 - b^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*
b^5)*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^
5 - b^6)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*
cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/
(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) + 3*((2*a^6 + 7*a^
5*b)*cos(f*x + e)^6 - 2*a^4*b^2 - 7*a^3*b^3 - (2*a^6 + 3*a^5*b - 14*a^4*b^2
)*cos(f*x + e)^4 - (4*a^5*b + 12*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^2)*sqrt(
a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(
f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a +
b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*
x + e)^2 + 1)) + 4*((3*a^6 + 3*a^5*b + 20*a^4*b^2 + 28*a^3*b^3 + 8*a^2*b^4)
*cos(f*x + e)^6 + 2*(3*a^5*b - 7*a^4*b^2 - 5*a^3*b^3 + 8*a^2*b^4 + 3*a*b^5)
*cos(f*x + e)^4 + 3*(a^4*b^2 - 5*a^3*b^3 - 8*a^2*b^4 - 2*a*b^5)*cos(f*x + e
)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^9 + 4*a^8*b + 6*a^7*b
^2 + 4*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2 - 8
*a^6*b^3 - 7*a^5*b^4 - 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b + 7*a^7*b^2 +

```
 8*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b^2 +
 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3*b^6)*f), 1/12*(3*((a^6 + 4*a^5*b +
 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^6 - a^4*b^2 - 4*a^3*b^3 - 6*
a^2*b^4 - 4*a*b^5 - b^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^
4 - 2*a*b^5)*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^4
- 2*a*b^5 - b^6)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4
+ 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 3*((2*a^
6 + 7*a^5*b)*cos(f*x + e)^6 - 2*a^4*b^2 - 7*a^3*b^3 - (2*a^6 + 3*a^5*b - 14
*a^4*b^2)*cos(f*x + e)^4 - (4*a^5*b + 12*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^
2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b +
 b^2)) + 2*((3*a^6 + 3*a^5*b + 20*a^4*b^2 + 28*a^3*b^3 + 8*a^2*b^4)*cos(f*x
 + e)^6 + 2*(3*a^5*b - 7*a^4*b^2 - 5*a^3*b^3 + 8*a^2*b^4 + 3*a*b^5)*cos(f*x
 + e)^4 + 3*(a^4*b^2 - 5*a^3*b^3 - 8*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^2)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a
^6*b^3 + a^5*b^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2 - 8*a^6*b^3
 - 7*a^5*b^4 - 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b + 7*a^7*b^2 + 8*a^6*b
^3 + 2*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b^2 + 4*a^6*b
^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3*b^6)*f)]
```

Sympy [F]

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(cot(e + f*x)**3/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(e+f\,x)^3}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

```
[In] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.433 $\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 268

$$\begin{aligned}\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx &= \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} \\ &\quad - \frac{(8a^2+36ab+63b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{9/2}f} \\ &\quad + \frac{b(12a^2+39ab-8b^2)}{24a(a+b)^3f\,(a+b\sec^2(e+fx))^{3/2}} + \frac{(4a+11b)\cot^2(e+fx)}{8(a+b)^2f\,(a+b\sec^2(e+fx))^{3/2}} \\ &\quad - \frac{\cot^4(e+fx)}{4(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} + \frac{b(4a^3+15a^2b-32ab^2-8b^3)}{8a^2(a+b)^4f\sqrt{a+b\sec^2(e+fx)}}\end{aligned}$$

```
[Out] arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f-1/8*(8*a^2+36*a*b+63*b^
2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(9/2)/f+1/24*b*(12*a
^2+39*a*b-8*b^2)/a/(a+b)^3/f/(a+b*sec(f*x+e)^2)^(3/2)+1/8*(4*a+11*b)*cot(f*
x+e)^2/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(3/2)-1/4*cot(f*x+e)^4/(a+b)/f/(a+b*sec
(f*x+e)^2)^(3/2)+1/8*b*(4*a^3+15*a^2*b-32*a*b^2-8*b^3)/a^2/(a+b)^4/f/(a+b*s
ec(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4224, 457, 105, 156, 157, 162, 65, 214}

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} - \frac{(8a^2+36ab+63b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{9/2}} + \frac{b(12a^2+39ab-8b^2)}{24af(a+b)^3\,(a+b\sec^2(e+fx))^{3/2}} + \frac{b(4a^3+15a^2b-32ab^2-8b^3)}{8a^2f(a+b)^4\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^4(e+fx)}{4f(a+b)\,(a+b\sec^2(e+fx))^{3/2}} + \frac{(4a+11b)\cot^2(e+fx)}{8f(a+b)^2\,(a+b\sec^2(e+fx))^{3/2}}$$

```
[In] Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f) - ((8*a^2 + 36*a*b
+ 63*b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(8*(a + b)^(9/2)
*f) + (b*(12*a^2 + 39*a*b - 8*b^2))/(24*a*(a + b)^3*f*(a + b*Sec[e + f*x]^2
)^(3/2)) + ((4*a + 11*b)*Cot[e + f*x]^2)/(8*(a + b)^2*f*(a + b*Sec[e + f*x]
^2)^(3/2)) - Cot[e + f*x]^4/(4*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)) + (b
*(4*a^3 + 15*a^2*b - 32*a*b^2 - 8*b^3))/(8*a^2*(a + b)^4*f*Sqrt[a + b*Sec[e
 + f*x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
```

```
[2*n, p])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x(-1+x^2)^3(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)^3x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^4(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{2(a+b)+\frac{7bx}{2}}{(-1+x)^2x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{4(a+b)f}$$

$$= \frac{(4a+11b)\cot^2(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^4(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{2(a+b)^2+\frac{5}{4}b(4a+11b)x}{(-1+x)x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{4(a+b)^2f}$$

$$= \frac{b(12a^2+39ab-8b^2)}{24a(a+b)^3f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{(4a+11b)\cot^2(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^4(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-3(a+b)^3-\frac{3}{8}b(12a^2+39ab-8b^2)x}{(-1+x)x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{6a(a+b)^3f}$$

$$= \frac{b(12a^2+39ab-8b^2)}{24a(a+b)^3f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{(4a+11b)\cot^2(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^4(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{b(4a^3+15a^2b-32ab^2-8b^3)}{8a^2(a+b)^4f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{\frac{3}{2}(a+b)^4+\frac{3}{16}b(4a^3+15a^2b-32ab^2-8b^3)x}{(-1+x)x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{3a^2(a+b)^4f}$$

$$= \frac{b(12a^2+39ab-8b^2)}{24a(a+b)^3f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{(4a+11b)\cot^2(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^4(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{b(4a^3+15a^2b-32ab^2-8b^3)}{8a^2(a+b)^4f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2a^2f} + \frac{(8a^2+36ab+63b^2)\,\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{16(a+b)^4f}$$

$$
\begin{aligned}
&= \frac{b(12a^2+39ab-8b^2)}{24a(a+b)^3 f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{(4a+11b)\cot^2(e+fx)}{8(a+b)^2 f\left(a+b\sec^2(e+fx)\right)^{3/2}} \\
&\quad - \frac{\cot^4(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{b(4a^3+15a^2b-32ab^2-8b^3)}{8a^2(a+b)^4 f\sqrt{a+b\sec^2(e+fx)}} \\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{a^2bf} \\
&\quad + \frac{(8a^2+36ab+63b^2)\,\text{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{8b(a+b)^4 f}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} - \frac{(8a^2+36ab+63b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{9/2}f} \\
&\quad + \frac{b(12a^2+39ab-8b^2)}{24a(a+b)^3 f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{(4a+11b)\cot^2(e+fx)}{8(a+b)^2 f\left(a+b\sec^2(e+fx)\right)^{3/2}} \\
&\quad - \frac{\cot^4(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{b(4a^3+15a^2b-32ab^2-8b^3)}{8a^2(a+b)^4 f\sqrt{a+b\sec^2(e+fx)}}
\end{aligned}
$$

Mathematica [F]

$$
\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx
$$

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
[Out] Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 77708 vs. 2(238) = 476.

Time = 10.43 (sec) , antiderivative size = 77709, normalized size of antiderivative = 289.96

method	result	size
default	Expression too large to display	77709

```
[In] int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1141 vs. $2(238) = 476$.

Time = 18.70 (sec) , antiderivative size = 4751, normalized size of antiderivative = 17.73

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/96*(12*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*
cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 +
b^7 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*cos(f*x
 + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 +
a*b^6 + b^7)*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5
- 4*a*b^6 - b^7)*cos(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a
^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2
+ b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f
*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)) + 3*((8*a^7 + 36*a^6*b + 63*a^5*b^2)*cos(f*x + e)^8 + 8*a^5*b^2 +
 36*a^4*b^3 + 63*a^3*b^4 - 2*(8*a^7 + 28*a^6*b + 27*a^5*b^2 - 63*a^4*b^3)*c
os(f*x + e)^6 + (8*a^7 + 4*a^6*b - 73*a^5*b^2 - 216*a^4*b^3 + 63*a^3*b^4)*c
os(f*x + e)^4 + 2*(8*a^6*b + 28*a^5*b^2 + 27*a^4*b^3 - 63*a^3*b^4)*cos(f*x
+ e)^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b
+ 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e
)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)
^4 - 2*cos(f*x + e)^2 + 1)) - 4*((18*a^7 + 69*a^6*b + 51*a^5*b^2 + 104*a^4*
b^3 + 136*a^3*b^4 + 32*a^2*b^5)*cos(f*x + e)^8 - (12*a^7 + 21*a^6*b - 93*a^
5*b^2 + 106*a^4*b^3 + 176*a^3*b^4 - 56*a^2*b^5 - 24*a*b^6)*cos(f*x + e)^6 -
 (24*a^6*b + 96*a^5*b^2 - 83*a^4*b^3 + 5*a^3*b^4 + 208*a^2*b^5 + 48*a*b^6)*
cos(f*x + e)^4 - 3*(4*a^5*b^2 + 19*a^4*b^3 - 17*a^3*b^4 - 40*a^2*b^5 - 8*a*
b^6)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^10 +
5*a^9*b + 10*a^8*b^2 + 10*a^7*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8 -
 2*(a^10 + 4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6)*f*cos(f*x
 + e)^6 + (a^10 + a^9*b - 9*a^8*b^2 - 25*a^7*b^3 - 25*a^6*b^4 - 9*a^5*b^5 +
 a^4*b^6 + a^3*b^7)*f*cos(f*x + e)^4 + 2*(a^9*b + 4*a^8*b^2 + 5*a^7*b^3 - 5
*a^5*b^5 - 4*a^4*b^6 - a^3*b^7)*f*cos(f*x + e)^2 + (a^8*b^2 + 5*a^7*b^3 + 1
0*a^6*b^4 + 10*a^5*b^5 + 5*a^4*b^6 + a^3*b^7)*f), 1/48*(3*((8*a^7 + 36*a^6*
b + 63*a^5*b^2)*cos(f*x + e)^8 + 8*a^5*b^2 + 36*a^4*b^3 + 63*a^3*b^4 - 2*(8
*a^7 + 28*a^6*b + 27*a^5*b^2 - 63*a^4*b^3)*cos(f*x + e)^6 + (8*a^7 + 4*a^6*
b - 73*a^5*b^2 - 216*a^4*b^3 + 63*a^3*b^4)*cos(f*x + e)^4 + 2*(8*a^6*b + 28
*a^5*b^2 + 27*a^4*b^3 - 63*a^3*b^4)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2
*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 6*((a^7 + 5*a^6*b
 + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*cos(f*x + e)^8 + a^5*b^2
```

```
+ 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7 - 2*(a^7 + 4*a^6*b +
5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*cos(f*x + e)^6 + (a^7 + a^6*b -
9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cos(f*x + e)
^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*cos(f*x
+ e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160
*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x
 + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x +
 e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 2*((18*a^7 +
69*a^6*b + 51*a^5*b^2 + 104*a^4*b^3 + 136*a^3*b^4 + 32*a^2*b^5)*cos(f*x + e
)^8 - (12*a^7 + 21*a^6*b - 93*a^5*b^2 + 106*a^4*b^3 + 176*a^3*b^4 - 56*a^2*
b^5 - 24*a*b^6)*cos(f*x + e)^6 - (24*a^6*b + 96*a^5*b^2 - 83*a^4*b^3 + 5*a^
3*b^4 + 208*a^2*b^5 + 48*a*b^6)*cos(f*x + e)^4 - 3*(4*a^5*b^2 + 19*a^4*b^3
- 17*a^3*b^4 - 40*a^2*b^5 - 8*a*b^6)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2))/((a^10 + 5*a^9*b + 10*a^8*b^2 + 10*a^7*b^3 + 5*a^6*b
^4 + a^5*b^5)*f*cos(f*x + e)^8 - 2*(a^10 + 4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4
- 4*a^5*b^5 - a^4*b^6)*f*cos(f*x + e)^6 + (a^10 + a^9*b - 9*a^8*b^2 - 25*a^
7*b^3 - 25*a^6*b^4 - 9*a^5*b^5 + a^4*b^6 + a^3*b^7)*f*cos(f*x + e)^4 + 2*(a
^9*b + 4*a^8*b^2 + 5*a^7*b^3 - 5*a^5*b^5 - 4*a^4*b^6 - a^3*b^7)*f*cos(f*x +
 e)^2 + (a^8*b^2 + 5*a^7*b^3 + 10*a^6*b^4 + 10*a^5*b^5 + 5*a^4*b^6 + a^3*b^
7)*f), -1/96*(24*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^
2*b^5)*cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a
*b^6 + b^7 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*
cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2
*b^5 + a*b^6 + b^7)*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a
^2*b^5 - 4*a*b^6 - b^7)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x
+ e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 3
*((8*a^7 + 36*a^6*b + 63*a^5*b^2)*cos(f*x + e)^8 + 8*a^5*b^2 + 36*a^4*b^3 +
 63*a^3*b^4 - 2*(8*a^7 + 28*a^6*b + 27*a^5*b^2 - 63*a^4*b^3)*cos(f*x + e)^6
 + (8*a^7 + 4*a^6*b - 73*a^5*b^2 - 216*a^4*b^3 + 63*a^3*b^4)*cos(f*x + e)^4
 + 2*(8*a^6*b + 28*a^5*b^2 + 27*a^4*b^3 - 63*a^3*b^4)*cos(f*x + e)^2)*sqrt(
a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(
f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a +
 b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*
x + e)^2 + 1)) + 4*((18*a^7 + 69*a^6*b + 51*a^5*b^2 + 104*a^4*b^3 + 136*a^3
*b^4 + 32*a^2*b^5)*cos(f*x + e)^8 - (12*a^7 + 21*a^6*b - 93*a^5*b^2 + 106*a
^4*b^3 + 176*a^3*b^4 - 56*a^2*b^5 - 24*a*b^6)*cos(f*x + e)^6 - (24*a^6*b +
96*a^5*b^2 - 83*a^4*b^3 + 5*a^3*b^4 + 208*a^2*b^5 + 48*a*b^6)*cos(f*x + e)^
4 - 3*(4*a^5*b^2 + 19*a^4*b^3 - 17*a^3*b^4 - 40*a^2*b^5 - 8*a*b^6)*cos(f*x
+ e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^10 + 5*a^9*b + 10*
a^8*b^2 + 10*a^7*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8 - 2*(a^10 + 4*
a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6)*f*cos(f*x + e)^6 + (a^
10 + a^9*b - 9*a^8*b^2 - 25*a^7*b^3 - 25*a^6*b^4 - 9*a^5*b^5 + a^4*b^6 + a^
3*b^7)*f*cos(f*x + e)^4 + 2*(a^9*b + 4*a^8*b^2 + 5*a^7*b^3 - 5*a^5*b^5 - 4*
a^4*b^6 - a^3*b^7)*f*cos(f*x + e)^2 + (a^8*b^2 + 5*a^7*b^3 + 10*a^6*b^4 + 1
```

```
0*a^5*b^5 + 5*a^4*b^6 + a^3*b^7)*f), -1/48*(12*((a^7 + 5*a^6*b + 10*a^5*b^2
 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 +
 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5
*a^3*b^4 - 4*a^2*b^5 - a*b^6)*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 2
5*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cos(f*x + e)^4 + 2*(a^6*b
 + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*cos(f*x + e)^2)*sqrt(
-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*
b*cos(f*x + e)^2 + a*b^2)) - 3*((8*a^7 + 36*a^6*b + 63*a^5*b^2)*cos(f*x + e
)^8 + 8*a^5*b^2 + 36*a^4*b^3 + 63*a^3*b^4 - 2*(8*a^7 + 28*a^6*b + 27*a^5*b^
2 - 63*a^4*b^3)*cos(f*x + e)^6 + (8*a^7 + 4*a^6*b - 73*a^5*b^2 - 216*a^4*b^
3 + 63*a^3*b^4)*cos(f*x + e)^4 + 2*(8*a^6*b + 28*a^5*b^2 + 27*a^4*b^3 - 63*
a^3*b^4)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2
+ b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*
cos(f*x + e)^2 + a*b + b^2)) + 2*((18*a^7 + 69*a^6*b + 51*a^5*b^2 + 104*a^4
*b^3 + 136*a^3*b^4 + 32*a^2*b^5)*cos(f*x + e)^8 - (12*a^7 + 21*a^6*b - 93*a
^5*b^2 + 106*a^4*b^3 + 176*a^3*b^4 - 56*a^2*b^5 - 24*a*b^6)*cos(f*x + e)^6
- (24*a^6*b + 96*a^5*b^2 - 83*a^4*b^3 + 5*a^3*b^4 + 208*a^2*b^5 + 48*a*b^6)
*cos(f*x + e)^4 - 3*(4*a^5*b^2 + 19*a^4*b^3 - 17*a^3*b^4 - 40*a^2*b^5 - 8*a
*b^6)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^10 +
 5*a^9*b + 10*a^8*b^2 + 10*a^7*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8
- 2*(a^10 + 4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6)*f*cos(f*
x + e)^6 + (a^10 + a^9*b - 9*a^8*b^2 - 25*a^7*b^3 - 25*a^6*b^4 - 9*a^5*b^5
+ a^4*b^6 + a^3*b^7)*f*cos(f*x + e)^4 + 2*(a^9*b + 4*a^8*b^2 + 5*a^7*b^3 -
5*a^5*b^5 - 4*a^4*b^6 - a^3*b^7)*f*cos(f*x + e)^2 + (a^8*b^2 + 5*a^7*b^3 +
10*a^6*b^4 + 10*a^5*b^5 + 5*a^4*b^6 + a^3*b^7)*f)]
```

Sympy [F]

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\cot^5{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

[In] integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)

[Out] Integral(cot(e + f*x)**5/(a + b*sec(e + f*x)**2)**(5/2), x)

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(fx+e)^5}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Hanged}$$

```
[In] int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] \text{Hanged}
```

3.434 $\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{5/2}f} - \frac{(a+b)\tan^3(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}}+\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tan(e+fx)}{f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f+arctanh(b^
(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(5/2)/f+(1/a^2-1/b^2)*tan(f*
x+e)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*(a+b)*tan(f*x+e)^3/a/b/f/(a+b+b*tan(f
*x+e)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4226, 2000, 481, 592, 537, 223, 212, 385, 209}

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f} + \frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tan(e+fx)}{f\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{b^{5/2}f} - \frac{(a+b)\tan^3(e+fx)}{3abf\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[In] Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f)
) + ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(b^(5/2)
*f) - ((a + b)*Tan[e + f*x]^3)/(3*a*b*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) +
 ((a^(-2) - b^(-2))*Tan[e + f*x])/(f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+b(1+x^2))^{5/2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{(a+b)\tan^3(e+fx)}{3abf\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)+3ax^2)}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3abf} \\
&= -\frac{(a+b)\tan^3(e+fx)}{3abf\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{(a^2-b^2)\tan(e+fx)}{a^2b^2f\sqrt{a+b+b\tan^2(e+fx)}} \\
&\quad - \frac{\text{Subst}\left(\int \frac{-3(a^2-b^2)-3a^2x^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{3a^2b^2f}
\end{aligned}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{(a^2-b^2)\tan(e+fx)}{a^2b^2f\sqrt{a+b+b\tan^2(e+fx)}}$$
$$- \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{a^2f}$$
$$+ \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{b^2f}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{(a^2-b^2)\tan(e+fx)}{a^2b^2f\sqrt{a+b+b\tan^2(e+fx)}}$$
$$- \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^2f} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^2f}$$

$$= -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} + \frac{\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{5/2}f}$$
$$- \frac{(a+b)\tan^3(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{(a^2-b^2)\tan(e+fx)}{a^2b^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 316 vs. $2(157) = 314$.

Time = 11.15 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.01

$$\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx =$$
$$-\frac{\left(\frac{b^2\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{a}} - \frac{a^2\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)(a+2b+a\cos(2e+2fx))^{5/2}\sec^5(e+fx)}{4\sqrt{2}a^2b^2f\left(a+b\sec^2(e+fx)\right)^{5/2}}$$
$$+\frac{(a+2b+a\cos(2e+2fx))^3\sec^5(e+fx)\left(\frac{-a^2\sin(e+fx)-2ab\sin(e+fx)-b^2\sin(e+fx)}{6a^2bf(a+2b+a\cos(2e+2fx))^2} + \frac{-3a^2\sin(e+fx)+ab\sin(e+fx)+4b^2 [illegible]}{12a^2b^2f(a+2b+a\cos(2e+2f [illegible]}\right.}{\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/4*(((b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/
Sqrt[a] - (a^2*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2
]])/Sqrt[b])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5)/(Sqrt[2]*
a^2*b^2*f*(a + b*Sec[e + f*x]^2)^(5/2)) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3
*Sec[e + f*x]^5*((-(a^2*Sin[e + f*x]) - 2*a*b*Sin[e + f*x] - b^2*Sin[e + f*
x])/(6*a^2*b*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2) + (-3*a^2*Sin[e + f*x] + a
*b*Sin[e + f*x] + 4*b^2*Sin[e + f*x])/(12*a^2*b^2*f*(a + 2*b + a*Cos[2*e +
2*f*x]))))/(a + b*Sec[e + f*x]^2)^(5/2)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1499 vs. $2(139) = 278$.

Time = 9.86 (sec) , antiderivative size = 1500, normalized size of antiderivative = 9.55

method	result	size
default	Expression too large to display	1500

```
[In] int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/f/b^(7/2)/(-a)^(1/2)/a^2*(12*(-a)^(1/2)*b^(7/2)*a*(1-cos(f*x+e))^5*csc
(f*x+e)^5+12*(-a)^(1/2)*b^(9/2)*(1-cos(f*x+e))^5*csc(f*x+e)^5-40*(1-cos(f*x
+e))^3*a*(-a)^(1/2)*b^(7/2)*csc(f*x+e)^3+24*(-a)^(1/2)*b^(9/2)*(1-cos(f*x+e
))^3*csc(f*x+e)^3-12*(-a)^(1/2)*b^(3/2)*a^3*(1-cos(f*x+e))^5*csc(f*x+e)^5-1
2*(-a)^(1/2)*b^(5/2)*a^2*(1-cos(f*x+e))^5*csc(f*x+e)^5-40*(1-cos(f*x+e))^3*
a^2*(-a)^(1/2)*b^(5/2)*csc(f*x+e)^3-6*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*
csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e
)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)
))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*b^(7/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e
)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(
1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(3/2)+12*(-a)^(1/2)*b^(7/2)*a*(csc(f*x+e)
-cot(f*x+e))+12*(-a)^(1/2)*b^(9/2)*(csc(f*x+e)-cot(f*x+e))+24*(-a)^(1/2)*b^
(3/2)*a^3*(1-cos(f*x+e))^3*csc(f*x+e)^3+3*(-a)^(1/2)*ln(4*(-a*(1-cos(f*x+e)
)^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2+b^(1/2)*(a*(1-cos(f*x+e))^
4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x
+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+
e))-a-b)/((1-cos(f*x+e))^2*csc(f*x+e)^2+2*csc(f*x+e)-2*cot(f*x+e)+1))*(a*(1
-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+
e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(3/2)*a^2*b+3*(-a
)^(1/2)*ln(4*(a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)
^2+b^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4
-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(
1/2)-2*a*(csc(f*x+e)-cot(f*x+e))+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-2*csc(
f*x+e)+2*cot(f*x+e)+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4
*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*
x+e)^2+a+b)^(3/2)*a^2*b-12*(-a)^(1/2)*b^(3/2)*a^3*(csc(f*x+e)-cot(f*x+e))-1
2*(-a)^(1/2)*b^(5/2)*a^2*(csc(f*x+e)-cot(f*x+e)))*(a*(1-cos(f*x+e))^4*csc(f
*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2
*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^5/(
(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos
(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/((1-cos(f*x+
e))^2*csc(f*x+e)^2-1)^2)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(139) = 278$.

Time = 2.15 (sec) , antiderivative size = 2035, normalized size of antiderivative = 12.96

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*(a^2*b^3*cos(f*x + e)^4 + 2*a*b^4*cos(f*x + e)^2 + b^5)*sqrt(-a)*
log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 -
 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*
b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a
^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b +
 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sq
rt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 6*(a^5*c
os(f*x + e)^4 + 2*a^4*b*cos(f*x + e)^2 + a^3*b^2)*sqrt(b)*log(((a^2 - 6*a*b
 + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x
+ e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 8*((3*a^4*b - a^3*b^2 - 4*a^2*b
^3)*cos(f*x + e)^3 + (4*a^3*b^2 + a^2*b^3 - 3*a*b^4)*cos(f*x + e))*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^5*b^3*f*cos(f*x + e)^4
 + 2*a^4*b^4*f*cos(f*x + e)^2 + a^3*b^5*f), 1/24*(12*(a^5*cos(f*x + e)^4 +
2*a^4*b*cos(f*x + e)^2 + a^3*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e
)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) - 3*(a^2*b^3*cos(f*x + e)^4 +
2*a*b^4*cos(f*x + e)^2 + b^5)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^
4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^
2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)
*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*
a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e)) - 8*((3*a^4*b - a^3*b^2 - 4*a^2*b^3)*cos(f*x +
 e)^3 + (4*a^3*b^2 + a^2*b^3 - 3*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^5*b^3*f*cos(f*x + e)^4 + 2*a^4*b^4*
f*cos(f*x + e)^2 + a^3*b^5*f), 1/12*(3*(a^2*b^3*cos(f*x + e)^4 + 2*a*b^4*co
s(f*x + e)^2 + b^5)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b
)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3
 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 3*(a^5*cos(f*x + e)^4 + 2*a^4*
b*cos(f*x + e)^2 + a^3*b^2)*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
 e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b
^2)/cos(f*x + e)^4) - 4*((3*a^4*b - a^3*b^2 - 4*a^2*b^3)*cos(f*x + e)^3 + (
```

```
4*a^3*b^2 + a^2*b^3 - 3*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*sin(f*x + e))/(a^5*b^3*f*cos(f*x + e)^4 + 2*a^4*b^4*f*cos(f*x
 + e)^2 + a^3*b^5*f), 1/12*(3*(a^2*b^3*cos(f*x + e)^4 + 2*a*b^4*cos(f*x + e
)^2 + b^5)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x
 + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*
b)*cos(f*x + e)^2)*sin(f*x + e))) + 6*(a^5*cos(f*x + e)^4 + 2*a^4*b*cos(f*x
 + e)^2 + a^3*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f
*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x
 + e)^2 + b^2)*sin(f*x + e))) - 4*((3*a^4*b - a^3*b^2 - 4*a^2*b^3)*cos(f*x
+ e)^3 + (4*a^3*b^2 + a^2*b^3 - 3*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^5*b^3*f*cos(f*x + e)^4 + 2*a^4*b^4
*f*cos(f*x + e)^2 + a^3*b^5*f)]
```

Sympy [F]

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan^6{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(tan(e + f*x)**6/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(fx+e)^6}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(fx+e)^6}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\tan(e+f\,x)^6}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

```
[In] int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.435 $\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 120

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{(a+b)\tan(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-3b)\tan(e+fx)}{3a^2bf\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f+1/3*(a-3*b)
*tan(f*x+e)/a^2/b/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*(a+b)*tan(f*x+e)/a/b/f/(
a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4226, 2000, 481, 541, 12, 385, 209}

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f} + \frac{(a-3b)\tan(e+fx)}{3a^2bf\sqrt{a+b\tan^2(e+fx)+b}} - \frac{(a+b)\tan(e+fx)}{3abf\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[In] Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f) -
 ((a + b)*Tan[e + f*x])/(3*a*b*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((a -
3*b)*Tan[e + f*x])/(3*a^2*b*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+b(1+x^2))^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{(a+b)\tan(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a+b+(a-2b)x^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3abf}\\
&= -\frac{(a+b)\tan(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-3b)\tan(e+fx)}{3a^2bf\sqrt{a+b+b\tan^2(e+fx)}}\\
&\quad + \frac{\text{Subst}\left(\int \frac{3b(a+b)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{3a^2b(a+b)f}\\
&= -\frac{(a+b)\tan(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-3b)\tan(e+fx)}{3a^2bf\sqrt{a+b+b\tan^2(e+fx)}}\\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{a^2f}\\
&= -\frac{(a+b)\tan(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-3b)\tan(e+fx)}{3a^2bf\sqrt{a+b+b\tan^2(e+fx)}}\\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^2f}\\
&= \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{(a+b)\tan(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}}\\
&\quad + \frac{(a-3b)\tan(e+fx)}{3a^2bf\sqrt{a+b+b\tan^2(e+fx)}}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 409 vs. $2(120) = 240$.

Time = 6.55 (sec) , antiderivative size = 409, normalized size of antiderivative = 3.41

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))^{5/2}\sec^4(e+fx)\left(\frac{\sqrt{2}\csc(e+fx)\sec(e+fx)\left(\frac{\sin^2(e+fx)}{a+b}+\cdots\right.}{\cdots}\right.}{\cdots}$$

[In] Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]

[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^(5/2)*Sec[e + f*x]^4*((Sqrt[2]*Csc[e + f*x]*Sec[e + f*x]*(Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*Sin[e + f*x]^2)/(a + b)^2 - (12*Sin[e + f*x]^4)/(a + b) + (16*(a + b - a*Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*x]^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b - a*Sin[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^3))/(a + b - a*Sin[e + f*x]^2)^(3/2) + (8*(2*a + 3*b + a*Cos[2*(e + f*x)])*Tan[e + f*x])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)) - (12*(b + (3*a + 2*b)*Cos[2*(e + f*x)])*Tan[e + f*x])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2))))/(384*f*(a + b*Sec[e + f*x]^2)^(5/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(106) = 212$.

Time = 4.59 (sec) , antiderivative size = 657, normalized size of antiderivative = 5.48

method	result
default	$-\frac{\left(a(1-\cos(fx+e))^4\csc(fx+e)^4+b(1-\cos(fx+e))^4\csc(fx+e)^4-2a(1-\cos(fx+e))^2\csc(fx+e)^2+2b(1-\cos(fx+e))^2\csc(fx+e)^2+\cdots\right.}{\cdots}$

[In] int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/3/f/a^2/(-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+

```
e)^2+a+b)*(-6*(-a)^(1/2)*a*(1-cos(f*x+e))^5*csc(f*x+e)^5-6*(-a)^(1/2)*b*(1-
cos(f*x+e))^5*csc(f*x+e)^5+20*(1-cos(f*x+e))^3*a*(-a)^(1/2)*csc(f*x+e)^3-12
*(-a)^(1/2)*b*(1-cos(f*x+e))^3*csc(f*x+e)^3+3*ln(4*((-a)^(1/2)*(a*(1-cos(f*
x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*c
sc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-co
t(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e
)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(
1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(3/2)-6*(-a)^(1/2)*a*(csc(f*x+e)-cot(f*x+
e))-6*(-a)^(1/2)*b*(csc(f*x+e)-cot(f*x+e)))/((a*(1-cos(f*x+e))^4*csc(f*x+e)
^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1
-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^2)^(5/2)
/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(106) = 212$.

Time = 0.95 (sec) , antiderivative size = 661, normalized size of antiderivative = 5.51

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \left[-\frac{3\left(a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2\right)\sqrt{-a}\log\left(128\,a^4\cos(fx+e)\right.}{}\right.$$

$$-\frac{3\left(a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\left(a^2-ab\right)\cos(fx+e)^3+\left(a^2-6\,ab+b^2\right)\cos(fx+e\right.}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-\left(a^3-3\,a^2b\right)\cos(fx+e)^2\right)}\right.}{12\left(a^5f\cos(fx+e)^4+2\,a^4bf\cos\right.}$$

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*log(12
8*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos
(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b
^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(4*a^2*cos(f
*x + e)^3 - (a^2 - 3*a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*sin(f*x + e))/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a
^3*b^2*f), -1/12*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(
a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 -
```

```
 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^
2)*sin(f*x + e))) + 4*(4*a^2*cos(f*x + e)^3 - (a^2 - 3*a*b)*cos(f*x + e))*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^5*f*cos(f*x + e
)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f)]
```

Sympy [F]

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan^4{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(tan(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan{(fx+e)}^4}{\left(b\sec{(fx+e)}^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(e+f\,x)^4}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

```
[In] int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.436 $\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 119

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} + \frac{\tan(e+fx)}{3af\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{(2a+3b)\tan(e+fx)}{3a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f+1/3*(2*a+3
*b)*tan(f*x+e)/a^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/3*tan(f*x+e)/a/f/(a
+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4226, 2000, 482, 541, 12, 385, 209}

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f} + \frac{(2a+3b)\tan(e+fx)}{3a^2f(a+b)\sqrt{a+b\tan^2(e+fx)+b}} + \frac{\tan(e+fx)}{3af\,(a+b\tan^2(e+fx)+b)^{3/2}}$$

```
[In] Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f)
) + Tan[e + f*x]/(3*a*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((2*a + 3*b)*Ta
n[e + f*x])/(3*a^2*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+b(1+x^2))^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{3af\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1-2x^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3af}$$

$$= \frac{\tan(e+fx)}{3af\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{(2a+3b)\tan(e+fx)}{3a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{3(a+b)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{3a^2(a+b)f}$$

$$= \frac{\tan(e+fx)}{3af\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{(2a+3b)\tan(e+fx)}{3a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{a^2f}$$

$$= \frac{\tan(e+fx)}{3af\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{(2a+3b)\tan(e+fx)}{3a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^2f}$$

$$= -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} + \frac{\tan(e+fx)}{3af\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{(2a+3b)\tan(e+fx)}{3a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 410 vs. $2(119) = 238$.

Time = 4.69 (sec) , antiderivative size = 410, normalized size of antiderivative = 3.45

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))^{5/2}\sec^4(e+fx)\left(-\frac{\sqrt{2}\csc(e+fx)\sec(e+fx)\left(\frac{\sin^2(e+fx)}{a+b}+\cdots\right.}{\cdots}\right.}{\cdots}$$

[In] Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^(5/2)*Sec[e + f*x]^4*(-((Sqrt[2]*Csc[e + f*
x]*Sec[e + f*x]*(Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*S
in[e + f*x]^2)/(a + b)^2 - (12*Sin[e + f*x]^4)/(a + b) + (16*(a + b - a*Sin
[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*x]^2
)/(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b - a*
Sin[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sq
rt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^3))/(
a + b - a*Sin[e + f*x]^2)^(3/2)) + (8*(2*a + 3*b + a*Cos[2*(e + f*x)])*Tan[
e + f*x])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)) - (4*(b + (3*a +
 2*b)*Cos[2*(e + f*x)])*Tan[e + f*x])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*
x)])^(3/2))))/(384*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1002 vs. $2(105) = 210$.

Time = 5.60 (sec) , antiderivative size = 1003, normalized size of antiderivative = 8.43

method	result	size
default	Expression too large to display	1003

[In] int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)

```
[Out] 1/3/f/(a+b)/a^2/(-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e)
)^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc
(f*x+e)^2+a+b)*(-6*(-a)^(1/2)*a^2*(1-cos(f*x+e))^5*csc(f*x+e)^5-12*(-a)^(1/
2)*a*b*(1-cos(f*x+e))^5*csc(f*x+e)^5-6*(-a)^(1/2)*b^2*(1-cos(f*x+e))^5*csc(
f*x+e)^5+12*(1-cos(f*x+e))^3*a^2*(-a)^(1/2)*csc(f*x+e)^3+8*(1-cos(f*x+e))^3
```

```
*a*(-a)^(1/2)*b*csc(f*x+e)^3-12*(-a)^(1/2)*b^2*(1-cos(f*x+e))^3*csc(f*x+e)^
3+3*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*cs
c(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e
)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e)^2+
1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1
-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(3/2)*a+
3*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(
f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^
2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1)
)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-c
os(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(3/2)*b-6*
(-a)^(1/2)*a^2*(csc(f*x+e)-cot(f*x+e))-12*(-a)^(1/2)*a*b*(csc(f*x+e)-cot(f*
x+e))-6*(-a)^(1/2)*b^2*(csc(f*x+e)-cot(f*x+e)))/((a*(1-cos(f*x+e))^4*csc(f*
x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*
b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^2)^(
5/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(105) = 210$.

Time = 0.82 (sec) , antiderivative size = 773, normalized size of antiderivative = 6.50

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \left[-\frac{3\left(\left(a^3+a^2b\right)\cos\left(fx+e\right)^4+ab^2+b^3+2\left(a^2b+ab^2\right)\cos\left(fx+e\right)^2\right)\sqrt{-a}}{}\right.$$

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^3 + a^2*b)*cos(f*x + e)^4 + a*b^2 + b^3 + 2*(a^2*b + a*b^2)*c
os(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(
f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^
3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*
cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5
 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2
 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
sin(f*x + e)) - 8*((3*a^3 + 4*a^2*b)*cos(f*x + e)^3 + (2*a^2*b + 3*a*b^2)*c
os(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^
6 + a^5*b)*f*cos(f*x + e)^4 + 2*(a^5*b + a^4*b^2)*f*cos(f*x + e)^2 + (a^4*b
^2 + a^3*b^3)*f), 1/12*(3*((a^3 + a^2*b)*cos(f*x + e)^4 + a*b^2 + b^3 + 2*(
a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8
*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqr
```

```
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a
*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*((3*a^3 + 4*a^2*b
)*cos(f*x + e)^3 + (2*a^2*b + 3*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^6 + a^5*b)*f*cos(f*x + e)^4 + 2*(a^
5*b + a^4*b^2)*f*cos(f*x + e)^2 + (a^4*b^2 + a^3*b^3)*f)]
```

Sympy [F]

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\tan^2\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(tan(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\tan\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(e+f\,x)^2}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

```
[In] int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.437 $\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 125

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\tan(e+fx)}{3a(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*(5*a+
3*b)*tan(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*b*tan(f*x+e)/a
/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4213, 425, 541, 12, 385, 209}

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f} - \frac{b(5a+3b)\tan(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}} - \frac{b\tan(e+fx)}{3af(a+b)\,(a+b\tan^2(e+fx)+b)^{3/2}}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f) -
 (b*Tan[e + f*x])/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*(5*
a + 3*b)*Tan[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a+b-2bx^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3a(a+b)f}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{3(a+b)^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{3a^2(a+b)^2f}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{a^2f}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^2f}$$

$$= \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 6.48 (sec) , antiderivative size = 1927, normalized size of antiderivative = 15.42

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{}{4\sqrt{2}f\left(a+b\sec^2(e+fx)\right)^{5/2}\left(a+b-a\sin^2(e+fx)\right)^{5/2}\left(3(a+b)\,\text{AppellF1}\left(\right.\right.}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^4*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5
/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2,
5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2,
-1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*
((15*a*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^
2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Sin[e + f*
x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*S
in[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x
]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1/
2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5
)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2,
5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
-2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Appel
lF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e +
 f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt[2]*(a + b - a*Sin[
e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a
*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos[e
 + f*x]^4*Sin[e + f*x]*((5*a*f*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - (4*f*A
ppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[
e + f*x]*Sin[e + f*x])/3))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3
*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
+ b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)
/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e
+ f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/
2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^4*Sin[e + f*x]*
(2*f*(5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
 + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f
*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a + b)*((5*a*f*AppellF1[3/2
, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Si
n[e + f*x])/(3*(a + b)) - (4*f*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3) + Sin[e + f*x]^2*
(5*a*((21*a*f*AppellF1[5/2, -2, 9/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b)) - (12*f*AppellF1[5/2, -1,
 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e +
 f*x])/5) - 4*(a + b)*((3*a*f*AppellF1[5/2, -1, 7/2, 7/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (6*(a + b)^
3*f*Cot[e + f*x]*Csc[e + f*x]^4*(-1 + (a*Sin[e + f*x]^2)/(a + b))^2*((Sqrt[
a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/(Sqrt[a + b]*Sq
```

```
rt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + (a^2*Sin[e + f*x]^4)/(3*(a + b)^2*(-1
 + (a*Sin[e + f*x]^2)/(a + b))^2) + (a*Sin[e + f*x]^2)/((a + b)*(-1 + (a*Si
n[e + f*x]^2)/(a + b)))))/(a^3*(1 - (a*Sin[e + f*x]^2)/(a + b))^(3/2))))))/
(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2,
5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
-2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Appel
lF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e +
 f*x]^2)^2)))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1262 vs. 2(111) = 222.

Time = 6.40 (sec) , antiderivative size = 1263, normalized size of antiderivative = 10.10

method	result	size
default	Expression too large to display	1263

```
[In] int(1/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f/(a+b)^2/a^2/(-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+
e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*c
sc(f*x+e)^2+a+b)*(12*(-a)^(1/2)*a^2*b*(1-cos(f*x+e))^5*csc(f*x+e)^5+18*(-a)
^(1/2)*a*b^2*(1-cos(f*x+e))^5*csc(f*x+e)^5+6*(-a)^(1/2)*b^3*(1-cos(f*x+e))^
5*csc(f*x+e)^5-24*(1-cos(f*x+e))^3*a^2*(-a)^(1/2)*b*csc(f*x+e)^3+4*(-a)^(1/
2)*a*b^2*(1-cos(f*x+e))^3*csc(f*x+e)^3+12*(-a)^(1/2)*b^3*(1-cos(f*x+e))^3*c
sc(f*x+e)^3-3*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e
)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b
)^(3/2)*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^
4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f
*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e
)^2+1))*a^2-6*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e
)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b
)^(3/2)*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^
4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f
*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e
)^2+1))*a*b-3*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e
)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b
)^(3/2)*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^
4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f
*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e
)^2+1))*b^2+12*(-a)^(1/2)*a^2*b*(csc(f*x+e)-cot(f*x+e))+18*(-a)^(1/2)*a*b^2
*(csc(f*x+e)-cot(f*x+e))+6*(-a)^(1/2)*b^3*(csc(f*x+e)-cot(f*x+e)))/((a*(1-c
os(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e)
)^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/((1-cos(f*x+e))^2*c
sc(f*x+e)^2-1)^2)^(5/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(111) = 222$.

Time = 0.73 (sec) , antiderivative size = 881, normalized size of antiderivative = 7.05

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \left[-\frac{3\left(\left(a^4+2\,a^3b+a^2b^2\right)\cos\left(fx+e\right)^4+a^2b^2+2\,ab^3+b^4+2\left(a^3b+2\,a^2b^2\right.\right.}{}\right.$$

$$-\frac{3\left(\left(a^4+2\,a^3b+a^2b^2\right)\cos\left(fx+e\right)^4+a^2b^2+2\,ab^3+b^4+2\left(a^3b+2\,a^2b^2+ab^3\right)\cos\left(fx+e\right)^2\right)\sqrt{a}\arctan\left(\right.}{12\left(\left(a^7+2\,a^6b+a^5b^2\right)\right.}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 2*a*b^3 + b
^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*(3*a^3*b + 2*a^2*b
^2)*cos(f*x + e)^3 + (5*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*
x + e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*
a^4*b^3 + a^3*b^4)*f), -1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 +
 a^2*b^2 + 2*a*b^3 + b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sq
rt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^
2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x +
e)^2)*sin(f*x + e))) + 4*(2*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (5*a^2*b
^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b + 2*a^5*b
^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*f)]
```

Sympy [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral((a + b*sec(e + f*x)**2)**(-5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

```
[In] int(1/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(1/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.438 $\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 174

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\cot(e+fx)}{3a(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{b(7a+3b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-3b)(3a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a^2(a+b)^3f}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*(7*a
+3*b)*cot(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*(a-3*b)*(3*a+
b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a^2/(a+b)^3/f-1/3*b*cot(f*x+e)/a/(
a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4226, 2000, 483, 593, 597, 12, 385, 209}

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f} - \frac{(a-3b)(3a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3a^2f(a+b)^3} - \frac{b(7a+3b)\cot(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}} - \frac{b\cot(e+fx)}{3af(a+b)\,(a+b\tan^2(e+fx)+b)^{3/2}}$$

```
[In] Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f)
) - (b*Cot[e + f*x])/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*
(7*a + 3*b)*Cot[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2]
) - ((a - 3*b)*(3*a + b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*a^
2*(a + b)^3*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+b(1+x^2))^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{b\cot(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a-b-4bx^2}{x^2(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3a(a+b)f}\\
&= -\frac{b\cot(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(7a+3b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}\\
&\quad + \frac{\text{Subst}\left(\int \frac{(a-3b)(3a+b)-2b(7a+3b)x^2}{x^2(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{3a^2(a+b)^2f}
\end{aligned}$$

$$= -\frac{b\cot(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(7a+3b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-3b)(3a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a^2(a+b)^3f} - \frac{\text{Subst}\left(\int \frac{3(a+b)^3}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{3a^2(a+b)^3f}$$

$$= -\frac{b\cot(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(7a+3b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-3b)(3a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a^2(a+b)^3f} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{a^2f}$$

$$= -\frac{b\cot(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(7a+3b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-3b)(3a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a^2(a+b)^3f} - \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^2f}$$

$$= -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\cot(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(7a+3b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-3b)(3a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a^2(a+b)^3f}$$

Mathematica [A] (verified)

Time = 7.77 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.42

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)(a+2b+a\cos(2e+2fx))^{5/2}\sec^5(e+fx)}{4\sqrt{2}a^{5/2}f\left(a+b\sec^2(e+fx)\right)^{5/2}} - \frac{(a+2b+a\cos(2(e+fx)))\left(3(3a^4+8a^3b+5a^2b^2-12ab^3-4b^4)+4(3a^4+6a^3b+8ab^3+3b^4)\cos(2(e+f\right.}{48a^2(a+b)^3f\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/4*(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2*
b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5)/(Sqrt[2]*a^(5/2)*f*(a + b*Sec
[e + f*x]^2)^(5/2)) - ((a + 2*b + a*Cos[2*(e + f*x)])*(3*(3*a^4 + 8*a^3*b +
 5*a^2*b^2 - 12*a*b^3 - 4*b^4) + 4*(3*a^4 + 6*a^3*b + 8*a*b^3 + 3*b^4)*Cos[
2*(e + f*x)] + a*(3*a^3 + 9*a*b^2 + 4*b^3)*Cos[4*(e + f*x)])*Csc[e + f*x]*S
ec[e + f*x]^5)/(48*a^2*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1861 vs. $2(156) = 312$.

Time = 7.59 (sec) , antiderivative size = 1862, normalized size of antiderivative = 10.70

method	result	size
default	Expression too large to display	1862

```
[In] int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/f/(a+b)^3/a^2/(-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+
e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*c
sc(f*x+e)^2+a+b)*(3*(-a)^(1/2)*a^4*(1-cos(f*x+e))^8*csc(f*x+e)^8+6*(-a)^(1/
2)*a^3*b*(1-cos(f*x+e))^8*csc(f*x+e)^8+3*(-a)^(1/2)*a^2*b^2*(1-cos(f*x+e))^
8*csc(f*x+e)^8-12*a^4*(1-cos(f*x+e))^6*(-a)^(1/2)*csc(f*x+e)^6-24*(-a)^(1/2
)*a^2*b^2*(1-cos(f*x+e))^6*csc(f*x+e)^6-48*(-a)^(1/2)*a*b^3*(1-cos(f*x+e))^
6*csc(f*x+e)^6-12*(-a)^(1/2)*b^4*(1-cos(f*x+e))^6*csc(f*x+e)^6+18*(-a)^(1/2
)*a^4*(1-cos(f*x+e))^4*csc(f*x+e)^4-12*(-a)^(1/2)*a^3*b*(1-cos(f*x+e))^4*cs
c(f*x+e)^4+90*(-a)^(1/2)*a^2*b^2*(1-cos(f*x+e))^4*csc(f*x+e)^4-32*(-a)^(1/2
)*a*b^3*(1-cos(f*x+e))^4*csc(f*x+e)^4-24*(-a)^(1/2)*b^4*(1-cos(f*x+e))^4*cs
c(f*x+e)^4+6*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x
+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*
csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(
f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)
^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)
^(3/2)*a^3*(csc(f*x+e)-cot(f*x+e))+18*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*
csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e
)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)
))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1
-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*
x+e))^2*csc(f*x+e)^2+a+b)^(3/2)*a^2*b*(csc(f*x+e)-cot(f*x+e))+18*ln(4*((-a)
^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a
*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)
-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-cos(
f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2
*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(3/2)*a*b^2*(csc(f*x+e
)-cot(f*x+e))+6*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(
f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))
```

```
^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*c
sc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x
+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a
+b)^(3/2)*b^3*(csc(f*x+e)-cot(f*x+e))-12*(-a)^(1/2)*a^4*(1-cos(f*x+e))^2*cs
c(f*x+e)^2-24*(-a)^(1/2)*a^2*b^2*(1-cos(f*x+e))^2*csc(f*x+e)^2-48*(-a)^(1/2
)*a*b^3*(1-cos(f*x+e))^2*csc(f*x+e)^2-12*(-a)^(1/2)*b^4*(1-cos(f*x+e))^2*cs
c(f*x+e)^2+3*(-a)^(1/2)*a^4+6*(-a)^(1/2)*a^3*b+3*(-a)^(1/2)*a^2*b^2)/((a*(1
-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+
e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/((1-cos(f*x+e))^2
*csc(f*x+e)^2-1)^2)^(5/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^5/(1-cos(f*x+e)
)*sin(f*x+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(156) = 312$.

Time = 2.15 (sec) , antiderivative size = 1097, normalized size of antiderivative = 6.30

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 3*a^3*b^2
 + a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(
f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x
 + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b
 + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos
(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 +
2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 -
b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))*sin(f*x + e) + 8*((3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*cos(f*x + e)^5
 + (6*a^4*b - 9*a^3*b^2 + 4*a^2*b^3 + 3*a*b^4)*cos(f*x + e)^3 + (3*a^3*b^2
- 8*a^2*b^3 - 3*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2))/(((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b
+ 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3
+ 3*a^4*b^4 + a^3*b^5)*f)*sin(f*x + e)), 1/12*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a
*b^4 + b^5 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*
b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^
2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f
*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*
x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*s
in(f*x + e) - 4*((3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*cos(f*x + e)^5 + (6*a^4*b
- 9*a^3*b^2 + 4*a^2*b^3 + 3*a*b^4)*cos(f*x + e)^3 + (3*a^3*b^2 - 8*a^2*b^3
- 3*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^
```

```
8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2
+ 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4
+ a^3*b^5)*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(cot(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(fx+e)^2}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(cot(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(fx+e)^2}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(e+f\,x)^2}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

```
[In] int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.439 $\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 236

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(3a+b)\cot^3(e+fx)}{a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a-b)\left(3a^2+14ab+3b^2\right)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a^2(a+b)^4f} - \frac{\left(a^2-10ab-3b^2\right)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a^2(a+b)^3f}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-b*(3*a+b)*c
ot(f*x+e)^3/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/3*(a-b)*(3*a^2+14*a*
b+3*b^2)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a^2/(a+b)^4/f-1/3*(a^2-10*a*
b-3*b^2)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/a^2/(a+b)^3/f-1/3*b*cot(f*
x+e)^3/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used

= {4226, 2000, 483, 593, 597, 12, 385, 209}

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f} - \frac{(a^2-10ab-3b^2)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3a^2f(a+b)^3} + \frac{(a-b)\,(3a^2+14ab+3b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3a^2f(a+b)^4} - \frac{b(3a+b)\cot^3(e+fx)}{a^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}} - \frac{b\cot^3(e+fx)}{3af(a+b)\,(a+b\tan^2(e+fx)+b)^{3/2}}$$

```
[In] Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f) -
 (b*Cot[e + f*x]^3)/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*(
3*a + b)*Cot[e + f*x]^3)/(a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2]) +
 ((a - b)*(3*a^2 + 14*a*b + 3*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]
^2])/(3*a^2*(a + b)^4*f) - ((a^2 - 10*a*b - 3*b^2)*Cot[e + f*x]^3*Sqrt[a +
b + b*Tan[e + f*x]^2])/(3*a^2*(a + b)^3*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
```

```
IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+b(1+x^2))^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3(a-b)-6bx^2}{x^4(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3a(a+b)f}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(3a+b)\cot^3(e+fx)}{a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{3(a^2-10ab-3b^2)-12b(3a+b)x^2}{x^4(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{3a^2(a+b)^2f}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(3a+b)\cot^3(e+fx)}{a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a^2-10ab-3b^2)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a^2(a+b)^3f} - \frac{\text{Subst}\left(\int \frac{3(a-b)(3a^2+14ab+3b^2)+6b(a^2-10ab-3b^2)x^2}{x^2(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{9a^2(a+b)^3f}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(3a+b)\cot^3(e+fx)}{a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a-b)\left(3a^2+14ab+3b^2\right)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a^2(a+b)^4f} - \frac{(a^2-10ab-3b^2)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a^2(a+b)^3f} + \frac{\text{Subst}\left(\int \frac{9(a+b)^4}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{9a^2(a+b)^4f}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(3a+b)\cot^3(e+fx)}{a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a-b)\left(3a^2+14ab+3b^2\right)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a^2(a+b)^4f} - \frac{(a^2-10ab-3b^2)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a^2(a+b)^3f} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{a^2f}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(3a+b)\cot^3(e+fx)}{a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$
$$+ \frac{(a-b)\left(3a^2+14ab+3b^2\right)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a^2(a+b)^4f}$$
$$- \frac{\left(a^2-10ab-3b^2\right)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a^2(a+b)^3f}$$
$$+ \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^2f}$$

$$= \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}$$
$$- \frac{b(3a+b)\cot^3(e+fx)}{a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$
$$+ \frac{(a-b)\left(3a^2+14ab+3b^2\right)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a^2(a+b)^4f}$$
$$- \frac{\left(a^2-10ab-3b^2\right)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a^2(a+b)^3f}$$

Mathematica [A] (verified)

Time = 11.19 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.99

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)(a+2b+a\cos(2e+2fx))^{5/2}\sec^5(e+fx)}{4\sqrt{2}a^{5/2}f\left(a+b\sec^2(e+fx)\right)^{5/2}}$$
$$- \frac{(a+2b+a\cos(2(e+fx)))^3\sec^5(e+fx)\left(-4(a+3b)\csc(e+fx)+(a+b)\csc^3(e+fx)+\frac{4b^3(6a^2+13ab+3b^2}{a^2(a}\right.}{24(a+b)^4f\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2*b + a
*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5)/(4*Sqrt[2]*a^(5/2)*f*(a + b*Sec[e
+ f*x]^2)^(5/2)) - ((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^5*(-4*(a
+ 3*b)*Csc[e + f*x] + (a + b)*Csc[e + f*x]^3 + (4*b^3*(6*a^2 + 13*a*b + 3*b
^2 + 2*a*(3*a + b)*Cos[2*(e + f*x)])*Sin[e + f*x])/(a^2*(a + 2*b + a*Cos[2*
(e + f*x)])^2)))/(24*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2611 vs. 2(216) = 432.

Time = 9.65 (sec) , antiderivative size = 2612, normalized size of antiderivative = 11.07

method	result	size
default	Expression too large to display	2612

```
[In] int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/f/(a+b)^4/a^2/(-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x
+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*
csc(f*x+e)^2+a+b)*(-18*(-a)^(1/2)*a^5*(1-cos(f*x+e))^2*csc(f*x+e)^2+48*(-a)
^(1/2)*b^5*(1-cos(f*x+e))^8*csc(f*x+e)^8+96*(-a)^(1/2)*b^5*(1-cos(f*x+e))^6
*csc(f*x+e)^6+48*(-a)^(1/2)*b^5*(1-cos(f*x+e))^4*csc(f*x+e)^4+(-a)^(1/2)*a^
5*(1-cos(f*x+e))^12*csc(f*x+e)^12-18*(-a)^(1/2)*a^5*(1-cos(f*x+e))^10*csc(f
*x+e)^10+63*(-a)^(1/2)*a^5*(1-cos(f*x+e))^8*csc(f*x+e)^8-92*a^5*(1-cos(f*x+
e))^6*(-a)^(1/2)*csc(f*x+e)^6+63*(-a)^(1/2)*a^5*(1-cos(f*x+e))^4*csc(f*x+e)
^4+(-a)^(1/2)*a^5+240*(-a)^(1/2)*a*b^4*(1-cos(f*x+e))^8*csc(f*x+e)^8+240*(-
a)^(1/2)*a*b^4*(1-cos(f*x+e))^4*csc(f*x+e)^4-42*(-a)^(1/2)*a^2*b^3*(1-cos(f
*x+e))^2*csc(f*x+e)^2+108*(-a)^(1/2)*a^3*b^2*(1-cos(f*x+e))^6*csc(f*x+e)^6-
652*(-a)^(1/2)*a^2*b^3*(1-cos(f*x+e))^6*csc(f*x+e)^6+224*(-a)^(1/2)*a*b^4*(
1-cos(f*x+e))^6*csc(f*x+e)^6-24*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*
x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*
b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1
-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f
*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^
2*csc(f*x+e)^2+a+b)^(3/2)*a^4*(1-cos(f*x+e))^3*csc(f*x+e)^3-24*ln(4*((-a)^(
1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(
1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2
*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-cos(f*
x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*c
sc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(3/2)*b^4*(1-cos(f*x+e))
^3*csc(f*x+e)^3+189*(-a)^(1/2)*a^4*b*(1-cos(f*x+e))^4*csc(f*x+e)^4-51*(-a)^
(1/2)*a^3*b^2*(1-cos(f*x+e))^4*csc(f*x+e)^4+15*(-a)^(1/2)*a^2*b^3*(1-cos(f*
x+e))^4*csc(f*x+e)^4-78*(-a)^(1/2)*a^4*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+3*(-
a)^(1/2)*a^4*b*(1-cos(f*x+e))^12*csc(f*x+e)^12+3*(-a)^(1/2)*a^3*b^2*(1-cos(
f*x+e))^12*csc(f*x+e)^12+(-a)^(1/2)*a^2*b^3*(1-cos(f*x+e))^12*csc(f*x+e)^12
-78*(-a)^(1/2)*a^4*b*(1-cos(f*x+e))^10*csc(f*x+e)^10-102*(-a)^(1/2)*a^3*b^2
*(1-cos(f*x+e))^10*csc(f*x+e)^10-42*(-a)^(1/2)*a^2*b^3*(1-cos(f*x+e))^10*cs
c(f*x+e)^10+189*(-a)^(1/2)*a^4*b*(1-cos(f*x+e))^8*csc(f*x+e)^8-51*(-a)^(1/2
)*a^3*b^2*(1-cos(f*x+e))^8*csc(f*x+e)^8+15*(-a)^(1/2)*a^2*b^3*(1-cos(f*x+e)
)^8*csc(f*x+e)^8-228*a^4*(1-cos(f*x+e))^6*(-a)^(1/2)*b*csc(f*x+e)^6-102*(-a
)^(1/2)*a^3*b^2*(1-cos(f*x+e))^2*csc(f*x+e)^2+3*(-a)^(1/2)*a^4*b+3*(-a)^(1/
2)*a^3*b^2+(-a)^(1/2)*a^2*b^3-96*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f
```

```
*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2
*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((
1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(
f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))
^2*csc(f*x+e)^2+a+b)^(3/2)*a^3*b*(1-cos(f*x+e))^3*csc(f*x+e)^3-144*ln(4*((-
a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2
*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/
2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-co
s(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))
^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(3/2)*a^2*b^2*(1-cos
(f*x+e))^3*csc(f*x+e)^3-96*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^
4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-
cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(
f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e)
)^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc
(f*x+e)^2+a+b)^(3/2)*a*b^3*(1-cos(f*x+e))^3*csc(f*x+e)^3)/((a*(1-cos(f*x+e)
)^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f
*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)
^2-1)^2)^(5/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^5/(1-cos(f*x+e))^3*sin(f*x
+e)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(216) = 432$.

Time = 7.36 (sec) , antiderivative size = 1579, normalized size of antiderivative = 6.69

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^6
 - a^4*b^2 - 4*a^3*b^3 - 6*a^2*b^4 - 4*a*b^5 - b^6 - (a^6 + 2*a^5*b - 2*a^4
*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b
^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*cos(f*x + e)^2)*sqrt(-a)*log(12
8*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos
(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b
^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) - 8
*(4*(a^6 + 3*a^5*b + 3*a^3*b^3 + a^2*b^4)*cos(f*x + e)^7 - 3*(a^6 + a^5*b -
 8*a^4*b^2 + 8*a^3*b^3 - a^2*b^4 - a*b^5)*cos(f*x + e)^5 - 6*(a^5*b + 3*a^4
*b^2 - 4*a^3*b^3 + 3*a^2*b^4 + a*b^5)*cos(f*x + e)^3 - (3*a^4*b^2 + 11*a^3*
```

```
b^3 - 11*a^2*b^4 - 3*a*b^5)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2))/(((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*f*cos(f*x +
 e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2 - 8*a^6*b^3 - 7*a^5*b^4 - 2*a^4*b^5)*f*c
os(f*x + e)^4 - (2*a^8*b + 7*a^7*b^2 + 8*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 -
a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 +
a^3*b^6)*f)*sin(f*x + e)), -1/12*(3*((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3
 + a^2*b^4)*cos(f*x + e)^6 - a^4*b^2 - 4*a^3*b^3 - 6*a^2*b^4 - 4*a*b^5 - b^
6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*cos(f*x +
 e)^4 - (2*a^5*b + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*cos(f
*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x
 + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*
b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e) - 4*(4*(a^6 + 3*a^5*b + 3*a^
3*b^3 + a^2*b^4)*cos(f*x + e)^7 - 3*(a^6 + a^5*b - 8*a^4*b^2 + 8*a^3*b^3 -
a^2*b^4 - a*b^5)*cos(f*x + e)^5 - 6*(a^5*b + 3*a^4*b^2 - 4*a^3*b^3 + 3*a^2*
b^4 + a*b^5)*cos(f*x + e)^3 - (3*a^4*b^2 + 11*a^3*b^3 - 11*a^2*b^4 - 3*a*b^
5)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^9 + 4*a^
8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b -
2*a^7*b^2 - 8*a^6*b^3 - 7*a^5*b^4 - 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b
+ 7*a^7*b^2 + 8*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2
 - (a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3*b^6)*f)*sin(f*x + e))
]
```

Sympy [F]

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(cot(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(fx+e)^4}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Hanged}$$

```
[In] int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] \text{Hanged}
```

3.440 $\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 315

$$\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(11a+3b)\cot^5(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(15a^4+70a^3b+128a^2b^2-70ab^3-15b^4)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a^2(a+b)^5f} + \frac{(5a^3+19a^2b-65ab^2-15b^3)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a^2(a+b)^4f} - \frac{(a^2-20ab-5b^2)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5a^2(a+b)^3f}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*(11*
a+3*b)*cot(f*x+e)^5/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/15*(15*a^4+7
0*a^3*b+128*a^2*b^2-70*a*b^3-15*b^4)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/
a^2/(a+b)^5/f+1/15*(5*a^3+19*a^2*b-65*a*b^2-15*b^3)*cot(f*x+e)^3*(a+b+b*tan
(f*x+e)^2)^(1/2)/a^2/(a+b)^4/f-1/5*(a^2-20*a*b-5*b^2)*cot(f*x+e)^5*(a+b+b*t
an(f*x+e)^2)^(1/2)/a^2/(a+b)^3/f-1/3*b*cot(f*x+e)^5/a/(a+b)/f/(a+b+b*tan(f*
x+e)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4226, 2000, 483, 593, 597, 12, 385, 209}

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f} - \frac{(a^2-20ab-5b^2)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5a^2f(a+b)^3} - \frac{b(11a+3b)\cot^5(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}} + \frac{(5a^3+19a^2b-65ab^2-15b^3)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15a^2f(a+b)^4} - \frac{(15a^4+70a^3b+128a^2b^2-70ab^3-15b^4)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15a^2f(a+b)^5} - \frac{b\cot^5(e+fx)}{3af(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[In] Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f)
) - (b*Cot[e + f*x]^5)/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (
b*(11*a + 3*b)*Cot[e + f*x]^5)/(3*a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*
x]^2]) - ((15*a^4 + 70*a^3*b + 128*a^2*b^2 - 70*a*b^3 - 15*b^4)*Cot[e + f*x
]*Sqrt[a + b + b*Tan[e + f*x]^2])/(15*a^2*(a + b)^5*f) + ((5*a^3 + 19*a^2*b
 - 65*a*b^2 - 15*b^3)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(15*a^
2*(a + b)^4*f) - ((a^2 - 20*a*b - 5*b^2)*Cot[e + f*x]^5*Sqrt[a + b + b*Tan[
e + f*x]^2])/(5*a^2*(a + b)^3*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
```

erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+b(1+x^2))^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a-5b-8bx^2}{x^6(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3a(a+b)f}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{b(11a+3b)\cot^5(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{3(a^2-20ab-5b^2)-6b(11a+3b)x^2}{x^6(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{3a^2(a+b)^2f}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{b(11a+3b)\cot^5(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a^2-20ab-5b^2)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5a^2(a+b)^3f} - \frac{\text{Subst}\left(\int \frac{3(5a^3+19a^2b-65ab^2-15b^3)+12b(a^2-20ab-5b^2)x^2}{x^4(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{15a^2(a+b)^3f}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{b(11a+3b)\cot^5(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5a^3+19a^2b-65ab^2-15b^3)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a^2(a+b)^4f} - \frac{(a^2-20ab-5b^2)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5a^2(a+b)^3f} + \frac{\text{Subst}\left(\int \frac{3(15a^4+70a^3b+128a^2b^2-70ab^3-15b^4)+6b(5a^3+19a^2b-65ab^2-15b^3)x^2}{x^2(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{45a^2(a+b)^4f}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(11a+3b)\cot^5(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$
$$-\frac{(15a^4+70a^3b+128a^2b^2-70ab^3-15b^4)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a^2(a+b)^5f}$$
$$+\frac{(5a^3+19a^2b-65ab^2-15b^3)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a^2(a+b)^4f}$$
$$-\frac{(a^2-20ab-5b^2)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5a^2(a+b)^3f}$$
$$-\frac{\text{Subst}\left(\int\frac{45(a+b)^5}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{45a^2(a+b)^5f}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(11a+3b)\cot^5(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$
$$-\frac{(15a^4+70a^3b+128a^2b^2-70ab^3-15b^4)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a^2(a+b)^5f}$$
$$+\frac{(5a^3+19a^2b-65ab^2-15b^3)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a^2(a+b)^4f}$$
$$-\frac{(a^2-20ab-5b^2)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5a^2(a+b)^3f}$$
$$-\frac{\text{Subst}\left(\int\frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{a^2f}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(11a+3b)\cot^5(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$
$$-\frac{(15a^4+70a^3b+128a^2b^2-70ab^3-15b^4)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a^2(a+b)^5f}$$
$$+\frac{(5a^3+19a^2b-65ab^2-15b^3)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a^2(a+b)^4f}$$
$$-\frac{(a^2-20ab-5b^2)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5a^2(a+b)^3f}$$
$$-\frac{\text{Subst}\left(\int\frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^2f}$$

$$= -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}$$
$$- \frac{b(11a+3b)\cot^5(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$
$$- \frac{(15a^4+70a^3b+128a^2b^2-70ab^3-15b^4)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a^2(a+b)^5f}$$
$$+ \frac{(5a^3+19a^2b-65ab^2-15b^3)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a^2(a+b)^4f}$$
$$- \frac{(a^2-20ab-5b^2)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5a^2(a+b)^3f}$$

Mathematica [A] (verified)

Time = 16.99 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.86

$$\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx =$$
$$-\frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)(a+2b+a\cos(2e+2fx))^{5/2}\sec^5(e+fx)}{4\sqrt{2}a^{5/2}f\left(a+b\sec^2(e+fx)\right)^{5/2}}$$
$$+\frac{(a+2b+a\cos(2(e+fx)))^3\left(-\frac{20b^5(a+b)}{a^2(a+2b+a\cos(2(e+fx)))^2}+\frac{10b^4(15a+4b)}{a^2(a+2b+a\cos(2(e+fx)))}-(23a^2+100ab+150b^2)\csc^2(\right.}{120(a+b)^5f\left(a+b\sec^2(\right.}$$

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/4*(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2*
b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5)/(Sqrt[2]*a^(5/2)*f*(a + b*Sec
[e + f*x]^2)^(5/2)) + ((a + 2*b + a*Cos[2*(e + f*x)])^3*((-20*b^5*(a + b))/
(a^2*(a + 2*b + a*Cos[2*(e + f*x)])^2) + (10*b^4*(15*a + 4*b))/(a^2*(a + 2*
b + a*Cos[2*(e + f*x)])) - (23*a^2 + 100*a*b + 150*b^2)*Csc[e + f*x]^2 + (a
 + b)*(11*a + 25*b)*Csc[e + f*x]^4 - 3*(a + b)^2*Csc[e + f*x]^6)*Sec[e + f*
x]^4*Tan[e + f*x])/(120*(a + b)^5*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3417 vs. 2(289) = 578.

Time = 11.21 (sec) , antiderivative size = 3418, normalized size of antiderivative = 10.85

method	result	size
default	Expression too large to display	3418

```
[In] int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/480/f/(a+b)^5/a^2/(-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*
x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2
*csc(f*x+e)^2+a+b)*(-960*(-a)^(1/2)*b^6*(1-cos(f*x+e))^10*csc(f*x+e)^10-192
0*(-a)^(1/2)*b^6*(1-cos(f*x+e))^8*csc(f*x+e)^8-960*(-a)^(1/2)*b^6*(1-cos(f*
x+e))^6*csc(f*x+e)^6+3*(-a)^(1/2)*a^6*(1-cos(f*x+e))^16*csc(f*x+e)^16-44*(-
a)^(1/2)*a^6*(1-cos(f*x+e))^14*csc(f*x+e)^14+444*(-a)^(1/2)*a^6*(1-cos(f*x+
e))^12*csc(f*x+e)^12-1428*(-a)^(1/2)*a^6*(1-cos(f*x+e))^10*csc(f*x+e)^10+20
50*a^6*(1-cos(f*x+e))^8*(-a)^(1/2)*csc(f*x+e)^8-1428*(-a)^(1/2)*a^6*(1-cos(
f*x+e))^6*csc(f*x+e)^6+444*(-a)^(1/2)*a^6*(1-cos(f*x+e))^4*csc(f*x+e)^4-44*
(-a)^(1/2)*a^6*(1-cos(f*x+e))^2*csc(f*x+e)^2+3*(-a)^(1/2)*a^6+12*(-a)^(1/2)
*a^5*b+18*(-a)^(1/2)*a^4*b^2+12*(-a)^(1/2)*a^3*b^3+2400*ln(4*((-a)^(1/2)*(a
*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f
*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc
(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4
*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+
e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(3/2)*a^4*b*(1-cos(f*x+e))^5*cs
c(f*x+e)^5+4800*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(
f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))
^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*c
sc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x
+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a
+b)^(3/2)*a^3*b^2*(1-cos(f*x+e))^5*csc(f*x+e)^5+4800*ln(4*((-a)^(1/2)*(a*(1
-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+
e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*
x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*cs
c(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^
2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(3/2)*a^2*b^3*(1-cos(f*x+e))^5*csc
(f*x+e)^5+2400*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f
*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^
2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*cs
c(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+
e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+
b)^(3/2)*a*b^4*(1-cos(f*x+e))^5*csc(f*x+e)^5+3*(-a)^(1/2)*a^2*b^4-76*(-a)^(
1/2)*a^2*b^4*(1-cos(f*x+e))^2*csc(f*x+e)^2+12*(-a)^(1/2)*a^5*b*(1-cos(f*x+e
))^16*csc(f*x+e)^16+18*(-a)^(1/2)*a^4*b^2*(1-cos(f*x+e))^16*csc(f*x+e)^16+1
```

2*(-a)^(1/2)*a^3*b^3*(1-cos(f*x+e))^16*csc(f*x+e)^16+3*(-a)^(1/2)*a^2*b^4*(
1-cos(f*x+e))^16*csc(f*x+e)^16-208*(-a)^(1/2)*a^5*b*(1-cos(f*x+e))^14*csc(f
x+e)^14-360(-a)^(1/2)*a^4*b^2*(1-cos(f*x+e))^14*csc(f*x+e)^14-272*(-a)^(1
/2)*a^3*b^3*(1-cos(f*x+e))^14*csc(f*x+e)^14-76*(-a)^(1/2)*a^2*b^4*(1-cos(f*
x+e))^14*csc(f*x+e)^14+2448*(-a)^(1/2)*a^5*b*(1-cos(f*x+e))^12*csc(f*x+e)^1
2+5448*(-a)^(1/2)*a^4*b^2*(1-cos(f*x+e))^12*csc(f*x+e)^12+5328*(-a)^(1/2)*a
^3*b^3*(1-cos(f*x+e))^12*csc(f*x+e)^12+1884*(-a)^(1/2)*a^2*b^4*(1-cos(f*x+e
))^12*csc(f*x+e)^12-6192*(-a)^(1/2)*a^5*b*(1-cos(f*x+e))^10*csc(f*x+e)^10-5
760*(-a)^(1/2)*a*b^5*(1-cos(f*x+e))^10*csc(f*x+e)^10-5760*(-a)^(1/2)*a*b^5*
(1-cos(f*x+e))^6*csc(f*x+e)^6-8280*(-a)^(1/2)*a^4*b^2*(1-cos(f*x+e))^10*csc
(f*x+e)^10+4752*(-a)^(1/2)*a^3*b^3*(1-cos(f*x+e))^10*csc(f*x+e)^10+3468*(-a
)^(1/2)*a^2*b^4*(1-cos(f*x+e))^10*csc(f*x+e)^10+7880*a^5*(1-cos(f*x+e))^8*(
-a)^(1/2)*b*csc(f*x+e)^8+10188*(-a)^(1/2)*a^4*b^2*(1-cos(f*x+e))^8*csc(f*x+
e)^8-1720*(-a)^(1/2)*a^3*b^3*(1-cos(f*x+e))^8*csc(f*x+e)^8+22210*(-a)^(1/2)
*a^2*b^4*(1-cos(f*x+e))^8*csc(f*x+e)^8-6400*(-a)^(1/2)*a*b^5*(1-cos(f*x+e))
^8*csc(f*x+e)^8+480*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-
cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x
+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))
^2*csc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc
(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)
^2+a+b)^(3/2)*a^5*(1-cos(f*x+e))^5*csc(f*x+e)^5+480*ln(4*((-a)^(1/2)*(a*(1-
cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e
))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x
+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*csc
(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2
+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(3/2)*b^5*(1-cos(f*x+e))^5*csc(f*x+
e)^5-6192*(-a)^(1/2)*a^5*b*(1-cos(f*x+e))^6*csc(f*x+e)^6-8280*(-a)^(1/2)*a^
4*b^2*(1-cos(f*x+e))^6*csc(f*x+e)^6+4752*(-a)^(1/2)*a^3*b^3*(1-cos(f*x+e))^
6*csc(f*x+e)^6+3468*(-a)^(1/2)*a^2*b^4*(1-cos(f*x+e))^6*csc(f*x+e)^6+2448*(
-a)^(1/2)*a^5*b*(1-cos(f*x+e))^4*csc(f*x+e)^4+5448*(-a)^(1/2)*a^4*b^2*(1-co
s(f*x+e))^4*csc(f*x+e)^4+5328*(-a)^(1/2)*a^3*b^3*(1-cos(f*x+e))^4*csc(f*x+e
)^4+1884*(-a)^(1/2)*a^2*b^4*(1-cos(f*x+e))^4*csc(f*x+e)^4-208*(-a)^(1/2)*a^
5*b*(1-cos(f*x+e))^2*csc(f*x+e)^2-360*(-a)^(1/2)*a^4*b^2*(1-cos(f*x+e))^2*c
sc(f*x+e)^2-272*(-a)^(1/2)*a^3*b^3*(1-cos(f*x+e))^2*csc(f*x+e)^2)/((a*(1-co
s(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))
^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/((1-cos(f*x+e))^2*cs
c(f*x+e)^2-1)^2)^(5/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^5/(1-cos(f*x+e))^5
*sin(f*x+e)^5

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(289) = 578$.

Time = 24.64 (sec) , antiderivative size = 2059, normalized size of antiderivative = 6.54

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
[Out] [-1/120*(15*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5
)*cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6
+ b^7 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*cos(f
*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5
+ a*b^6 + b^7)*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^
5 - 4*a*b^6 - b^7)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 25
6*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x
+ e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b +
 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a
^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3
 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) + 8*((23*a^7 + 100*a^6*b + 1
50*a^5*b^2 + 75*a^3*b^4 + 20*a^2*b^5)*cos(f*x + e)^9 - (35*a^7 + 118*a^6*b
+ 75*a^5*b^2 - 300*a^4*b^3 + 225*a^3*b^4 - 10*a^2*b^5 - 15*a*b^6)*cos(f*x +
 e)^7 + 3*(5*a^7 - 59*a^5*b^2 - 150*a^4*b^3 + 125*a^3*b^4 - 50*a^2*b^5 - 15
*a*b^6)*cos(f*x + e)^5 + (30*a^6*b + 105*a^5*b^2 + 92*a^4*b^3 - 350*a^3*b^4
 + 190*a^2*b^5 + 45*a*b^6)*cos(f*x + e)^3 + (15*a^5*b^2 + 70*a^4*b^3 + 128*
a^3*b^4 - 70*a^2*b^5 - 15*a*b^6)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2))/(((a^10 + 5*a^9*b + 10*a^8*b^2 + 10*a^7*b^3 + 5*a^6*b^4 +
a^5*b^5)*f*cos(f*x + e)^8 - 2*(a^10 + 4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a
^5*b^5 - a^4*b^6)*f*cos(f*x + e)^6 + (a^10 + a^9*b - 9*a^8*b^2 - 25*a^7*b^3
 - 25*a^6*b^4 - 9*a^5*b^5 + a^4*b^6 + a^3*b^7)*f*cos(f*x + e)^4 + 2*(a^9*b
+ 4*a^8*b^2 + 5*a^7*b^3 - 5*a^5*b^5 - 4*a^4*b^6 - a^3*b^7)*f*cos(f*x + e)^2
 + (a^8*b^2 + 5*a^7*b^3 + 10*a^6*b^4 + 10*a^5*b^5 + 5*a^4*b^6 + a^3*b^7)*f)
*sin(f*x + e)), 1/60*(15*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*
b^4 + a^2*b^5)*cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b
^5 + 5*a*b^6 + b^7 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 -
 a*b^6)*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4
 - 9*a^2*b^5 + a*b^6 + b^7)*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b
^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*c
os(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x
+ e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x +
 e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(
f*x + e) - 4*((23*a^7 + 100*a^6*b + 150*a^5*b^2 + 75*a^3*b^4 + 20*a^2*b^5)*
cos(f*x + e)^9 - (35*a^7 + 118*a^6*b + 75*a^5*b^2 - 300*a^4*b^3 + 225*a^3*b
```

```
^4 - 10*a^2*b^5 - 15*a*b^6)*cos(f*x + e)^7 + 3*(5*a^7 - 59*a^5*b^2 - 150*a^
4*b^3 + 125*a^3*b^4 - 50*a^2*b^5 - 15*a*b^6)*cos(f*x + e)^5 + (30*a^6*b + 1
05*a^5*b^2 + 92*a^4*b^3 - 350*a^3*b^4 + 190*a^2*b^5 + 45*a*b^6)*cos(f*x + e
)^3 + (15*a^5*b^2 + 70*a^4*b^3 + 128*a^3*b^4 - 70*a^2*b^5 - 15*a*b^6)*cos(f
*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^10 + 5*a^9*b + 1
0*a^8*b^2 + 10*a^7*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8 - 2*(a^10 +
4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6)*f*cos(f*x + e)^6 + (
a^10 + a^9*b - 9*a^8*b^2 - 25*a^7*b^3 - 25*a^6*b^4 - 9*a^5*b^5 + a^4*b^6 +
a^3*b^7)*f*cos(f*x + e)^4 + 2*(a^9*b + 4*a^8*b^2 + 5*a^7*b^3 - 5*a^5*b^5 -
4*a^4*b^6 - a^3*b^7)*f*cos(f*x + e)^2 + (a^8*b^2 + 5*a^7*b^3 + 10*a^6*b^4 +
 10*a^5*b^5 + 5*a^4*b^6 + a^3*b^7)*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(cot(e + f*x)**6/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(fx+e)^6}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Hanged}$$

```
[In] int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] \text{Hanged}
```

3.441 $\int (a+b\sec^2(e+fx))^p (d\tan(e+fx))^m\,dx$

Optimal result

Integrand size = 25, antiderivative size = 105

$$\int (a+b\sec^2(e+fx))^p (d\tan(e+fx))^m\,dx$$
$$= \frac{\text{AppellF1}\left(\frac{1+m}{2},1,-p,\frac{3+m}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)(d\tan(e+fx))^{1+m}\left(a+b+b\tan^2(e+fx)\right)^p \left(1\right.}{df(1+m)}$$

```
[Out] AppellF1(1/2+1/2*m,1,-p,3/2+1/2*m,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*(d*t
an(f*x+e))^(1+m)*(a+b+b*tan(f*x+e)^2)^p/d/f/(1+m)/((1+b*tan(f*x+e)^2/(a+b))
^p)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4226, 2000, 525, 524}

$$\int (a+b\sec^2(e+fx))^p (d\tan(e+fx))^m\,dx$$
$$= \frac{(d\tan(e+fx))^{m+1}\left(a+b\tan^2(e+fx)+b\right)^p \left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\left(\frac{m+1}{2},1,-p,\frac{m+3}{2},-\tan^2(e+f\right.}{df(m+1)}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*(d*Tan[e + f*x])^m,x]
```

```
[Out] (AppellF1[(1 + m)/2, 1, -p, (3 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/(a + b))]*(d*Tan[e + f*x])^(1 + m)*(a + b + b*Tan[e + f*x]^2)^p)/(d*f*(1
+ m)*(1 + (b*Tan[e + f*x]^2)/(a + b))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(dx)^m\left(a+b\left(1+x^2\right)\right)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{(dx)^m\left(a+b+bx^2\right)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{(dx)^m\left(1+\frac{bx^2}{a+b}\right)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{AppellF1}\left(\frac{1+m}{2}, 1, -p, \frac{3+m}{2}, -\tan^2(e+fx), -\frac{b\tan^2(e+fx)}{a+b}\right)(d\tan(e+fx))^{1+m}\left(a+b+b\tan^2(e+\right.}{df(1+m)}
\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 259 vs. $2(105) = 210$.

Time = 5.25 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.47

$$\int \left(a+b\sec^2(e+fx)\right)^p (d\tan(e+fx))^m\,dx$$
$$= \frac{\text{AppellF1}\left(\frac{1+m}{2},-p,1,\frac{3+m}{2},-\frac{b\tan^2(e+fx)}{a+b},-\tan^2(e+fx)\right)\cos(e+fx)\,(a+b}{f(1+m)\left(\text{AppellF1}\left(\frac{1+m}{2},-p,1,\frac{3+m}{2},-\frac{b\tan^2(e+fx)}{a+b},-\tan^2(e+fx)\right)+\frac{2\left(bp\,\text{AppellF1}\left(\frac{3+m}{2},1-p,1,\frac{5+m}{2},-\frac{b\tan^2(e+f}{a+b}\right.\right.}{}\right.}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*(d*Tan[e + f*x])^m,x]
```

```
[Out] (AppellF1[(1 + m)/2, -p, 1, (3 + m)/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[
e + f*x]^2]*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]*(d*Tan[e + f
*x])^m)/(f*(1 + m)*(AppellF1[(1 + m)/2, -p, 1, (3 + m)/2, -((b*Tan[e + f*x]
^2)/(a + b)), -Tan[e + f*x]^2] + (2*(b*p*AppellF1[(3 + m)/2, 1 - p, 1, (5 +
 m)/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[(
3 + m)/2, -p, 2, (5 + m)/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]
)*Tan[e + f*x]^2)/((a + b)*(3 + m))))
```

Maple [F]

$$\int \left(a+b\sec\left(fx+e\right)^2\right)^p \left(d\tan\left(fx+e\right)\right)^m dx$$

```
[In] int((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x)
```

Fricas [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p (d\tan(e+fx))^m\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \left(d\tan\left(fx+e\right)\right)^m\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^p (d\tan(e+fx))^m\,dx=\text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e)**2)**p*(d*tan(f*x+e))**m,x)
[Out] Timed out
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p (d\tan(e+fx))^m\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p (d\tan\left(fx+e\right))^m\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p (d\tan(e+fx))^m\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p (d\tan\left(fx+e\right))^m\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^p (d\tan(e+fx))^m\,dx=\int (d\tan(e+f\,x))^m\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

```
[In] int((d*tan(e + f*x))^m*(a + b/cos(e + f*x)^2)^p,x)
[Out] int((d*tan(e + f*x))^m*(a + b/cos(e + f*x)^2)^p, x)
```

3.442 $\int (a+b\sec^2(e+fx))^p \tan^5(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 122

$$\begin{aligned}\int (a+b\sec^2(e+fx))^p \tan^5(e+fx)\,dx &= -\frac{(a+2b)\,(a+b\sec^2(e+fx))^{1+p}}{2b^2f(1+p)} \\ &\quad - \frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^{1+p}}{2af(1+p)} \\ &\quad + \frac{(a+b\sec^2(e+fx))^{2+p}}{2b^2f(2+p)}\end{aligned}$$

```
[Out] -1/2*(a+2*b)*(a+b*sec(f*x+e)^2)^(p+1)/b^2/f/(p+1)-1/2*hypergeom([1, p+1],[2
+p],1+b*sec(f*x+e)^2/a)*(a+b*sec(f*x+e)^2)^(p+1)/a/f/(p+1)+1/2*(a+b*sec(f*x
+e)^2)^(2+p)/b^2/f/(2+p)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4224, 457, 90, 67}

$$\begin{aligned}\int (a+b\sec^2(e+fx))^p \tan^5(e+fx)\,dx &= -\frac{(a+2b)\,(a+b\sec^2(e+fx))^{p+1}}{2b^2f(p+1)} + \frac{(a+b\sec^2(e+fx))^{p+2}}{2b^2f(p+2)} \\ &\quad - \frac{(a+b\sec^2(e+fx))^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)}{a}+1\right)}{2af(p+1)}\end{aligned}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^5,x]
```

```
[Out] -1/2*((a + 2*b)*(a + b*Sec[e + f*x]^2)^(1 + p))/(b^2*f*(1 + p)) - (Hypergeo
metric2F1[1, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e + f*x]^2)
^(1 + p))/(2*a*f*(1 + p)) + (a + b*Sec[e + f*x]^2)^(2 + p)/(2*b^2*f*(2 + p)
)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)^2(a+bx^2)^p}{x}\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{(-1+x)^2(a+bx)^p}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= \frac{\text{Subst}\left(\int \left(\frac{(-a-2b)(a+bx)^p}{b}+\frac{(a+bx)^p}{x}+\frac{(a+bx)^{1+p}}{b}\right)dx, x, \sec^2(e+fx)\right)}{2f}
\end{aligned}
$$

$$= -\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2b^2f(1+p)} + \frac{\left(a+b\sec^2(e+fx)\right)^{2+p}}{2b^2f(2+p)} + \frac{\operatorname{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2b^2f(1+p)} - \frac{\text{Hypergeometric2F1}\left(1, 1+p, 2+p, 1+\frac{b\sec^2(e+fx)}{a}\right)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2af(1+p)} + \frac{\left(a+b\sec^2(e+fx)\right)^{2+p}}{2b^2f(2+p)}$$

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.77

$$\int \left(a+b\sec^2(e+fx)\right)^p \tan^5(e+fx)\,dx = -\frac{\left(a+b\sec^2(e+fx)\right)^{1+p}\left(b^2(2+p)\,\text{Hypergeometric2F1}\left(1, 1+p, 2+p, 1+\frac{b\sec^2(e+fx)}{a}\right) + a(a+2b(2+p\right.}{2ab^2f(1+p)(2+p)}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^5,x]
```

```
[Out] -1/2*((a + b*Sec[e + f*x]^2)^(1 + p)*(b^2*(2 + p)*Hypergeometric2F1[1, 1 +
p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a] + a*(a + 2*b*(2 + p) - b*(1 + p)*Sec[e
+ f*x]^2)))/(a*b^2*f*(1 + p)*(2 + p))
```

Maple [F]

$$\int \left(a+b\sec(fx+e)^2\right)^p \tan(fx+e)^5\,dx$$

```
[In] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x)
```

Fricas [F]

$$\int (a+b\sec^2(e+fx))^p\tan^5(e+fx)\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^5\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p\tan^5(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e)**5,x)
[Out] Timed out
```

Maxima [F]

$$\int (a+b\sec^2(e+fx))^p\tan^5(e+fx)\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^5\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

Giac [F]

$$\int (a+b\sec^2(e+fx))^p\tan^5(e+fx)\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^5\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p\tan^5(e+fx)\,dx=\int \tan(e+f\,x)^5\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

```
[In] int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^p, x)
```

3.443 $\int (a+b\sec^2(e+fx))^p \tan^3(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int (a+b\sec^2(e+fx))^p \tan^3(e+fx)\,dx = \frac{(a+b\sec^2(e+fx))^{1+p}}{2bf(1+p)} + \frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^{1+p}}{2af(1+p)}$$

```
[Out] 1/2*(a+b*sec(f*x+e)^2)^(p+1)/b/f/(p+1)+1/2*hypergeom([1, p+1],[2+p],1+b*sec
(f*x+e)^2/a)*(a+b*sec(f*x+e)^2)^(p+1)/a/f/(p+1)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4224, 457, 81, 67}

$$\int (a+b\sec^2(e+fx))^p \tan^3(e+fx)\,dx = \frac{(a+b\sec^2(e+fx))^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)}{a}+1\right)}{2af(p+1)} + \frac{(a+b\sec^2(e+fx))^{p+1}}{2bf(p+1)}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^3,x]
```

```
[Out] (a + b*Sec[e + f*x]^2)^(1 + p)/(2*b*f*(1 + p)) + (Hypergeometric2F1[1, 1 +
p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e + f*x]^2)^(1 + p))/(2*a*f*
(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a+bx^2)^p}{x}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(-1+x)(a+bx)^p}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{(a+b\sec^2(e+fx))^{1+p}}{2bf(1+p)} - \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{(a+b\sec^2(e+fx))^{1+p}}{2bf(1+p)} + \frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^{1+p}}{2af(1+p)}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.71

$$\int (a+b\sec^2(e+fx))^p \tan^3(e+fx)\,dx = \frac{\left(a+b\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sec^2(e+fx)}{a}\right)\right)(a+b\sec^2(e+fx))^{1+p}}{2abf(1+p)}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^3,x]
[Out] ((a + b*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a])*(a +
b*Sec[e + f*x]^2)^(1 + p))/(2*a*b*f*(1 + p))
```

Maple [F]

$$\int \left(a+b\sec(fx+e)^2\right)^p \tan(fx+e)^3\,dx$$

```
[In] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x)
[Out] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x)
```

Fricas [F]

$$\int (a+b\sec^2(e+fx))^p \tan^3(e+fx)\,dx = \int \left(b\sec(fx+e)^2+a\right)^p \tan(fx+e)^3\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

Sympy [F]

$$\int (a+b\sec^2(e+fx))^p \tan^3(e+fx)\,dx=\int \left(a+b\sec^2{(e+fx)}\right)^p \tan^3{(e+fx)}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e)**3,x)
[Out] Integral((a + b*sec(e + f*x)**2)**p*tan(e + f*x)**3, x)
```

Maxima [F]

$$\int (a+b\sec^2(e+fx))^p \tan^3(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p \tan\left(fx+e\right)^3\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

Giac [F]

$$\int (a+b\sec^2(e+fx))^p \tan^3(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p \tan\left(fx+e\right)^3\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p \tan^3(e+fx)\,dx=\int \mathrm{tan}(e+f\,x)^3\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

```
[In] int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^p, x)
```

3.444 $\int (a+b\sec^2(e+fx))^p \tan(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int (a+b\sec^2(e+fx))^p \tan(e+fx)\,dx = -\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^{1+p}}{2af(1+p)}$$

```
[Out] -1/2*hypergeom([1, p+1],[2+p],1+b*sec(f*x+e)^2/a)*(a+b*sec(f*x+e)^2)^(p+1)/
a/f/(p+1)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4224, 272, 67}

$$\int (a+b\sec^2(e+fx))^p \tan(e+fx)\,dx = -\frac{(a+b\sec^2(e+fx))^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)}{a}+1\right)}{2af(p+1)}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x],x]
```

```
[Out] -1/2*(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*S
ec[e + f*x]^2)^(1 + p))/(a*f*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
```

```
d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{x}\,dx, x, \sec(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}\\ &= -\frac{\text{Hypergeometric2F1}\left(1, 1+p, 2+p, 1+\frac{b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^{1+p}}{2af(1+p)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int (a+b\sec^2(e+fx))^p\tan(e+fx)\,dx = -\frac{\text{Hypergeometric2F1}\left(1, 1+p, 2+p, 1+\frac{b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^{1+p}}{2af(1+p)}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x],x]
[Out] -1/2*(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*S
ec[e + f*x]^2)^(1 + p))/(a*f*(1 + p))
```

Maple [F]

$$\int \left(a+b\sec\left(fx+e\right)^{2}\right)^{p}\tan\left(fx+e\right)dx$$

```
[In] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x)
[Out] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x)
```

Fricas [F]

$$\int (a+b\sec^2(e+fx))^p\tan(e+fx)\,dx = \int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\tan\left(fx+e\right)\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

Sympy [F]

$$\int (a+b\sec^2(e+fx))^p\tan(e+fx)\,dx = \int \left(a+b\sec^{2}\left(e+fx\right)\right)^{p}\tan\left(e+fx\right)dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e),x)
[Out] Integral((a + b*sec(e + f*x)**2)**p*tan(e + f*x), x)
```

Maxima [F]

$$\int (a+b\sec^2(e+fx))^p\tan(e+fx)\,dx = \int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\tan\left(fx+e\right)\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

Giac [F]

$$\int (a+b\sec^2(e+fx))^p \tan(e+fx)\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \tan\left(fx+e\right)\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p \tan(e+fx)\,dx = \int \tan(e+f\,x)\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

```
[In] int(tan(e + f*x)*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(tan(e + f*x)*(a + b/cos(e + f*x)^2)^p, x)
```

3.445 $\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 114

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = -\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sec^2(e+fx)}{a+b}\right)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2(a+b)f(1+p)} + \frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sec^2(e+fx)}{a}\right)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2af(1+p)}$$

```
[Out] -1/2*hypergeom([1, p+1],[2+p],(a+b*sec(f*x+e)^2)/(a+b))*(a+b*sec(f*x+e)^2)^
(p+1)/(a+b)/f/(p+1)+1/2*hypergeom([1, p+1],[2+p],1+b*sec(f*x+e)^2/a)*(a+b*s
ec(f*x+e)^2)^(p+1)/a/f/(p+1)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4224, 457, 88, 70, 67}

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)}{a}+1\right)}{2af(p+1)} - \frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)+a}{a+b}\right)}{2f(p+1)(a+b)}$$

```
[In] Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -1/2*(Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sec[e + f*x]^2)/(a + b)]*(a
 + b*Sec[e + f*x]^2)^(1 + p))/((a + b)*f*(1 + p)) + (Hypergeometric2F1[1, 1
 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e + f*x]^2)^(1 + p))/(2*a
*f*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{x(-1+x^2)}\,dx, x, \sec(e+fx)\right)}{f}$$

$$
\begin{aligned}
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{(-1+x)x}\,dx, x, \sec^2(e+fx)\right)}{2f} \\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{-1+x}\,dx, x, \sec^2(e+fx)\right)}{2f} - \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, \sec^2(e+fx)\right)}{2f} \\
&= -\frac{\text{Hypergeometric2F1}\left(1, 1+p, 2+p, \frac{a+b\sec^2(e+fx)}{a+b}\right)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2(a+b)f(1+p)} \\
&\quad + \frac{\text{Hypergeometric2F1}\left(1, 1+p, 2+p, 1+\frac{b\sec^2(e+fx)}{a}\right)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2af(1+p)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.90 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01

$$
\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx
$$

$$
= \frac{(a+2b+a\cos(2(e+fx)))\left((a+b)\,\text{Hypergeometric2F1}\left(1, 1+p, 2+p, \frac{a+b+b\tan^2(e+fx)}{a}\right) - a\,\text{Hypergeom}\right.}{4a(a+b)f(1+p)}
$$

```
[In] Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*((a + b)*Hypergeometric2F1[1, 1 + p, 2 + p,
 (a + b + b*Tan[e + f*x]^2)/a] - a*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (
b*Tan[e + f*x]^2)/(a + b)])*Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p)/(4*a*(
a + b)*f*(1 + p))
```

Maple [F]

$$
\int \cot\left(fx+e\right)\left(a+b\sec\left(fx+e\right)^{2}\right)^{p}dx
$$

```
[In] int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)\,dx$$

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*cot(f*x + e), x)
```

Sympy [F]

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^p\cot\left(e+fx\right)dx$$

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2)**p,x)
[Out] Integral((a + b*sec(e + f*x)**2)**p*cot(e + f*x), x)
```

Maxima [F]

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)\,dx$$

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e), x)
```

Giac [F]

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)\,dx$$

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \cot(e+f\,x)\,\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)^p\,dx$$

```
[In] int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^p, x)
```

3.446 $\int \cot^3(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 157

$$\int \cot^3(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = -\frac{\cot^2(e+fx) \left(a+b\sec^2(e+fx)\right)^{1+p}}{2(a+b)f} + \frac{(a+b-bp) \text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sec^2(e+fx)}{a+b}\right) \left(a+b\sec^2(e+fx)\right)^{1+p}}{2(a+b)^2 f(1+p)} - \frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sec^2(e+fx)}{a}\right) \left(a+b\sec^2(e+fx)\right)^{1+p}}{2af(1+p)}$$

```
[Out] -1/2*cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(p+1)/(a+b)/f+1/2*(-b*p+a+b)*hypergeom
([1, p+1],[2+p],(a+b*sec(f*x+e)^2)/(a+b))*(a+b*sec(f*x+e)^2)^(p+1)/(a+b)^2/
f/(p+1)-1/2*hypergeom([1, p+1],[2+p],1+b*sec(f*x+e)^2/a)*(a+b*sec(f*x+e)^2)
^(p+1)/a/f/(p+1)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

$= \{4224, 457, 105, 162, 70, 67\}$

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$= \frac{(a-bp+b)\left(a+b\sec^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)+a}{a+b}\right)}{2f(p+1)(a+b)^2} - \frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)}{a}+1\right)}{2af(p+1)} - \frac{\cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{p+1}}{2f(a+b)}$$

```
[In] Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -1/2*(Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(1 + p))/((a + b)*f) + ((a + b
- b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sec[e + f*x]^2)/(a + b)]*(
a + b*Sec[e + f*x]^2)^(1 + p))/(2*(a + b)^2*f*(1 + p)) - (Hypergeometric2F1
[1, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e + f*x]^2)^(1 + p))
/(2*a*f*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{x(-1+x^2)^2}\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{(-1+x)^2x}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= -\frac{\cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2(a+b)f} - \frac{\text{Subst}\left(\int \frac{(a+bx)^p(a+b-bpx)}{(-1+x)x}\,dx, x, \sec^2(e+fx)\right)}{2(a+b)f}\\
&= -\frac{\cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2(a+b)f} + \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&\quad - \frac{(a+b-bp)\text{Subst}\left(\int \frac{(a+bx)^p}{-1+x}\,dx, x, \sec^2(e+fx)\right)}{2(a+b)f}\\
&= -\frac{\cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2(a+b)f}\\
&\quad + \frac{(a+b-bp)\,\text{Hypergeometric2F1}\left(1, 1+p, 2+p, \frac{a+b\sec^2(e+fx)}{a+b}\right)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2(a+b)^2f(1+p)}\\
&\quad - \frac{\text{Hypergeometric2F1}\left(1, 1+p, 2+p, 1+\frac{b\sec^2(e+fx)}{a}\right)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2af(1+p)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.96 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = -\frac{(b+(a+b)\cot^2(e+fx))\left(a(a+b)(1+p)\cot^2(e+fx)+(a+b)^2\,\text{Hypergeometric2F1}\left(1,1+p,2+p\right.\right.}{}$$

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -1/2*((b + (a + b)*Cot[e + f*x]^2)*(a*(a + b)*(1 + p)*Cot[e + f*x]^2 + (a +
 b)^2*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b + b*Tan[e + f*x]^2)/a] - a*
(a + b - b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Tan[e + f*x]^2)/(a
+ b)])*(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^2)/(a*(a + b)^2*f*(1 + p))
```

Maple [F]

$$\int \cot(fx+e)^3\left(a+b\sec(fx+e)^2\right)^p dx$$

```
[In] int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec(fx+e)^2+a\right)^p\cot(fx+e)^3\,dx$$

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Timed out}$$

```
[In] integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^3\,dx$$

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^3, x)
```

Giac [F]

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^3\,dx$$

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \cot(e+f\,x)^3\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

```
[In] int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^p, x)
```

3.447 $\int (a+b\sec^2(e+fx))^p \tan^4(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int (a+b\sec^2(e+fx))^p \tan^4(e+fx)\,dx$$
$$= \frac{\operatorname{AppellF1}\left(\frac{5}{2},1,-p,\frac{7}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^5(e+fx)\,(a+b+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+\ldots}{a+b}\right.}{5f}$$

```
[Out] 1/5*AppellF1(5/2,1,-p,7/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)^5
*(a+b+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4226, 2000, 525, 524}

$$\int (a+b\sec^2(e+fx))^p \tan^4(e+fx)\,dx$$
$$= \frac{\tan^5(e+fx)\,(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{5}{2},1,-p,\frac{7}{2},-\tan^2(e+fx),-\frac{b\tan^2(\ldots}{a+\ldots}\right.}{5f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^4,x]
```

```
[Out] (AppellF1[5/2, 1, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^p)/(5*f*(1 + (b*Tan[e + f*x]^2)/(
a + b))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4(a+b(1+x^2))^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4(a+b+bx^2)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((a+b+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^4\left(1+\frac{bx^2}{a+b}\right)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{AppellF1}\left(\frac{5}{2}, 1, -p, \frac{7}{2}, -\tan^2(e+fx), -\frac{b\tan^2(e+fx)}{a+b}\right)\tan^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+b\right.}{5f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2777 vs. $2(88) = 176$.

Time = 18.57 (sec) , antiderivative size = 2777, normalized size of antiderivative = 31.56

$$\int (a + b\sec^2(e + fx))^p \tan^4(e + fx)\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^4,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*(a + b*Sec[e + f*x]^2)
^p*Tan[e + f*x]^5*((9*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1
, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/
2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)
*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])
*Tan[e + f*x]^2) + (-3*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)
/(a + b))] + Hypergeometric2F1[3/2, -p, 5/2, -((b*Tan[e + f*x]^2)/(a + b))]
*Tan[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(3*f*(((a + 2*b + a*C
os[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(1 + p)*((9*(a + b)*AppellF1[1/2, -p, 1
, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(3*(
a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*
x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)),
-Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(
a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (-3*Hypergeometric2F1[1/2, -p,
 3/2, -((b*Tan[e + f*x]^2)/(a + b))] + Hypergeometric2F1[3/2, -p, 5/2, -((b
*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))
^p))/3 - (2*a*p*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^p*
Sin[2*(e + f*x)]*Tan[e + f*x]*((9*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Ta
n[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(3*(a + b)*AppellF
1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p
*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^
2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[
e + f*x]^2])*Tan[e + f*x]^2) + (-3*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan
[e + f*x]^2)/(a + b))] + Hypergeometric2F1[3/2, -p, 5/2, -((b*Tan[e + f*x]^
2)/(a + b))]*Tan[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/3 + (2*p*
(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Tan[e + f*x]^2*((9*(a +
 b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^
2]*Cos[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan
[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2,
 -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (-3*Hyp
ergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))] + Hypergeometri
c2F1[3/2, -p, 5/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2)/(1 + (b*T
an[e + f*x]^2)/(a + b))^p))/3 + ((a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e +
```

f*x]^2)^p*Tan[e + f*x]*((-18*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e +
 f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)*A
ppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] +
2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e +
 f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)),
 -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (9*(a + b)*Cos[e + f*x]^2*((2*b*p*Appe
llF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Se
c[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*
Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3))
/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
 + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a +
b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]
^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (2*b*p*Sec[e + f*x]^2*Tan
[e + f*x]*(1 + (b*Tan[e + f*x]^2)/(a + b))^(-1 - p)*(-3*Hypergeometric2F1[1
/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))] + Hypergeometric2F1[3/2, -p, 5/
2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2))/(a + b) - (9*(a + b)*App
ellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[
e + f*x]^2*(4*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b
)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^
2)/(a + b)), -Tan[e + f*x]^2])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*
b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*
x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/
2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f
*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[
e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5 - (6*
b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[
e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b))) - (a + b)*((6*b*p*App
ellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*S
ec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (12*AppellF1[5/2, -p, 3, 7/2, -((
b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5
))))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -T
an[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(
a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e +
f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)^2 + (2*Hypergeometric2F
1[3/2, -p, 5/2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]
- 3*Csc[e + f*x]*Sec[e + f*x]*(-Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e
+ f*x]^2)/(a + b))] + (1 + (b*Tan[e + f*x]^2)/(a + b))^p) + 3*Sec[e + f*x]^
2*Tan[e + f*x]*(-Hypergeometric2F1[3/2, -p, 5/2, -((b*Tan[e + f*x]^2)/(a +
b))] + (1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(1 + (b*Tan[e + f*x]^2)/(a + b)
)^p))/3))

Maple [F]

$$\int \left(a+b\sec \left(fx+e\right)^{2}\right)^{p} \tan \left(fx+e\right)^{4}dx$$

```
[In] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x)
[Out] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x)
```

Fricas [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p \tan^4(e+fx)\,dx = \int \left(b\sec \left(fx+e\right)^{2}+a\right)^{p} \tan \left(fx+e\right)^{4}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^p \tan^4(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e)**4,x)
[Out] Timed out
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p \tan^4(e+fx)\,dx = \int \left(b\sec \left(fx+e\right)^{2}+a\right)^{p} \tan \left(fx+e\right)^{4}\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^4, x)
```

Giac [F]

$$\int (a+b\sec^2(e+fx))^p\tan^4(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^4\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p\tan^4(e+fx)\,dx=\int \tan(e+f\,x)^4\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

```
[In] int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^p, x)
```

3.448 $\int (a+b\sec^2(e+fx))^p\tan^2(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int (a+b\sec^2(e+fx))^p\tan^2(e+fx)\,dx = \frac{\operatorname{AppellF1}\left(\frac{3}{2},1,-p,\frac{5}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+}{a+b}\right.}{3f}$$

```
[Out] 1/3*AppellF1(3/2,1,-p,5/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)^3
*(a+b+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4226, 2000, 525, 524}

$$\int (a+b\sec^2(e+fx))^p\tan^2(e+fx)\,dx = \frac{\tan^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{2},1,-p,\frac{5}{2},-\tan^2(e+fx),-\frac{b\tan^2(}{a+}\right.}{3f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^2,x]
```

```
[Out] (AppellF1[3/2, 1, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2)/(
a + b))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2(a+b(1+x^2))^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2(a+b+bx^2)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((a+b+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^2\left(1+\frac{bx^2}{a+b}\right)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{AppellF1}\left(\frac{3}{2}, 1, -p, \frac{5}{2}, -\tan^2(e+fx), -\frac{b\tan^2(e+fx)}{a+b}\right)\tan^3(e+fx)\,(a+b+b\tan^2(e+fx))^p\left(1+\frac{b\right.}{}}{3f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2465 vs. 2(88) = 176.

Time = 17.52 (sec) , antiderivative size = 2465, normalized size of antiderivative = 28.01

$$\int (a+b\sec^2(e+fx))^p\tan^2(e+fx)\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*(a + b*Sec[e + f*x]^2)
^p*Tan[e + f*x]^3*(Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a
+ b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p - (3*(a + b)*AppellF1[1/2, -p, 1,
 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(3*(a
 + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x
]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -
Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a
 + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)))/(f*((a + 2*b + a*Cos[2*(e + f*x
)])^p*(Sec[e + f*x]^2)^(1 + p)*(Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e
+ f*x]^2)/(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p - (3*(a + b)*AppellF
1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e +
f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)),
 -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e
 + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) - 2*a*p*(a + 2*b +
a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^p*Sin[2*(e + f*x)]*Tan[e + f*
x]*(Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]/(1 + (b*
Tan[e + f*x]^2)/(a + b))^p - (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[
e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(3*(a + b)*AppellF1[
1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*A
ppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]
 - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2])*Tan[e + f*x]^2)) + 2*p*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e +
 f*x]^2)^p*Tan[e + f*x]^2*(Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x
]^2)/(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p - (3*(a + b)*AppellF1[1/2
, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^
2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan
[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a
+ b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*
x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) + (a + 2*b + a*Cos[2*(e
+ f*x)])^p*(Sec[e + f*x]^2)^p*Tan[e + f*x]*((-2*b*p*Hypergeometric2F1[1/2,
-p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]*(1 + (b
*Tan[e + f*x]^2)/(a + b))^(-1 - p))/(a + b) + (6*(a + b)*AppellF1[1/2, -p,
1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*Sin[e
+ f*x])/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)),
```

```
 -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e
 + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a + b)*Cos[e +
 f*x]^2*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)),
 -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*AppellF1[3/
2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]
^2*Tan[e + f*x])/3))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]
^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*T
an[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/
2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (Csc[
e + f*x]*Sec[e + f*x]*(-Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2
)/(a + b))] + (1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(1 + (b*Tan[e + f*x]^2)/
(a + b))^p + (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a +
 b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2*(4*(b*p*AppellF1[3/2, 1 - p, 1, 5/2,
-((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p,
 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Sec[e + f*x]^2*Ta
n[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*
x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) -
 (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^
2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*AppellF1[5/
2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f
*x]^2*Tan[e + f*x])/5 - (6*b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -((b*Tan[
e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a +
 b))) - (a + b)*((6*b*p*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(
a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (12*Ap
pellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec
[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*T
an[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1,
5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2
, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]
^2)^2)))
```

Maple [F]

$$\int \left(a + b\sec \left(fx + e\right)^{2}\right)^{p} \tan \left(fx + e\right)^{2} dx$$

[In] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x)

[Out] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x)

Fricas [F]

$$\int (a+b\sec^2(e+fx))^p \tan^2(e+fx)\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \tan\left(fx+e\right)^2\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^2, x)
```

Sympy [F]

$$\int (a+b\sec^2(e+fx))^p \tan^2(e+fx)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^p \tan^2\left(e+fx\right)dx$$

```
[In] integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e)**2,x)
[Out] Integral((a + b*sec(e + f*x)**2)**p*tan(e + f*x)**2, x)
```

Maxima [F]

$$\int (a+b\sec^2(e+fx))^p \tan^2(e+fx)\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \tan\left(fx+e\right)^2\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^2, x)
```

Giac [F]

$$\int (a+b\sec^2(e+fx))^p \tan^2(e+fx)\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \tan\left(fx+e\right)^2\,dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p\tan^2(e+fx)\,dx=\int \tan(e+f\,x)^2\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

```
[In] int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^p, x)
```

3.449 $\int (a+b\sec^2(e+fx))^p\,dx$

Optimal result

Integrand size = 14, antiderivative size = 83

$$\int (a+b\sec^2(e+fx))^p\,dx = \frac{\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+f\ldots}{a+b}\right.}{f}$$

```
[Out] AppellF1(1/2,1,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b+
b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4213, 441, 440}

$$\int (a+b\sec^2(e+fx))^p\,dx = \frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e\ldots}{a+b}\right.}{f}$$

```
[In] Int[(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((a+b+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{AppellF1}\left(\frac{1}{2}, 1, -p, \frac{3}{2}, -\tan^2(e+fx), -\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\,(a+b+b\tan^2(e+fx))^p\left(1+\frac{b\,\mathrm{ta}}{}\right.}{f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2137 vs. $2(83) = 166$.

Time = 6.25 (sec) , antiderivative size = 2137, normalized size of antiderivative = 25.75

$$\int (a+b\sec^2(e+fx))^p\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*
(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, -p, 1, 3
```

/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-1 + p))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (6*(a + b)*p*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (6*a*(a + b)*p*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^p*Sin[e + f*x]*Sin[2*(e + f*x)])/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (3*(a + b)*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]*(4*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5 - (6*b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -(

```
(b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/
(5*(a + b))) - (a + b)*((6*b*p*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*
x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) -
 (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]
^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2,
 -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 -
 p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*Appel
lF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e
 + f*x]^2)^2))
```

Maple [F]

$$\int \left(a+b\sec \left(fx+e\right)^{2}\right)^{p}dx$$

[In] int((a+b*sec(f*x+e)^2)^p,x)

[Out] int((a+b*sec(f*x+e)^2)^p,x)

Fricas [F]

$$\int \left(a+b\sec ^2(e+fx)\right)^p\,dx=\int \left(b\sec \left(fx+e\right)^2+a\right)^{p}dx$$

[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")

[Out] integral((b*sec(f*x + e)^2 + a)^p, x)

Sympy [F]

$$\int \left(a+b\sec ^2(e+fx)\right)^p\,dx=\int \left(a+b\sec ^{2}\left(e+fx\right)\right)^{p}\,dx$$

[In] integrate((a+b*sec(f*x+e)**2)**p,x)

[Out] Integral((a + b*sec(e + f*x)**2)**p, x)

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \, dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p, x)
```

Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \, dx$$

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p \, dx$$

```
[In] int((a + b/cos(e + f*x)^2)^p,x)
[Out] int((a + b/cos(e + f*x)^2)^p, x)
```

3.450 $\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = -\frac{\text{AppellF1}\left(-\frac{1}{2},1,-p,\frac{1}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{f}$$

```
[Out] -AppellF1(-1/2,1,-p,1/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*cot(f*x+e)*(a+
b+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4226, 2000, 525, 524}

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = -\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\left(-\frac{1}{2},1,-p,\frac{1}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

```
[In] Int[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[-1/2, 1, -p, 1/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
)]*Cot[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a
 + b))^p))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+b(1+x^2))^p}{x^2(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^p}{x^2(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((a+b+b\tan^2(e+fx))^p \left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right)^p}{x^2(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\text{AppellF1}\left(-\frac{1}{2}, 1, -p, \frac{1}{2}, -\tan^2(e+fx), -\frac{b\tan^2(e+fx)}{a+b}\right)\cot(e+fx)\,(a+b+b\tan^2(e+fx))^p \left(1\right.}{f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2469 vs. $2(84) = 168$.

Time = 17.48 (sec) , antiderivative size = 2469, normalized size of antiderivative = 29.39

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Result too large to show}$$

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^p*Cot[e + f*x]^3*(Sec[e + f*x]^2)^p*(a + b*
Sec[e + f*x]^2)^p*(-(Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/
(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p) - (3*(a + b)*AppellF1[1/2, -p
, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sin[e + f*x]^2)/(
3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e +
 f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)
), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)))/(f*(2*p*(a + 2*b + a*Cos[2*
(e + f*x)])^p*(Sec[e + f*x]^2)^p*(-(Hypergeometric2F1[-1/2, -p, 1/2, -((b*T
an[e + f*x]^2)/(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p) - (3*(a + b)*A
ppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Si
n[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a
+ b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e +
f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b
*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) - (a + 2*b +
a*Cos[2*(e + f*x)])^p*Csc[e + f*x]^2*(Sec[e + f*x]^2)^p*(-(Hypergeometric2F
1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a
+ b))^p) - (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b
)), -Tan[e + f*x]^2]*Sin[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -
((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p
, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF
1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e +
 f*x]^2)) - 2*a*p*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*Cot[e + f*x]*(Sec
[e + f*x]^2)^p*Sin[2*(e + f*x)]*(-(Hypergeometric2F1[-1/2, -p, 1/2, -((b*Ta
n[e + f*x]^2)/(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p) - (3*(a + b)*Ap
pellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sin
[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a +
 b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f
*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*
Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) + (a + 2*b + a
*Cos[2*(e + f*x)])^p*Cot[e + f*x]*(Sec[e + f*x]^2)^p*((2*b*p*Hypergeometric
2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*
x]*(1 + (b*Tan[e + f*x]^2)/(a + b))^(-1 - p))/(a + b) - (6*(a + b)*AppellF1
[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f
*x]*Sin[e + f*x])/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)
```

```
/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[
e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2,
-((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a +
b)*Sin[e + f*x]^2*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)
/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*A
ppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Se
c[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Ta
n[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5
/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2,
 -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^
2) - (Csc[e + f*x]*Sec[e + f*x]*(Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[
e + f*x]^2)/(a + b))] - (1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(1 + (b*Tan[e
+ f*x]^2)/(a + b))^p + (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*
x]^2)/(a + b)), -Tan[e + f*x]^2]*Sin[e + f*x]^2*(4*(b*p*AppellF1[3/2, 1 - p
, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF
1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Sec[e +
 f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*
Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*
(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan
[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*A
ppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]
*Sec[e + f*x]^2*Tan[e + f*x])/5 - (6*b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2,
 -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x
])/(5*(a + b))) - (a + b)*((6*b*p*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e +
 f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)
) - (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f
*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*AppellF1[1/2, -p, 1, 3
/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2,
1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*Ap
pellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Ta
n[e + f*x]^2)^2)))
```

Maple [F]

$$\int \cot (fx+e)^2 \left(a+b\sec (fx+e)^2\right)^p dx$$

```
[In] int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^2\,dx$$

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2)**p,x)
[Out] Timed out
```

Maxima [F]

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^2\,dx$$

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

Giac [F]

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^2\,dx$$

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \cot(e+f\,x)^2\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

```
[In] int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^p, x)
```

3.451 $\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = -\frac{\text{AppellF1}\left(-\frac{3}{2},1,-p,-\frac{1}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan\cdots}{}\right.}{3f}$$

```
[Out] -1/3*AppellF1(-3/2,1,-p,-1/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*cot(f*x+e
)^3*(a+b+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4226, 2000, 525, 524}

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = -\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\left(-\frac{3}{2},1,-p,-\frac{1}{2},-\tan^2(e+fx),-\frac{b\cdots}{}\right.}{3f}$$

```
[In] Int[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -1/3*(AppellF1[-3/2, 1, -p, -1/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
+ b))]*Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]
^2)/(a + b))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+b(1+x^2))^p}{x^4(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^p}{x^4(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((a+b+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right)^p}{x^4(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\text{AppellF1}\left(-\frac{3}{2}, 1, -p, -\frac{1}{2}, -\tan^2(e+fx), -\frac{b\tan^2(e+fx)}{a+b}\right)\cot^3(e+fx)\,(a+b+b\tan^2(e+fx))^p}{3f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3033 vs. $2(88) = 176$.

Time = 18.98 (sec) , antiderivative size = 3033, normalized size of antiderivative = 34.47

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Result too large to show}$$

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^p*Cot[e + f*x]^7*(Sec[e + f*x]^2)^p*(a + b*
Sec[e + f*x]^2)^p*((9*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(3*(a + b)*Appe
llF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(
b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*
x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -T
an[e + f*x]^2])*Tan[e + f*x]^2) - (Hypergeometric2F1[-3/2, -p, -1/2, -((b*T
an[e + f*x]^2)/(a + b))] - 3*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e +
f*x]^2)/(a + b))]*Tan[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(3*f
*((2*p*(a + 2*b + a*Cos[2*(e + f*x)])^p*Cot[e + f*x]^2*(Sec[e + f*x]^2)^p*(
(9*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/
2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1
 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*App
ellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan
[e + f*x]^2) - (Hypergeometric2F1[-3/2, -p, -1/2, -((b*Tan[e + f*x]^2)/(a +
 b))] - 3*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]*T
an[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/3 - (a + 2*b + a*Cos[2*
(e + f*x)])^p*Cot[e + f*x]^2*Csc[e + f*x]^2*(Sec[e + f*x]^2)^p*((9*(a + b)*
AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*S
in[e + f*x]^2*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan
[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/
2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2,
-p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2
) - (Hypergeometric2F1[-3/2, -p, -1/2, -((b*Tan[e + f*x]^2)/(a + b))] - 3*H
ypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]
^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))^p) - (2*a*p*(a + 2*b + a*Cos[2*(e + f*
x)])^(-1 + p)*Cot[e + f*x]^3*(Sec[e + f*x]^2)^p*Sin[2*(e + f*x)]*((9*(a + b
)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]
*Sin[e + f*x]^2*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*T
an[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1,
5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2
, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]
^2) - (Hypergeometric2F1[-3/2, -p, -1/2, -((b*Tan[e + f*x]^2)/(a + b))] - 3
*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*
```

x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/3 + ((a + 2*b + a*Cos[2*(e + f*x
)])^p*Cot[e + f*x]^3*(Sec[e + f*x]^2)^p*((18*(a + b)*AppellF1[1/2, -p, 1, 3
/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sin[e + f*x]^2*Tan[e +
f*x])/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -
Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/
(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e +
 f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (18*(a + b)*AppellF1
[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Tan[e + f
x]^3)/(3(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)),
-Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)
/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e
+ f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (9*(a + b)*Sin[e +
f*x]^2*Tan[e + f*x]^2*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x
]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) -
(2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2
]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((
b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p,
1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[
3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f
*x]^2) + (2*b*p*Sec[e + f*x]^2*Tan[e + f*x]*(1 + (b*Tan[e + f*x]^2)/(a + b)
)^(-1 - p)*(Hypergeometric2F1[-3/2, -p, -1/2, -((b*Tan[e + f*x]^2)/(a + b))
] - 3*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e
 + f*x]^2))/(a + b) - (9*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x
]^2)/(a + b)), -Tan[e + f*x]^2]*Sin[e + f*x]^2*Tan[e + f*x]^2*(4*(b*p*Appel
lF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (
a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*
x]^2])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1 - p,
 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan
[e + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/
(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e + f*x]
^2*(b*p*((-6*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -T
an[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5 - (6*b*(1 - p)*AppellF1[5/2,
2 - p, 1, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]
^2*Tan[e + f*x])/(5*(a + b))) - (a + b)*((6*b*p*AppellF1[5/2, 1 - p, 2, 7/2
, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*
x])/(5*(a + b)) - (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/(a + b
)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*AppellF1
[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*
AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2
] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
 + f*x]^2])*Tan[e + f*x]^2)^2 - (-6*Hypergeometric2F1[-1/2, -p, 1/2, -((b*T
an[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x] - 3*Sec[e + f*x]^2*Tan
[e + f*x]*(Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]
- (1 + (b*Tan[e + f*x]^2)/(a + b))^p) - 3*Csc[e + f*x]*Sec[e + f*x]*(-Hyper
geometric2F1[-3/2, -p, -1/2, -((b*Tan[e + f*x]^2)/(a + b))] + (1 + (b*Tan[e

```
+ f*x]^2)/(a + b))^p))/(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/3))
```

Maple [F]

$$\int \cot(fx+e)^4 \left(a+b\sec(fx+e)^2\right)^p dx$$

```
[In] int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(b\sec(fx+e)^2+a\right)^p \cot(fx+e)^4 \, dx$$

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p \, dx = \text{Timed out}$$

```
[In] integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2)**p,x)
[Out] Timed out
```

Maxima [F]

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(b\sec(fx+e)^2+a\right)^p \cot(fx+e)^4 \, dx$$

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

Giac [F]

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^4\,dx$$

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \cot(e+f\,x)^4\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

```
[In] int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^p, x)
```

3.452 $\int (a+b\sec^3(e+fx))\tan^5(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 92

$$\int (a+b\sec^3(e+fx))\tan^5(e+fx)\,dx = -\frac{a\log(\cos(e+fx))}{f}-\frac{a\sec^2(e+fx)}{f}+\frac{b\sec^3(e+fx)}{3f}+\frac{a\sec^4(e+fx)}{4f}-\frac{2b\sec^5(e+fx)}{5f}+\frac{b\sec^7(e+fx)}{7f}$$

```
[Out] -a*ln(cos(f*x+e))/f-a*sec(f*x+e)^2/f+1/3*b*sec(f*x+e)^3/f+1/4*a*sec(f*x+e)^
4/f-2/5*b*sec(f*x+e)^5/f+1/7*b*sec(f*x+e)^7/f
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4223, 1816}

$$\int (a+b\sec^3(e+fx))\tan^5(e+fx)\,dx = \frac{a\sec^4(e+fx)}{4f}-\frac{a\sec^2(e+fx)}{f}-\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^7(e+fx)}{7f}-\frac{2b\sec^5(e+fx)}{5f}+\frac{b\sec^3(e+fx)}{3f}$$

```
[In] Int[(a + b*Sec[e + f*x]^3)*Tan[e + f*x]^5,x]
```

```
[Out] -((a*Log[Cos[e + f*x]])/f) - (a*Sec[e + f*x]^2)/f + (b*Sec[e + f*x]^3)/(3*f
) + (a*Sec[e + f*x]^4)/(4*f) - (2*b*Sec[e + f*x]^5)/(5*f) + (b*Sec[e + f*x]
^7)/(7*f)
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2(b+ax^3)}{x^8}\,dx, x, \cos(e+fx)\right)}{f} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{b}{x^8}-\frac{2b}{x^6}+\frac{a}{x^5}+\frac{b}{x^4}-\frac{2a}{x^3}+\frac{a}{x}\right)\,dx, x, \cos(e+fx)\right)}{f} \\
&= -\frac{a\log(\cos(e+fx))}{f}-\frac{a\sec^2(e+fx)}{f}+\frac{b\sec^3(e+fx)}{3f} \\
&\quad +\frac{a\sec^4(e+fx)}{4f}-\frac{2b\sec^5(e+fx)}{5f}+\frac{b\sec^7(e+fx)}{7f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

$$\begin{aligned}
&\int (a+b\sec^3(e+fx))\tan^5(e+fx)\,dx \\
&= \frac{b\sec^3(e+fx)}{3f}-\frac{2b\sec^5(e+fx)}{5f}+\frac{b\sec^7(e+fx)}{7f} \\
&\quad -\frac{a(4\log(\cos(e+fx))+2\tan^2(e+fx)-\tan^4(e+fx))}{4f}
\end{aligned}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^3)*Tan[e + f*x]^5,x]
[Out] (b*Sec[e + f*x]^3)/(3*f) - (2*b*Sec[e + f*x]^5)/(5*f) + (b*Sec[e + f*x]^7)/
(7*f) - (a*(4*Log[Cos[e + f*x]] + 2*Tan[e + f*x]^2 - Tan[e + f*x]^4))/(4*f)
```

Maple [A] (verified)

Time = 3.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{\frac{b\sec(fx+e)^7}{7}-\frac{2b\sec(fx+e)^5}{5}+\frac{a\sec(fx+e)^4}{4}+\frac{b\sec(fx+e)^3}{3}-a\sec(fx+e)^2+a\ln(\sec(fx+e))}{f}$
default	$\frac{\frac{b\sec(fx+e)^7}{7}-\frac{2b\sec(fx+e)^5}{5}+\frac{a\sec(fx+e)^4}{4}+\frac{b\sec(fx+e)^3}{3}-a\sec(fx+e)^2+a\ln(\sec(fx+e))}{f}$
parts	$\frac{a\left(\frac{\tan(fx+e)^4}{4}-\frac{\tan(fx+e)^2}{2}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}+\frac{b\left(\frac{\sec(fx+e)^7}{7}-\frac{2\sec(fx+e)^5}{5}+\frac{\sec(fx+e)^3}{3}\right)}{f}$
risch	$iax+\frac{2iae}{f}-\frac{4\left(105a\,e^{12i(fx+e)}-70b\,e^{11i(fx+e)}+420a\,e^{10i(fx+e)}+56b\,e^{9i(fx+e)}+735a\,e^{8i(fx+e)}-228b\,e^{7i(fx+e)}+73\right.}{105f\left(e^{2i(fx+e)}+1\right)^7}$

```
[In] int((a+b*sec(f*x+e)^3)*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/7*b*sec(f*x+e)^7-2/5*b*sec(f*x+e)^5+1/4*a*sec(f*x+e)^4+1/3*b*sec(f*x
+e)^3-a*sec(f*x+e)^2+a*ln(sec(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

$$\int \left(a+b\sec^3(e+fx)\right)\tan^5(e+fx)\,dx = -\frac{420\,a\cos\left(fx+e\right)^7\log\left(-\cos\left(fx+e\right)\right)+420\,a\cos\left(fx+e\right)^5-140\,b\cos\left(fx+e\right)^4-105\,a\cos\left(fx+e\right)^3}{420\,f\cos\left(fx+e\right)^7}$$

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] -1/420*(420*a*cos(f*x + e)^7*log(-cos(f*x + e)) + 420*a*cos(f*x + e)^5 - 14
0*b*cos(f*x + e)^4 - 105*a*cos(f*x + e)^3 + 168*b*cos(f*x + e)^2 - 60*b)/(f
*cos(f*x + e)^7)
```

Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.29

$$\int \left(a+b\sec^3(e+fx)\right)\tan^5(e+fx)\,dx = \begin{cases} \frac{a\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a\tan^4\left(e+fx\right)}{4f}-\frac{a\tan^2\left(e+fx\right)}{2f}+\frac{b\tan^4\left(e+fx\right)\sec^3\left(e+fx\right)}{7f}-\frac{4b\tan^2\left(e+fx\right)\sec^3\left(e+fx\right)}{35f}+\frac{8b\sec^3\left(e+fx\right)}{105f} \\ x\left(a+b\sec^3\left(e\right)\right)\tan^5\left(e\right) \end{cases}$$

```
[In] integrate((a+b*sec(f*x+e)**3)*tan(f*x+e)**5,x)
```

```
[Out] Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**4/(4*f) - a*t
an(e + f*x)**2/(2*f) + b*tan(e + f*x)**4*sec(e + f*x)**3/(7*f) - 4*b*tan(e
+ f*x)**2*sec(e + f*x)**3/(35*f) + 8*b*sec(e + f*x)**3/(105*f), Ne(f, 0)),
(x*(a + b*sec(e)**3)*tan(e)**5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.79

$$\int (a+b\sec^3(e+fx))\tan^5(e+fx)\,dx$$
$$= -\frac{420\,a\log\left(\cos\left(fx+e\right)\right) + \frac{420\,a\cos\left(fx+e\right)^5-140\,b\cos\left(fx+e\right)^4-105\,a\cos\left(fx+e\right)^3+168\,b\cos\left(fx+e\right)^2-60\,b}{\cos\left(fx+e\right)^7}}{420\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -1/420*(420*a*log(cos(f*x + e)) + (420*a*cos(f*x + e)^5 - 140*b*cos(f*x + e
)^4 - 105*a*cos(f*x + e)^3 + 168*b*cos(f*x + e)^2 - 60*b)/cos(f*x + e)^7)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. 2(84) = 168.

Time = 1.63 (sec) , antiderivative size = 340, normalized size of antiderivative = 3.70

$$\int (a+b\sec^3(e+fx))\tan^5(e+fx)\,dx$$
$$= \frac{420\,a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right) - 420\,a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-1\right|\right) + \frac{1089\,a+64\,b+\frac{8463\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{448\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}}{}}{}$$

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] 1/420*(420*a*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)) - 420*a*l
og(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 1)) + (1089*a + 64*b + 8463
*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 448*b*(cos(f*x + e) - 1)/(cos(f*
x + e) + 1) + 28749*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 1344*b*(c
os(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 51555*a*(cos(f*x + e) - 1)^3/(cos
(f*x + e) + 1)^3 - 2240*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 51555
*a*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 4480*b*(cos(f*x + e) - 1)^4/
(cos(f*x + e) + 1)^4 + 28749*a*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 +
8463*a*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6 + 1089*a*(cos(f*x + e) - 1
)^7/(cos(f*x + e) + 1)^7)/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)^7)/f
```

Mupad [B] (verification not implemented)

Time = 24.34 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.47

$$\int (a + b\sec^3(e + fx))\tan^5(e + fx)\,dx = \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right)^2\right)}{f} - \frac{2\,a\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right)^{12} - 14\,a\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right)^{10} + \left(32\,a + \frac{32\,b}{3}\right)\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right)^8 + \left(\frac{16\,b}{3} - 32\,a\right)\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right)^6 + (1}{f\,\left(\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right)^{14} - 7\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right)^{12} + 21\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right)^{10} - 35\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right)^8 + 35\tan\left(\frac{e}{2} + \right.\right.}$$

```
[In] int(tan(e + f*x)^5*(a + b/cos(e + f*x)^3),x)
```

```
[Out] (2*a*atanh(tan(e/2 + (f*x)/2)^2))/f - ((16*b)/105 - tan(e/2 + (f*x)/2)^2*(2
*a + (16*b)/15) + tan(e/2 + (f*x)/2)^4*(14*a + (16*b)/5) - tan(e/2 + (f*x)/
2)^6*(32*a - (16*b)/3) + tan(e/2 + (f*x)/2)^8*(32*a + (32*b)/3) - 14*a*tan(
e/2 + (f*x)/2)^10 + 2*a*tan(e/2 + (f*x)/2)^12)/(f*(7*tan(e/2 + (f*x)/2)^2 -
 21*tan(e/2 + (f*x)/2)^4 + 35*tan(e/2 + (f*x)/2)^6 - 35*tan(e/2 + (f*x)/2)^
8 + 21*tan(e/2 + (f*x)/2)^10 - 7*tan(e/2 + (f*x)/2)^12 + tan(e/2 + (f*x)/2)
^14 - 1))
```

3.453 $\int (a+b\sec^3(e+fx))\tan^3(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int (a+b\sec^3(e+fx))\tan^3(e+fx)\,dx=\frac{a\log(\cos(e+fx))}{f}+\frac{a\sec^2(e+fx)}{2f}-\frac{b\sec^3(e+fx)}{3f}+\frac{b\sec^5(e+fx)}{5f}$$

```
[Out] a*ln(cos(f*x+e))/f+1/2*a*sec(f*x+e)^2/f-1/3*b*sec(f*x+e)^3/f+1/5*b*sec(f*x+
e)^5/f
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4223, 1816}

$$\int (a+b\sec^3(e+fx))\tan^3(e+fx)\,dx=\frac{a\sec^2(e+fx)}{2f}+\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^5(e+fx)}{5f}-\frac{b\sec^3(e+fx)}{3f}$$

```
[In] Int[(a + b*Sec[e + f*x]^3)*Tan[e + f*x]^3,x]
```

```
[Out] (a*Log[Cos[e + f*x]])/f + (a*Sec[e + f*x]^2)/(2*f) - (b*Sec[e + f*x]^3)/(3*
f) + (b*Sec[e + f*x]^5)/(5*f)
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)(b+ax^3)}{x^6}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{b}{x^6}-\frac{b}{x^4}+\frac{a}{x^3}-\frac{a}{x}\right)\,dx, x, \cos(e+fx)\right)}{f}\\
&= \frac{a\log(\cos(e+fx))}{f}+\frac{a\sec^2(e+fx)}{2f}-\frac{b\sec^3(e+fx)}{3f}+\frac{b\sec^5(e+fx)}{5f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \left(a+b\sec^3(e+fx)\right)\tan^3(e+fx)\,dx = -\frac{b\sec^3(e+fx)}{3f}+\frac{b\sec^5(e+fx)}{5f}+\frac{a(2\log(\cos(e+fx))+\tan^2(e+fx))}{2f}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^3)*Tan[e + f*x]^3,x]
```

```
[Out] -1/3*(b*Sec[e + f*x]^3)/f + (b*Sec[e + f*x]^5)/(5*f) + (a*(2*Log[Cos[e + f*
x]] + Tan[e + f*x]^2))/(2*f)
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{\frac{b\sec(fx+e)^5}{5}-\frac{b\sec(fx+e)^3}{3}+\frac{a\sec(fx+e)^2}{2}-a\ln(\sec(fx+e))}{f}$
default	$\frac{\frac{b\sec(fx+e)^5}{5}-\frac{b\sec(fx+e)^3}{3}+\frac{a\sec(fx+e)^2}{2}-a\ln(\sec(fx+e))}{f}$
parts	$\frac{a\left(\frac{\tan(fx+e)^2}{2}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}+\frac{b\left(\frac{\sec(fx+e)^5}{5}-\frac{\sec(fx+e)^3}{3}\right)}{f}$
risch	$-iax-\frac{2iae}{f}+\frac{2a\,e^{8i(fx+e)}-\frac{8b\,e^{7i(fx+e)}}{3}+6a\,e^{6i(fx+e)}+\frac{16b\,e^{5i(fx+e)}}{15}+6a\,e^{4i(fx+e)}-\frac{8b\,e^{3i(fx+e)}}{3}+2a\,e^{2i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)^5}$

```
[In] int((a+b*sec(f*x+e)^3)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
[Out] 1/f*(1/5*b*sec(f*x+e)^5-1/3*b*sec(f*x+e)^3+1/2*a*sec(f*x+e)^2-a*ln(sec(f*x+
e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int (a+b\sec^3(e+fx))\tan^3(e+fx)\,dx = \frac{30\,a\cos(fx+e)^5\log(-\cos(fx+e))+15\,a\cos(fx+e)^3-10\,b\cos(fx+e)^2+6\,b}{30\,f\cos(fx+e)^5}$$

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^3,x, algorithm="fricas")
[Out] 1/30*(30*a*cos(f*x + e)^5*log(-cos(f*x + e)) + 15*a*cos(f*x + e)^3 - 10*b*c
os(f*x + e)^2 + 6*b)/(f*cos(f*x + e)^5)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.34

$$\int (a+b\sec^3(e+fx))\tan^3(e+fx)\,dx = \begin{cases} -\frac{a\log(\tan^2(e+fx)+1)}{2f}+\frac{a\tan^2(e+fx)}{2f}+\frac{b\tan^2(e+fx)\sec^3(e+fx)}{5f}-\frac{2b\sec^3(e+fx)}{15f} & \text{for } f\neq 0 \\ x(a+b\sec^3(e))\tan^3(e) & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*sec(f*x+e)**3)*tan(f*x+e)**3,x)
[Out] Piecewise((-a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**2/(2*f) + b*
tan(e + f*x)**2*sec(e + f*x)**3/(5*f) - 2*b*sec(e + f*x)**3/(15*f), Ne(f, 0
)), (x*(a + b*sec(e)**3)*tan(e)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int (a+b\sec^3(e+fx))\tan^3(e+fx)\,dx = \frac{30\,a\log\left(\cos\left(fx+e\right)\right)+\frac{15\,a\cos(fx+e)^3-10\,b\cos(fx+e)^2+6\,b}{\cos(fx+e)^5}}{30\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^3,x, algorithm="maxima")
[Out] 1/30*(30*a*log(cos(f*x + e)) + (15*a*cos(f*x + e)^3 - 10*b*cos(f*x + e)^2 +
 6*b)/cos(f*x + e)^5)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(55) = 110.

Time = 0.69 (sec) , antiderivative size = 271, normalized size of antiderivative = 4.44

$$\int (a+b\sec^3(e+fx))\tan^3(e+fx)\,dx =$$

$$-\frac{60\,a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)-60\,a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-1\right|\right)+\frac{137\,a+16\,b+\frac{805\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{80\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{173}{(}}{}}{60\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^3,x, algorithm="giac")
[Out] -1/60*(60*a*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)) - 60*a*log
(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 1)) + (137*a + 16*b + 805*a*(
cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 80*b*(cos(f*x + e) - 1)/(cos(f*x + e
) + 1) + 1730*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 80*b*(cos(f*x +
 e) - 1)^2/(cos(f*x + e) + 1)^2 + 1730*a*(cos(f*x + e) - 1)^3/(cos(f*x + e)
 + 1)^3 + 240*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 805*a*(cos(f*x
+ e) - 1)^4/(cos(f*x + e) + 1)^4 + 137*a*(cos(f*x + e) - 1)^5/(cos(f*x + e)
 + 1)^5)/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)^5)/f
```

Mupad [B] (verification not implemented)

Time = 24.52 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.74

$$\int (a+b\sec^3(e+fx))\tan^3(e+fx)\,dx$$

$$= \frac{2\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+(-6\,a-4\,b)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+\left(6\,a-\frac{4\,b}{3}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+\left(-2\,a-\frac{4\,b}{3}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}$$

$$-\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\right)}{f}$$

```
[In] int(tan(e + f*x)^3*(a + b/cos(e + f*x)^3),x)
```

```
[Out] ((4*b)/15 - tan(e/2 + (f*x)/2)^2*(2*a + (4*b)/3) - tan(e/2 + (f*x)/2)^6*(6*
a + 4*b) + tan(e/2 + (f*x)/2)^4*(6*a - (4*b)/3) + 2*a*tan(e/2 + (f*x)/2)^8)
/(f*(5*tan(e/2 + (f*x)/2)^2 - 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/
2)^6 - 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 - 1)) - (2*a*atanh(ta
n(e/2 + (f*x)/2)^2))/f
```

3.454 $\int (a+b\sec^3(e+fx))\tan(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 30

$$\int (a+b\sec^3(e+fx))\tan(e+fx)\,dx=-\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^3(e+fx)}{3f}$$

```
[Out] -a*ln(cos(f*x+e))/f+1/3*b*sec(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4223, 14}

$$\int (a+b\sec^3(e+fx))\tan(e+fx)\,dx=\frac{b\sec^3(e+fx)}{3f}-\frac{a\log(\cos(e+fx))}{f}$$

```
[In] Int[(a + b*Sec[e + f*x]^3)*Tan[e + f*x],x]
```

```
[Out] -((a*Log[Cos[e + f*x]])/f) + (b*Sec[e + f*x]^3)/(3*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
```

```
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{b+ax^3}{x^4}\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{b}{x^4}+\frac{a}{x}\right)\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^3(e+fx)}{3f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \left(a+b\sec^3(e+fx)\right)\tan(e+fx)\,dx = -\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^3(e+fx)}{3f}$$

```
[In] Integrate[(a + b*Sec[e + f*x]^3)*Tan[e + f*x],x]
[Out] -((a*Log[Cos[e + f*x]])/f) + (b*Sec[e + f*x]^3)/(3*f)
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{\frac{b\sec(fx+e)^3}{3}+a\ln(\sec(fx+e))}{f}$	26
default	$\frac{\frac{b\sec(fx+e)^3}{3}+a\ln(\sec(fx+e))}{f}$	26
parts	$\frac{a\ln\left(1+\tan(fx+e)^2\right)}{2f}+\frac{b\sec(fx+e)^3}{3f}$	33
risch	$iax+\frac{2iae}{f}+\frac{8b\,e^{3i(fx+e)}}{3f\left(e^{2i(fx+e)}+1\right)^3}-\frac{a\ln\left(e^{2i(fx+e)}+1\right)}{f}$	61

```
[In] int((a+b*sec(f*x+e)^3)*tan(f*x+e),x,method=_RETURNVERBOSE)
[Out] 1/f*(1/3*b*sec(f*x+e)^3+a*ln(sec(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \left(a+b\sec^3(e+fx)\right)\tan(e+fx)\,dx=-\frac{3\,a\cos\left(fx+e\right)^3\log\left(-\cos\left(fx+e\right)\right)-b}{3\,f\cos\left(fx+e\right)^3}$$

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e),x, algorithm="fricas")
[Out] -1/3*(3*a*cos(f*x + e)^3*log(-cos(f*x + e)) - b)/(f*cos(f*x + e)^3)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \left(a+b\sec^3(e+fx)\right)\tan(e+fx)\,dx=\begin{cases}\frac{a\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{b\sec^3\left(e+fx\right)}{3f} & \text{for } f\neq 0\\ x(a+b\sec^3\left(e\right))\tan\left(e\right) & \text{otherwise}\end{cases}$$

```
[In] integrate((a+b*sec(f*x+e)**3)*tan(f*x+e),x)
[Out] Piecewise(((a*log(tan(e + f*x)**2 + 1)/(2*f) + b*sec(e + f*x)**3/(3*f), Ne(f
, 0)), (x*(a + b*sec(e)**3)*tan(e), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \left(a+b\sec^3(e+fx)\right)\tan(e+fx)\,dx=-\frac{a\log\left(\cos\left(fx+e\right)^3\right)-\frac{b}{\cos(fx+e)^3}}{3\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e),x, algorithm="maxima")
[Out] -1/3*(a*log(cos(f*x + e)^3) - b/cos(f*x + e)^3)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(28) = 56$.

Time = 0.35 (sec) , antiderivative size = 181, normalized size of antiderivative = 6.03

$$\int (a + b\sec^3(e + fx))\tan(e + fx)\,dx$$
$$= \frac{6\,a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right) - 6\,a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-1\right|\right) + \frac{11\,a+4\,b+\frac{33\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{33\,a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{12\,b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{11\,a(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}}{\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right)^3}}{6\,f}$$

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] 1/6*(6*a*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)) - 6*a*log(abs
(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 1)) + (11*a + 4*b + 33*a*(cos(f*x
 + e) - 1)/(cos(f*x + e) + 1) + 33*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1
)^2 + 12*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 11*a*(cos(f*x + e) -
 1)^3/(cos(f*x + e) + 1)^3)/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)^3)/
f
```

Mupad [B] (verification not implemented)

Time = 20.76 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.77

$$\int (a + b\sec^3(e + fx))\tan(e + fx)\,dx$$
$$= \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\right)}{f} - \frac{2\,b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+\frac{2\,b}{3}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}$$

```
[In] int(tan(e + f*x)*(a + b/cos(e + f*x)^3),x)
```

```
[Out] (2*a*atanh(tan(e/2 + (f*x)/2)^2))/f - ((2*b)/3 + 2*b*tan(e/2 + (f*x)/2)^4)/
(f*(3*tan(e/2 + (f*x)/2)^2 - 3*tan(e/2 + (f*x)/2)^4 + tan(e/2 + (f*x)/2)^6
- 1))
```

3.455 $\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx=\frac{(a+b)\log(1-\cos(e+fx))}{2f}+\frac{(a-b)\log(1+\cos(e+fx))}{2f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] 1/2*(a+b)*ln(1-cos(f*x+e))/f+1/2*(a-b)*ln(1+cos(f*x+e))/f+b*sec(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4223, 1816}

$$\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx=\frac{(a+b)\log(1-\cos(e+fx))}{2f}+\frac{(a-b)\log(\cos(e+fx)+1)}{2f}+\frac{b\sec(e+fx)}{f}$$

```
[In] Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^3),x]
```

```
[Out] ((a + b)*Log[1 - Cos[e + f*x]])/(2*f) + ((a - b)*Log[1 + Cos[e + f*x]])/(2*
f) + (b*Sec[e + f*x])/f
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{b+ax^3}{x^2(1-x^2)}\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{-a-b}{2(-1+x)}+\frac{b}{x^2}+\frac{-a+b}{2(1+x)}\right)\,dx, x, \cos(e+fx)\right)}{f}\\ &= \frac{(a+b)\log(1-\cos(e+fx))}{2f}+\frac{(a-b)\log(1+\cos(e+fx))}{2f}+\frac{b\sec(e+fx)}{f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.28

$$\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx = -\frac{b\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{a\log(\cos(e+fx))}{f}+\frac{b\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{a\log(\tan(e+fx))}{f}+\frac{b\sec(e+fx)}{f}$$

```
[In] Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^3),x]
[Out] -((b*Log[Cos[(e + f*x)/2]])/f) + (a*Log[Cos[e + f*x]])/f + (b*Log[Sin[(e +
f*x)/2]])/f + (a*Log[Tan[e + f*x]])/f + (b*Sec[e + f*x])/f
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{a\ln(\sin(fx+e))+b\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)}{f}$
default	$\frac{a\ln(\sin(fx+e))+b\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)}{f}$
risch	$-iax-\frac{2iae}{f}+\frac{2b\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)}+\frac{\ln\left(e^{i(fx+e)}-1\right)a}{f}+\frac{\ln\left(e^{i(fx+e)}-1\right)b}{f}+\frac{\ln\left(e^{i(fx+e)}+1\right)a}{f}-\frac{\ln\left(e^{i(fx+e)}+1\right)}{f}$

```
[In] int(cot(f*x+e)*(a+b*sec(f*x+e)^3),x,method=_RETURNVERBOSE)
[Out] 1/f*(a*ln(sin(f*x+e))+b*(1/cos(f*x+e)+ln(csc(f*x+e)-cot(f*x+e))))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13

$$\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx = \frac{(a-b)\cos(fx+e)\log\left(\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)+(a+b)\cos(fx+e)\log\left(-\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)+2\,b}{2\,f\cos(fx+e)}$$

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^3),x, algorithm="fricas")
[Out] 1/2*((a - b)*cos(f*x + e)*log(1/2*cos(f*x + e) + 1/2) + (a + b)*cos(f*x + e
)*log(-1/2*cos(f*x + e) + 1/2) + 2*b)/(f*cos(f*x + e))
```

Sympy [F]

$$\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx = \int \left(a+b\sec^3(e+fx)\right)\cot(e+fx)\,dx$$

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)**3),x)
[Out] Integral((a + b*sec(e + f*x)**3)*cot(e + f*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx = \frac{(a-b)\log\left(\cos\left(fx+e\right)+1\right)+(a+b)\log\left(\cos\left(fx+e\right)-1\right)+\frac{2b}{\cos(fx+e)}}{2f}$$

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

```
[Out] 1/2*((a - b)*log(cos(f*x + e) + 1) + (a + b)*log(cos(f*x + e) - 1) + 2*b/co
s(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.61

$$\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx = \frac{(a+b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)-2a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)+\frac{4b}{\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1}}{2f}$$

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

```
[Out] 1/2*((a + b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - 2*a*log(ab
s(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)) + 4*b/((cos(f*x + e) - 1)/(c
os(f*x + e) + 1) + 1))/f
```

Mupad [B] (verification not implemented)

Time = 20.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

$$\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx = \frac{a\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}-\frac{a\ln\left({\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^2+1\right)}{f}-\frac{2b}{f\left({\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^2-1\right)}+\frac{b\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}$$

```
[In] int(cot(e + f*x)*(a + b/cos(e + f*x)^3),x)
```

```
[Out] (a*log(tan(e/2 + (f*x)/2)))/f - (a*log(tan(e/2 + (f*x)/2)^2 + 1))/f - (2*b)
/(f*(tan(e/2 + (f*x)/2)^2 - 1)) + (b*log(tan(e/2 + (f*x)/2)))/f
```

3.456 $\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx=-\frac{(a+b\cos(e+fx))\csc^2(e+fx)}{2f}-\frac{(2a-b)\log(1-\cos(e+fx))}{4f}-\frac{(2a+b)\log(1+\cos(e+fx))}{4f}$$

```
[Out] -1/2*(a+b*cos(f*x+e))*csc(f*x+e)^2/f-1/4*(2*a-b)*ln(1-cos(f*x+e))/f-1/4*(2*
a+b)*ln(1+cos(f*x+e))/f
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4223, 1828, 647, 31}

$$\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx=-\frac{(2a-b)\log(1-\cos(e+fx))}{4f}-\frac{(2a+b)\log(\cos(e+fx)+1)}{4f}-\frac{\csc^2(e+fx)(a+b\cos(e+fx))}{2f}$$

```
[In] Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^3),x]
```

```
[Out] -1/2*((a + b*Cos[e + f*x])*Csc[e + f*x]^2)/f - ((2*a - b)*Log[1 - Cos[e + f
*x]])/(4*f) - ((2*a + b)*Log[1 + Cos[e + f*x]])/(4*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{b+ax^3}{(1-x^2)^2}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{(a+b\cos(e+fx))\csc^2(e+fx)}{2f}+\frac{\text{Subst}\left(\int \frac{-b+2ax}{1-x^2}\,dx, x, \cos(e+fx)\right)}{2f}\\
&= -\frac{(a+b\cos(e+fx))\csc^2(e+fx)}{2f}+\frac{(2a-b)\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \cos(e+fx)\right)}{4f}\\
&\quad+\frac{(2a+b)\text{Subst}\left(\int \frac{1}{-1-x}\,dx, x, \cos(e+fx)\right)}{4f}\\
&= -\frac{(a+b\cos(e+fx))\csc^2(e+fx)}{2f}\\
&\quad-\frac{(2a-b)\log(1-\cos(e+fx))}{4f}-\frac{(2a+b)\log(1+\cos(e+fx))}{4f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.58

$$\begin{aligned}&\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx\\&=-\frac{b\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f}-\frac{b\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2f}+\frac{b\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f}\\&\quad-\frac{a(\cot^2(e+fx)+2\log(\cos(e+fx))+2\log(\tan(e+fx)))}{2f}+\frac{b\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f}\end{aligned}$$

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^3),x]
```

```
[Out] -1/8*(b*Csc[(e + f*x)/2]^2)/f - (b*Log[Cos[(e + f*x)/2]])/(2*f) + (b*Log[Si
n[(e + f*x)/2]])/(2*f) - (a*(Cot[e + f*x]^2 + 2*Log[Cos[e + f*x]] + 2*Log[T
an[e + f*x]]))/(2*f) + (b*Sec[(e + f*x)/2]^2)/(8*f)
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{a\left(-\frac{\cot(fx+e)^2}{2}-\ln(\sin(fx+e))\right)+b\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)}{f}$
default	$\frac{a\left(-\frac{\cot(fx+e)^2}{2}-\ln(\sin(fx+e))\right)+b\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)}{f}$
risch	$iax+\frac{2iae}{f}+\frac{b\,e^{3i(fx+e)}+2a\,e^{2i(fx+e)}+b\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}-1\right)^2}-\frac{\ln\left(e^{i(fx+e)}+1\right)a}{f}-\frac{\ln\left(e^{i(fx+e)}+1\right)b}{2f}-\frac{\ln\left(e^{i(fx+e)}-1\right)a}{f}+$

```
[In] int(cot(f*x+e)^3*(a+b*sec(f*x+e)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*(-1/2*cot(f*x+e)^2-ln(sin(f*x+e)))+b*(-1/2*csc(f*x+e)*cot(f*x+e)+1/2
*ln(csc(f*x+e)-cot(f*x+e))))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.38

$$\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$$
$$=\frac{2\,b\cos\left(fx+e\right)-\left((2\,a+b)\cos\left(fx+e\right)^2-2\,a-b\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-\left((2\,a-b)\cos\left(fx+e\right)^2-\right.}{4\left(f\cos\left(fx+e\right)^2-f\right)}$$

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^3),x, algorithm="fricas")
[Out] 1/4*(2*b*cos(f*x + e) - ((2*a + b)*cos(f*x + e)^2 - 2*a - b)*log(1/2*cos(f*
x + e) + 1/2) - ((2*a - b)*cos(f*x + e)^2 - 2*a + b)*log(-1/2*cos(f*x + e)
+ 1/2) + 2*a)/(f*cos(f*x + e)^2 - f)
```

Sympy [F]

$$\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx=\int\left(a+b\sec^3\left(e+fx\right)\right)\cot^3\left(e+fx\right)dx$$

```
[In] integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**3),x)
[Out] Integral((a + b*sec(e + f*x)**3)*cot(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$$
$$=-\frac{(2\,a+b)\log\left(\cos\left(fx+e\right)+1\right)+(2\,a-b)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\,(b\cos(fx+e)+a)}{\cos(fx+e)^2-1}}{4\,f}$$

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^3),x, algorithm="maxima")
[Out] -1/4*((2*a + b)*log(cos(f*x + e) + 1) + (2*a - b)*log(cos(f*x + e) - 1) - 2
*(b*cos(f*x + e) + a)/(cos(f*x + e)^2 - 1))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(66) = 132$.

Time = 0.30 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.39

$$\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx = -\frac{2\,(2\,a-b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)-8\,a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)-\frac{\left(a+b+\frac{4\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)(\cos(fx+e)\cdots}{\cos(fx+e)-1}}{8\,f}$$

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

```
[Out] -1/8*(2*(2*a - b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - 8*a*l
og(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)) - (a + b + 4*a*(cos(f*x
 + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))*
(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - a*(cos(f*x + e) - 1)/(cos(f*x + e)
+ 1) + b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/f
```

Mupad [B] (verification not implemented)

Time = 20.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.19

$$\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx = \frac{a\,\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}{f} - \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\left(\frac{a}{8}-\frac{b}{8}\right)}{f} - \frac{\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\left(\frac{a}{8}+\frac{b}{8}\right)}{f} - \frac{\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(a-\frac{b}{2}\right)}{f}$$

```
[In] int(cot(e + f*x)^3*(a + b/cos(e + f*x)^3),x)
```

```
[Out] (a*log(tan(e/2 + (f*x)/2)^2 + 1))/f - (tan(e/2 + (f*x)/2)^2*(a/8 - b/8))/f
- (cot(e/2 + (f*x)/2)^2*(a/8 + b/8))/f - (log(tan(e/2 + (f*x)/2))*(a - b/2)
)/f
```

3.457 $\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 219

$$\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\left(a^{2/3}+2b^{2/3}\right)\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{4/3}f} - \frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}b^{4/3}f} + \frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+a^{2/3}\cos^2(e+fx)\right)}{6\sqrt[3]{a}b^{4/3}f} - \frac{\log\left(b+a\cos^3(e+fx)\right)}{3af} + \frac{\sec(e+fx)}{bf}$$

```
[Out] -1/3*(a^(2/3)-2*b^(2/3))*ln(b^(1/3)+a^(1/3)*cos(f*x+e))/a^(1/3)/b^(4/3)/f+1
/6*(a^(2/3)-2*b^(2/3))*ln(b^(2/3)-a^(1/3)*b^(1/3)*cos(f*x+e)+a^(2/3)*cos(f*
x+e)^2)/a^(1/3)/b^(4/3)/f-1/3*ln(b+a*cos(f*x+e)^3)/a/f+sec(f*x+e)/b/f-1/3*(
a^(2/3)+2*b^(2/3))*arctan(1/3*(b^(1/3)-2*a^(1/3)*cos(f*x+e))/b^(1/3)*3^(1/2
))/a^(1/3)/b^(4/3)/f*3^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4223, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx$$
$$=-\frac{\left(a^{2/3}+2b^{2/3}\right)\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{4/3}f}+\frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}\right)}{6\sqrt[3]{a}b^{4/3}f}-\frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(\sqrt[3]{a}\cos(e+fx)+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{4/3}f}-\frac{\log\left(a\cos^3(e+fx)+b\right)}{3af}+\frac{\sec(e+fx)}{bf}$$

```
[In] Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] -(((a^(2/3) + 2*b^(2/3))*ArcTan[(b^(1/3) - 2*a^(1/3)*Cos[e + f*x])/(Sqrt[3]
*b^(1/3))])/(Sqrt[3]*a^(1/3)*b^(4/3)*f)) - ((a^(2/3) - 2*b^(2/3))*Log[b^(1/
3) + a^(1/3)*Cos[e + f*x]])/(3*a^(1/3)*b^(4/3)*f) + ((a^(2/3) - 2*b^(2/3))*
Log[b^(2/3) - a^(1/3)*b^(1/3)*Cos[e + f*x] + a^(2/3)*Cos[e + f*x]^2])/(6*a^
(1/3)*b^(4/3)*f) - Log[b + a*Cos[e + f*x]^3]/(3*a*f) + Sec[e + f*x]/(b*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^2(b+ax^3)}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{bx^2} + \frac{-2b-ax+bx^2}{b(b+ax^3)}\right)\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{\sec(e+fx)}{bf} - \frac{\text{Subst}\left(\int \frac{-2b-ax+bx^2}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{bf}$$

$$= \frac{\sec(e+fx)}{bf} - \frac{\text{Subst}\left(\int \frac{x^2}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{f} - \frac{\text{Subst}\left(\int \frac{-2b-ax}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{bf}$$

$$= -\frac{\log\left(b+a\cos^3(e+fx)\right)}{3af} + \frac{\sec(e+fx)}{bf} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}\left(-a\sqrt[3]{b}-4\sqrt[3]{a}b\right)+\sqrt[3]{a}\left(-a\sqrt[3]{b}+2\sqrt[3]{a}b\right)x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx, x, \cos(e+fx)\right)}{3\sqrt[3]{a}b^{5/3}f} - \frac{\left(a^{2/3}-2b^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}x}\,dx, x, \cos(e+fx)\right)}{3b^{4/3}f}$$

$$= -\frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}b^{4/3}f} - \frac{\log\left(b+a\cos^3(e+fx)\right)}{3af} + \frac{\sec(e+fx)}{bf} + \frac{\left(a^{2/3}-2b^{2/3}\right)\text{Subst}\left(\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx, x, \cos(e+fx)\right)}{6\sqrt[3]{a}b^{4/3}f} + \frac{\left(a^{2/3}+2b^{2/3}\right)\text{Subst}\left(\int \frac{1}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx, x, \cos(e+fx)\right)}{2bf}$$

$$= -\frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}b^{4/3}f} + \frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+a^{2/3}\cos^2(e+fx)\right)}{6\sqrt[3]{a}b^{4/3}f} - \frac{\log\left(b+a\cos^3(e+fx)\right)}{3af} + \frac{\sec(e+fx)}{bf} + \frac{\left(a^{2/3}+2b^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{a}\cos(e+fx)}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}b^{4/3}f}$$

$$= -\frac{\left(a^{2/3}+2b^{2/3}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{a}\cos(e+fx)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}b^{4/3}f} - \frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}b^{4/3}f} + \frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+a^{2/3}\cos^2(e+fx)\right)}{6\sqrt[3]{a}b^{4/3}f} - \frac{\log\left(b+a\cos^3(e+fx)\right)}{3af} + \frac{\sec(e+fx)}{bf}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.89 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.15

$$\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx = \frac{3b\log\left(\sec^2\left(\frac{1}{2}(e+fx)\right)\right) - \text{RootSum}\left[-8a+12a\#1-6a\#1^2+a\#1^3-b\#1^3\&, \frac{-4a^2\log\left(1-\#1+\tan^2\left(\frac{1}{2}(e+f\right.\right.}{}\right.}{}$$

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] (3*b*Log[Sec[(e + f*x)/2]^2] - RootSum[-8*a + 12*a*#1 - 6*a*#1^2 + a*#1^3 -
 b*#1^3 & , (-4*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2] + 4*a*b*Log[1 - #1 + T
an[(e + f*x)/2]^2] + 2*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1 - 8*a*b*Log[
1 - #1 + Tan[(e + f*x)/2]^2]*#1 + a*b*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1^2
 - b^2*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1^2)/(4*a - 4*a*#1 + a*#1^2 - b*#1
^2) & ] + 3*a*Sec[e + f*x])/(3*a*b*f)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.02 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.06

method	result
risch	$\frac{ix}{a}+\frac{2ie}{af}+\frac{2\,\mathrm{e}^{i(fx+e)}}{fb\left(\mathrm{e}^{2i(fx+e)}+1\right)}-i\left(\sum_{\_R=\mathrm{RootOf}\left(27a^3b^4f^3\_Z^3+27ia^2b^4f^2\_Z^2+\left(-18a^3b^2f-9a\,b^4f\right)\_Z-ia^4+2ia^2\right.}\right.$
derivativedivides	$\frac{1}{b\cos(fx+e)}+\frac{2b\left(\frac{\ln\left(\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\cos(fx+e)}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}\right)+a}{}$
default	$\frac{1}{b\cos(fx+e)}+\frac{2b\left(\frac{\ln\left(\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\cos(fx+e)}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}\right)+a}{}$

```
[In] int(tan(f*x+e)^5/(a+b*sec(f*x+e)^3),x,method=_RETURNVERBOSE)
```

```
[Out] I*x/a+2*I/a/f*e+2*exp(I*(f*x+e))/f/b/(exp(2*I*(f*x+e))+1)-I*sum(_R*ln(exp(2
*I*(f*x+e))+(-18/(a^3+8*a*b^2)*a^2*b^3*f^2*_R^2-36*I/(a^3+8*a*b^2)*b^3*f*a*
_R+8/(a^3+8*a*b^2)*a^2*b+10/(a^3+8*a*b^2)*b^3)*exp(I*(f*x+e))+1),_R=RootOf(
27*a^3*b^4*f^3*_Z^3+27*I*a^2*b^4*f^2*_Z^2+(-18*a^3*b^2*f-9*a*b^4*f)*_Z-I*a^
4+2*I*a^2*b^2-I*b^4))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 4427, normalized size of antiderivative = 20.21

$$\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

```
[Out] -1/36*(2*((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/
27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2
```

f^3) - 1/54(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) +
1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(
a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))
*a*b*f*cos(f*x + e)*log(1/36*((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)
/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2
*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/
3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) +
1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^
3))^(1/3) + 6/(a*f))^2*a^2*b^3*f^2 - ((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^
2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) +
 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f
^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4
f^3) + 1/18(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^
3*b^4*f^3))^(1/3) + 6/(a*f))*a*b^3*f + 4*a^2*b + 5*b^3 + (a^3 + 8*a*b^2)*co
s(f*x + e)) - (((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2)
)/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a
^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt
(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 +
b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/
(a*f))*a*b*f*cos(f*x + e) + 3*sqrt(1/3)*a*b*f*sqrt(-(((-I*sqrt(3) + 1)*(1/(
a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^
2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 +
 b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2
 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^
2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))^2*a^2*b^2*f^2 - 12*((-I*sqrt(3
) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*
(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 -
2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3)
+ 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*
(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))*a*b^2*f + 288*a^2 +
 36*b^2)/(a^2*b^2*f^2))*cos(f*x + e) - 18*b*cos(f*x + e))*log(1/36*((-I*sqr
t(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/
54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4
 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^
3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/
54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))^2*a^2*b^3*f^2 -
((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f
^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1
/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27
/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f
^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))*a*b^3*f
+ 4*a^2*b + 5*b^3 - 1/12*sqrt(1/3)*(((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2
 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) +
1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^
3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*

f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))*a^2*b^3*f^2 + 18*a*b^3*f)*sqrt(-(((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))^2*a^2*b^2*f^2 - 12*((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))*a*b^2*f + 288*a^2 + 36*b^2)/(a^2*b^2*f^2)) - 2*(a^3 + 8*a*b^2)*cos(f*x + e)) - (((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))*a*b*f*cos(f*x + e) - 3*sqrt(1/3)*a*b*f*sqrt(-(((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))^2*a^2*b^2*f^2 - 12*((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))*a*b^2*f + 288*a^2 + 36*b^2)/(a^2*b^2*f^2))*cos(f*x + e) - 18*b*cos(f*x + e))*log(-1/36*((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))^2*a^2*b^3*f^2 + ((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))*a*b^3*f - 4*a^2*b - 5*b^3 - 1/12*sqrt(1/3)*(((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6

```
/(a*f))*a^2*b^3*f^2 + 18*a*b^3*f)*sqrt(-(((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (
2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^
3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b
^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a
*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)
/(a^3*b^4*f^3))^(1/3) + 6/(a*f))^2*a^2*b^2*f^2 - 12*((-I*sqrt(3) + 1)*(1/(a
^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2
)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 +
b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2
+ 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2
*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))*a*b^2*f + 288*a^2 + 36*b^2)/(a^
2*b^2*f^2)) + 2*(a^3 + 8*a*b^2)*cos(f*x + e)) - 36*a)/(a*b*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx = \int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx$$

```
[In] integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**3),x)
```

```
[Out] Integral(tan(e + f*x)**5/(a + b*sec(e + f*x)**3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00

$$\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx = \frac{\frac{2\sqrt{3}\left(2ab\left(3\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{b}{a}\right)+3a^2\left(\frac{b}{a}\right)^{\frac{2}{3}}+2b^2\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}-2\cos(fx+e)\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{ab^2}-\frac{3\left(2b\left(\left(\frac{b}{a}\right)^{\frac{2}{3}}+1\right)-a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)\log\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cdots\right.}{ab\left(\frac{b}{a}\right)^{\frac{2}{3}}}}{18f}$$

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

```
[Out] 1/18*(2*sqrt(3)*(2*a*b*(3*(b/a)^(1/3) - b/a) + 3*a^2*(b/a)^(2/3) + 2*b^2)*a
rctan(-1/3*sqrt(3)*((b/a)^(1/3) - 2*cos(f*x + e))/(b/a)^(1/3))/(a*b^2) - 3*
(2*b*((b/a)^(2/3) + 1) - a*(b/a)^(1/3))*log(cos(f*x + e)^2 - (b/a)^(1/3)*co
s(f*x + e) + (b/a)^(2/3))/(a*b*(b/a)^(2/3)) - 6*(b*((b/a)^(2/3) - 2) + a*(b
/a)^(1/3))*log((b/a)^(1/3) + cos(f*x + e))/(a*b*(b/a)^(2/3)) + 18/(b*cos(f*
x + e)))/f
```

Giac [F]

$$\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx=\int \frac{\tan(fx+e)^5}{b\sec(fx+e)^3+a}\,dx$$

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 22.95 (sec) , antiderivative size = 7402, normalized size of antiderivative = 33.80

$$\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx=\text{Too large to display}$$

```
[In] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^3),x)
```

```
[Out] symsum(log(-(262144*(148*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z +
 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*b^17 - 1920*a*b^15 - 156*b^16*
cos(e + f*x) + 300*b^16 + 16*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4
*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*b^18 + 5232*a^2*b^14 - 7
872*a^3*b^13 + 7080*a^4*b^12 - 3840*a^5*b^11 + 1200*a^6*b^10 - 192*a^7*b^9
+ 12*a^8*b^8 - 5916*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a
^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^2*b^15 + 4820*root(27*a^3*b^4*z^3
 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)
*a^3*b^14 + 5933*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*
b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^4*b^13 - 12882*root(27*a^3*b^4*z^3 +
 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a
^5*b^12 + 8891*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^
2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^6*b^11 - 2872*root(27*a^3*b^4*z^3 + 27
*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^7*
b^10 + 447*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z
- 2*a^2*b^2 + b^4 + a^4, z, k)*a^8*b^9 - 26*root(27*a^3*b^4*z^3 + 27*a^2*b^
4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^9*b^8 + r
oot(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2
+ b^4 + a^4, z, k)*a^10*b^7 + 1396*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9
*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a*b^17 + 192*root(
27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^
4 + a^4, z, k)^3*a*b^18 - 3936*a^2*b^14*cos(e + f*x) + 7152*a^3*b^13*cos(e
+ f*x) - 7800*a^4*b^12*cos(e + f*x) + 5136*a^5*b^11*cos(e + f*x) - 1920*a^6
*b^10*cos(e + f*x) + 336*a^7*b^9*cos(e + f*x) - 12*a^8*b^8*cos(e + f*x) - 7
68*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*
b^2 + b^4 + a^4, z, k)^2*a^2*b^16 + 4772*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z
```

^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^3*b^15 - 1
3924*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^
2*b^2 + b^4 + a^4, z, k)^2*a^4*b^14 + 6927*root(27*a^3*b^4*z^3 + 27*a^2*b^4
*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^5*b^13 +
 5747*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a
^2*b^2 + b^4 + a^4, z, k)^2*a^6*b^12 - 5944*root(27*a^3*b^4*z^3 + 27*a^2*b^
4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^7*b^11
+ 2004*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*
a^2*b^2 + b^4 + a^4, z, k)^2*a^8*b^10 - 239*root(27*a^3*b^4*z^3 + 27*a^2*b^
4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^9*b^9 +
 13*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2
*b^2 + b^4 + a^4, z, k)^2*a^10*b^8 + 4296*root(27*a^3*b^4*z^3 + 27*a^2*b^4*
z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^2*b^17 -
11856*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a
^2*b^2 + b^4 + a^4, z, k)^3*a^3*b^16 + 16956*root(27*a^3*b^4*z^3 + 27*a^2*b
^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^4*b^15
 - 17916*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z -
2*a^2*b^2 + b^4 + a^4, z, k)^3*a^5*b^14 + 11175*root(27*a^3*b^4*z^3 + 27*a^
2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^6*b
^13 - 4608*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z
- 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^7*b^12 + 2118*root(27*a^3*b^4*z^3 + 27*a
^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^8*
b^11 - 372*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z
- 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^9*b^10 + 15*root(27*a^3*b^4*z^3 + 27*a^2
*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^10*b
^9 + 864*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z -
2*a^2*b^2 + b^4 + a^4, z, k)^4*a^2*b^18 + 3240*root(27*a^3*b^4*z^3 + 27*a^2
*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^3*b^
17 - 12996*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z
- 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^4*b^16 + 4140*root(27*a^3*b^4*z^3 + 27*a
^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^5*
b^15 + 16668*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*
z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^6*b^14 - 16011*root(27*a^3*b^4*z^3 + 2
7*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a
^7*b^13 + 4959*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^
2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^8*b^12 - 873*root(27*a^3*b^4*z^3 + 2
7*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a
^9*b^11 + 9*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z
 - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^10*b^10 + 1728*root(27*a^3*b^4*z^3 + 27
*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^5*a^
3*b^18 - 5724*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2
*z - 2*a^2*b^2 + b^4 + a^4, z, k)^5*a^4*b^17 + 6912*root(27*a^3*b^4*z^3 + 2
7*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^5*a
^5*b^16 - 3024*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^
2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^5*a^6*b^15 - 1080*root(27*a^3*b^4*z^3 +

27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^5*
a^7*b^14 + 1836*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b
^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^5*a^8*b^13 - 648*root(27*a^3*b^4*z^3 +
27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^5*
a^9*b^12 + 1296*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b
^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^6*a^4*b^18 - 7452*root(27*a^3*b^4*z^3 +
27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^6
*a^5*b^17 + 14904*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3
*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^6*a^6*b^16 - 12960*root(27*a^3*b^4*z^
3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k
)^6*a^7*b^15 + 4536*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a
^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^6*a^8*b^14 - 324*root(27*a^3*b^4*z^
3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k
)^6*a^9*b^13 + 1456*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a
^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a*b^16 - 52*root(27*a^3*b^4*z^3 + 2
7*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*b^1
7*cos(e + f*x) + 1200*a*b^15*cos(e + f*x) - 880*root(27*a^3*b^4*z^3 + 27*a^
2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a*b^16*
cos(e + f*x) + 4764*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a
^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^2*b^15*cos(e + f*x) - 6932*root(2
7*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4
+ a^4, z, k)*a^3*b^14*cos(e + f*x) - 1109*root(27*a^3*b^4*z^3 + 27*a^2*b^4
*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^4*b^13*cos
(e + f*x) + 12234*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3
*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^5*b^12*cos(e + f*x) - 12299*root(27
*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4
+ a^4, z, k)*a^6*b^11*cos(e + f*x) + 5032*root(27*a^3*b^4*z^3 + 27*a^2*b^4*
z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^7*b^10*cos(
e + f*x) - 807*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^
2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^8*b^9*cos(e + f*x) + 50*root(27*a^3*b^
4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4,
z, k)*a^9*b^8*cos(e + f*x) - root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4
*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^10*b^7*cos(e + f*x) - 54
8*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b
^2 + b^4 + a^4, z, k)^2*a*b^17*cos(e + f*x) + 160*root(27*a^3*b^4*z^3 + 27*
a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^2
*b^16*cos(e + f*x) - 1380*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z
+ 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^3*b^15*cos(e + f*x) + 121
40*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*
b^2 + b^4 + a^4, z, k)^2*a^4*b^14*cos(e + f*x) - 14767*root(27*a^3*b^4*z^3
+ 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^
2*a^5*b^13*cos(e + f*x) - 1659*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b
^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^6*b^12*cos(e + f*x)
+ 9272*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*
a^2*b^2 + b^4 + a^4, z, k)^2*a^7*b^11*cos(e + f*x) - 3691*root(27*a^3*b^4*z

^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^8*b^10*cos(e + f*x) + 510*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^9*b^9*cos(e + f*x) - 38*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^10*b^8*cos(e + f*x) + root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^11*b^7*cos(e + f*x) - 1992*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^2*b^17*cos(e + f*x) + 8112*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^3*b^16*cos(e + f*x) - 18300*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^4*b^15*cos(e + f*x) + 19788*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^5*b^14*cos(e + f*x) - 10095*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^6*b^13*cos(e + f*x) + 6000*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^7*b^12*cos(e + f*x) - 4134*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^8*b^11*cos(e + f*x) + 660*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^9*b^10*cos(e + f*x) - 39*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^10*b^9*cos(e + f*x) - 2376*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^3*b^17*cos(e + f*x) + 11124*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^4*b^16*cos(e + f*x) - 10044*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^5*b^15*cos(e + f*x) - 10260*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^6*b^14*cos(e + f*x) + 19899*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^7*b^13*cos(e + f*x) - 10287*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^8*b^12*cos(e + f*x) + 2025*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^9*b^11*cos(e + f*x) - 81*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^10*b^10*cos(e + f*x) + 1404*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^5*a^4*b^17*cos(e + f*x) - 6048*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^5*a^5*b^16*cos(e + f*x) + 8208*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^5*a^6*b^15*cos(e + f*x) - 2376*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^5*a^7*b^14*cos(e + f*x) - 2700*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^5*a^8*b^13*cos(e + f*x) + 1512*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^5*a^9*b^12*cos(e + f*x) +

```
3564*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^
2*b^2 + b^4 + a^4, z, k)^6*a^5*b^17*cos(e + f*x) - 12312*root(27*a^3*b^4*z^
3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k
)^6*a^6*b^16*cos(e + f*x) + 15552*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*
a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^6*a^7*b^15*cos(e + f*
x) - 8424*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z -
 2*a^2*b^2 + b^4 + a^4, z, k)^6*a^8*b^14*cos(e + f*x) + 1620*root(27*a^3*b^
4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4,
z, k)^6*a^9*b^13*cos(e + f*x)))/cos(e/2 + (f*x)/2)^2)*root(27*a^3*b^4*z^3 +
 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k),
k, 1, 3)/f + log(1/cos(e/2 + (f*x)/2)^2)/(a*f) + 1/(b*f*cos(e + f*x))
```

3.458 $\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 166

$$\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = \frac{\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}f} - \frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}b^{2/3}f} + \frac{\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+a^{2/3}\cos^2(e+fx)\right)}{6\sqrt[3]{a}b^{2/3}f} + \frac{\log\left(b+a\cos^3(e+fx)\right)}{3af}$$

```
[Out] -1/3*ln(b^(1/3)+a^(1/3)*cos(f*x+e))/a^(1/3)/b^(2/3)/f+1/6*ln(b^(2/3)-a^(1/3
)*b^(1/3)*cos(f*x+e)+a^(2/3)*cos(f*x+e)^2)/a^(1/3)/b^(2/3)/f+1/3*ln(b+a*cos
(f*x+e)^3)/a/f+1/3*arctan(1/3*(b^(1/3)-2*a^(1/3)*cos(f*x+e))/b^(1/3)*3^(1/2
))/a^(1/3)/b^(2/3)/f*3^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {4223, 1885, 206, 31, 648, 631, 210, 642, 266}

$$\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = \frac{\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}\right)}{6\sqrt[3]{a}b^{2/3}f} + \frac{\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}f} - \frac{\log\left(\sqrt[3]{a}\cos(e+fx)+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}f} + \frac{\log\left(a\cos^3(e+fx)+b\right)}{3af}$$

```
[In] Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] ArcTan[(b^(1/3) - 2*a^(1/3)*Cos[e + f*x])/(Sqrt[3]*b^(1/3))]/(Sqrt[3]*a^(1/
3)*b^(2/3)*f) - Log[b^(1/3) + a^(1/3)*Cos[e + f*x]]/(3*a^(1/3)*b^(2/3)*f) +
 Log[b^(2/3) - a^(1/3)*b^(1/3)*Cos[e + f*x] + a^(2/3)*Cos[e + f*x]^2]/(6*a^
(1/3)*b^(2/3)*f) + Log[b + a*Cos[e + f*x]^3]/(3*a*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1-x^2}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{f} + \frac{\text{Subst}\left(\int \frac{x^2}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{\log(b+a\cos^3(e+fx))}{3af} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}x}dx, x, \cos(e+fx)\right)}{3b^{2/3}f} - \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}dx, x, \cos(e+fx)\right)}{3b^{2/3}f}$$

$$= -\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}b^{2/3}f} + \frac{\log(b+a\cos^3(e+fx))}{3af} + \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}dx, x, \cos(e+fx)\right)}{6\sqrt[3]{a}b^{2/3}f} - \frac{\text{Subst}\left(\int \frac{1}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}dx, x, \cos(e+fx)\right)}{2\sqrt[3]{b}f}$$

$$= -\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}b^{2/3}f} + \frac{\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+a^{2/3}\cos^2(e+fx)\right)}{6\sqrt[3]{a}b^{2/3}f} + \frac{\log(b+a\cos^3(e+fx))}{3af} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}dx, x, 1-\frac{2\sqrt[3]{a}\cos(e+fx)}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}b^{2/3}f}$$

$$= \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{a}\cos(e+fx)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}f} - \frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}b^{2/3}f} + \frac{\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+a^{2/3}\cos^2(e+fx)\right)}{6\sqrt[3]{a}b^{2/3}f} + \frac{\log(b+a\cos^3(e+fx))}{3af}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.64 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.46

$$\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}dx$$

$$= \frac{-3\log\left(\sec^2\left(\frac{1}{2}(e+fx)\right)\right) + \text{RootSum}\left[-a-b+3a\#1-3b\#1-3a\#1^2-3b\#1^2+a\#1^3-b\#1^3\&, \frac{-a\log}{}\right.}{}$$

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^3),x]
```

[Out] (-3*Log[Sec[(e + f*x)/2]^2] + RootSum[-a - b + 3*a*#1 - 3*b*#1 - 3*a*#1^2 - 3*b*#1^2 + a*#1^3 - b*#1^3 & , (-(a*Log[-#1 + Tan[(e + f*x)/2]^2]) - b*Log[-#1 + Tan[(e + f*x)/2]^2] - 4*a*Log[-#1 + Tan[(e + f*x)/2]^2]*#1 - 2*b*Log[-#1 + Tan[(e + f*x)/2]^2]*#1 + a*Log[-#1 + Tan[(e + f*x)/2]^2]*#1^2 - b*Log[-#1 + Tan[(e + f*x)/2]^2]*#1^2)/(a - b - 2*a*#1 - 2*b*#1 + a*#1^2 - b*#1^2) &])/(3*a*f)

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.38 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{ix}{a}-\frac{2ie}{af}+i\left(\sum_{\_R=\text{RootOf}\left(27b^2a^3f^3\_Z^3+27ib^2a^2f^2\_Z^2-9\_Z a\, b^2f+ia^2-ib^2\right)}\_R\ln\left(e^{2i(fx+e)}+\left(-6\right.\right.\right.$
derivativedivides	$\frac{-\frac{\ln\left(\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\ln\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\cos(fx+e)}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\ln\left(b+a\cos(fx+e\right.}{3a}}{f}$
default	$\frac{-\frac{\ln\left(\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\ln\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\cos(fx+e)}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\ln\left(b+a\cos(fx+e\right.}{3a}}{f}$

[In] int(tan(f*x+e)^3/(a+b*sec(f*x+e)^3),x,method=_RETURNVERBOSE)

[Out] -I*x/a-2*I/a/f*e+I*sum(_R*ln(exp(2*I*(f*x+e))+(-6*I*b*f*_R+2*b/a)*exp(I*(f*x+e))+1),_R=RootOf(27*b^2*a^3*f^3*_Z^3+27*I*b^2*a^2*f^2*_Z^2-9*_Z*a*b^2*f+I*a^2-I*b^2))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 1052, normalized size of antiderivative = 6.34

$$\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = \text{Too large to display}$$

[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="fricas")

```
[Out] -1/12*(2*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2
 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f*log(-1/2*(3*(I*sqrt(3) + 1)*(-1
/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) -
2/(a*f))*a*b*f - a*cos(f*x + e) - b) - ((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3)
 + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f
+ 3*sqrt(1/3)*a*f*sqrt(-((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*
f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*f^2 + 4*(3*(I
*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b
^2*f^3))^(1/3) - 2/(a*f))*a*f + 4)/(a^2*f^2)) + 6)*log(1/2*(3*(I*sqrt(3) +
1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1
/3) - 2/(a*f))*a*b*f + 3/2*sqrt(1/3)*a*b*f*sqrt(-((3*(I*sqrt(3) + 1)*(-1/54
/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(
a*f))^2*a^2*f^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3)
- 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f + 4)/(a^2*f^2)) - 2*
a*cos(f*x + e) + b) - ((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^
3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f - 3*sqrt(1/3)*a*f
*sqrt(-((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2
- b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*f^2 + 4*(3*(I*sqrt(3) + 1)*(-1
/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) -
2/(a*f))*a*f + 4)/(a^2*f^2)) + 6)*log(-1/2*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f
^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a
*b*f + 3/2*sqrt(1/3)*a*b*f*sqrt(-((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/5
4/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*f^2
+ 4*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^
2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f + 4)/(a^2*f^2)) + 2*a*cos(f*x + e) -
 b))/(a*f)
```

Sympy [F]

$$\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx=\int \frac{\tan^3{(e+fx)}}{a+b\sec^3{(e+fx)}}\,dx$$

```
[In] integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**3),x)
```

```
[Out] Integral(tan(e + f*x)**3/(a + b*sec(e + f*x)**3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.96

$$\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\frac{2\sqrt{3}\left(a\left(3\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{2b}{a}\right)+2b\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}-2\cos(fx+e)\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{ab} - \frac{3\left(2\left(\frac{b}{a}\right)^{\frac{2}{3}}+1\right)\log\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{a\left(\frac{b}{a}\right)^{\frac{2}{3}}} - \cdots}{18\,f}$$

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

```
[Out] -1/18*(2*sqrt(3)*(a*(3*(b/a)^(1/3) - 2*b/a) + 2*b)*arctan(-1/3*sqrt(3)*((b/
a)^(1/3) - 2*cos(f*x + e))/(b/a)^(1/3))/(a*b) - 3*(2*(b/a)^(2/3) + 1)*log(c
os(f*x + e)^2 - (b/a)^(1/3)*cos(f*x + e) + (b/a)^(2/3))/(a*(b/a)^(2/3)) - 6
*((b/a)^(2/3) - 1)*log((b/a)^(1/3) + cos(f*x + e))/(a*(b/a)^(2/3)))/f
```

Giac [F]

$$\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = \int \frac{\tan(fx+e)^3}{b\sec(fx+e)^3+a}\,dx$$

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 24.32 (sec) , antiderivative size = 1620, normalized size of antiderivative = 9.76

$$\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = \text{Too large to display}$$

```
[In] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^3),x)
```

```
[Out] symsum(log(262144*(a - b)^2*(8*a - 8*b + 4*root(27*a^3*b^2*z^3 - 27*a^2*b^2
*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*a^2 + 4*root(27*a^3*b^2*z^3 - 27*a^2*b^
2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*b^2 - 3*root(27*a^3*b^2*z^3 - 27*a^2*b
^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a^3 - 24*root(27*a^3*b^2*z^3 - 27*a
^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a*b^2 - 36*root(27*a^3*b^2*z^3
```

```
- 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^3*a^3*b + 28*root(27*a^3*b^
2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*a*b + 36*root(27*a^3*
b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^3*a^2*b^2)*(16*a^2*
tan(e/2 + (f*x)/2)^2 + 32*b^2*tan(e/2 + (f*x)/2)^2 - 4*root(27*a^3*b^2*z^3
- 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*a^3 - 4*root(27*a^3*b^2*z^3
 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*b^3 - 8*a^2 + 8*b^2 + 3*ro
ot(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a^4 - 3
*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a^4*
tan(e/2 + (f*x)/2)^2 + 24*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z
+ a^2 - b^2, z, k)^2*a*b^3 + 3*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b
^2*z + a^2 - b^2, z, k)^2*a^3*b + 36*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 +
 9*a*b^2*z + a^2 - b^2, z, k)^3*a^4*b - 48*a*b*tan(e/2 + (f*x)/2)^2 + 24*ro
ot(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a^2*b^2
 - 36*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^3
*a^2*b^3 + 14*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2,
 z, k)*a^3*tan(e/2 + (f*x)/2)^2 - 4*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 +
9*a*b^2*z + a^2 - b^2, z, k)*b^3*tan(e/2 + (f*x)/2)^2 - 32*root(27*a^3*b^2*
z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*a*b^2 - 32*root(27*a^3*
b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*a^2*b - 146*root(27
*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*a*b^2*tan(e/2
+ (f*x)/2)^2 + 64*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 -
b^2, z, k)*a^2*b*tan(e/2 + (f*x)/2)^2 + 24*root(27*a^3*b^2*z^3 - 27*a^2*b^2
*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a*b^3*tan(e/2 + (f*x)/2)^2 + 57*root(
27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a^3*b*tan(
e/2 + (f*x)/2)^2 - 54*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^
2 - b^2, z, k)^3*a^4*b*tan(e/2 + (f*x)/2)^2 + 84*root(27*a^3*b^2*z^3 - 27*a
^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a^2*b^2*tan(e/2 + (f*x)/2)^2 -
36*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^3*a^
2*b^3*tan(e/2 + (f*x)/2)^2 + 198*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a
*b^2*z + a^2 - b^2, z, k)^3*a^3*b^2*tan(e/2 + (f*x)/2)^2))*root(27*a^3*b^2*
z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k), k, 1, 3)/f - log(tan(e
/2 + (f*x)/2)^2 + 1)/(a*f)
```

3.459 $\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 23

$$\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\log\left(b+a\cos^3(e+fx)\right)}{3af}$$

```
[Out] -1/3*ln(b+a*cos(f*x+e)^3)/a/f
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4223, 266}

$$\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\log\left(a\cos^3(e+fx)+b\right)}{3af}$$

```
[In] Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] -1/3*Log[b + a*Cos[e + f*x]^3]/(a*f)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
```

```
x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^2}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\log\left(b+a\cos^3(e+fx)\right)}{3af}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\log\left(b+a\cos^3(e+fx)\right)}{3af}$$

```
[In] Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^3),x]
[Out] -1/3*Log[b + a*Cos[e + f*x]^3]/(a*f)
```

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

method	result	size
derivativedivides	$\frac{\frac{\ln(\sec(fx+e))}{a}-\frac{\ln\left(a+b\sec(fx+e)^3\right)}{3a}}{f}$	35
default	$\frac{\frac{\ln(\sec(fx+e))}{a}-\frac{\ln\left(a+b\sec(fx+e)^3\right)}{3a}}{f}$	35
risch	$\frac{ix}{a}+\frac{2ie}{af}-\frac{\ln\left(e^{6i(fx+e)}+3\,e^{4i(fx+e)}+\frac{8b\,e^{3i(fx+e)}}{a}+3\,e^{2i(fx+e)}+1\right)}{3af}$	76

```
[In] int(tan(f*x+e)/(a+b*sec(f*x+e)^3),x,method=_RETURNVERBOSE)
[Out] 1/f*(1/a*ln(sec(f*x+e))-1/3/a*ln(a+b*sec(f*x+e)^3))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\log\left(a\cos\left(fx+e\right)^3+b\right)}{3\,af}$$

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="fricas")
[Out] -1/3*log(a*cos(f*x + e)^3 + b)/(a*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(19) = 38.

Time = 18.05 (sec) , antiderivative size = 139, normalized size of antiderivative = 6.04

$$\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx$$

$$= \begin{cases} \frac{\tilde{\infty}x\tan(e)}{\sec^3(e)} & \text{for } a=0 \wedge b=0 \\ -\frac{1}{3bf\sec^3(e+fx)} & \text{for } a=0 \\ \frac{\log(\tan^2(e+fx)+1)}{2af} & \text{for } b=0 \\ \frac{x\tan(e)}{a+b\sec^3(e)} & \text{for } f=0 \\ -\frac{\log\left(-\sqrt[3]{-\frac{a}{b}}+\sec(e+fx)\right)}{3af}+\frac{\log(\tan^2(e+fx)+1)}{2af}-\frac{\log\left(4\left(-\frac{a}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-\frac{a}{b}}\sec(e+fx)+4\sec^2(e+fx)\right)}{3af} & \text{otherwise} \end{cases}$$

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)**3),x)
[Out] Piecewise((zoo*x*tan(e)/sec(e)**3, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (-1/(3*
b*f*sec(e + f*x)**3), Eq(a, 0)), (log(tan(e + f*x)**2 + 1)/(2*a*f), Eq(b, 0
)), (x*tan(e)/(a + b*sec(e)**3), Eq(f, 0)), (-log(-(-a/b)**(1/3) + sec(e +
f*x))/(3*a*f) + log(tan(e + f*x)**2 + 1)/(2*a*f) - log(4*(-a/b)**(2/3) + 4*
(-a/b)**(1/3)*sec(e + f*x) + 4*sec(e + f*x)**2)/(3*a*f), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\log\left(a\cos\left(fx+e\right)^3+b\right)}{3\,af}$$

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
[Out] -1/3*log(a*cos(f*x + e)^3 + b)/(a*f)
```

Giac [F]

$$\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx = \int \frac{\tan\left(fx+e\right)}{b\sec\left(fx+e\right)^3+a}\,dx$$

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 20.76 (sec) , antiderivative size = 114, normalized size of antiderivative = 4.96

$$\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx = \frac{3\,\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)-\ln\left(a+b-3\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+3\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+3\,b\tan\left(\frac{e}{2}\right.\right.}{3\,a\,f}$$

```
[In] int(tan(e + f*x)/(a + b/cos(e + f*x)^3),x)
[Out] (3*log(tan(e/2 + (f*x)/2)^2 + 1) - log(a + b - 3*a*tan(e/2 + (f*x)/2)^2 + 3
*a*tan(e/2 + (f*x)/2)^4 - a*tan(e/2 + (f*x)/2)^6 + 3*b*tan(e/2 + (f*x)/2)^2
 + 3*b*tan(e/2 + (f*x)/2)^4 + b*tan(e/2 + (f*x)/2)^6))/(3*a*f)
```

3.460 $\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 295

$$\begin{aligned}\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx &= -\frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}\left(a^{4/3}+a^{2/3}b^{2/3}+b^{4/3}\right)f}+\frac{\log(1-\cos(e+fx))}{2(a+b)f}\\&+\frac{\log(1+\cos(e+fx))}{2(a-b)f}-\frac{\left(a^{2/3}+b^{2/3}\right)b^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}\left(a^2-b^2\right)f}\\&+\frac{\left(a^{2/3}+b^{2/3}\right)b^{2/3}\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+a^{2/3}\cos^2(e+fx)\right)}{6\sqrt[3]{a}\left(a^2-b^2\right)f}\\&-\frac{b^2\log\left(b+a\cos^3(e+fx)\right)}{3a\left(a^2-b^2\right)f}\end{aligned}$$

```
[Out] 1/2*ln(1-cos(f*x+e))/(a+b)/f+1/2*ln(1+cos(f*x+e))/(a-b)/f-1/3*(a^(2/3)+b^(2
/3))*b^(2/3)*ln(b^(1/3)+a^(1/3)*cos(f*x+e))/a^(1/3)/(a^2-b^2)/f+1/6*(a^(2/3
)+b^(2/3))*b^(2/3)*ln(b^(2/3)-a^(1/3)*b^(1/3)*cos(f*x+e)+a^(2/3)*cos(f*x+e)
^2)/a^(1/3)/(a^2-b^2)/f-1/3*b^2*ln(b+a*cos(f*x+e)^3)/a/(a^2-b^2)/f-1/3*b^(2
/3)*arctan(1/3*(b^(1/3)-2*a^(1/3)*cos(f*x+e))/b^(1/3)*3^(1/2))/a^(1/3)/(a^(
4/3)+a^(2/3)*b^(2/3)+b^(4/3))/f*3^(1/2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {4223, 6857, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}f\left(a^{2/3}b^{2/3}+a^{4/3}+b^{4/3}\right)}-\frac{b^2\log\left(a\cos^3(e+fx)+b\right)}{3af\left(a^2-b^2\right)}+\frac{b^{2/3}\left(a^{2/3}+b^{2/3}\right)\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}\right)}{6\sqrt[3]{a}f\left(a^2-b^2\right)}-\frac{b^{2/3}\left(a^{2/3}+b^{2/3}\right)\log\left(\sqrt[3]{a}\cos(e+fx)+\sqrt[3]{b}\right)}{3\sqrt[3]{a}f\left(a^2-b^2\right)}+\frac{\log(1-\cos(e+fx))}{2f(a+b)}+\frac{\log(\cos(e+fx)+1)}{2f(a-b)}$$

```
[In] Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] -((b^(2/3)*ArcTan[(b^(1/3) - 2*a^(1/3)*Cos[e + f*x])/(Sqrt[3]*b^(1/3))])/(S
qrt[3]*a^(1/3)*(a^(4/3) + a^(2/3)*b^(2/3) + b^(4/3))*f)) + Log[1 - Cos[e +
f*x]]/(2*(a + b)*f) + Log[1 + Cos[e + f*x]]/(2*(a - b)*f) - ((a^(2/3) + b^(
2/3))*b^(2/3)*Log[b^(1/3) + a^(1/3)*Cos[e + f*x]])/(3*a^(1/3)*(a^2 - b^2)*f
) + ((a^(2/3) + b^(2/3))*b^(2/3)*Log[b^(2/3) - a^(1/3)*b^(1/3)*Cos[e + f*x]
 + a^(2/3)*Cos[e + f*x]^2])/(6*a^(1/3)*(a^2 - b^2)*f) - (b^2*Log[b + a*Cos[
e + f*x]^3])/(3*a*(a^2 - b^2)*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

[n, 0]

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(b+ax^3)}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{2(a+b)(-1+x)} - \frac{1}{2(a-b)(1+x)} - \frac{b(b-ax+bx^2)}{(-a^2+b^2)(b+ax^3)}\right)\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{\log(1-\cos(e+fx))}{2(a+b)f} + \frac{\log(1+\cos(e+fx))}{2(a-b)f} - \frac{b\text{Subst}\left(\int \frac{b-ax+bx^2}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{(a^2-b^2)\,f}$$

$$= \frac{\log(1-\cos(e+fx))}{2(a+b)f} + \frac{\log(1+\cos(e+fx))}{2(a-b)f} - \frac{b\text{Subst}\left(\int \frac{b-ax}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{(a^2-b^2)\,f} - \frac{b^2\text{Subst}\left(\int \frac{x^2}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{(a^2-b^2)\,f}$$

$$= \frac{\log(1-\cos(e+fx))}{2(a+b)f} + \frac{\log(1+\cos(e+fx))}{2(a-b)f} - \frac{b^2\log\left(b+a\cos^3(e+fx)\right)}{3a\,(a^2-b^2)\,f} - \frac{\sqrt[3]{b}\text{Subst}\left(\int \frac{\sqrt[3]{b}\left(-a\sqrt[3]{b}+2\sqrt[3]{a}b\right)+\sqrt[3]{a}\left(-a\sqrt[3]{b}-\sqrt[3]{a}b\right)x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx, x, \cos(e+fx)\right)}{3\sqrt[3]{a}\,(a^2-b^2)\,f} - \frac{\left(\left(a^{2/3}+b^{2/3}\right)b^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}x}\,dx, x, \cos(e+fx)\right)}{3\,(a^2-b^2)\,f}$$

$$= \frac{\log(1-\cos(e+fx))}{2(a+b)f} + \frac{\log(1+\cos(e+fx))}{2(a-b)f} - \frac{\left(a^{2/3}+b^{2/3}\right)b^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}\,(a^2-b^2)\,f} - \frac{b^2\log\left(b+a\cos^3(e+fx)\right)}{3a\,(a^2-b^2)\,f} + \frac{b\text{Subst}\left(\int \frac{1}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx, x, \cos(e+fx)\right)}{2\left(a^{4/3}+a^{2/3}b^{2/3}+b^{4/3}\right)f} + \frac{\left(\left(a^{2/3}+b^{2/3}\right)b^{2/3}\right)\text{Subst}\left(\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx, x, \cos(e+fx)\right)}{6\sqrt[3]{a}\,(a^2-b^2)\,f}$$

$$= \frac{\log(1-\cos(e+fx))}{2(a+b)f} + \frac{\log(1+\cos(e+fx))}{2(a-b)f} - \frac{(a^{2/3}+b^{2/3})\, b^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}\,(a^2-b^2)\, f} + \frac{(a^{2/3}+b^{2/3})\, b^{2/3}\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+a^{2/3}\cos^2(e+fx)\right)}{6\sqrt[3]{a}\,(a^2-b^2)\, f} - \frac{b^2\log\left(b+a\cos^3(e+fx)\right)}{3a\,(a^2-b^2)\, f} + \frac{b^{2/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{a}\cos(e+fx)}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}\,(a^{4/3}+a^{2/3}b^{2/3}+b^{4/3})\, f}$$

$$= -\frac{b^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}\cos(e+fx)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\,(a^{4/3}+a^{2/3}b^{2/3}+b^{4/3})\, f} + \frac{\log(1-\cos(e+fx))}{2(a+b)f} + \frac{\log(1+\cos(e+fx))}{2(a-b)f} - \frac{(a^{2/3}+b^{2/3})\, b^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}\,(a^2-b^2)\, f} + \frac{(a^{2/3}+b^{2/3})\, b^{2/3}\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+a^{2/3}\cos^2(e+fx)\right)}{6\sqrt[3]{a}\,(a^2-b^2)\, f} - \frac{b^2\log\left(b+a\cos^3(e+fx)\right)}{3a\,(a^2-b^2)\, f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.69 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.73

$$\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx = \frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{(a-b)f} + \frac{\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(a+b)f} + \frac{b\left(3b\log\left(\sec^2\left(\frac{1}{2}(e+fx)\right)\right) + (-a+b)\text{RootSum}\left[-8a+12a\#1-6a\#1^2+a\#1^3-b\#1^3\&, \frac{-4a\log(1-\#\cdots}{\cdots}\right.\right.}{3\,(a^3-ab^2)\, f}$$

```
[In] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] Log[Cos[(e + f*x)/2]]/((a - b)*f) + Log[Sin[(e + f*x)/2]]/((a + b)*f) + (b*
(3*b*Log[Sec[(e + f*x)/2]^2] + (-a + b)*RootSum[-8*a + 12*a*#1 - 6*a*#1^2 +
 a*#1^3 - b*#1^3 & , (-4*a*Log[1 - #1 + Tan[(e + f*x)/2]^2] + 2*a*Log[1 - #
1 + Tan[(e + f*x)/2]^2]*#1 + b*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1^2)/(4*a
- 4*a*#1 + a*#1^2 - b*#1^2) & ]))/(3*(a^3 - a*b^2)*f)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.43 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.98

method	result
risch	$\frac{ix}{a}-\frac{ix}{a+b}-\frac{ie}{f(a+b)}-\frac{ix}{a-b}-\frac{ie}{f(a-b)}-\frac{2ia^2b^2f^3x}{-a^5f^3+a^3b^2f^3}-\frac{2ia^2b^2f^2e}{-a^5f^3+a^3b^2f^3}+\frac{\ln(e^{i(fx+e)}-1)}{f(a+b)}+\frac{\ln(e^{i(fx+e)}}{f(a-b)}$
derivativedivides	$\frac{\frac{\ln(1+\cos(fx+e))}{2a-2b}+\frac{\ln(-1+\cos(fx+e))}{2a+2b}+\frac{\left(-b\left(\frac{\ln\left(\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\right.}{}\right.\right.}{}}{}$
default	$\frac{\frac{\ln(1+\cos(fx+e))}{2a-2b}+\frac{\ln(-1+\cos(fx+e))}{2a+2b}+\frac{\left(-b\left(\frac{\ln\left(\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\right.}{}\right.\right.}{}}{}$

```
[In] int(cot(f*x+e)/(a+b*sec(f*x+e)^3),x,method=_RETURNVERBOSE)
```

```
[Out] I*x/a-I/(a+b)*x-I/f/(a+b)*e-I/(a-b)*x-I/f/(a-b)*e-2*I*a^2*b^2*f^3/(-a^5*f^3
+a^3*b^2*f^3)*x-2*I*a^2*b^2*f^2/(-a^5*f^3+a^3*b^2*f^3)*e+1/f/(a+b)*ln(exp(I
*(f*x+e))-1)+1/f/(a-b)*ln(exp(I*(f*x+e))+1)+I*sum(_R*ln(exp(2*I*(f*x+e))+((
-18*a^3/b*f^2+18*a*b*f^2)*_R^2+18*I*b*f*_R-4*b/a)*exp(I*(f*x+e))+1),_R=Root
Of((27*a^5*f^3-27*a^3*b^2*f^3)*_Z^3-27*I*b^2*a^2*f^2*_Z^2+9*_Z*a*b^2*f+I*b^
2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 6482, normalized size of antiderivative = 21.97

$$\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx = \int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx$$

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)**3),x)
[Out] Integral(cot(e + f*x)/(a + b*sec(e + f*x)**3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.04

$$\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\frac{2\sqrt{3}\left(ab^2\left(3\left(\frac{b}{a}\right)^{\frac{1}{3}}+\frac{2b}{a}\right)-3a^2b\left(\frac{b}{a}\right)^{\frac{2}{3}}-2b^3\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}-2\cos(fx+e)\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{\left(a^4\left(\frac{b}{a}\right)^{\frac{2}{3}}-a^2b^2\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}}+\frac{3\left(b^2\left(2\left(\frac{b}{a}\right)^{\frac{2}{3}}-1\right)-ab\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)\log\left(\cos(fx+e)^2-\right.}{a^3\left(\frac{b}{a}\right)^{\frac{2}{3}}-ab^2\left(\frac{b}{a}\right)^{\frac{2}{3}}}}{18}$$

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
[Out] -1/18*(2*sqrt(3)*(a*b^2*(3*(b/a)^(1/3) + 2*b/a) - 3*a^2*b*(b/a)^(2/3) - 2*b
^3)*arctan(-1/3*sqrt(3)*((b/a)^(1/3) - 2*cos(f*x + e))/(b/a)^(1/3))/((a^4*(
b/a)^(2/3) - a^2*b^2*(b/a)^(2/3))*(b/a)^(1/3)) + 3*(b^2*(2*(b/a)^(2/3) - 1)
 - a*b*(b/a)^(1/3))*log(cos(f*x + e)^2 - (b/a)^(1/3)*cos(f*x + e) + (b/a)^(
2/3))/(a^3*(b/a)^(2/3) - a*b^2*(b/a)^(2/3)) + 6*(b^2*((b/a)^(2/3) + 1) + a*
b*(b/a)^(1/3))*log((b/a)^(1/3) + cos(f*x + e))/(a^3*(b/a)^(2/3) - a*b^2*(b/
a)^(2/3)) - 9*log(cos(f*x + e) + 1)/(a - b) - 9*log(cos(f*x + e) - 1)/(a +
b))/f
```

Giac [F]

$$\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx = \int \frac{\cot(fx+e)}{b\sec(fx+e)^3+a}\,dx$$

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 23.78 (sec) , antiderivative size = 11182, normalized size of antiderivative = 37.91

$$\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx = \text{Too large to display}$$

```
[In] int(cot(e + f*x)/(a + b/cos(e + f*x)^3),x)
```

```
[Out] log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2))/(f*(a + b)) - log(1/cos(e/2 + (f
*x)/2)^2)/(f*(a + b)) + (a*symsum(log((262144*(832*root(27*a^3*b^2*z^3 - 27
*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*b^7 - 22*a*b^5 - 840*b^6
*cos(e + f*x) + 440*b^6 - 264*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2
*z^2 + 9*a*b^2*z - b^2, z, k)^2*b^8 + 16*root(27*a^3*b^2*z^3 - 27*a^5*z^3 -
 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*b^9 + 1823*root(27*a^3*b^2*z^3 -
 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*a^2*b^5 - 21*root(27*
a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*a^3*b^4
- 8864*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2,
 z, k)^2*a*b^7 + 3092*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9
*a*b^2*z - b^2, z, k)^3*a*b^8 - 192*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a
^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a*b^9 + 88*root(27*a^3*b^2*z^3 - 27*a
^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*b^8*cos(e + f*x) - a^2*b
^4*cos(e + f*x) + 65221*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 +
 9*a*b^2*z - b^2, z, k)^2*a^2*b^6 - 32708*root(27*a^3*b^2*z^3 - 27*a^5*z^3
- 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a^3*b^5 + 2859*root(27*a^3*b^2*
z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a^4*b^4 - 9*ro
ot(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*
a^5*b^3 + 26274*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2
*z - b^2, z, k)^3*a^2*b^7 - 212230*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^
2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^3*b^6 + 216667*root(27*a^3*b^2*z^3 -
 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^4*b^5 - 44745*roo
t(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a
^5*b^4 + 1584*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z
 - b^2, z, k)^3*a^6*b^3 - 12720*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b
^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^2*b^8 + 14028*root(27*a^3*b^2*z^3 - 27*
```

a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^3*b^7 + 156387*root(2
7*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^4*
b^6 - 457125*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z
- b^2, z, k)^4*a^5*b^5 + 228117*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b
^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^6*b^4 - 24723*root(27*a^3*b^2*z^3 - 27*
a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^7*b^3 + 486*root(27*a
^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^8*b^2
 + 864*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2,
 z, k)^5*a^2*b^9 + 18792*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2
+ 9*a*b^2*z - b^2, z, k)^5*a^3*b^8 - 151488*root(27*a^3*b^2*z^3 - 27*a^5*z^
3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^4*b^7 + 577008*root(27*a^3*
b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^5*b^6 -
414504*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2,
 z, k)^5*a^6*b^5 - 144432*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2
 + 9*a*b^2*z - b^2, z, k)^5*a^7*b^4 + 63702*root(27*a^3*b^2*z^3 - 27*a^5*z^
3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^8*b^3 + 486*root(27*a^3*b^2
*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^9*b^2 - 172
8*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k
)^6*a^3*b^9 + 3672*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*
b^2*z - b^2, z, k)^6*a^4*b^8 + 69444*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*
a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^5*b^7 - 637794*root(27*a^3*b^2*z^3
 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^6*b^6 + 1468908
*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)
^6*a^7*b^5 - 1112400*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*
a*b^2*z - b^2, z, k)^6*a^8*b^4 + 210384*root(27*a^3*b^2*z^3 - 27*a^5*z^3 -
27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^9*b^3 - 486*root(27*a^3*b^2*z^3
 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^10*b^2 + 1296*r
oot(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7
*a^4*b^9 - 23004*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^
2*z - b^2, z, k)^7*a^5*b^8 + 195534*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a
^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^6*b^7 - 778734*root(27*a^3*b^2*z^3
- 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^7*b^6 + 1175796*
root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^
7*a^8*b^5 - 690768*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*
b^2*z - b^2, z, k)^7*a^9*b^4 + 120366*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27
*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^10*b^3 - 486*root(27*a^3*b^2*z^3
- 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^11*b^2 - 8702*ro
ot(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*a*
b^6 - 272*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b
^2, z, k)*b^7*cos(e + f*x) + 62*a*b^5*cos(e + f*x) + 13774*root(27*a^3*b^2*
z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*a*b^6*cos(e + f*
x) - 4098*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b
^2, z, k)*a^2*b^5*cos(e + f*x) + 122*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*
a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*a^3*b^4*cos(e + f*x) + 2088*root(27*a^
3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a*b^7*co

s(e + f*x) - 980*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a*b^8*cos(e + f*x) - 85013*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a^2*b^6*cos(e + f*x) + 55956*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a^3*b^5*cos(e + f*x) - 8075*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a^4*b^4*cos(e + f*x) + 117*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a^5*b^3*cos(e + f*x) + 818*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^2*b^7*cos(e + f*x) + 217434*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^3*b^6*cos(e + f*x) - 285091*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^4*b^5*cos(e + f*x) + 82633*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^5*b^4*cos(e + f*x) - 6984*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^6*b^3*cos(e + f*x) + 3792*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^2*b^8*cos(e + f*x) - 42132*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^3*b^7*cos(e + f*x) - 54423*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^4*b^6*cos(e + f*x) + 435417*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^5*b^5*cos(e + f*x) - 280113*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^6*b^4*cos(e + f*x) + 49239*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^7*b^3*cos(e + f*x) - 3402*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^8*b^2*cos(e + f*x) - 4968*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^3*b^8*cos(e + f*x) + 99864*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^4*b^7*cos(e + f*x) - 643536*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^5*b^6*cos(e + f*x) + 636552*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^6*b^5*cos(e + f*x) - 936*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^7*b^4*cos(e + f*x) - 28170*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^8*b^3*cos(e + f*x) - 3402*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^9*b^2*cos(e + f*x) - 2376*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^4*b^8*cos(e + f*x) + 972*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^5*b^7*cos(e + f*x) + 457758*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^6*b^6*cos(e + f*x) - 1352916*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^7*b^5*cos(e + f*x) + 1122336*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^8*b^4*cos(e + f*x) - 229176*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^9*b^3*cos(e + f*x) + 3402*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^10*b^2*cos(e + f*x) + 7452*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a

^5*b^8*cos(e + f*x) - 139482*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^6*b^7*cos(e + f*x) + 729810*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^7*b^6*cos(e + f*x) - 1208844*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^8*b^5*cos(e + f*x) + 752328*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^9*b^4*cos(e + f*x) - 144666*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^10*b^3*cos(e + f*x) + 3402*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^11*b^2*cos(e + f*x)))/cos(e/2 + (f*x)/2)^2)*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k), k, 1, 3))/(f*(a + b)) + (b*symsum(log((262144*(832*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*b^7 - 22*a*b^5 - 840*b^6*cos(e + f*x) + 440*b^6 - 264*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*b^8 + 16*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*b^9 + 1823*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*a^2*b^5 - 21*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*a^3*b^4 - 8864*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a*b^7 + 3092*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a*b^8 - 192*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a*b^9 + 88*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*b^8*cos(e + f*x) - a^2*b^4*cos(e + f*x) + 65221*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a^2*b^6 - 32708*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a^3*b^5 + 2859*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a^4*b^4 - 9*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a^5*b^3 + 26274*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^2*b^7 - 212230*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^3*b^6 + 216667*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^4*b^5 - 44745*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^5*b^4 + 1584*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^6*b^3 - 12720*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^2*b^8 + 14028*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^3*b^7 + 156387*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^4*b^6 - 457125*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^5*b^5 + 228117*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^6*b^4 - 24723*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^7*b^3 + 486*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^8*b^2 + 864*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^2*b^9 + 18792*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^3*b^8 - 151488*root(27*a^3*b^2*z^

3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^4*b^7 + 577008
*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)
^5*a^5*b^6 - 414504*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a
*b^2*z - b^2, z, k)^5*a^6*b^5 - 144432*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 2
7*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^7*b^4 + 63702*root(27*a^3*b^2*z^
3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^8*b^3 + 486*ro
ot(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*
a^9*b^2 - 1728*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*
z - b^2, z, k)^6*a^3*b^9 + 3672*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b
^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^4*b^8 + 69444*root(27*a^3*b^2*z^3 - 27*
a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^5*b^7 - 637794*root(2
7*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^6*
b^6 + 1468908*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z
 - b^2, z, k)^6*a^7*b^5 - 1112400*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2
*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^8*b^4 + 210384*root(27*a^3*b^2*z^3 -
27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^9*b^3 - 486*root(2
7*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^10
*b^2 + 1296*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z -
 b^2, z, k)^7*a^4*b^9 - 23004*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2
*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^5*b^8 + 195534*root(27*a^3*b^2*z^3 - 27*a
^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^6*b^7 - 778734*root(27
*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^7*b
^6 + 1175796*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z
- b^2, z, k)^7*a^8*b^5 - 690768*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b
^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^9*b^4 + 120366*root(27*a^3*b^2*z^3 - 27
*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^10*b^3 - 486*root(27
*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^11*
b^2 - 8702*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z -
b^2, z, k)*a*b^6 - 272*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 +
9*a*b^2*z - b^2, z, k)*b^7*cos(e + f*x) + 62*a*b^5*cos(e + f*x) + 13774*roo
t(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*a*b
^6*cos(e + f*x) - 4098*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 +
9*a*b^2*z - b^2, z, k)*a^2*b^5*cos(e + f*x) + 122*root(27*a^3*b^2*z^3 - 27*
a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*a^3*b^4*cos(e + f*x) + 20
88*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z,
k)^2*a*b^7*cos(e + f*x) - 980*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2
*z^2 + 9*a*b^2*z - b^2, z, k)^3*a*b^8*cos(e + f*x) - 85013*root(27*a^3*b^2*
z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a^2*b^6*cos(e
+ f*x) + 55956*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*
z - b^2, z, k)^2*a^3*b^5*cos(e + f*x) - 8075*root(27*a^3*b^2*z^3 - 27*a^5*z
^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a^4*b^4*cos(e + f*x) + 117*r
oot(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2
*a^5*b^3*cos(e + f*x) + 818*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z
^2 + 9*a*b^2*z - b^2, z, k)^3*a^2*b^7*cos(e + f*x) + 217434*root(27*a^3*b^2
*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^3*b^6*cos(e

+ f*x) - 285091*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^4*b^5*cos(e + f*x) + 82633*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^5*b^4*cos(e + f*x) - 6984*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^6*b^3*cos(e + f*x) + 3792*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^2*b^8*cos(e + f*x) - 42132*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^3*b^7*cos(e + f*x) - 54423*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^4*b^6*cos(e + f*x) + 435417*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^5*b^5*cos(e + f*x) - 280113*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^6*b^4*cos(e + f*x) + 49239*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^7*b^3*cos(e + f*x) - 3402*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^8*b^2*cos(e + f*x) - 4968*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^3*b^8*cos(e + f*x) + 99864*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^4*b^7*cos(e + f*x) - 643536*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^5*b^6*cos(e + f*x) + 636552*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^6*b^5*cos(e + f*x) - 936*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^7*b^4*cos(e + f*x) - 28170*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^8*b^3*cos(e + f*x) - 3402*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^9*b^2*cos(e + f*x) - 2376*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^4*b^8*cos(e + f*x) + 972*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^5*b^7*cos(e + f*x) + 457758*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^6*b^6*cos(e + f*x) - 1352916*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^7*b^5*cos(e + f*x) + 1122336*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^8*b^4*cos(e + f*x) - 229176*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^9*b^3*cos(e + f*x) + 3402*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^10*b^2*cos(e + f*x) + 7452*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^5*b^8*cos(e + f*x) - 139482*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^6*b^7*cos(e + f*x) + 729810*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^7*b^6*cos(e + f*x) - 1208844*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^8*b^5*cos(e + f*x) + 752328*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^9*b^4*cos(e + f*x) - 144666*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^10*b^3*cos(e + f*x) + 3402*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^11*b^2*cos(e + f*x)))/cos(e/2 + (f*x)/2)^2)*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2

```
+ 9*a*b^2*z - b^2, z, k), k, 1, 3))/(f*(a + b)) - (b*log(1/cos(e/2 + (f*x)/
2)^2))/(a*f*(a + b))
```

3.461 $\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 393

$$\begin{aligned}\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx &= \frac{b^{4/3}\left(a^2-3a^{2/3}b^{4/3}+2b^2\right)\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}\left(a^2-b^2\right)^2 f}\\
&\quad-\frac{1}{4(a+b)f(1-\cos(e+fx))}-\frac{1}{4(a-b)f(1+\cos(e+fx))}\\
&\quad-\frac{(2a+5b)\log(1-\cos(e+fx))}{4(a+b)^2 f}-\frac{(2a-5b)\log(1+\cos(e+fx))}{4(a-b)^2 f}\\
&\quad-\frac{b^{4/3}\left(a^2+3a^{2/3}b^{4/3}+2b^2\right)\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}\left(a^2-b^2\right)^2 f}\\
&\quad+\frac{b^{4/3}\left(a^2+3a^{2/3}b^{4/3}+2b^2\right)\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+a^{2/3}\cos^2(e+fx)\right)}{6\sqrt[3]{a}\left(a^2-b^2\right)^2 f}\\
&\quad-\frac{b^2\left(2a^2+b^2\right)\log\left(b+a\cos^3(e+fx)\right)}{3a\left(a^2-b^2\right)^2 f}\end{aligned}$$

```
[Out] -1/4/(a+b)/f/(1-cos(f*x+e))-1/4/(a-b)/f/(1+cos(f*x+e))-1/4*(2*a+5*b)*ln(1-c
os(f*x+e))/(a+b)^2/f-1/4*(2*a-5*b)*ln(1+cos(f*x+e))/(a-b)^2/f-1/3*b^(4/3)*(
a^2+3*a^(2/3)*b^(4/3)+2*b^2)*ln(b^(1/3)+a^(1/3)*cos(f*x+e))/a^(1/3)/(a^2-b^
2)^2/f+1/6*b^(4/3)*(a^2+3*a^(2/3)*b^(4/3)+2*b^2)*ln(b^(2/3)-a^(1/3)*b^(1/3)
*cos(f*x+e)+a^(2/3)*cos(f*x+e)^2)/a^(1/3)/(a^2-b^2)^2/f-1/3*b^2*(2*a^2+b^2)
*ln(b+a*cos(f*x+e)^3)/a/(a^2-b^2)^2/f+1/3*b^(4/3)*(a^2-3*a^(2/3)*b^(4/3)+2*
b^2)*arctan(1/3*(b^(1/3)-2*a^(1/3)*cos(f*x+e))/b^(1/3)*3^(1/2))/a^(1/3)/(a^
2-b^2)^2/f*3^(1/2)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4223, 6857, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx$$
$$= -\frac{b^2(2a^2+b^2)\log\left(a\cos^3(e+fx)+b\right)}{3af\left(a^2-b^2\right)^2} + \frac{b^{4/3}\left(-3a^{2/3}b^{4/3}+a^2+2b^2\right)\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}f\left(a^2-b^2\right)^2} + \frac{b^{4/3}\left(3a^{2/3}b^{4/3}+a^2+2b^2\right)\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}\right)}{6\sqrt[3]{a}f\left(a^2-b^2\right)^2} - \frac{b^{4/3}\left(3a^{2/3}b^{4/3}+a^2+2b^2\right)\log\left(\sqrt[3]{a}\cos(e+fx)+\sqrt[3]{b}\right)}{3\sqrt[3]{a}f\left(a^2-b^2\right)^2} - \frac{1}{4f(a+b)(1-\cos(e+fx))} - \frac{1}{4f(a-b)(\cos(e+fx)+1)} - \frac{(2a+5b)\log(1-\cos(e+fx))}{4f(a+b)^2} - \frac{(2a-5b)\log(\cos(e+fx)+1)}{4f(a-b)^2}$$

```
[In] Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] (b^(4/3)*(a^2 - 3*a^(2/3)*b^(4/3) + 2*b^2)*ArcTan[(b^(1/3) - 2*a^(1/3)*Cos[
e + f*x])/(Sqrt[3]*b^(1/3))])/(Sqrt[3]*a^(1/3)*(a^2 - b^2)^2*f) - 1/(4*(a +
 b)*f*(1 - Cos[e + f*x])) - 1/(4*(a - b)*f*(1 + Cos[e + f*x])) - ((2*a + 5*
b)*Log[1 - Cos[e + f*x]])/(4*(a + b)^2*f) - ((2*a - 5*b)*Log[1 + Cos[e + f*
x]])/(4*(a - b)^2*f) - (b^(4/3)*(a^2 + 3*a^(2/3)*b^(4/3) + 2*b^2)*Log[b^(1/
3) + a^(1/3)*Cos[e + f*x]])/(3*a^(1/3)*(a^2 - b^2)^2*f) + (b^(4/3)*(a^2 + 3
*a^(2/3)*b^(4/3) + 2*b^2)*Log[b^(2/3) - a^(1/3)*b^(1/3)*Cos[e + f*x] + a^(2
/3)*Cos[e + f*x]^2])/(6*a^(1/3)*(a^2 - b^2)^2*f) - (b^2*(2*a^2 + b^2)*Log[b
 + a*Cos[e + f*x]^3])/(3*a*(a^2 - b^2)^2*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
```

```
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^2(b+ax^3)}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{4(a+b)(-1+x)^2} + \frac{2a+5b}{4(a+b)^2(-1+x)} - \frac{1}{4(a-b)(1+x)^2} + \frac{2a-5b}{4(a-b)^2(1+x)} + \frac{b^2(a^2+2b^2-3abx+(2a^2+b^2)x^2)}{(a^2-b^2)^2(b+ax^3)}\right) d\right.}{f}$$

$$= -\frac{1}{4(a+b)f(1-\cos(e+fx))} - \frac{1}{4(a-b)f(1+\cos(e+fx))} - \frac{(2a+5b)\log(1-\cos(e+fx))}{4(a+b)^2f} - \frac{(2a-5b)\log(1+\cos(e+fx))}{4(a-b)^2f} - \frac{b^2\text{Subst}\left(\int \frac{a^2+2b^2-3abx+(2a^2+b^2)x^2}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{(a^2-b^2)^2 f}$$

$$= -\frac{1}{4(a+b)f(1-\cos(e+fx))} - \frac{1}{4(a-b)f(1+\cos(e+fx))} - \frac{(2a+5b)\log(1-\cos(e+fx))}{4(a+b)^2f} - \frac{(2a-5b)\log(1+\cos(e+fx))}{4(a-b)^2f} - \frac{b^2\text{Subst}\left(\int \frac{a^2+2b^2-3abx}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{(a^2-b^2)^2 f} - \frac{(b^2(2a^2+b^2))\,\text{Subst}\left(\int \frac{x^2}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{(a^2-b^2)^2 f}$$

$$= -\frac{1}{4(a+b)f(1-\cos(e+fx))} - \frac{1}{4(a-b)f(1+\cos(e+fx))} - \frac{(2a+5b)\log(1-\cos(e+fx))}{4(a+b)^2f} - \frac{(2a-5b)\log(1+\cos(e+fx))}{4(a-b)^2f} - \frac{b^2(2a^2+b^2)\log\left(b+a\cos^3(e+fx)\right)}{3a\left(a^2-b^2\right)^2f} - \frac{b^{4/3}\text{Subst}\left(\int\frac{\sqrt[3]{b}\left(-3ab^{4/3}+2\sqrt[3]{a}(a^2+2b^2)\right)+\sqrt[3]{a}\left(-3ab^{4/3}-\sqrt[3]{a}(a^2+2b^2)\right)x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}dx,x,\cos(e+fx)\right)}{3\sqrt[3]{a}\left(a^2-b^2\right)^2f} - \frac{\left(b^{4/3}\left(a^2+3a^{2/3}b^{4/3}+2b^2\right)\right)\text{Subst}\left(\int\frac{1}{\sqrt[3]{b}+\sqrt[3]{a}x}dx,x,\cos(e+fx)\right)}{3\left(a^2-b^2\right)^2f}$$

$$= -\frac{1}{4(a+b)f(1-\cos(e+fx))} - \frac{1}{4(a-b)f(1+\cos(e+fx))} - \frac{(2a+5b)\log(1-\cos(e+fx))}{4(a+b)^2f} - \frac{(2a-5b)\log(1+\cos(e+fx))}{4(a-b)^2f} - \frac{b^{4/3}\left(a^2+3a^{2/3}b^{4/3}+2b^2\right)\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}\left(a^2-b^2\right)^2f} - \frac{b^2(2a^2+b^2)\log\left(b+a\cos^3(e+fx)\right)}{3a\left(a^2-b^2\right)^2f} - \frac{\left(b^{5/3}\left(a^2-3a^{2/3}b^{4/3}+2b^2\right)\right)\text{Subst}\left(\int\frac{1}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}dx,x,\cos(e+fx)\right)}{2\left(a^2-b^2\right)^2f} + \frac{\left(b^{4/3}\left(a^2+3a^{2/3}b^{4/3}+2b^2\right)\right)\text{Subst}\left(\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}dx,x,\cos(e+fx)\right)}{6\sqrt[3]{a}\left(a^2-b^2\right)^2f}$$

$$= -\frac{1}{4(a+b)f(1-\cos(e+fx))} - \frac{1}{4(a-b)f(1+\cos(e+fx))} - \frac{(2a+5b)\log(1-\cos(e+fx))}{4(a+b)^2f} - \frac{(2a-5b)\log(1+\cos(e+fx))}{4(a-b)^2f} - \frac{b^{4/3}\left(a^2+3a^{2/3}b^{4/3}+2b^2\right)\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}\left(a^2-b^2\right)^2 f} + \frac{b^{4/3}\left(a^2+3a^{2/3}b^{4/3}+2b^2\right)\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+a^{2/3}\cos^2(e+fx)\right)}{6\sqrt[3]{a}\left(a^2-b^2\right)^2 f} - \frac{b^2(2a^2+b^2)\log\left(b+a\cos^3(e+fx)\right)}{3a\left(a^2-b^2\right)^2 f} - \frac{\left(b^{4/3}\left(a^2-3a^{2/3}b^{4/3}+2b^2\right)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{a}\cos(e+fx)}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}\left(a^2-b^2\right)^2 f}$$

$$= \frac{b^{4/3}\left(a^2-3a^{2/3}b^{4/3}+2b^2\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{a}\cos(e+fx)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\left(a^2-b^2\right)^2 f} - \frac{1}{4(a+b)f(1-\cos(e+fx))} - \frac{1}{4(a-b)f(1+\cos(e+fx))} - \frac{(2a+5b)\log(1-\cos(e+fx))}{4(a+b)^2f} - \frac{(2a-5b)\log(1+\cos(e+fx))}{4(a-b)^2f} - \frac{b^{4/3}\left(a^2+3a^{2/3}b^{4/3}+2b^2\right)\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}\left(a^2-b^2\right)^2 f} + \frac{b^{4/3}\left(a^2+3a^{2/3}b^{4/3}+2b^2\right)\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+a^{2/3}\cos^2(e+fx)\right)}{6\sqrt[3]{a}\left(a^2-b^2\right)^2 f} - \frac{b^2(2a^2+b^2)\log\left(b+a\cos^3(e+fx)\right)}{3a\left(a^2-b^2\right)^2 f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.65 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.85

$$\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx$$

$$= \frac{-\frac{3\csc^2\left(\frac{1}{2}(e+fx)\right)}{a+b} + \frac{12(-2a+5b)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{(a-b)^2} - \frac{12(2a+5b)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(a+b)^2} + \frac{8b^2\left(3(2a^2+b^2)\log\left(\sec^2\left(\frac{1}{2}(e+fx)\right)\right)+(-a+b)\text{Ro}\right.}{}}{}$$

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] ((-3*Csc[(e + f*x)/2]^2)/(a + b) + (12*(-2*a + 5*b)*Log[Cos[(e + f*x)/2]])/
(a - b)^2 - (12*(2*a + 5*b)*Log[Sin[(e + f*x)/2]])/(a + b)^2 + (8*b^2*(3*(2
*a^2 + b^2)*Log[Sec[(e + f*x)/2]^2] + (-a + b)*RootSum[-8*a + 12*a*#1 - 6*a
*#1^2 + a*#1^3 - b*#1^3 & , (8*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2] - 4*a*b
*Log[1 - #1 + Tan[(e + f*x)/2]^2] - 6*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2]*
#1 + 2*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1^2 + b^2*Log[1 - #1 + Tan[(e
+ f*x)/2]^2]*#1^2)/(4*a - 4*a*#1 + a*#1^2 - b*#1^2) & ]))/(a*(a^2 - b^2)^2)
 - (3*Sec[(e + f*x)/2]^2)/(a - b))/(24*f)
```

Maple [A] (verified)

Time = 8.55 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\frac{1}{(4a+4b)(-1+\cos(fx+e))}+\frac{(-2a-5b)\ln(-1+\cos(fx+e))}{4(a+b)^2}-\frac{\left((a^2+2b^2)\left(\frac{\ln\left(\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+\right.}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}\right.\right.}{}}{}$
default	$\frac{\frac{1}{(4a+4b)(-1+\cos(fx+e))}+\frac{(-2a-5b)\ln(-1+\cos(fx+e))}{4(a+b)^2}-\frac{\left((a^2+2b^2)\left(\frac{\ln\left(\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+\right.}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}\right.\right.}{}}{}$
risch	$\frac{iae}{f(a^2+2ab+b^2)}+\frac{4ia^4b^2f^2e}{a^7f^3-2a^5b^2f^3+a^3b^4f^3}+\frac{5ibe}{2f(a^2+2ab+b^2)}+\frac{iae}{f(a^2-2ab+b^2)}-\frac{5ibe}{2f(a^2-2ab+b^2)}-i\Big(\sum_{\_R=\mathrm{R}}$

```
[In] int(cot(f*x+e)^3/(a+b*sec(f*x+e)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(4*a+4*b)/(-1+cos(f*x+e))+1/4/(a+b)^2*(-2*a-5*b)*ln(-1+cos(f*x+e))-(
(a^2+2*b^2)*(1/3/a/(b/a)^(2/3)*ln(cos(f*x+e)+(b/a)^(1/3))-1/6/a/(b/a)^(2/3)
*ln(cos(f*x+e)^2-(b/a)^(1/3)*cos(f*x+e)+(b/a)^(2/3))+1/3/a/(b/a)^(2/3)*3^(1
/2)*arctan(1/3*3^(1/2)*(2/(b/a)^(1/3)*cos(f*x+e)-1)))-3*a*b*(-1/3/a/(b/a)^(
```

```
1/3)*ln(cos(f*x+e)+(b/a)^(1/3))+1/6/a/(b/a)^(1/3)*ln(cos(f*x+e)^2-(b/a)^(1/
3)*cos(f*x+e)+(b/a)^(2/3))+1/3*3^(1/2)/a/(b/a)^(1/3)*arctan(1/3*3^(1/2)*(2/
(b/a)^(1/3)*cos(f*x+e)-1)))+1/3*(2*a^2+b^2)/a*ln(b+a*cos(f*x+e)^3))*b^2/(a-
b)^2/(a+b)^2-1/(4*a-4*b)/(1+cos(f*x+e))+1/4/(a-b)^2*(-2*a+5*b)*ln(1+cos(f*x
+e)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.21 (sec) , antiderivative size = 10746, normalized size of antiderivative = 27.34

$$\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = \int \frac{\cot^3{(e+fx)}}{a+b\sec^3{(e+fx)}}\,dx$$

```
[In] integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**3),x)
[Out] Integral(cot(e + f*x)**3/(a + b*sec(e + f*x)**3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.24

$$\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx$$
$$= \frac{\frac{4\sqrt{3}\left(a^2b^3\left(9\left(\frac{b}{a}\right)^{\frac{2}{3}}+4\right)-a^3b^2\left(3\left(\frac{b}{a}\right)^{\frac{1}{3}}+\frac{4b}{a}\right)-2ab^4\left(3\left(\frac{b}{a}\right)^{\frac{1}{3}}+\frac{b}{a}\right)+2b^5\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}-2\cos(fx+e)\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{\left(a^6\left(\frac{b}{a}\right)^{\frac{2}{3}}-2a^4b^2\left(\frac{b}{a}\right)^{\frac{2}{3}}+a^2b^4\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}} - \frac{6\left(a^2b^2\left(4\left(\frac{b}{a}\right)^{\frac{2}{3}}-1\right)+2}{}}{}$$

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

```
[Out] 1/36*(4*sqrt(3)*(a^2*b^3*(9*(b/a)^(2/3) + 4) - a^3*b^2*(3*(b/a)^(1/3) + 4*b
/a) - 2*a*b^4*(3*(b/a)^(1/3) + b/a) + 2*b^5)*arctan(-1/3*sqrt(3)*((b/a)^(1/
3) - 2*cos(f*x + e))/(b/a)^(1/3))/((a^6*(b/a)^(2/3) - 2*a^4*b^2*(b/a)^(2/3)
 + a^2*b^4*(b/a)^(2/3))*(b/a)^(1/3)) - 6*(a^2*b^2*(4*(b/a)^(2/3) - 1) + 2*b
^4*((b/a)^(2/3) - 1) - 3*a*b^3*(b/a)^(1/3))*log(cos(f*x + e)^2 - (b/a)^(1/3
)*cos(f*x + e) + (b/a)^(2/3))/(a^5*(b/a)^(2/3) - 2*a^3*b^2*(b/a)^(2/3) + a*
b^4*(b/a)^(2/3)) - 12*(a^2*b^2*(2*(b/a)^(2/3) + 1) + b^4*((b/a)^(2/3) + 2)
+ 3*a*b^3*(b/a)^(1/3))*log((b/a)^(1/3) + cos(f*x + e))/(a^5*(b/a)^(2/3) - 2
*a^3*b^2*(b/a)^(2/3) + a*b^4*(b/a)^(2/3)) - 9*(2*a - 5*b)*log(cos(f*x + e)
+ 1)/(a^2 - 2*a*b + b^2) - 9*(2*a + 5*b)*log(cos(f*x + e) - 1)/(a^2 + 2*a*b
 + b^2) - 18*(b*cos(f*x + e) - a)/((a^2 - b^2)*cos(f*x + e)^2 - a^2 + b^2))
/f
```

Giac [F]

$$\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = \int \frac{\cot\left(fx+e\right)^3}{b\sec\left(fx+e\right)^3+a}\,dx$$

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 35.83 (sec) , antiderivative size = 58699, normalized size of antiderivative = 149.36

$$\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = \text{Too large to display}$$

```
[In] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^3),x)
```

```
[Out] -(a^3*cos(e/2 + (f*x)/2)^4 + a^3*sin(e/2 + (f*x)/2)^4 - a*b^2*cos(e/2 + (f*
x)/2)^4 + a*b^2*sin(e/2 + (f*x)/2)^4 + 2*a^2*b*sin(e/2 + (f*x)/2)^4 - 8*a^3
*log((cos(e/2 + (f*x)/2)^2 + sin(e/2 + (f*x)/2)^2)/cos(e/2 + (f*x)/2)^2)*co
s(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^2 + 8*b^3*log((cos(e/2 + (f*x)/2)^2 +
 sin(e/2 + (f*x)/2)^2)/cos(e/2 + (f*x)/2)^2)*cos(e/2 + (f*x)/2)^2*sin(e/2 +
 (f*x)/2)^2 + 8*a^3*cos(e/2 + (f*x)/2)^2*log(sin(e/2 + (f*x)/2)/cos(e/2 + (
f*x)/2))*sin(e/2 + (f*x)/2)^2 - 8*a^4*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/
2)^2*symsum(log((131072*(980*b^11*cos(e/2 + (f*x)/2)^2 + 336*b^11*sin(e/2 +
 (f*x)/2)^2 + 1764*a^2*b^9*cos(e/2 + (f*x)/2)^2 + 392*a^3*b^8*cos(e/2 + (f*
x)/2)^2 + 640*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^
2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*b^13*sin(e/2 + (f*x)/2)^2
 + 32*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
```

27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*b^14*sin(e/2 + (f*x)/2)^2 - 1176*
a^2*b^9*sin(e/2 + (f*x)/2)^2 - 784*a^3*b^8*sin(e/2 + (f*x)/2)^2 + 952*root(
54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*
z^2 - 9*a*b^4*z - b^4, z, k)*b^12*cos(e/2 + (f*x)/2)^2 + 2352*a*b^10*cos(e/
2 + (f*x)/2)^2 + 1944*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 5
4*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*b^12*sin(e/2 + (f*x
)/2)^2 - 56*a*b^10*sin(e/2 + (f*x)/2)^2 + 304*root(54*a^5*b^2*z^3 - 27*a^3*
b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z
, k)^2*b^13*cos(e/2 + (f*x)/2)^2 + 32*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3
- 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*b
^14*cos(e/2 + (f*x)/2)^2 + 1032*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a
^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a*b^11*si
n(e/2 + (f*x)/2)^2 + 39032*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^
3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^2*b^10*cos(e
/2 + (f*x)/2)^2 + 30296*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 -
54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^3*b^9*cos(e/2 +
(f*x)/2)^2 + 7420*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a
^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^4*b^8*cos(e/2 + (f*x
)/2)^2 - 252*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2
*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^5*b^7*cos(e/2 + (f*x)/2)^2
- 168*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^6*b^6*cos(e/2 + (f*x)/2)^2 + 142
40*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*
a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a*b^12*cos(e/2 + (f*x)/2)^2 + 4064*r
oot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*
b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a*b^13*cos(e/2 + (f*x)/2)^2 + 384*root(5
4*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z
^2 - 9*a*b^4*z - b^4, z, k)^4*a*b^14*cos(e/2 + (f*x)/2)^2 - 27888*root(54*a
^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
- 9*a*b^4*z - b^4, z, k)*a^2*b^10*sin(e/2 + (f*x)/2)^2 - 55576*root(54*a^5*
b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9
*a*b^4*z - b^4, z, k)*a^3*b^9*sin(e/2 + (f*x)/2)^2 - 32174*root(54*a^5*b^2*
z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b
^4*z - b^4, z, k)*a^4*b^8*sin(e/2 + (f*x)/2)^2 - 3318*root(54*a^5*b^2*z^3 -
27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z
- b^4, z, k)*a^5*b^7*sin(e/2 + (f*x)/2)^2 + 252*root(54*a^5*b^2*z^3 - 27*a^
3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4,
z, k)*a^6*b^6*sin(e/2 + (f*x)/2)^2 + 24840*root(54*a^5*b^2*z^3 - 27*a^3*b^
4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z,
k)^2*a*b^12*sin(e/2 + (f*x)/2)^2 + 7856*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^
3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3
*a*b^13*sin(e/2 + (f*x)/2)^2 + 384*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 2
7*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a*b^
14*sin(e/2 + (f*x)/2)^2 + 107772*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*
a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^2*b^

11*cos(e/2 + (f*x)/2)^2 + 156216*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^3*b^10*cos(e/2 + (f*x)/2)^2 + 55448*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^4*b^9*cos(e/2 + (f*x)/2)^2 + 21772*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^5*b^8*cos(e/2 + (f*x)/2)^2 + 35364*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^6*b^7*cos(e/2 + (f*x)/2)^2 + 3588*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^7*b^6*cos(e/2 + (f*x)/2)^2 - 3051*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^8*b^5*cos(e/2 + (f*x)/2)^2 + 18*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^9*b^4*cos(e/2 + (f*x)/2)^2 + 73528*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^2*b^12*cos(e/2 + (f*x)/2)^2 + 222176*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^3*b^11*cos(e/2 + (f*x)/2)^2 - 101192*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^4*b^10*cos(e/2 + (f*x)/2)^2 - 567064*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^5*b^9*cos(e/2 + (f*x)/2)^2 - 125428*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^6*b^8*cos(e/2 + (f*x)/2)^2 + 278436*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^7*b^7*cos(e/2 + (f*x)/2)^2 + 66894*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^8*b^6*cos(e/2 + (f*x)/2)^2 - 26928*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^9*b^5*cos(e/2 + (f*x)/2)^2 - 3042*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^10*b^4*cos(e/2 + (f*x)/2)^2 + 648*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^11*b^3*cos(e/2 + (f*x)/2)^2 + 19104*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^2*b^13*cos(e/2 + (f*x)/2)^2 + 128832*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^3*b^12*cos(e/2 + (f*x)/2)^2 - 191988*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^4*b^11*cos(e/2 + (f*x)/2)^2 - 899856*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^5*b^10*cos(e/2 + (f*x)/2)^2 + 183204*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^6*b^9*cos(e/2 + (f*x)/2)^2 + 1173036*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a

^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^7*b^8*cos(e/2 + (f*x)/2)^2 - 519612
*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^
2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^8*b^7*cos(e/2 + (f*x)/2)^2 - 666384*
root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2
*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^9*b^6*cos(e/2 + (f*x)/2)^2 + 368028*r
oot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*
b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^10*b^5*cos(e/2 + (f*x)/2)^2 + 46260*ro
ot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b
^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^11*b^4*cos(e/2 + (f*x)/2)^2 - 46332*roo
t(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^
4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^12*b^3*cos(e/2 + (f*x)/2)^2 + 5832*root(
54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*
z^2 - 9*a*b^4*z - b^4, z, k)^4*a^13*b^2*cos(e/2 + (f*x)/2)^2 + 1728*root(54
*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^
2 - 9*a*b^4*z - b^4, z, k)^5*a^2*b^14*cos(e/2 + (f*x)/2)^2 + 34560*root(54*
a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
 - 9*a*b^4*z - b^4, z, k)^5*a^3*b^13*cos(e/2 + (f*x)/2)^2 - 51480*root(54*a
^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
- 9*a*b^4*z - b^4, z, k)^5*a^4*b^12*cos(e/2 + (f*x)/2)^2 - 677880*root(54*a
^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
- 9*a*b^4*z - b^4, z, k)^5*a^5*b^11*cos(e/2 + (f*x)/2)^2 + 773640*root(54*a
^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
- 9*a*b^4*z - b^4, z, k)^5*a^6*b^10*cos(e/2 + (f*x)/2)^2 + 1207440*root(54*
a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
 - 9*a*b^4*z - b^4, z, k)^5*a^7*b^9*cos(e/2 + (f*x)/2)^2 - 1363176*root(54*
a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
 - 9*a*b^4*z - b^4, z, k)^5*a^8*b^8*cos(e/2 + (f*x)/2)^2 - 82728*root(54*a^
5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
 9*a*b^4*z - b^4, z, k)^5*a^9*b^7*cos(e/2 + (f*x)/2)^2 + 738792*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^5*a^10*b^6*cos(e/2 + (f*x)/2)^2 - 412704*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^5*a^11*b^5*cos(e/2 + (f*x)/2)^2 + 11304*root(54*a^5*
b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9
*a*b^4*z - b^4, z, k)^5*a^12*b^4*cos(e/2 + (f*x)/2)^2 + 36288*root(54*a^5*b
^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*
a*b^4*z - b^4, z, k)^5*a^13*b^3*cos(e/2 + (f*x)/2)^2 - 5832*root(54*a^5*b^2
*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*
b^4*z - b^4, z, k)^5*a^14*b^2*cos(e/2 + (f*x)/2)^2 + 3456*root(54*a^5*b^2*z
^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^
4*z - b^4, z, k)^6*a^3*b^14*cos(e/2 + (f*x)/2)^2 + 7344*root(54*a^5*b^2*z^3
 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*
z - b^4, z, k)^6*a^4*b^13*cos(e/2 + (f*x)/2)^2 - 225504*root(54*a^5*b^2*z^3
 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*
z - b^4, z, k)^6*a^5*b^12*cos(e/2 + (f*x)/2)^2 + 450468*root(54*a^5*b^2*z^3

- 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*
z - b^4, z, k)^6*a^6*b^11*cos(e/2 + (f*x)/2)^2 + 360072*root(54*a^5*b^2*z^3
- 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*
z - b^4, z, k)^6*a^7*b^10*cos(e/2 + (f*x)/2)^2 - 879984*root(54*a^5*b^2*z^3
- 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*
z - b^4, z, k)^6*a^8*b^9*cos(e/2 + (f*x)/2)^2 - 183600*root(54*a^5*b^2*z^3
- 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z
- b^4, z, k)^6*a^9*b^8*cos(e/2 + (f*x)/2)^2 + 431352*root(54*a^5*b^2*z^3 -
27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z
- b^4, z, k)^6*a^10*b^7*cos(e/2 + (f*x)/2)^2 + 165888*root(54*a^5*b^2*z^3 -
27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z
- b^4, z, k)^6*a^11*b^6*cos(e/2 + (f*x)/2)^2 - 61344*root(54*a^5*b^2*z^3 -
27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z -
b^4, z, k)^6*a^12*b^5*cos(e/2 + (f*x)/2)^2 - 114480*root(54*a^5*b^2*z^3 -
27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z -
b^4, z, k)^6*a^13*b^4*cos(e/2 + (f*x)/2)^2 + 52164*root(54*a^5*b^2*z^3 - 2
7*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z -
b^4, z, k)^6*a^14*b^3*cos(e/2 + (f*x)/2)^2 - 5832*root(54*a^5*b^2*z^3 - 27*
a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^
4, z, k)^6*a^15*b^2*cos(e/2 + (f*x)/2)^2 + 2592*root(54*a^5*b^2*z^3 - 27*a^
3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4,
z, k)^7*a^4*b^14*cos(e/2 + (f*x)/2)^2 - 28512*root(54*a^5*b^2*z^3 - 27*a^3
*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4,
z, k)^7*a^5*b^13*cos(e/2 + (f*x)/2)^2 + 75816*root(54*a^5*b^2*z^3 - 27*a^3*
b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z
, k)^7*a^6*b^12*cos(e/2 + (f*x)/2)^2 + 71280*root(54*a^5*b^2*z^3 - 27*a^3*b
^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z,
k)^7*a^7*b^11*cos(e/2 + (f*x)/2)^2 - 323352*root(54*a^5*b^2*z^3 - 27*a^3*b
^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z,
k)^7*a^8*b^10*cos(e/2 + (f*x)/2)^2 + 483408*root(54*a^5*b^2*z^3 - 27*a^3*b
^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z,
k)^7*a^10*b^8*cos(e/2 + (f*x)/2)^2 - 142560*root(54*a^5*b^2*z^3 - 27*a^3*b
^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z,
k)^7*a^11*b^7*cos(e/2 + (f*x)/2)^2 - 307152*root(54*a^5*b^2*z^3 - 27*a^3*b
^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z,
k)^7*a^12*b^6*cos(e/2 + (f*x)/2)^2 + 142560*root(54*a^5*b^2*z^3 - 27*a^3*b
^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z,
k)^7*a^13*b^5*cos(e/2 + (f*x)/2)^2 + 62856*root(54*a^5*b^2*z^3 - 27*a^3*b^
4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z,
k)^7*a^14*b^4*cos(e/2 + (f*x)/2)^2 - 42768*root(54*a^5*b^2*z^3 - 27*a^3*b^4
*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k
)^7*a^15*b^3*cos(e/2 + (f*x)/2)^2 + 5832*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z
^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^
7*a^16*b^2*cos(e/2 + (f*x)/2)^2 - 43760*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^
3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2

*a^2*b^11*sin(e/2 + (f*x)/2)^2 - 401720*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^
3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2
*a^3*b^10*sin(e/2 + (f*x)/2)^2 - 563860*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^
3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2
*a^4*b^9*sin(e/2 + (f*x)/2)^2 - 249110*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3
 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*
a^5*b^8*sin(e/2 + (f*x)/2)^2 - 59988*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 -
 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^
6*b^7*sin(e/2 + (f*x)/2)^2 - 35586*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 2
7*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^7*
b^6*sin(e/2 + (f*x)/2)^2 + 5751*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a
^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^8*b^5
*sin(e/2 + (f*x)/2)^2 - 18*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^
3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^9*b^4*sin(
e/2 + (f*x)/2)^2 + 95632*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3
 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^2*b^12*sin(e
/2 + (f*x)/2)^2 - 439696*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3
 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^3*b^11*sin(e
/2 + (f*x)/2)^2 - 1471448*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3
 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^4*b^10*sin(
e/2 + (f*x)/2)^2 - 606964*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3
 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^5*b^9*sin(e
/2 + (f*x)/2)^2 + 521558*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3
 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^6*b^8*sin(e/
2 + (f*x)/2)^2 - 415182*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 -
 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^7*b^7*sin(e/2
 + (f*x)/2)^2 - 598284*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 -
54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^8*b^6*sin(e/2
+ (f*x)/2)^2 + 24606*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54
*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^9*b^5*sin(e/2 +
(f*x)/2)^2 + 20592*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a
^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^10*b^4*sin(e/2 + (
f*x)/2)^2 - 756*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*
b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^11*b^3*sin(e/2 + (f*x
)/2)^2 + 33984*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b
^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^2*b^13*sin(e/2 + (f*x)
/2)^2 + 32784*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^
2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^3*b^12*sin(e/2 + (f*x)/
2)^2 - 1159584*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b
^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^4*b^11*sin(e/2 + (f*x)
/2)^2 + 567024*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b
^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^5*b^10*sin(e/2 + (f*x)
/2)^2 + 6779964*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*
b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^6*b^9*sin(e/2 + (f*x)
/2)^2 + 4475790*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*

b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^7*b^8*sin(e/2 + (f*x)
/2)^2 - 2069340*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*
b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^8*b^7*sin(e/2 + (f*x)
/2)^2 - 421956*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b
^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^9*b^6*sin(e/2 + (f*x)/
2)^2 + 382248*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^
2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^10*b^5*sin(e/2 + (f*x)/
2)^2 - 554778*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^
2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^11*b^4*sin(e/2 + (f*x)/
2)^2 + 146880*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^
2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^12*b^3*sin(e/2 + (f*x)/
2)^2 - 7776*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*
z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^13*b^2*sin(e/2 + (f*x)/2)
^2 + 1728*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^
2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^2*b^14*sin(e/2 + (f*x)/2)^2
 + 54432*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2
 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^3*b^13*sin(e/2 + (f*x)/2)^2
- 422856*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2
 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^4*b^12*sin(e/2 + (f*x)/2)^2
+ 625176*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2
 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^5*b^11*sin(e/2 + (f*x)/2)^2
+ 6126696*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^
2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^6*b^10*sin(e/2 + (f*x)/2)^2
 - 2480004*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z
^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^7*b^9*sin(e/2 + (f*x)/2)^2
 - 15505344*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*
z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^8*b^8*sin(e/2 + (f*x)/2)^
2 - 346572*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z
^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^9*b^7*sin(e/2 + (f*x)/2)^2
 + 9474120*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z
^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^10*b^6*sin(e/2 + (f*x)/2)^
2 + 24660*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^
2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^11*b^5*sin(e/2 + (f*x)/2)^2
 - 1571688*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z
^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^12*b^4*sin(e/2 + (f*x)/2)^
2 + 232740*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z
^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^13*b^3*sin(e/2 + (f*x)/2)^
2 + 7776*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2
 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^14*b^2*sin(e/2 + (f*x)/2)^2
+ 3456*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^3*b^14*sin(e/2 + (f*x)/2)^2 -
1728*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 2
7*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^4*b^13*sin(e/2 + (f*x)/2)^2 - 31
9896*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 2
7*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^5*b^12*sin(e/2 + (f*x)/2)^2 + 32

46912*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^6*b^11*sin(e/2 + (f*x)/2)^2 - 5322240*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^7*b^10*sin(e/2 + (f*x)/2)^2 - 9560160*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^8*b^9*sin(e/2 + (f*x)/2)^2 + 16055280*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^9*b^8*sin(e/2 + (f*x)/2)^2 + 8485344*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^10*b^7*sin(e/2 + (f*x)/2)^2 - 14873760*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^11*b^6*sin(e/2 + (f*x)/2)^2 - 1269216*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^12*b^5*sin(e/2 + (f*x)/2)^2 + 4449384*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^13*b^4*sin(e/2 + (f*x)/2)^2 - 901152*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^14*b^3*sin(e/2 + (f*x)/2)^2 + 7776*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^15*b^2*sin(e/2 + (f*x)/2)^2 + 2592*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^4*b^14*sin(e/2 + (f*x)/2)^2 - 58320*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^5*b^13*sin(e/2 + (f*x)/2)^2 + 603936*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^6*b^12*sin(e/2 + (f*x)/2)^2 - 2230416*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^7*b^11*sin(e/2 + (f*x)/2)^2 + 536544*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^8*b^10*sin(e/2 + (f*x)/2)^2 + 6518880*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^9*b^9*sin(e/2 + (f*x)/2)^2 - 5251392*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^10*b^8*sin(e/2 + (f*x)/2)^2 - 5590944*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^11*b^7*sin(e/2 + (f*x)/2)^2 + 6456672*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^12*b^6*sin(e/2 + (f*x)/2)^2 + 838512*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^13*b^5*sin(e/2 + (f*x)/2)^2 - 2340576*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^14*b^4*sin(e/2 + (f*x)/2)^2 + 522288*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^15*b^3*sin(e/2 + (f*x)/2)^2 - 7776*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z

^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^16*b^2*sin(e/2 + (f*x)/2)^
2 + 17192*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^
2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a*b^11*cos(e/2 + (f*x)/2)^2))/(
cos(e/2 + (f*x)/2)^2*(a + b)^3*(a^2 - 2*a*b + b^2)))*root(54*a^5*b^2*z^3 -
27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z -
b^4, z, k), k, 1, 3) + 8*a^2*b^2*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^2
symsum(log((131072(980*b^11*cos(e/2 + (f*x)/2)^2 + 336*b^11*sin(e/2 + (f*
x)/2)^2 + 1764*a^2*b^9*cos(e/2 + (f*x)/2)^2 + 392*a^3*b^8*cos(e/2 + (f*x)/2
)^2 + 640*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^
2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*b^13*sin(e/2 + (f*x)/2)^2 + 3
2*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a
^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*b^14*sin(e/2 + (f*x)/2)^2 - 1176*a^2*
b^9*sin(e/2 + (f*x)/2)^2 - 784*a^3*b^8*sin(e/2 + (f*x)/2)^2 + 952*root(54*a
^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
- 9*a*b^4*z - b^4, z, k)*b^12*cos(e/2 + (f*x)/2)^2 + 2352*a*b^10*cos(e/2 +
(f*x)/2)^2 + 1944*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^
4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*b^12*sin(e/2 + (f*x)/2)
^2 - 56*a*b^10*sin(e/2 + (f*x)/2)^2 + 304*root(54*a^5*b^2*z^3 - 27*a^3*b^4*
z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)
^2*b^13*cos(e/2 + (f*x)/2)^2 + 32*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27
*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*b^14*
cos(e/2 + (f*x)/2)^2 + 1032*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z
^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a*b^11*sin(e/
2 + (f*x)/2)^2 + 39032*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 -
54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^2*b^10*cos(e/2 +
(f*x)/2)^2 + 30296*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*
a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^3*b^9*cos(e/2 + (f*
x)/2)^2 + 7420*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b
^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^4*b^8*cos(e/2 + (f*x)/2)
^2 - 252*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2
- 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^5*b^7*cos(e/2 + (f*x)/2)^2 - 1
68*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*
a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^6*b^6*cos(e/2 + (f*x)/2)^2 + 14240*r
oot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*
b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a*b^12*cos(e/2 + (f*x)/2)^2 + 4064*root(
54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*
z^2 - 9*a*b^4*z - b^4, z, k)^3*a*b^13*cos(e/2 + (f*x)/2)^2 + 384*root(54*a^
5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^4*a*b^14*cos(e/2 + (f*x)/2)^2 - 27888*root(54*a^5*b
^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*
a*b^4*z - b^4, z, k)*a^2*b^10*sin(e/2 + (f*x)/2)^2 - 55576*root(54*a^5*b^2*
z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b
^4*z - b^4, z, k)*a^3*b^9*sin(e/2 + (f*x)/2)^2 - 32174*root(54*a^5*b^2*z^3
- 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z
- b^4, z, k)*a^4*b^8*sin(e/2 + (f*x)/2)^2 - 3318*root(54*a^5*b^2*z^3 - 27*

a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^5*b^7*sin(e/2 + (f*x)/2)^2 + 252*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^6*b^6*sin(e/2 + (f*x)/2)^2 + 24840*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a*b^12*sin(e/2 + (f*x)/2)^2 + 7856*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a*b^13*sin(e/2 + (f*x)/2)^2 + 384*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a*b^14*sin(e/2 + (f*x)/2)^2 + 107772*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^2*b^11*cos(e/2 + (f*x)/2)^2 + 156216*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^3*b^10*cos(e/2 + (f*x)/2)^2 + 55448*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^4*b^9*cos(e/2 + (f*x)/2)^2 + 21772*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^5*b^8*cos(e/2 + (f*x)/2)^2 + 35364*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^6*b^7*cos(e/2 + (f*x)/2)^2 + 3588*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^7*b^6*cos(e/2 + (f*x)/2)^2 - 3051*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^8*b^5*cos(e/2 + (f*x)/2)^2 + 18*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^9*b^4*cos(e/2 + (f*x)/2)^2 + 73528*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^2*b^12*cos(e/2 + (f*x)/2)^2 + 222176*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^3*b^11*cos(e/2 + (f*x)/2)^2 - 101192*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^4*b^10*cos(e/2 + (f*x)/2)^2 - 567064*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^5*b^9*cos(e/2 + (f*x)/2)^2 - 125428*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^6*b^8*cos(e/2 + (f*x)/2)^2 + 278436*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^7*b^7*cos(e/2 + (f*x)/2)^2 + 66894*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^8*b^6*cos(e/2 + (f*x)/2)^2 - 26928*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^9*b^5*cos(e/2 + (f*x)/2)^2 - 3042*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^10*b^4*cos(e/2 + (f*x)/2)^2 + 648*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^11*b^3*cos(e/2 + (f*x)/2)^2 + 19104*ro

ot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b
^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^2*b^13*cos(e/2 + (f*x)/2)^2 + 128832*ro
ot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b
^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^3*b^12*cos(e/2 + (f*x)/2)^2 - 191988*ro
ot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b
^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^4*b^11*cos(e/2 + (f*x)/2)^2 - 899856*ro
ot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b
^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^5*b^10*cos(e/2 + (f*x)/2)^2 + 183204*ro
ot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b
^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^6*b^9*cos(e/2 + (f*x)/2)^2 + 1173036*ro
ot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b
^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^7*b^8*cos(e/2 + (f*x)/2)^2 - 519612*roo
t(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^
4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^8*b^7*cos(e/2 + (f*x)/2)^2 - 666384*root
(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4
*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^9*b^6*cos(e/2 + (f*x)/2)^2 + 368028*root(
54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*
z^2 - 9*a*b^4*z - b^4, z, k)^4*a^10*b^5*cos(e/2 + (f*x)/2)^2 + 46260*root(5
4*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z
^2 - 9*a*b^4*z - b^4, z, k)^4*a^11*b^4*cos(e/2 + (f*x)/2)^2 - 46332*root(54
*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^
2 - 9*a*b^4*z - b^4, z, k)^4*a^12*b^3*cos(e/2 + (f*x)/2)^2 + 5832*root(54*a
^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
- 9*a*b^4*z - b^4, z, k)^4*a^13*b^2*cos(e/2 + (f*x)/2)^2 + 1728*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^5*a^2*b^14*cos(e/2 + (f*x)/2)^2 + 34560*root(54*a^5*
b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9
*a*b^4*z - b^4, z, k)^5*a^3*b^13*cos(e/2 + (f*x)/2)^2 - 51480*root(54*a^5*b
^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*
a*b^4*z - b^4, z, k)^5*a^4*b^12*cos(e/2 + (f*x)/2)^2 - 677880*root(54*a^5*b
^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*
a*b^4*z - b^4, z, k)^5*a^5*b^11*cos(e/2 + (f*x)/2)^2 + 773640*root(54*a^5*b
^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*
a*b^4*z - b^4, z, k)^5*a^6*b^10*cos(e/2 + (f*x)/2)^2 + 1207440*root(54*a^5*
b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9
*a*b^4*z - b^4, z, k)^5*a^7*b^9*cos(e/2 + (f*x)/2)^2 - 1363176*root(54*a^5*
b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9
*a*b^4*z - b^4, z, k)^5*a^8*b^8*cos(e/2 + (f*x)/2)^2 - 82728*root(54*a^5*b^
2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a
*b^4*z - b^4, z, k)^5*a^9*b^7*cos(e/2 + (f*x)/2)^2 + 738792*root(54*a^5*b^2
*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*
b^4*z - b^4, z, k)^5*a^10*b^6*cos(e/2 + (f*x)/2)^2 - 412704*root(54*a^5*b^2
*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*
b^4*z - b^4, z, k)^5*a^11*b^5*cos(e/2 + (f*x)/2)^2 + 11304*root(54*a^5*b^2*
z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b

^4*z - b^4, z, k)^5*a^12*b^4*cos(e/2 + (f*x)/2)^2 + 36288*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^13*b^3*cos(e/2 + (f*x)/2)^2 - 5832*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^14*b^2*cos(e/2 + (f*x)/2)^2 + 3456*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^3*b^14*cos(e/2 + (f*x)/2)^2 + 7344*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^4*b^13*cos(e/2 + (f*x)/2)^2 - 225504*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^5*b^12*cos(e/2 + (f*x)/2)^2 + 450468*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^6*b^11*cos(e/2 + (f*x)/2)^2 + 360072*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^7*b^10*cos(e/2 + (f*x)/2)^2 - 879984*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^8*b^9*cos(e/2 + (f*x)/2)^2 - 183600*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^9*b^8*cos(e/2 + (f*x)/2)^2 + 431352*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^10*b^7*cos(e/2 + (f*x)/2)^2 + 165888*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^11*b^6*cos(e/2 + (f*x)/2)^2 - 61344*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^12*b^5*cos(e/2 + (f*x)/2)^2 - 114480*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^13*b^4*cos(e/2 + (f*x)/2)^2 + 52164*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^14*b^3*cos(e/2 + (f*x)/2)^2 - 5832*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^15*b^2*cos(e/2 + (f*x)/2)^2 + 2592*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^4*b^14*cos(e/2 + (f*x)/2)^2 - 28512*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^5*b^13*cos(e/2 + (f*x)/2)^2 + 75816*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^6*b^12*cos(e/2 + (f*x)/2)^2 + 71280*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^7*b^11*cos(e/2 + (f*x)/2)^2 - 323352*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^8*b^10*cos(e/2 + (f*x)/2)^2 + 483408*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^10*b^8*cos(e/2 + (f*x)/2)^2 - 142560*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^11*b^7*cos(e/2 + (f*x)/2)^2 - 307152*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z

^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^
7*a^12*b^6*cos(e/2 + (f*x)/2)^2 + 142560*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z
^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^
7*a^13*b^5*cos(e/2 + (f*x)/2)^2 + 62856*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^
3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7
*a^14*b^4*cos(e/2 + (f*x)/2)^2 - 42768*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3
 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*
a^15*b^3*cos(e/2 + (f*x)/2)^2 + 5832*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 -
 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^
16*b^2*cos(e/2 + (f*x)/2)^2 - 43760*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 -
27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^2
*b^11*sin(e/2 + (f*x)/2)^2 - 401720*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 -
27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^3
*b^10*sin(e/2 + (f*x)/2)^2 - 563860*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 -
27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^4
*b^9*sin(e/2 + (f*x)/2)^2 - 249110*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 2
7*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^5*
b^8*sin(e/2 + (f*x)/2)^2 - 59988*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*
a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^6*b^
7*sin(e/2 + (f*x)/2)^2 - 35586*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^
7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^7*b^6*
sin(e/2 + (f*x)/2)^2 + 5751*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z
^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^8*b^5*sin
(e/2 + (f*x)/2)^2 - 18*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 -
54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^9*b^4*sin(e/2
+ (f*x)/2)^2 + 95632*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54
*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^2*b^12*sin(e/2 +
 (f*x)/2)^2 - 439696*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54
*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^3*b^11*sin(e/2 +
 (f*x)/2)^2 - 1471448*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 5
4*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^4*b^10*sin(e/2
+ (f*x)/2)^2 - 606964*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 5
4*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^5*b^9*sin(e/2 +
 (f*x)/2)^2 + 521558*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54
*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^6*b^8*sin(e/2 +
(f*x)/2)^2 - 415182*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*
a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^7*b^7*sin(e/2 + (
f*x)/2)^2 - 598284*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a
^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^8*b^6*sin(e/2 + (f
*x)/2)^2 + 24606*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4
*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^9*b^5*sin(e/2 + (f*x
)/2)^2 + 20592*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b
^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^10*b^4*sin(e/2 + (f*x)
/2)^2 - 756*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*
z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^11*b^3*sin(e/2 + (f*x)/2)

^2 + 33984*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z
^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^2*b^13*sin(e/2 + (f*x)/2)^
2 + 32784*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^
2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^3*b^12*sin(e/2 + (f*x)/2)^2
 - 1159584*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z
^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^4*b^11*sin(e/2 + (f*x)/2)^
2 + 567024*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z
^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^5*b^10*sin(e/2 + (f*x)/2)^
2 + 6779964*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*
z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^6*b^9*sin(e/2 + (f*x)/2)^
2 + 4475790*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*
z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^7*b^8*sin(e/2 + (f*x)/2)^
2 - 2069340*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*
z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^8*b^7*sin(e/2 + (f*x)/2)^
2 - 421956*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z
^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^9*b^6*sin(e/2 + (f*x)/2)^2
 + 382248*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^
2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^10*b^5*sin(e/2 + (f*x)/2)^2
 - 554778*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^
2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^11*b^4*sin(e/2 + (f*x)/2)^2
 + 146880*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^
2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^12*b^3*sin(e/2 + (f*x)/2)^2
 - 7776*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2
- 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^13*b^2*sin(e/2 + (f*x)/2)^2 +
 1728*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^2*b^14*sin(e/2 + (f*x)/2)^2 + 5
4432*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 2
7*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^3*b^13*sin(e/2 + (f*x)/2)^2 - 42
2856*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 2
7*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^4*b^12*sin(e/2 + (f*x)/2)^2 + 62
5176*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 2
7*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^5*b^11*sin(e/2 + (f*x)/2)^2 + 61
26696*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^6*b^10*sin(e/2 + (f*x)/2)^2 - 2
480004*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^7*b^9*sin(e/2 + (f*x)/2)^2 - 1
5505344*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2
- 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^8*b^8*sin(e/2 + (f*x)/2)^2 -
346572*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^9*b^7*sin(e/2 + (f*x)/2)^2 + 9
474120*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^10*b^6*sin(e/2 + (f*x)/2)^2 +
24660*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^11*b^5*sin(e/2 + (f*x)/2)^2 - 1
571688*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -

27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^12*b^4*sin(e/2 + (f*x)/2)^2 +
232740*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^13*b^3*sin(e/2 + (f*x)/2)^2 +
7776*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 2
7*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^14*b^2*sin(e/2 + (f*x)/2)^2 + 34
56*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*
a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^3*b^14*sin(e/2 + (f*x)/2)^2 - 1728
*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^
2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^4*b^13*sin(e/2 + (f*x)/2)^2 - 319896
*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^
2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^5*b^12*sin(e/2 + (f*x)/2)^2 + 324691
2*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a
^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^6*b^11*sin(e/2 + (f*x)/2)^2 - 53222
40*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*
a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^7*b^10*sin(e/2 + (f*x)/2)^2 - 9560
160*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27
*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^8*b^9*sin(e/2 + (f*x)/2)^2 + 1605
5280*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 2
7*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^9*b^8*sin(e/2 + (f*x)/2)^2 + 848
5344*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 2
7*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^10*b^7*sin(e/2 + (f*x)/2)^2 - 14
873760*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^11*b^6*sin(e/2 + (f*x)/2)^2 -
1269216*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2
- 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^12*b^5*sin(e/2 + (f*x)/2)^2 +
4449384*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2
- 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^13*b^4*sin(e/2 + (f*x)/2)^2
- 901152*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2
- 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^14*b^3*sin(e/2 + (f*x)/2)^2
+ 7776*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^15*b^2*sin(e/2 + (f*x)/2)^2 +
2592*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 2
7*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^4*b^14*sin(e/2 + (f*x)/2)^2 - 58
320*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27
*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^5*b^13*sin(e/2 + (f*x)/2)^2 + 603
936*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27
*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^6*b^12*sin(e/2 + (f*x)/2)^2 - 223
0416*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 2
7*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^7*b^11*sin(e/2 + (f*x)/2)^2 + 53
6544*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 2
7*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^8*b^10*sin(e/2 + (f*x)/2)^2 + 65
18880*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^9*b^9*sin(e/2 + (f*x)/2)^2 - 52
51392*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^10*b^8*sin(e/2 + (f*x)/2)^2 - 5

590944*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^11*b^7*sin(e/2 + (f*x)/2)^2 + 6456672*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^12*b^6*sin(e/2 + (f*x)/2)^2 + 838512*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^13*b^5*sin(e/2 + (f*x)/2)^2 - 2340576*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^14*b^4*sin(e/2 + (f*x)/2)^2 + 522288*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^15*b^3*sin(e/2 + (f*x)/2)^2 - 7776*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^16*b^2*sin(e/2 + (f*x)/2)^2 + 17192*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a*b^11*cos(e/2 + (f*x)/2)^2))/(cos(e/2 + (f*x)/2)^2*(a + b)^3*(a^2 - 2*a*b + b^2)))*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k), k, 1, 3) + 8*a*b^2*log((cos(e/2 + (f*x)/2)^2 + sin(e/2 + (f*x)/2)^2)/cos(e/2 + (f*x)/2)^2)*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^2 - 8*a^2*b*log((cos(e/2 + (f*x)/2)^2 + sin(e/2 + (f*x)/2)^2)/cos(e/2 + (f*x)/2)^2)*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^2 - 20*a*b^2*cos(e/2 + (f*x)/2)^2*log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2))*sin(e/2 + (f*x)/2)^2 + 12*a^2*b*cos(e/2 + (f*x)/2)^2*log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2))*sin(e/2 + (f*x)/2)^2 + 8*a*b^3*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^2*symsum(log((131072*(980*b^11*cos(e/2 + (f*x)/2)^2 + 336*b^11*sin(e/2 + (f*x)/2)^2 + 1764*a^2*b^9*cos(e/2 + (f*x)/2)^2 + 392*a^3*b^8*cos(e/2 + (f*x)/2)^2 + 640*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*b^13*sin(e/2 + (f*x)/2)^2 + 32*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*b^14*sin(e/2 + (f*x)/2)^2 - 1176*a^2*b^9*sin(e/2 + (f*x)/2)^2 - 784*a^3*b^8*sin(e/2 + (f*x)/2)^2 + 952*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*b^12*cos(e/2 + (f*x)/2)^2 + 2352*a*b^10*cos(e/2 + (f*x)/2)^2 + 1944*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*b^12*sin(e/2 + (f*x)/2)^2 - 56*a*b^10*sin(e/2 + (f*x)/2)^2 + 304*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*b^13*cos(e/2 + (f*x)/2)^2 + 32*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*b^14*cos(e/2 + (f*x)/2)^2 + 1032*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a*b^11*sin(e/2 + (f*x)/2)^2 + 39032*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^2*b^10*cos(e/2 + (f*x)/2)^2 + 30296*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^3*b^9*cos(e/2 + (f*x)/2)^2 + 7420*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2* b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^4*b^8*cos(e/2 + (f*x)/2)^2 - 252*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^5*b^7*cos(e/2 + (f*x)/2)^2 - 168*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^6*b^6*cos(e/2 + (f*x)/2)^2 + 14240*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a*b^12*cos(e/2 + (f*x)/2)^2 + 4064*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a*b^13*cos(e/2 + (f*x)/2)^2 + 384*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a*b^14*cos(e/2 + (f*x)/2)^2 - 27888*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^2*b^10*sin(e/2 + (f*x)/2)^2 - 55576*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^3*b^9*sin(e/2 + (f*x)/2)^2 - 32174*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^4*b^8*sin(e/2 + (f*x)/2)^2 - 3318*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^5*b^7*sin(e/2 + (f*x)/2)^2 + 252*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^6*b^6*sin(e/2 + (f*x)/2)^2 + 24840*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a*b^12*sin(e/2 + (f*x)/2)^2 + 7856*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a*b^13*sin(e/2 + (f*x)/2)^2 + 384*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a*b^14*sin(e/2 + (f*x)/2)^2 + 107772*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^2*b^11*cos(e/2 + (f*x)/2)^2 + 156216*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^3*b^10*cos(e/2 + (f*x)/2)^2 + 55448*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^4*b^9*cos(e/2 + (f*x)/2)^2 + 21772*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^5*b^8*cos(e/2 + (f*x)/2)^2 + 35364*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^6*b^7*cos(e/2 + (f*x)/2)^2 + 3588*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^7*b^6*cos(e/2 + (f*x)/2)^2 - 3051*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^8*b^5*cos(e/2 + (f*x)/2)^2 + 18*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^9*b^4*cos(e/2 + (f*x)/2)^2 + 73528*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^2*b^12*cos(e/2 + (f*x)/2)^2 + 222176*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*

z^2 - 9*a*b^4*z - b^4, z, k)^3*a^3*b^11*cos(e/2 + (f*x)/2)^2 - 101192*root(
54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*
z^2 - 9*a*b^4*z - b^4, z, k)^3*a^4*b^10*cos(e/2 + (f*x)/2)^2 - 567064*root(
54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*
z^2 - 9*a*b^4*z - b^4, z, k)^3*a^5*b^9*cos(e/2 + (f*x)/2)^2 - 125428*root(5
4*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z
^2 - 9*a*b^4*z - b^4, z, k)^3*a^6*b^8*cos(e/2 + (f*x)/2)^2 + 278436*root(54
*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^
2 - 9*a*b^4*z - b^4, z, k)^3*a^7*b^7*cos(e/2 + (f*x)/2)^2 + 66894*root(54*a
^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
- 9*a*b^4*z - b^4, z, k)^3*a^8*b^6*cos(e/2 + (f*x)/2)^2 - 26928*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^3*a^9*b^5*cos(e/2 + (f*x)/2)^2 - 3042*root(54*a^5*b^
2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a
*b^4*z - b^4, z, k)^3*a^10*b^4*cos(e/2 + (f*x)/2)^2 + 648*root(54*a^5*b^2*z
^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^
4*z - b^4, z, k)^3*a^11*b^3*cos(e/2 + (f*x)/2)^2 + 19104*root(54*a^5*b^2*z^
3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4
*z - b^4, z, k)^4*a^2*b^13*cos(e/2 + (f*x)/2)^2 + 128832*root(54*a^5*b^2*z^
3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4
*z - b^4, z, k)^4*a^3*b^12*cos(e/2 + (f*x)/2)^2 - 191988*root(54*a^5*b^2*z^
3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4
*z - b^4, z, k)^4*a^4*b^11*cos(e/2 + (f*x)/2)^2 - 899856*root(54*a^5*b^2*z^
3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4
*z - b^4, z, k)^4*a^5*b^10*cos(e/2 + (f*x)/2)^2 + 183204*root(54*a^5*b^2*z^
3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4
*z - b^4, z, k)^4*a^6*b^9*cos(e/2 + (f*x)/2)^2 + 1173036*root(54*a^5*b^2*z^
3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4
*z - b^4, z, k)^4*a^7*b^8*cos(e/2 + (f*x)/2)^2 - 519612*root(54*a^5*b^2*z^3
 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*
z - b^4, z, k)^4*a^8*b^7*cos(e/2 + (f*x)/2)^2 - 666384*root(54*a^5*b^2*z^3
- 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z
 - b^4, z, k)^4*a^9*b^6*cos(e/2 + (f*x)/2)^2 + 368028*root(54*a^5*b^2*z^3 -
 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z
- b^4, z, k)^4*a^10*b^5*cos(e/2 + (f*x)/2)^2 + 46260*root(54*a^5*b^2*z^3 -
27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z -
 b^4, z, k)^4*a^11*b^4*cos(e/2 + (f*x)/2)^2 - 46332*root(54*a^5*b^2*z^3 - 2
7*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z -
b^4, z, k)^4*a^12*b^3*cos(e/2 + (f*x)/2)^2 + 5832*root(54*a^5*b^2*z^3 - 27*
a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^
4, z, k)^4*a^13*b^2*cos(e/2 + (f*x)/2)^2 + 1728*root(54*a^5*b^2*z^3 - 27*a^
3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4,
 z, k)^5*a^2*b^14*cos(e/2 + (f*x)/2)^2 + 34560*root(54*a^5*b^2*z^3 - 27*a^3
*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4,
z, k)^5*a^3*b^13*cos(e/2 + (f*x)/2)^2 - 51480*root(54*a^5*b^2*z^3 - 27*a^3*

b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z
, k)^5*a^4*b^12*cos(e/2 + (f*x)/2)^2 - 677880*root(54*a^5*b^2*z^3 - 27*a^3*
b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z
, k)^5*a^5*b^11*cos(e/2 + (f*x)/2)^2 + 773640*root(54*a^5*b^2*z^3 - 27*a^3*
b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z
, k)^5*a^6*b^10*cos(e/2 + (f*x)/2)^2 + 1207440*root(54*a^5*b^2*z^3 - 27*a^3
*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4,
z, k)^5*a^7*b^9*cos(e/2 + (f*x)/2)^2 - 1363176*root(54*a^5*b^2*z^3 - 27*a^3
*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4,
z, k)^5*a^8*b^8*cos(e/2 + (f*x)/2)^2 - 82728*root(54*a^5*b^2*z^3 - 27*a^3*b
^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z,
 k)^5*a^9*b^7*cos(e/2 + (f*x)/2)^2 + 738792*root(54*a^5*b^2*z^3 - 27*a^3*b^
4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z,
k)^5*a^10*b^6*cos(e/2 + (f*x)/2)^2 - 412704*root(54*a^5*b^2*z^3 - 27*a^3*b^
4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z,
k)^5*a^11*b^5*cos(e/2 + (f*x)/2)^2 + 11304*root(54*a^5*b^2*z^3 - 27*a^3*b^4
*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k
)^5*a^12*b^4*cos(e/2 + (f*x)/2)^2 + 36288*root(54*a^5*b^2*z^3 - 27*a^3*b^4*
z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)
^5*a^13*b^3*cos(e/2 + (f*x)/2)^2 - 5832*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^
3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5
*a^14*b^2*cos(e/2 + (f*x)/2)^2 + 3456*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3
- 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a
^3*b^14*cos(e/2 + (f*x)/2)^2 + 7344*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 -
27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^4
*b^13*cos(e/2 + (f*x)/2)^2 - 225504*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 -
27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^5
*b^12*cos(e/2 + (f*x)/2)^2 + 450468*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 -
27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^6
*b^11*cos(e/2 + (f*x)/2)^2 + 360072*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 -
27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^7
*b^10*cos(e/2 + (f*x)/2)^2 - 879984*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 -
27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^8
*b^9*cos(e/2 + (f*x)/2)^2 - 183600*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 2
7*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^9*
b^8*cos(e/2 + (f*x)/2)^2 + 431352*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27
*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^10*
b^7*cos(e/2 + (f*x)/2)^2 + 165888*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27
*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^11*
b^6*cos(e/2 + (f*x)/2)^2 - 61344*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*
a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^12*b
^5*cos(e/2 + (f*x)/2)^2 - 114480*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*
a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^13*b
^4*cos(e/2 + (f*x)/2)^2 + 52164*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a
^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^14*b^

3*cos(e/2 + (f*x)/2)^2 - 5832*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7
*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^15*b^2*
cos(e/2 + (f*x)/2)^2 + 2592*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z
^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^4*b^14*co
s(e/2 + (f*x)/2)^2 - 28512*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^
3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^5*b^13*cos
(e/2 + (f*x)/2)^2 + 75816*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3
 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^6*b^12*cos(
e/2 + (f*x)/2)^2 + 71280*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3
- 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^7*b^11*cos(e
/2 + (f*x)/2)^2 - 323352*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3
- 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^8*b^10*cos(e
/2 + (f*x)/2)^2 + 483408*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3
- 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^10*b^8*cos(e
/2 + (f*x)/2)^2 - 142560*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3
- 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^11*b^7*cos(e
/2 + (f*x)/2)^2 - 307152*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3
- 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^12*b^6*cos(e
/2 + (f*x)/2)^2 + 142560*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3
- 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^13*b^5*cos(e
/2 + (f*x)/2)^2 + 62856*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 -
 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^14*b^4*cos(e/
2 + (f*x)/2)^2 - 42768*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 -
54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^15*b^3*cos(e/2
 + (f*x)/2)^2 + 5832*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54
*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^16*b^2*cos(e/2 +
 (f*x)/2)^2 - 43760*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*
a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^2*b^11*sin(e/2 +
(f*x)/2)^2 - 401720*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*
a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^3*b^10*sin(e/2 +
(f*x)/2)^2 - 563860*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*
a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^4*b^9*sin(e/2 + (
f*x)/2)^2 - 249110*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a
^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^5*b^8*sin(e/2 + (f
*x)/2)^2 - 59988*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4
*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^6*b^7*sin(e/2 + (f*x
)/2)^2 - 35586*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b
^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^7*b^6*sin(e/2 + (f*x)/
2)^2 + 5751*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*
z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^8*b^5*sin(e/2 + (f*x)/2)^
2 - 18*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^9*b^4*sin(e/2 + (f*x)/2)^2 + 9
5632*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 2
7*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^2*b^12*sin(e/2 + (f*x)/2)^2 - 43
9696*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 2

7*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^3*b^11*sin(e/2 + (f*x)/2)^2 - 14
71448*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^4*b^10*sin(e/2 + (f*x)/2)^2 - 6
06964*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^5*b^9*sin(e/2 + (f*x)/2)^2 + 52
1558*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 2
7*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^6*b^8*sin(e/2 + (f*x)/2)^2 - 415
182*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27
*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^7*b^7*sin(e/2 + (f*x)/2)^2 - 5982
84*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*
a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^8*b^6*sin(e/2 + (f*x)/2)^2 + 24606
*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^
2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^9*b^5*sin(e/2 + (f*x)/2)^2 + 20592*r
oot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*
b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^10*b^4*sin(e/2 + (f*x)/2)^2 - 756*root
(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4
*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^11*b^3*sin(e/2 + (f*x)/2)^2 + 33984*root(
54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*
z^2 - 9*a*b^4*z - b^4, z, k)^4*a^2*b^13*sin(e/2 + (f*x)/2)^2 + 32784*root(5
4*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z
^2 - 9*a*b^4*z - b^4, z, k)^4*a^3*b^12*sin(e/2 + (f*x)/2)^2 - 1159584*root(
54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*
z^2 - 9*a*b^4*z - b^4, z, k)^4*a^4*b^11*sin(e/2 + (f*x)/2)^2 + 567024*root(
54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*
z^2 - 9*a*b^4*z - b^4, z, k)^4*a^5*b^10*sin(e/2 + (f*x)/2)^2 + 6779964*root
(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4
*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^6*b^9*sin(e/2 + (f*x)/2)^2 + 4475790*root
(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4
*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^7*b^8*sin(e/2 + (f*x)/2)^2 - 2069340*root
(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4
*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^8*b^7*sin(e/2 + (f*x)/2)^2 - 421956*root(
54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*
z^2 - 9*a*b^4*z - b^4, z, k)^4*a^9*b^6*sin(e/2 + (f*x)/2)^2 + 382248*root(5
4*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z
^2 - 9*a*b^4*z - b^4, z, k)^4*a^10*b^5*sin(e/2 + (f*x)/2)^2 - 554778*root(5
4*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z
^2 - 9*a*b^4*z - b^4, z, k)^4*a^11*b^4*sin(e/2 + (f*x)/2)^2 + 146880*root(5
4*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z
^2 - 9*a*b^4*z - b^4, z, k)^4*a^12*b^3*sin(e/2 + (f*x)/2)^2 - 7776*root(54*
a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
 - 9*a*b^4*z - b^4, z, k)^4*a^13*b^2*sin(e/2 + (f*x)/2)^2 + 1728*root(54*a^
5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
 9*a*b^4*z - b^4, z, k)^5*a^2*b^14*sin(e/2 + (f*x)/2)^2 + 54432*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^5*a^3*b^13*sin(e/2 + (f*x)/2)^2 - 422856*root(54*a^5

*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^5*a^4*b^12*sin(e/2 + (f*x)/2)^2 + 625176*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^5*a^5*b^11*sin(e/2 + (f*x)/2)^2 + 6126696*root(54*a^
5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
 9*a*b^4*z - b^4, z, k)^5*a^6*b^10*sin(e/2 + (f*x)/2)^2 - 2480004*root(54*a
^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
 - 9*a*b^4*z - b^4, z, k)^5*a^7*b^9*sin(e/2 + (f*x)/2)^2 - 15505344*root(54*
a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
 - 9*a*b^4*z - b^4, z, k)^5*a^8*b^8*sin(e/2 + (f*x)/2)^2 - 346572*root(54*a
^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
 - 9*a*b^4*z - b^4, z, k)^5*a^9*b^7*sin(e/2 + (f*x)/2)^2 + 9474120*root(54*a
^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
 - 9*a*b^4*z - b^4, z, k)^5*a^10*b^6*sin(e/2 + (f*x)/2)^2 + 24660*root(54*a^
5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
 9*a*b^4*z - b^4, z, k)^5*a^11*b^5*sin(e/2 + (f*x)/2)^2 - 1571688*root(54*a
^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
 - 9*a*b^4*z - b^4, z, k)^5*a^12*b^4*sin(e/2 + (f*x)/2)^2 + 232740*root(54*a
^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
 - 9*a*b^4*z - b^4, z, k)^5*a^13*b^3*sin(e/2 + (f*x)/2)^2 + 7776*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^5*a^14*b^2*sin(e/2 + (f*x)/2)^2 + 3456*root(54*a^5*b
^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*
a*b^4*z - b^4, z, k)^6*a^3*b^14*sin(e/2 + (f*x)/2)^2 - 1728*root(54*a^5*b^2
*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*
b^4*z - b^4, z, k)^6*a^4*b^13*sin(e/2 + (f*x)/2)^2 - 319896*root(54*a^5*b^2
*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*
b^4*z - b^4, z, k)^6*a^5*b^12*sin(e/2 + (f*x)/2)^2 + 3246912*root(54*a^5*b^
2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a
*b^4*z - b^4, z, k)^6*a^6*b^11*sin(e/2 + (f*x)/2)^2 - 5322240*root(54*a^5*b
^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*
a*b^4*z - b^4, z, k)^6*a^7*b^10*sin(e/2 + (f*x)/2)^2 - 9560160*root(54*a^5*
b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9
*a*b^4*z - b^4, z, k)^6*a^8*b^9*sin(e/2 + (f*x)/2)^2 + 16055280*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^6*a^9*b^8*sin(e/2 + (f*x)/2)^2 + 8485344*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^6*a^10*b^7*sin(e/2 + (f*x)/2)^2 - 14873760*root(54*a
^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
 - 9*a*b^4*z - b^4, z, k)^6*a^11*b^6*sin(e/2 + (f*x)/2)^2 - 1269216*root(54*
a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
 - 9*a*b^4*z - b^4, z, k)^6*a^12*b^5*sin(e/2 + (f*x)/2)^2 + 4449384*root(54
*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^
2 - 9*a*b^4*z - b^4, z, k)^6*a^13*b^4*sin(e/2 + (f*x)/2)^2 - 901152*root(54
*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^

2 - 9*a*b^4*z - b^4, z, k)^6*a^14*b^3*sin(e/2 + (f*x)/2)^2 + 7776*root(54*a
^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
- 9*a*b^4*z - b^4, z, k)^6*a^15*b^2*sin(e/2 + (f*x)/2)^2 + 2592*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^7*a^4*b^14*sin(e/2 + (f*x)/2)^2 - 58320*root(54*a^5*
b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9
*a*b^4*z - b^4, z, k)^7*a^5*b^13*sin(e/2 + (f*x)/2)^2 + 603936*root(54*a^5*
b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9
*a*b^4*z - b^4, z, k)^7*a^6*b^12*sin(e/2 + (f*x)/2)^2 - 2230416*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^7*a^7*b^11*sin(e/2 + (f*x)/2)^2 + 536544*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^7*a^8*b^10*sin(e/2 + (f*x)/2)^2 + 6518880*root(54*a^
5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
 9*a*b^4*z - b^4, z, k)^7*a^9*b^9*sin(e/2 + (f*x)/2)^2 - 5251392*root(54*a^
5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
 9*a*b^4*z - b^4, z, k)^7*a^10*b^8*sin(e/2 + (f*x)/2)^2 - 5590944*root(54*a
^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
- 9*a*b^4*z - b^4, z, k)^7*a^11*b^7*sin(e/2 + (f*x)/2)^2 + 6456672*root(54*
a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
 - 9*a*b^4*z - b^4, z, k)^7*a^12*b^6*sin(e/2 + (f*x)/2)^2 + 838512*root(54*
a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
 - 9*a*b^4*z - b^4, z, k)^7*a^13*b^5*sin(e/2 + (f*x)/2)^2 - 2340576*root(54
*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^
2 - 9*a*b^4*z - b^4, z, k)^7*a^14*b^4*sin(e/2 + (f*x)/2)^2 + 522288*root(54
*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^
2 - 9*a*b^4*z - b^4, z, k)^7*a^15*b^3*sin(e/2 + (f*x)/2)^2 - 7776*root(54*a
^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
- 9*a*b^4*z - b^4, z, k)^7*a^16*b^2*sin(e/2 + (f*x)/2)^2 + 17192*root(54*a^
5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
 9*a*b^4*z - b^4, z, k)*a*b^11*cos(e/2 + (f*x)/2)^2))/(cos(e/2 + (f*x)/2)^2
(a + b)^3(a^2 - 2*a*b + b^2)))*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*
a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k), k, 1, 3
) - 8*a^3*b*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^2*symsum(log((131072*(9
80*b^11*cos(e/2 + (f*x)/2)^2 + 336*b^11*sin(e/2 + (f*x)/2)^2 + 1764*a^2*b^9
*cos(e/2 + (f*x)/2)^2 + 392*a^3*b^8*cos(e/2 + (f*x)/2)^2 + 640*root(54*a^5*
b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9
*a*b^4*z - b^4, z, k)^2*b^13*sin(e/2 + (f*x)/2)^2 + 32*root(54*a^5*b^2*z^3
- 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z
 - b^4, z, k)^3*b^14*sin(e/2 + (f*x)/2)^2 - 1176*a^2*b^9*sin(e/2 + (f*x)/2)
^2 - 784*a^3*b^8*sin(e/2 + (f*x)/2)^2 + 952*root(54*a^5*b^2*z^3 - 27*a^3*b^
4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z,
k)*b^12*cos(e/2 + (f*x)/2)^2 + 2352*a*b^10*cos(e/2 + (f*x)/2)^2 + 1944*root
(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4
*z^2 - 9*a*b^4*z - b^4, z, k)*b^12*sin(e/2 + (f*x)/2)^2 - 56*a*b^10*sin(e/2

+ (f*x)/2)^2 + 304*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*b^13*cos(e/2 + (f*x)/2)^2 + 32*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*b^14*cos(e/2 + (f*x)/2)^2 + 1032*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a*b^11*sin(e/2 + (f*x)/2)^2 + 39032*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^2*b^10*cos(e/2 + (f*x)/2)^2 + 30296*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^3*b^9*cos(e/2 + (f*x)/2)^2 + 7420*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^4*b^8*cos(e/2 + (f*x)/2)^2 - 252*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^5*b^7*cos(e/2 + (f*x)/2)^2 - 168*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^6*b^6*cos(e/2 + (f*x)/2)^2 + 14240*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a*b^12*cos(e/2 + (f*x)/2)^2 + 4064*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a*b^13*cos(e/2 + (f*x)/2)^2 + 384*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a*b^14*cos(e/2 + (f*x)/2)^2 - 27888*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^2*b^10*sin(e/2 + (f*x)/2)^2 - 55576*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^3*b^9*sin(e/2 + (f*x)/2)^2 - 32174*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^4*b^8*sin(e/2 + (f*x)/2)^2 - 3318*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^5*b^7*sin(e/2 + (f*x)/2)^2 + 252*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^6*b^6*sin(e/2 + (f*x)/2)^2 + 24840*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a*b^12*sin(e/2 + (f*x)/2)^2 + 7856*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a*b^13*sin(e/2 + (f*x)/2)^2 + 384*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a*b^14*sin(e/2 + (f*x)/2)^2 + 107772*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^2*b^11*cos(e/2 + (f*x)/2)^2 + 156216*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^3*b^10*cos(e/2 + (f*x)/2)^2 + 55448*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^4*b^9*cos(e/2 + (f*x)/2)^2 + 21772*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^5*b^8*cos(e/2 + (f*x)/2)^2 + 3536

4*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^6*b^7*cos(e/2 + (f*x)/2)^2 + 3588*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^7*b^6*cos(e/2 + (f*x)/2)^2 - 3051*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^8*b^5*cos(e/2 + (f*x)/2)^2 + 18*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^9*b^4*cos(e/2 + (f*x)/2)^2 + 73528*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^2*b^12*cos(e/2 + (f*x)/2)^2 + 222176*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^3*b^11*cos(e/2 + (f*x)/2)^2 - 101192*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^4*b^10*cos(e/2 + (f*x)/2)^2 - 567064*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^5*b^9*cos(e/2 + (f*x)/2)^2 - 125428*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^6*b^8*cos(e/2 + (f*x)/2)^2 + 278436*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^7*b^7*cos(e/2 + (f*x)/2)^2 + 66894*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^8*b^6*cos(e/2 + (f*x)/2)^2 - 26928*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^9*b^5*cos(e/2 + (f*x)/2)^2 - 3042*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^10*b^4*cos(e/2 + (f*x)/2)^2 + 648*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^11*b^3*cos(e/2 + (f*x)/2)^2 + 19104*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^2*b^13*cos(e/2 + (f*x)/2)^2 + 128832*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^3*b^12*cos(e/2 + (f*x)/2)^2 - 191988*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^4*b^11*cos(e/2 + (f*x)/2)^2 - 899856*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^5*b^10*cos(e/2 + (f*x)/2)^2 + 183204*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^6*b^9*cos(e/2 + (f*x)/2)^2 + 1173036*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^7*b^8*cos(e/2 + (f*x)/2)^2 - 519612*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^8*b^7*cos(e/2 + (f*x)/2)^2 - 666384*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^9*b^6*cos(e/2 + (f*x)/2)^2 + 368028*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4,

z, k)^4*a^10*b^5*cos(e/2 + (f*x)/2)^2 + 46260*root(54*a^5*b^2*z^3 - 27*a^3
*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4,
z, k)^4*a^11*b^4*cos(e/2 + (f*x)/2)^2 - 46332*root(54*a^5*b^2*z^3 - 27*a^3*
b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z
, k)^4*a^12*b^3*cos(e/2 + (f*x)/2)^2 + 5832*root(54*a^5*b^2*z^3 - 27*a^3*b^
4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z,
k)^4*a^13*b^2*cos(e/2 + (f*x)/2)^2 + 1728*root(54*a^5*b^2*z^3 - 27*a^3*b^4*
z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)
^5*a^2*b^14*cos(e/2 + (f*x)/2)^2 + 34560*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z
^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^
5*a^3*b^13*cos(e/2 + (f*x)/2)^2 - 51480*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^
3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5
*a^4*b^12*cos(e/2 + (f*x)/2)^2 - 677880*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^
3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5
*a^5*b^11*cos(e/2 + (f*x)/2)^2 + 773640*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^
3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5
*a^6*b^10*cos(e/2 + (f*x)/2)^2 + 1207440*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z
^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^
5*a^7*b^9*cos(e/2 + (f*x)/2)^2 - 1363176*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z
^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^
5*a^8*b^8*cos(e/2 + (f*x)/2)^2 - 82728*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3
 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*
a^9*b^7*cos(e/2 + (f*x)/2)^2 + 738792*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3
- 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a
^10*b^6*cos(e/2 + (f*x)/2)^2 - 412704*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3
- 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a
^11*b^5*cos(e/2 + (f*x)/2)^2 + 11304*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 -
 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^
12*b^4*cos(e/2 + (f*x)/2)^2 + 36288*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 -
27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^1
3*b^3*cos(e/2 + (f*x)/2)^2 - 5832*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27
*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^14*
b^2*cos(e/2 + (f*x)/2)^2 + 3456*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a
^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^3*b^1
4*cos(e/2 + (f*x)/2)^2 + 7344*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7
*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^4*b^13*
cos(e/2 + (f*x)/2)^2 - 225504*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7
*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^5*b^12*
cos(e/2 + (f*x)/2)^2 + 450468*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7
*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^6*b^11*
cos(e/2 + (f*x)/2)^2 + 360072*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7
*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^7*b^10*
cos(e/2 + (f*x)/2)^2 - 879984*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7
*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^8*b^9*c
os(e/2 + (f*x)/2)^2 - 183600*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*

z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^9*b^8*co
s(e/2 + (f*x)/2)^2 + 431352*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z
^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^10*b^7*co
s(e/2 + (f*x)/2)^2 + 165888*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z
^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^11*b^6*co
s(e/2 + (f*x)/2)^2 - 61344*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^
3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^12*b^5*cos
(e/2 + (f*x)/2)^2 - 114480*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^
3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^13*b^4*cos
(e/2 + (f*x)/2)^2 + 52164*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3
 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^14*b^3*cos(
e/2 + (f*x)/2)^2 - 5832*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 -
 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^15*b^2*cos(e/
2 + (f*x)/2)^2 + 2592*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 5
4*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^4*b^14*cos(e/2
+ (f*x)/2)^2 - 28512*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54
*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^5*b^13*cos(e/2 +
 (f*x)/2)^2 + 75816*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*
a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^6*b^12*cos(e/2 +
(f*x)/2)^2 + 71280*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a
^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^7*b^11*cos(e/2 + (
f*x)/2)^2 - 323352*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a
^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^8*b^10*cos(e/2 + (
f*x)/2)^2 + 483408*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a
^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^10*b^8*cos(e/2 + (
f*x)/2)^2 - 142560*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a
^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^11*b^7*cos(e/2 + (
f*x)/2)^2 - 307152*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a
^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^12*b^6*cos(e/2 + (
f*x)/2)^2 + 142560*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a
^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^13*b^5*cos(e/2 + (
f*x)/2)^2 + 62856*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^
4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^14*b^4*cos(e/2 + (f
*x)/2)^2 - 42768*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4
*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^15*b^3*cos(e/2 + (f*
x)/2)^2 + 5832*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b
^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^16*b^2*cos(e/2 + (f*x)
/2)^2 - 43760*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^
2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^2*b^11*sin(e/2 + (f*x)/
2)^2 - 401720*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^
2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^3*b^10*sin(e/2 + (f*x)/
2)^2 - 563860*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^
2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^4*b^9*sin(e/2 + (f*x)/2
)^2 - 249110*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2
*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^5*b^8*sin(e/2 + (f*x)/2)

^2 - 59988*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z
^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^6*b^7*sin(e/2 + (f*x)/2)^2
 - 35586*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2
 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^7*b^6*sin(e/2 + (f*x)/2)^2 +
 5751*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^8*b^5*sin(e/2 + (f*x)/2)^2 - 18
*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^
2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^9*b^4*sin(e/2 + (f*x)/2)^2 + 95632*r
oot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*
b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^2*b^12*sin(e/2 + (f*x)/2)^2 - 439696*r
oot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*
b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^3*b^11*sin(e/2 + (f*x)/2)^2 - 1471448*
root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2
*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^4*b^10*sin(e/2 + (f*x)/2)^2 - 606964*
root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2
*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^5*b^9*sin(e/2 + (f*x)/2)^2 + 521558*r
oot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*
b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^6*b^8*sin(e/2 + (f*x)/2)^2 - 415182*ro
ot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b
^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^7*b^7*sin(e/2 + (f*x)/2)^2 - 598284*roo
t(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^
4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^8*b^6*sin(e/2 + (f*x)/2)^2 + 24606*root(
54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*
z^2 - 9*a*b^4*z - b^4, z, k)^3*a^9*b^5*sin(e/2 + (f*x)/2)^2 + 20592*root(54
*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^
2 - 9*a*b^4*z - b^4, z, k)^3*a^10*b^4*sin(e/2 + (f*x)/2)^2 - 756*root(54*a^
5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
 9*a*b^4*z - b^4, z, k)^3*a^11*b^3*sin(e/2 + (f*x)/2)^2 + 33984*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^4*a^2*b^13*sin(e/2 + (f*x)/2)^2 + 32784*root(54*a^5*
b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9
*a*b^4*z - b^4, z, k)^4*a^3*b^12*sin(e/2 + (f*x)/2)^2 - 1159584*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^4*a^4*b^11*sin(e/2 + (f*x)/2)^2 + 567024*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^4*a^5*b^10*sin(e/2 + (f*x)/2)^2 + 6779964*root(54*a^
5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
 9*a*b^4*z - b^4, z, k)^4*a^6*b^9*sin(e/2 + (f*x)/2)^2 + 4475790*root(54*a^
5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
 9*a*b^4*z - b^4, z, k)^4*a^7*b^8*sin(e/2 + (f*x)/2)^2 - 2069340*root(54*a^
5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
 9*a*b^4*z - b^4, z, k)^4*a^8*b^7*sin(e/2 + (f*x)/2)^2 - 421956*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^4*a^9*b^6*sin(e/2 + (f*x)/2)^2 + 382248*root(54*a^5*
b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9

*a*b^4*z - b^4, z, k)^4*a^10*b^5*sin(e/2 + (f*x)/2)^2 - 554778*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^11*b^4*sin(e/2 + (f*x)/2)^2 + 146880*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^12*b^3*sin(e/2 + (f*x)/2)^2 - 7776*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^13*b^2*sin(e/2 + (f*x)/2)^2 + 1728*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^2*b^14*sin(e/2 + (f*x)/2)^2 + 54432*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^3*b^13*sin(e/2 + (f*x)/2)^2 - 422856*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^4*b^12*sin(e/2 + (f*x)/2)^2 + 625176*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^5*b^11*sin(e/2 + (f*x)/2)^2 + 6126696*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^6*b^10*sin(e/2 + (f*x)/2)^2 - 2480004*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^7*b^9*sin(e/2 + (f*x)/2)^2 - 15505344*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^8*b^8*sin(e/2 + (f*x)/2)^2 - 346572*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^9*b^7*sin(e/2 + (f*x)/2)^2 + 9474120*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^10*b^6*sin(e/2 + (f*x)/2)^2 + 24660*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^11*b^5*sin(e/2 + (f*x)/2)^2 - 1571688*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^12*b^4*sin(e/2 + (f*x)/2)^2 + 232740*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^13*b^3*sin(e/2 + (f*x)/2)^2 + 7776*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^14*b^2*sin(e/2 + (f*x)/2)^2 + 3456*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^3*b^14*sin(e/2 + (f*x)/2)^2 - 1728*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^4*b^13*sin(e/2 + (f*x)/2)^2 - 319896*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^5*b^12*sin(e/2 + (f*x)/2)^2 + 3246912*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^6*b^11*sin(e/2 + (f*x)/2)^2 - 5322240*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^7*b^10*sin(e/2 + (f*x)/2)^2 - 9560160*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^8*b^9*sin(e/2 + (f*x)/2)^2 + 16055280*root(54*a^5*b^2*z

^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^
4*z - b^4, z, k)^6*a^9*b^8*sin(e/2 + (f*x)/2)^2 + 8485344*root(54*a^5*b^2*z
^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^
4*z - b^4, z, k)^6*a^10*b^7*sin(e/2 + (f*x)/2)^2 - 14873760*root(54*a^5*b^2
*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*
b^4*z - b^4, z, k)^6*a^11*b^6*sin(e/2 + (f*x)/2)^2 - 1269216*root(54*a^5*b^
2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a
*b^4*z - b^4, z, k)^6*a^12*b^5*sin(e/2 + (f*x)/2)^2 + 4449384*root(54*a^5*b
^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*
a*b^4*z - b^4, z, k)^6*a^13*b^4*sin(e/2 + (f*x)/2)^2 - 901152*root(54*a^5*b
^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*
a*b^4*z - b^4, z, k)^6*a^14*b^3*sin(e/2 + (f*x)/2)^2 + 7776*root(54*a^5*b^2
*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*
b^4*z - b^4, z, k)^6*a^15*b^2*sin(e/2 + (f*x)/2)^2 + 2592*root(54*a^5*b^2*z
^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^
4*z - b^4, z, k)^7*a^4*b^14*sin(e/2 + (f*x)/2)^2 - 58320*root(54*a^5*b^2*z^
3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4
*z - b^4, z, k)^7*a^5*b^13*sin(e/2 + (f*x)/2)^2 + 603936*root(54*a^5*b^2*z^
3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4
*z - b^4, z, k)^7*a^6*b^12*sin(e/2 + (f*x)/2)^2 - 2230416*root(54*a^5*b^2*z
^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^
4*z - b^4, z, k)^7*a^7*b^11*sin(e/2 + (f*x)/2)^2 + 536544*root(54*a^5*b^2*z
^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^
4*z - b^4, z, k)^7*a^8*b^10*sin(e/2 + (f*x)/2)^2 + 6518880*root(54*a^5*b^2*
z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b
^4*z - b^4, z, k)^7*a^9*b^9*sin(e/2 + (f*x)/2)^2 - 5251392*root(54*a^5*b^2*
z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b
^4*z - b^4, z, k)^7*a^10*b^8*sin(e/2 + (f*x)/2)^2 - 5590944*root(54*a^5*b^2
*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*
b^4*z - b^4, z, k)^7*a^11*b^7*sin(e/2 + (f*x)/2)^2 + 6456672*root(54*a^5*b^
2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a
*b^4*z - b^4, z, k)^7*a^12*b^6*sin(e/2 + (f*x)/2)^2 + 838512*root(54*a^5*b^
2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a
*b^4*z - b^4, z, k)^7*a^13*b^5*sin(e/2 + (f*x)/2)^2 - 2340576*root(54*a^5*b
^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*
a*b^4*z - b^4, z, k)^7*a^14*b^4*sin(e/2 + (f*x)/2)^2 + 522288*root(54*a^5*b
^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*
a*b^4*z - b^4, z, k)^7*a^15*b^3*sin(e/2 + (f*x)/2)^2 - 7776*root(54*a^5*b^2
*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*
b^4*z - b^4, z, k)^7*a^16*b^2*sin(e/2 + (f*x)/2)^2 + 17192*root(54*a^5*b^2*
z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b
^4*z - b^4, z, k)*a*b^11*cos(e/2 + (f*x)/2)^2))/(cos(e/2 + (f*x)/2)^2*(a +
b)^3*(a^2 - 2*a*b + b^2)))*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^
3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k), k, 1, 3))/(8*
a*f*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^2*(a + b)^2*(a - b))

3.462 $\int (a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int (a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m dx = \text{Int}((a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m, x)$$

```
[Out] Unintegrable((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m dx = \int (a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m dx$$

```
[In] Int[(a + b*(c*Sec[e + f*x])^n)^p*(d*Tan[e + f*x])^m,x]
[Out] Defer[Int][(a + b*(c*Sec[e + f*x])^n)^p*(d*Tan[e + f*x])^m, x]
```

Rubi steps

$$\text{integral} = \int (a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m dx$$

Mathematica [N/A]

Not integrable

Time = 5.73 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m \, dx = \int (a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m \, dx$$

```
[In] Integrate[(a + b*(c*Sec[e + f*x])^n)^p*(d*Tan[e + f*x])^m,x]
[Out] Integrate[(a + b*(c*Sec[e + f*x])^n)^p*(d*Tan[e + f*x])^m, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.83 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int (a+b(c\sec(fx+e))^n)^p (d\tan(fx+e))^m \, dx$$

```
[In] int((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x)
[Out] int((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m \, dx = \int ((c\sec(fx+e))^n b + a)^p (d\tan(fx+e))^m \, dx$$

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x, algorithm="fricas")
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)
```

Sympy [N/A]

Not integrable

Time = 104.48 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int (a+b(c\sec(e+fx))^n)^p\,(d\tan(e+fx))^m\,dx = \int (d\tan{(e+fx)})^m\,(a+b(c\sec{(e+fx)})^n)^p\,dx$$

```
[In] integrate((a+b*(c*sec(f*x+e))**n)**p*(d*tan(f*x+e))**m,x)
[Out] Integral((d*tan(e + f*x))**m*(a + b*(c*sec(e + f*x))**n)**p, x)
```

Maxima [N/A]

Not integrable

Time = 5.46 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (a+b(c\sec(e+fx))^n)^p\,(d\tan(e+fx))^m\,dx = \int ((c\sec{(fx+e)})^n\,b+a)^p(d\tan{(fx+e)})^m\,dx$$

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x, algorithm="maxima")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)
```

Giac [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (a+b(c\sec(e+fx))^n)^p\,(d\tan(e+fx))^m\,dx = \int ((c\sec{(fx+e)})^n\,b+a)^p(d\tan{(fx+e)})^m\,dx$$

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x, algorithm="giac")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)
```

Mupad [N/A]

Not integrable

Time = 21.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int (a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m \, dx = \int (d\tan(e+f\,x))^m \left(a+b\left(\frac{c}{\cos(e+f\,x)}\right)^n\right)^p \, dx$$

```
[In] int((d*tan(e + f*x))^m*(a + b*(c/cos(e + f*x))^n)^p,x)
[Out] int((d*tan(e + f*x))^m*(a + b*(c/cos(e + f*x))^n)^p, x)
```

3.463 $\int (a+b(c\sec(e+fx))^n)^p \tan^5(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 226

$$\begin{aligned}\int (a+b(c\sec(e+fx))^n)^p \tan^5(e+fx)\,dx &= -\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b(c\sec(e+fx))^n}{a}\right)(a+b(c\sec(e+fx))^n)^{1+p}}{afn(1+p)} \\ &\quad - \frac{\text{Hypergeometric2F1}\left(\frac{2}{n},-p,\frac{2+n}{n},-\frac{b(c\sec(e+fx))^n}{a}\right)\sec^2(e+fx)\,(a+b(c\sec(e+fx))^n)^p\left(1+\frac{b(c\sec(e+fx)}{a}\right.}{f} \\ &\quad + \frac{\text{Hypergeometric2F1}\left(\frac{4}{n},-p,\frac{4+n}{n},-\frac{b(c\sec(e+fx))^n}{a}\right)\sec^4(e+fx)\,(a+b(c\sec(e+fx))^n)^p\left(1+\frac{b(c\sec(e+fx)}{a}\right.}{4f}\end{aligned}$$

```
[Out] -hypergeom([1, p+1],[2+p],1+b*(c*sec(f*x+e))^n/a)*(a+b*(c*sec(f*x+e))^n)^(p
+1)/a/f/n/(p+1)-hypergeom([-p, 2/n],[(2+n)/n],-b*(c*sec(f*x+e))^n/a)*sec(f*
x+e)^2*(a+b*(c*sec(f*x+e))^n)^p/f/((1+b*(c*sec(f*x+e))^n/a)^p)+1/4*hypergeo
m([-p, 4/n],[(4+n)/n],-b*(c*sec(f*x+e))^n/a)*sec(f*x+e)^4*(a+b*(c*sec(f*x+e
))^n)^p/f/((1+b*(c*sec(f*x+e))^n/a)^p)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used

$= \{4224, 6874, 374, 12, 272, 67, 372, 371\}$

$$\int (a+b(c\sec(e+fx))^n)^p \tan^5(e+fx)\,dx$$
$$= \frac{\sec^4(e+fx)\,(a+b(c\sec(e+fx))^n)^p\left(\frac{b(c\sec(e+fx))^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{4}{n},-p,\frac{n+4}{n},-\frac{b(c\sec(e+fx))^n}{a}\right.}{4f}$$
$$-\frac{\sec^2(e+fx)\,(a+b(c\sec(e+fx))^n)^p\left(\frac{b(c\sec(e+fx))^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{n},-p,\frac{n+2}{n},-\frac{b(c\sec(e+fx\right.}{f}$$
$$-\frac{(a+b(c\sec(e+fx))^n)^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b(c\sec(e+fx))^n}{a}+1\right)}{afn(p+1)}$$

```
[In] Int[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^5,x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sec[e + f*x])^n)/a]*(a + b*
(c*Sec[e + f*x])^n)^(1 + p))/(a*f*n*(1 + p))) - (Hypergeometric2F1[2/n, -p,
 (2 + n)/n, -((b*(c*Sec[e + f*x])^n)/a)]*Sec[e + f*x]^2*(a + b*(c*Sec[e + f
*x])^n)^p)/(f*(1 + (b*(c*Sec[e + f*x])^n)/a)^p) + (Hypergeometric2F1[4/n, -
p, (4 + n)/n, -((b*(c*Sec[e + f*x])^n)/a)]*Sec[e + f*x]^4*(a + b*(c*Sec[e +
 f*x])^n)^p)/(4*f*(1 + (b*(c*Sec[e + f*x])^n)/a)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)^2(a+b(cx)^n)^p}{x}\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(\frac{(a+b(cx)^n)^p}{x} - 2x(a+b(cx)^n)^p + x^3(a+b(cx)^n)^p\right)\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{(a+b(cx)^n)^p}{x}\,dx, x, \sec(e+fx)\right)}{f}\\
&\quad+ \frac{\text{Subst}\left(\int x^3(a+b(cx)^n)^p\,dx, x, \sec(e+fx)\right)}{f}\\
&\quad- \frac{2\text{Subst}\left(\int x(a+b(cx)^n)^p\,dx, x, \sec(e+fx)\right)}{f}
\end{aligned}
$$

$$=\frac{\text{Subst}\left(\int \frac{c(a+bx^n)^p}{x}\,dx,x,c\sec(e+fx)\right)}{cf}+\frac{\text{Subst}\left(\int \frac{x^3(a+bx^n)^p}{c^3}\,dx,x,c\sec(e+fx)\right)}{cf} - \frac{2\text{Subst}\left(\int \frac{x(a+bx^n)^p}{c}\,dx,x,c\sec(e+fx)\right)}{cf}$$

$$=\frac{\text{Subst}\left(\int \frac{(a+bx^n)^p}{x}\,dx,x,c\sec(e+fx)\right)}{f}+\frac{\text{Subst}\left(\int x^3(a+bx^n)^p\,dx,x,c\sec(e+fx)\right)}{c^4f} - \frac{2\text{Subst}\left(\int x(a+bx^n)^p\,dx,x,c\sec(e+fx)\right)}{c^2f}$$

$$=\frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,(c\sec(e+fx))^n\right)}{fn} + \frac{\left((a+b(c\sec(e+fx))^n)^p\left(1+\frac{b(c\sec(e+fx))^n}{a}\right)^{-p}\right)\text{Subst}\left(\int x^3\left(1+\frac{bx^n}{a}\right)^p\,dx,x,c\sec(e+fx)\right)}{c^4f} - \frac{\left(2(a+b(c\sec(e+fx))^n)^p\left(1+\frac{b(c\sec(e+fx))^n}{a}\right)^{-p}\right)\text{Subst}\left(\int x\left(1+\frac{bx^n}{a}\right)^p\,dx,x,c\sec(e+fx)\right)}{c^2f}$$

$$=-\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b(c\sec(e+fx))^n}{a}\right)(a+b(c\sec(e+fx))^n)^{1+p}}{afn(1+p)} - \frac{\text{Hypergeometric2F1}\left(\frac{2}{n},-p,\frac{2+n}{n},-\frac{b(c\sec(e+fx))^n}{a}\right)\sec^2(e+fx)\,(a+b(c\sec(e+fx))^n)^p\left(1+\frac{b(c\text{se}\right.}{f} + \frac{\text{Hypergeometric2F1}\left(\frac{4}{n},-p,\frac{4+n}{n},-\frac{b(c\sec(e+fx))^n}{a}\right)\sec^4(e+fx)\,(a+b(c\sec(e+fx))^n)^p\left(1+\frac{b(c\text{se}\right.}{4f}$$

Mathematica [A] (warning: unable to verify)

Time = 13.76 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.08

$$\int (a+b(c\sec(e+fx))^n)^p\tan^5(e+fx)\,dx$$

$$=\frac{(a+b(c\sec(e+fx))^n)^p\left(1+\frac{b\left(c\sqrt{\sec^2(e+fx)}\right)^n}{a}\right)^{-p}\left(-4an(1+p)\,\text{Hypergeometric2F1}\left(\frac{2}{n},-p,\frac{2+n}{n},-\frac{b\left(c\sqrt{\text{se}}\right.}{}\right.\right.}{}$$

```
[In] Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^5,x]
```

```
[Out] ((a + b*(c*Sec[e + f*x])^n)^p*(-4*a*n*(1 + p)*Hypergeometric2F1[2/n, -p, (2
 + n)/n, -((b*(c*Sqrt[Sec[e + f*x]^2])^n)/a)]*Sec[e + f*x]^2 + a*n*(1 + p)*
```

```
Hypergeometric2F1[4/n, -p, (4 + n)/n, -((b*(c*Sqrt[Sec[e + f*x]^2])^n)/a)]*
Sec[e + f*x]^4 - 4*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sqrt[Sec[e
+ f*x]^2])^n)/a]*(a + b*(c*Sqrt[Sec[e + f*x]^2])^n)*(1 + (b*(c*Sqrt[Sec[e +
 f*x]^2])^n)/a)^p))/(4*a*f*n*(1 + p)*(1 + (b*(c*Sqrt[Sec[e + f*x]^2])^n)/a)
^p)
```

Maple [F]

$$\int (a+b(c\sec(fx+e))^n)^p \tan(fx+e)^5\,dx$$

```
[In] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x)
[Out] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x)
```

Fricas [F]

$$\int (a+b(c\sec(e+fx))^n)^p \tan^5(e+fx)\,dx = \int ((c\sec(fx+e))^n b + a)^p \tan(fx+e)^5\,dx$$

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x, algorithm="fricas")
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b(c\sec(e+fx))^n)^p \tan^5(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate((a+b*(c*sec(f*x+e))**n)**p*tan(f*x+e)**5,x)
[Out] Timed out
```

Maxima [F]

$$\int (a+b(c\sec(e+fx))^n)^p \tan^5(e+fx)\,dx = \int ((c\sec(fx+e))^n b + a)^p \tan(fx+e)^5\,dx$$

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x, algorithm="maxima")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^5, x)
```

Giac [F]

$$\int (a+b(c\sec(e+fx))^n)^p \tan^5(e+fx)\,dx = \int ((c\sec(fx+e))^n b + a)^p \tan(fx+e)^5\,dx$$

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x, algorithm="giac")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b(c\sec(e+fx))^n)^p \tan^5(e+fx)\,dx = \int \tan(e+f\,x)^5 \left(a+b\left(\frac{c}{\cos(e+f\,x)}\right)^n\right)^p dx$$

```
[In] int(tan(e + f*x)^5*(a + b*(c/cos(e + f*x))^n)^p,x)
[Out] int(tan(e + f*x)^5*(a + b*(c/cos(e + f*x))^n)^p, x)
```

3.464 $\int (a+b(c\sec(e+fx))^n)^p \tan^3(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 143

$$\int (a+b(c\sec(e+fx))^n)^p \tan^3(e+fx)\,dx = \frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b(c\sec(e+fx))^n}{a}\right)(a+b(c\sec(e+fx))^n)^{1+p}}{afn(1+p)} + \frac{\text{Hypergeometric2F1}\left(\frac{2}{n},-p,\frac{2+n}{n},-\frac{b(c\sec(e+fx))^n}{a}\right)\sec^2(e+fx)(a+b(c\sec(e+fx))^n)^p\left(1+\frac{b(c\sec(e+fx))^n}{a}\right)^{-p}}{2f}$$

```
[Out] hypergeom([1, p+1],[2+p],1+b*(c*sec(f*x+e))^n/a)*(a+b*(c*sec(f*x+e))^n)^(p+
1)/a/f/n/(p+1)+1/2*hypergeom([-p, 2/n],[(2+n)/n],-b*(c*sec(f*x+e))^n/a)*sec
(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p/f/((1+b*(c*sec(f*x+e))^n/a)^p)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4224, 6874, 374, 12, 272, 67, 372, 371}

$$\int (a+b(c\sec(e+fx))^n)^p \tan^3(e+fx)\,dx = \frac{\sec^2(e+fx)(a+b(c\sec(e+fx))^n)^p\left(\frac{b(c\sec(e+fx))^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{n},-p,\frac{n+2}{n},-\frac{b(c\sec(e+fx))^n}{a}\right)}{2f} + \frac{(a+b(c\sec(e+fx))^n)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b(c\sec(e+fx))^n}{a}+1\right)}{afn(p+1)}$$

```
[In] Int[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^3,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sec[e + f*x])^n)/a]*(a + b*(c
*Sec[e + f*x])^n)^(1 + p))/(a*f*n*(1 + p)) + (Hypergeometric2F1[2/n, -p, (2
 + n)/n, -((b*(c*Sec[e + f*x])^n)/a)]*Sec[e + f*x]^2*(a + b*(c*Sec[e + f*x]
)^n)^p)/(2*f*(1 + (b*(c*Sec[e + f*x])^n)/a)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
```

```
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a+b(cx)^n)^p}{x}\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{(a+b(cx)^n)^p}{x} + x(a+b(cx)^n)^p\right)\,dx, x, \sec(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \frac{(a+b(cx)^n)^p}{x}\,dx, x, \sec(e+fx)\right)}{f} + \frac{\text{Subst}\left(\int x(a+b(cx)^n)^p\,dx, x, \sec(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \frac{c(a+bx^n)^p}{x}\,dx, x, c\sec(e+fx)\right)}{cf} + \frac{\text{Subst}\left(\int \frac{x(a+bx^n)^p}{c}\,dx, x, c\sec(e+fx)\right)}{cf}\\
&= -\frac{\text{Subst}\left(\int \frac{(a+bx^n)^p}{x}\,dx, x, c\sec(e+fx)\right)}{f} + \frac{\text{Subst}\left(\int x(a+bx^n)^p\,dx, x, c\sec(e+fx)\right)}{c^2f}\\
&= -\frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, (c\sec(e+fx))^n\right)}{fn}\\
&\quad + \frac{\left((a+b(c\sec(e+fx))^n)^p\left(1+\frac{b(c\sec(e+fx))^n}{a}\right)^{-p}\right)\text{Subst}\left(\int x\left(1+\frac{bx^n}{a}\right)^p\,dx, x, c\sec(e+fx)\right)}{c^2f}\\
&= \frac{\text{Hypergeometric2F1}\left(1, 1+p, 2+p, 1+\frac{b(c\sec(e+fx))^n}{a}\right)(a+b(c\sec(e+fx))^n)^{1+p}}{afn(1+p)}\\
&\quad + \frac{\text{Hypergeometric2F1}\left(\frac{2}{n}, -p, \frac{2+n}{n}, -\frac{b(c\sec(e+fx))^n}{a}\right)\sec^2(e+fx)(a+b(c\sec(e+fx))^n)^p\left(1+\frac{b(c\cdots}{}\right.}{2f}\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 6.43 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.13

$$\int (a+b(c\sec(e+fx))^n)^p\tan^3(e+fx)\,dx$$
$$=\frac{(a+b(c\sec(e+fx))^n)^p\left(\frac{2\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\left(c\sqrt{\sec^2(e+fx)}\right)^n}{a}\right)\left(a+b\left(c\sqrt{\sec^2(e+fx)}\right)^n\right)}{an(1+p)}+\text{Hypergeometr}\right.}{2f}$$

```
[In] Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^3,x]
[Out] ((a + b*(c*Sec[e + f*x])^n)^p*((2*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b
*(c*Sqrt[Sec[e + f*x]^2])^n)/a]*(a + b*(c*Sqrt[Sec[e + f*x]^2])^n))/(a*n*(1
 + p)) + (Hypergeometric2F1[2/n, -p, (2 + n)/n, -((b*(c*Sqrt[Sec[e + f*x]^2
])^n)/a)]*Sec[e + f*x]^2)/(1 + (b*(c*Sqrt[Sec[e + f*x]^2])^n)/a)^p))/(2*f)
```

Maple [F]

$$\int (a+b(c\sec(fx+e))^n)^p\tan(fx+e)^3\,dx$$

```
[In] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x)
[Out] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x)
```

Fricas [F]

$$\int (a+b(c\sec(e+fx))^n)^p\tan^3(e+fx)\,dx=\int ((c\sec(fx+e))^n\,b+a)^p\tan(fx+e)^3\,dx$$

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x, algorithm="fricas")
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^3, x)
```

Sympy [F]

$$\int (a+b(c\sec(e+fx))^n)^p\tan^3(e+fx)\,dx = \int (a+b(c\sec{(e+fx)})^n)^p\tan^3{(e+fx)}\,dx$$

```
[In] integrate((a+b*(c*sec(f*x+e))**n)**p*tan(f*x+e)**3,x)
[Out] Integral((a + b*(c*sec(e + f*x))**n)**p*tan(e + f*x)**3, x)
```

Maxima [F]

$$\int (a+b(c\sec(e+fx))^n)^p\tan^3(e+fx)\,dx = \int ((c\sec{(fx+e)})^n b+a)^p\tan{(fx+e)}^3\,dx$$

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x, algorithm="maxima")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^3, x)
```

Giac [F]

$$\int (a+b(c\sec(e+fx))^n)^p\tan^3(e+fx)\,dx = \int ((c\sec{(fx+e)})^n b+a)^p\tan{(fx+e)}^3\,dx$$

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x, algorithm="giac")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b(c\sec(e+fx))^n)^p\tan^3(e+fx)\,dx = \int \tan(e+f\,x)^3\left(a+b\left(\frac{c}{\cos{(e+f\,x)}}\right)^n\right)^p\,dx$$

```
[In] int(tan(e + f*x)^3*(a + b*(c/cos(e + f*x))^n)^p,x)
[Out] int(tan(e + f*x)^3*(a + b*(c/cos(e + f*x))^n)^p, x)
```

3.465 $\int (a+b(c\sec(e+fx))^n)^p \tan(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 59

$$\int (a+b(c\sec(e+fx))^n)^p \tan(e+fx)\,dx = -\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b(c\sec(e+fx))^n}{a}\right)(a+b(c\sec(e+fx))^n)^{1+p}}{afn(1+p)}$$

```
[Out] -hypergeom([1, p+1],[2+p],1+b*(c*sec(f*x+e))^n/a)*(a+b*(c*sec(f*x+e))^n)^(p
+1)/a/f/n/(p+1)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4224, 374, 12, 272, 67}

$$\int (a+b(c\sec(e+fx))^n)^p \tan(e+fx)\,dx = -\frac{(a+b(c\sec(e+fx))^n)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b(c\sec(e+fx))^n}{a}+1\right)}{afn(p+1)}$$

```
[In] Int[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x],x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sec[e + f*x])^n)/a]*(a + b*
(c*Sec[e + f*x])^n)^(1 + p))/(a*f*n*(1 + p)))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b(cx)^n)^p}{x}\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{c(a+bx^n)^p}{x}\,dx, x, c\sec(e+fx)\right)}{cf}\\
&= \frac{\text{Subst}\left(\int \frac{(a+bx^n)^p}{x}\,dx, x, c\sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, (c\sec(e+fx))^n\right)}{fn}\\
&= -\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b(c\sec(e+fx))^n}{a}\right)(a+b(c\sec(e+fx))^n)^{1+p}}{afn(1+p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int (a+b(c\sec(e+fx))^n)^p\tan(e+fx)\,dx = -\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b(c\sec(e+fx))^n}{a}\right)(a+b(c\sec(e+fx))^n)^{1+p}}{afn(1+p)}$$

```
[In] Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x],x]
[Out] -((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sec[e + f*x])^n)/a]*(a + b*
(c*Sec[e + f*x])^n)^(1 + p))/(a*f*n*(1 + p)))
```

Maple [F]

$$\int (a+b(c\sec(fx+e))^n)^p\tan(fx+e)\,dx$$

```
[In] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x)
[Out] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x)
```

Fricas [F]

$$\int (a+b(c\sec(e+fx))^n)^p\tan(e+fx)\,dx = \int ((c\sec(fx+e))^n b+a)^p\tan(fx+e)\,dx$$

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x, algorithm="fricas")
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e), x)
```

Sympy [F]

$$\int (a+b(c\sec(e+fx))^n)^p\tan(e+fx)\,dx = \int (a+b(c\sec(e+fx))^n)^p\tan(e+fx)\,dx$$

```
[In] integrate((a+b*(c*sec(f*x+e))**n)**p*tan(f*x+e),x)
[Out] Integral((a + b*(c*sec(e + f*x))**n)**p*tan(e + f*x), x)
```

Maxima [F]

$$\int (a+b(c\sec(e+fx))^n)^p \tan(e+fx)\,dx = \int ((c\sec(fx+e))^n b + a)^p \tan(fx+e)\,dx$$

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x, algorithm="maxima")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e), x)
```

Giac [F]

$$\int (a+b(c\sec(e+fx))^n)^p \tan(e+fx)\,dx = \int ((c\sec(fx+e))^n b + a)^p \tan(fx+e)\,dx$$

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x, algorithm="giac")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b(c\sec(e+fx))^n)^p \tan(e+fx)\,dx = \int \tan(e+f\,x)\,\left(a+b\,{\left(\frac{c}{\cos(e+f\,x)}\right)}^n\right)^p\,dx$$

```
[In] int(tan(e + f*x)*(a + b*(c/cos(e + f*x))^n)^p,x)
[Out] int(tan(e + f*x)*(a + b*(c/cos(e + f*x))^n)^p, x)
```

3.466 $\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\text{Int}\left(\cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

```
[In] Int[Cot[e + f*x]*(a + b*(c*Sec[e + f*x])^n)^p,x]
[Out] Defer[Int][Cot[e + f*x]*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Rubi steps

$$\text{integral}=\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Mathematica [N/A]

Not integrable

Time = 4.53 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx=\int \cot(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$$

```
[In] Integrate[Cot[e + f*x]*(a + b*(c*Sec[e + f*x])^n)^p,x]
[Out] Integrate[Cot[e + f*x]*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.60 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \cot\left(fx+e\right)\left(a+b(c\sec\left(fx+e\right))^n\right)^p dx$$

```
[In] int(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x)
[Out] int(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx=\int \left((c\sec\left(fx+e\right))^n b+a\right)^p\cot\left(fx+e\right)\,dx$$

```
[In] integrate(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e), x)
```

Sympy [N/A]

Not integrable

Time = 12.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \cot(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx=\int \left(a+b(c\sec\left(e+fx\right))^n\right)^p\cot\left(e+fx\right)dx$$

```
[In] integrate(cot(f*x+e)*(a+b*(c*sec(f*x+e))**n)**p,x)
[Out] Integral((a + b*(c*sec(e + f*x))**n)**p*cot(e + f*x), x)
```

Maxima [N/A]

Not integrable

Time = 3.78 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int ((c\sec{(fx+e)})^n b+a)^p \cot{(fx+e)}\,dx$$

```
[In] integrate(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int ((c\sec{(fx+e)})^n b+a)^p \cot{(fx+e)}\,dx$$

```
[In] integrate(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e), x)
```

Mupad [N/A]

Not integrable

Time = 20.71 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int \cot(e+f\,x)\,\left(a+b\,{\left(\frac{c}{\cos{(e+f\,x)}}\right)}^n\right)^p\,dx$$

```
[In] int(cot(e + f*x)*(a + b*(c/cos(e + f*x))^n)^p,x)
[Out] int(cot(e + f*x)*(a + b*(c/cos(e + f*x))^n)^p, x)
```

3.467 $\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\mathrm{Int}\left(\cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

```
[In] Int[Cot[e + f*x]^3*(a + b*(c*Sec[e + f*x])^n)^p,x]
[Out] Defer[Int][Cot[e + f*x]^3*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Rubi steps

$$\text{integral}=\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Mathematica [N/A]

Not integrable

Time = 31.51 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

```
[In] Integrate[Cot[e + f*x]^3*(a + b*(c*Sec[e + f*x])^n)^p,x]
[Out] Integrate[Cot[e + f*x]^3*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.62 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \cot\left(fx+e\right)^3\left(a+b(c\sec\left(fx+e\right))^n\right)^p dx$$

```
[In] int(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x)
[Out] int(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int \left((c\sec\left(fx+e\right))^n b+a\right)^p\cot\left(fx+e\right)^3\,dx$$

```
[In] integrate(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(cot(f*x+e)**3*(a+b*(c*sec(f*x+e))**n)**p,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 9.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int\left(\left(c\sec\left(fx+e\right)\right)^n b+a\right)^p\cot\left(fx+e\right)^3\,dx$$

```
[In] integrate(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^3, x)
```

Giac [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int\left(\left(c\sec\left(fx+e\right)\right)^n b+a\right)^p\cot\left(fx+e\right)^3\,dx$$

```
[In] integrate(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^3, x)
```

Mupad [N/A]

Not integrable

Time = 21.97 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int\cot(e+f\,x)^3\left(a+b\left(\frac{c}{\cos\left(e+f\,x\right)}\right)^n\right)^p\,dx$$

```
[In] int(cot(e + f*x)^3*(a + b*(c/cos(e + f*x))^n)^p,x)
[Out] int(cot(e + f*x)^3*(a + b*(c/cos(e + f*x))^n)^p, x)
```

3.468 $\int (a+b(c\sec(e+fx))^n)^p \tan^2(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (a+b(c\sec(e+fx))^n)^p \tan^2(e+fx)\,dx = \text{Int}\left((a+b(c\sec(e+fx))^n)^p \tan^2(e+fx),x\right)$$

```
[Out] Unintegrable((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b(c\sec(e+fx))^n)^p \tan^2(e+fx)\,dx = \int (a+b(c\sec(e+fx))^n)^p \tan^2(e+fx)\,dx$$

```
[In] Int[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^2,x]
```

```
[Out] Defer[Int][(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^2, x]
```

Rubi steps

$$\text{integral} = \int (a+b(c\sec(e+fx))^n)^p \tan^2(e+fx)\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (a+b(c\sec(e+fx))^n)^p \tan^2(e+fx)\,dx = \int (a+b(c\sec(e+fx))^n)^p \tan^2(e+fx)\,dx$$

```
[In] Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^2,x]
[Out] Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a+b(c\sec(fx+e))^n)^p \tan(fx+e)^2\,dx$$

```
[In] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x)
[Out] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (a+b(c\sec(e+fx))^n)^p \tan^2(e+fx)\,dx = \int ((c\sec(fx+e))^n b+a)^p \tan(fx+e)^2\,dx$$

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x, algorithm="fricas")
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^2, x)
```

Sympy [N/A]

Not integrable

Time = 19.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int (a+b(c\sec(e+fx))^n)^p \tan^2(e+fx)\,dx = \int (a+b(c\sec(e+fx))^n)^p \tan^2(e+fx)\,dx$$

```
[In] integrate((a+b*(c*sec(f*x+e))**n)**p*tan(f*x+e)**2,x)
[Out] Integral((a + b*(c*sec(e + f*x))**n)**p*tan(e + f*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 5.86 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (a+b(c\sec(e+fx))^n)^p\tan^2(e+fx)\,dx = \int ((c\sec(fx+e))^n b + a)^p \tan(fx+e)^2\,dx$$

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x, algorithm="maxima")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^2, x)
```

Giac [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (a+b(c\sec(e+fx))^n)^p\tan^2(e+fx)\,dx = \int ((c\sec(fx+e))^n b + a)^p \tan(fx+e)^2\,dx$$

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x, algorithm="giac")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^2, x)
```

Mupad [N/A]

Not integrable

Time = 21.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int (a+b(c\sec(e+fx))^n)^p\tan^2(e+fx)\,dx = \int \tan(e+f\,x)^2\left(a+b\left(\frac{c}{\cos(e+f\,x)}\right)^n\right)^p\,dx$$

```
[In] int(tan(e + f*x)^2*(a + b*(c/cos(e + f*x))^n)^p,x)
[Out] int(tan(e + f*x)^2*(a + b*(c/cos(e + f*x))^n)^p, x)
```

3.469 $\int (a+b(c\sec(e+fx))^n)^p\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (a+b(c\sec(e+fx))^n)^p\,dx = \text{Int}\left((a+b(c\sec(e+fx))^n)^p,x\right)$$

```
[Out] Unintegrable((a+b*(c*sec(f*x+e))^n)^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b(c\sec(e+fx))^n)^p\,dx = \int (a+b(c\sec(e+fx))^n)^p\,dx$$

```
[In] Int[(a + b*(c*Sec[e + f*x])^n)^p,x]
```

```
[Out] Defer[Int][(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Rubi steps

$$\text{integral} = \int (a+b(c\sec(e+fx))^n)^p\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.74 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (a+b(c\sec(e+fx))^n)^p \, dx = \int (a+b(c\sec(e+fx))^n)^p \, dx$$

```
[In] Integrate[(a + b*(c*Sec[e + f*x])^n)^p,x]
[Out] Integrate[(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.48 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (a+b(c\sec(fx+e))^n)^p \, dx$$

```
[In] int((a+b*(c*sec(f*x+e))^n)^p,x)
[Out] int((a+b*(c*sec(f*x+e))^n)^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (a+b(c\sec(e+fx))^n)^p \, dx = \int ((c\sec(fx+e))^n b + a)^p \, dx$$

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*sec(f*x + e))^n*b + a)^p, x)
```

Sympy [N/A]

Not integrable

Time = 1.73 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (a+b(c\sec(e+fx))^n)^p \, dx = \int (a+b(c\sec(e+fx))^n)^p \, dx$$

```
[In] integrate((a+b*(c*sec(f*x+e))**n)**p,x)
[Out] Integral((a + b*(c*sec(e + f*x))**n)**p, x)
```

Maxima [N/A]

Not integrable

Time = 3.95 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (a+b(c\sec(e+fx))^n)^p \, dx = \int ((c\sec(fx+e))^n b + a)^p \, dx$$

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (a+b(c\sec(e+fx))^n)^p \, dx = \int ((c\sec(fx+e))^n b + a)^p \, dx$$

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 20.78 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int (a+b(c\sec(e+fx))^n)^p \, dx = \int \left(a+b\left(\frac{c}{\cos(e+f\,x)}\right)^n\right)^p \, dx$$

```
[In] int((a + b*(c/cos(e + f*x))^n)^p,x)
[Out] int((a + b*(c/cos(e + f*x))^n)^p, x)
```

3.470 $\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\operatorname{Int}\left(\cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

```
[In] Int[Cot[e + f*x]^2*(a + b*(c*Sec[e + f*x])^n)^p,x]
```

```
[Out] Defer[Int][Cot[e + f*x]^2*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Rubi steps

$$\text{integral}=\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.79 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

```
[In] Integrate[Cot[e + f*x]^2*(a + b*(c*Sec[e + f*x])^n)^p,x]
[Out] Integrate[Cot[e + f*x]^2*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.60 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \cot\left(fx+e\right)^2\left(a+b\left(c\sec\left(fx+e\right)\right)^n\right)^p dx$$

```
[In] int(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x)
[Out] int(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int \left(\left(c\sec\left(fx+e\right)\right)^n b+a\right)^p\cot\left(fx+e\right)^2\,dx$$

```
[In] integrate(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^2, x)
```

Sympy [N/A]

Not integrable

Time = 73.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int \left(a+b\left(c\sec\left(e+fx\right)\right)^n\right)^p\cot^2\left(e+fx\right)dx$$

```
[In] integrate(cot(f*x+e)**2*(a+b*(c*sec(f*x+e))**n)**p,x)
[Out] Integral((a + b*(c*sec(e + f*x))**n)**p*cot(e + f*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 6.35 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int \left(\left(c\sec\left(fx+e\right)\right)^n b+a\right)^p\cot\left(fx+e\right)^2\,dx$$

```
[In] integrate(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int \left(\left(c\sec\left(fx+e\right)\right)^n b+a\right)^p\cot\left(fx+e\right)^2\,dx$$

```
[In] integrate(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^2, x)
```

Mupad [N/A]

Not integrable

Time = 21.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int \cot(e+f\,x)^2\left(a+b\left(\frac{c}{\cos\left(e+f\,x\right)}\right)^n\right)^p\,dx$$

```
[In] int(cot(e + f*x)^2*(a + b*(c/cos(e + f*x))^n)^p,x)
[Out] int(cot(e + f*x)^2*(a + b*(c/cos(e + f*x))^n)^p, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                   end if
               else #result contains complex but optimal is not
                   if debug then
                           print("result contains complex but optimal is not");
                   fi;
                   return "C","Result contains complex when optimal does not.";
               fi;
           else # result do not contain complex
                # this assumes optimal do not as well. No check is needed here.
               if debug then
                      print("result do not contain complex, this assumes optimal do not as well")
               fi;
               if leaf_count_result<=2*leaf_count_optimal then
                   if debug then
                       print("leaf_count_result<=2*leaf_count_optimal");
                   fi;
                   return "A"," ";
               else
                   if debug then
                       print("leaf_count_result>2*leaf_count_optimal");
                   fi;
                   return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                    convert(leaf_count_result,string),"$ vs. $2(",
                                    convert(leaf_count_optimal,string),")=",convert(2*leaf_count
               fi;
           fi;
       else #ExpnType(result) > ExpnType(optimal)
           if debug then
               print("ExpnType(result) > ExpnType(optimal)");
           fi;
           return "C",cat("Result contains higher order function than in optimal. Order ",
                            convert(ExpnType_result,string)," vs. order ",
                            convert(ExpnType_optimal,string),".");
       fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-ty
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation ="none"
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_result


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```